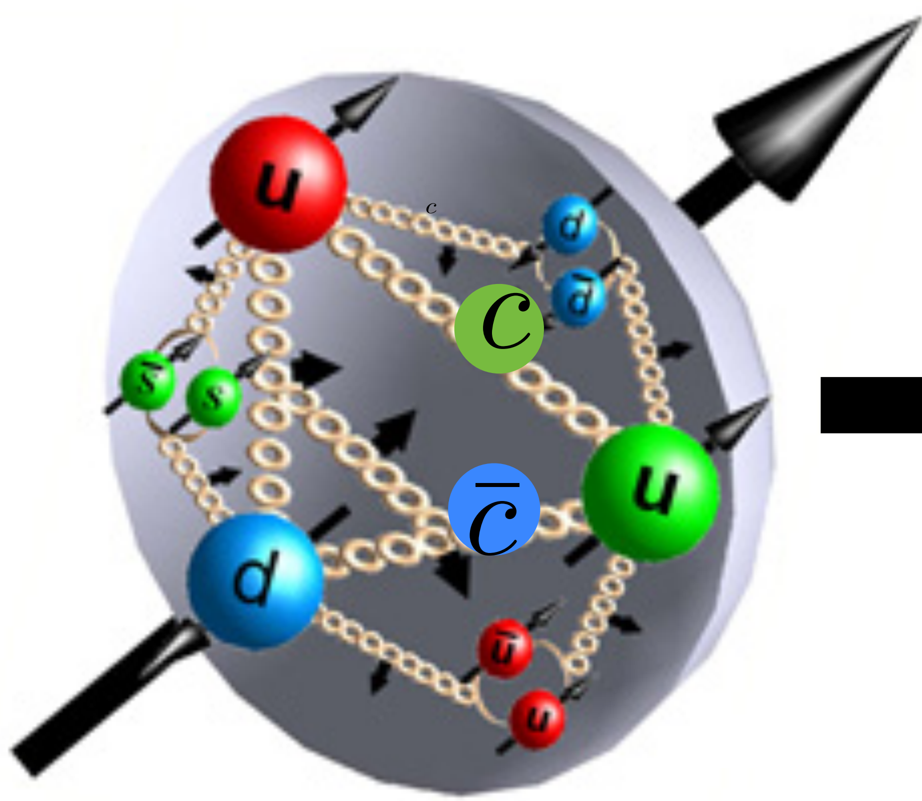
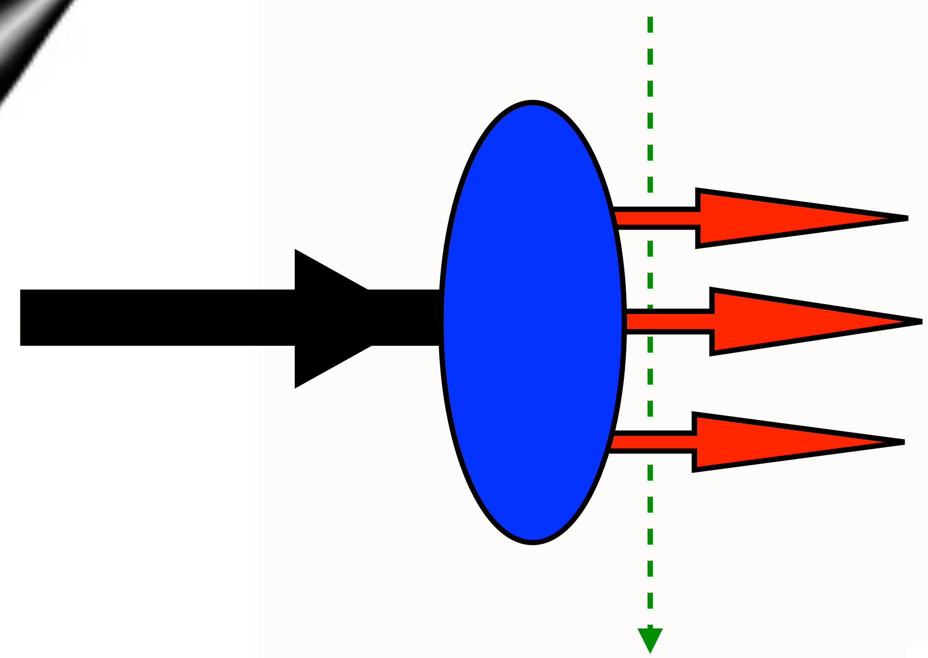
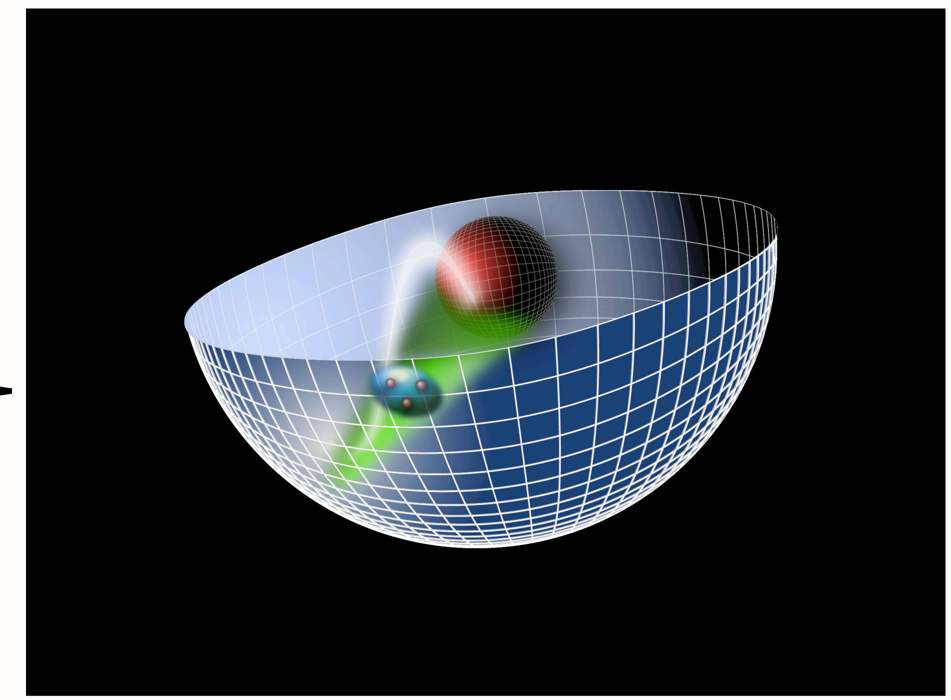
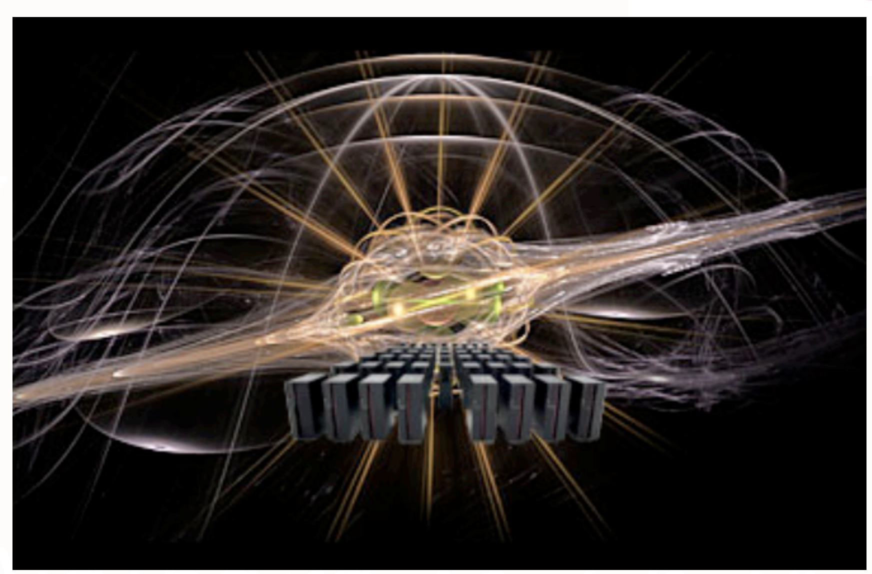
Light-Front Quantum Chromodynamics







Stan Brodsky



May 14, 2013

Honoring James Vary

International Conference on Nuclear Theory in the Supercomputing Era

QCD Lagrangian

Fundamental Theory of Hadron and Nuclear Physics

gluon dynamics

quark kinetic energy +
quark-gluon dynamics

quark mass term

$$\boxed{\mathcal{L}_{QCD} = -\frac{1}{4} Tr(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f}$$

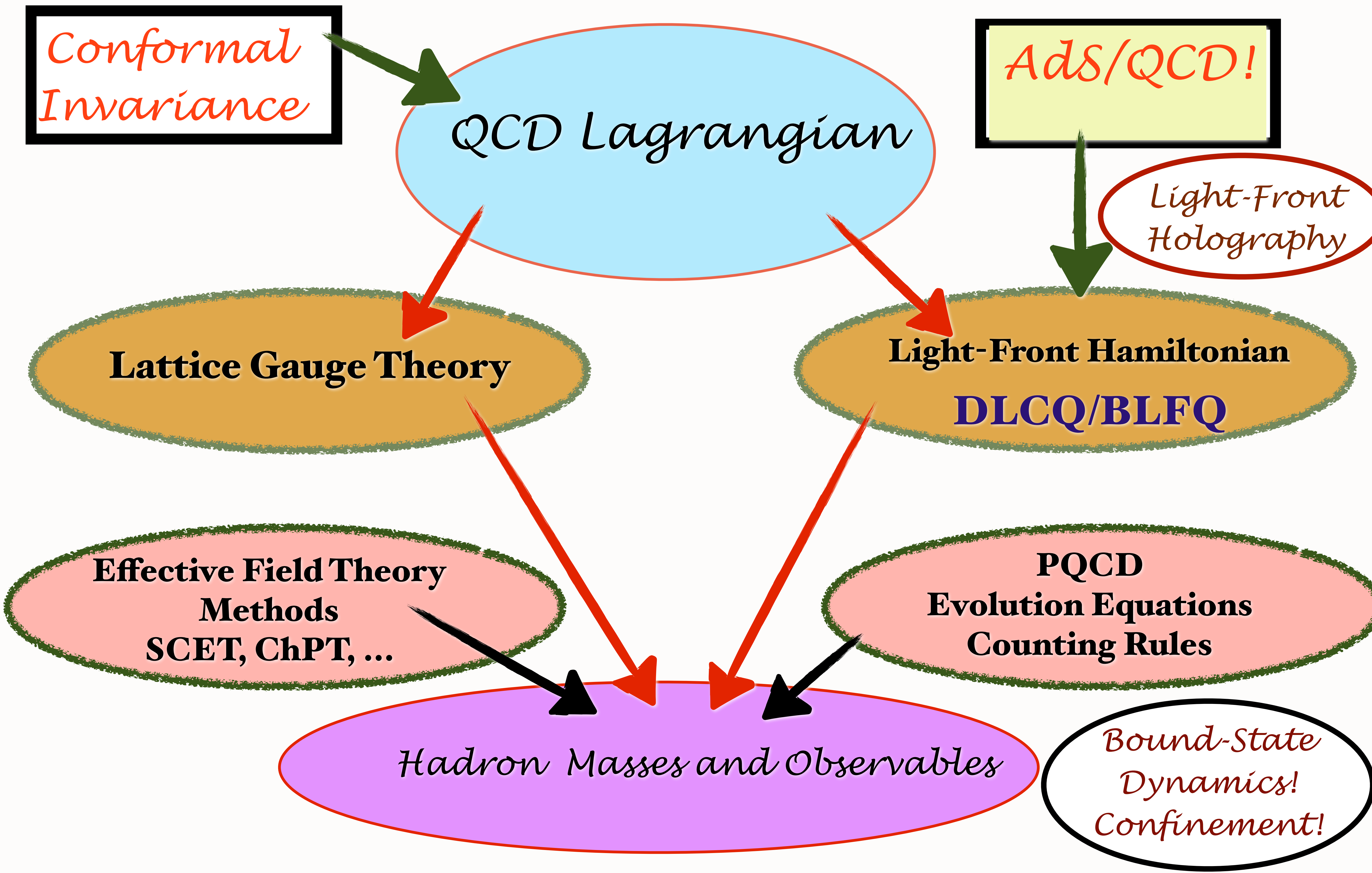
$$iD^\mu = i\partial^\mu - gA^\mu \quad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Yang Mills Gauge Principle: Color Rotation and Phase Invariance at Every Point of Space and Time

Scale-Invariant Coupling
Renormalizable
Asymptotic Freedom
Color Confinement
Conformal if m_q=0

QCD Mass Scale from Confinement not Explicit

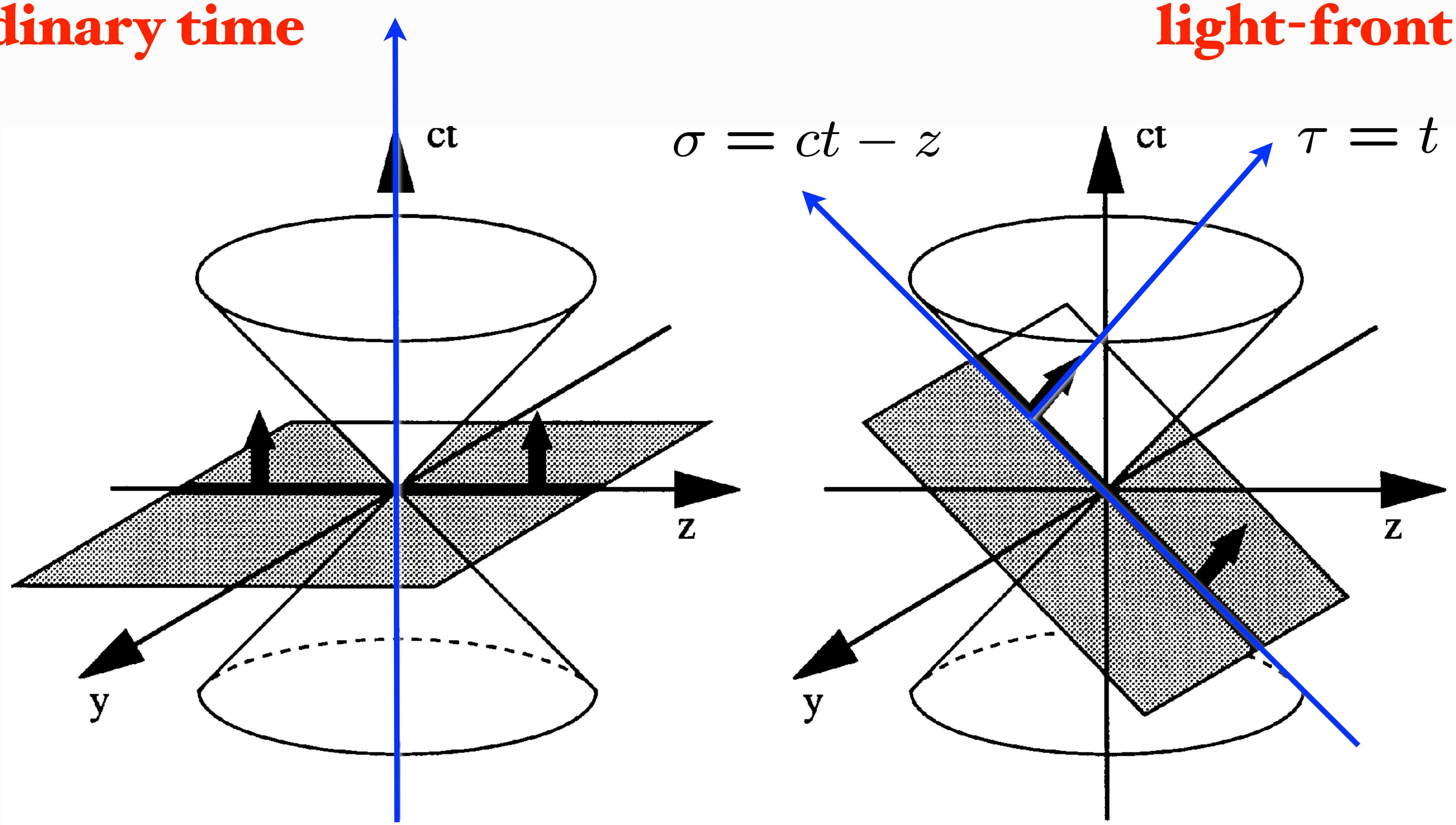
Predict Hadron Properties from First Principles!



Dirac's Amazing Idea: The Front Form

Evolve in ordinary time

Evolve in light-front time!



Instant Form **Front Form**

Each element of flash photograph illuminated at same LF time

$$\tau = t + z/c$$

Evolve in LF time

$$P^- = i \frac{d}{d\tau}$$

Eigenvalue

$$P^- = \frac{\mathcal{M}^2 + \vec{P}_\perp^2}{P^+}$$

$$H^{QCD}_{LF} |\Psi_h> = \mathcal{M}^2_h |\Psi_h>$$

HELEN BRADLEY - PHOTOGRAPHY

- LF coordinates

$x^+ = x^0 + x^3$	light-front time	$P^+ = P^0 + P^3$	longitudinal momentum
$x^- = x^0 - x^3$	longitudinal space variable	$P^- = P^0 - P^3$	light-front Hamiltonian
$\mathbf{x}_\perp = (x^1, x^2)$	transverse space variable	$\mathbf{P}_\perp = (P^1, P^2)$	transverse momentum

- On shell relation $P_\mu P^\mu = P^- P^+ - \mathbf{P}_\perp^2 = \mathcal{M}^2$ leads to dispersion relation for LF Hamilnotian P^-

$$P^- = \frac{\mathbf{P}_\perp^2 + M^2}{P^+}, \qquad P^+ > 0$$

- Hamiltonian equation for the relativistic bound state

$$i \frac{\partial}{\partial x^+} |\psi(P)\rangle = P^- |\psi(P)\rangle = \frac{M^2 + \mathbf{P}_\perp^2}{P^+} |\psi(P)\rangle$$

where P^- is derived from the QCD Lagrangian: kinetic energy of partons plus confining interaction

- Construct LF Lorentz invariant Hamiltonian $P^2 = P^- P^+ - \mathbf{P}_\perp^2$

$$P_\mu P^\mu |\psi(P)\rangle = M^2 |\psi(P)\rangle$$

- LF quantization is the ideal framework to describe hadronic structure in terms of constituents: simple vacuum structure allows unambiguous definition of partonic content of a hadron

Light-Front QCD

Physical gauge: $A^+ = 0$

Exact frame-independent formulation of nonperturbative QCD!

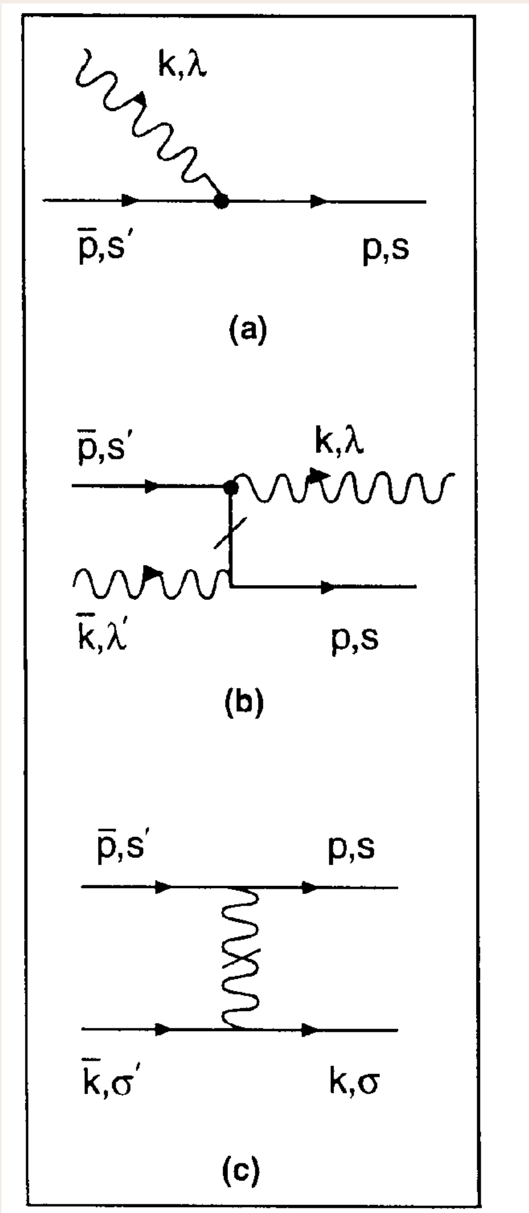
$$L^{QCD} \rightarrow H^{QCD}_{LF}$$

$$H^{QCD}_{LF} = \sum_i [\frac{m^2 + k^2_\perp}{x}]_i + H^{int}_{LF}$$

H^{int}_{LF}: Matrix in Fock Space

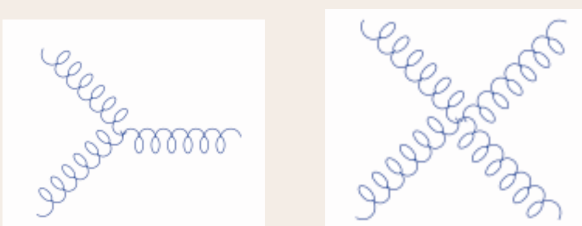
$$H^{QCD}_{LF} |\Psi_h> = \mathcal{M}^2_h |\Psi_h>$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i>$$



Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions

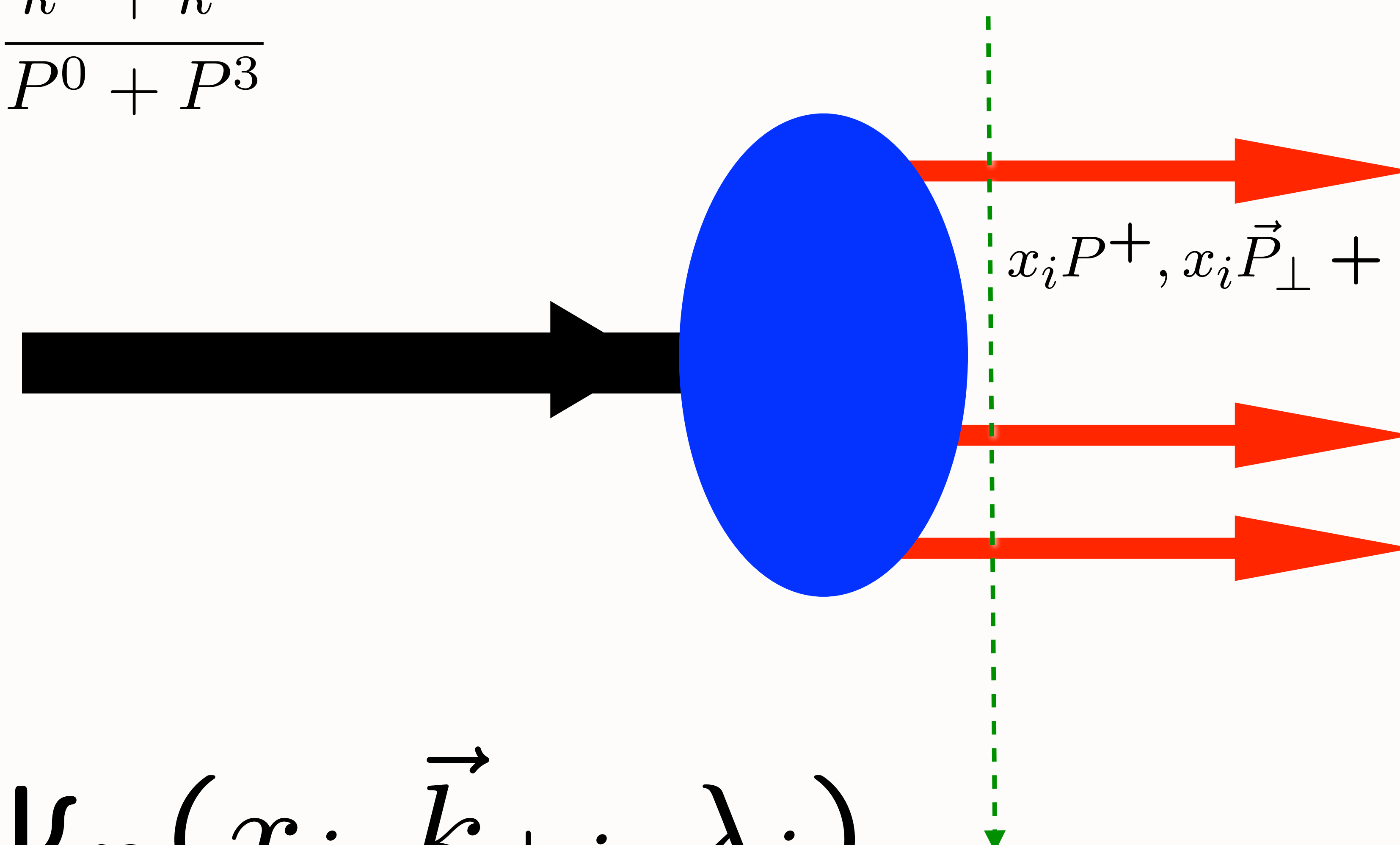
LFWFs: Off-shell in P- and invariant mass



H^{int}_{LF}

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

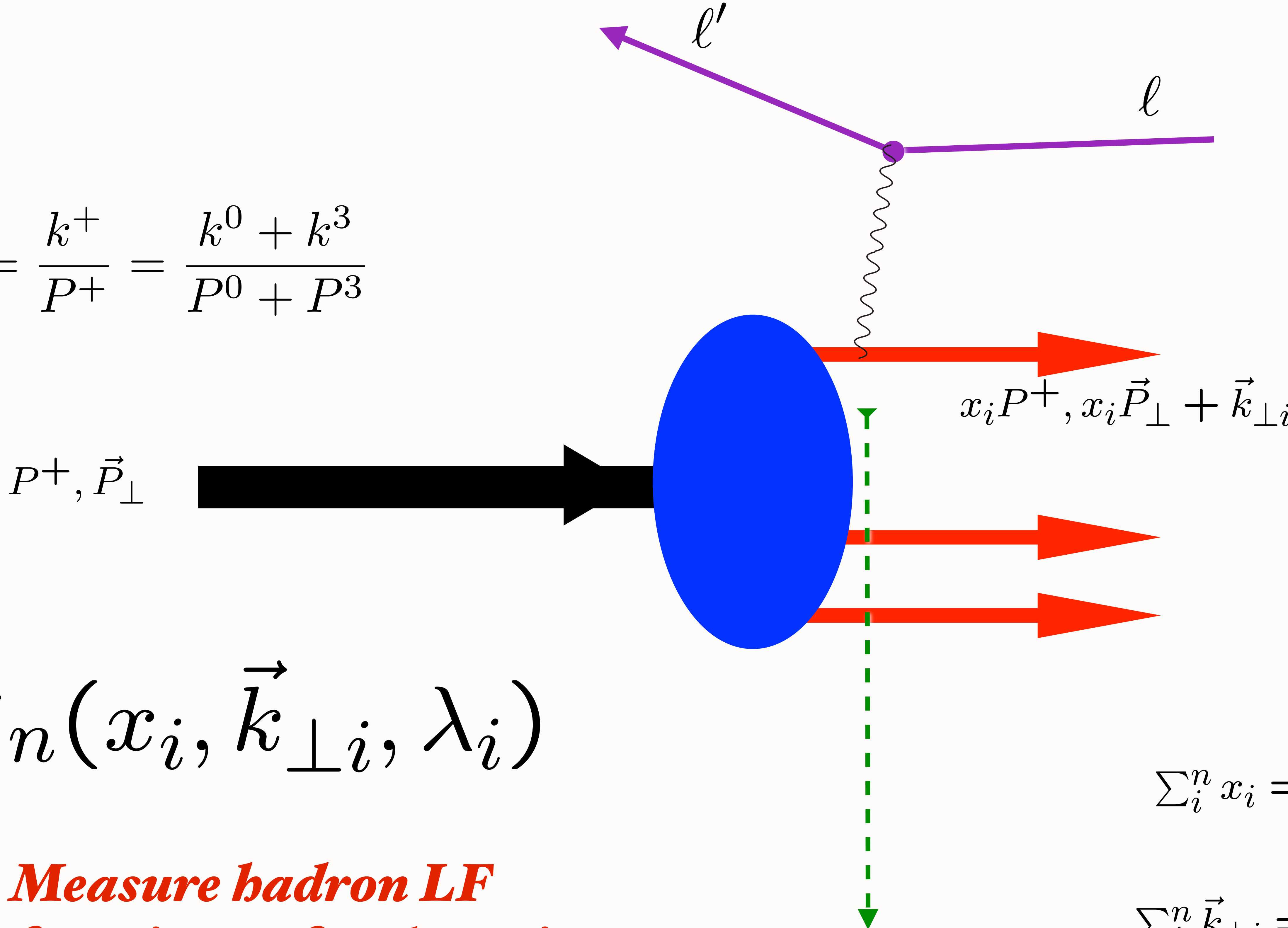
$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

Bethe-Salpeter WF integrated over k^-

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Measure hadron LF wavefunction at fixed LF time

Fixed $\tau = t + z/c$

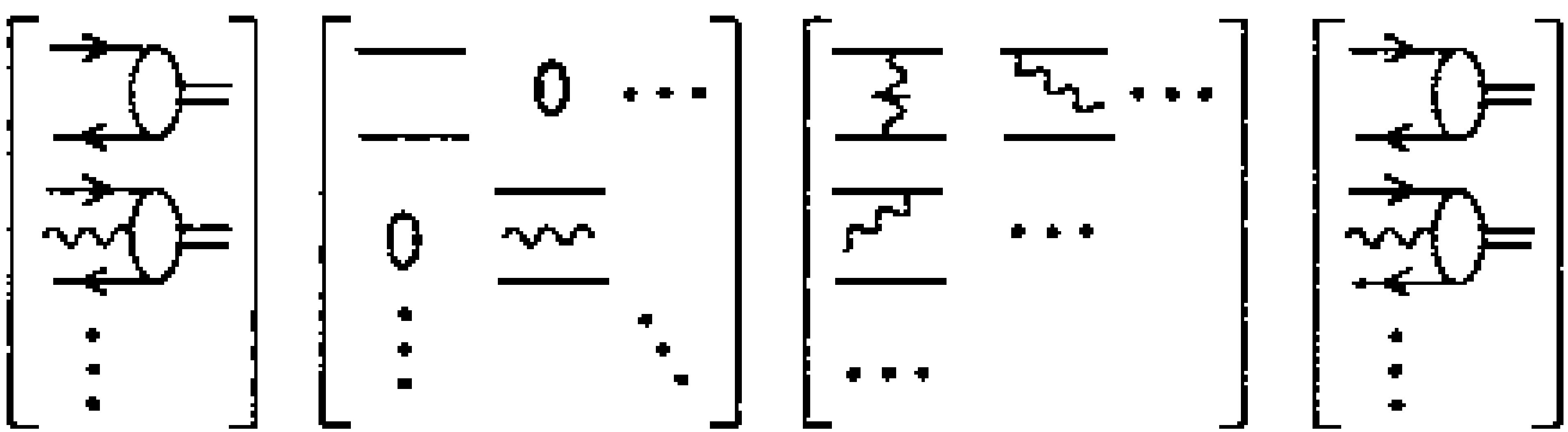
Like a flash photograph

LIGHT-FRONT MATRIX EQUATION

Rigorous Method for Solving Non-Perturbative QCD!

$$\left(M_\pi^2 - \sum_i \frac{\vec{k}_{\perp i}^{\,2} + m_i^2}{x_i}\right) \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix} = \begin{bmatrix} \langle q\bar{q}|\, V\, |q\bar{q}\rangle & \langle q\bar{q}|\, V\, |q\bar{q}g\rangle & \cdots \\ \langle q\bar{q}g|\, V\, |q\bar{q}\rangle & \langle q\bar{q}g|\, V\, |q\bar{q}g\rangle & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix}$$

$$A^+ = 0$$



Minkowski space; frame-independent; no fermion doubling; no ghosts

- *Light-Front Vacuum = vacuum of free Hamiltonian!*

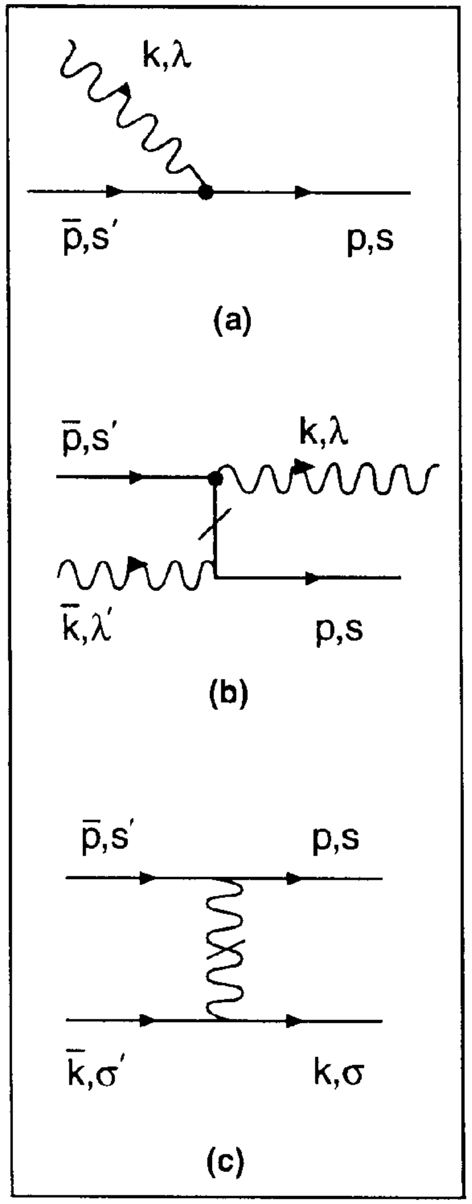
Light-Front QCD

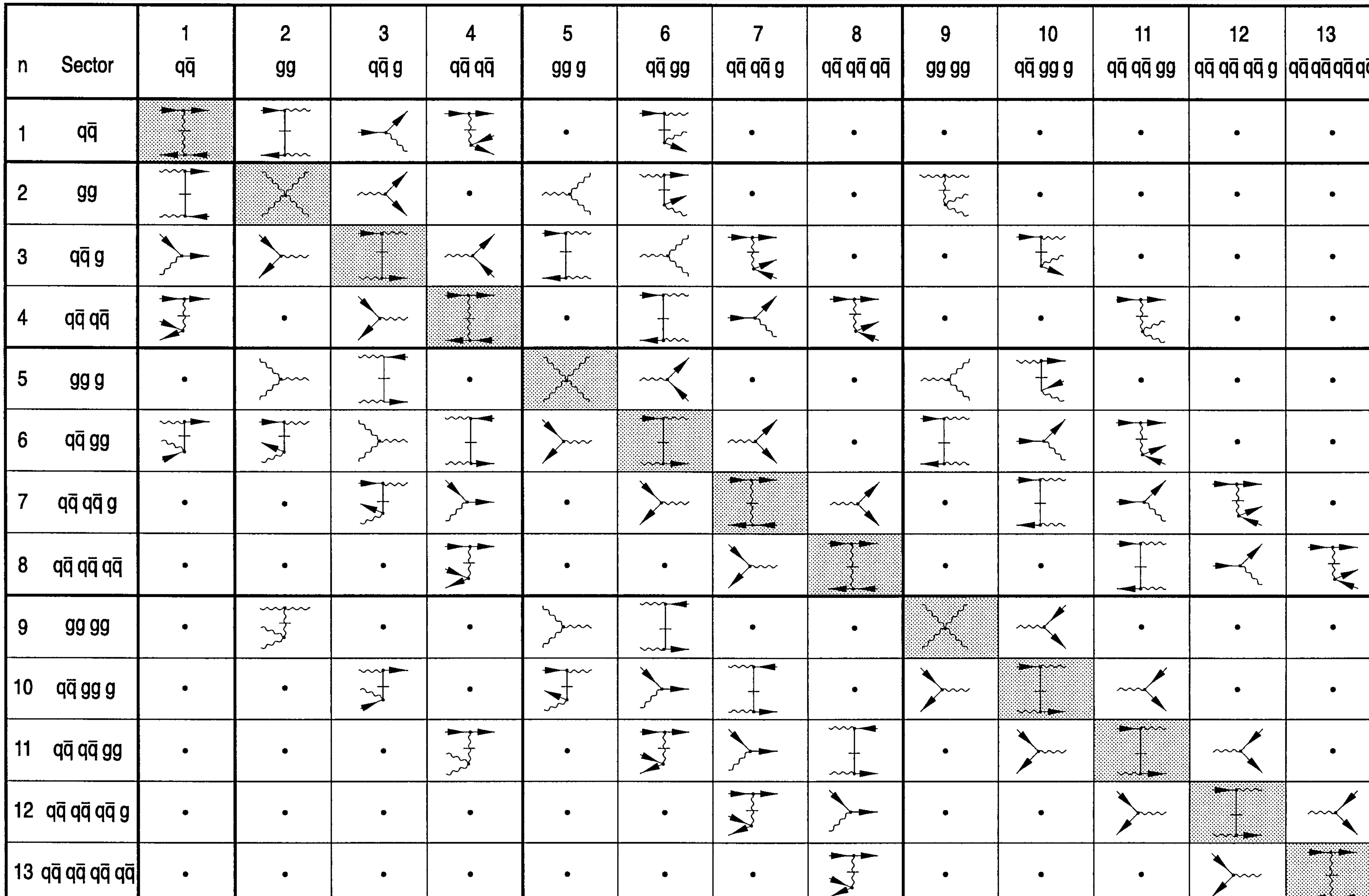
Heisenberg Equation

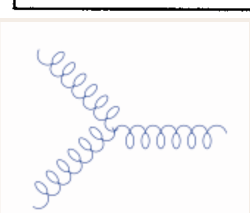
$$H_{LC}^{QCD}|\Psi_h\rangle = \mathcal{M}_h^2\ |\Psi_h\rangle$$

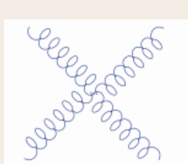
DLCQ: Solve QCD(1+1) for any quark mass and flavors

