Inhomogeneous Neutrons and Cold Atoms

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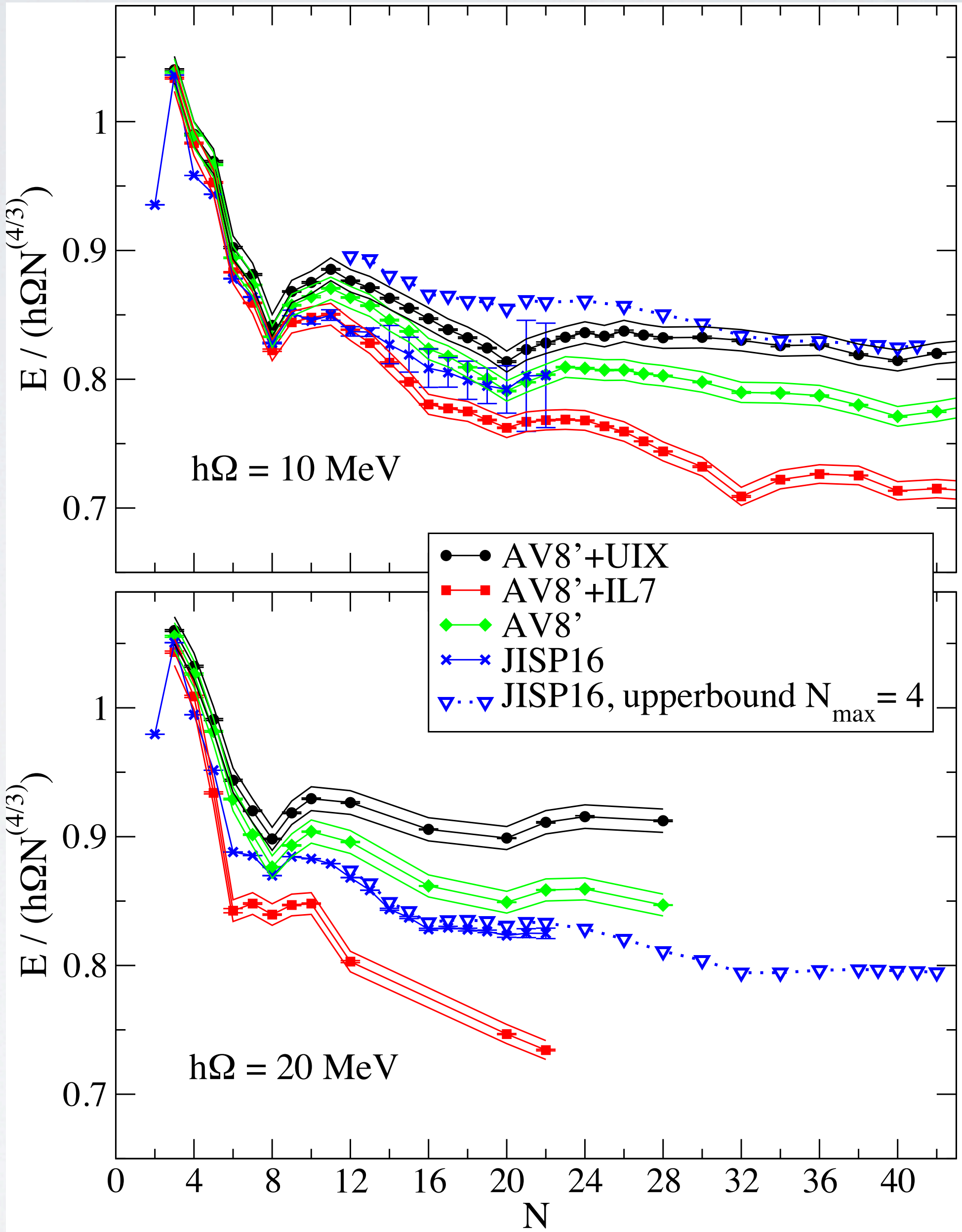
Ab-initio calculations $\Leftrightarrow$ Density Functional and Finite Systems

- Shell Closure and Fermi Quasiparticles
- Phonons and Gradient Terms
- Superfluid Pairing
- Spin-orbit Splitting
- ....

Neutron Drops

Extreme Isospin Limit
of
Nuclear Density
Functional

Applications:
Neutron-rich Nuclei
Neutron Star Crust



Maris, et al, PRC 2013

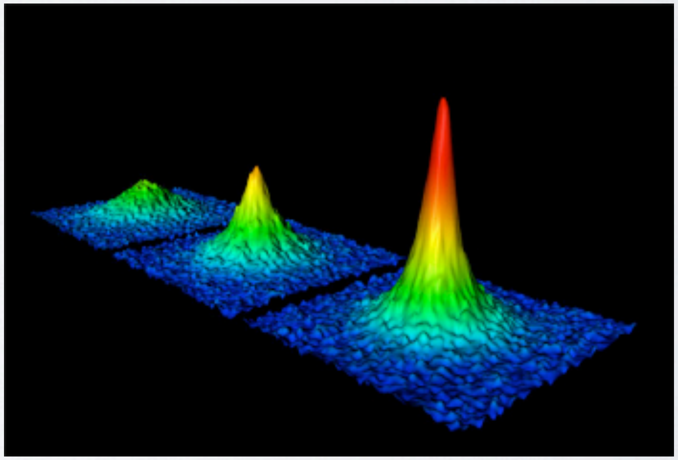
Trapped Cold Atoms

Simple Interaction tunable from BCS (nearly free fermions to BEC (nearly free bosons)

$$H = \sum_i T_i + \sum_{i<j} V_0 \, \delta^3(r_i - r_j)$$

Cold-Atom Experiments w/ External Potentials

Linear Response
3D to 2D transition
Optical Lattices
\`Exotic' Superfluid States
Drops (2D)



Inhomogeneous Cold Atoms: What do we expect?

Scale-free Interaction:

Ground-State Energy $$E = \xi \frac{3}{5} \frac{\hbar^2}{2m} [3\pi^2 \rho]^{2/3}$$

Scale Invariance and large pairing gap place very strong constraints on the density functional.

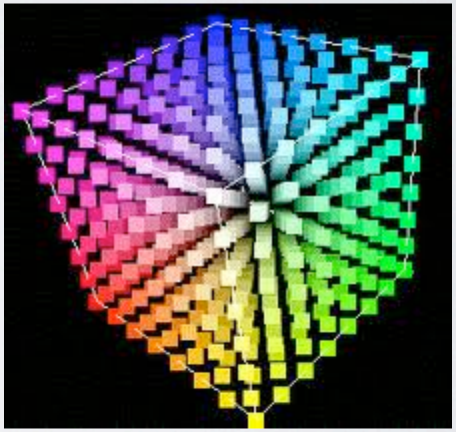
Energy Density $$\mathcal{E}\ [\rho(r)] = \mathcal{E}\ [\rho(xr)]\ x^{5/3}$$

Computational Method: Auxiliary Field MC

Branching random walk; evolving single-particle orbitals
Each `configuration': $L^3 \times N$ amplitudes

Importance sampling and overlap with BCS wave function (critical)
Each step requires 2N 3D FFTs:

