Generalized Parton Distributions for The Proton

Dipankar Chakrabarti
IIT Kanpur, India.

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Plan of the talk:

- Introduction to GPDs.
- GPDs in QDS/QCD
- GPDs in position space
- A model for proton GPDs
- comparison of the GPDs
- Conclusions

Introduction to GPDs

- Deep Inelastic Scatterings(DIS): => parton distribution functions(PDFs)
- PDFs : probablity of finding partons carrying x fraction of the hadron's total momentum. => help to understand the structure of hadrons.
- PDFs are functions of x only.

- Deeply Virtual Compton Scattering(DVCS)/ Meson productions involve Generalized Parton Distributions(GPDs).
- GPDs are functions of x, \zeta: longitudinal momentum fraction transferred(skewness), t: total momentum transferred square. Contain more info than PDFs.
- GPDs: off-forward matrix elements=>no probabilistic interpretation

- Ji sum rule relates the orbital angular momentum and the moments of GPDs.

[X. Ji, Phys. Rev. Lett. 78 (1997) 610.]

- M. Burkardt showed that GPDs in transverse impact parameter space have probabilistic interpretation and give info about spatial structure.

[M. Burkardt, Int. J. Mod. Phy. A18 (2003) 127]

- GPDs provide both angular mom. and spatial info.

$$\lim_{t\to 0}\int_{-1}^{1} dx\ x[H_f(x,\zeta,t)+E_f(x,\zeta,t)] = 2\langle J_f\rangle$$

[X. Ji, Phys. Rev. Lett. 78 (1997) 610.]

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- DVCS amplitude can be defined by light-cone time ordered product of currents:

$$M^{\mu\nu} = i \int d^4y \; e^{-iq\cdot y} \langle P' \mid T(J^\mu(y) J^\nu(0)) \mid P \rangle$$

- In deeply virtual region

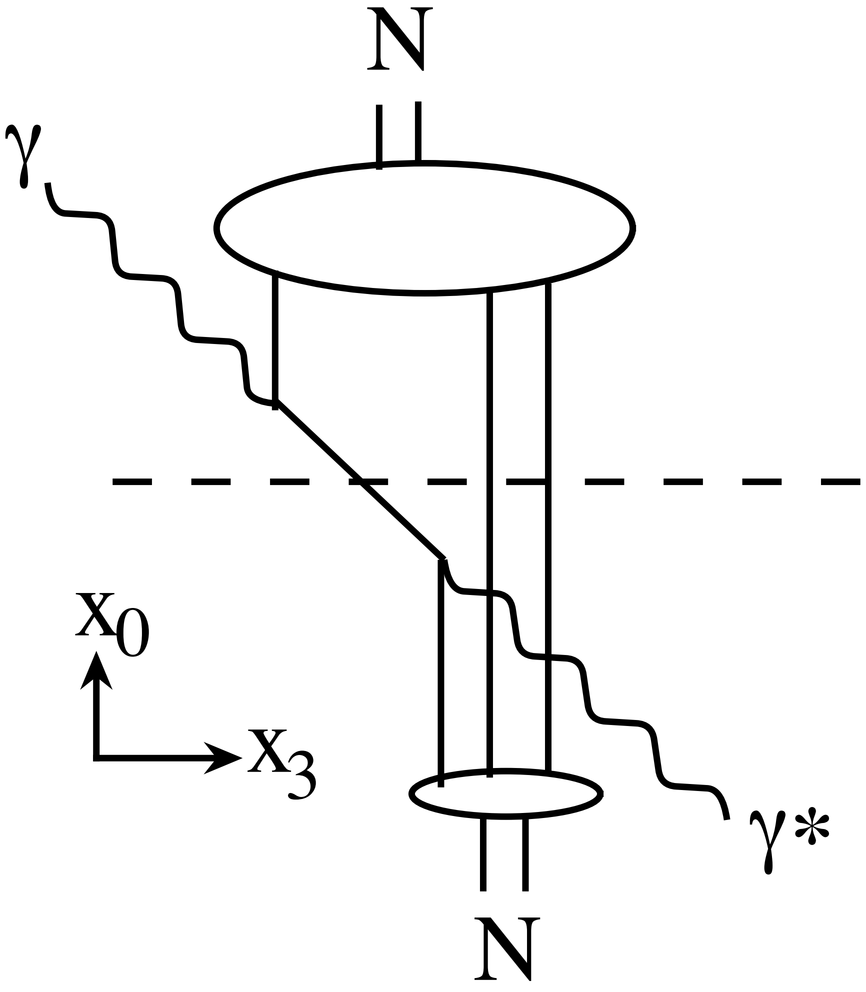
photon polarization

$$M^{IJ}(q_\perp, \Delta_\perp, \zeta) = \quad \epsilon^I_\mu \epsilon^{*J}_\nu M^{\mu\nu} = -e^2 \frac{1}{2\bar{P}^+} \int_{\zeta-1}^{1} dx$$
$$t^{IJ}(x,\zeta) \bar{U}(P') \Big[H(x,\zeta,t)\gamma^+ + E(x,\zeta,t) i\sigma^{+i}(-\Delta_i)/(2M) \Big] U(P)$$