Hornbostel, Pauli, sjb



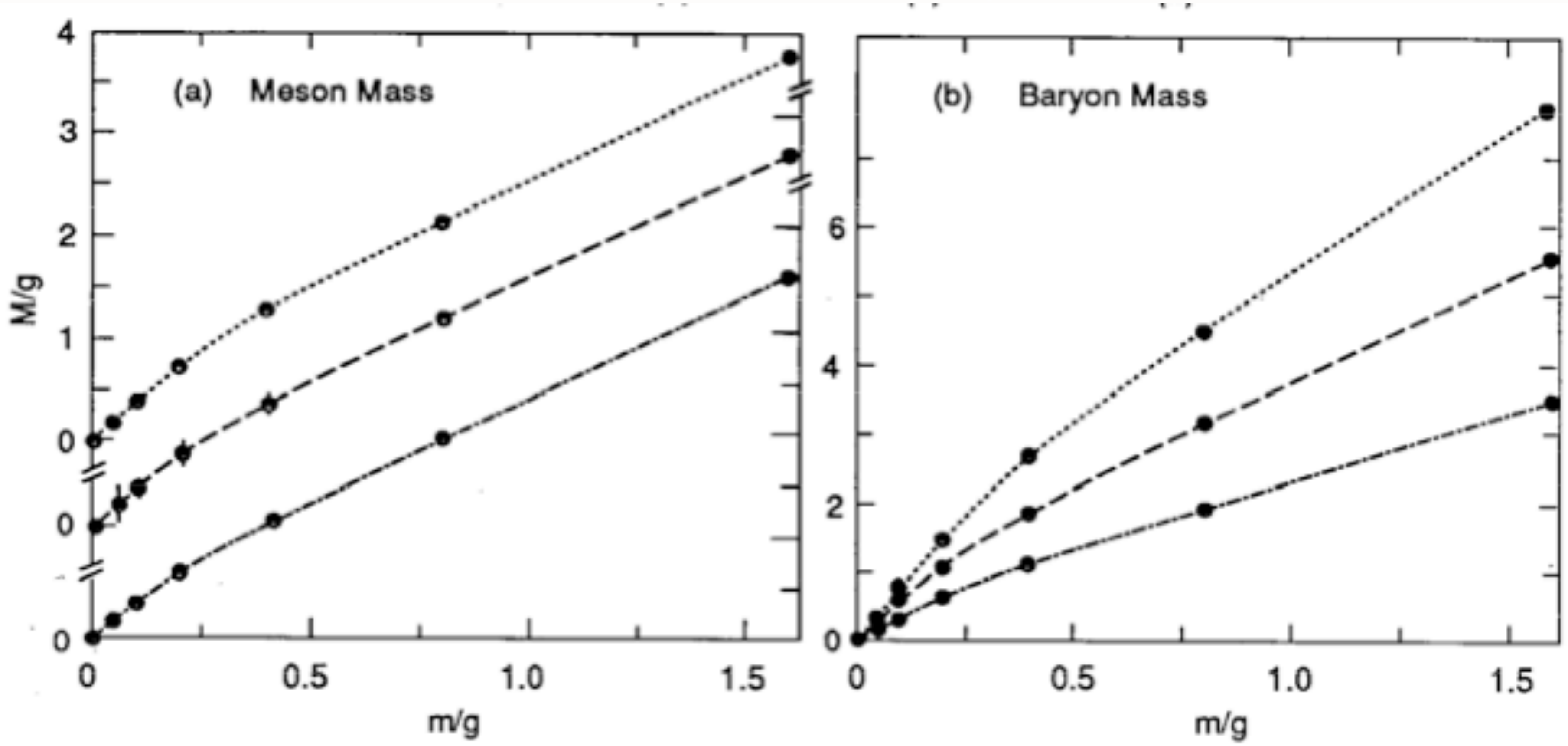




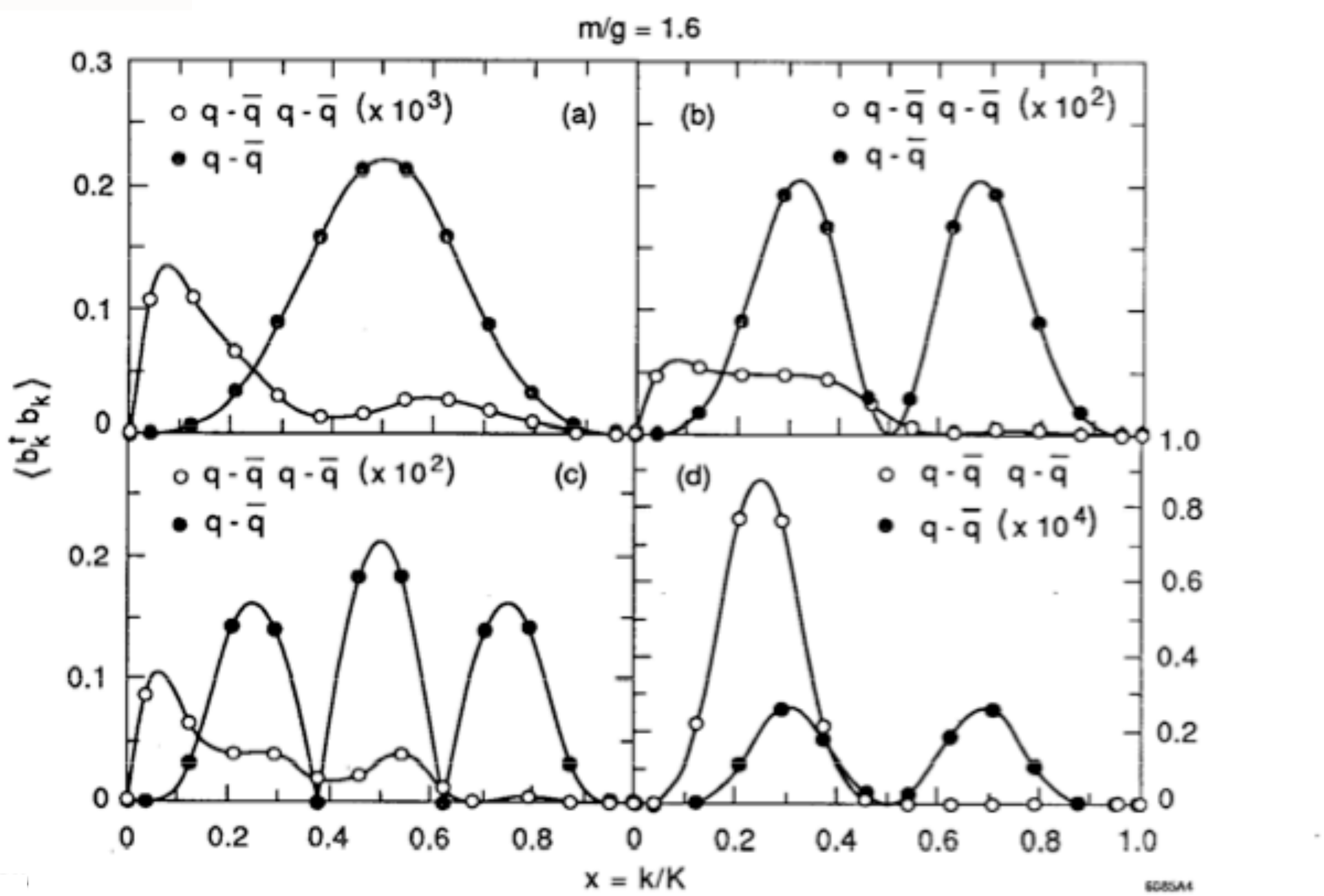


Minkowski space; frame-independent; no fermion doubling; no ghosts

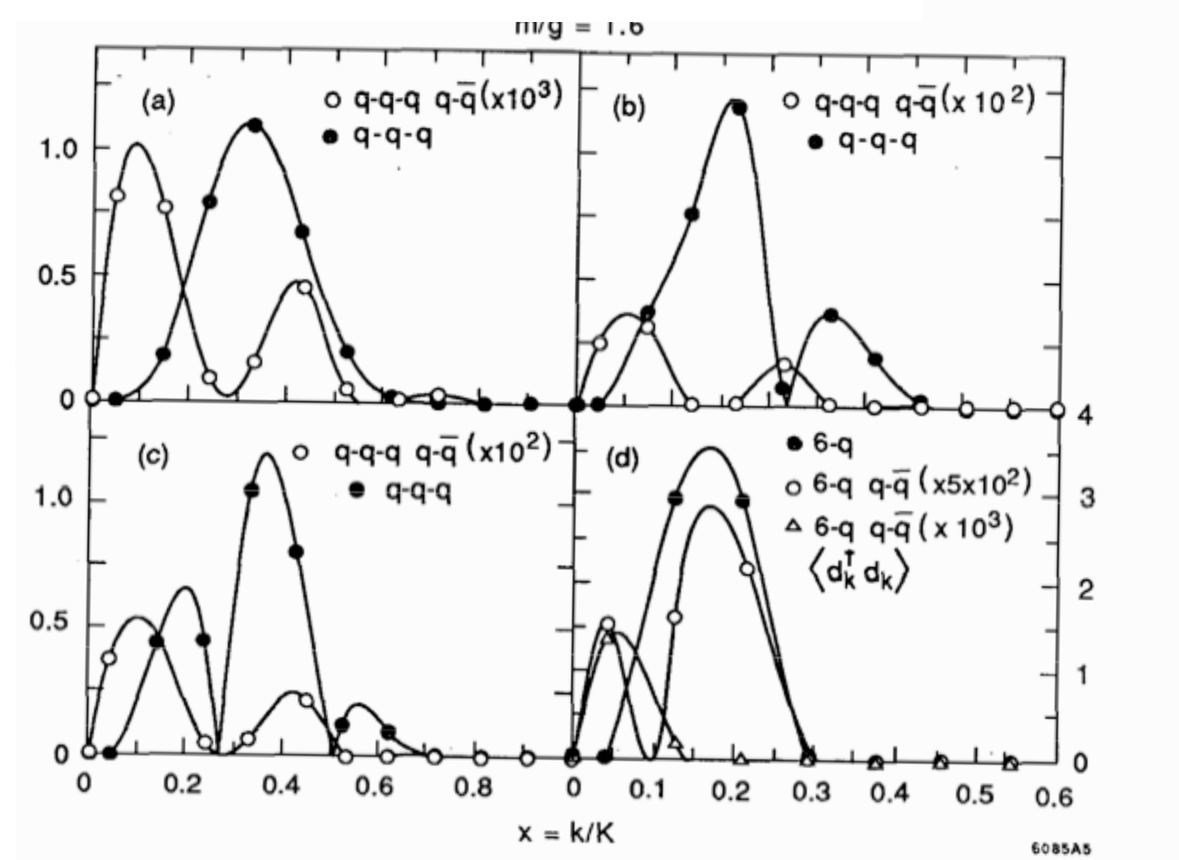
DLCQ: Solve QCD(1+1) for any quark mass and flavors



Extrapolated masses for $N = 2, 3$ and 4 meson and baryon.



a-c) First three states in $N = 3$ meson spectrum for $m/g = 1.6$, 2K=24. d) Eleventh state.



a-c) First three states in $N = 3$ baryon spectrum, 2K=21. d) First $B = 2$ state.

Hornbostel, Pauli, sjb

Angular Momentum on the Light-Front

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z.$$

Conserved
LF Fock-State by Fock-State
Every Vertex

$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

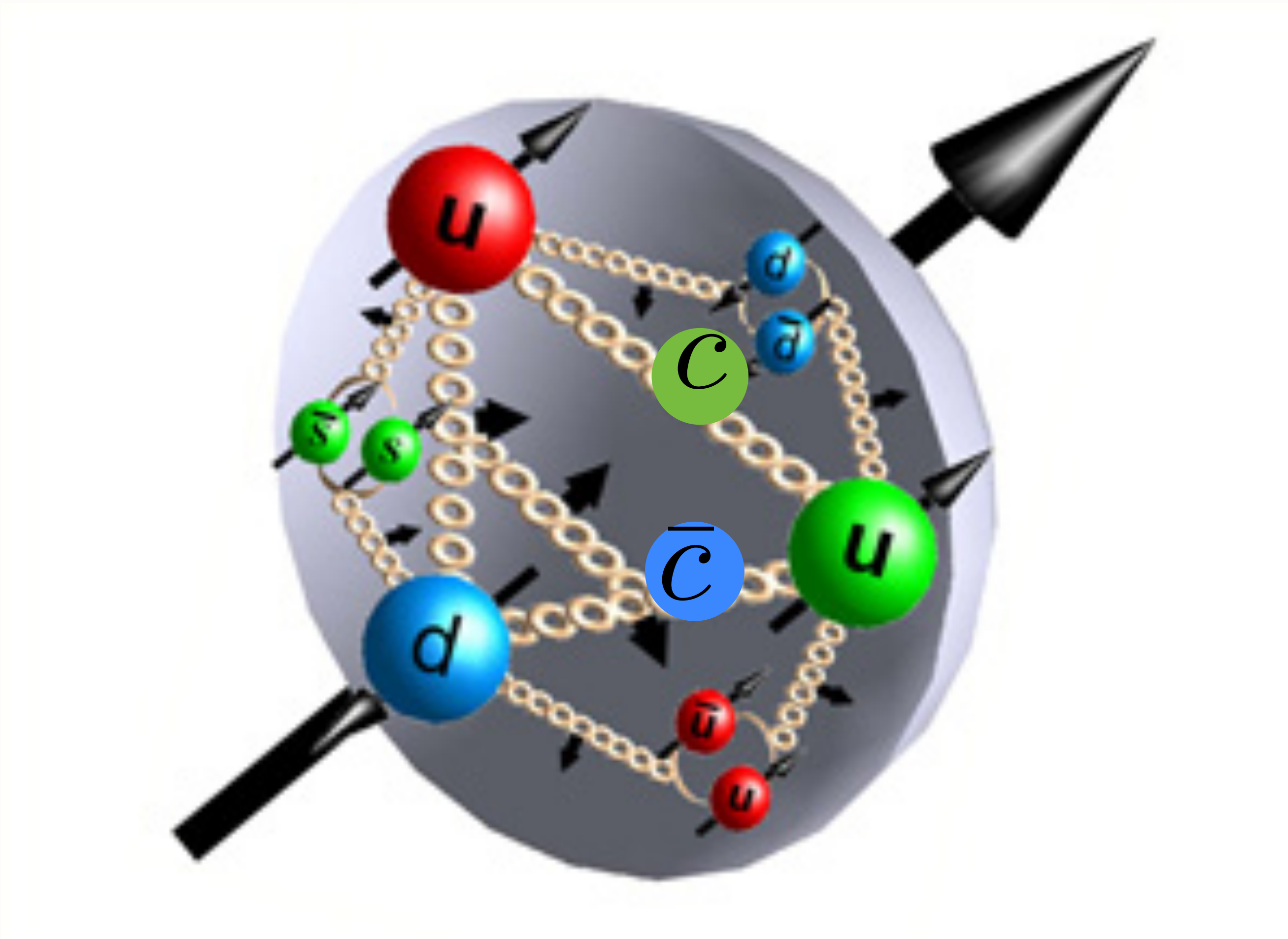
n-1 orbital angular momenta

Parke-Taylor Amplitudes **Stasto**

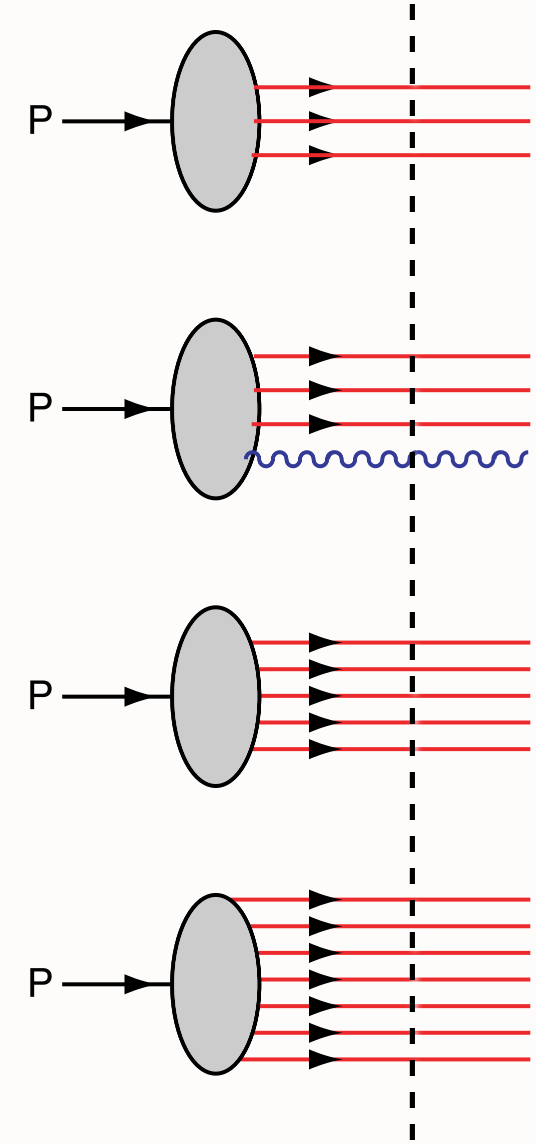
Nonzero Anomalous Moment <--> Nonzero orbital angular momentum
Drell, sjb

Wavefunction at fixed LF time: Arbitrarily Off-Shell in Invariant Mass

Eigenstate: all Fock states contribute



Higher Fock States of the Proton



Fixed LF time

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; \vec{k}_{\perp_i}, \lambda_i>$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

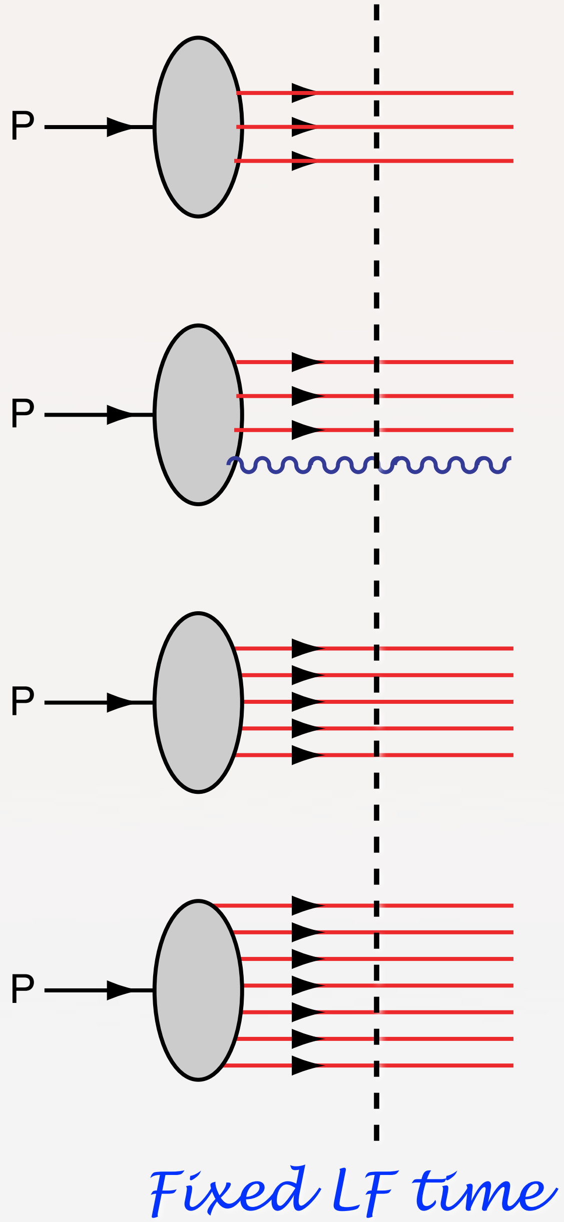
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \sum_i^n x_i = 1, \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Intrinsic heavy quarks

s(x), c(x), b(x) at high x !

$\bar{s}(x) \neq s(x)$

$\bar{u}(x) \neq \bar{d}(x)$

Mueller: gluon Fock states

BFKL Pomeron

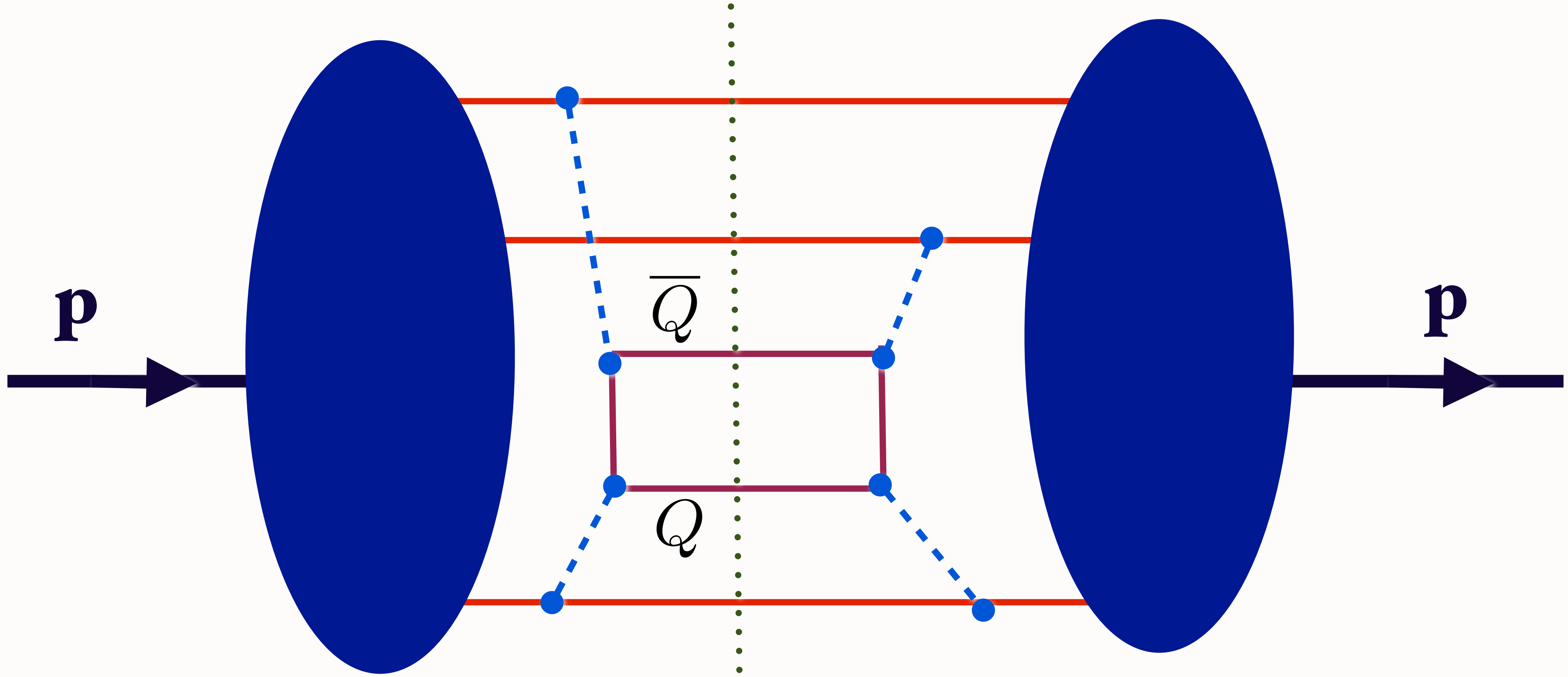
Hidden Color

Fixed LF time

Proton Self Energy

Intrinsic Heavy Quarks

$$x_Q \propto (m_Q^2 + k_\perp^2)^{1/2}$$



Probability (QED) $\propto \frac{1}{M_\ell^4}$

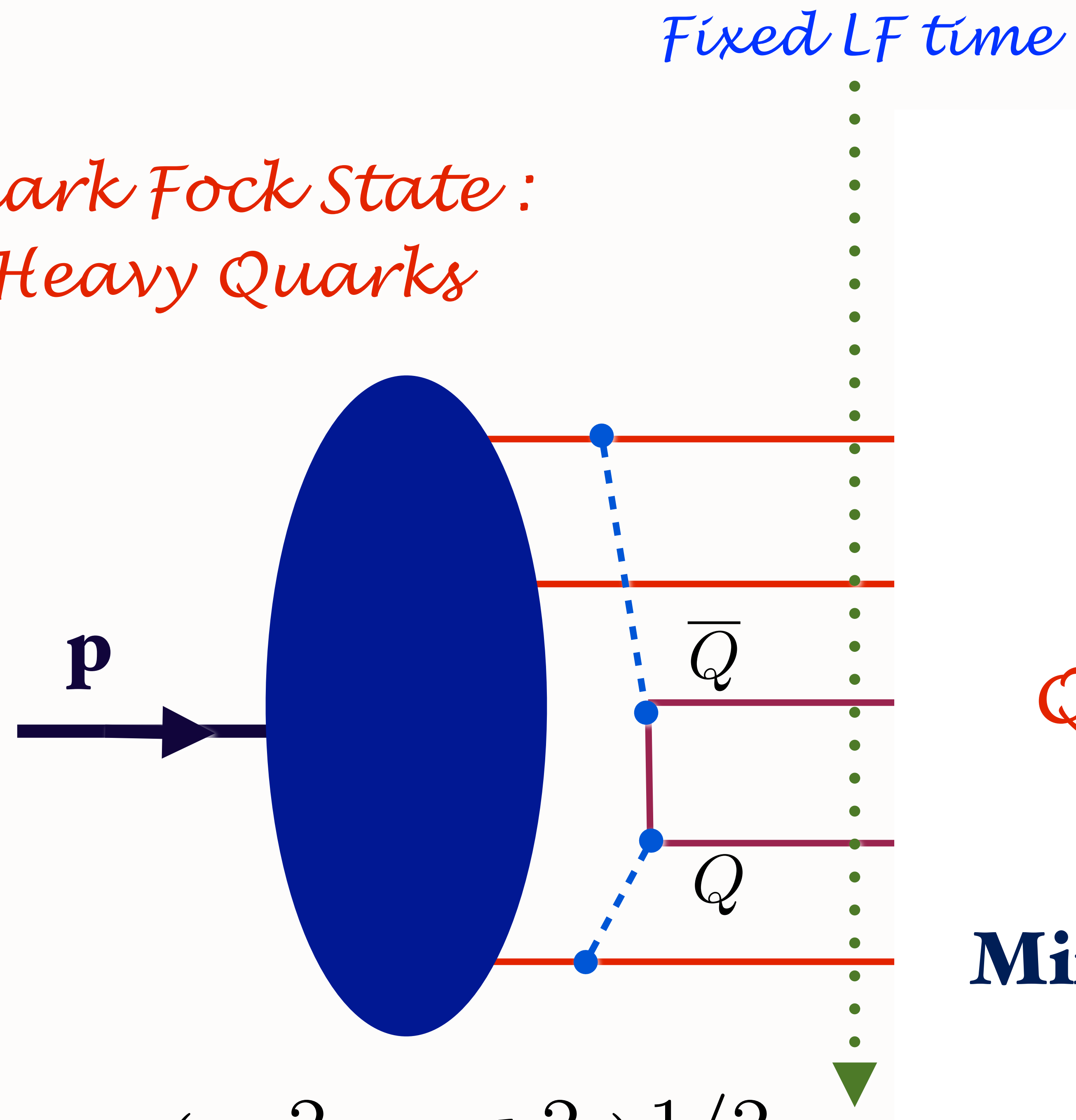
Probability (QCD) $\propto \frac{1}{M_Q^2}$

Collins, Ellis, Gunion, Mueller, sjb

M. Polyakov

Proton 5-quark Fock State :

Intrinsic Heavy Quarks



QCD predicts Intrinsic Heavy Quarks at high x!

Minimal off-shellness

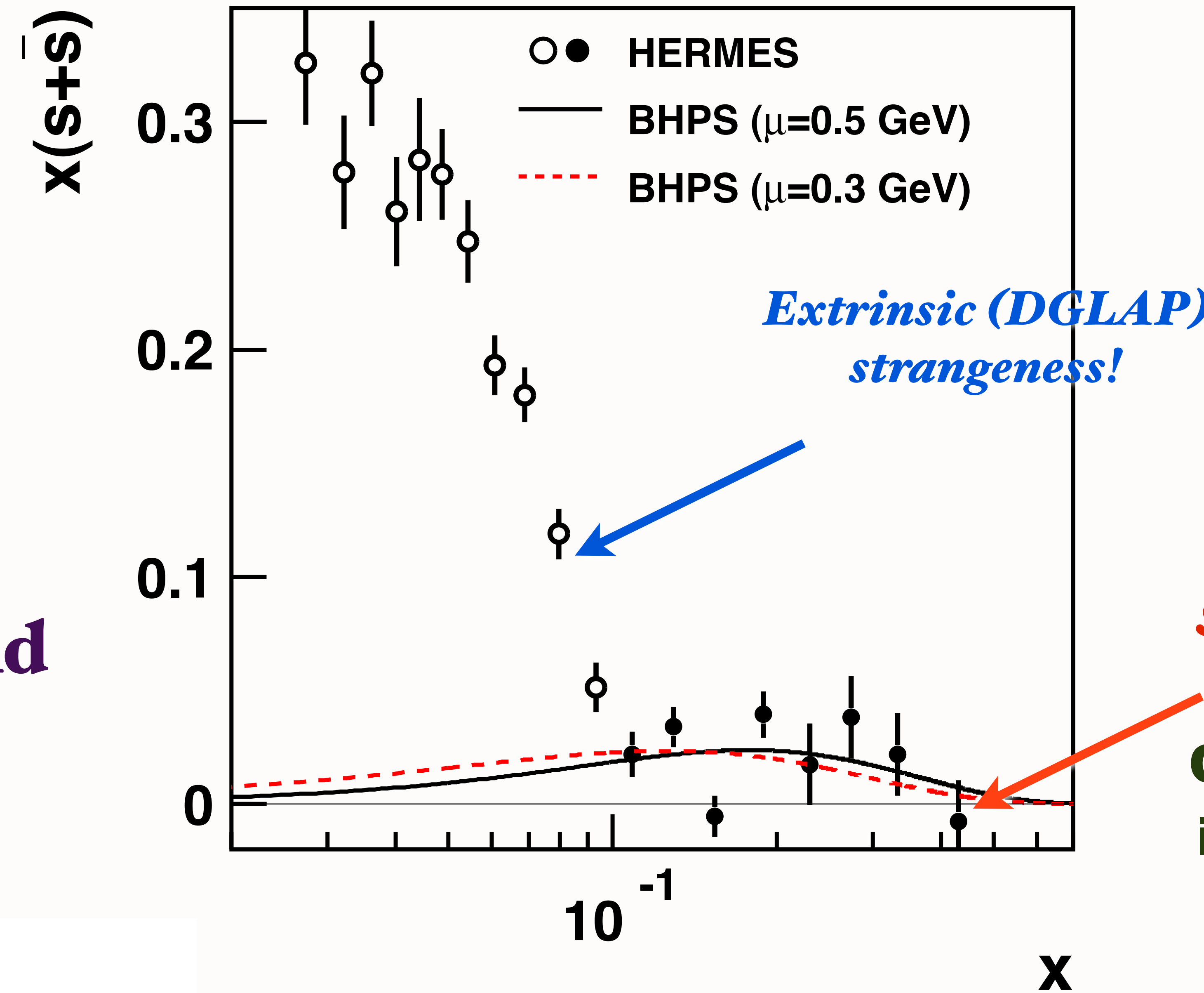
$$x_Q \propto (m_Q^2 + k_\perp^2)^{1/2}$$

$$\text{Probability (QED)} \propto \frac{1}{M_\ell^4} \qquad \text{Probability (QCD)} \propto \frac{1}{M_Q^2}$$

Collins, Ellis, Gunion, Mueller, sjb

M. Polyakov

HERMES: Two components to s(x,Q²)!



Intrinsic strangeness!

Consistent with intrinsic charm data

QCD: $\frac{1}{M_Q^2}$ scaling

W. C. Chang and J.-C. Peng

arXiv:1105.2381

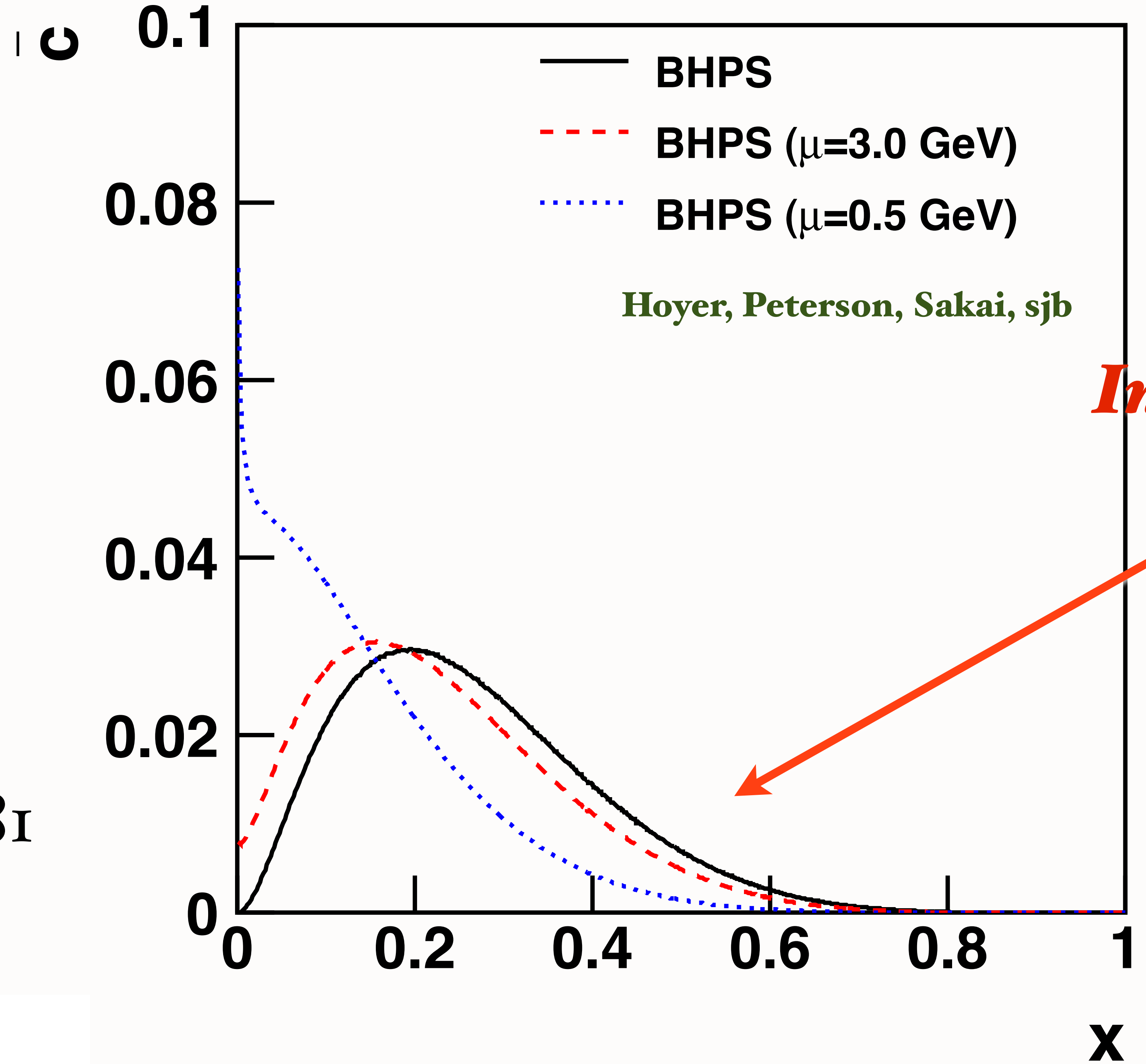
Comparison of the HERMES $x(s(x)+\bar{s}(x))$ data with the calculations based on the BHPS model. The solid and dashed curves are obtained by evolving the BHPS result to $Q^2 = 2.5$ GeV^2 using $\mu = 0.5$ GeV and $\mu = 0.3$ GeV, respectively. The normalizations of the calculations are adjusted to fit the data at $x > 0.1$ with statistical errors only, denoted by solid circles.

$$s(x,Q^2) = s(x,Q^2)_{\text{extrinsic}} + s(x,Q^2)_{\text{intrinsic}}$$

QCD ($1/m_Q^2$) scaling: predict IC !

W. C. Chang and J.-C. Peng

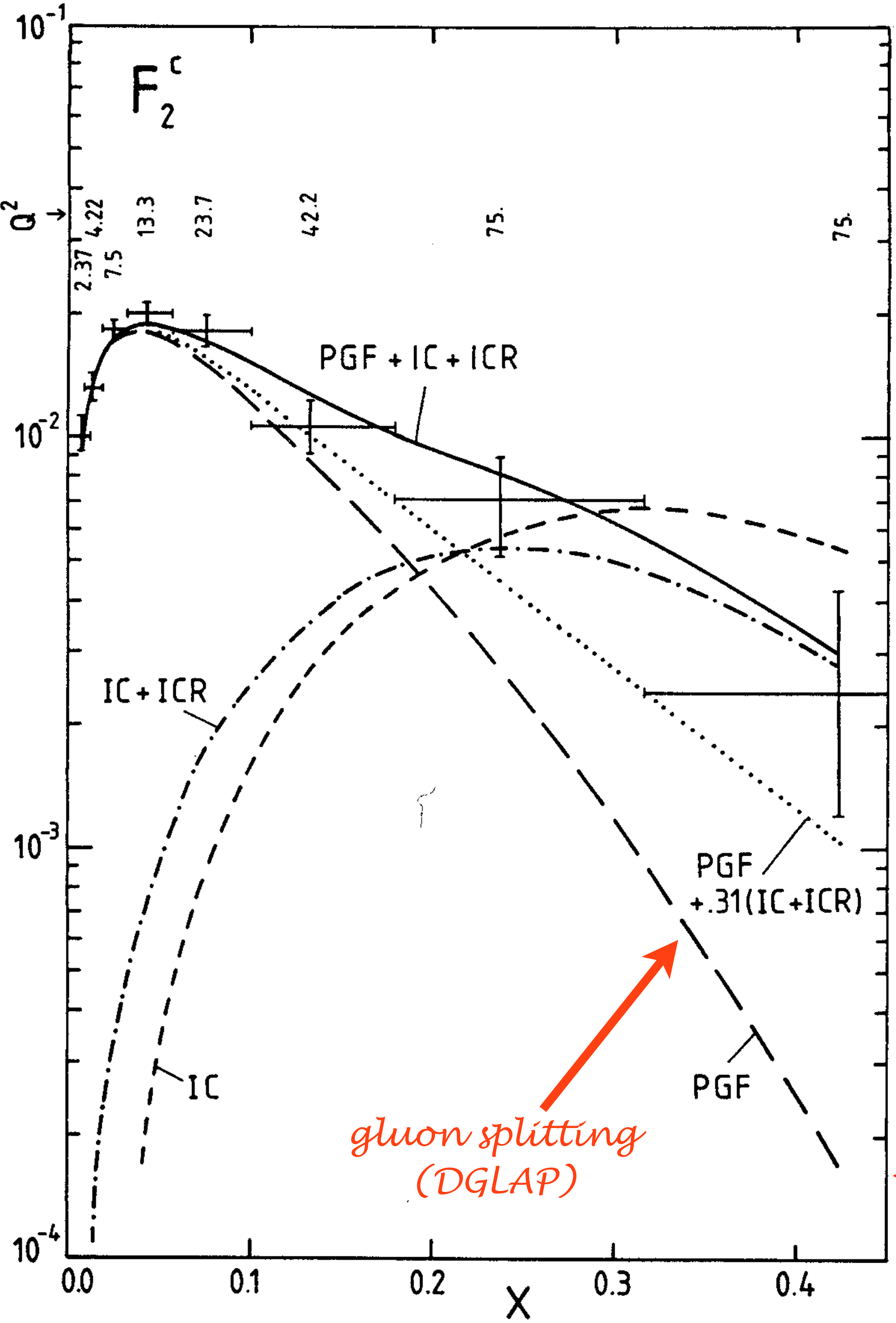
arXiv:1105.2381



Intrinsic Charm

Calculations of the $\bar{c}(x)$ distributions based on the BHPS model. The solid curve corresponds to the calculation using Eq. 1 and the dashed and dotted curves are obtained by evolving the BHPS result to $Q^2 = 75$ GeV^2 using $\mu = 3.0$ GeV, and $\mu = 0.5$ GeV, respectively. The normalization is set at $\mathcal{P}_5^{c\bar{c}} = 0.01$.

Consistent with EMC



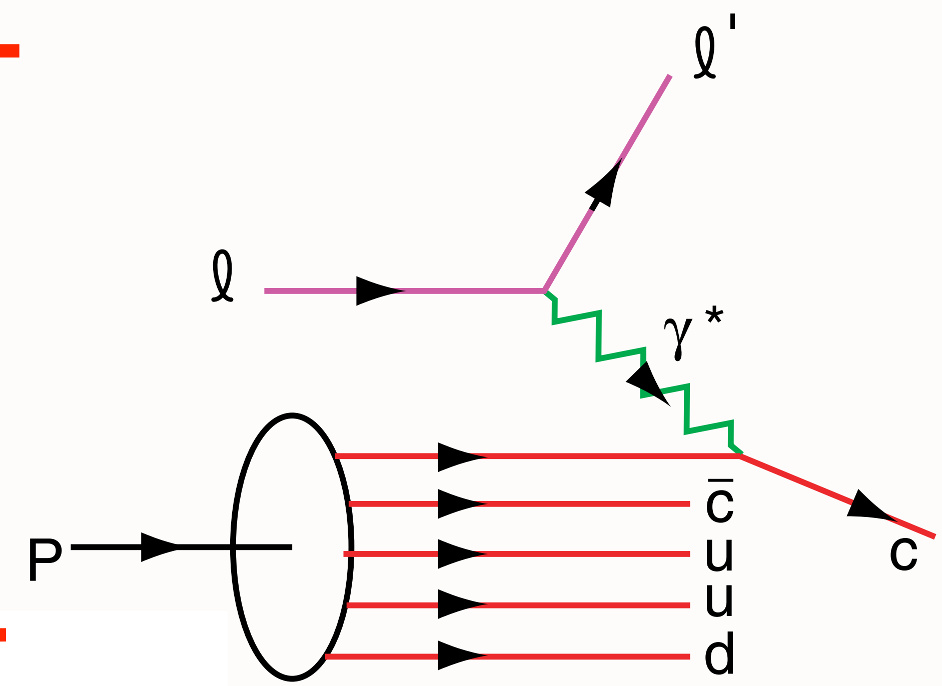
Measurement of Charm Structure Function!

J. J. Aubert et al. [European Muon Collaboration], "Production Of Charmed Particles In 250-Gev Mu+ - Iron Interactions," Nucl. Phys. B 213, 31 (1983).

First Evidence for Intrinsic Charm

Hoyer, Peterson, Sakai, sjb

factor of 30 !