    Kinetic energy in momentum space
    Potential energy in coordinate space

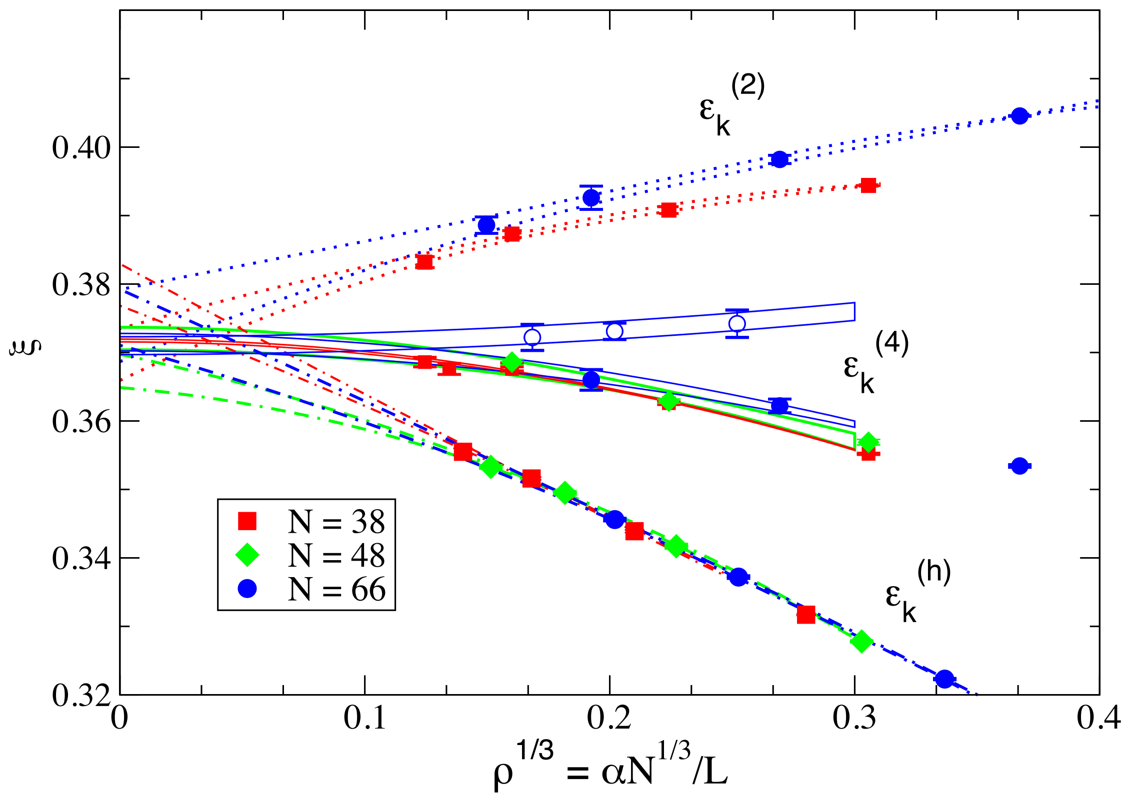


Typically L=24, N=66, Nt (time steps) ~ 10000)
~10,000 configurations
10-100 GB total memory for amplitudes

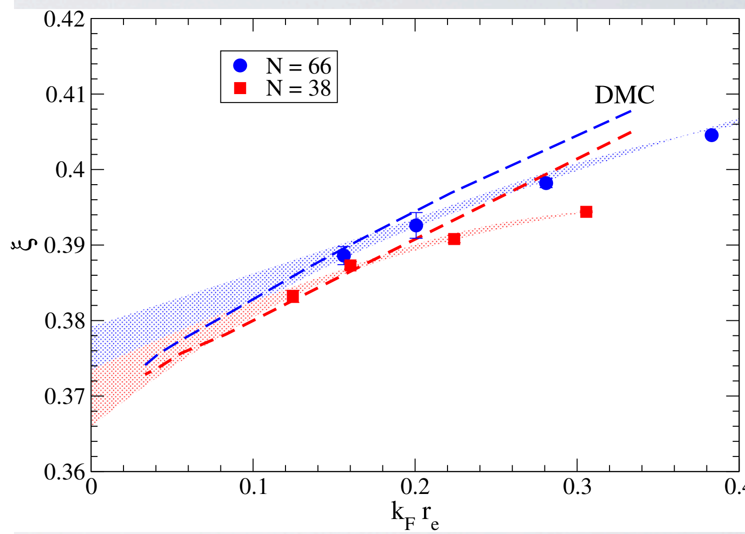
Time step governed by interaction: ~ .005 [depends upon algorithm]
Total time evolution governed by low-lying modes ~ 5-10 ω

Ground State: Homogeneous Cold Atoms

Equation of state: cold atoms



Effective Range



$E / E_{FG} = \xi + S\, k_F r_e + \ldots$

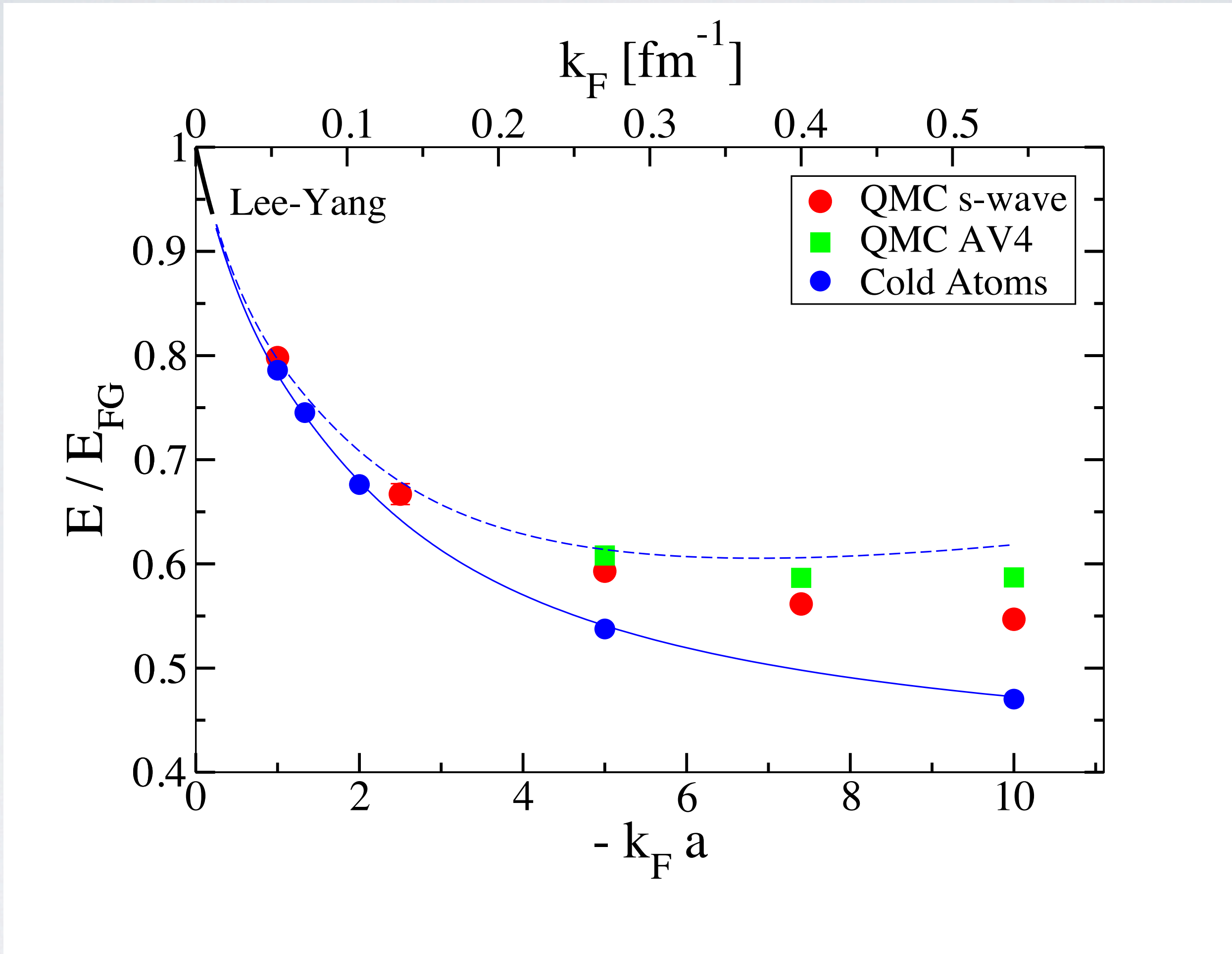
ξ = 0.372(5) QMC

0.375(5) Expt (MIT)