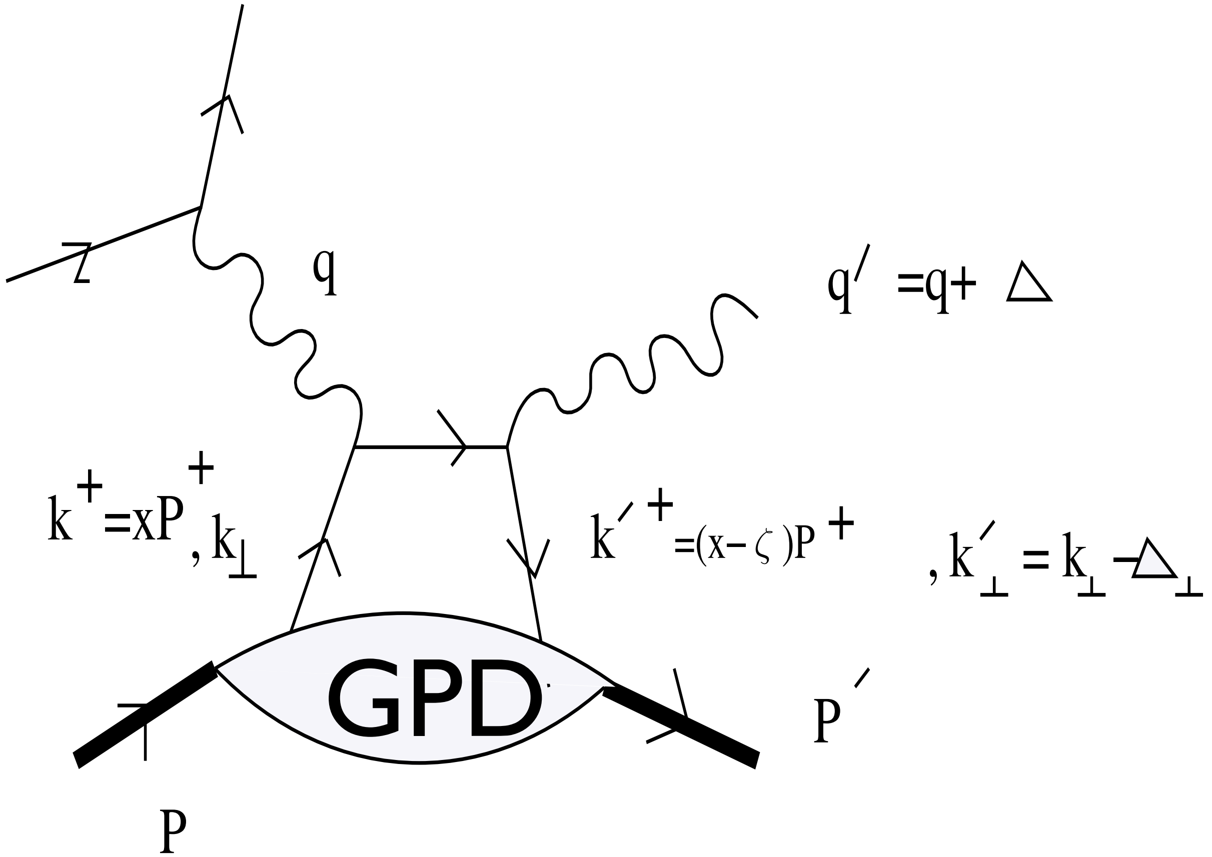
GPDS

DVCS in position space

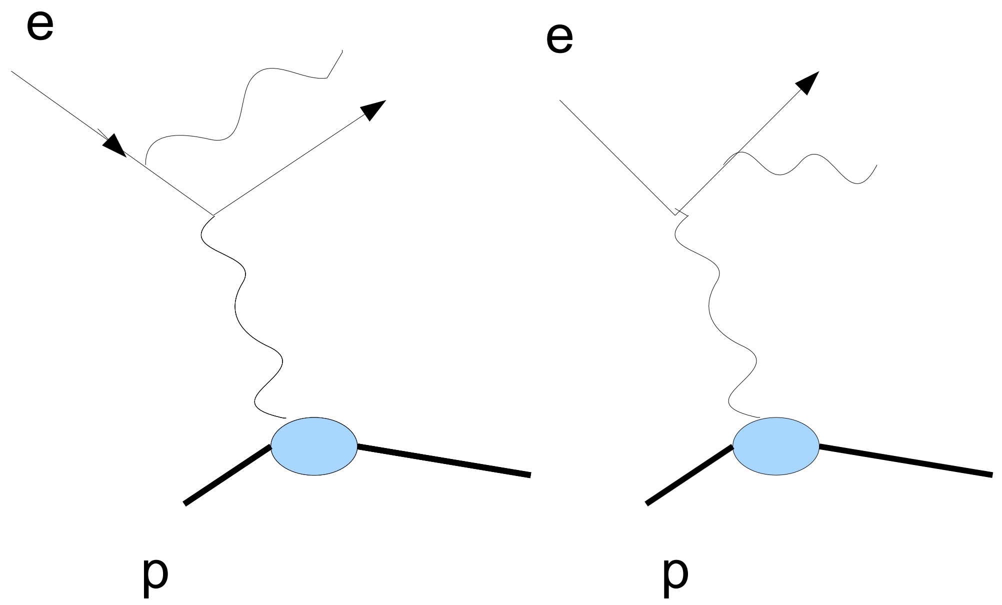
(the position of struck quark shifts in x^-)



Handbag diagram for DVCS



Bethe-Heitler process



Experimental studies of DVCS/GPDs

- experiments observe the overlap of DVCS and Bethe-Heitler processes. BH subtracted as background.
- HERA (H1, ZEUS, HARMES): DVCS experiments with wide range of energy.
- JLAB, Hall A and Hall B.
- COMPASS at CERN

Lightcone wavefunction formalism for DVCS/GPDs

- GPDs : nonperturbative quantities...lattice/ model dependent calculations.
- DVCS/GPDs can be evaluated from Fock state representation of incoming and outgoing proton.
- Lightcone wavefunctions encode all the bound state quark/gluon properties of hadrons.
- Lightcone boosts are kinematical=> frame independent amplitudes.

AdS/QCD

Brodsky and Teramond,
PLB 582,211(2004);
PRL 96, 201601(2006)

- Light front holographic mapping of string modes in AdS on the QCD excitations on the boundary.
- $AdS_{4+1} \rightarrow QCD_{3+1}$ but QCD is not a conformal theory
- Hard wall model: put a boundary on the AdS space where the wavefunctions vanish.
- In soft wall model, a confining potential is introduced in the AdS space.
  => LFWF for the hadrons

- AdS action in soft wall model:

$$S=\int d^4xdz\sqrt{g}\Big(\frac{i}{2}\bar{\Psi}e^M_A\Gamma^A D_M\Psi-\frac{i}{2}(D_M\bar{\Psi})e^M_A\Gamma^A\Psi-\mu\bar{\Psi}\Psi-V(z)\bar{\Psi}\Psi\Big).$$

- where $e^M_A=(z/R)\delta^M_A$ is the inverse veilbein.
- V(z) is the confining potential(linear confinement)
- In d=4 dimensions: $\Gamma_A=\{\gamma_\mu,-i\gamma_5\}.$

- With $z \to \zeta$, and $U(\zeta) = (R/\zeta)V(\zeta) = \kappa^2\zeta$, the light front wave equations can be written as

$$\Big(-\frac{d^2}{d\zeta^2} - \frac{1-4\nu^2}{4\zeta^2} + \kappa^4\zeta^2 + 2(\nu+1)\kappa^2\Big)\psi_+(\zeta) = \mathcal{M}^2\psi_+(\zeta)$$
$$\Big(-\frac{d^2}{d\zeta^2} - \frac{1-4(\nu+1)^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\nu\kappa^2\Big)\psi_-(\zeta) = \mathcal{M}^2\psi_-(\zeta)$$

- Solution:

$$\psi_+(\zeta) \sim \zeta^{\nu+1/2} e^{-\kappa^2\zeta^2/2} L_n^{\nu}(\kappa^2\zeta^2)$$
$$\psi_-(\zeta) \sim \zeta^{\nu+3/2} e^{-\kappa^2\zeta^2/2} L_n^{\nu+1}(\kappa^2\zeta^2)$$

- with $\mathcal{M}^2 = 4\kappa^2(n+\nu+1).$

GPDs from AdS/QCD

- GPDs in AdS/QCD can be derived from nucleon form factors:

- The Dirac form factors :