DGLAP / Photon-Gluon Fusion: factor of 30 too small

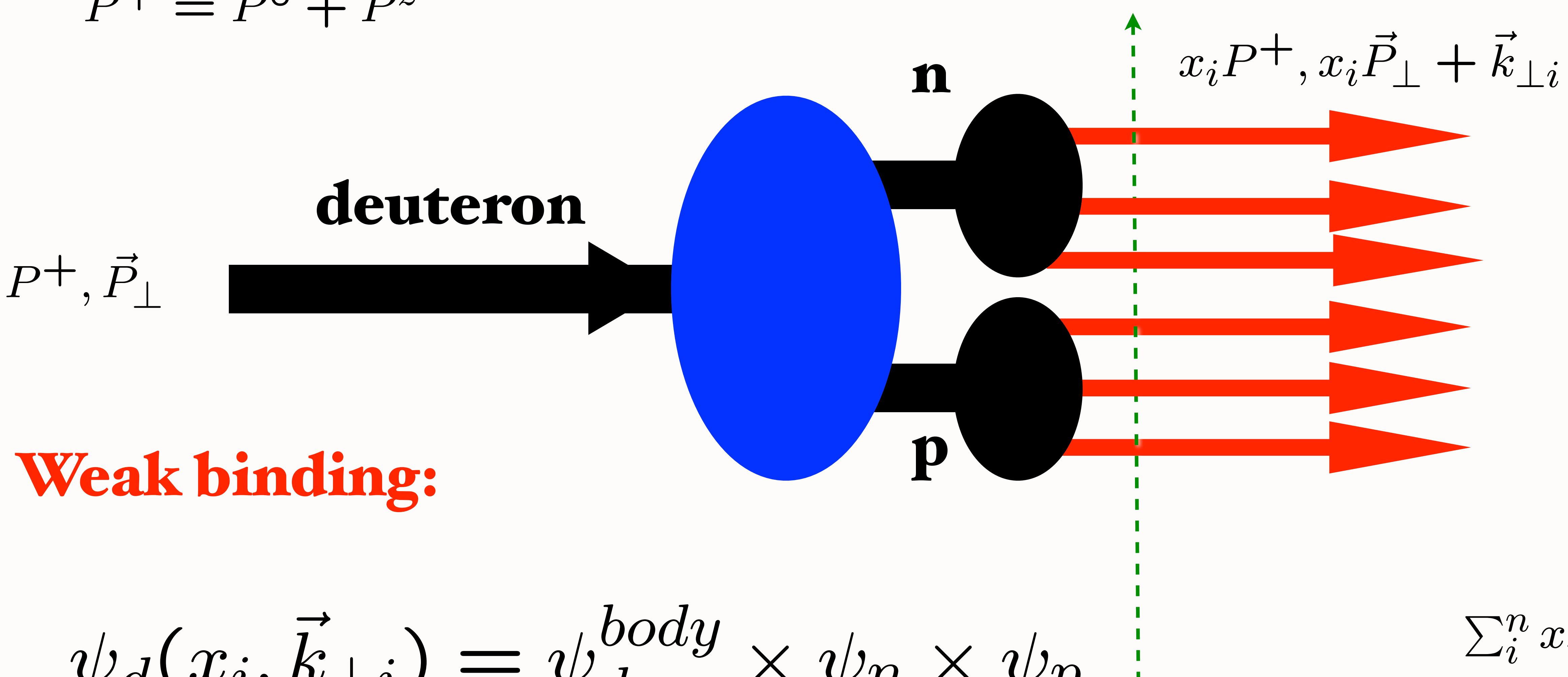
Two Components (separate evolution):

$$c(x, Q^2) = c(x, Q^2)_{\text{extrinsic}} + c(x, Q^2)_{\text{intrinsic}}$$

Deuteron Light-Front Wavefunction

$P^+ = P^0 + P^z$

Fixed $\tau = t + z/c$



Weak binding:

$$\psi_d(x_i, \vec{k}_{\perp i}) = \psi_d^{body} \times \psi_n \times \psi_p$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Two color-singlet combinations of three 3_C

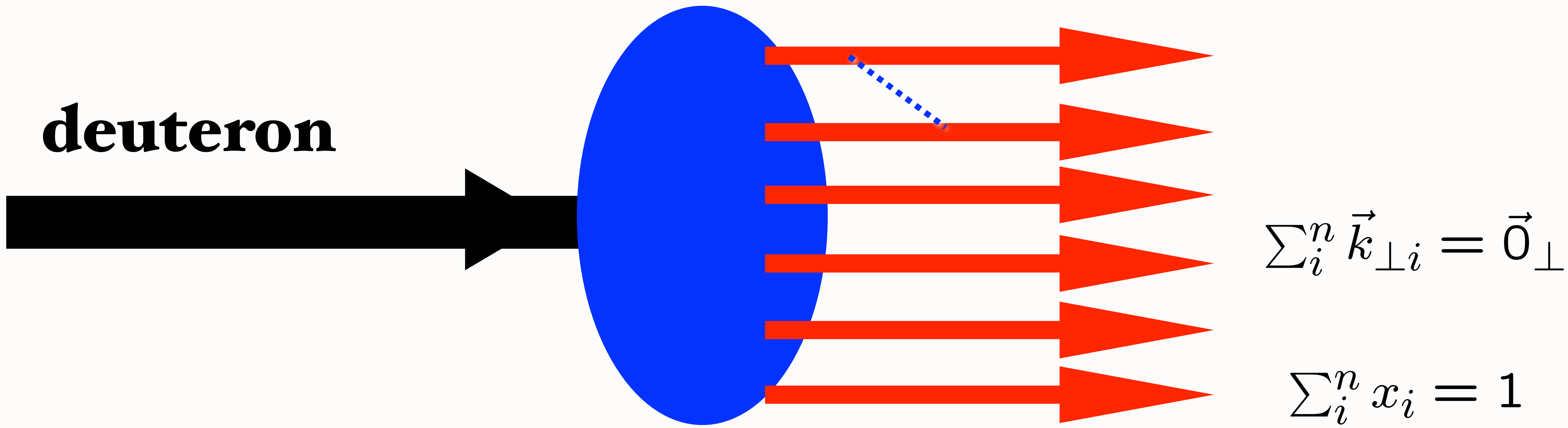
C. Ji, sjb

Properties of Deuteron Light-Front Wavefunction

- Cluster Decomposition Theorem for relativistic systems
- Factorization of LFWF in weak binding limit
- Reduced Nuclear Form Factor
- No Wigner Boosts - Melosh factors built in
- Low energy theorems
- $\psi_d(x_i, \vec{k}_{\perp i}) = \psi_d^{body} \times \psi_n \times \psi_p$

Evolution of 5 color-singlet Fock states

$$\Psi^{\mathbf{d}}_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



$$\Phi_n(x_i, Q) = \int^{k^2_{\perp i} < Q^2} \Pi' d^2 k_{\perp j} \psi_n(x_i, \vec{k}_{\perp j})$$

5 X 5 Matrix Evolution Equation for deuteron distribution amplitude

The evolution equation for six-quark systems in which the constituents have the light-cone longitudinal momentum fractions x_i ($i=1,2,\ldots,6$) can be obtained from a generalization of the proton (three-quark) case.[2] A nontrivial extension is the calculation of the color factor, C_d, of six-quark systems[5] (see below). Since in leading order only pairwise interactions, with transverse momentum Q, occur between quarks, the evolution equation for the six-quark system becomes $\{[dy] = \delta(1-\sum_{i=1}^{6} y_i)\prod_{i=1}^{6} dy_i$, $C_F = (n_c^2-1)/2n_c = \frac{4}{3}$, $\beta = 11 - \frac{2}{3}n_f$, and n_f is the effective number of flavors$\}$

$$\prod_{k=1}^{6} x_k \left[\frac{\partial}{\partial \xi} + \frac{3C_F}{\beta}\right] \tilde{\Phi}(x_i, Q) = -\frac{C_d}{\beta}\int_0^1 [dy] V(x_i, y_i)\, \tilde{\Phi}(y_i, Q),$$

$$\xi(Q^2) = \frac{\beta}{4\pi}\int_{Q_0^2}^{Q^2} \frac{dk^2}{k^2}\alpha_s(k^2) \sim \ln\left(\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)}\right).$$

$$V(x_i, y_i) = 2\prod_{k=1}^{6} x_k \sum_{i\neq j}^{6} \theta(y_i - x_i) \prod_{l\neq i,j}^{6} \delta(x_l - y_l)\frac{y_j}{x_j}\left(\frac{\delta_{h_i\bar{h}_j}}{x_i + x_j} + \frac{\Delta}{y_i - x_i}\right)$$

where $\delta_{h_i\bar{h}_j} = 1$ (0) when the helicities of the constituents $\{i, j\}$ are antiparallel (parallel). The infrared singularity at $x_i = y_i$ is cancelled by the factor $\Delta\tilde{\Phi}(y_i, Q) = \tilde{\Phi}(y_i, Q) - \tilde{\Phi}(x_i, Q)$ since the deuteron is a color singlet.

Hidden Color in QCD

Lepage, Ji, sjb

- **Deuteron six-quark wavefunction**
- **5 color-singlet combinations of 6 color-triplets -- only one state is |n p>**
- **Components evolve towards equality at short distances**
- **Hidden color states dominate deuteron form factor and photodisintegration at high momentum transfer**

$$\boxed{R = \frac{\frac{d\sigma}{dt}(\gamma d \to \Delta^{++}\Delta^{--})}{\frac{d\sigma}{dt}(\gamma d \to pn)}}$$

Ratio predicted to approach 2:5

QCD Prediction for Deuteron Form Factor

$$F_d(Q^2) = \left[\frac{\alpha_s(Q^2)}{Q^2}\right]^5 \sum_{m,n} d_{mn} \left(\ln \frac{Q^2}{\Lambda^2}\right)^{-\gamma_n^d - \gamma_m^d} \left[1 + O\left(\alpha_s(Q^2), \frac{m}{Q}\right)\right]$$

Define “Reduced” Form Factor

$$f_d(Q^2) \equiv \frac{F_d(Q^2)}{F_N{}^2(Q^2/4)}\,.$$

Same large momentum transfer behavior as pion form factor

$$f_d(Q^2) \sim \frac{\alpha_s(Q^2)}{Q^2}\left(\ln \frac{Q^2}{\Lambda^2}\right)^{-(2/5)\, C_F/\beta}$$

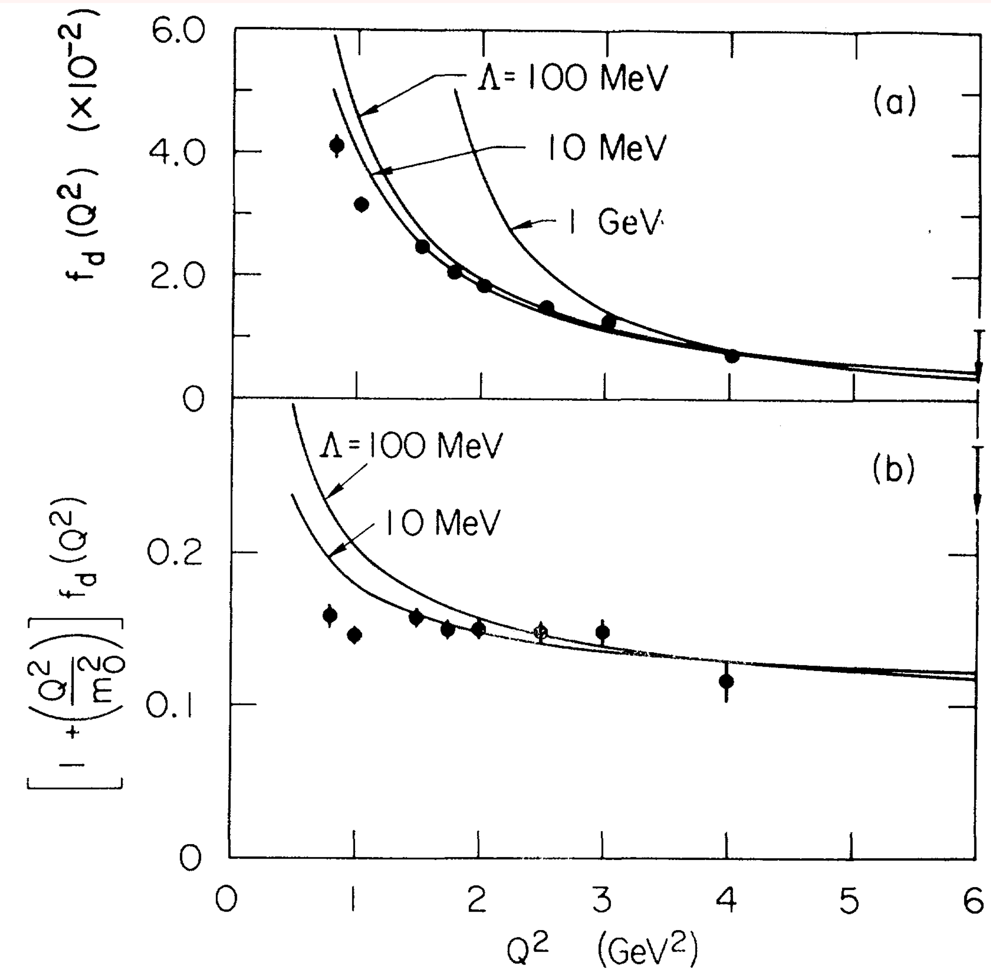
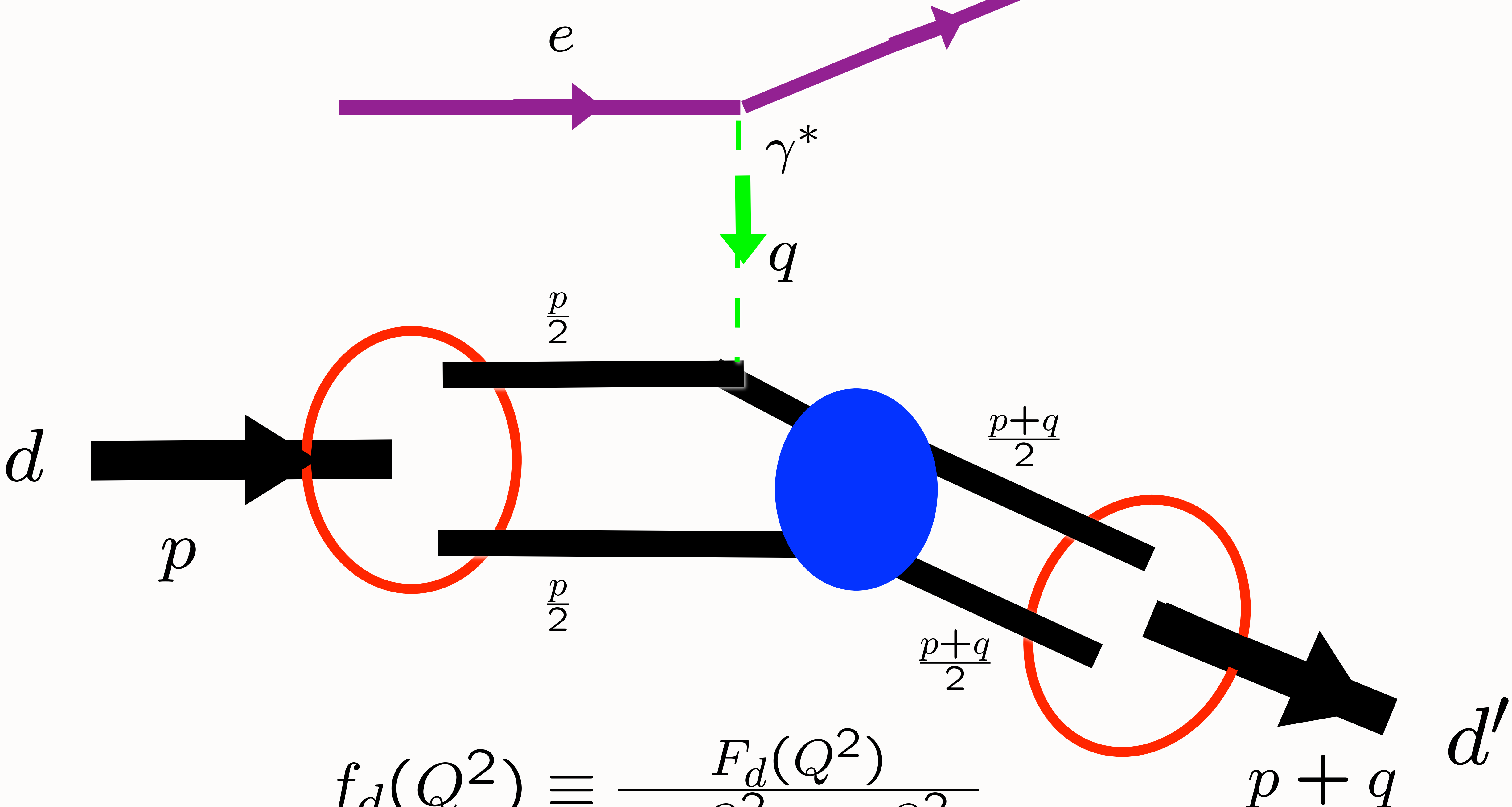


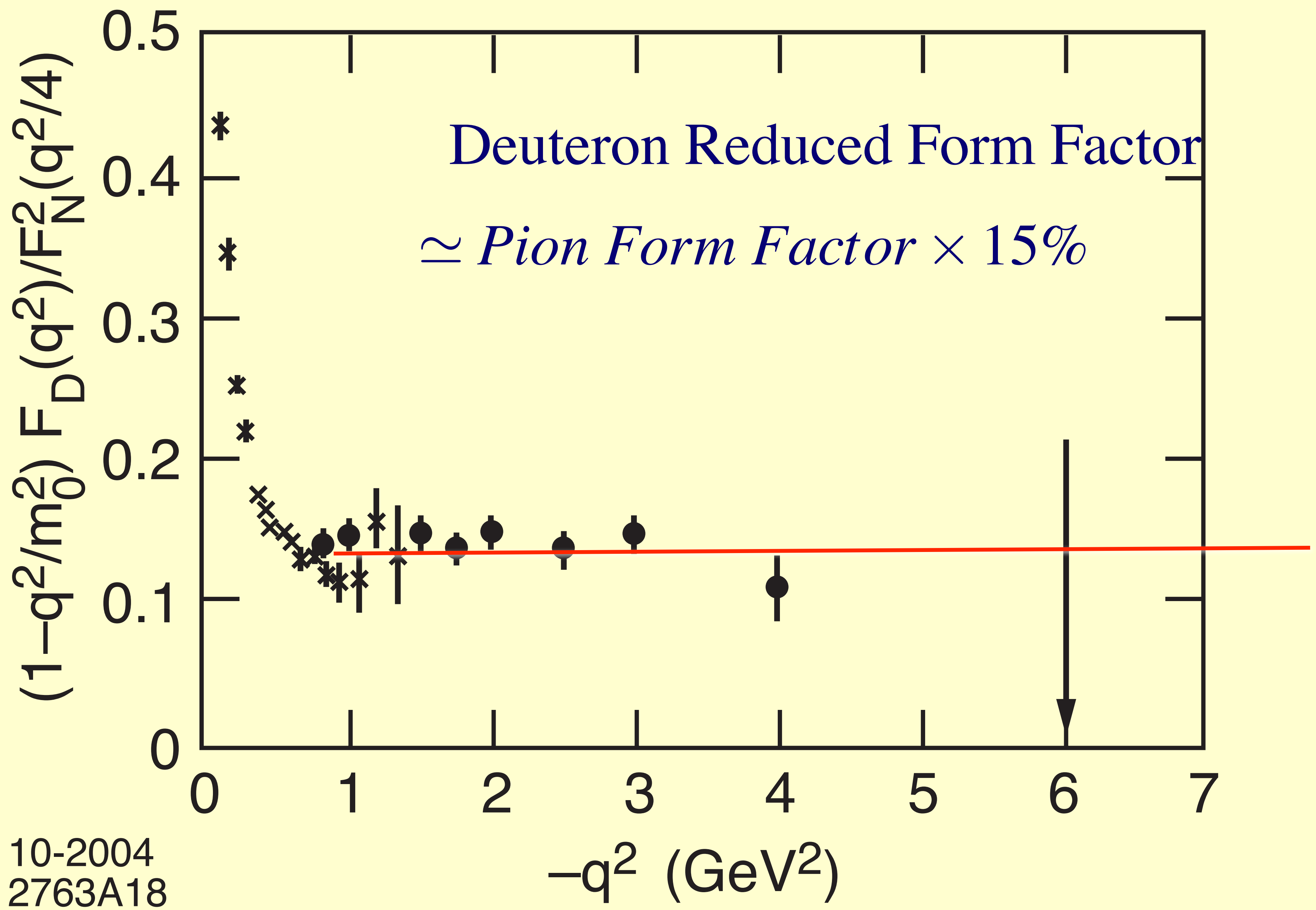
FIG. 2. (a) Comparison of the asymptotic QCD prediction $f_d(Q^2) \propto (1/Q^2)[\ln(Q^2/\Lambda^2)]^{-1-(2/5)C_F/\beta}$ with final data of Ref. 10 for the reduced deuteron form factor, where $F_N(Q^2) = [1 + Q^2/(0.71\ \mathrm{GeV}^2)]^{-2}$. The normalization is fixed at the $Q^2 = 4\ \mathrm{GeV}^2$ data point. (b) Comparison of the prediction $[1 + (Q^2/m_0{}^2)]\, f_d(Q^2) \propto [\ln(Q^2/\Lambda^2)]^{-1-(2/5)C_F/\beta}$ with the above data. The value $m_0{}^2 = 0.28\ \mathrm{GeV}^2$ is used (Ref. 8).

Define "Reduced" Form Factor



$$f_d(Q^2) \equiv \frac{F_d(Q^2)}{F_p(\frac{Q^2}{4})F_n(\frac{Q^2}{4})}$$

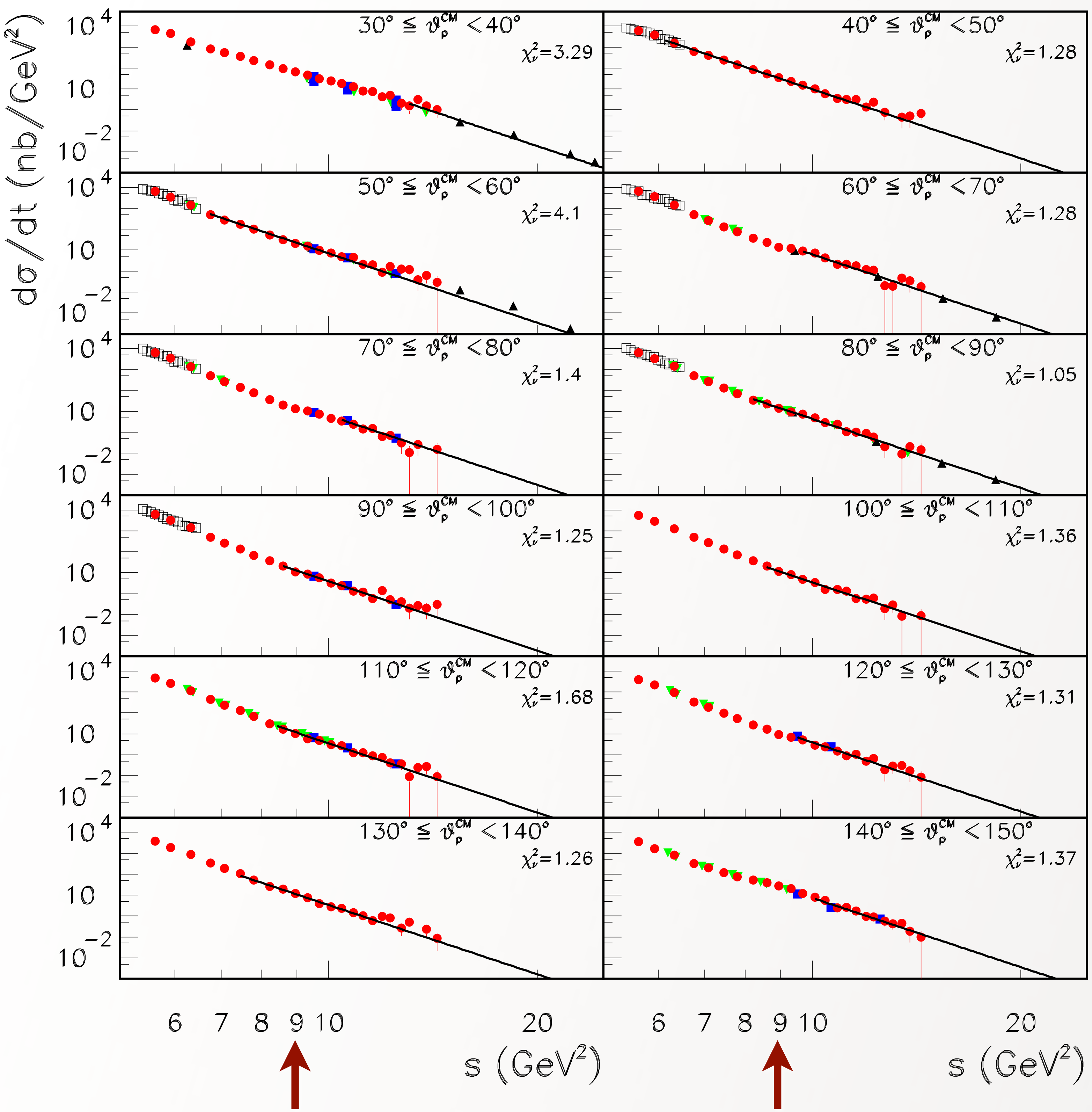
Elastic electron-deuteron scattering



- ***Large Magnitude: Evidence for Hidden Color in the Deuteron***

Deuteron Photodisintegration and Dimensional Counting

P.Rossi et al, P.R.L. 94, 012301 (2005)



PQCD and AdS/CFT:

$$s^{n_{tot}-2}\frac{d\sigma}{dt}(A+B\to C+D) =$$
$$F_{A+B\to C+D}(\theta_{CM})$$

$$s^{11}\frac{d\sigma}{dt}(\gamma d\to np) = F(\theta_{CM})$$

$$n_{tot}-2 =$$
$$(1 + 6 + 3 + 3) - 2 = 11$$

$$\gamma d \to (uudddus\bar{s}) \to np$$

at $s \simeq 9$ GeV^2

$$\gamma d \to (uudduc\bar{c}) \to np$$

at $s \simeq 25$ GeV^2

$\gamma d \rightarrow np$

$\gamma d \rightarrow (uuddd us\bar{s}) \rightarrow np$ at $s = 9$ GeV^2

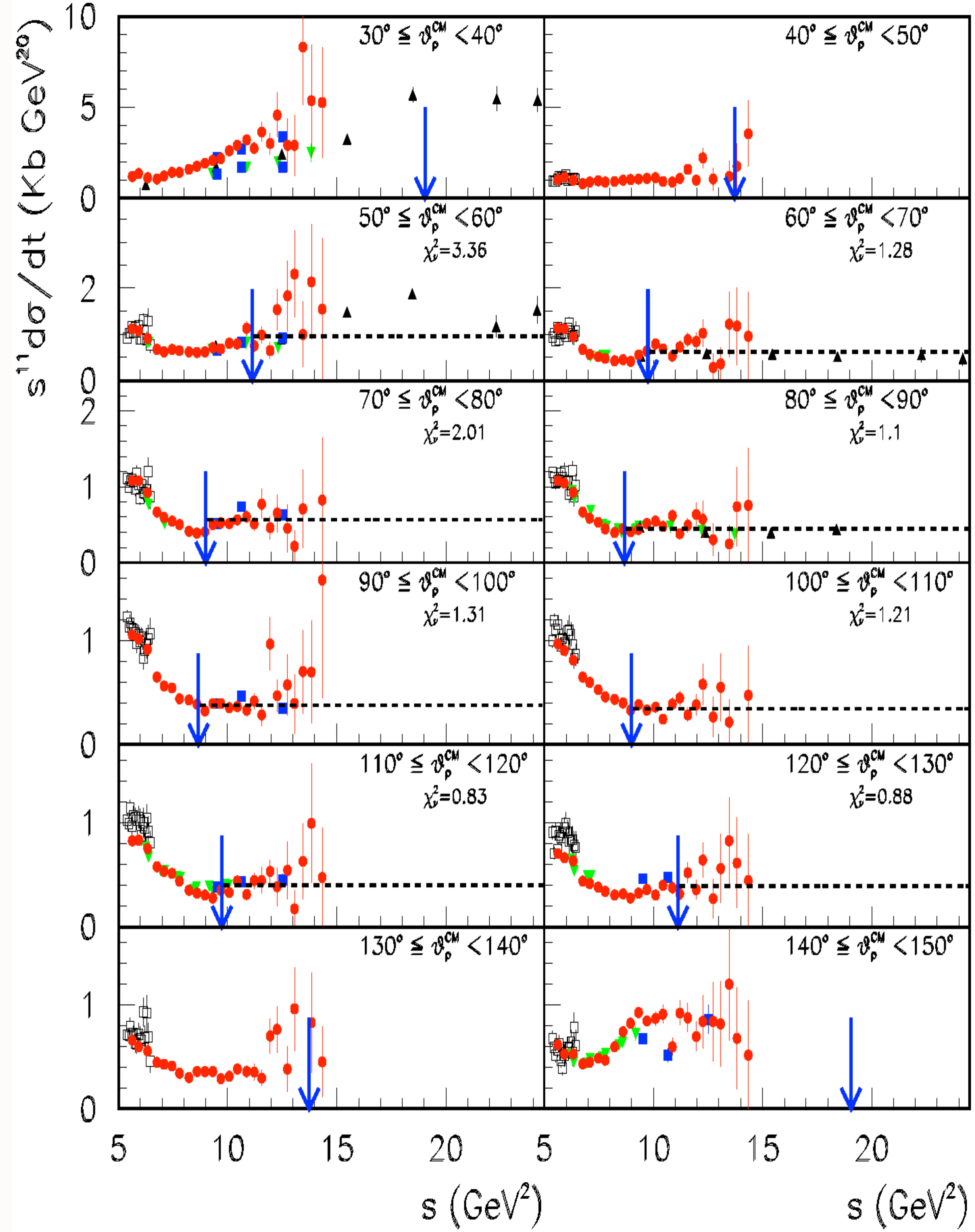
Fit of dσ/dt data for the central angles and $P_T \geq 1.1$ GeV/c with $A\ s^{-11}$

For all but two of the fits $\chi^2 \leq 1.34$

- Better χ^2 at 55° and 75° if different data sets are renormalized to each other
- No data at $P_T \geq 1.1$ GeV/c at forward and backward angles
- Clear s^{-11} behaviour for last 3 points at 35°

Data consistent with CCR

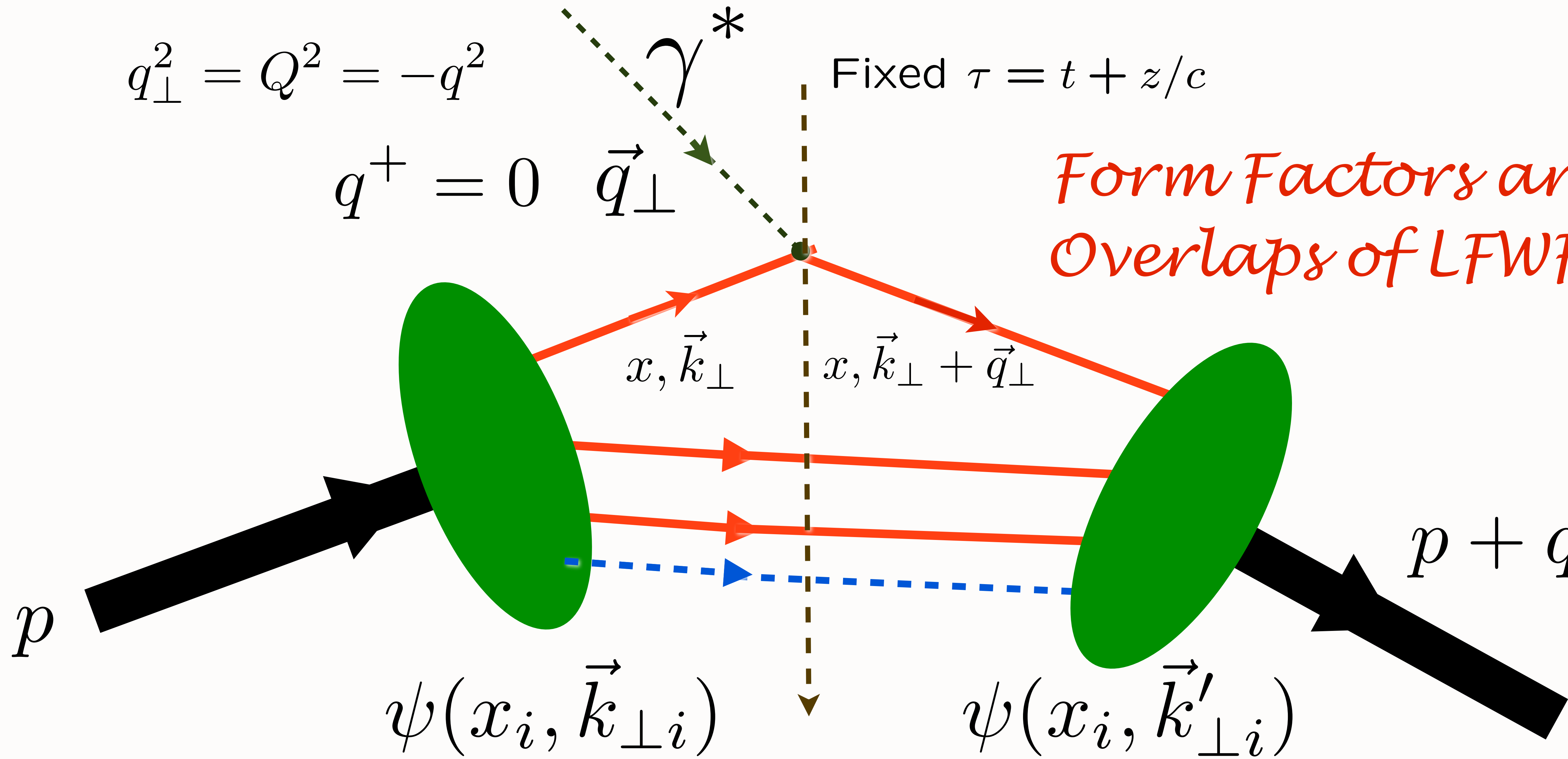
P.Rossi et al, P.R.L. 94, 012301 (2005)



$$< p+q|j^+(0)|p >= 2p^+F(q^2)$$

Interaction picture

Form Factors are Overlaps of LFWFs



struck $\quad \vec k'_{\perp i} = \vec k_{\perp i} + (1 - x_i)\vec q_\perp$

spectators $\quad \vec k'_{\perp i} = \vec k_{\perp i} - x_i \vec q_\perp$

Drell &Yan, West
Exact LF formula!

Exact LF Formula for Pauli Form Factor

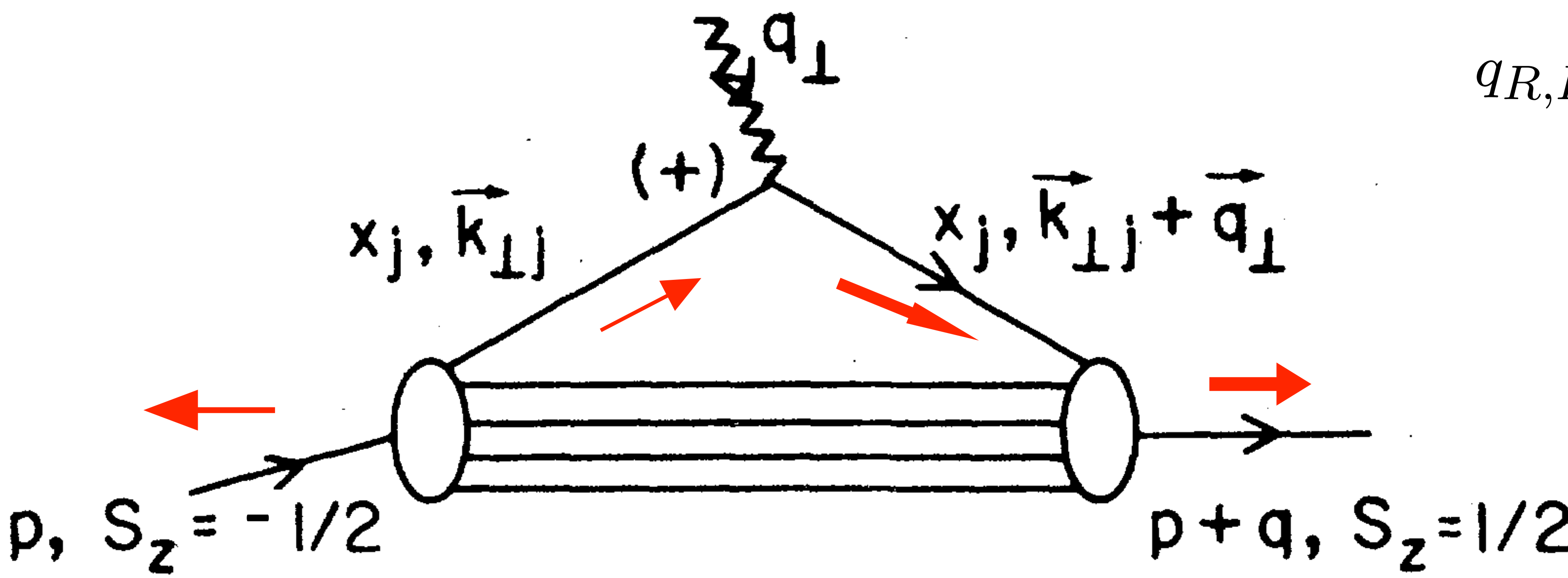
$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

Drell, sjb

$$\left[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

$$q_{R,L} = q^x \pm i q^y$$

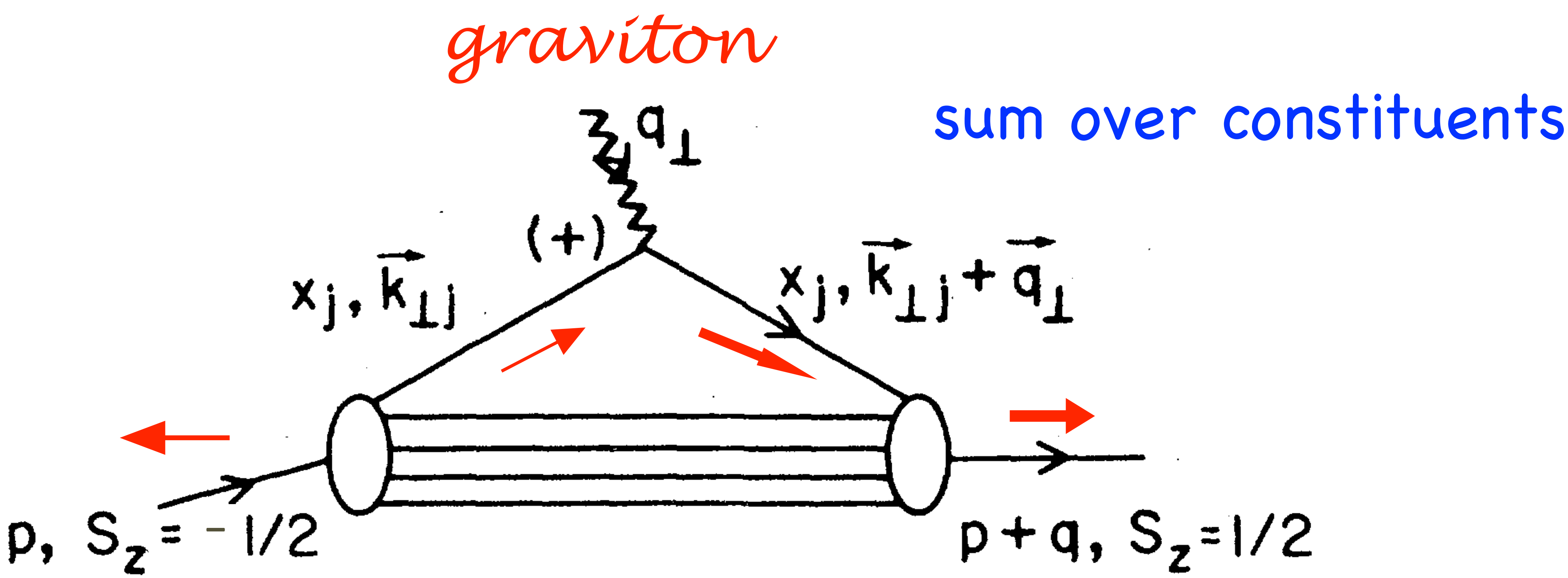


$$\boxed{\text{Must have } \Delta\ell_z = \pm 1 \text{ to have nonzero } F_2(q^2)}$$

Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum

Vanishing Anomalous gravitomagnetic moment B(0)

Terayev, Okun, et al: *B(0) Must vanish because of Equivalence Theorem*



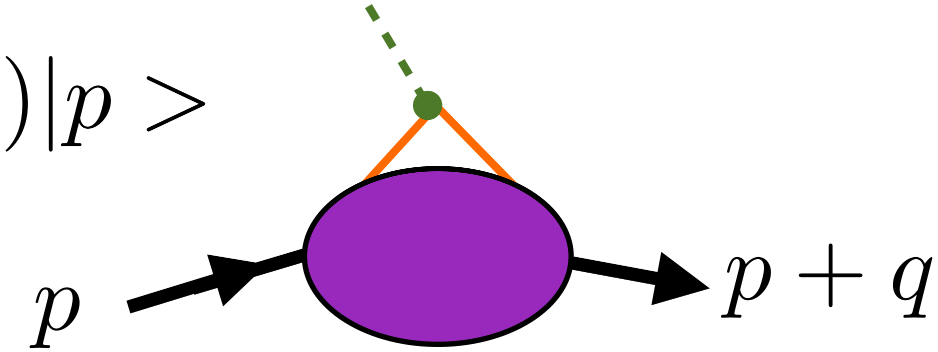
Hwang, Schmidt, sjb; Holstein et al

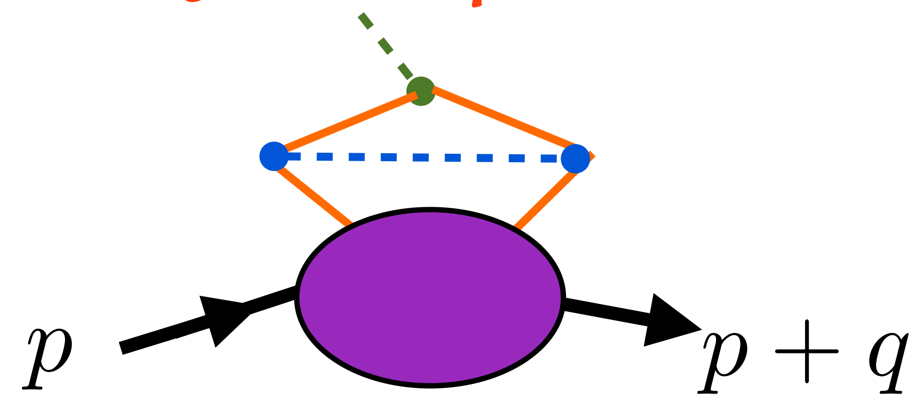
$$\boxed{B(0) = 0}$$

Each Fock State

Calculation of proton form factor in Instant Form

$$< p+q|J^{\mu}(0)|p >$$



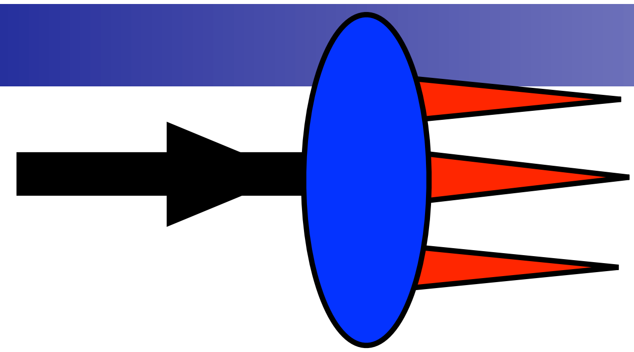


- **Need to boost proton wavefunction from p to p +q: Extremely complicated dynamical problem; particle number changes**
- **Need to couple to all currents arising from vacuum!! Remains even after normal-ordering**
- **Each time-ordered contribution is frame-dependent**
- **Divide by disconnected vacuum diagrams**