S = 0.12(3) QMC

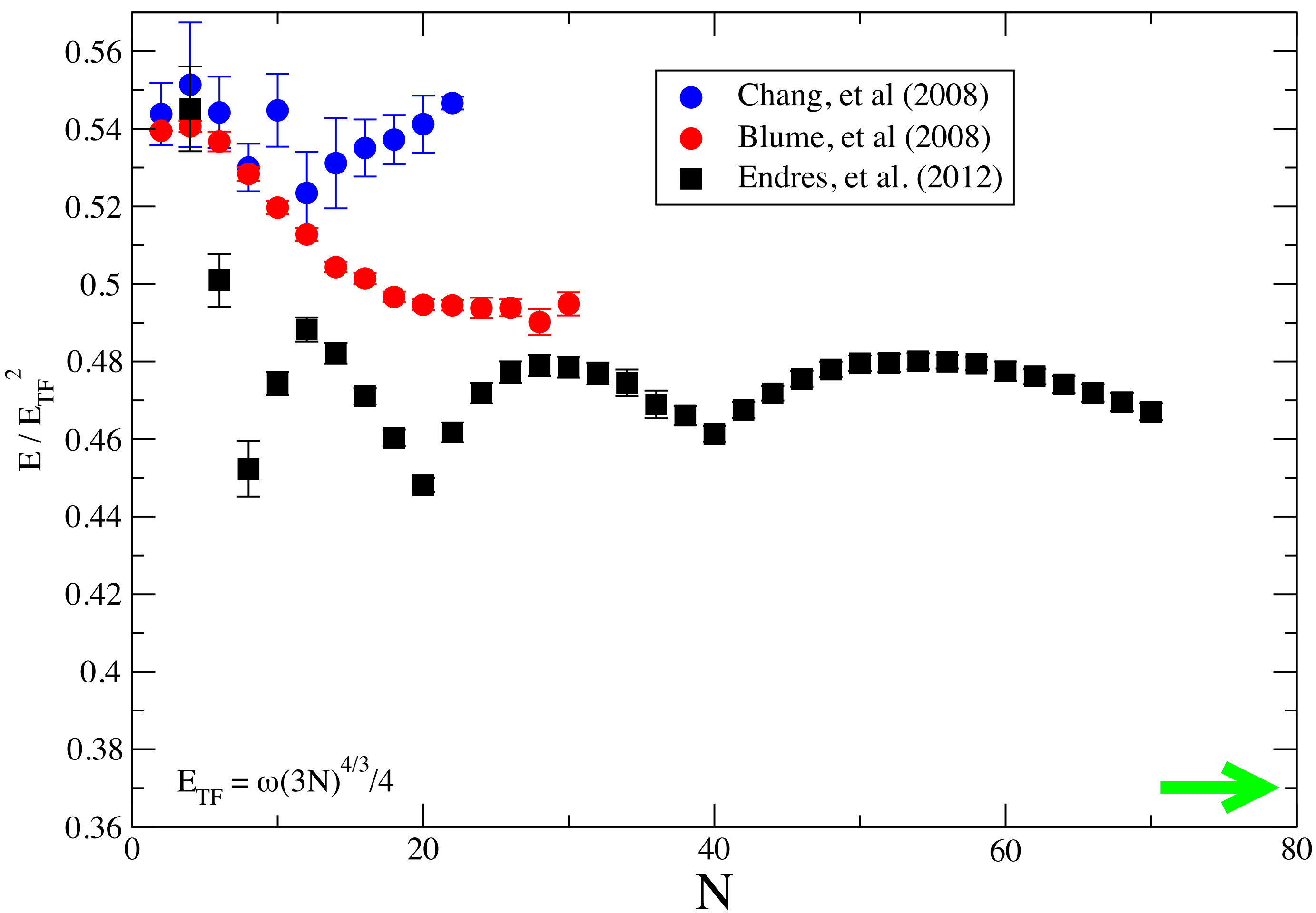
Carlson, Gandolfi, Schmidt, Gezerlis (PRA 2011)

Comparison of Cold Atoms to Neutrons : Bulk



Carlson, Gandolfi and Gezerlis; PTEP (2012)

Previous Results for Trapped Unitary Fermions



Convergence to Bulk?

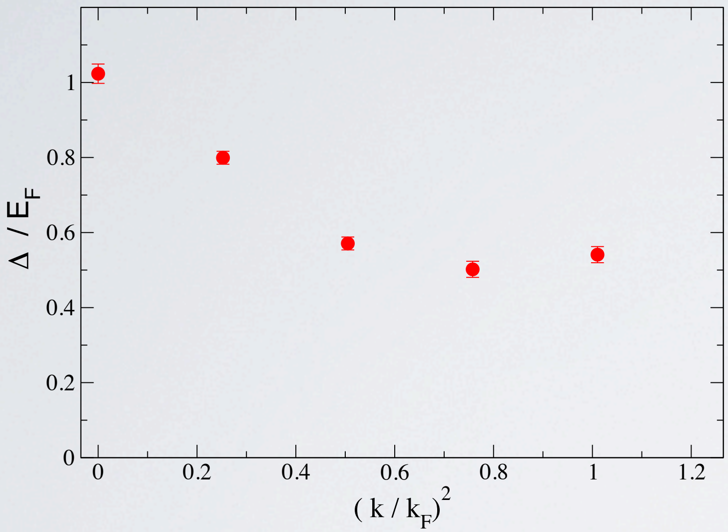
Closed Shells?

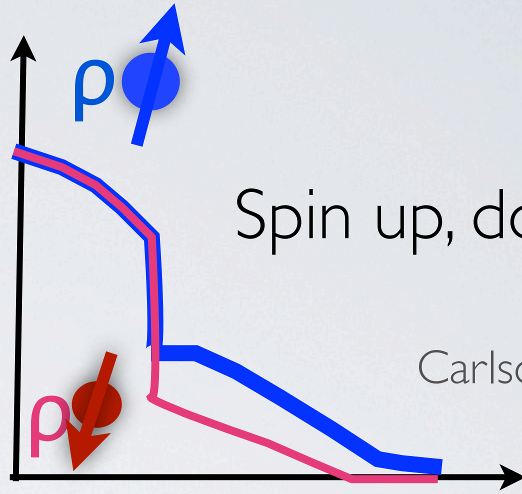
Role of Phonons?

Polarized Systems?

Large Pairing Gap: Low Energy Excitations are Bosons

Quasiparticle Dispersion





Spin up, down densities in a trap

Carlson and Reddy (2007)
MIT data

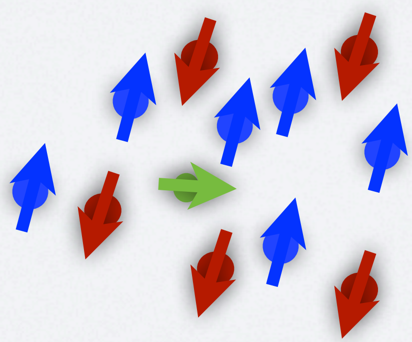
δ = 0.45 (05)

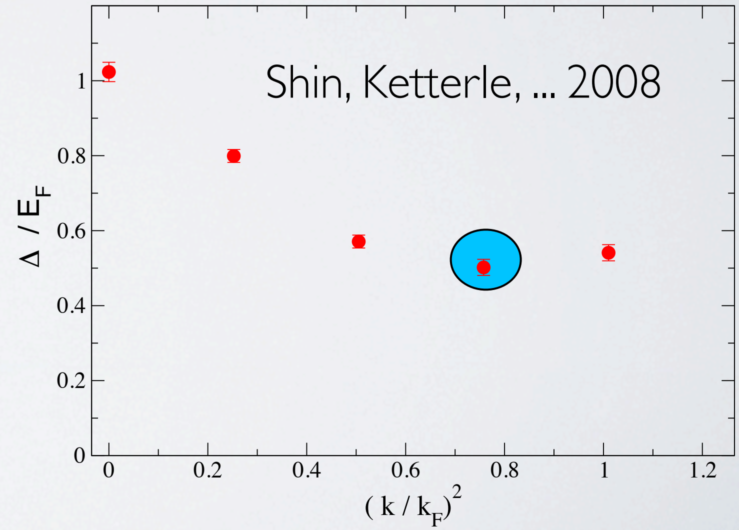
RF Spectroscopy

Carlson and Reddy (2005)

$$\Delta = \delta \frac{\hbar^2 k_F^2}{2m}$$

$$\delta = 0.50\ (03)$$





δ = 0.45 (05)