Brodsky, Teramond
arXiv:1203.4125

$$F_1^p(Q^2) = R^4 \int \frac{dz}{z^4} V(Q^2, z) \psi_+^2(z)$$
$$F_1^n(Q^2) = -\frac{1}{3} R^4 \int \frac{dz}{z^4} V(q^2, z)(\psi_+^2(z) - \psi_-^2(z))$$

with $F_1^{p/n}(0) = e_{p/n}$

- Pauli form factors: $F_2^{p/n}(Q^2) \sim \int \frac{dz}{z^3} \psi_+(z) V(Q^2, z) \psi_-(z),$

with $F_2^{p/n}(0) = \chi_{p/n}$ =anomalous mag moment

- The bulk to boundary propagator for the soft wall model can be written in a simple integral form:

$$V(Q^2, z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2} x^{Q^2/(4\kappa^2)} e^{-\kappa^2 z^2 x/(1-x)}$$

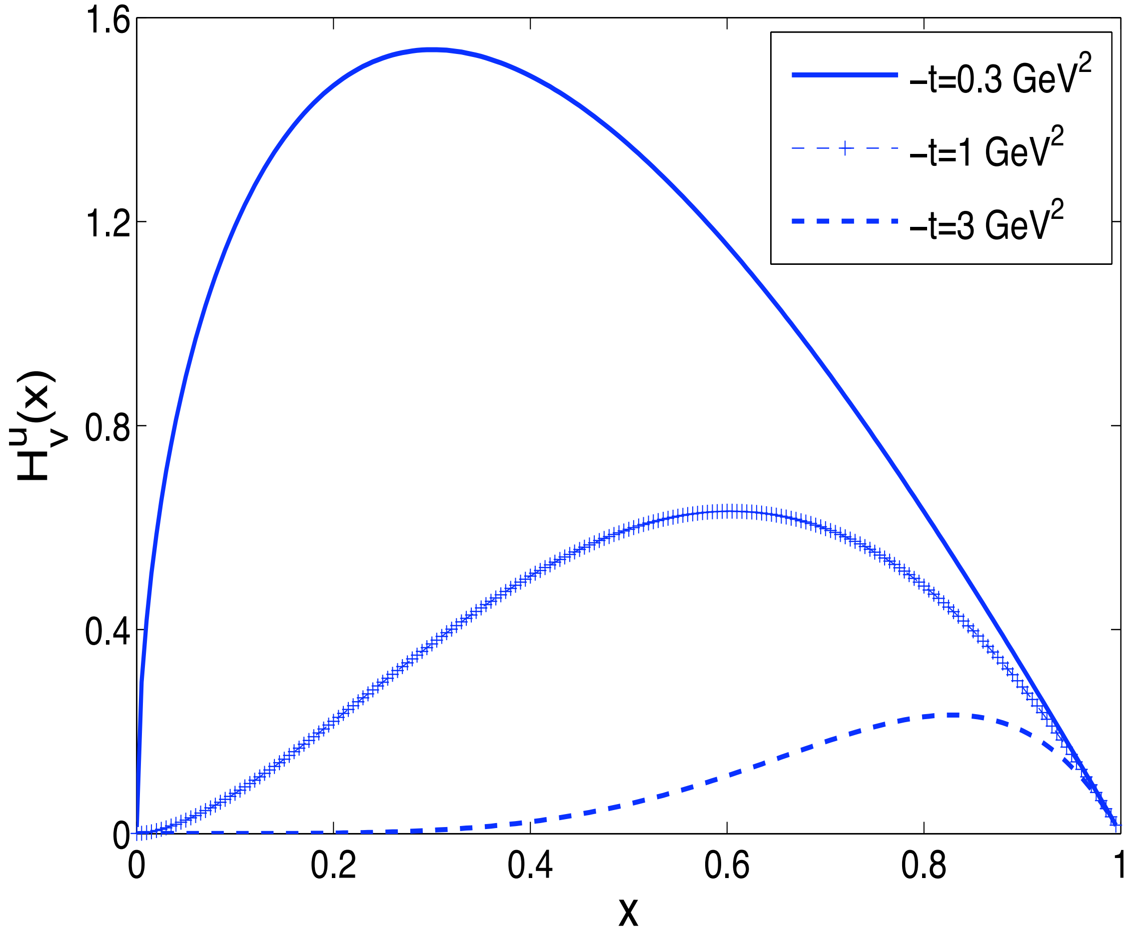
- On the other hand the form factors are related with the valence GPDs by

Diehl, Eur. Phy. J C39,1(2005)

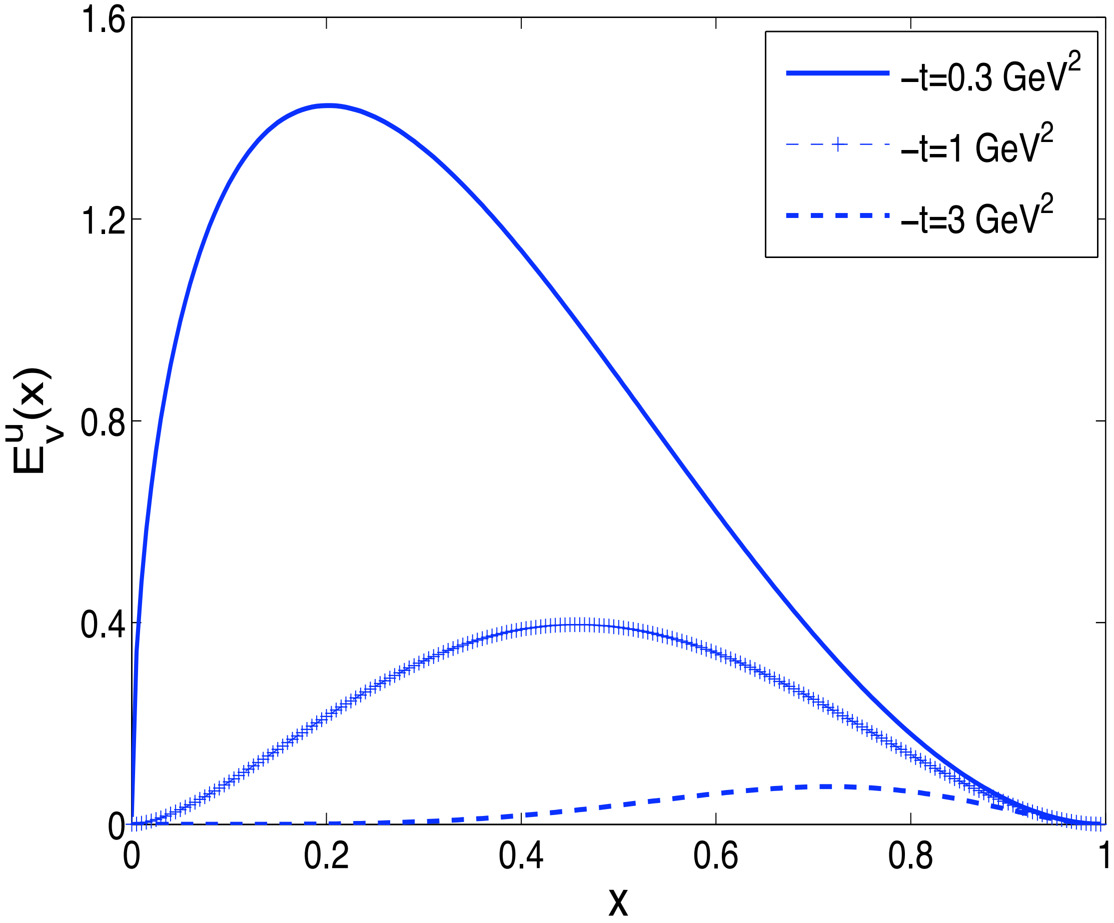
$$\begin{aligned}
F_1^p(t) &= \int_0^1 dx(\frac{2}{3}H_v^u(x,t) - \frac{1}{3}H_v^d(x,t)), \\
F_1^n(t) &= \int_0^1 dx(\frac{2}{3}H_v^d(x,t) - \frac{1}{3}H_v^u(x,t)), \\
F_2^p(t) &= \int_0^1 dx(\frac{2}{3}E_v^u(x,t) - \frac{1}{3}E_v^d(x,t)), \\
F_2^n(t) &= \int_0^1 dx(\frac{2}{3}E_v^d(x,t) - \frac{1}{3}E_v^u(x,t)).
\end{aligned}$$