Advantages of the Dirac's Front Form for Hadron Physics

- Measurements are made at fixed τ
- Causality is automatic
- Structure Functions are squares of LFWFs
- Form Factors are overlap of LFWFs
- LFWFs are frame-independent -- no boosts
- No dependence on observer's frame
- Dual to AdS/QCD
- LF Vacuum trivial -- no condensates

- *Light Front Wavefunctions:*



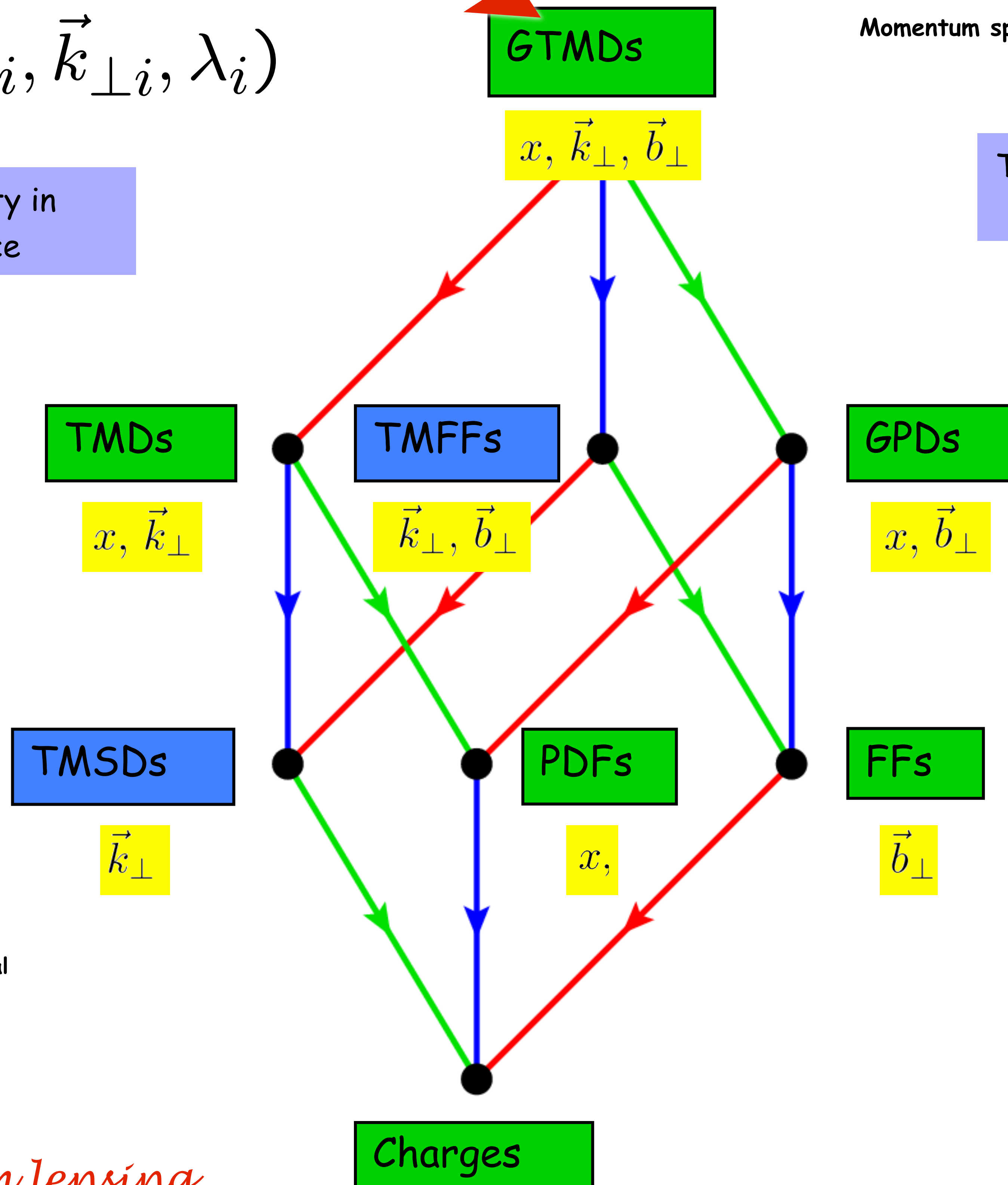
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

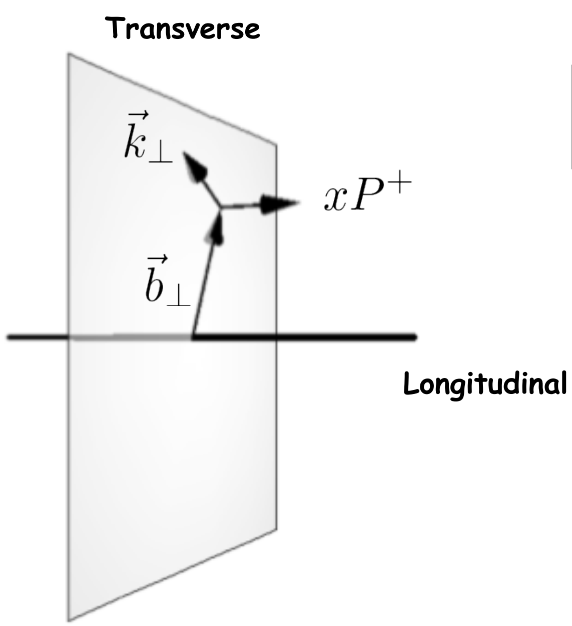
Transverse density in momentum space

Momentum space $\vec{k}_\perp \leftrightarrow \vec{z}_\perp$ **Position space**

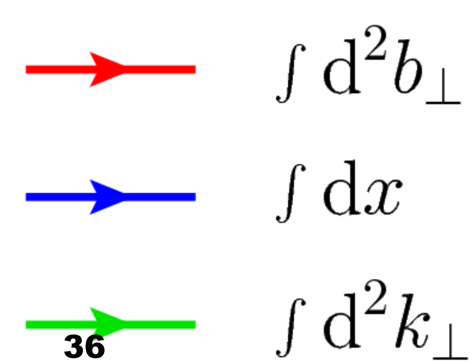
$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$

Transverse density in position space



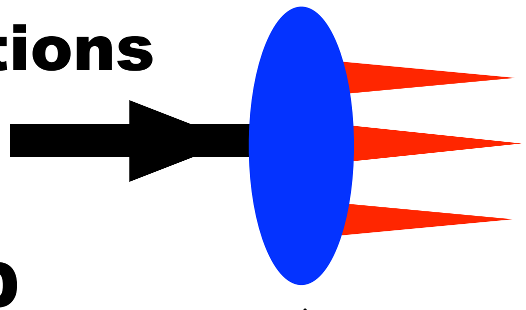


Lorce



Sivers, T-odd from lensing

- **LF wavefunctions play the role of Schrödinger wavefunctions in Atomic Physics**



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

- **LFWFs=Hadron Eigensolutions: Direct Connection to QCD Lagrangian**

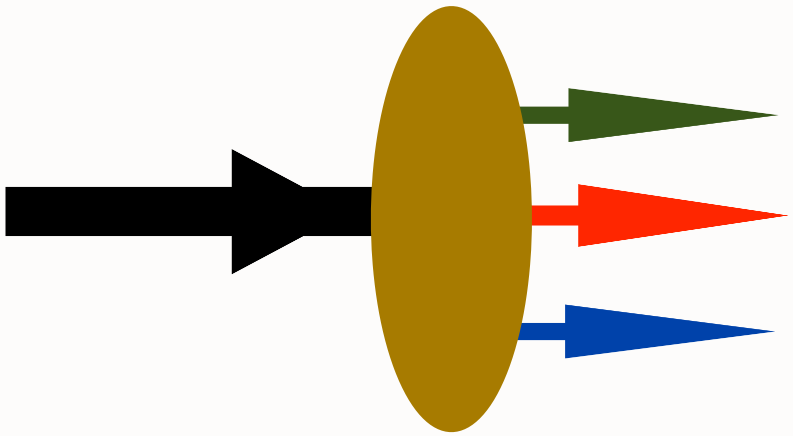
- **Relativistic, frame-independent: no boosts, no disc contraction, Melosh built into LF spinors**

- **Hadronic observables computed from LFWFs: Form factors, Structure Functions, Distribution Amplitudes, GPDs, TMDs, Weak Decays, ... modulo `lensing' from ISIs, FSIs**

- **Cannot compute current matrix elements using instant or point form from eigensolutions alone -- need to include vacuum currents!**

- **Hadron Physics without LFWFs is like Biology without DNA!**

- *Hadron Physics without LFWFs is like Biology without DNA!*



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

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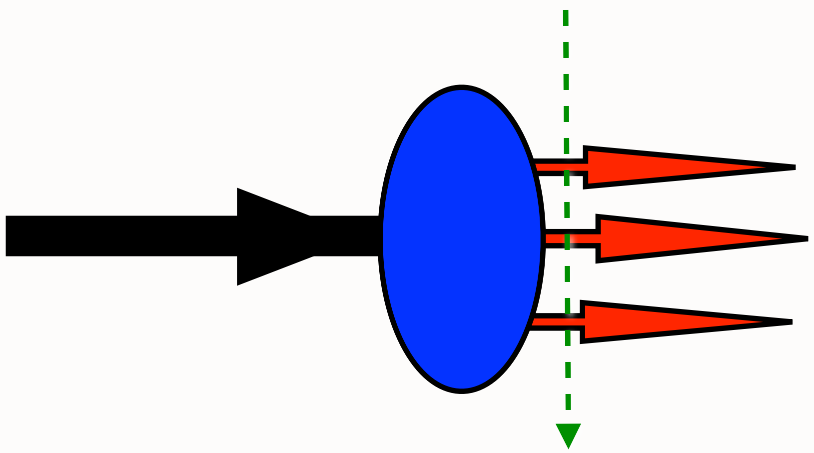
Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x_i = \frac{k_i^+}{P^+}$$

Invariant under boosts. Independent of P^μ

$$\mathrm{H}_{LF}^{QCD}|\psi> = M^2|\psi>$$

Direct connection to QCD Lagrangian

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

$$H_{QED}$$

QED atoms: positronium and muonium

$$(H_0 + H_{int})\ |\Psi>= E\ |\Psi>$$

Coupled Fock states

$$[-\frac{\Delta^2}{2m_{\rm red}} + V_{\rm eff}(\vec{S},\vec{r})]\ \psi(\vec{r}) = E\ \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$[-\frac{1}{2m_{\rm red}}\frac{d^2}{dr^2} + \frac{1}{2m_{\rm red}}\frac{\ell(\ell+1)}{r^2} + V_{\rm eff}(r,S,\ell)]\ \psi(r) = E\ \psi(r)$$

Spherical Basis r, θ, ϕ

$$\boxed{V_{eff} \to V_C(r) = -\frac{\alpha}{r}}$$

Coulomb potential

Semiclassical first approximation to QED --> **Bohr Spectrum**

Light-Front QCD

$$H^{LF}_{QCD}$$

$$(H^0_{LF} + H^I_{LF})|\Psi> = M^2|\Psi>$$

Coupled Fock states

$$[\frac{\vec{k}^2_\perp + m^2}{x(1-x)} + V^{LF}_{\rm eff}]\ \psi_{LF}(x, \vec{k}_\perp) = M^2\ \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{4\,\zeta^2} + U(\zeta, S, L)]\ \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)$$

$$\boxed{\zeta^2 = x(1-x)b^2_\perp}$$

Azimuthal Basis ζ, ϕ

$$\boxed{U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)}$$

Confining AdS/QCD potential!

Semiclassical first approximation to QCD

Light-Front Schrödinger Equation

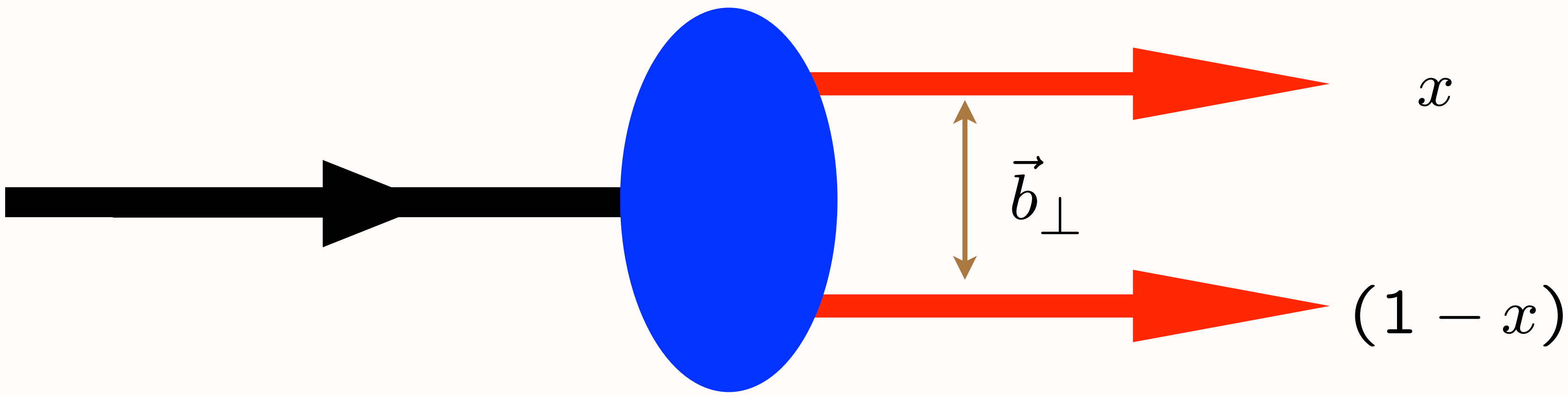
G. de Teramond, sjb

Relativistic LF single-variable radial equation for QCD & QED

Frame Independent!

$$[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{4\zeta^2} + U(\zeta, S, L)]\ \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)$$

$$\zeta^2 = x(1-x)\mathrm{b}^2_\perp .$$



AdS/QCD:

$$\boxed{U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)}$$

Light-Front Schrödinger Equation

G. de Teramond, sjb

Relativistic LF single-variable radial equation for QCD & QED

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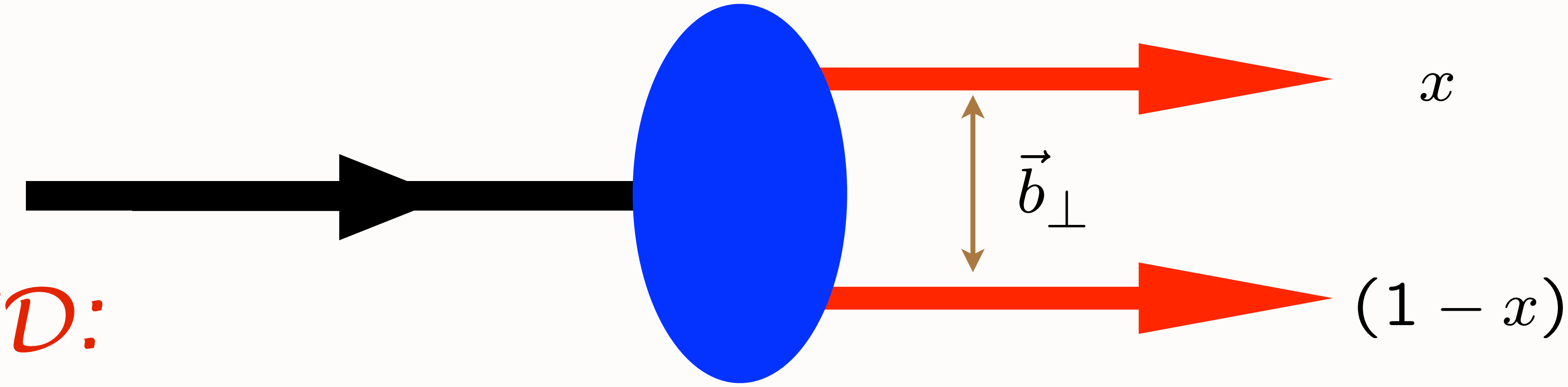
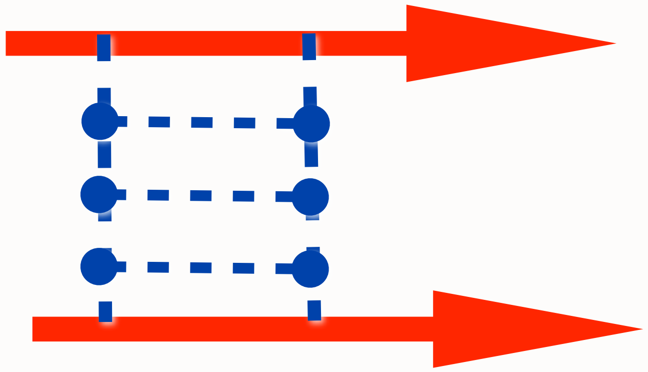
$$[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{4\zeta^2} + U(\zeta, S, L)]\ \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)$$

$$\zeta^2 = x(1-x)\mathrm{b}_\perp^2 .$$



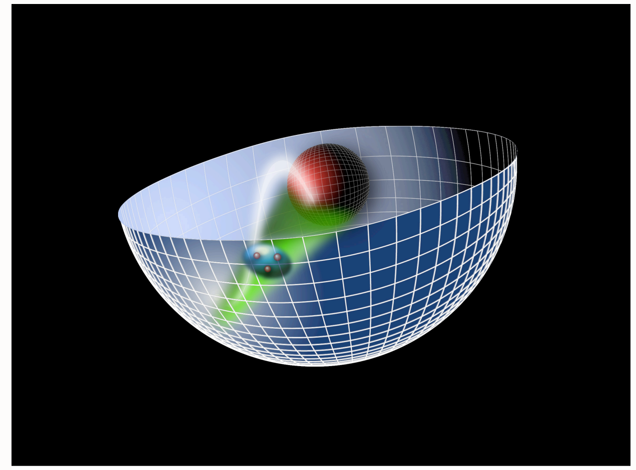
AdS/QCD:

U is the exact QCD potential
Conjecture: 'H'-diagrams generate



$$\boxed{U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)}$$

de Teramond, Dosch, sjb



AdS/QCD
Soft-Wall Model

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Conformal Symmetry
of the action

Confinement scale: $\kappa \simeq 0.5\ GeV$

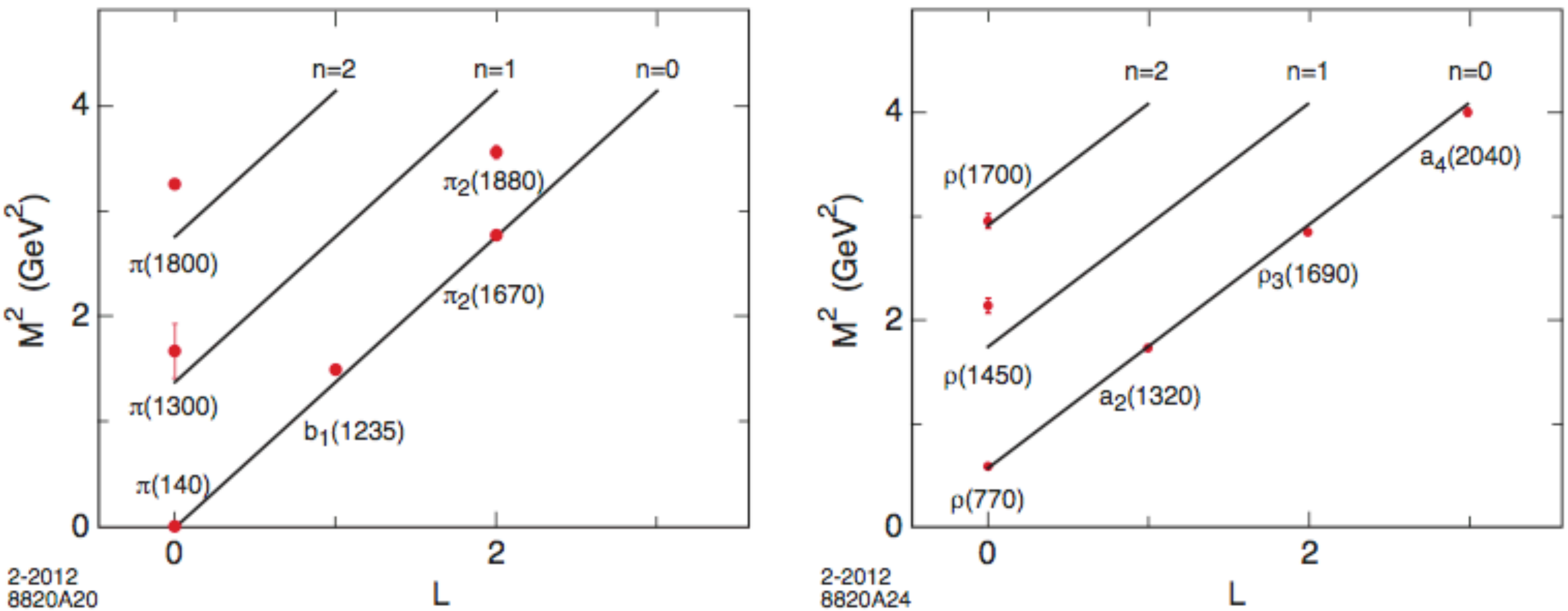
$$1/\kappa \simeq 0.4\ fm$$

$m_q = 0$

- $J = L + S, I = 1$ meson families $\boxed{\mathcal{M}^2_{n,L,S} = 4\kappa^2\,(n + L + S/2)}$

$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$

Same slope in n and L! ***Massless pion in Chiral Limit!***



I=1 orbital and radial excitations for the π ($\kappa = 0.59$ GeV) and the ρ-meson families ($\kappa = 0.54$ GeV)

- Triplet splitting for the $I = 1$, $L = 1$, $J = 0, 1, 2$, vector meson a-states

$$\mathcal{M}_{a_2(1320)} > \mathcal{M}_{a_1(1260)} > \mathcal{M}_{a_0(980)}$$

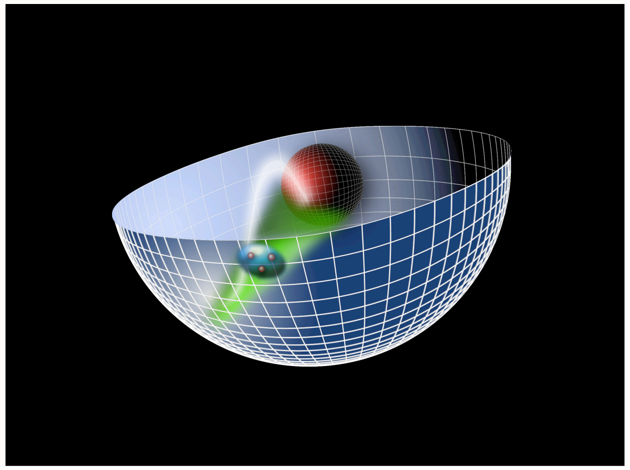
Mass ratio of the ρ and the a_1 mesons: coincides with Weinberg sum rules

Remarkable Features of Light-Front Schrödinger Equation

- *Relativistic, frame-independent*
- *QCD scale appears spontaneously - unique LF potential*
- *Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter*
- *Zero-mass pion for zero mass quarks!*
- *Regge slope same for n and L -- not usual HO*
- *Splitting in L persists to high mass   -- contradicts conventional wisdom based on breakdown of chiral symmetry*
- *Phenomenology: LFWFs, Form factors, electroproduction*
- *Extension to heavy quarks*

$$\boxed{U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)}$$

de Teramond, Dosch, sjb



AdS/QCD
Soft-Wall Model

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2}+\frac{1-4L^2}{4\zeta^2}+U(\zeta)\right]\psi(\zeta)=\mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

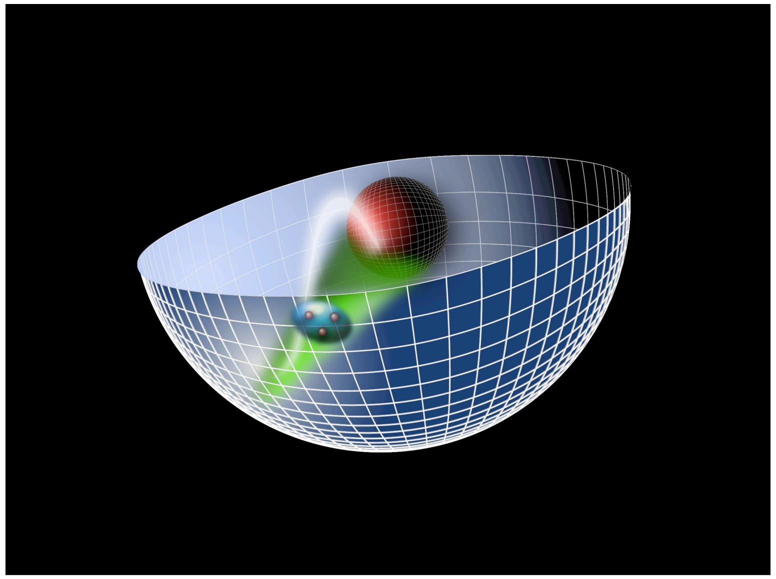
$$U(\zeta)=\kappa^4\zeta^2+2\kappa^2(L+S-1)$$

Conformal Symmetry
of the action

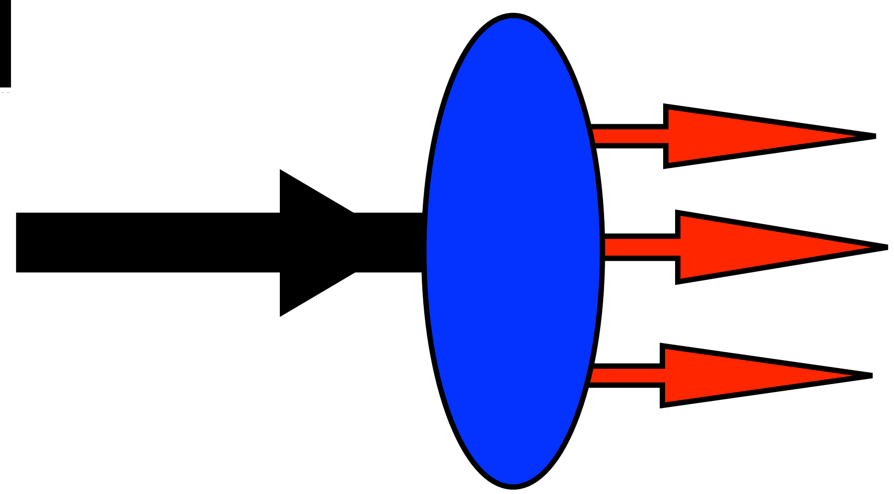
Confinement scale: $\kappa \simeq 0.5\ GeV$

$1/\kappa \simeq 0.4\ fm$

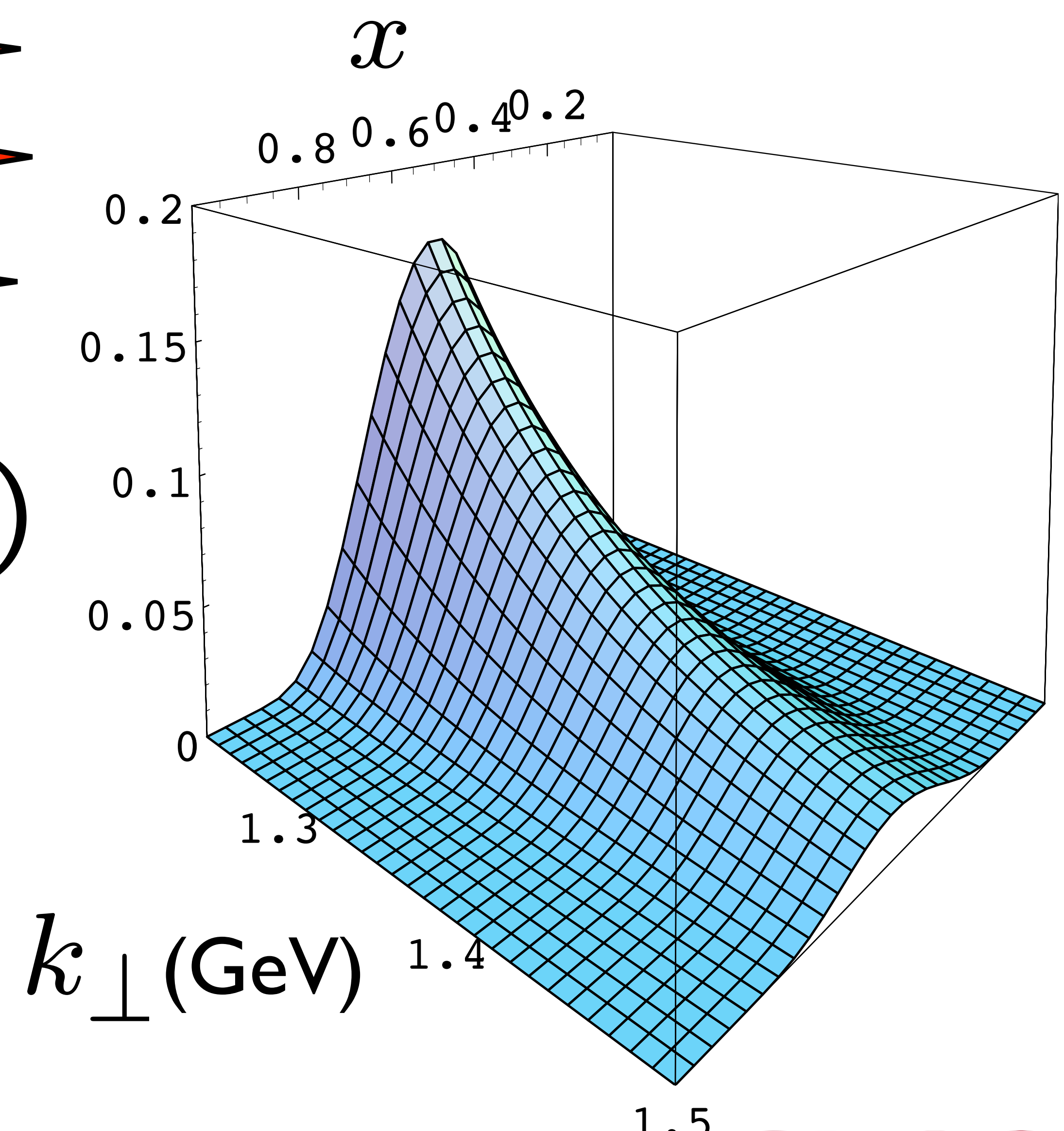
$\phi(z)$



- *Light-Front Holography*

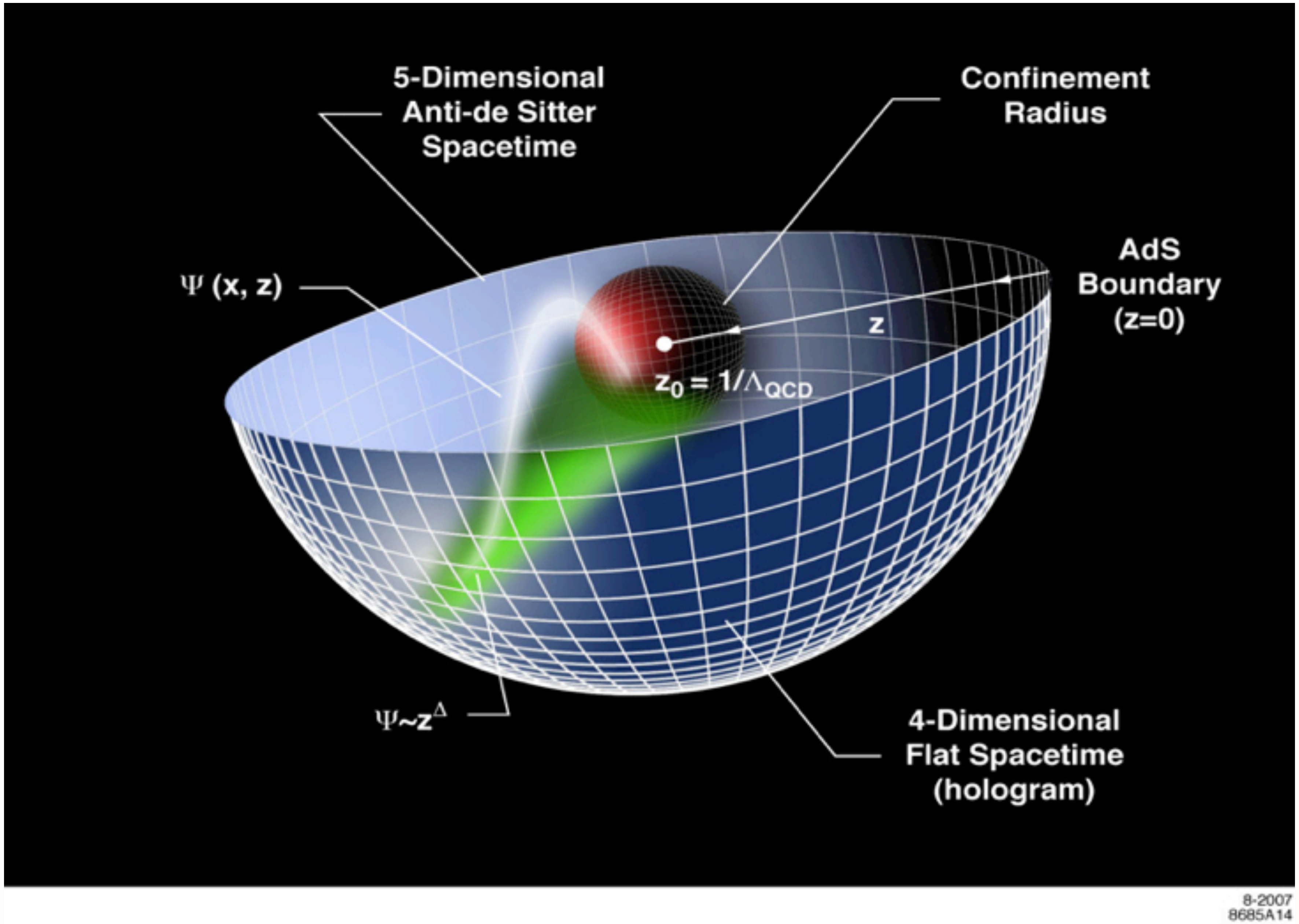


$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



- *Light Front Wavefunctions:*

  Schrödinger Wavefunctions of Hadron Physics



Changes in physical length scale mapped to evolution in the 5th dimension z

Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\rm QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).

Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).

AdS/CFT

- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

invariant measure

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2),$$

$x^\mu \to \lambda x^\mu, \ z \to \lambda z$, maps scale transformations into the holographic coordinate z.

- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.

Bosonic Solutions: Hard Wall Model

- Conformal metric: $ds^2 = g_{\ell m} dx^\ell dx^m$. $x^\ell = (x^\mu, z)$, $g_{\ell m} \to \left(R^2/z^2\right) \eta_{\ell m}$.
- Action for massive scalar modes on AdS$_{d+1}$:

$$S[\Phi] = \frac{1}{2} \int d^{d+1}x \sqrt{g}\, \tfrac{1}{2} \left[g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right], \quad \sqrt{g} \to (R/z)^{d+1}.$$

- Equation of motion

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left(\sqrt{g}\, g^{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0.$$

- Factor out dependence along x^μ-coordinates , $\Phi_P(x, z) = e^{-iP\cdot x}\, \Phi(z)$, $P_\mu P^\mu = \mathcal{M}^2$:

$$\boxed{\left[z^2 \partial_z^2 - (d-1) z\, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi(z) = 0.}$$

- Solution: $\Phi(z) \to z^\Delta$ as $z \to 0$,

$$\Phi(z) = C z^{d/2} J_{\Delta - d/2}(z \mathcal{M}) \qquad \Delta = \tfrac{1}{2}\left(d + \sqrt{d^2 + 4\mu^2 R^2} \right).$$

$$\Delta = 2 + L \qquad d = 4 \qquad (\mu R)^2 = L^2 - 4$$

$$ds^2 = \frac{R^2}{z^2} \boxed{e^{\varphi(z)}} \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

$$\boxed{z \Leftrightarrow \zeta, \quad \Phi_P(z) \Leftrightarrow |\psi(P)\rangle}$$

- Upon substitution $z \rightarrow \zeta$ and $\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\varphi(z)/2}\, \Phi_J(\zeta)$ in AdS WE

$$\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}} \partial_z \left(\frac{e^{\varphi(z)}}{z^{d-1-2J}} \partial_z\right) + \left(\frac{\mu R}{z}\right)^2\right] \Phi_J(z) = \mathcal{M}^2 \Phi_J(z)$$

find LFWE $(d = 4)$

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta)\right) \phi_J(\zeta) = M^2 \phi_J(\zeta)$$

with

$$\boxed{U(\zeta) = \frac{1}{2}\varphi''(z) + \frac{1}{4}\varphi'(z)^2 + \frac{2J-3}{2z}\varphi'(z)}$$

and $(\mu R)^2 = -(2 - J)^2 + L^2$

- AdS Breitenlohner-Freedman bound $(\mu R)^2 \geq -4$ equivalent to LF QM stability condition $L^2 \geq 0$

- Scaling dimension τ of AdS mode $\hat{\Phi}_J$ is $\tau = 2 + L$ in agreement with twist scaling dimension of a two parton bound state in QCD and determined by QM stability condition

Higher Spin Wave Equations in AdS Space and LF Holographic Mapping

H. G. Dosch, G. de Teramond, sjb   PRD 87 (2013)

- Description of higher spin modes in AdS space (Frondsal, Fradkin and Vasiliev)

- Integer spin-J fields in AdS conveniently described by tensor field $\Phi_{N_1 \cdots N_J}$ with effective action

$$S_{\it eff} = \int d^d x\, dz \sqrt{|g|}\, e^{\varphi(z)}\, g^{N_1 N_1'} \cdots g^{N_J N_J'} \Big(g^{M M'} D_M \Phi^*_{N_1 \ldots N_J}\, D_{M'} \Phi_{N_1' \ldots N_J'} - \mu^2_{\it eff}(z)\, \Phi^*_{N_1 \ldots N_J}\, \Phi_{N_1' \ldots N_J'} \Big)$$

  where D_M is the covariant derivative which includes affine connection

- The z-dependent effective AdS mass $\mu_{\it eff}(z)$ can absorb the contribution from different contractions in the action and is *a priori* unknown

- Effective mass $\mu_{\it eff}(z)$ allows a separation of kinematical and dynamical effects and is determined by precise mapping to light-front physics

- Non-trivial geometry of pure AdS encodes the kinematics and the additional deformations of AdS encode the dynamics, including confinement

General-Spin Hadrons

- Obtain spin-J mode $\Phi_{\mu_1 \cdots \mu_J}$ with all indices along 3+1 coordinates from Φ by shifting dimensions

$$\Phi_J(z) = \left(\frac{z}{R}\right)^{-J} \Phi(z)$$

$$\boxed{e^{\varphi(z)} = e^{+\kappa^2 z^2}}$$

- Substituting in the AdS scalar wave equation for Φ

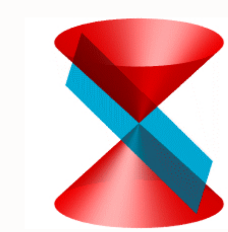
$$\left[z^2 \partial_z^2 - \left(3 - 2J - 2\kappa^2 z^2\right) z\, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2\right] \Phi_J = 0$$

- Upon substitution $z \to \zeta$

$$\phi_J(\zeta) \sim \zeta^{-3/2+J} e^{\kappa^2 \zeta^2/2}\, \Phi_J(\zeta)$$

we find the LF wave equation

$$\boxed{\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2 (L+S-1)\right) \phi_{\mu_1 \cdots \mu_J} = \mathcal{M}^2 \phi_{\mu_1 \cdots \mu_J}}$$



with $(\mu R)^2 = -(2-J)^2 + L^2$

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

Positive-sign dilaton

- de Teramond, sjb

AdS Soft-Wall Schrodinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z)\right]\Phi(z) = \mathcal{M}^2\Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$$

Derived from variation of Action for Dilaton-Modified AdS_5

Identical to Light-Front Bound State Equation!