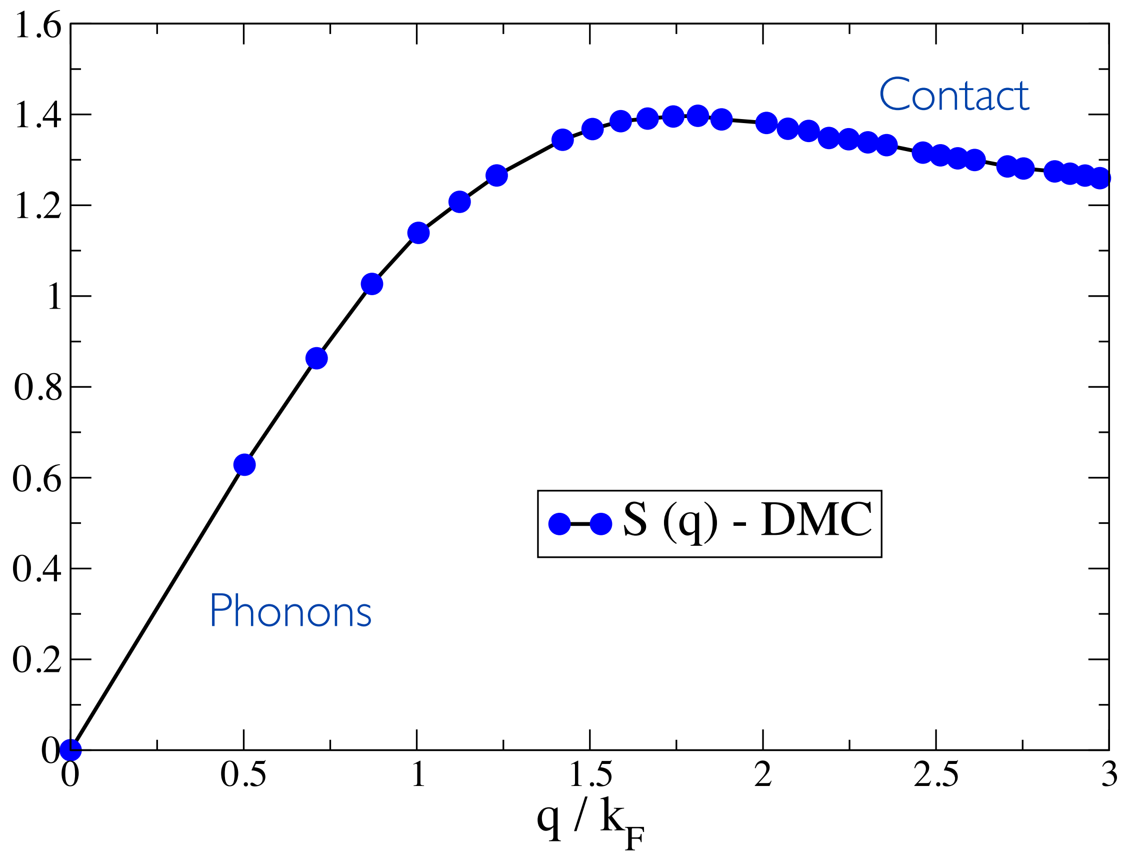
Long-wavelength excitations (phonons)

$$\mathcal{E} \;=\; \rho(r)\;\xi\;\frac{3}{5}\;\frac{\hbar^2}{2m}\;[3\pi^2\rho]^{2/3} + \frac{c}{m}\;\left(\frac{\nabla\rho\cdot\nabla\rho}{\rho}\right)$$

Son and Wingate (Ann Phys 2006)
Rupak and Schaefer (NPA 2009)

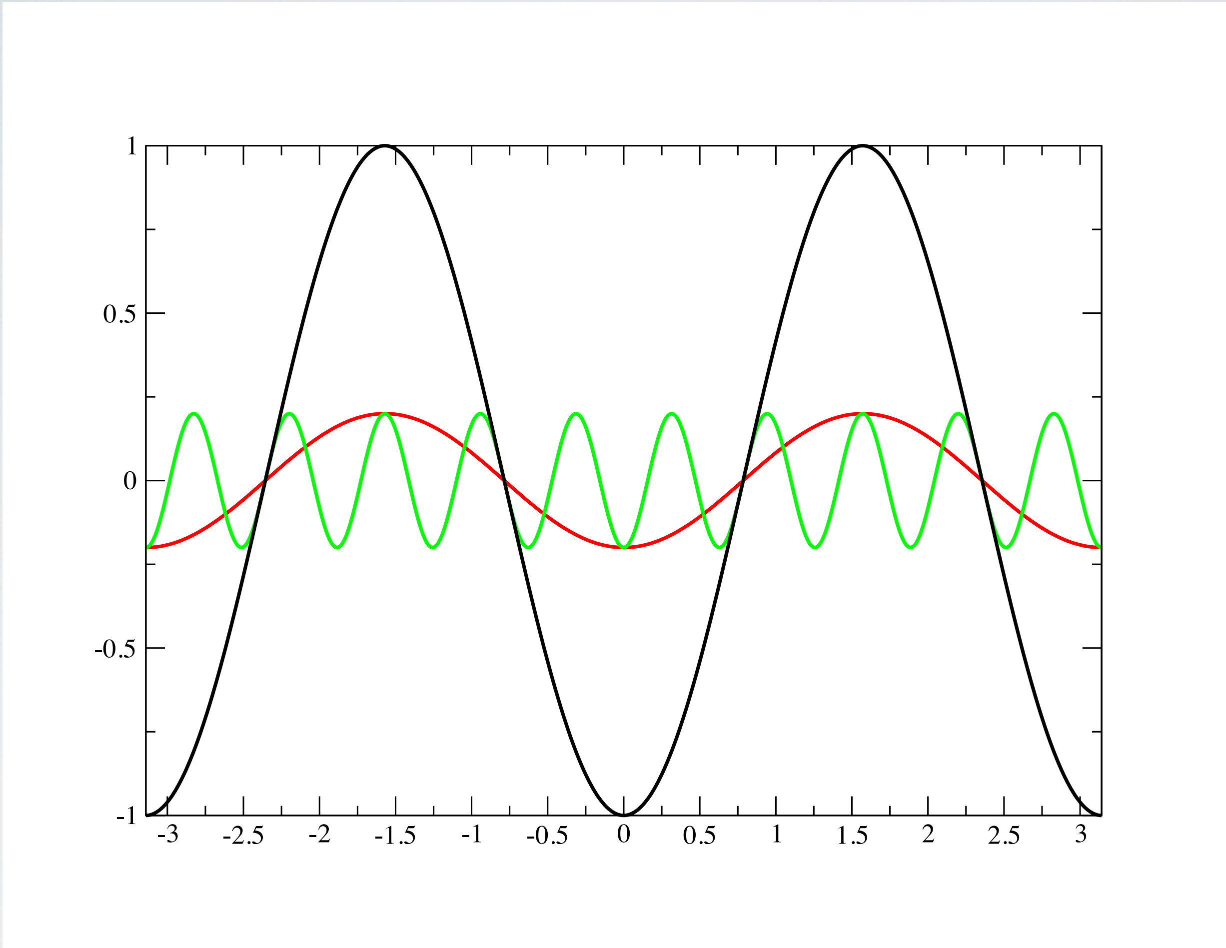
For free fermions
c = 1/36
Extended
Thomas-Fermi model