GPDs for u quark

GPD H(x,t)

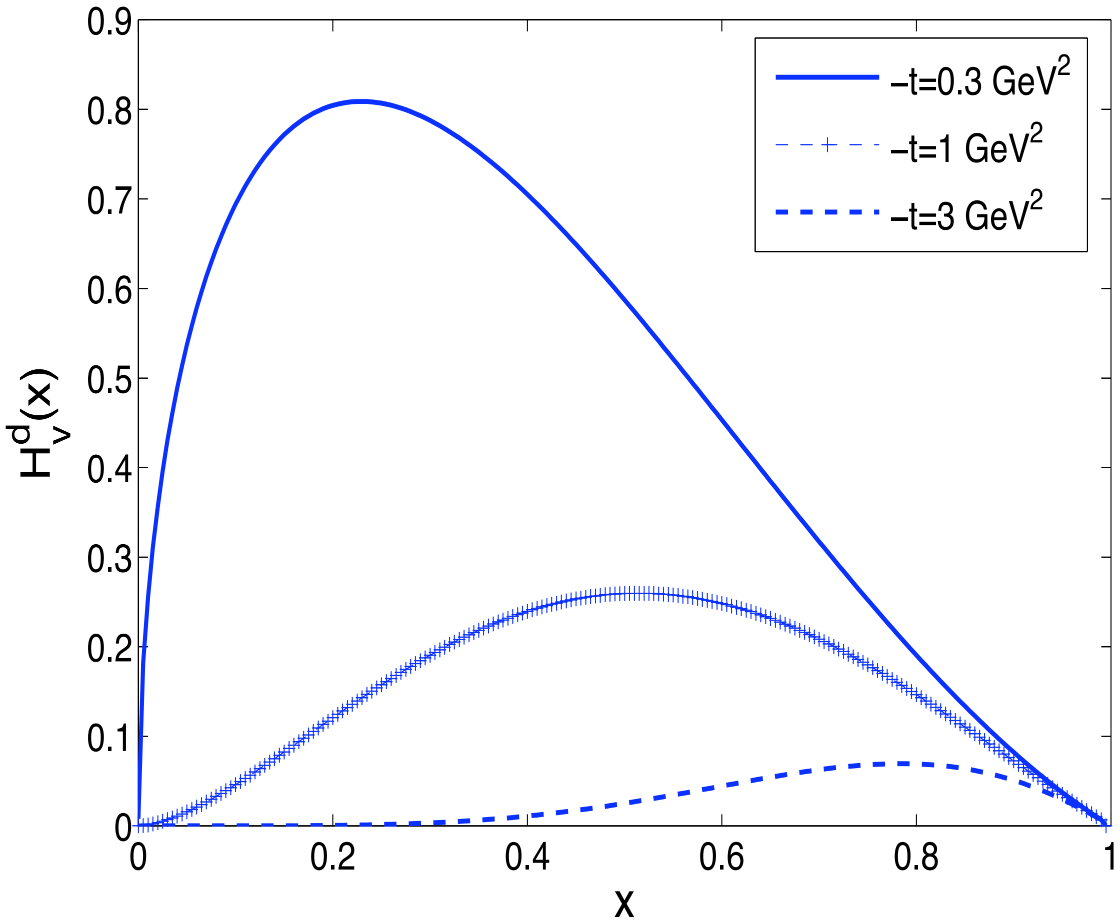


GPD E(x,t)