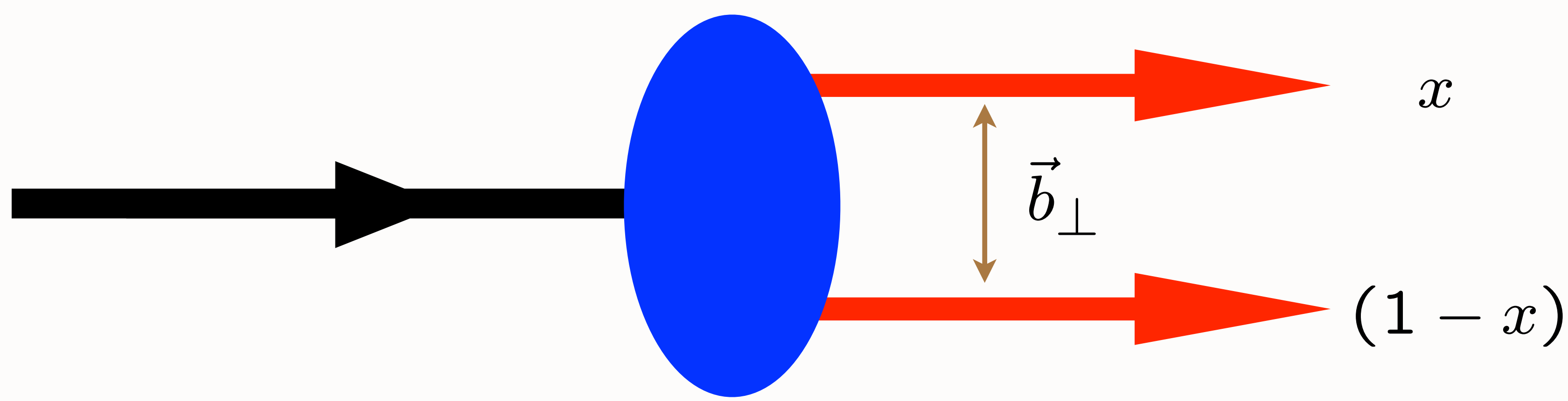
$$z \quad \longleftrightarrow \quad \zeta = \sqrt{x(1-x)\vec{b}_\perp^{\,2}}$$

de Teramond, sjb

$LF(3+1) \longleftrightarrow AdS_5$

$$\psi(x,\vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$



$$\psi(x,\zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

Light Front Holography: Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion

Light-Front Schrödinger Equation

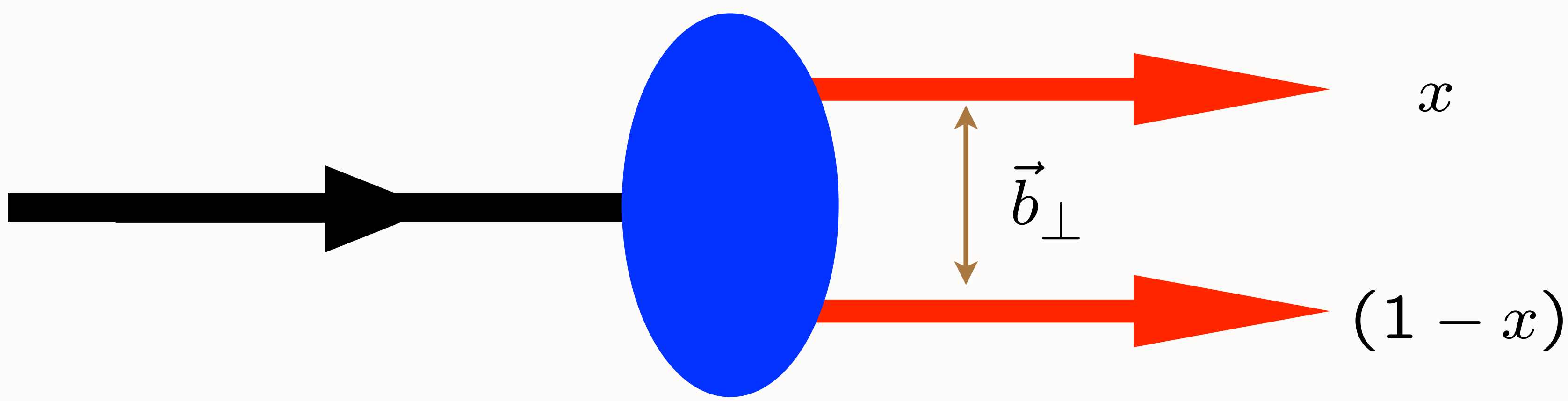
G. de Teramond, sjb

Relativistic LF single-variable radial equation for QCD & QED

Frame Independent!

$$[-\frac{d^2}{d\zeta^2} + \frac{m^2}{x(1-x)} + \frac{-1+4L^2}{\zeta^2} + U(\zeta, S, L)]\ \psi_{LF}(\zeta) = M^2\ \psi_{LF}(\zeta)$$

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2 .$$



AdS/QCD:

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Confining AdS/QCD potential

Semiclassical first approximation to QCD

Meson Spectrum in Soft Wall Model

- Dilaton profile $\varphi(z) = +\kappa^2 z^2$ $\qquad z \to \zeta$
- Effective potential: $U(\zeta^2) = \kappa^4 \zeta^2 + 2\kappa^2 (J-1)$
- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4 \zeta^2 + 2\kappa^2 (J-1)\right) \phi_J(\zeta) = M^2 \phi_J(\zeta)$$

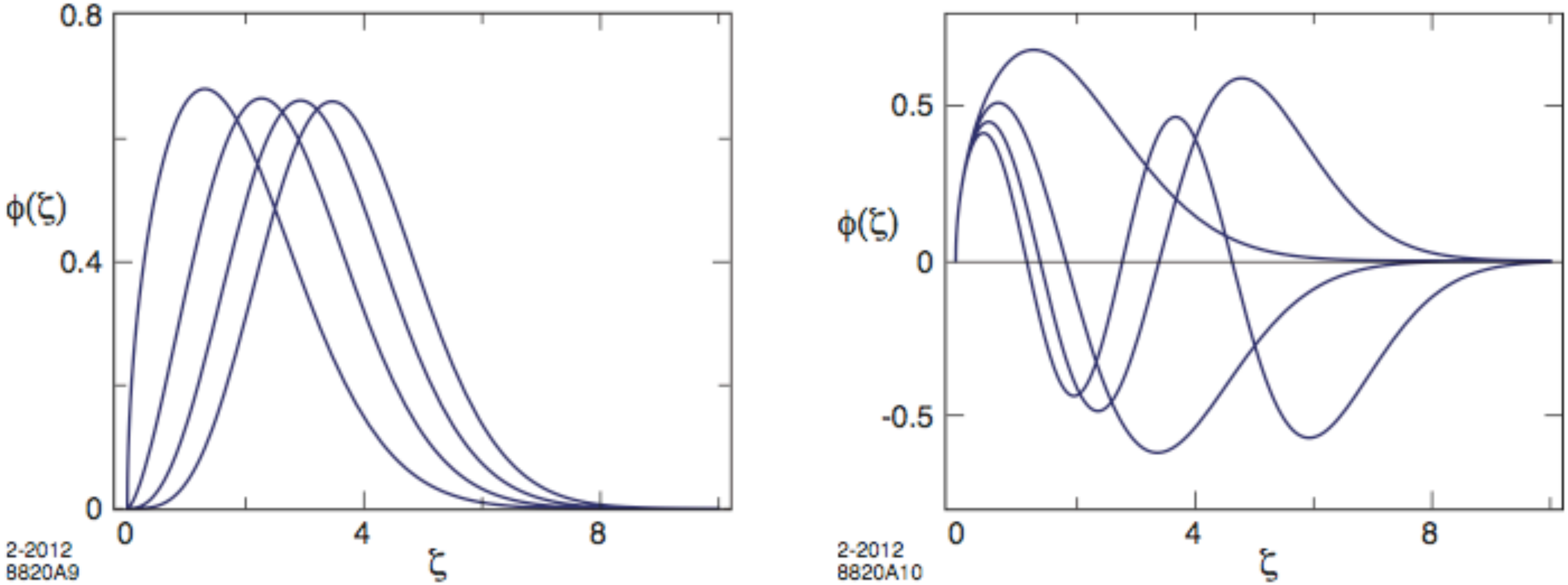
- Normalized eigenfunctions $\langle \phi | \phi \rangle = \int d\zeta \, \phi^2(z)^2 = 1$

$$\phi_{n,L}(\zeta) = \kappa^{1+L} \sqrt{\frac{2n!}{(n+L)!}} \, \zeta^{1/2+L} e^{-\kappa^2 \zeta^2/2} L_n^L(\kappa^2 \zeta^2)$$

- Eigenvalues

$$\mathcal{M}^2_{n,J,L} = 4\kappa^2 \left(n + \frac{J+L}{2}\right)$$

G. de Teramond, sjb



LFWFs $\phi_{n,L}(\zeta)$ in physical space-time: (L) orbital modes and (R) radial modes

Quark separation
increases with L

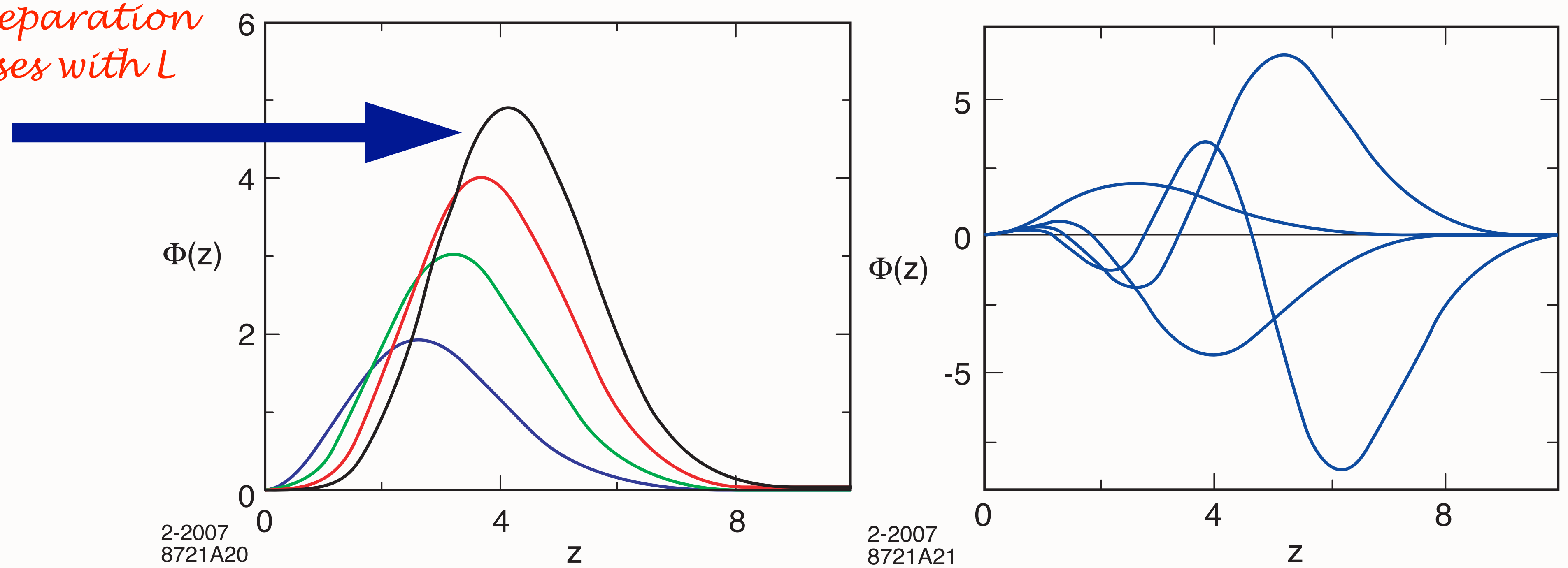
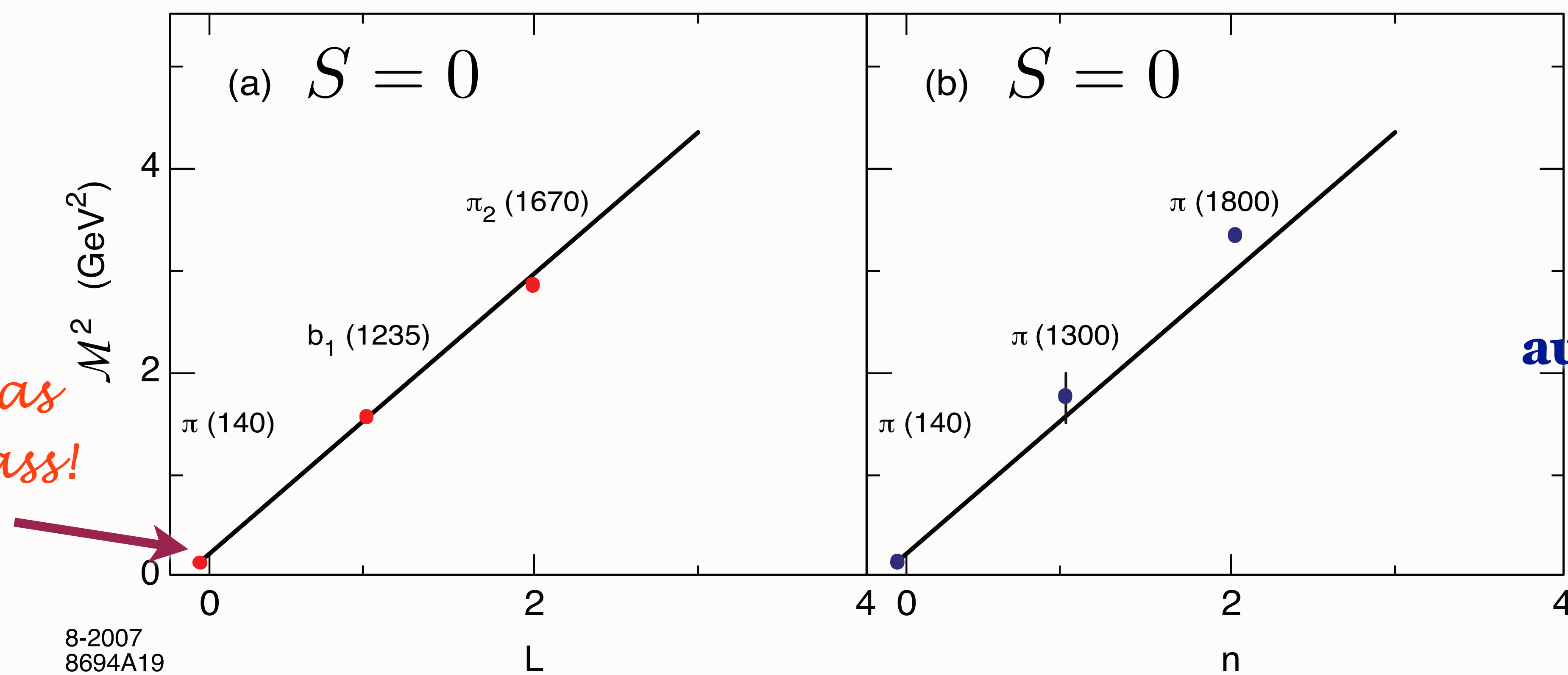


Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .

Soft Wall
Model



Pion has
zero mass!

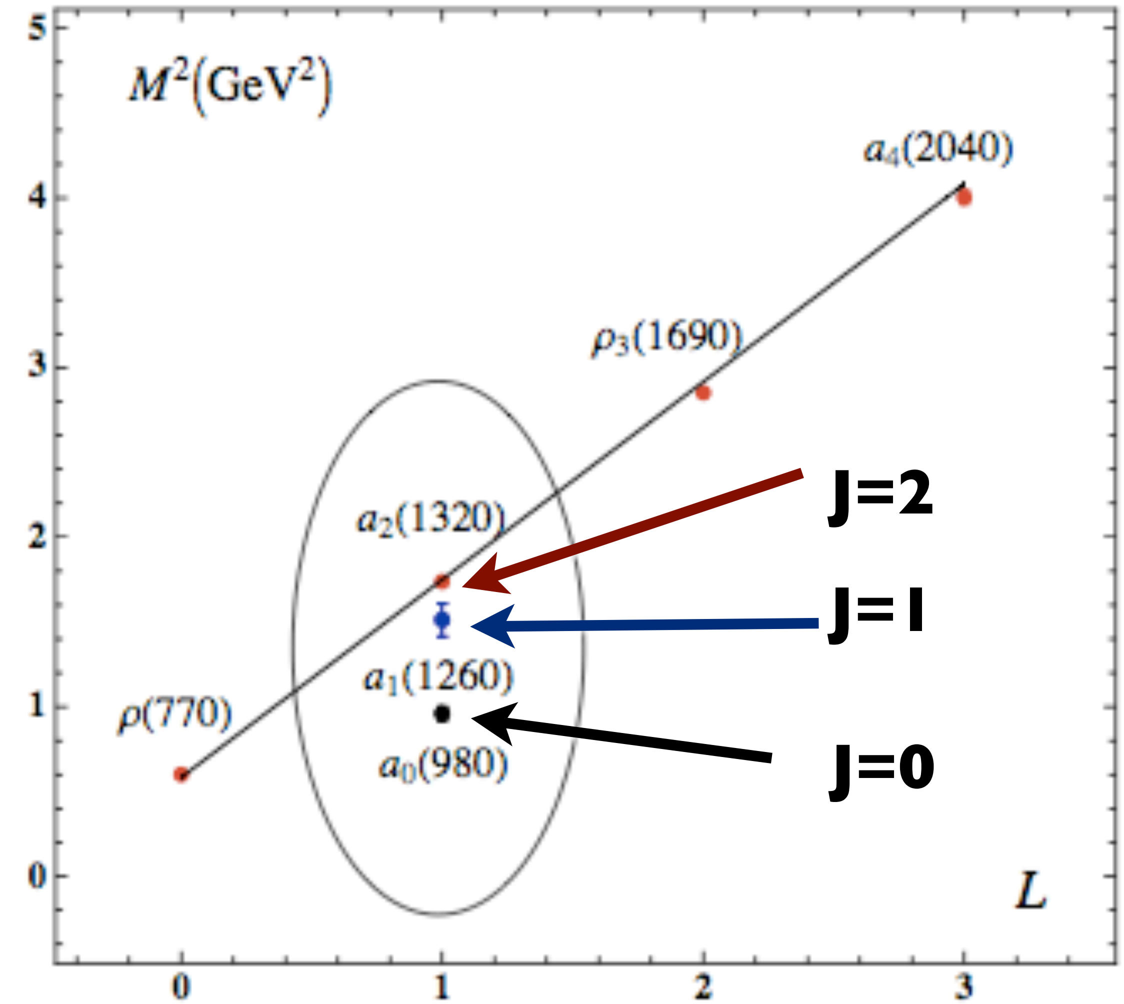
Pion mass automatically zero!

$$m_q = 0$$

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

$$\lambda = \kappa^2$$

$$\mathcal{M}^2_{n,J,L} = 4\lambda\left(n + \frac{J+L}{2}\right)$$



- Triplet splitting for the $L = 1$, $J = 0, 1, 2$ vector meson a-states

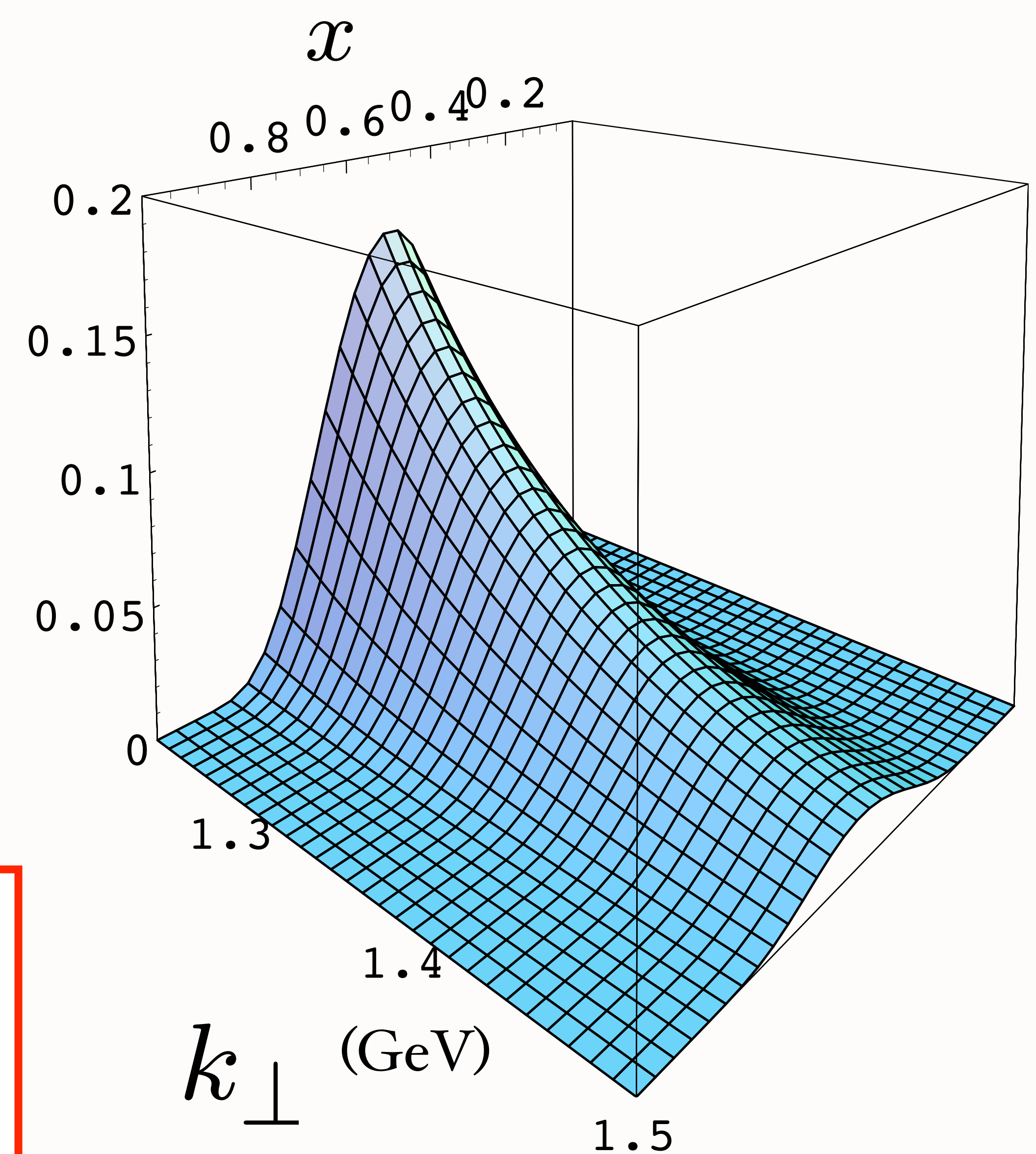
$$\mathcal{M}_{a_2(1320)} > \mathcal{M}_{a_1(1260)} > \mathcal{M}_{a_0(980)}$$

- Systematics of light meson spectra – orbital and radial excitations as well as important $J - L$ splitting, well described by light-front harmonic confinement model

- Linear Regge trajectories, a massless pion and relation between the ρ and a_1 mass $M_{a_1}/M_\rho = \sqrt{2}$ usually obtained from Weinberg sum rules [Weinberg (1967)]

Prediction from AdS/CFT: Meson LFWF

$$\psi_M(x, k_\perp^2)$$



de Teramond, sjb

"Soft Wall" model

$$\kappa = 0.375 \text{ GeV}$$

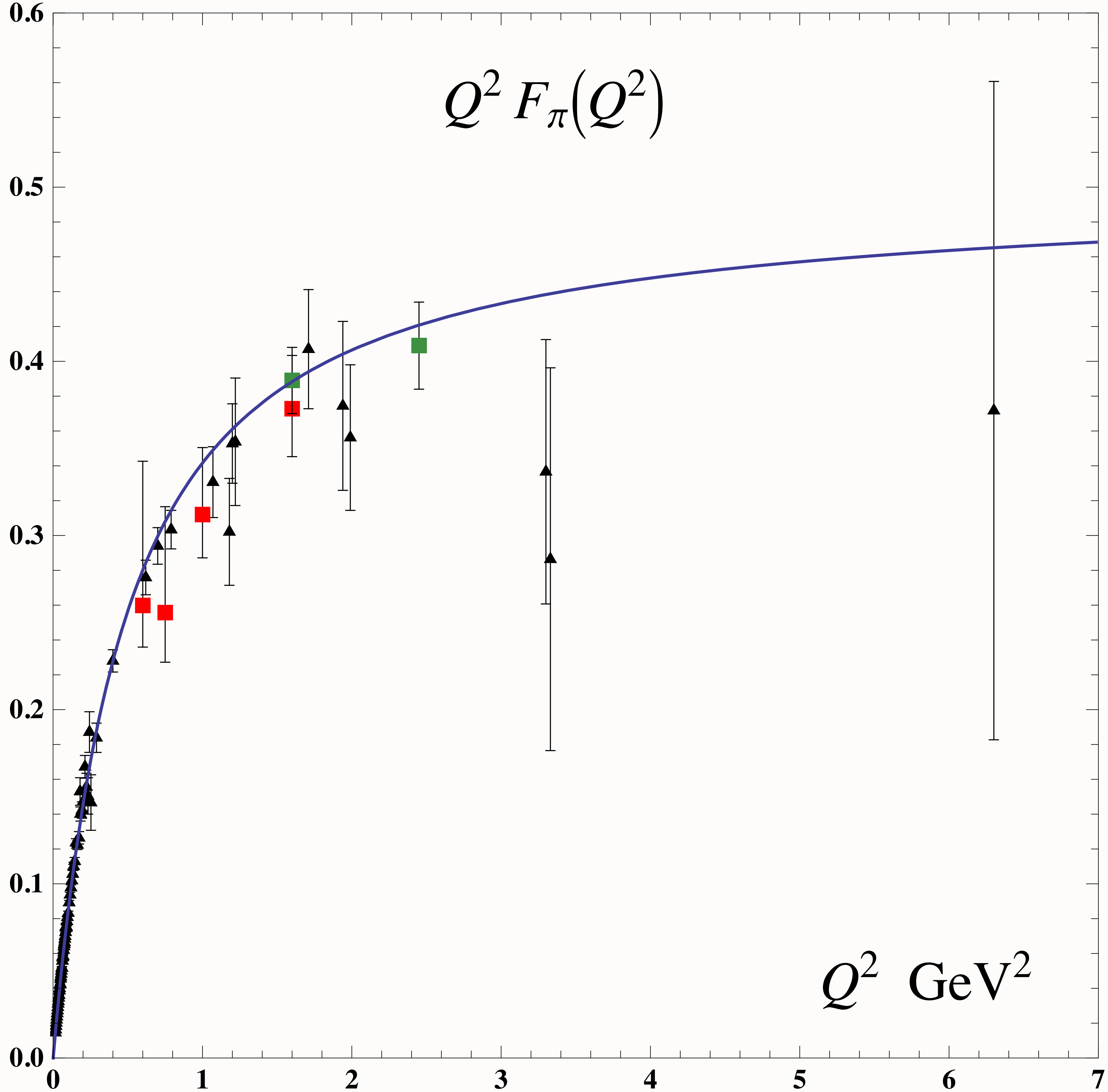
massless quarks

Note coupling
$$k_\perp^2, x$$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\boxed{\phi_M(x, Q_0) \propto \sqrt{x(1-x)}}$$

Provides Connection of Confinement to TMDs



$Q^2\,F_\pi(Q^2)$
Q^2 GeV^2
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0
1
2
3
4
5
6
7

PRL **109,** 081601 (2012) PHYSICAL REVIEW LETTERS week ending 24 AUGUST 2012

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction

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(Received 5 April 2012; published 20 August 2012)

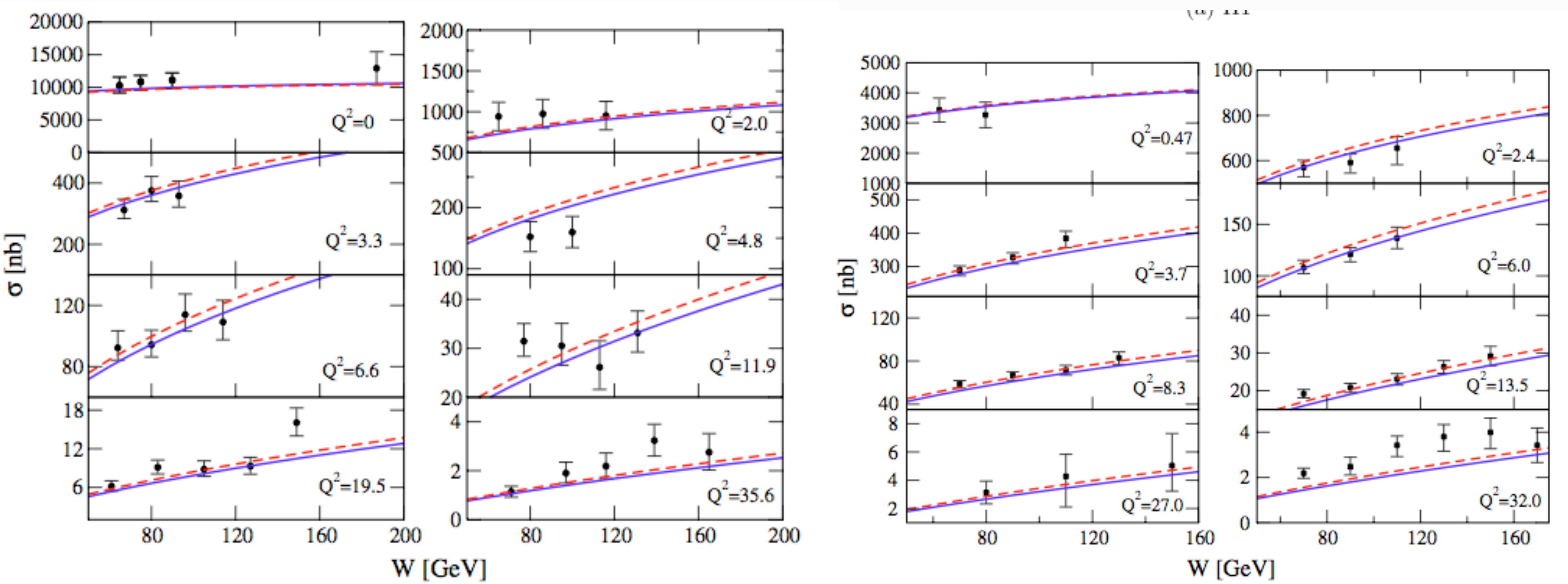
We show that anti–de Sitter/quantum chromodynamics generates predictions for the rate of diffractive ρ-meson electroproduction that are in agreement with data collected at the Hadron Electron Ring Accelerator electron–proton collider.

$$\phi(x,\zeta) = \mathcal{N}\frac{\kappa}{\sqrt{\pi}}\sqrt{x(1-x)}\exp\left(-\frac{\kappa^2\zeta^2}{2}\right),$$

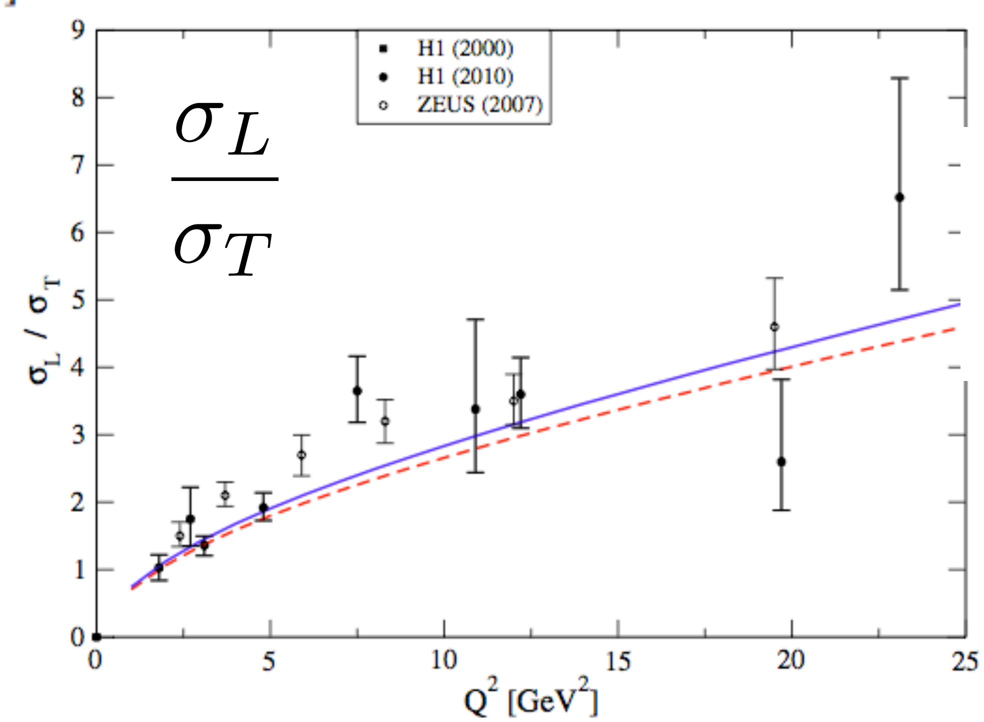
$$\tilde{\phi}(x,k) \propto \frac{1}{\sqrt{x(1-x)}}\exp\left(-\frac{M_{q\bar{q}}^2}{2\kappa^2}\right),$$

PRL **109,** 081601 (2012) PHYSICAL REVIEW LETTERS week ending 24 AUGUST 2012

AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction



(a) H (b) ZEUS



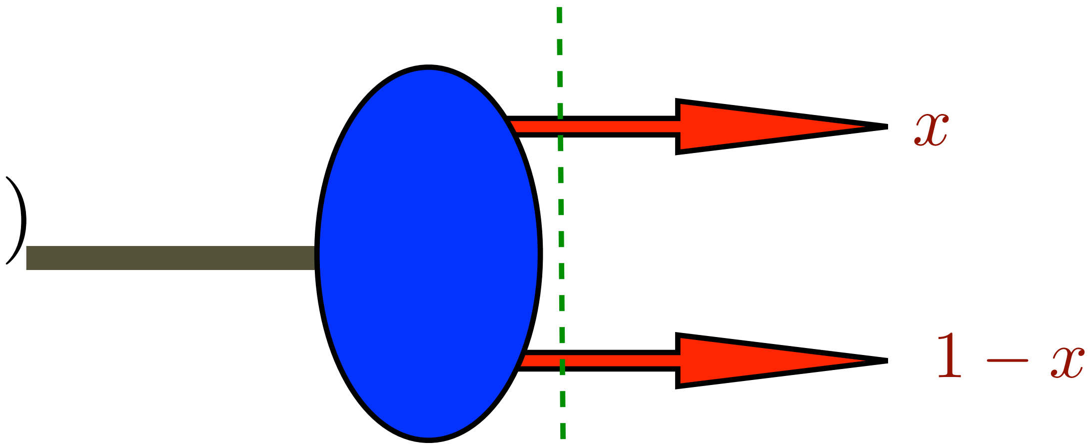
J. R. Forshaw, R. Sandapen

$$\gamma^* p \to \rho^0 p'$$

$$\tilde{\phi}(x, k) \propto \frac{1}{\sqrt{x(1-x)}} \exp\left(-\frac{M_{q\bar{q}}^2}{2\kappa^2}\right),$$

Hadron Distribution Amplitudes

$$\phi_M(x,Q) = \int^Q d^2\vec{k}\ \psi_{q\bar{q}}(x,\vec{k}_\perp)$$



$k_\perp^2 < Q^2$

$\sum_i x_i = 1$

Fixed $\tau = t + z/c$

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

Lepage, sjb

Lepage, sjb

Efremov, Radyushkin

- Evolution Equations from PQCD, OPE

Sachrajda, Frishman Lepage, sjb

- Conformal Expansions

Braun, Gardi

- Compute from valence light-front wavefunction in light-cone gauge

Second Moment of Pion Distribution Amplitude

$$< \xi^2 >= \int_{-1}^{1} d\xi \; \xi^2 \phi(\xi)$$

$$\xi = 1 - 2x$$

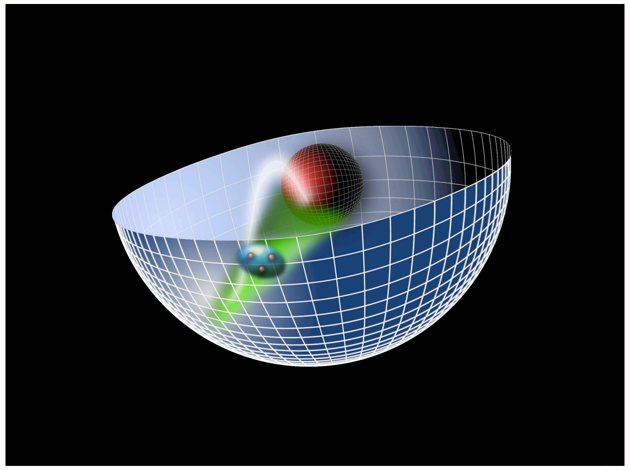
$$< \xi^2 >_\pi = 1/5 = 0.20 \qquad \phi_{asympt} \propto x(1-x)$$

$$< \xi^2 >_\pi = 1/4 = 0.25 \qquad \phi_{AdS/QCD} \propto \sqrt{x(1-x)}$$

Lattice (I) $< \xi^2 >_\pi = 0.28 \pm 0.03$ Donnellan et al.

Lattice (II) $< \xi^2 >_\pi = 0.269 \pm 0.039$ Braun et al.

AdS/QCD
Soft-Wall Model



Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Conformal Symmetry
of the action

Confinement scale: $\kappa \simeq 0.5\ GeV$

$$1/\kappa \simeq 0.4\ fm$$

- **de Alfaro, Fubini, Furlan**

$$G|\psi(\tau)>= i\frac{\partial}{\partial\tau}|\psi(\tau)>$$

$$G = uH + vD + wK$$

$$G = H_\tau = \frac{1}{2}\Big(-\frac{d^2}{dx^2} + \frac{g}{x^2} + \frac{4uw - v^2}{4}x^2\Big)$$

Retains conformal invariance of action despite mass scale!

$$4uw - v^2 = \kappa^4 = [M]^4$$

Identical to LF Hamiltonian with unique potential and dilaton!

- **Dosch, de Teramond, sjb**

$$\Big[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\Big]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Uniqueness

de Teramond, Dosch, sjb

- ζ^2 confinement potential and dilaton profile unique!
- Linear Regge trajectories in n and L: same slope!
- Massless pion in chiral limit!   No vacuum condensate!
- Derive from conformal invariance: conformally invariant action for massless quarks despite mass scale
- Same principle, equation of motion as de Alfaro, Fubini, Furlan
- <u>Conformal Invariance in Quantum Mechanics</u> Nuovo Cim. A34 (1976) 569

What determines the QCD mass scale Λ_{QCD}?

- Mass scale does not appear in the QCD Lagrangian (massless quarks)
- Dimensional Transmutation? Requires external constraint such as $\alpha_s(M_Z)$
- dAFF: Confinement Scale κ appears spontaneously via the Hamiltonian: $G = uH + vD + wK$ $\quad 4uw - v^2 = \kappa^4 = [M]^4$
- The confinement scale regulates infrared divergences, connects Λ_{QCD} to the confinement scale κ
- Only dimensionless mass ratios (and M times R) predicted
- Mass and time units [GeV] and [sec] from physics external to QCD
- New feature: bounded frame-independent relative time between constituents

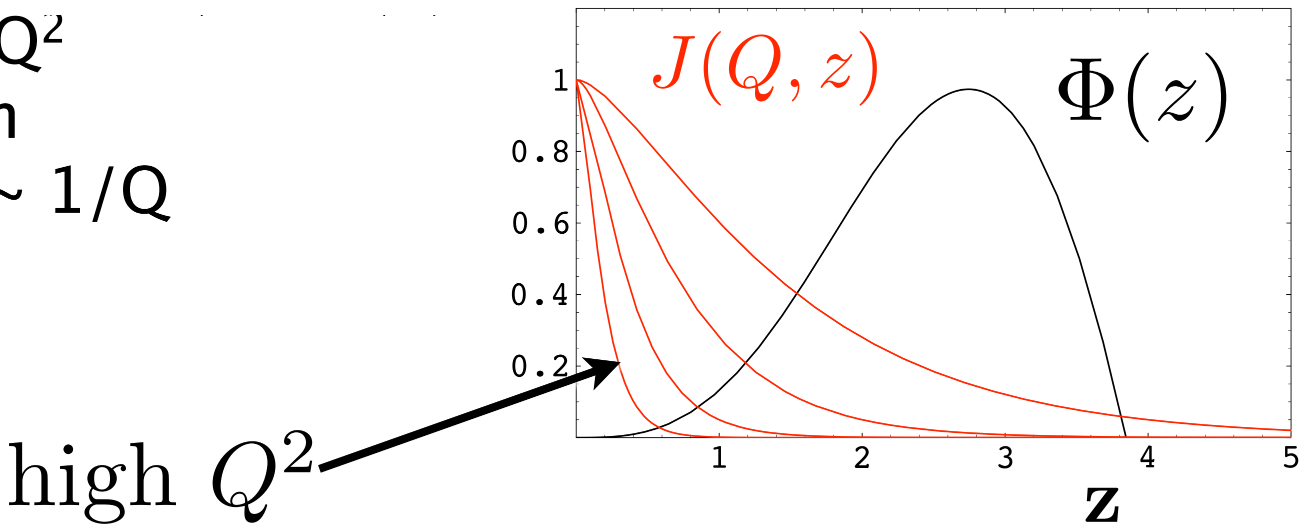
Hadron Form Factors from AdS/QCD

Propagation of external perturbation suppressed inside AdS.

$$J(Q,z) = zQK_1(zQ)$$

$$F(Q^2)_{I\to F} = \int \frac{dz}{z^3}\Phi_F(z)J(Q,z)\Phi_I(z)$$

High Q^2
from
small z ~ 1/Q



Polchinski, Strassler
de Teramond, sjb

Consider a specific AdS mode $\Phi^{(n)}$ dual to an n partonic Fock state $|n\rangle$. At small z, Φ scales as $\Phi^{(n)} \sim z^{\Delta_n}$. Thus:

$$F(Q^2) \to \left[\frac{1}{Q^2}\right]^{\tau-1},$$

Dimensional Quark Counting Rules:
General result from
AdS/CFT and Conformal Invariance

where $\tau = \Delta_n - \sigma_n$, $\sigma_n = \sum_{i=1}^{n}\sigma_i$. The twist is equal to the number of partons, $\tau = n$.