$$S(q) \;=\; \langle 0|\;\rho^\dagger(q)\;\rho(q)\;|0\rangle$$



Simple BCS treatment is very poor

Static Response: Energy Shift w/ external periodic potential



$$V_{ext} = -V_0 \cos\left[q \cdot r\right]$$

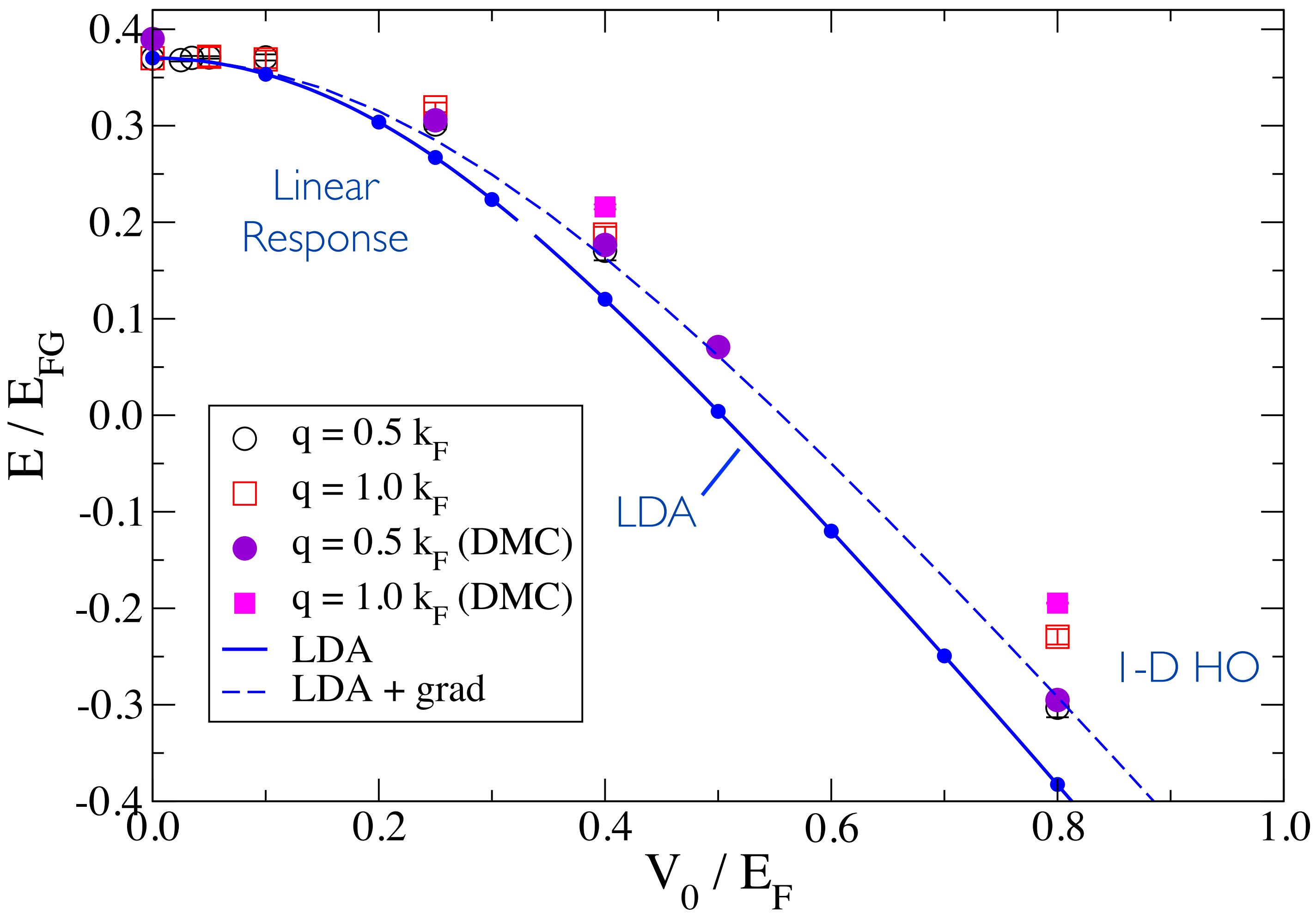