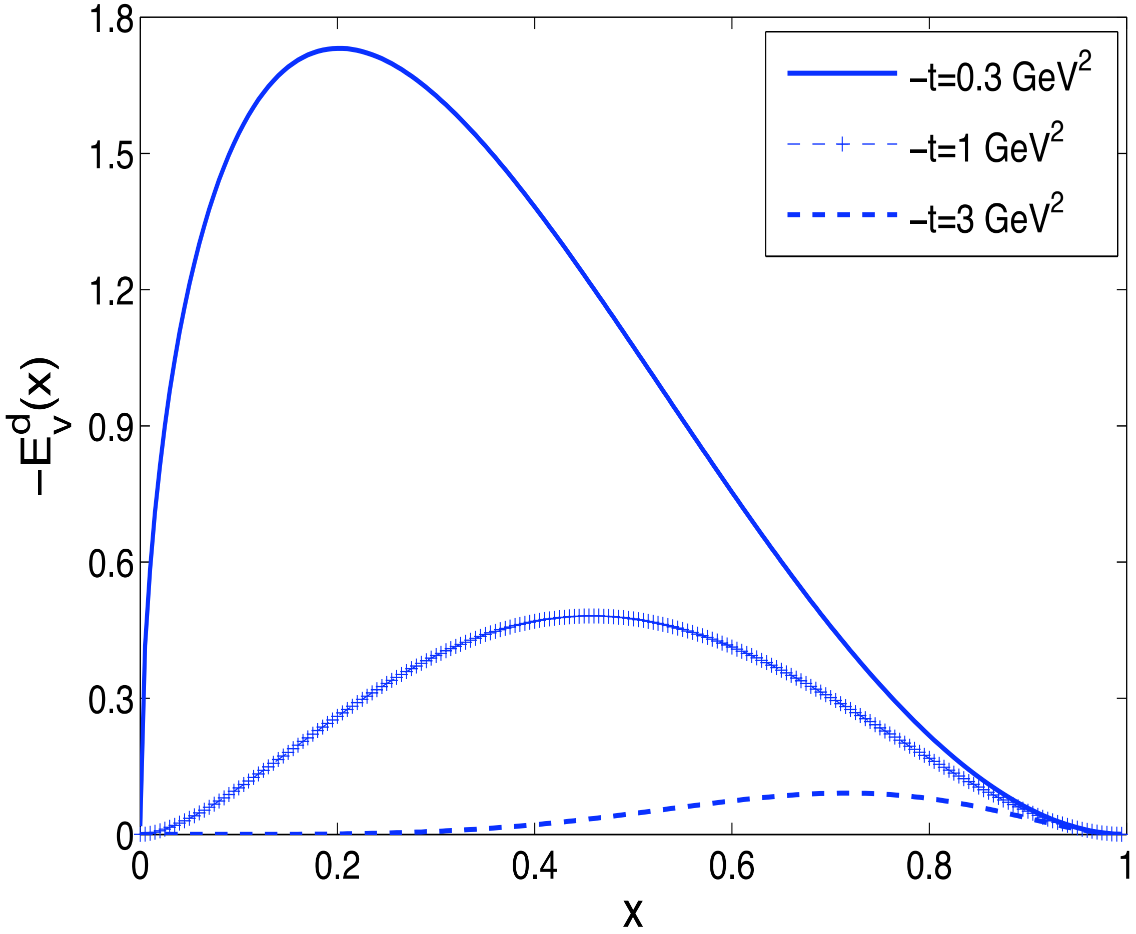


GPDs for d-quark

GPD H(x,t)



GPD E(x,t)

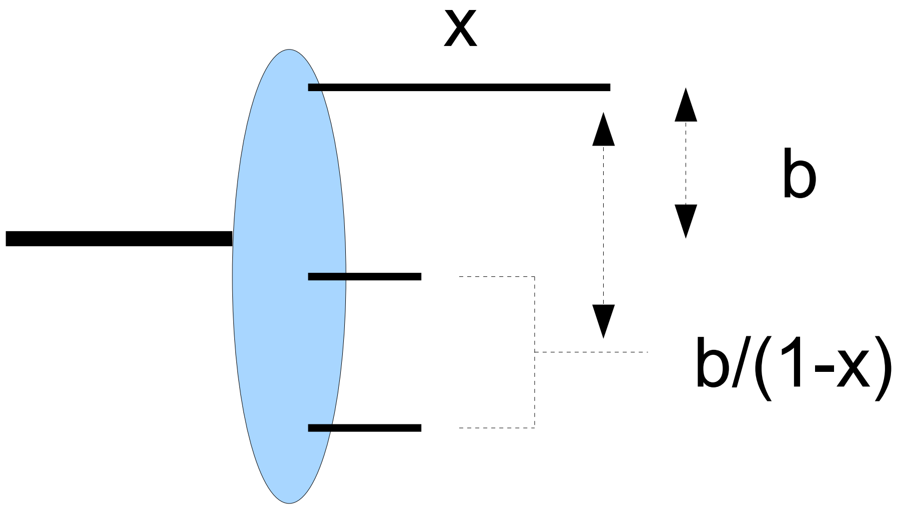


GPDs in impact parameter space

- Fourier transform w r t the transverse momentum transfer => GPDs in transverse position space.

$$q(x,b) = \frac{1}{4\pi^2}\int d^2\Delta e^{-i\Delta^\perp \cdot b^\perp} H(x,t)$$
$$e(x,b) = \frac{1}{4\pi^2}\int d^2\Delta e^{-i\Delta^\perp \cdot b^\perp} E(x,t).$$

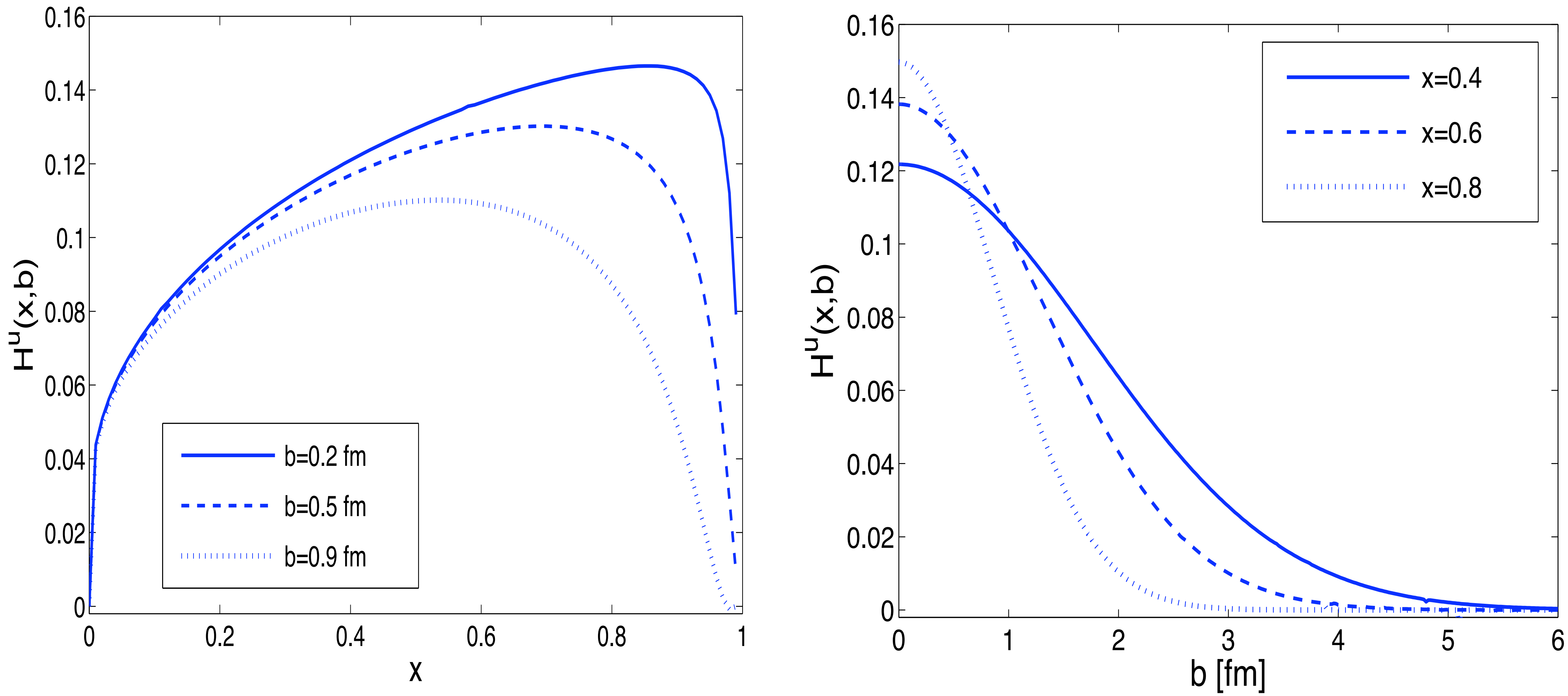
- Physically: probability of finding the quark of long. mom. fraction x at a transverse position $b_\perp$
- The relative distance $\frac{b}{1-x}$ betn the quark and the spectator gives the size of the system.