Holographic Mapping of AdS Modes to QCD LFWFs

Drell-Yan-West: Form Factors are Convolution of LFWFs

- Integrate Soper formula over angles:

$$F(q^2) = 2\pi \int_0^1 dx \, \frac{(1-x)}{x} \int \zeta d\zeta J_0\left(\zeta q \sqrt{\frac{1-x}{x}}\right) \tilde{\rho}(x,\zeta),$$

with $\widetilde{\rho}(x,\zeta)$ QCD effective transverse charge density.

- Transversality variable

$$\boxed{\zeta = \sqrt{x(1-x)\vec{b}_\perp^2}}$$

- Compare AdS and QCD expressions of FFs for arbitrary Q using identity:

$$\int_0^1 dx J_0\left(\zeta Q \sqrt{\frac{1-x}{x}}\right) = \zeta Q K_1(\zeta Q),$$

the solution for $J(Q,\zeta) = \zeta Q K_1(\zeta Q)$!

de Teramond, sjb

Identical to Polchinski-Strassler Convolution of AdS Amplitudes

Current Matrix Elements in AdS Space (SW)

sjb and GdT
Grigoryan and Radyushkin

- Propagation of external current inside AdS space described by the AdS wave equation

$$\left[z^2\partial_z^2 - z\left(1 + 2\kappa^2 z^2\right)\partial_z - Q^2 z^2\right] J_\kappa(Q, z) = 0.$$

- Solution bulk-to-boundary propagator

$$J_\kappa(Q, z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right),$$

Dressed Current in Soft-Wall Model

where $U(a, b, c)$ is the confluent hypergeometric function

$$\Gamma(a)U(a, b, z) = \int_0^\infty e^{-zt} t^{a-1}(1 + t)^{b-a-1} dt.$$

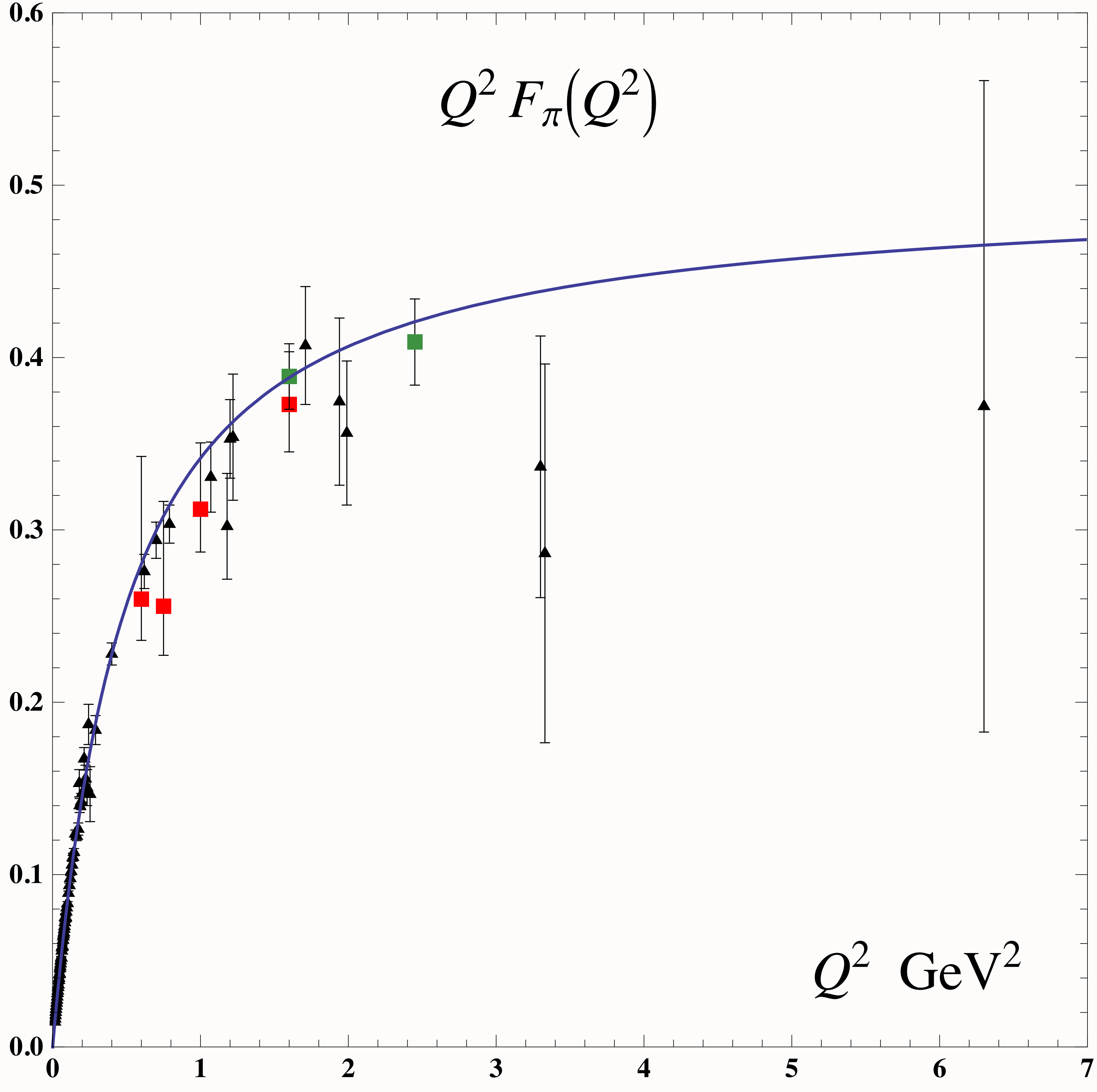
- Form factor in presence of the dilaton background $\varphi = \kappa^2 z^2$

$$F(Q^2) = R^3 \int \frac{dz}{z^3}\, e^{-\kappa^2 z^2} \Phi(z) J_\kappa(Q, z) \Phi(z).$$

- For large $Q^2 \gg 4\kappa^2$

$$J_\kappa(Q, z) \to zQK_1(zQ) = J(Q, z),$$

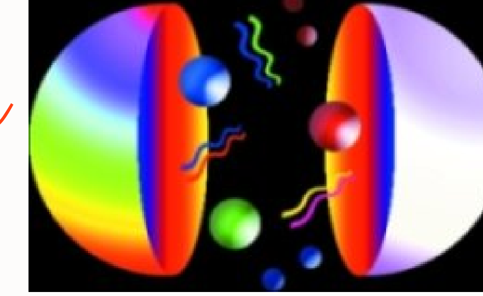
the external current decouples from the dilaton field.


$Q^2 F_\pi(Q^2)$
Q^2 GeV^2
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0
1
2
3
4
5
6
7

Fermionic Modes and Baryon Spectrum

GdT and sjb, PRL 94, 201601 (2005)

Yukawa interaction in 5 dimensions



From Nick Evans

- Action for Dirac field in AdS$_{d+1}$ in presence of dilaton background $\varphi(z)$ [Abidin and Carlson (2009)]

$$S = \int d^{d+1}\sqrt{g}\, e^{\varphi(z)} \left(i\overline{\Psi} e^M_A \Gamma^A D_M \Psi + h.c + \varphi(z)\overline{\Psi}\Psi - \mu \overline{\Psi}\Psi\right)$$

- Factor out plane waves along 3+1: $\Psi_P(x^\mu, z) = e^{-iP\cdot x}\Psi(z)$

$$\left[i\left(z\eta^{\ell m}\Gamma_\ell \partial_m + 2\Gamma_z\right) + \mu R + \kappa^2 z\right]\Psi(x^\ell) = 0.$$

- Solution $(\nu = \mu R - \frac{1}{2},\ \nu = L + 1)$

$$\Psi_+(z) \sim z^{\frac{5}{2}+\nu} e^{-\kappa^2 z^2/2} L_n^\nu(\kappa^2 z^2), \quad \Psi_-(z) \sim z^{\frac{7}{2}+\nu} e^{-\kappa^2 z^2/2} L_n^{\nu+1}(\kappa^2 z^2)$$

- Eigenvalues (how to fix the overall energy scale, see arXiv:1001.5193)

$$\mathcal{M}^2 = 4\kappa^2(n + L + 1)$$ positive parity

- Obtain spin-J mode $\Phi_{\mu_1 \cdots \mu_{J-1/2}}$, $J > \frac{1}{2}$, with all indices along 3+1 from Ψ by shifting dimensions
- Large N_C: $\mathcal{M}^2 = 4\kappa^2(N_C + n + L - 2) \implies \mathcal{M} \sim \sqrt{N_C}\,\Lambda_{\rm QCD}$

Light-Front Mapping

- A physical baryon satisfies the Rarita-Schwinger equation for spinors in physical space-time

$$(i\gamma^\mu \partial_\mu - M)\, u_{\nu_1 \cdots \nu_T}(P) = 0, \qquad \gamma^\nu u_{\nu \nu_2 \cdots \nu_T}(P) = 0.$$

- Upon substitution in AdS wave equation for spin J $\;(u^\pm$ chiral spinors)

$$\Psi^\pm_{\nu_1 \cdots \nu_T}(x,z) = e^{iP\cdot x} \left(\frac{R}{z}\right)^{T-d/2} \psi^\pm_T(z)\, u^\pm_{\nu_1 \cdots \nu_T}(P),$$

and $z \to \zeta$ find LFWE

$$\begin{aligned} -\frac{d}{d\zeta}\psi_- - \frac{\nu+\frac{1}{2}}{\zeta}\psi_- - V(\zeta)\psi_- &= M\psi_+, \\ \frac{d}{d\zeta}\psi_+ - \frac{\nu+\frac{1}{2}}{\zeta}\psi_+ - V(\zeta)\psi_+ &= M\psi_- \end{aligned}$$

provided that $|\mu R| = \nu + \frac{1}{2}$ and $\psi^\pm_T = \psi_\pm$ with effective LF potential

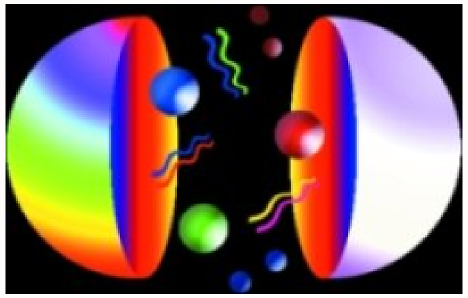
$$V(\zeta) = \frac{R}{\zeta}\rho(\zeta),$$

a J-independent potential – No spin-orbit coupling !

Fermionic Modes and Baryon Spectrum

[Hard wall model: GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[Soft wall model: GdT and S. J. Brodsky, (2005), arXiv:1001.5193]



From Nick Evans

- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{3/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+1}\left(\kappa^2\zeta^2\right)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L}\frac{1}{\sqrt{n+L+2}}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{5/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+2}\left(\kappa^2\zeta^2\right)$$

- Normalization

$$\int d\zeta\, \psi_+^2(\zeta) = \int d\zeta\, \psi_-^2(\zeta) = 1$$

- Eigenvalues

$$\mathcal{M}^2_{n,L,S=1/2} = 4\kappa^2\left(n+L+1\right)$$

- “Chiral partners”

$$\frac{\mathcal{M}_{N(1535)}}{\mathcal{M}_{N(940)}} = \sqrt{2}$$

Table 1: $SU(6)$ classification of confirmed baryons listed by the PDG. The labels S, L and n refer to the internal spin, orbital angular momentum and radial quantum number respectively. The $\Delta\frac{5}{2}^-(1930)$ does not fit the $SU(6)$ classification since its mass is too low compared to other members **70**-multiplet for $n = 0$, $L = 3$.

$SU(6)$	S	L	n	Baryon State
56	$\frac{1}{2}$	0	0	$N\frac{1}{2}^+(940)$
	$\frac{1}{2}$	0	1	$N\frac{1}{2}^+(1440)$
	$\frac{1}{2}$	0	2	$N\frac{1}{2}^+(1710)$
	$\frac{3}{2}$	0	0	$\Delta\frac{3}{2}^+(1232)$
	$\frac{3}{2}$	0	1	$\Delta\frac{3}{2}^+(1600)$
70	$\frac{1}{2}$	1	0	$N\frac{1}{2}^-(1535)$ $N\frac{3}{2}^-(1520)$
	$\frac{3}{2}$	1	0	$N\frac{1}{2}^-(1650)$ $N\frac{3}{2}^-(1700)$ $N\frac{5}{2}^-(1675)$
	$\frac{3}{2}$	1	1	$N\frac{1}{2}^-$ $N\frac{3}{2}^-(1875)$ $N\frac{5}{2}^-$
	$\frac{1}{2}$	1	0	$\Delta\frac{1}{2}^-(1620)$ $\Delta\frac{3}{2}^-(1700)$
56	$\frac{1}{2}$	2	0	$N\frac{3}{2}^+(1720)$ $N\frac{5}{2}^+(1680)$
	$\frac{1}{2}$	2	1	$N\frac{3}{2}^+(1900)$ $N\frac{5}{2}^+$
	$\frac{3}{2}$	2	0	$\Delta\frac{1}{2}^+(1910)$ $\Delta\frac{3}{2}^+(1920)$ $\Delta\frac{5}{2}^+(1905)$ $\Delta\frac{7}{2}^+(1950)$
70	$\frac{1}{2}$	3	0	$N\frac{5}{2}^-$ $N\frac{7}{2}^-$
	$\frac{3}{2}$	3	0	$N\frac{3}{2}^-$ $N\frac{5}{2}^-$ $N\frac{7}{2}^-(2190)$ $N\frac{9}{2}^-(2250)$
	$\frac{1}{2}$	3	0	$\Delta\frac{5}{2}^-$ $\Delta\frac{7}{2}^-$
56	$\frac{1}{2}$	4	0	$N\frac{7}{2}^+$ $N\frac{9}{2}^+(2220)$
	$\frac{3}{2}$	4	0	$\Delta\frac{5}{2}^+$ $\Delta\frac{7}{2}^+$ $\Delta\frac{9}{2}^+$ $\Delta\frac{11}{2}^+(2420)$
70	$\frac{1}{2}$	5	0	$N\frac{9}{2}^-$ $N\frac{11}{2}^-$
	$\frac{3}{2}$	5	0	$N\frac{7}{2}^-$ $N\frac{9}{2}^-$ $N\frac{11}{2}^-(2600)$ $N\frac{13}{2}^-$

PDG 2012

Baryon Spectrum

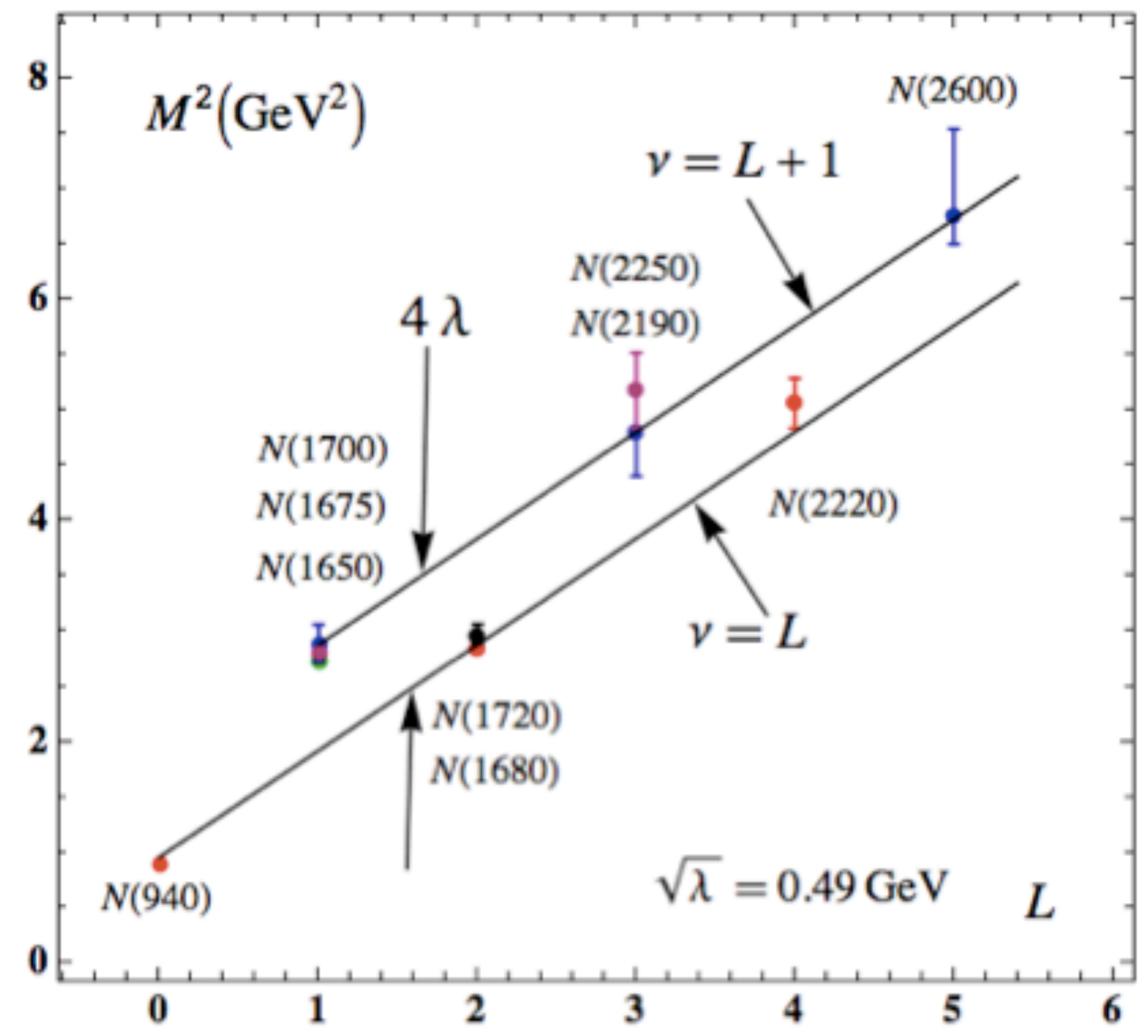
- Choose linear potential $V = \lambda\,\zeta, \quad \lambda > 0$
- Eigenfunctions

$$\psi_+(\zeta) \sim \zeta^{\frac{1}{2}+\nu} e^{-\lambda\zeta^2/2} L_n^\nu(\lambda\zeta^2), \qquad \psi_-(\zeta) \sim \zeta^{\frac{3}{2}+\nu} e^{-\lambda\zeta^2/2} L_n^{\nu+1}(\lambda\zeta^2)$$

- Eigenvalues

$$M^2 = 4\lambda(n + \nu + 1)$$

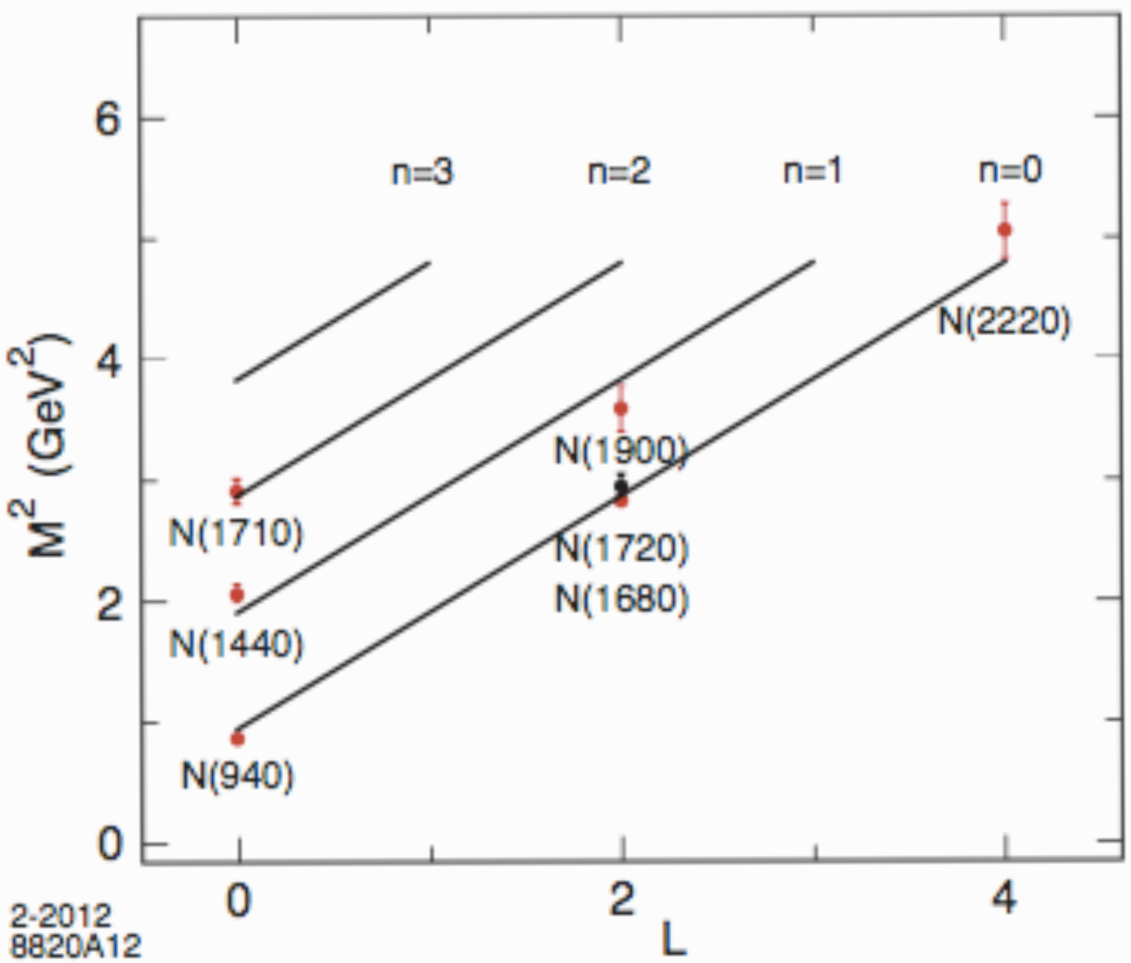
- Gap scale 4λ determines trajectory slope and spectrum gap between plus-parity spin-$\frac{1}{2}$ and minus-parity spin-$\frac{3}{2}$ nucleon families !
- For nucleons $\nu^+_{1/2} = L, \quad \nu^-_{3/2} = L + 1,$ where L is the relative LF angular momentum between the active quark and spectator cluster
- For $\lambda < 0$ no solution possible

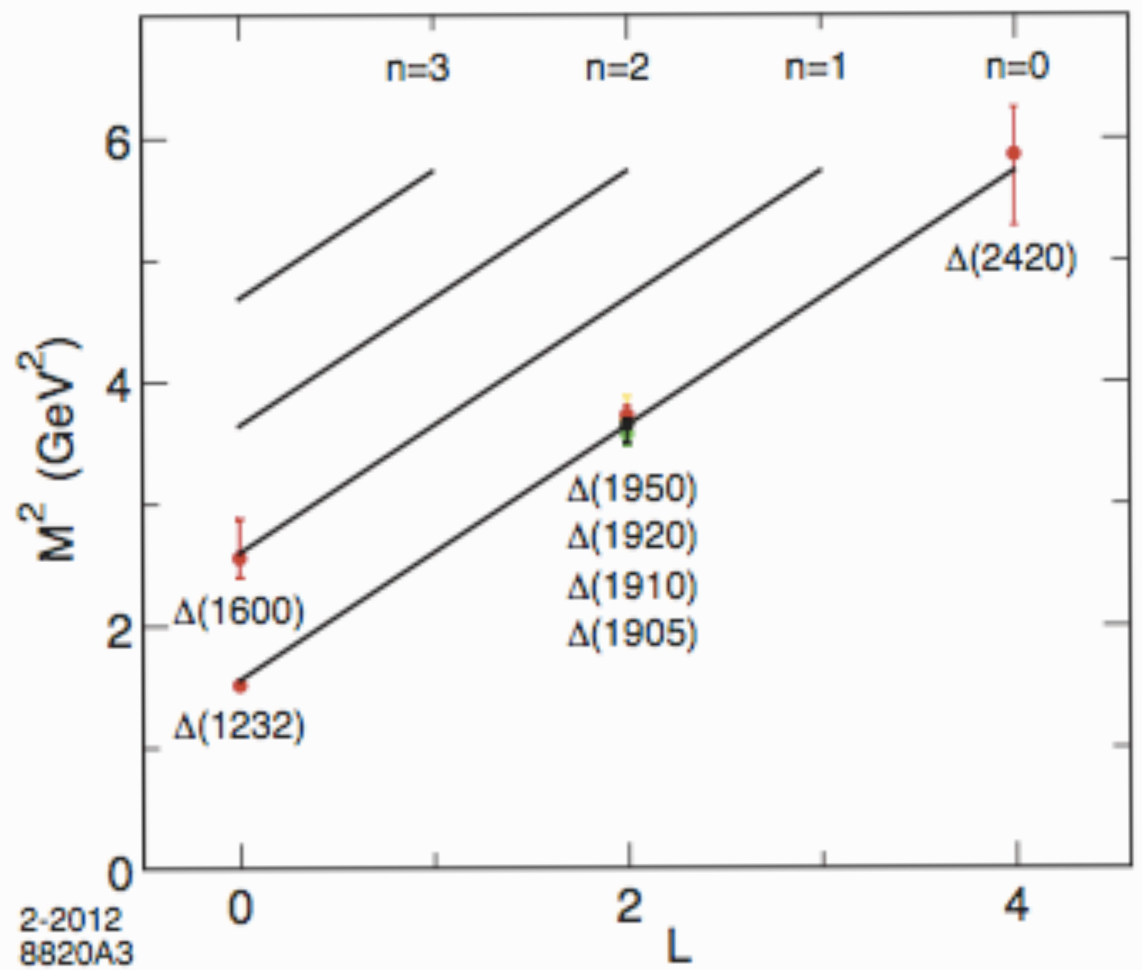


Identify L with ν

- Phenomenological identification to describe the full baryon spectrum: plus and negative sectors have internal spin $S = \frac{1}{2}$ and $S = \frac{3}{2}$

$$\nu^+_{1/2} = L, \qquad \nu^+_{3/2} = L + 1/2$$
$$\nu^-_{1/2} = L + 1/2, \qquad \nu^-_{3/2} = L + 1$$





Example: Orbital and radial excitations for positive parity N and Δ baryon families ($\sqrt{\lambda} \simeq 0.5$ GeV)

Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$
\begin{aligned}
F_+(Q^2) &= g_+ \int d\zeta \, J(Q,\zeta) |\psi_+(\zeta)|^2, \\
F_-(Q^2) &= g_- \int d\zeta \, J(Q,\zeta) |\psi_-(\zeta)|^2,
\end{aligned}
$$

  where the effective charges g_+ and g_- are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

- For $SU(6)$ spin-flavor symmetry

$$
\begin{aligned}
F_1^p(Q^2) &= \int d\zeta \, J(Q,\zeta) |\psi_+(\zeta)|^2, \\
F_1^n(Q^2) &= -\frac{1}{3} \int d\zeta \, J(Q,\zeta) \left[|\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2 \right],
\end{aligned}
$$

  where $F_1^p(0) = 1, \; F_1^n(0) = 0$.

- Compute Dirac proton form factor using SU(6) flavor symmetry

$$F_1^p(Q^2) = R^4 \int \frac{dz}{z^4} V(Q,z) \Psi_+^2(z)$$

- Nucleon AdS wave function

$$\Psi_+(z) = \frac{\kappa^{2+L}}{R^2} \sqrt{\frac{2n!}{(n+L)!}} z^{7/2+L} L_n^{L+1}\left(\kappa^2 z^2\right) e^{-\kappa^2 z^2/2}$$

- Normalization $\quad (F_1{}^p(0) = 1, \quad V(Q=0,z) = 1)$

$$R^4 \int \frac{dz}{z^4} \Psi_+^2(z) = 1$$

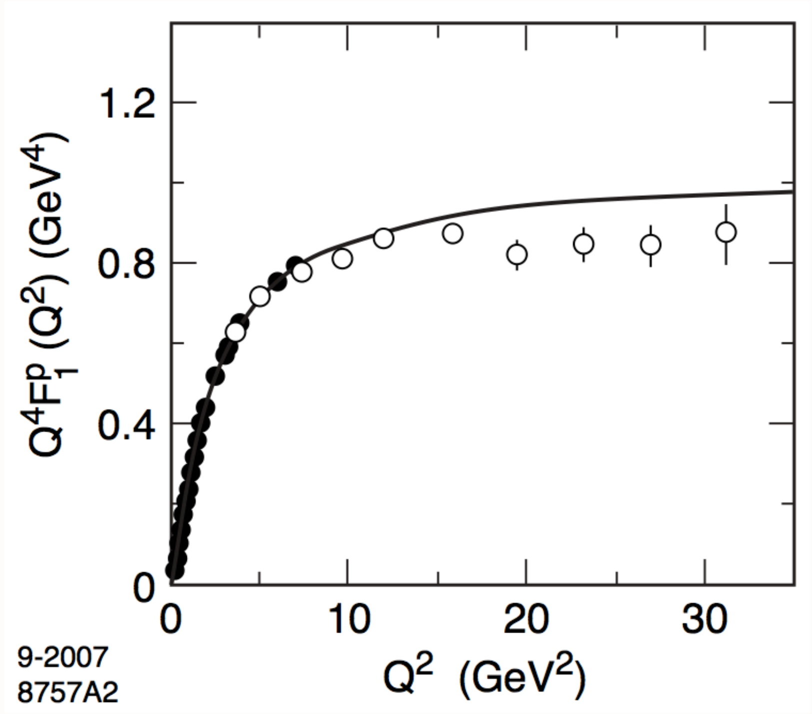
- Bulk-to-boundary propagator [Grigoryan and Radyushkin (2007)]

$$V(Q,z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2} \, x^{\frac{Q^2}{4\kappa^2}} e^{-\kappa^2 z^2 x/(1-x)}$$

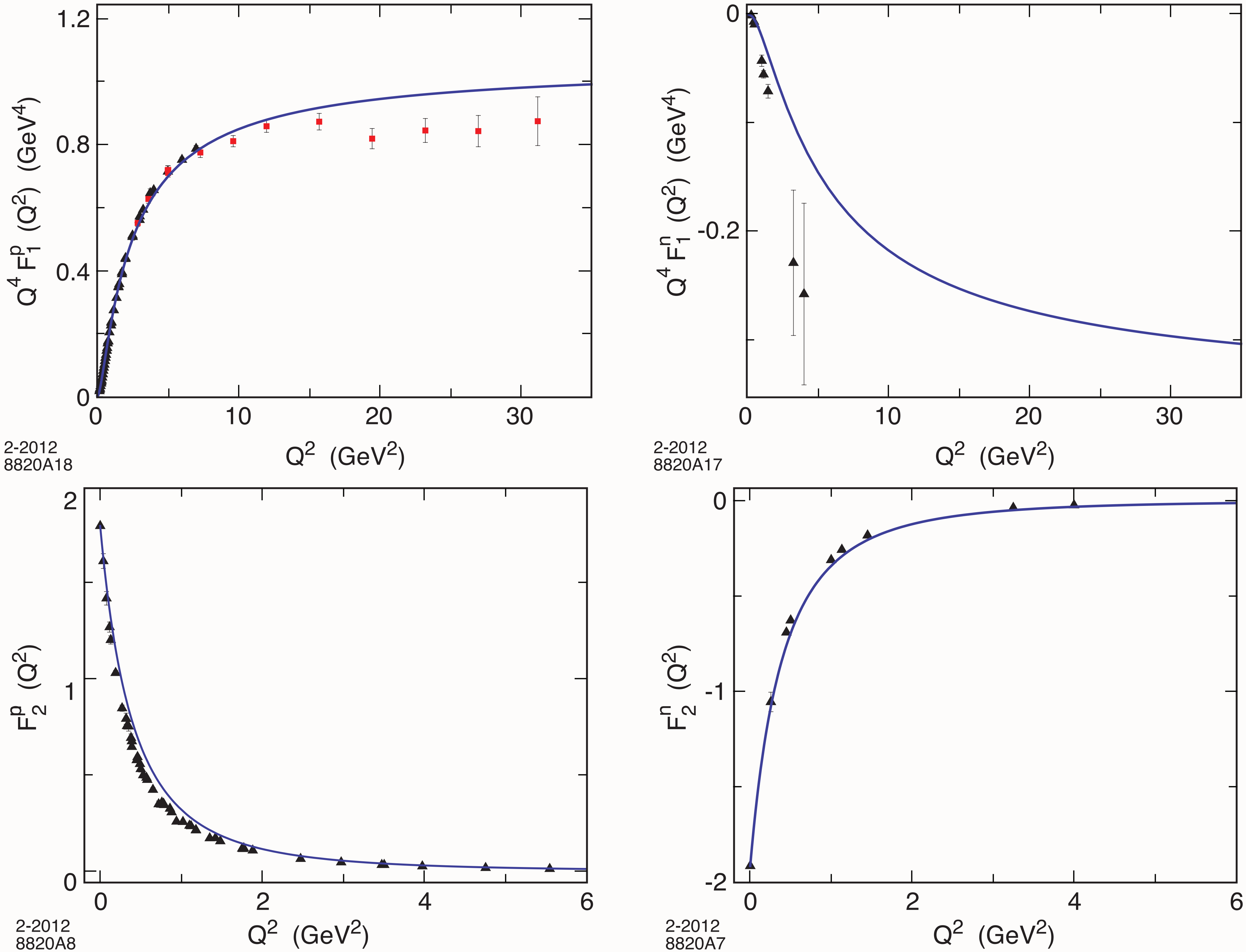
- Find

$$F_1^p(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}$$

with $\mathcal{M}_{\rho_n}^2 \to 4\kappa^2(n+1/2)$



Using $SU(6)$ flavor symmetry and normalization to static quantities



Nucleon Transition Form Factors

- Compute spin non-flip EM transition $N(940) \to N^*(1440)$: $\Psi_+^{n=0,L=0} \to \Psi_+^{n=1,L=0}$
- Transition form factor

$$F_{1\,N\to N^*}^{\,p}(Q^2) = R^4 \int \frac{dz}{z^4}\, \Psi_+^{n=1,L=0}(z) V(Q,z) \Psi_+^{n=0,L=0}(z)$$

- Orthonormality of Laguerre functions $\left(F_{1\,N\to N^*}^{\,p}(0) = 0, \quad V(Q=0,z) = 1\right)$

$$R^4 \int \frac{dz}{z^4}\, \Psi_+^{n',L}(z) \Psi_+^{n,L}(z) = \delta_{n,n'}$$

- Find

$$F_{1\,N\to N^*}^{\,p}(Q^2) = \frac{2\sqrt{2}}{3} \frac{\frac{Q^2}{M_P^2}}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)}$$

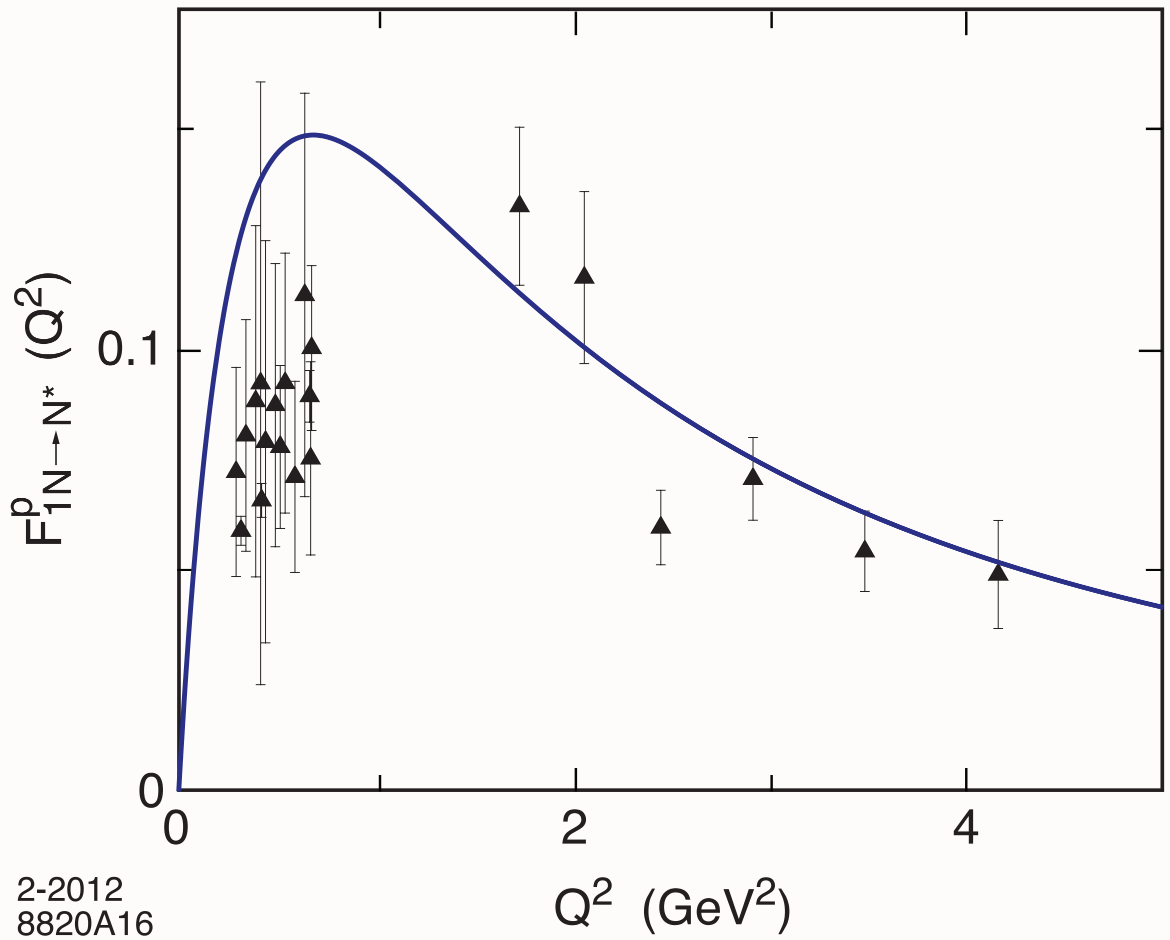
with $\mathcal{M}_{\rho_n}^{\,2} \to 4\kappa^2(n+1/2)$

de Teramond, sjb

Consistent with counting rule, twist 3

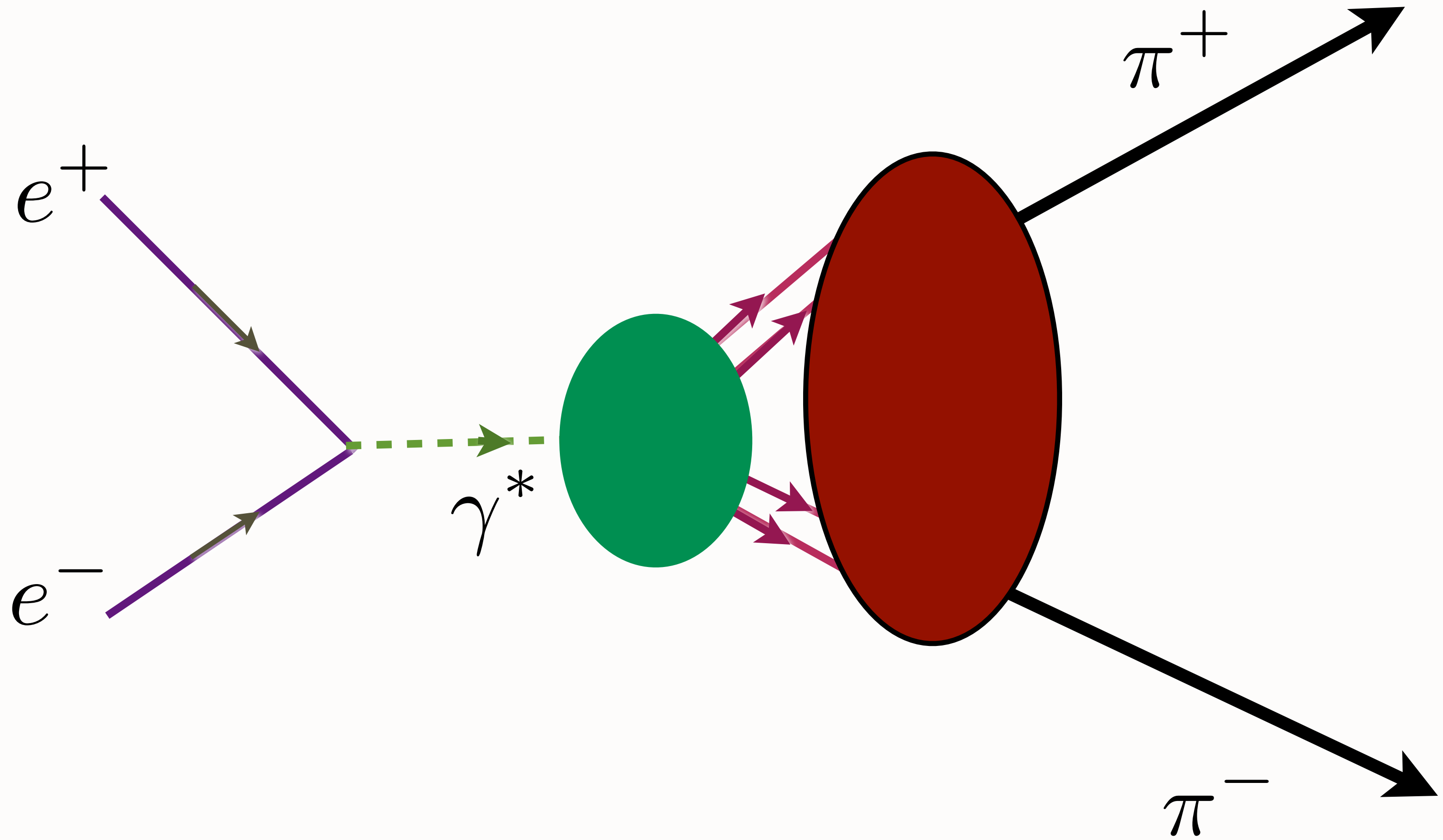
Nucleon Transition Form Factors

$$F^p_{1\,N\to N^*}(Q^2) = \frac{\sqrt{2}}{3}\frac{\frac{Q^2}{\mathcal{M}^2_\rho}}{\left(1+\frac{Q^2}{\mathcal{M}^2_\rho}\right)\left(1+\frac{Q^2}{\mathcal{M}^2_{\rho'}}\right)\left(1+\frac{Q^2}{\mathcal{M}^2_{\rho''}}\right)}.$$