small V_0 : linear response

Large V_0 : 2D system

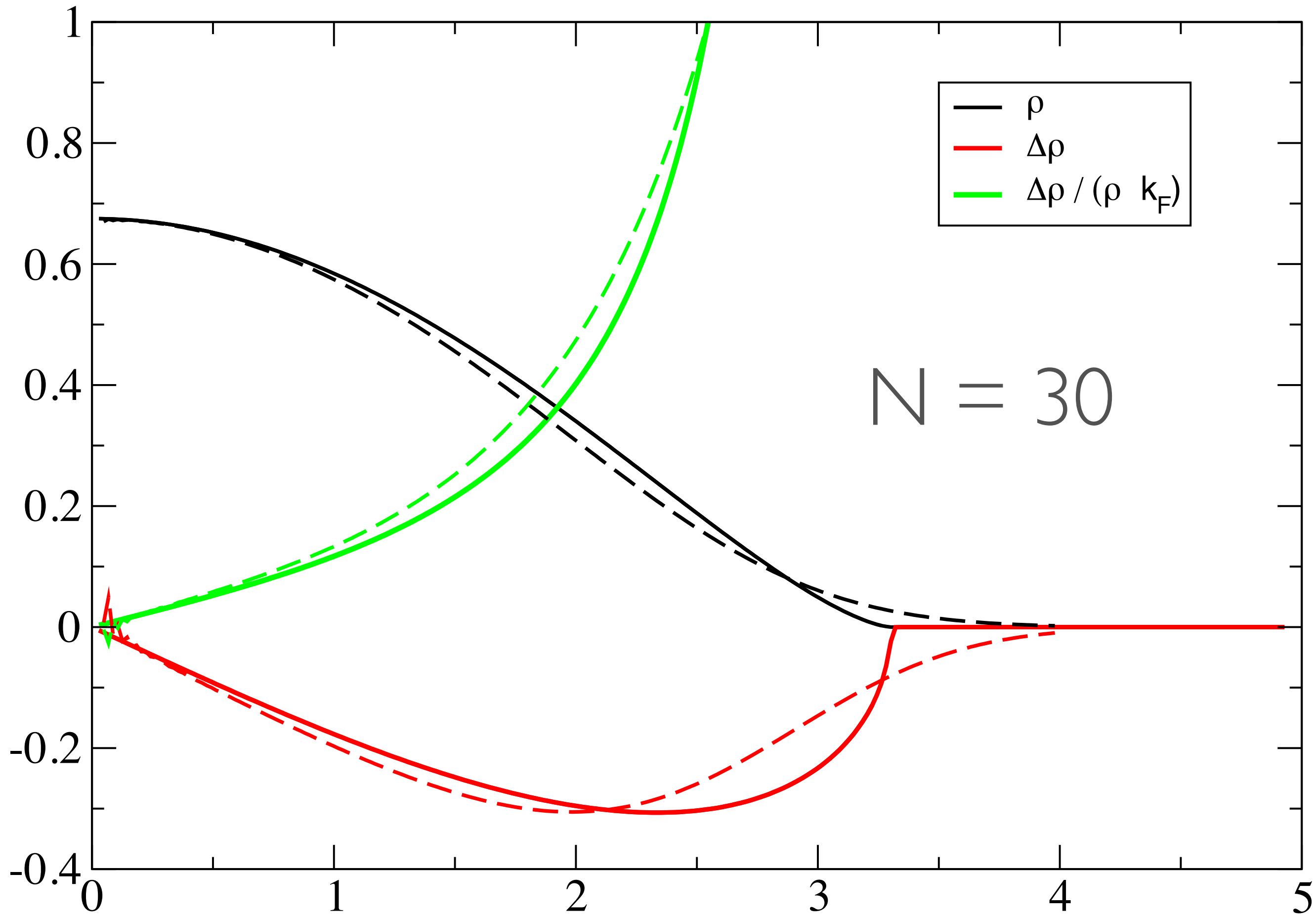
low q: phonons

high q: contact

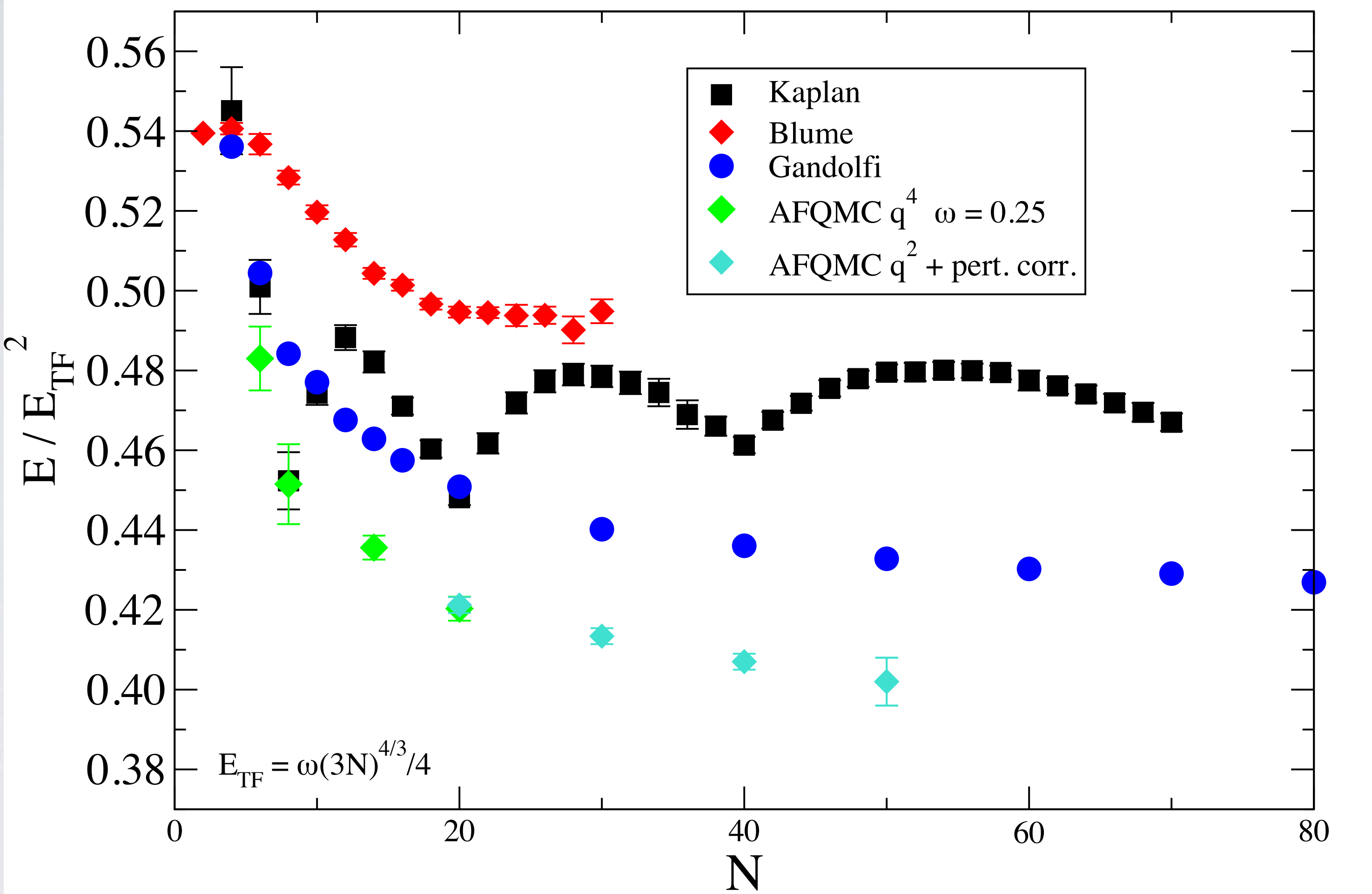
QMC Results for Static Response



lowest-order gradient insufficient for N=30 3D HO