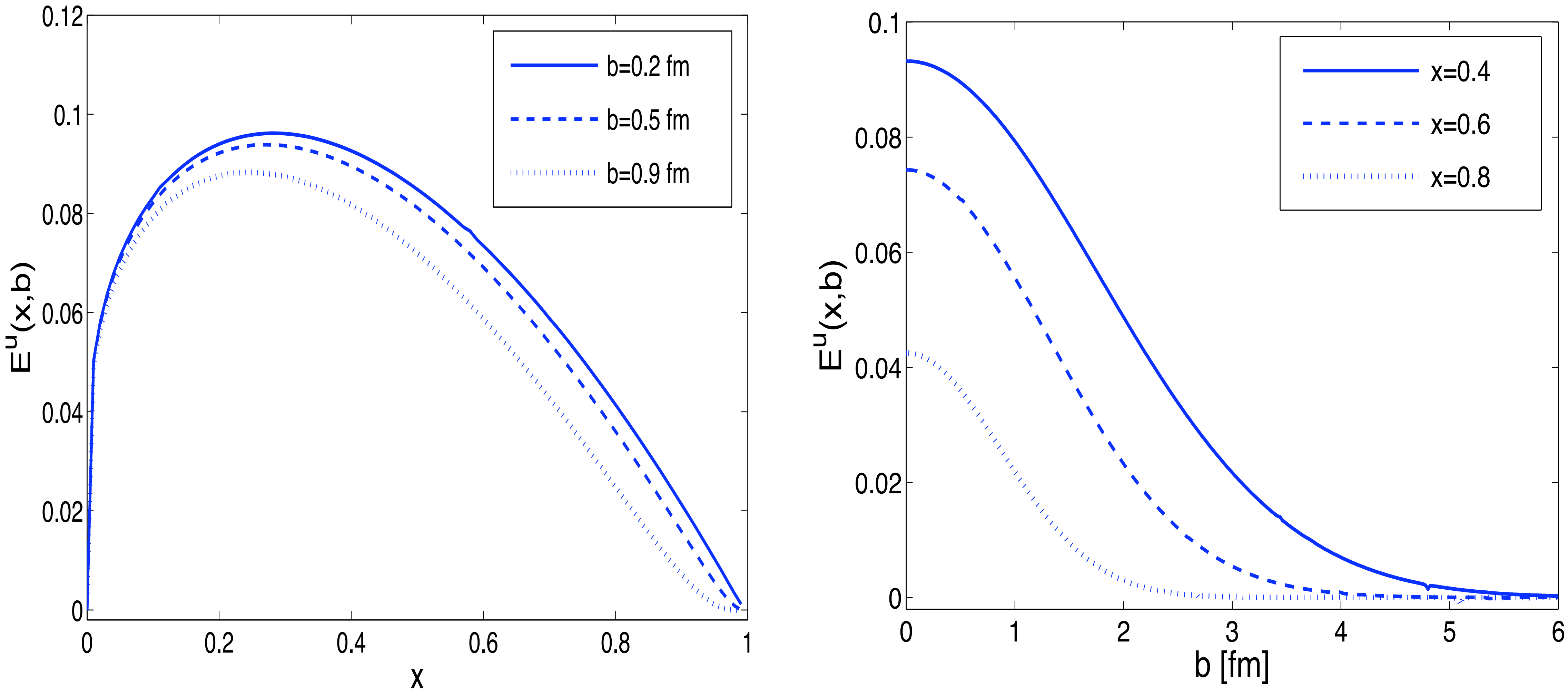
[adopted from: M. Diehl et al, Eur. Phys. J. C39(2005) 1.]

Impact parameter dependent GPDs

H(x,b) for u quark



GPD E(x,b) for u-quark



model for proton GPDs

- We consider here the phenomenological model for proton GPDs proposed by[S. Ahmad et al., Phys. rev. D 75, 094003(2007); Eur, Phys, J. C63, 407(2009)]
- model: spectator model+Regge term(for low x behavior)

$$H^I(x,t) \; = \; G^{\lambda^I}_{M^I_x}(x,t)\, x^{-\alpha^I - \beta^I_1 (1-x)^{p^I_1} t}$$

$$E^I(x,t) \; = \; \kappa\, G^{\lambda^I}_{M^I_x}(x,t)\, x^{-\alpha^I - \beta^I_2 (1-x)^{p^I_2} t}$$

- parameters are fixed by fitting proton electric and magnetic form factors.
- additional constrains: for zero skewness:

$$\int_0^1 dx E_q(x, t=0) = \kappa^q$$

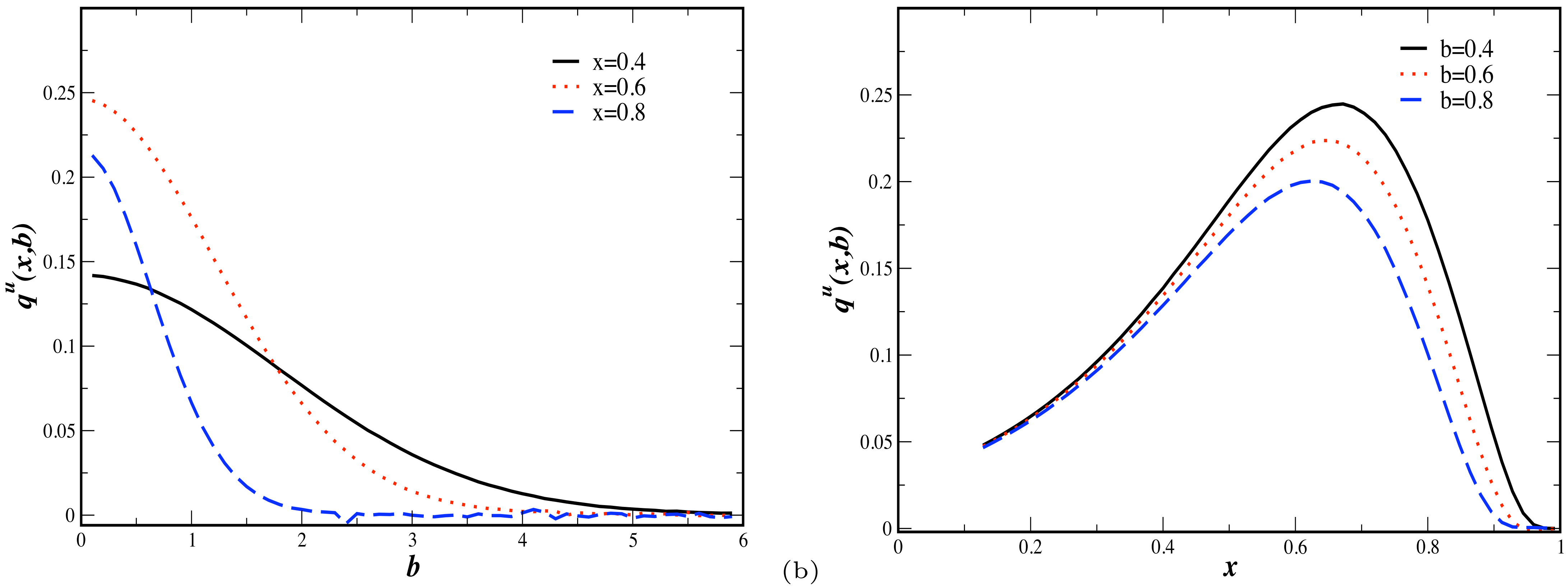
κ^q = anomalous magnetic moment

- For nonzero skewness, GPDs have to satisfy the polynomiality condition.
- Two different set of parameters satisfy the fits (Set-I, Set-II)