Proton transition form factor to the first radial excited state. Data from JLab

Dressed soft-wall current brings in higher Fock states and more vector meson poles

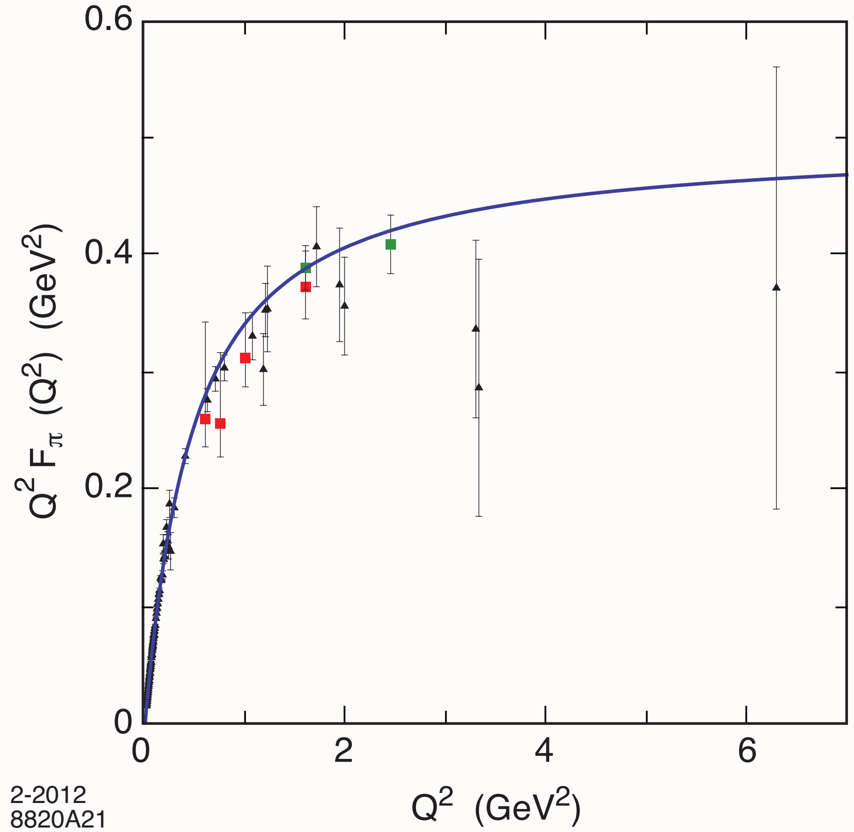


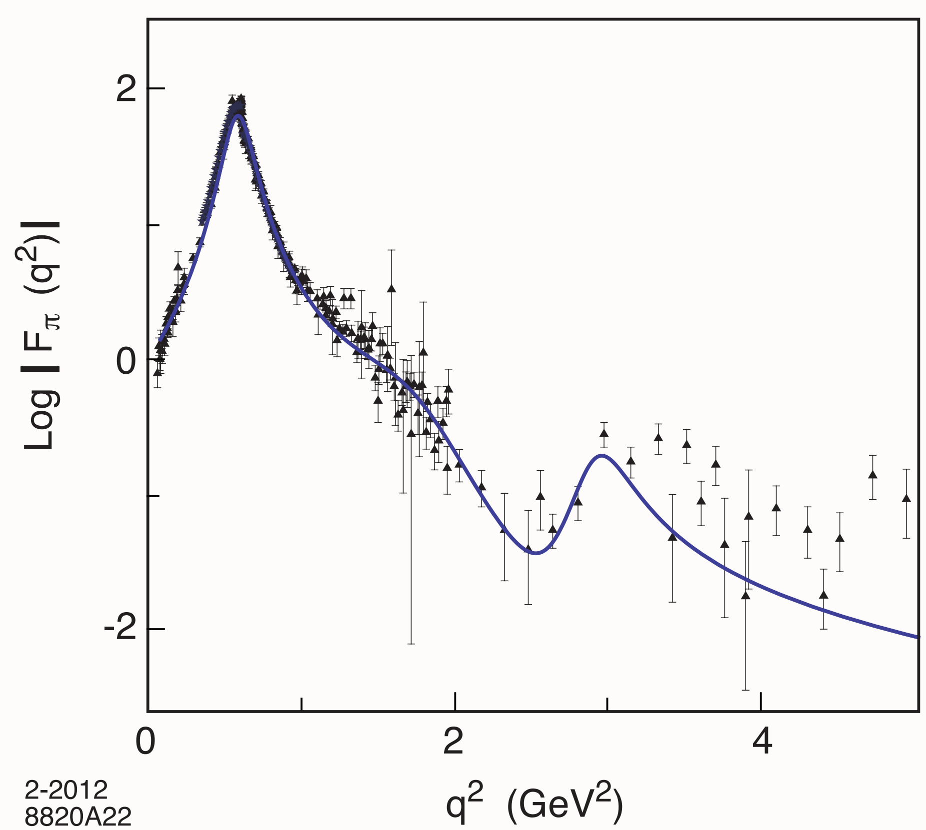
Higher Fock Components in LF Holographic QCD

- Effective interaction leads to $qq \to qq$, $q\overline{q} \to q\overline{q}$ but also to $q \to qq\overline{q}$ and $\overline{q} \to \overline{q}q\overline{q}$
- Higher Fock states can have any number of extra $q\overline{q}$ pairs, but surprisingly no dynamical gluons
- Example of relevance of higher Fock states and the absence of dynamical gluons at the hadronic scale

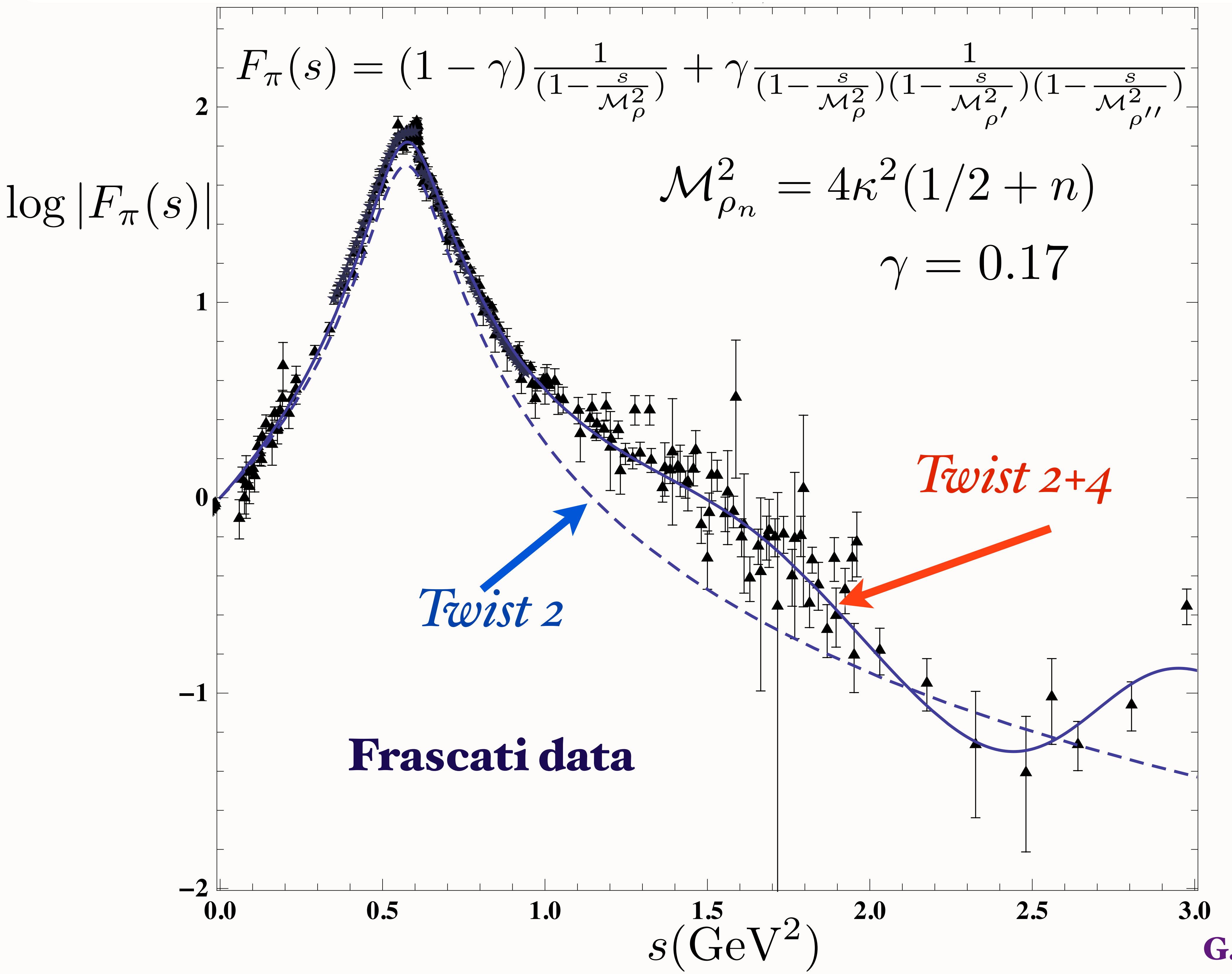
$$|\pi\rangle = \psi_{q\overline{q}/\pi}|q\overline{q}\rangle_{\tau=2} + \psi_{q\overline{q}q\overline{q}}|q\overline{q}q\overline{q}\rangle_{\tau=4} + \cdots$$

- Modify form factor formula introducing finite width: $q^2 \to q^2 + \sqrt{2}i\mathcal{M}\Gamma$ $\quad (P_{q\overline{q}q\overline{q}}$ = 13 %)





Timelike Pion Form Factor from AdS/QCD and Light-Front Holography



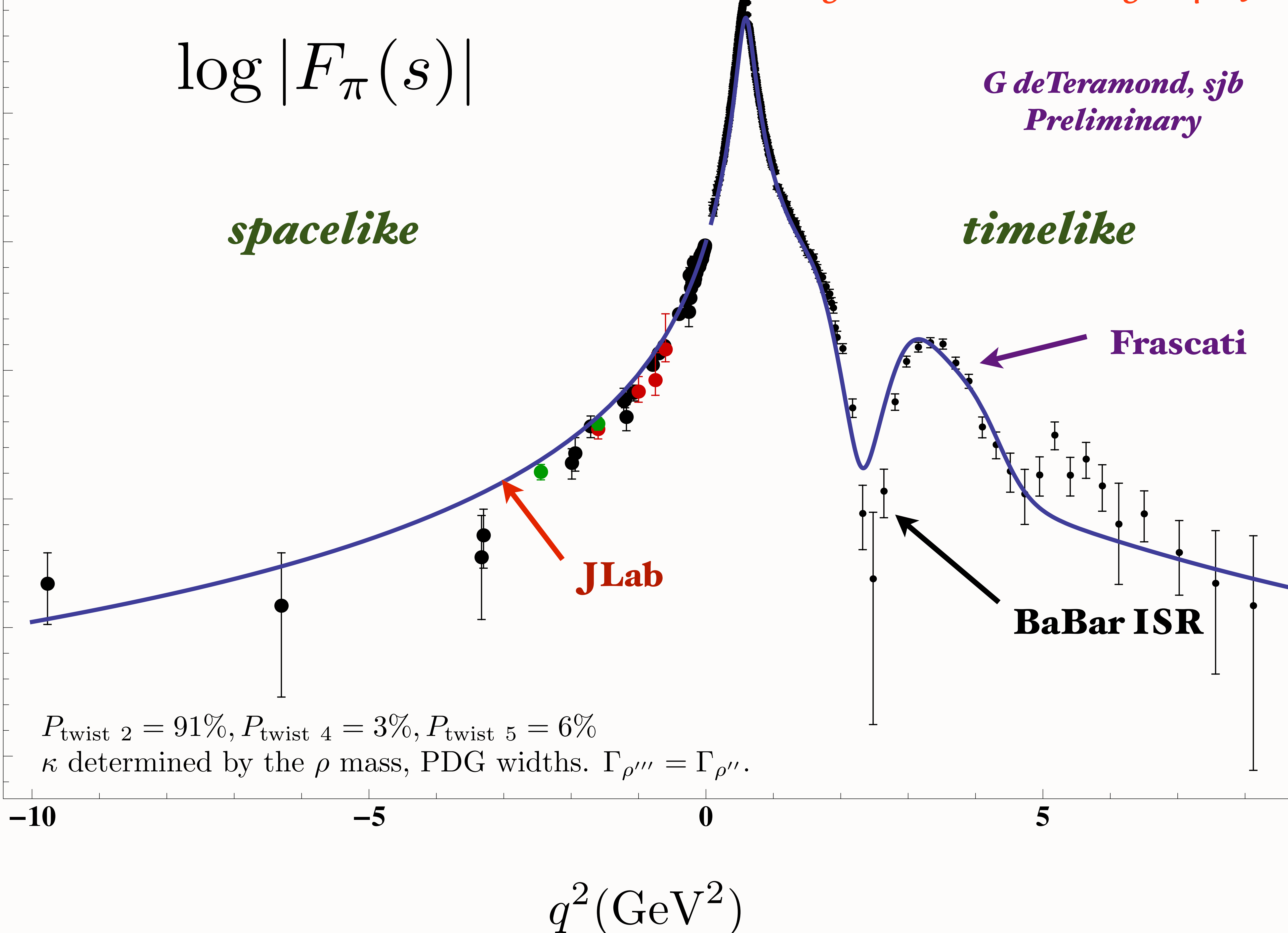
Prescription for Timelike poles :

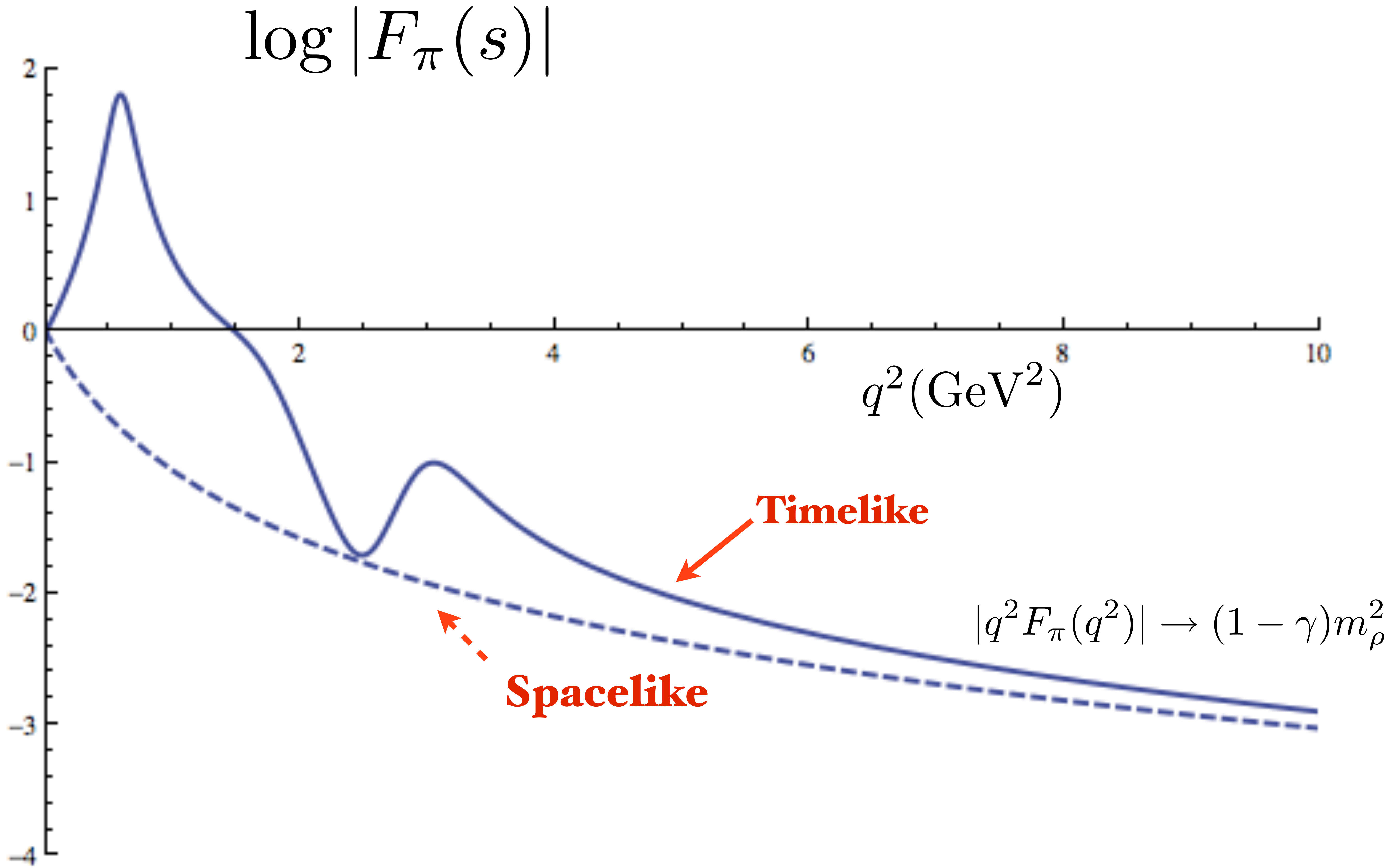
$$\frac{1}{s - M^2 + i\sqrt{s}\Gamma}$$

14% four-quark probability

G. de Teramond & sjb

Pion Form Factor from AdS/QCD and Light-Front Holography





$\log|F_\pi(s)|$
2
1
0
-1
-2
-3
-4
2
4
6
8
10
$q^2(\mathrm{GeV}^2)$
Timelike
$|q^2 F_\pi(q^2)| \to (1-\gamma)m_\rho^2$
Spacelike

Chiral Features of Soft-Wall AdS/QCD Model

- **Boost Invariant**
- **Trivial LF vacuum! No condensate, but consistent with GMOR**
- **Massless Pion**
- **Hadron Eigenstates have LF Fock components of different L^z**
- **Proton: equal probability** $S^z = +1/2, L^z = 0; S^z = -1/2, L^z = +1$

$$\boxed{J^z = +1/2 :< L^z >= 1/2, < S_q^z = 0 >}$$

- **Self-Dual Massive Eigenstates: Proton is its own chiral partner.**
- **Label State by minimum L as in Atomic Physics**
- **Minimum L dominates at short distances**
- **AdS/QCD Dictionary: Match to Interpolating Operator Twist at z=0.**

Gell-Mann Oakes Renner Formula in QCD

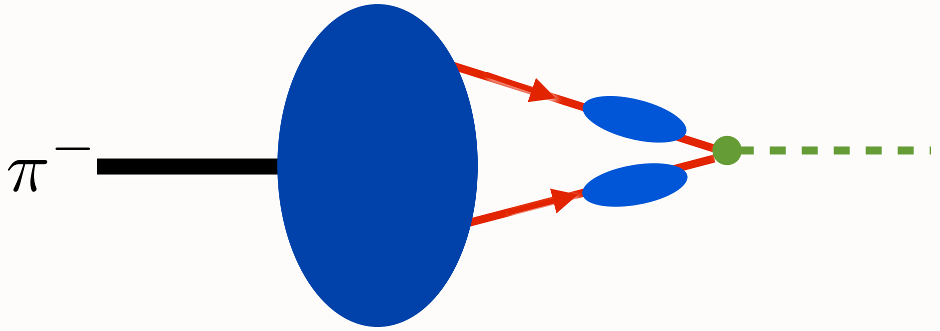
$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi^2} < 0|\bar{q}q|0 >$$

current algebra: effective pion field

$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi} < 0|i\bar{q}\gamma_5 q|\pi >$$

QCD: composite pion Bethe-Salpeter Eq.

vacuum condensate actually is normal pion decay matrix element



$$< 0|\bar{q}\gamma_5 q|\pi >$$

Maris, Roberts, Tandy

AdS/QCD and Light-Front Holography

- AdS/QCD: Incorporates scale transformations characteristic of QCD with a single scale -- RGE
- Light-Front Holography; unique connection of AdS_5 to Front-Form
- Profound connection between gravity in 5th dimension and physical 3+1 space time at fixed LF time τ
- Gives unique interpretation of z in AdS to physical variable ζ in 3+1 space-time

Basis Light-Front Quantization Approach to Quantum Field Theory

BLFQ

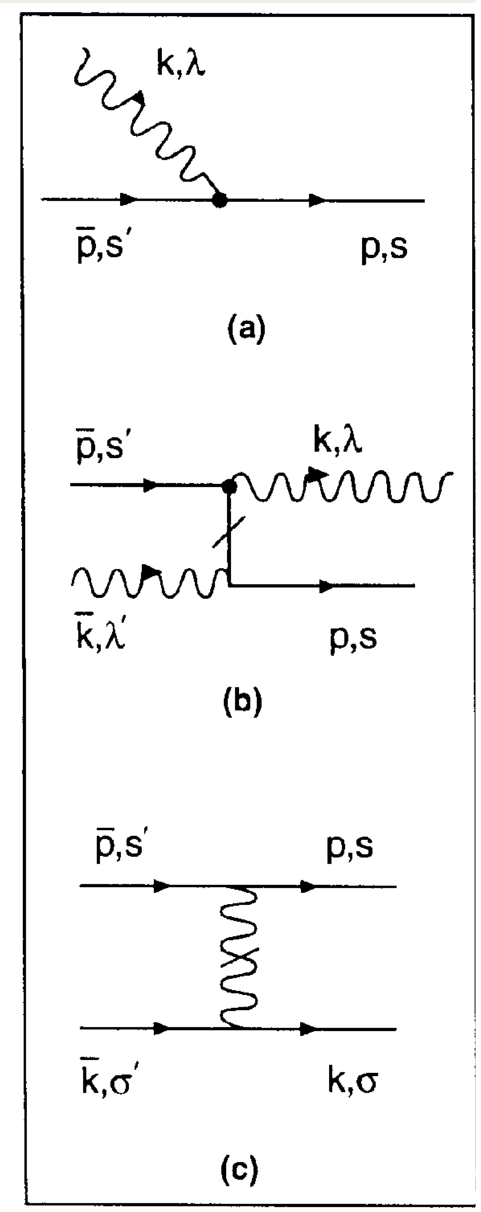
Use AdS/QCD basis functions!

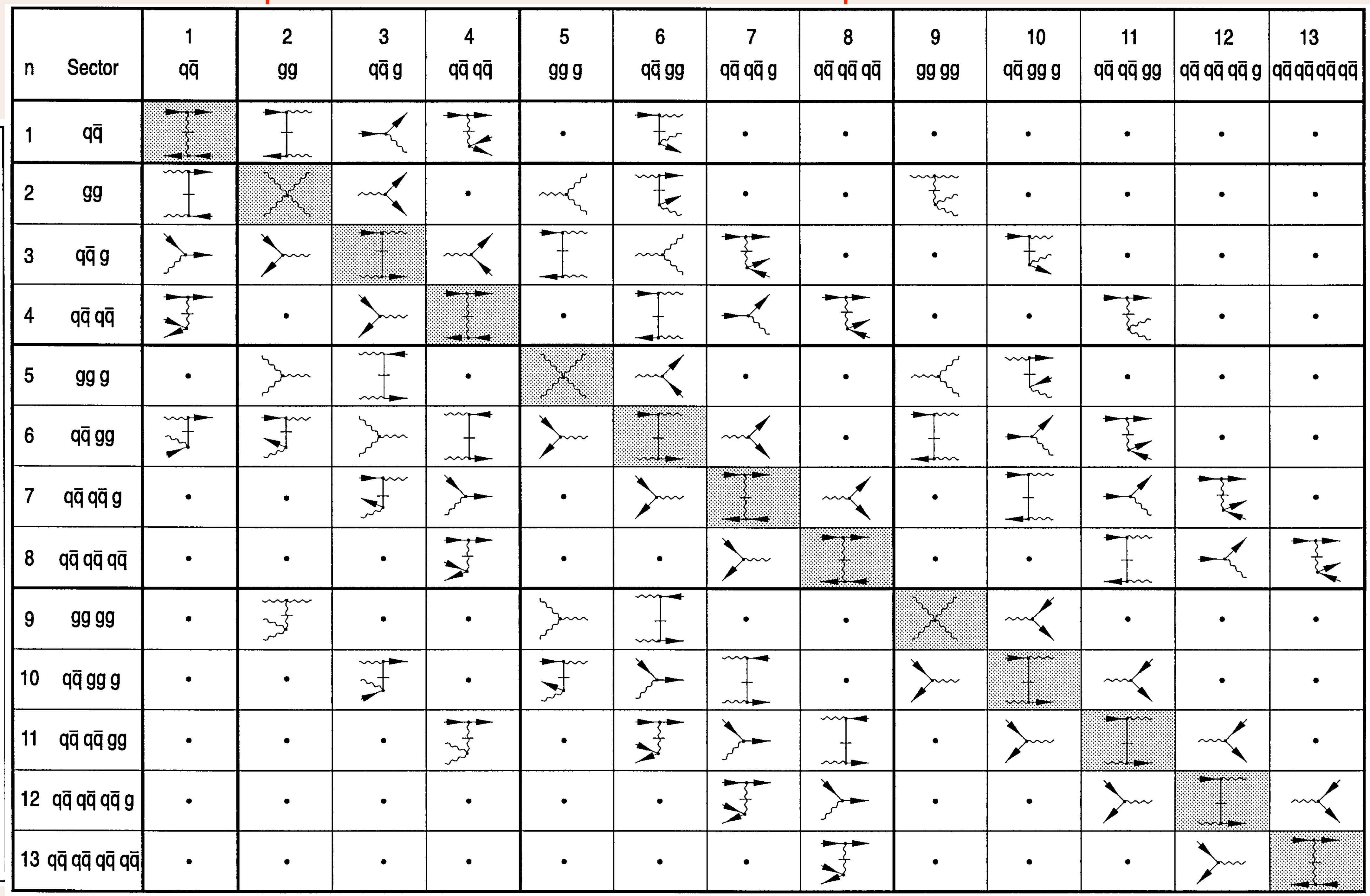
Xingbo Zhao
With Anton Ilderton,
Heli Honkanen, Pieter Maris,
James Vary, Stan Brodsky

Department of Physics and Astronomy
Iowa State University
Ames, USA

Light-Front QCD Heisenberg Equation

$$H_{LC}^{QCD}|\Psi_h\rangle = \mathcal{M}_h^2 \, |\Psi_h\rangle$$





BLFQ: Use AdS/QCD basis functions!

BLFQ

Use AdS/CFT orthonormal Light Front Wavefunctions as a basis for diagonalizing the QCD LF Hamiltonian

- Good initial approximation
- Better than plane wave basis
- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion
- Similar to Shell Model calculations
- **Hamiltonian light-front field theory within an AdS/QCD basis.**
  J.P. Vary, H. Honkanen, Jun Li, P. Maris, A. Harindranath,

  G.F. de Teramond, P. Sternberg, X. Zhao, E.G. Ng, C. Yang, sjb

Basis functions

- HO basis for transverse momentum states:

$$\Phi_{n,m}(p^{\perp}) = \Phi_{n,m}(\rho,\phi) = \sqrt{2\pi}\frac{1}{b}\sqrt{\frac{2n!}{(|m|+n)!}}e^{im\phi}\rho^{|m|}e^{-\rho^2/2}L_n^{|m|}(\rho^2),$$

with

$$\rho = \frac{|p^{\perp}|}{b}, \quad b = \sqrt{\mathbf{M_0\Omega}}$$

- Discretize longitudinal momentum:

$$\psi_k(x^-) = \frac{1}{\sqrt{2L}}\,\mathrm{e}^{i\,\frac{\pi}{L}k\,x^-},$$

$$k = \begin{cases} k = 1,2,3,\ldots\,\text{(periodic boundary condition for bosons)}, \\ \\ k = \frac{1}{2},\frac{3}{2},\ldots\,\text{(antiperiodic boundary condition for fermions)} \end{cases}$$

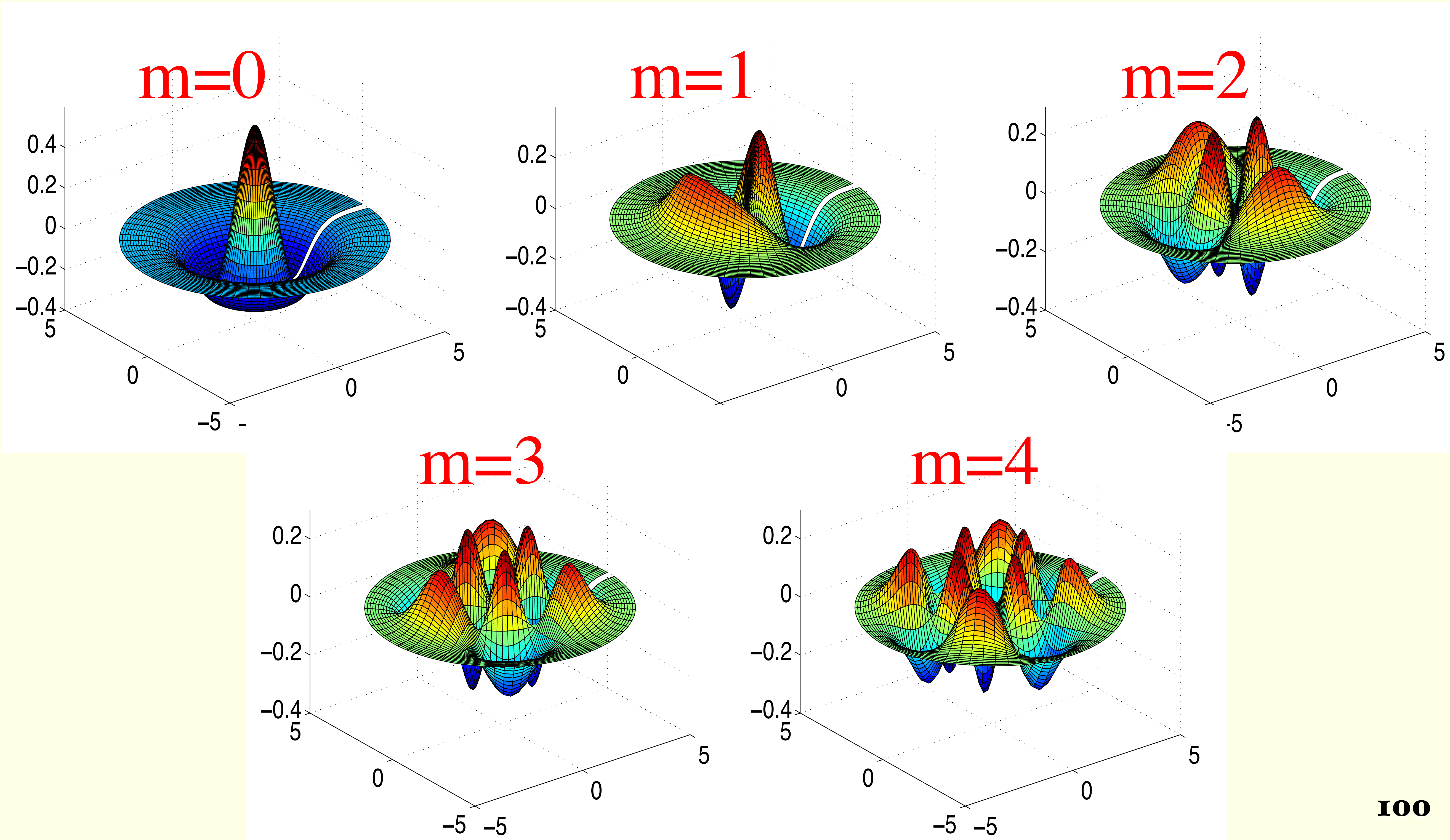
- Full 3-D:

$$\Psi_{k,n,m}(x^-,\rho,\phi) = \psi_k(x^-)\Phi_{n,m}(\rho,\phi). \tag{1}$$

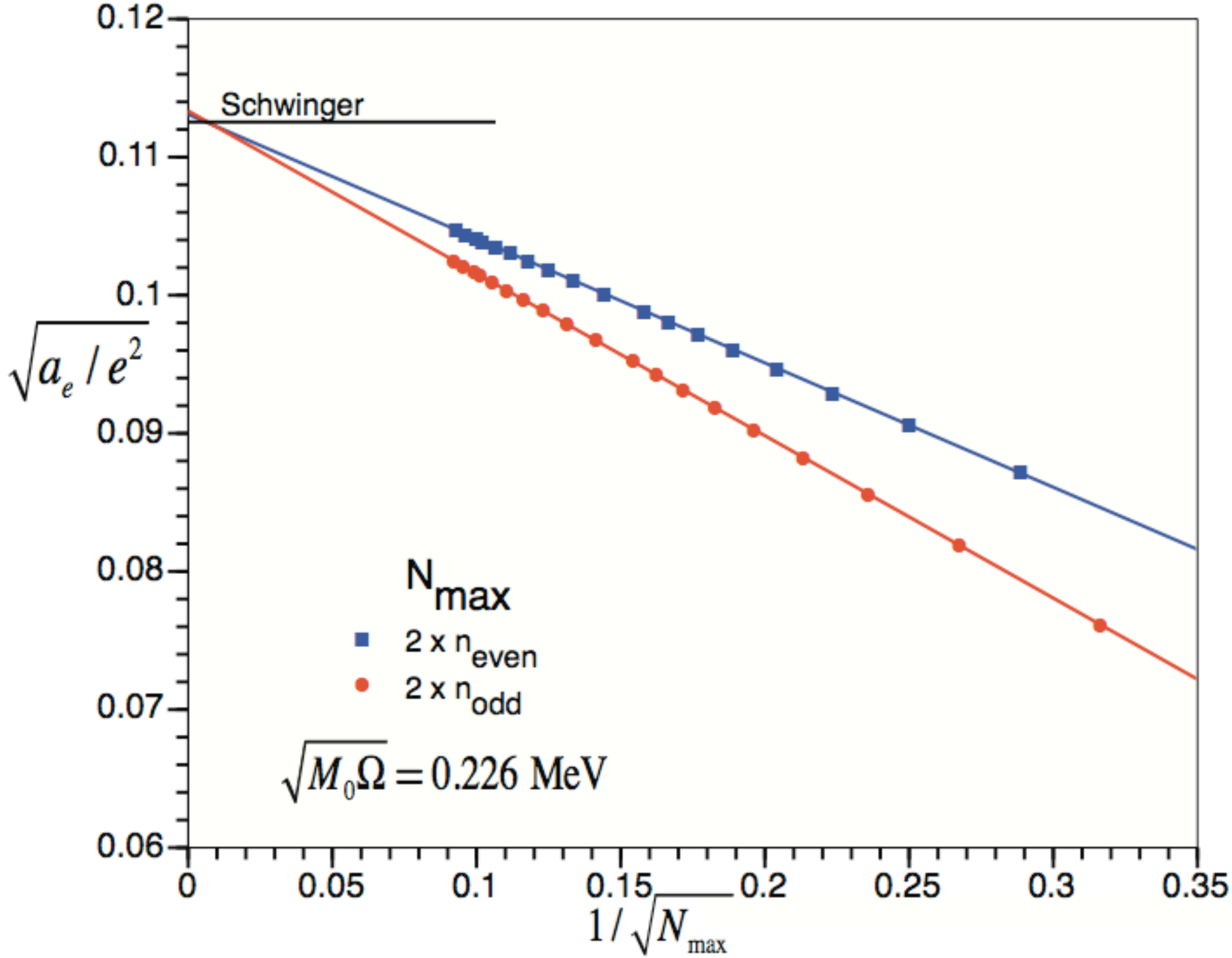
- 2-D harmonic trap with the basis function scale

Set of transverse 2D HO modes for $n = 1$

J.P. Vary, H. Honkanen, Jun Li, P. Maris, S.J. Brodsky, A. Harindranath, G.F. de Teramond, P. Sternberg, E.G. Ng, C. Yang, PRC



Numerical Results for Electron g-2



[Zhao, Honkanen, Maris, Vary, Brodsky, 2012]

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS$_5$ space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4}\int d^4x\, dz\, \sqrt{g}\, e^{\varphi(z)}\, \frac{1}{g_5^2}\, G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)}\frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$

- Coupling measured at momentum scale Q

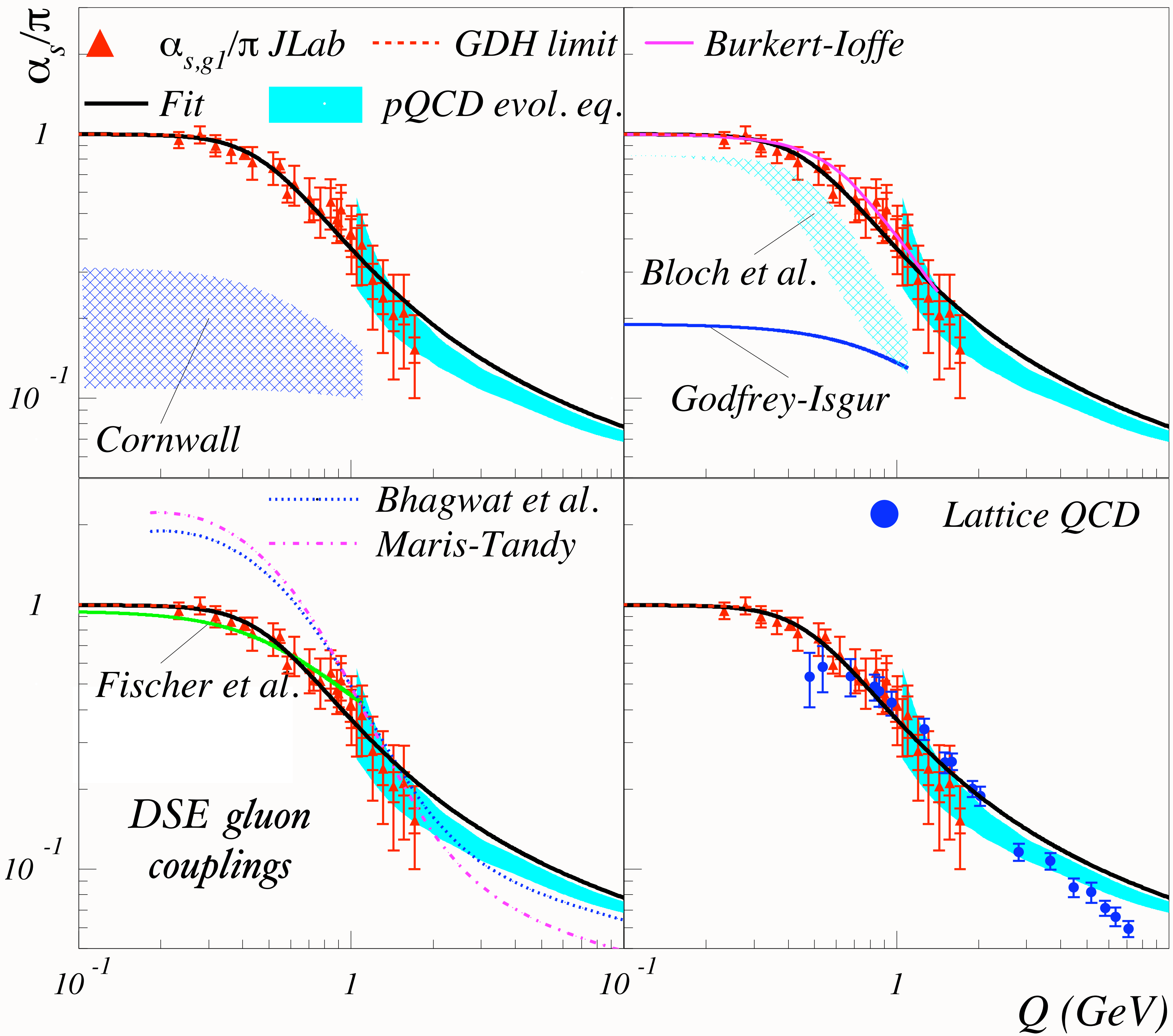
$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta\, J_0(\zeta Q)\, \alpha_s^{AdS}(\zeta)$$

- Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0)\, e^{-Q^2/4\kappa^2}.$$

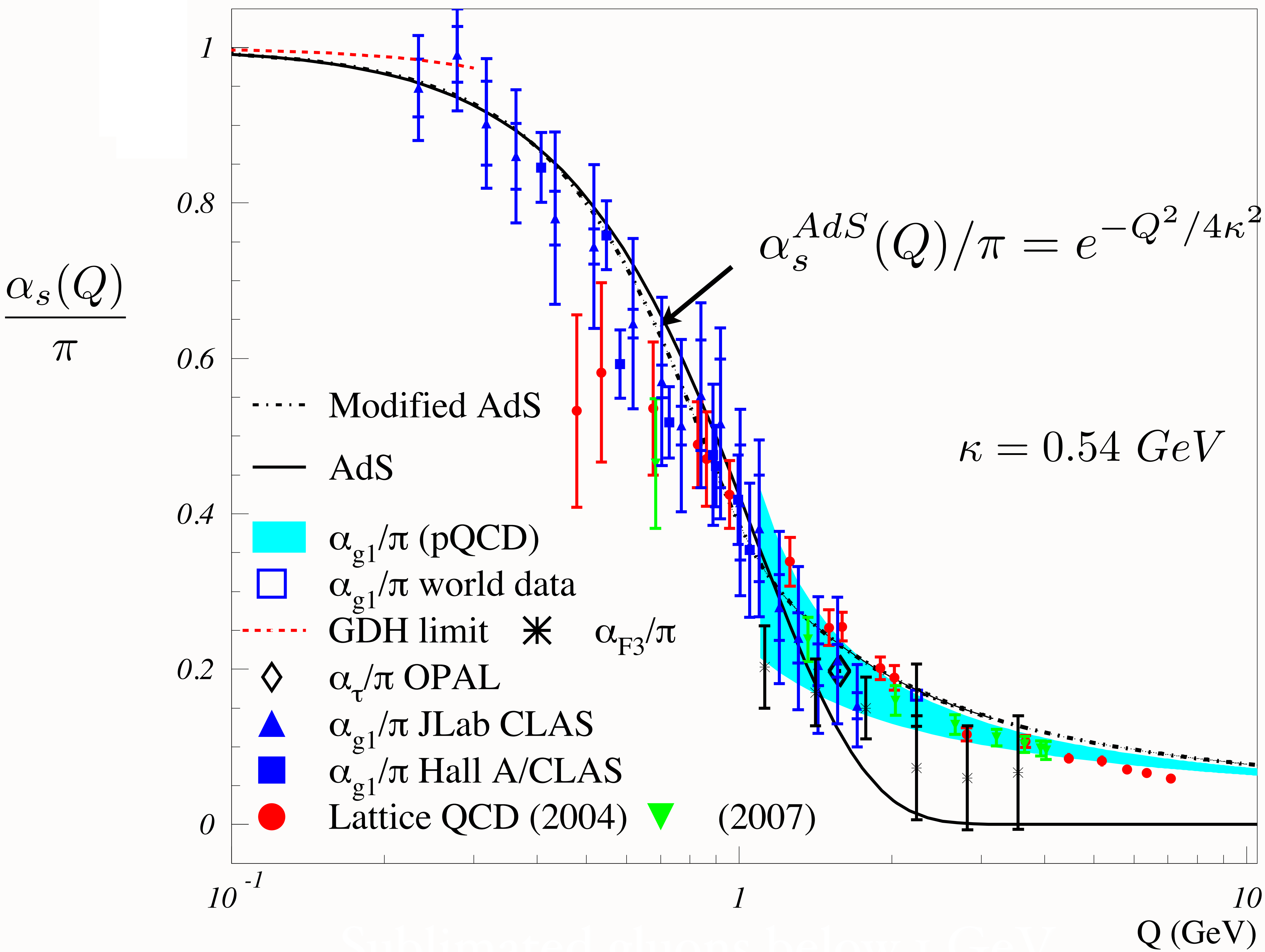
where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

Deur, Korsch, et al.