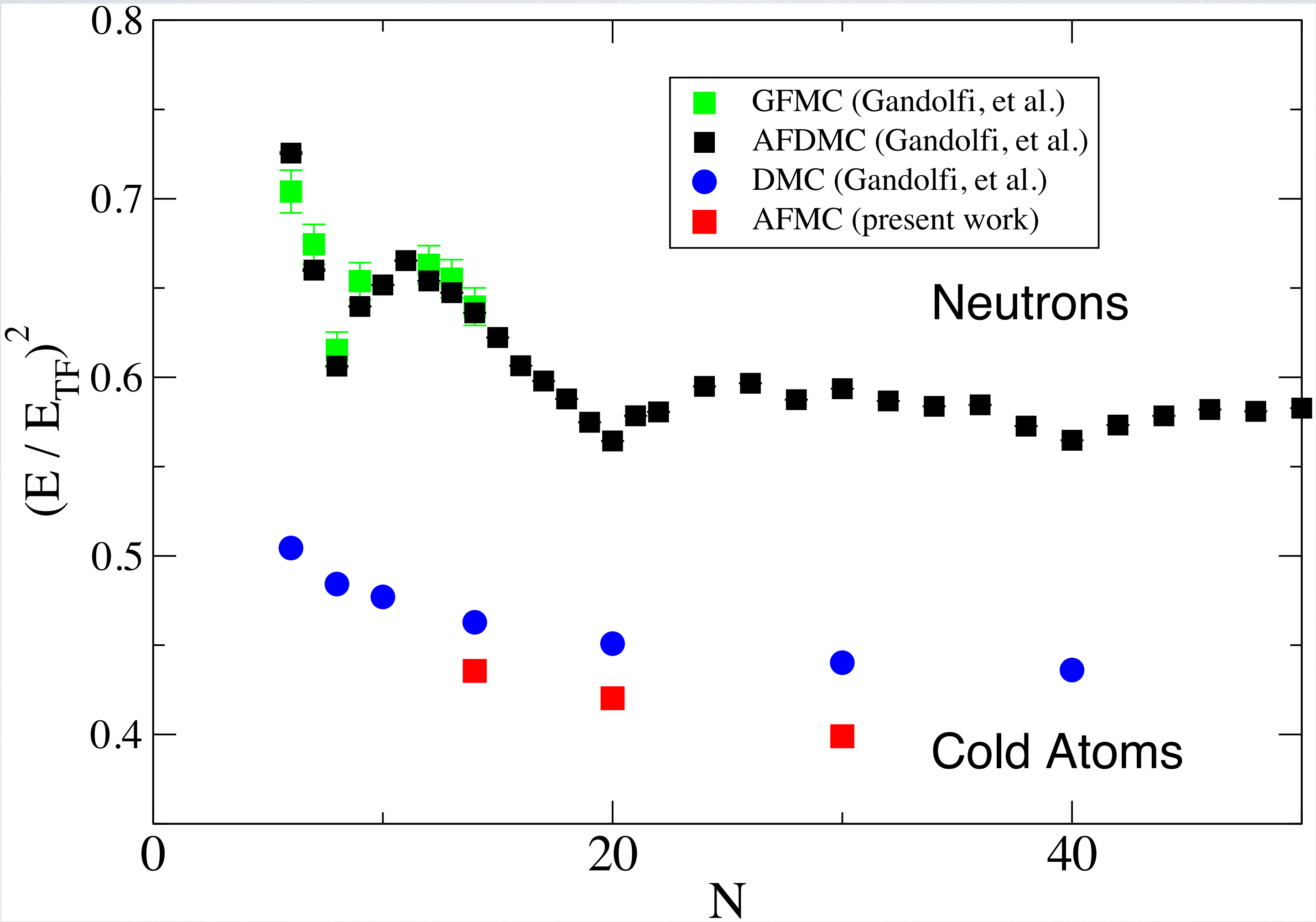


3D Harmonic Oscillator

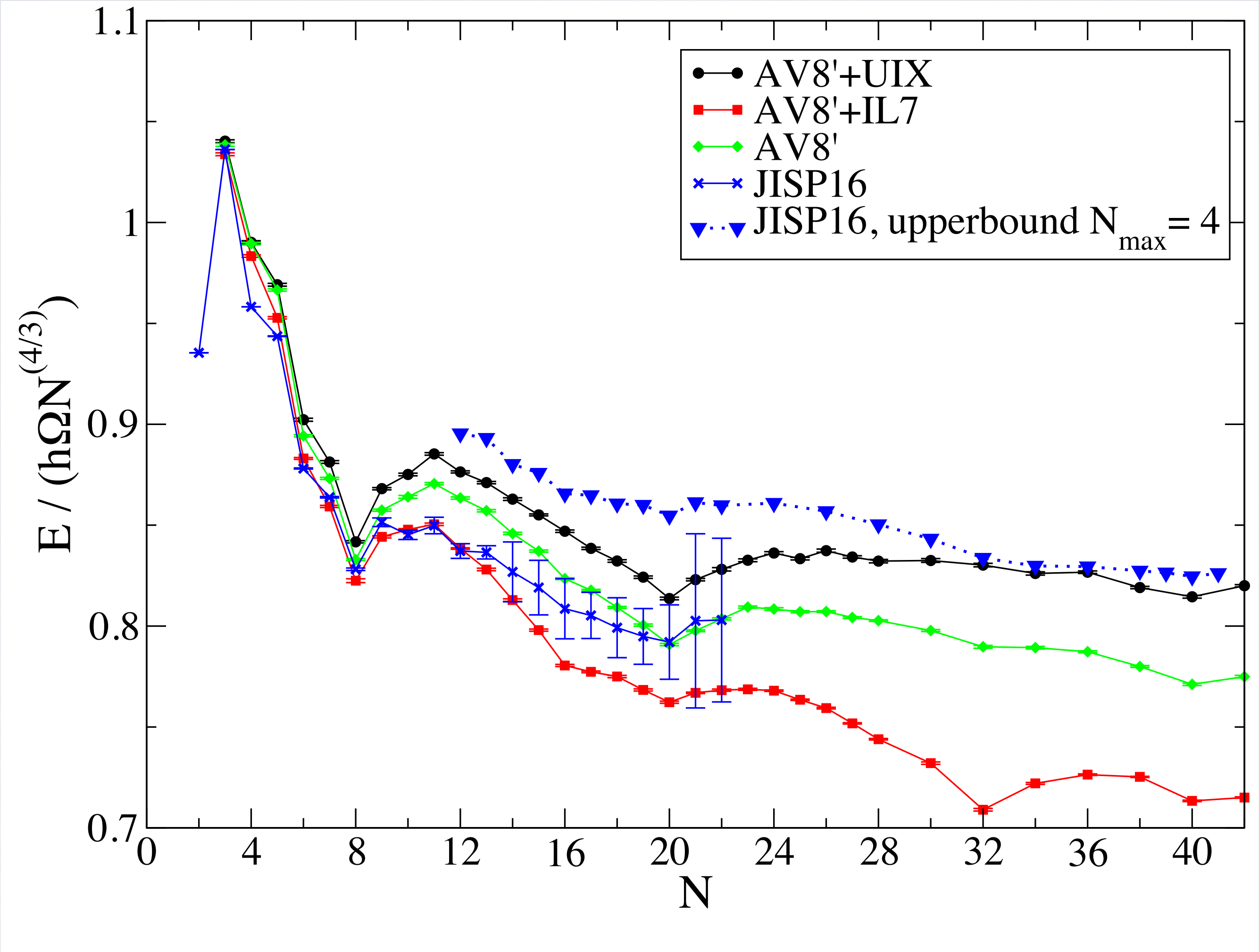


No shell effects - no fermionic quasiparticles
Extrapolates to bulk correctly

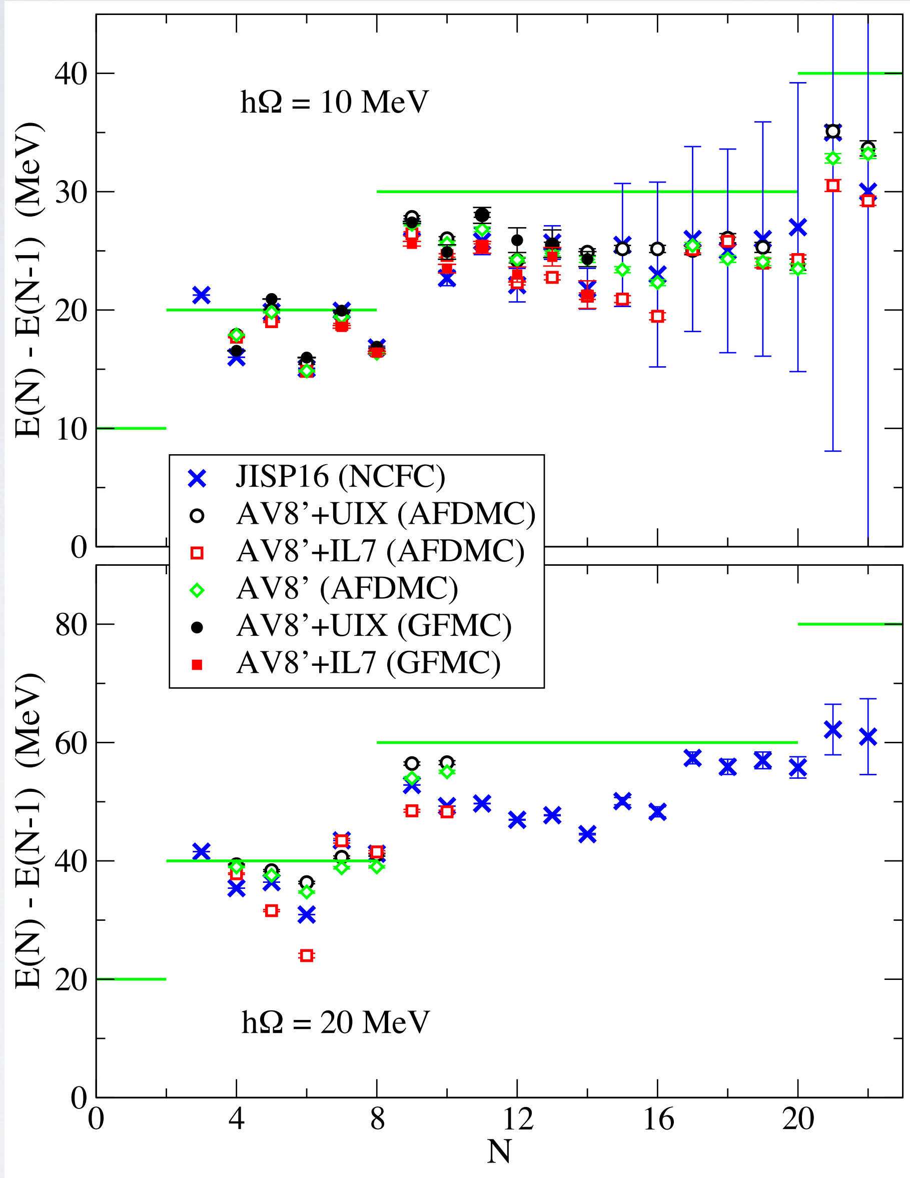
Comparison of Cold Atoms to Neutrons : Drops