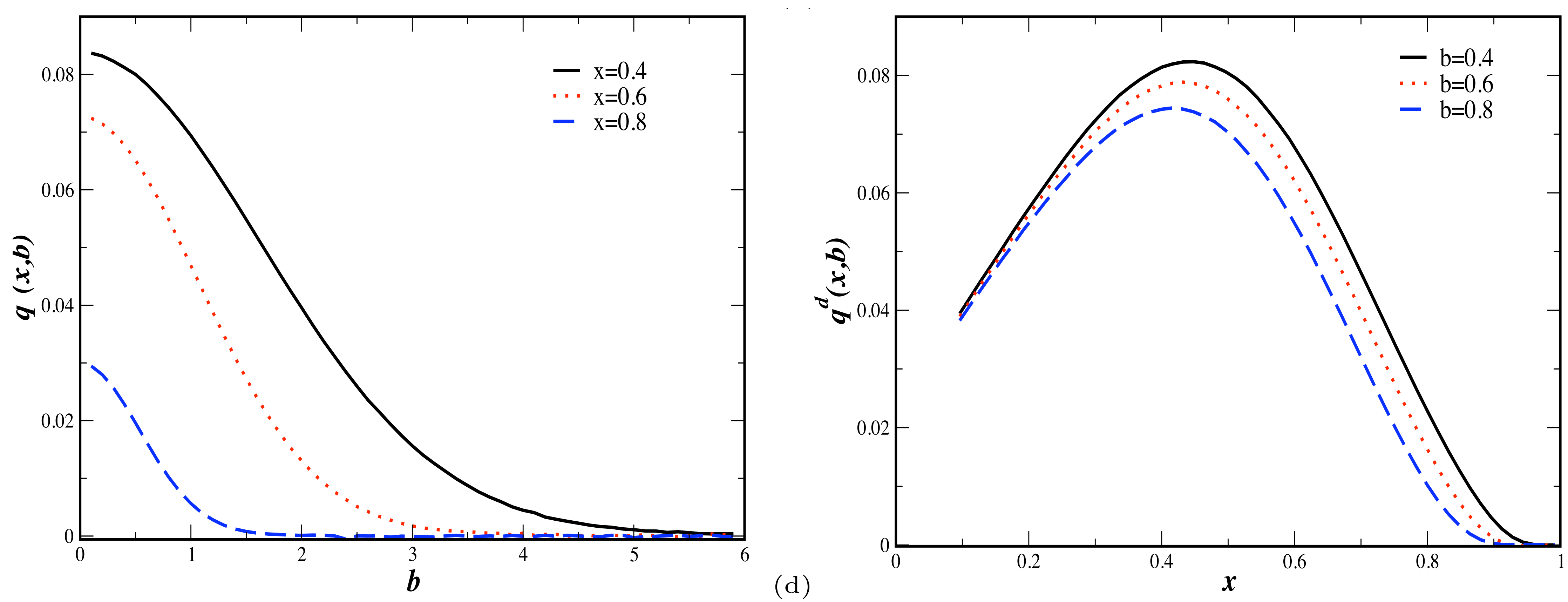
Zero skewness

[DC, R. Manohar, A. Mukherjee, Phy. lett. B682,428(2010)]