Running Coupling from Light-Front Holography and AdS/QCD

Analytic, defined at all scales, IR Fixed Point



Deur, de Teramond, sjb

Light and heavy mesons in a soft-wall holographic model

Valery E. Lyubovitskij[*1†], **Tanja Branz**[1], **Thomas Gutsche**[1], **Ivan Schmidt**[2], **Alfredo Vega**[2]

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Kepler Center for Astro and Particle Physics,
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[2]*Departamento de Física y Centro Científico Tecnológico de Valparaíso (CCTVal),*
Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, Chile

We study the spectrum and decay constants of light and heavy mesons in a soft-wall holographic approach, using the correspondence of string theory in Anti-de Sitter space and conformal field theory in physical space-time.

$$|\pi^+ >= |u\bar{d} >$$

$$m_u = 2\ MeV$$
$$m_d = 5\ MeV$$

$$|K^+ >= |u\bar{s} >$$

$$m_s = 95\ MeV$$

$$|D^+ >= |c\bar{d} >$$

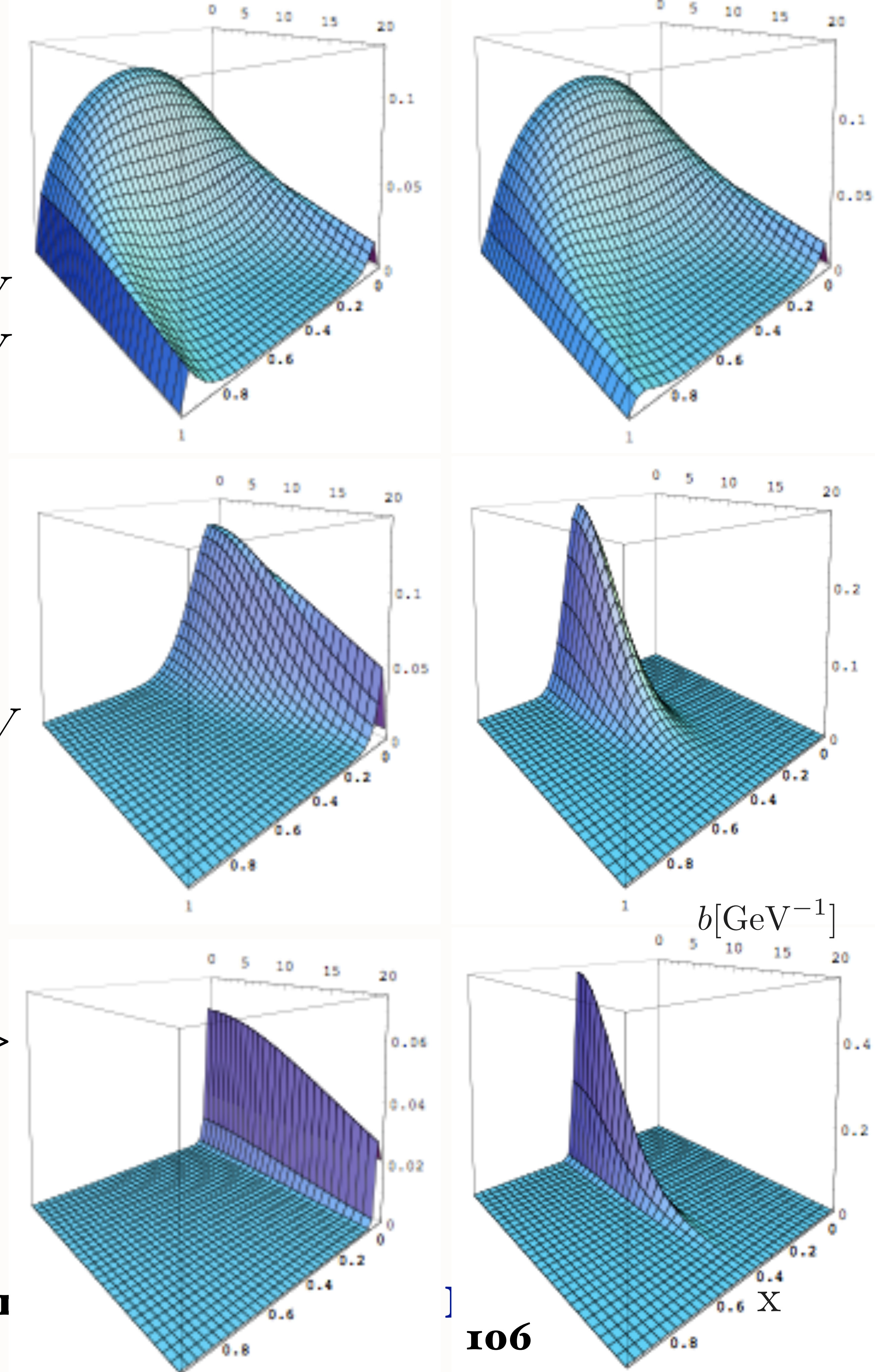
$$m_c = 1.25\ GeV$$

$$|\eta_c >= |c\bar{c} >$$

$$|B^+ >= |u\bar{b} >$$

$$m_b = 4.2\ GeV$$

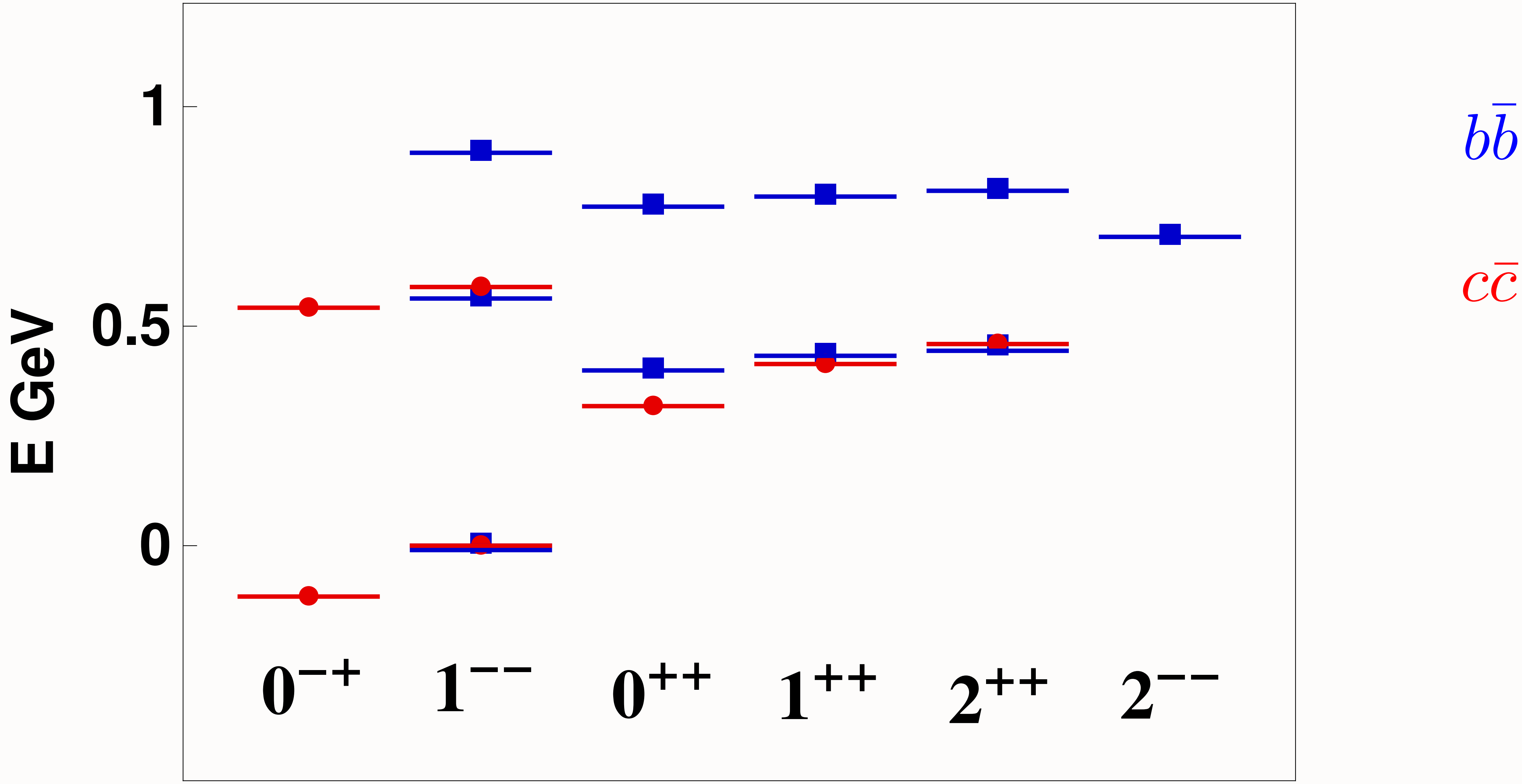
$$|\eta_b >= |b\bar{b} >$$



$$\kappa = 375\ MeV$$

Quigg and Rosner (1979:

Excitation energies of quarkonia appear to be flavor-independent



logarithmic potential?

Heavy Quark Systems and Conformal Invariance

H.G. Dosch, G. de Teramond, sjb

- Structure of excitations for heavy quark bound states is largely independent of the reduced mass
- Quigg and Rosner (1979) "For what form of the quark-antiquark potential is the level spacing independent of the reduced mass ?" : $V(r) = C \ln(r/r_0)$
- New perspective from non-relativistic realization of the dAFF construction!
- Consider the operator

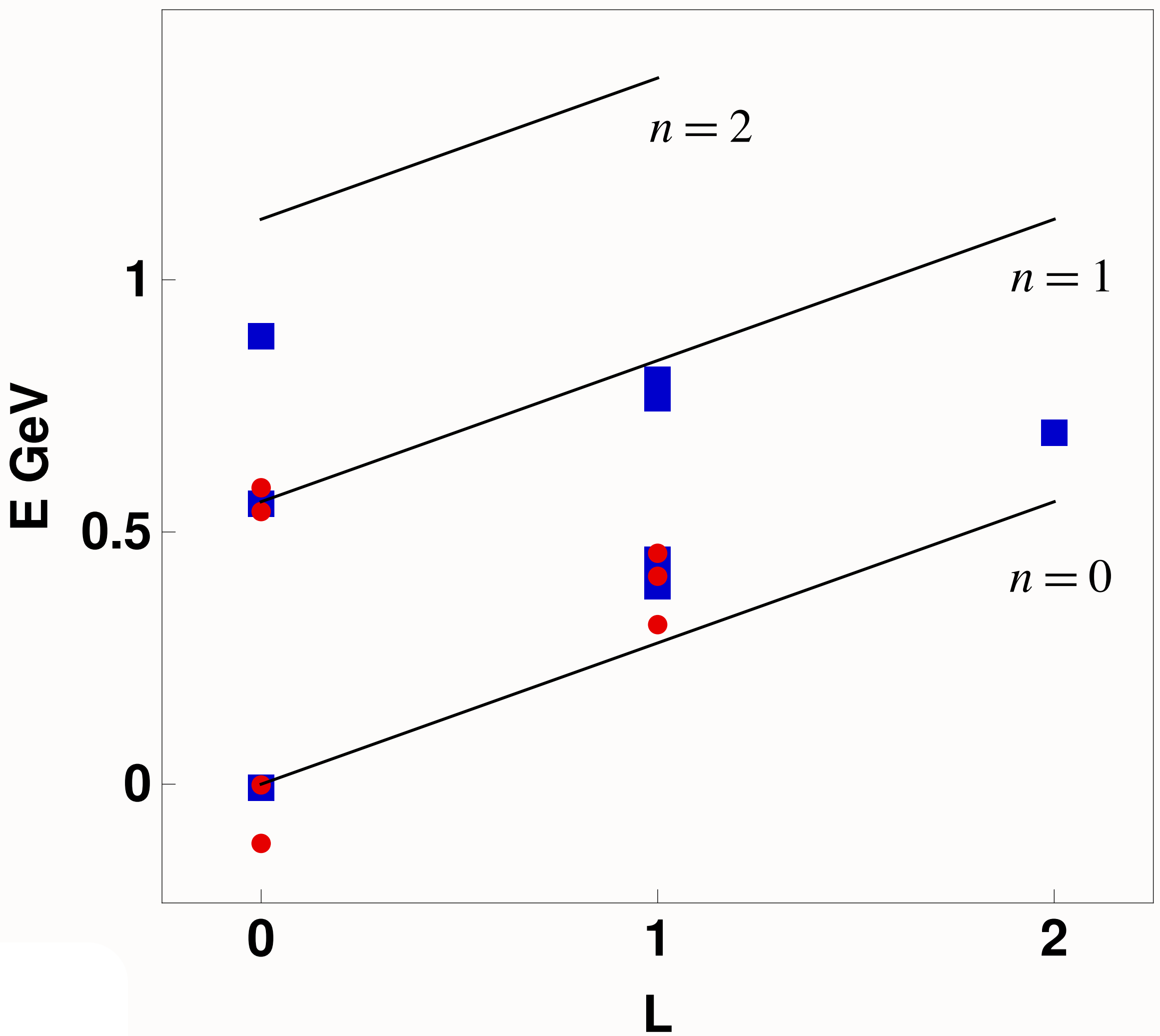
$$G_{NR} = a\, H_t + b\, D + c\, K,$$

- The corresponding dAFF NR Hamiltonian is

$$H_{NR} = \tfrac{1}{2}\left(\dot{q}^2 + \frac{g}{q^2} + \frac{4\,ac - b^2}{4} q^2\right),$$

$$q \to \sqrt{m}r, \quad \dot{q} \to \frac{1}{\sqrt{m}} i \frac{d}{dr} \qquad [q(t), \dot{q}(t)] = i$$

$$H = -\frac{1}{2m}\left(\frac{1}{r^2}\frac{\partial}{\partial r}r^2\frac{\partial}{\partial r} - \frac{l(l+1)}{r^2}\right) + \frac{1}{2}m\,\Lambda^2 r^2,$$



$$E_{n\ell} = \left(2\,n + \ell + \frac{3}{2}\right)\,\Lambda$$

$$\Lambda = 0.28\ GeV$$

Flavor independent

Universal confinement time!

Excitation energies of $c\bar{c}$ (red boxes) and $b\bar{b}$ (blue diamonds) with different values of angular momentum ℓ. Only well established states below open flavour threshold are shown.

]

TABLE III: Predictions for bottom-charmed mesons.

ℓ	J^P	$n=0$	$n=1$
0	$0^-, 1^-$	6.28	6.84
1	$0^+, 1^+, 2^+$	6.56	7.12
2	$1^-, 2^-, 3^-$	6.94	7.50

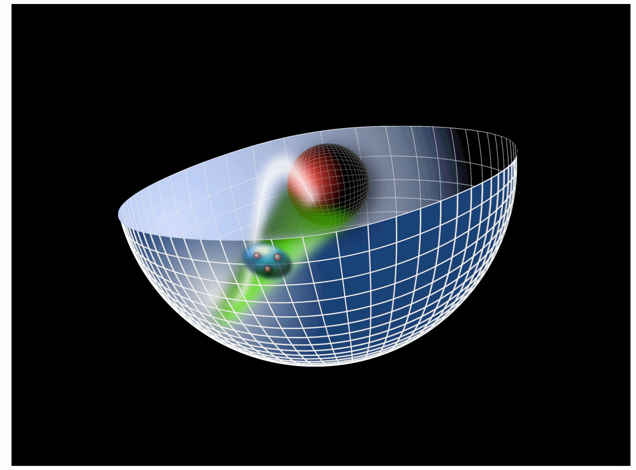
TABLE IV: Comparison of predictions and data for the ground states of heavy-strange mesons.

	D_s	D_s^*	B_s	B_s^*
Exp.	1.968	2.112	5.367	5.415
Predict.	2.06	2.06	5.24	5.24

Flavor independent

$$< r^2 > \sim \frac{1}{m_{reduced}} \qquad < \vec{p}^{\,2} > \sim m_{reduced}$$

Universal confinement time!



AdS/QCD Soft-Wall Model

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Conformal Symmetry of the action

Confinement scale: $\kappa \simeq 0.5\ GeV$

$1/\kappa \simeq 0.4\ fm$

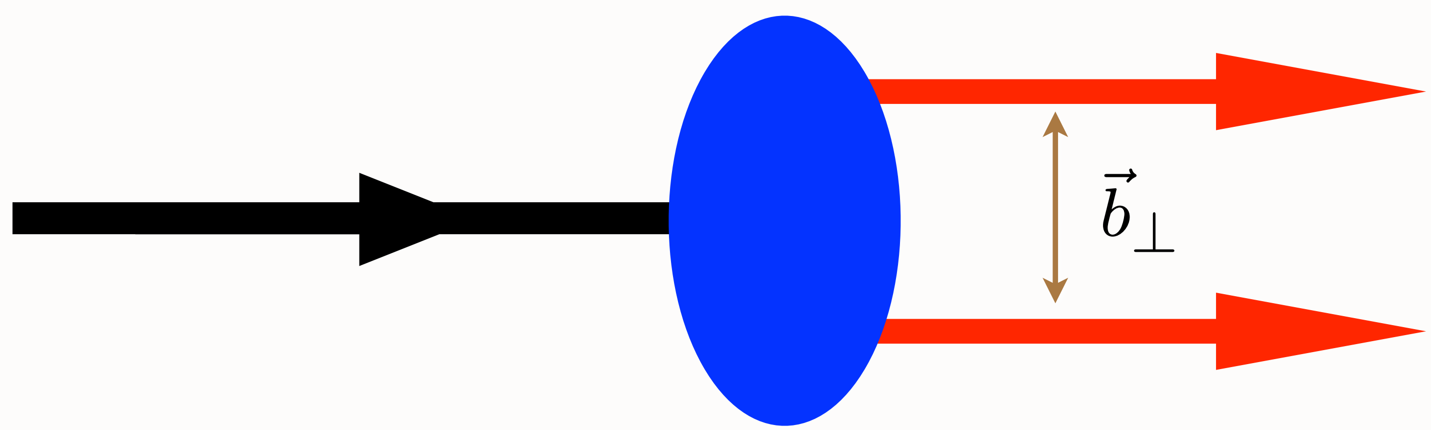
de Teramond, sjb

$LF(3+1) \longleftrightarrow AdS_5$

Light-Front Holography

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$



$$\psi(x, \zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

Light Front Holography: Unique mapping derived from equality of LF and AdS formulae for bound-states and form factors

String Theory

Goal: First Approximant to QCD

AdS/CFT

Mapping of Poincare' and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space

Counting rules for Hard Exclusive Scattering
Regge Trajectories

AdS/QCD

Conformal behavior at short distances + Confinement at large distance

QCD at the Amplitude Level

Semi-Classical QCD / Wave Equations

Holography

Boost Invariant 3+1 Light-Front Wave Equations

J =0,1,1/2,3/2 plus L

Integrable!

Hadron Spectra, Wavefunctions, Dynamics

An analytic first approximation to QCD

AdS/QCD + Light-Front Holography

- **As Simple as Schrödinger Theory in Atomic Physics**
- **LF radial variable ζ conjugate to invariant mass squared**
- **Relativistic, Frame-Independent, Color-Confining**
- **Unique confining potential!**
- **QCD Coupling at all scales: Essential for Gauge Link phenomena**
- **Hadron Spectroscopy and Dynamics from one parameter**
- **Wave Functions, Form Factors, Hadronic Observables, Constituent Counting Rules**
- **Insight into QCD Condensates: Zero cosmological constant!**
- **Systematically improvable with DLCQ-BLFQ Methods**

Light-Front Schrödinger Equation

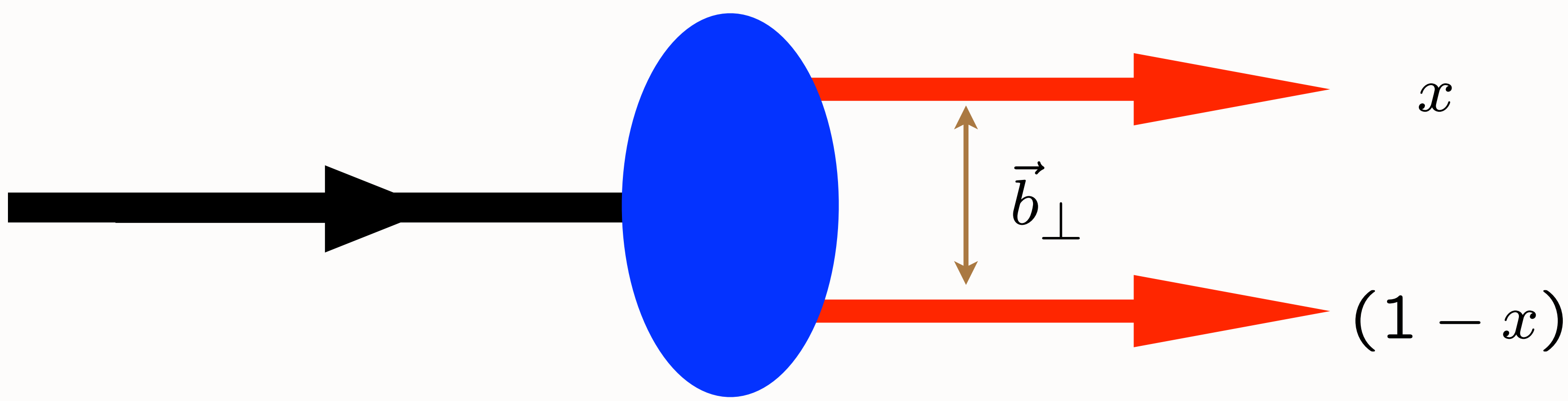
G. de Teramond, sjb

Relativistic LF single-variable radial equation for QCD & QED

Frame Independent!

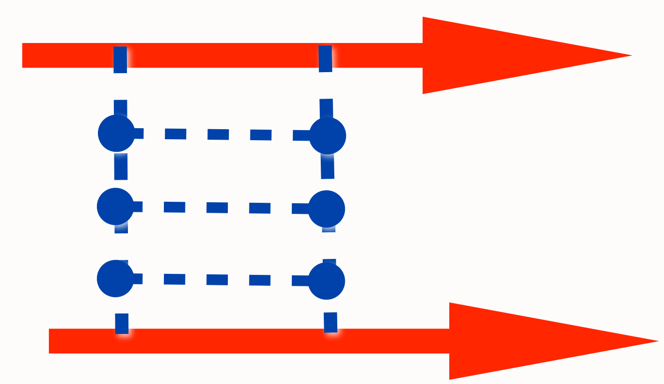
$$\left[-\frac{d^2}{d\zeta^2}+\frac{4L^2-1}{4\zeta^2}+U(\zeta^2,J,L,M^2)\right]\Psi_{J,L}(\zeta^2)=M^2\Psi_{J,L}(\zeta^2)$$

$$\zeta^2 = x(1-x)\mathrm{b}_\perp^2.$$

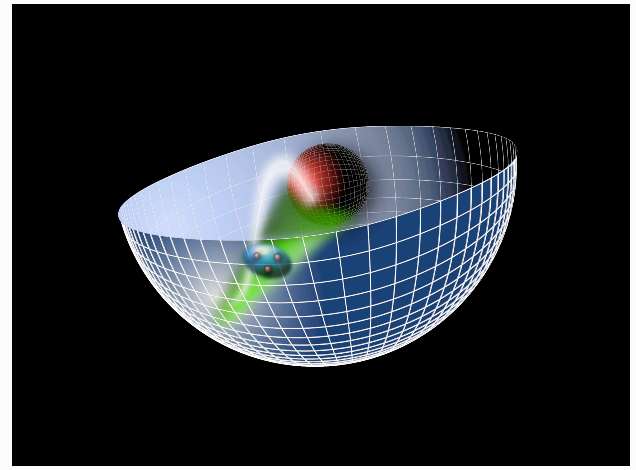


U is the exact QCD potential
Conjecture: 'H'-diagrams generate

$$U(\zeta,S,L)=\kappa^2\zeta^2+\kappa^2(L+S-1/2)$$



AdS/QCD
Soft-Wall Model



Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2}+\frac{1-4L^2}{4\zeta^2}+U(\zeta)\right]\psi(\zeta)=\mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta)=\kappa^4\zeta^2+2\kappa^2(L+S-1)$$

Conformal Symmetry
of the action

Confinement scale: $\kappa \simeq 0.5 \ GeV$

$$1/\kappa \simeq 0.4 \ fm$$

Basis Light-Front Quantization Approach to Quantum Field Theory

Use AdS/QCD orthonormal Light Front Wavefunctions as a basis for diagonalizing the QCD LF Hamiltonian

BLFQ

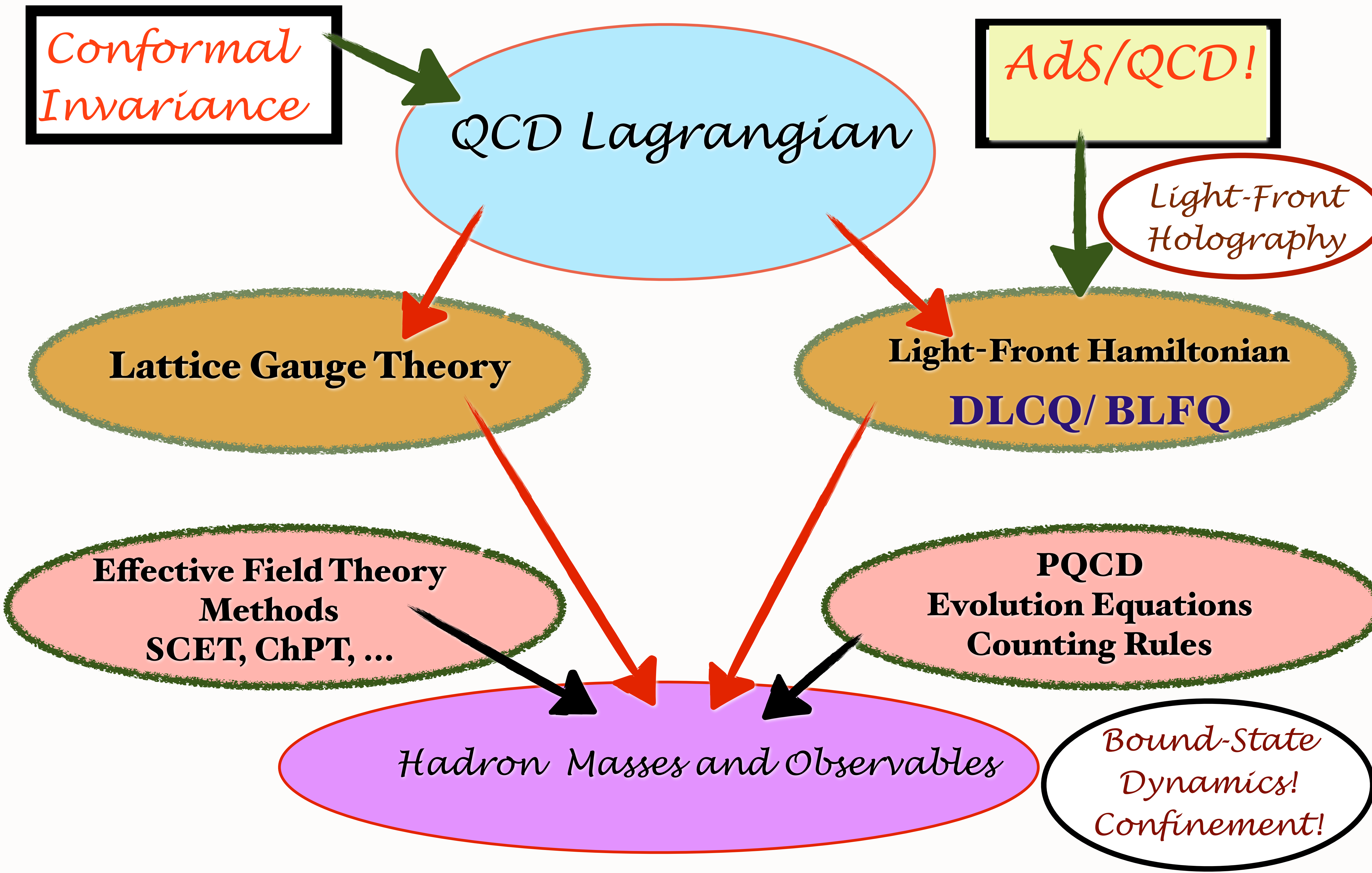
- Good initial approximation
- Better than plane wave basis
- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion
- Similar to Shell Model calculations
- **Hamiltonian light-front field theory within an AdS/QCD basis.**

- Xingbo Zhao
- Anton Ilderton,
- Heli Honkanen
- Pieter Maris,
- James Vary
- Stan Brodsky

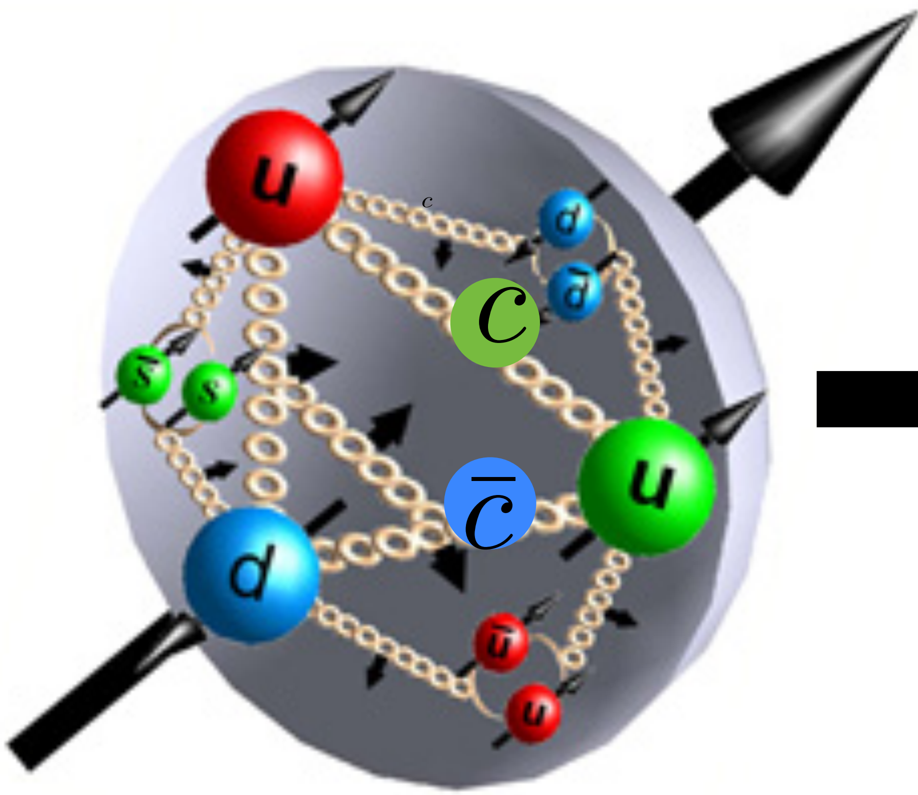
New Directions

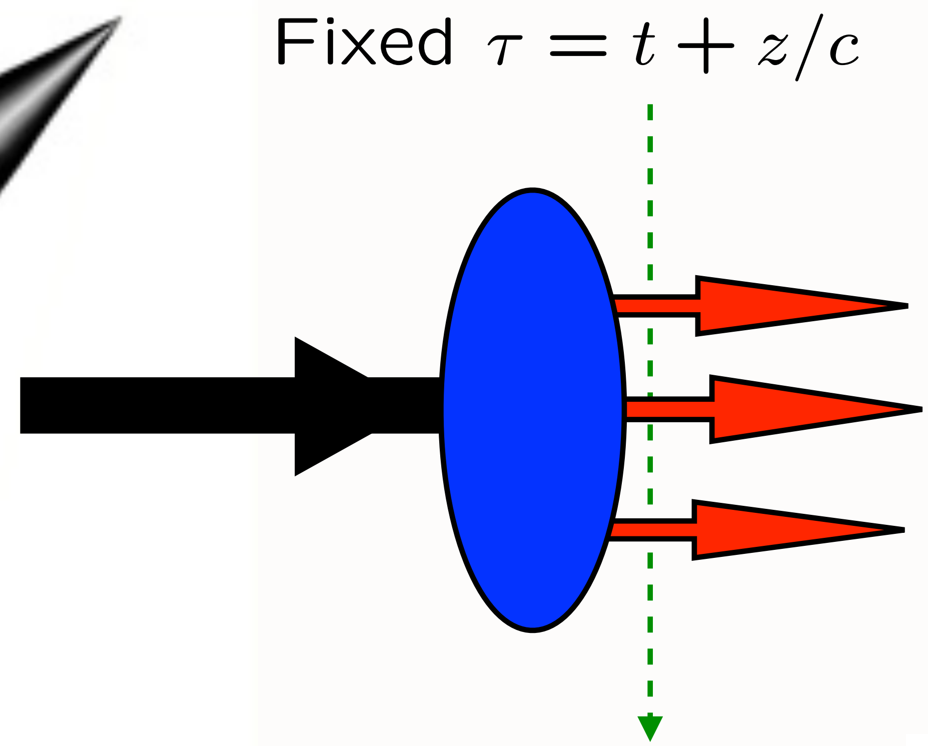
- Hadronization at the Amplitude Level
- LF Confinement potential and LFWFs predicted
- Eliminate Factorization Scale: Fracture function determines off-shellness
- Eliminate Renormalization Scale Ambiguity: Principle of Maximal Conformality (PMC)
- Exclusive Channels: PQCD Gluon exchange versus Soft Interactions
- Massive quark spectroscopy
- Sublimated Gluons: Gluons appear at high virtuality
- Hidden Color of Nuclear Wavefunctions
- Duality: connection to DIS at high x

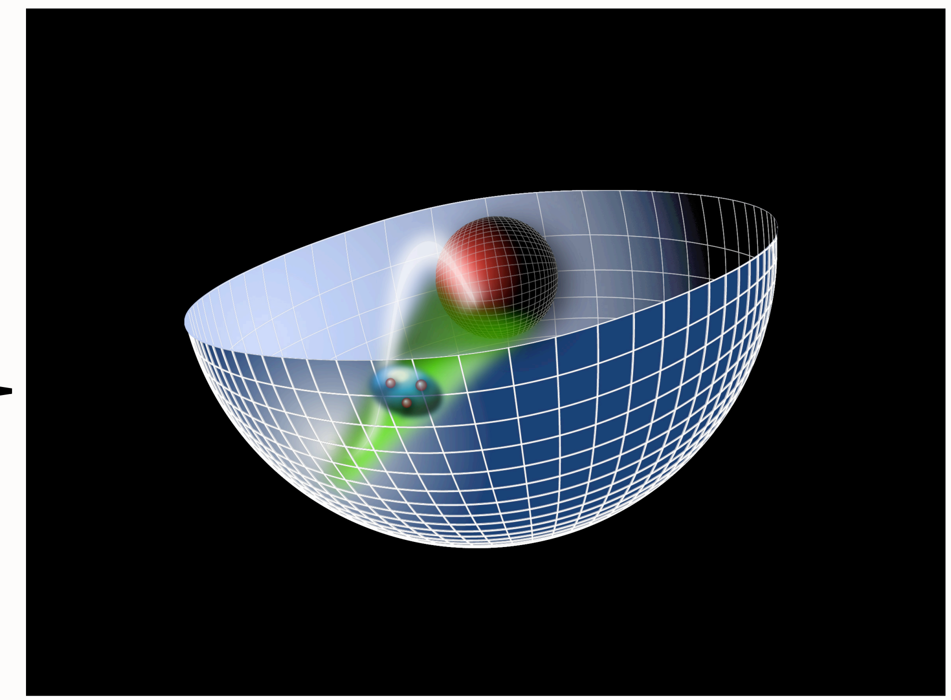
Predict Hadron Properties from First Principles!



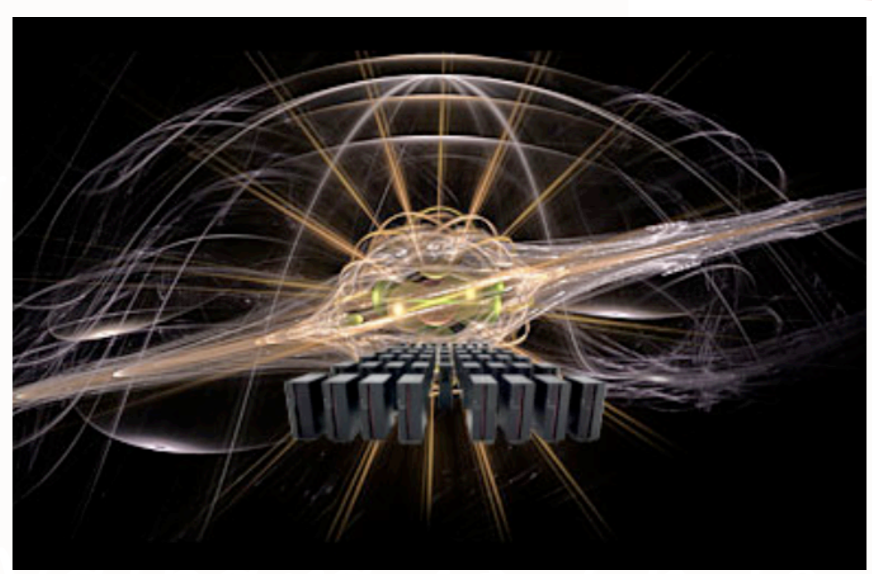
Light-Front Quantum Chromodynamics







Stan Brodsky



James Vary -- Happy Birthday!

May 14, 2013

International Conference on Nuclear Theory in the Supercomputing Era