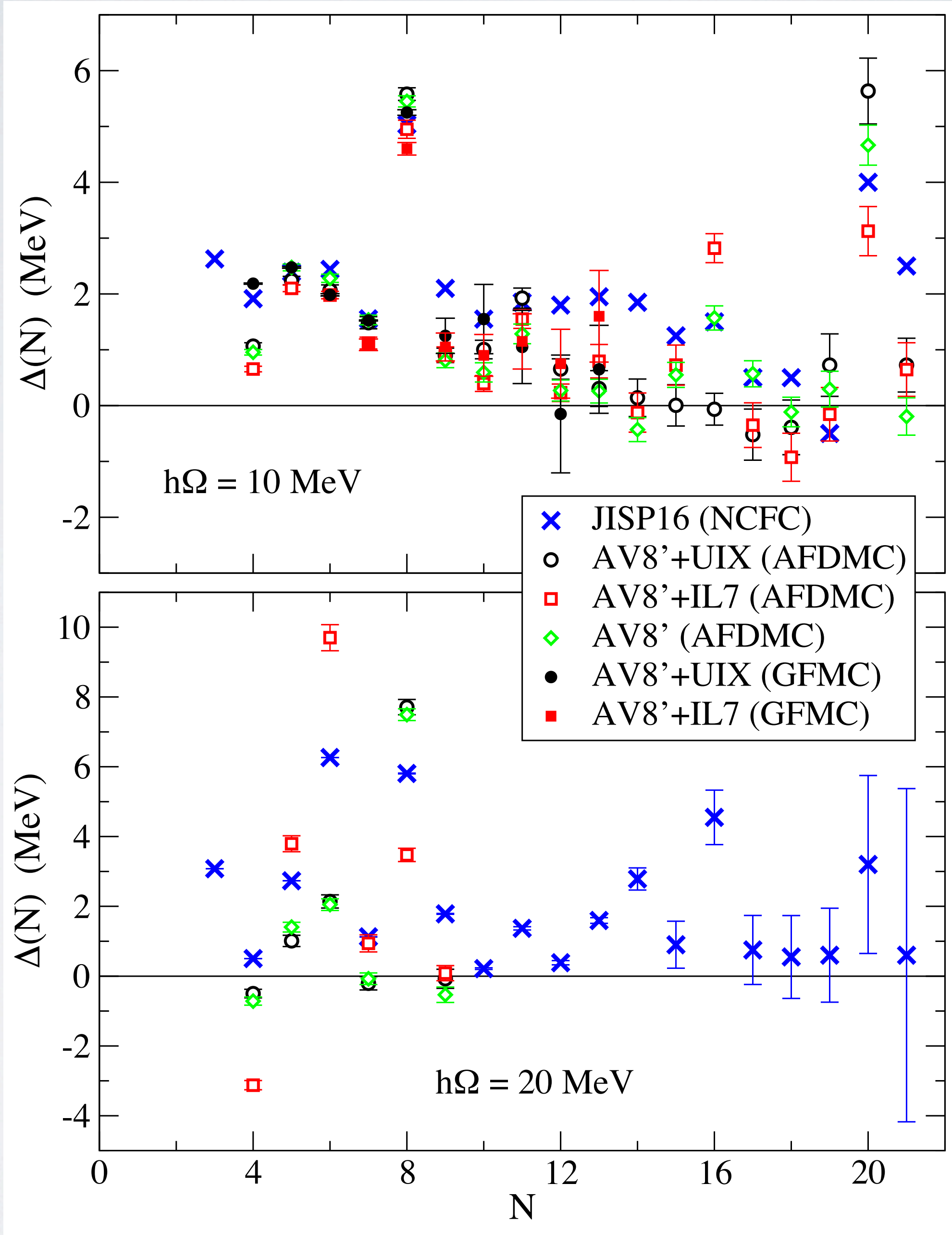
Neutron Drops: Different Interactions



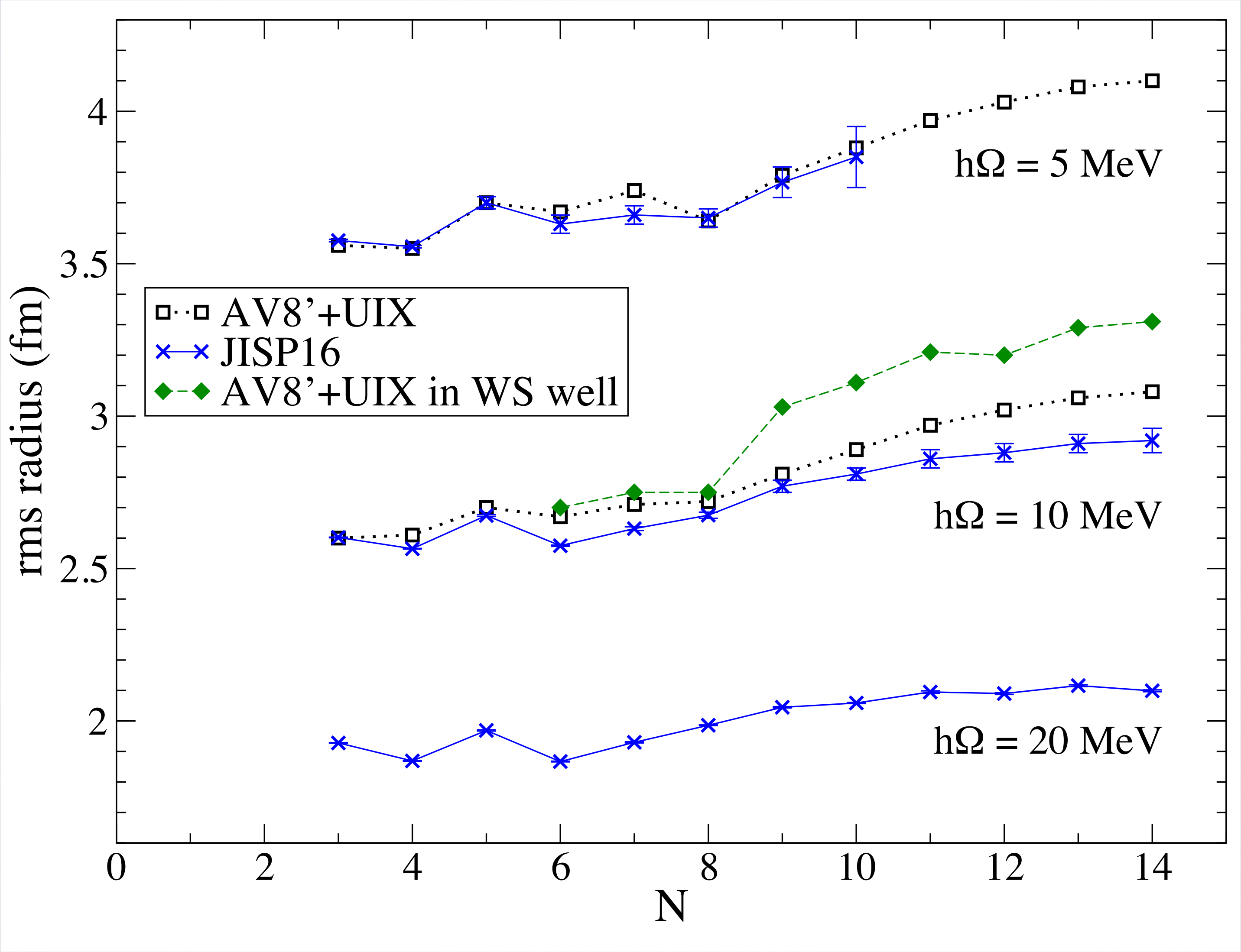
Shell Closures in Neutron Drops



Pairing Gaps in Neutron Drops



RMS radii of neutron drops



Future Efforts

Cold atoms: 2d drops -comparison w/ experiment
Spin and Density Response
Exotic states in 2d-3d

Neutron Drops, Matter:

Add few (10%) protons
Different interactions, methods
Smooth interpolation from
neutron rich nuclei, drops
Weak response of nuclei, matter