H(x,b) for u-quark

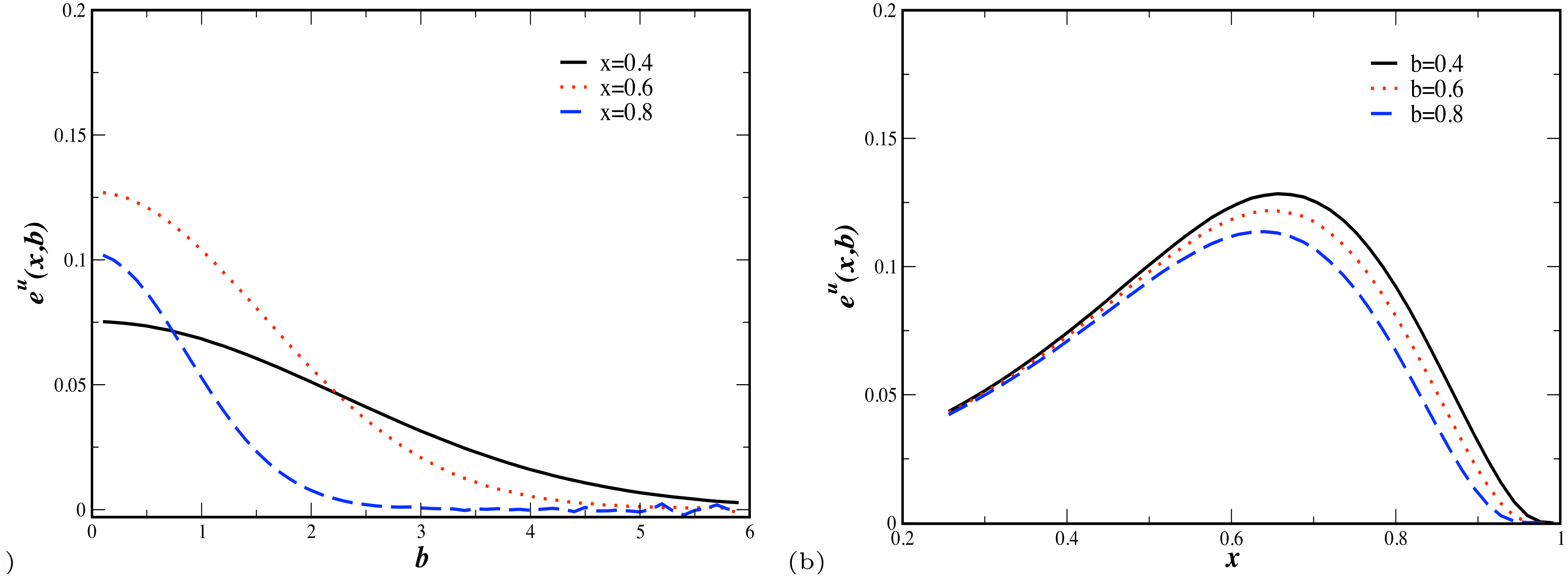


H(x,b) for d-quark

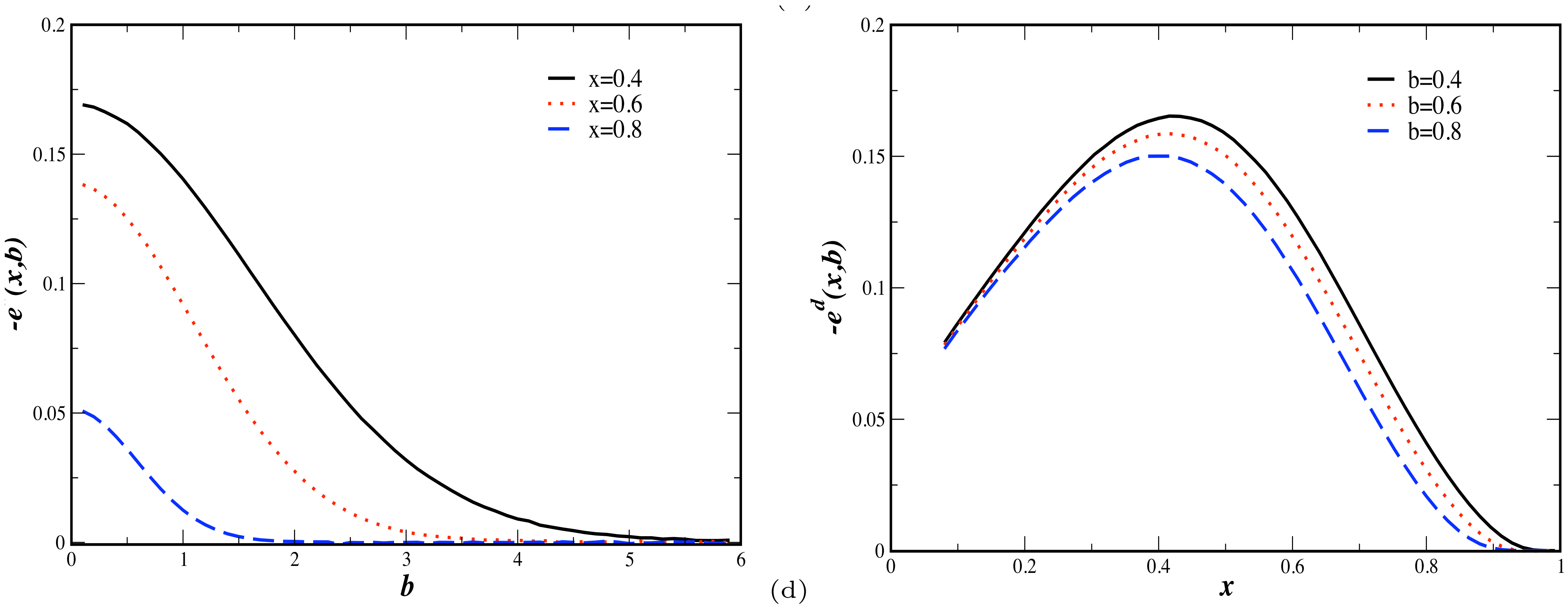


d-quark is smaller than u-quark =>
u-quark contribution dominates helicity
nonflip distribution.

E(x,b) for u-quark



E(x,b) for d-quark



For small and medium b, d-quark is larger than u-quark.=> d-quark dominates proton helicity flip distribution.

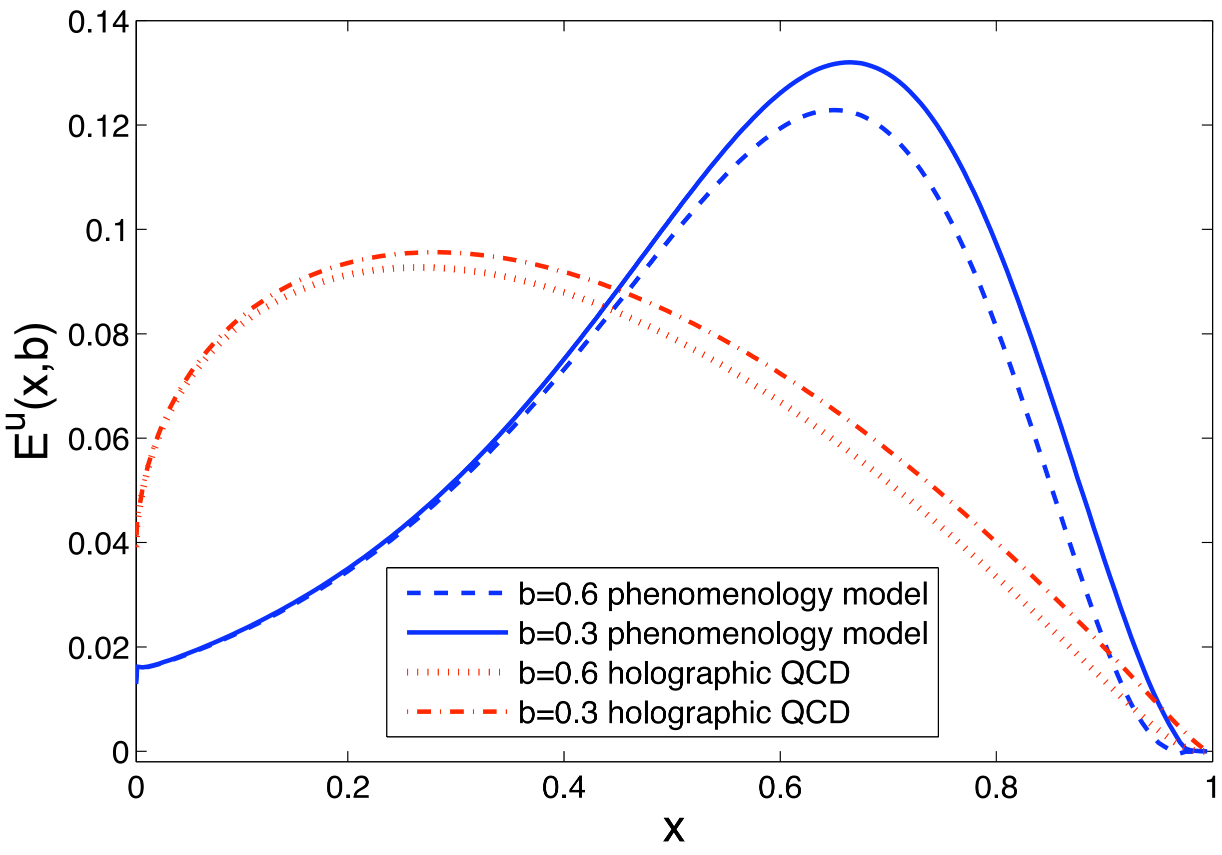
- FT of H(x,t) for a transversely polarised proton is given by

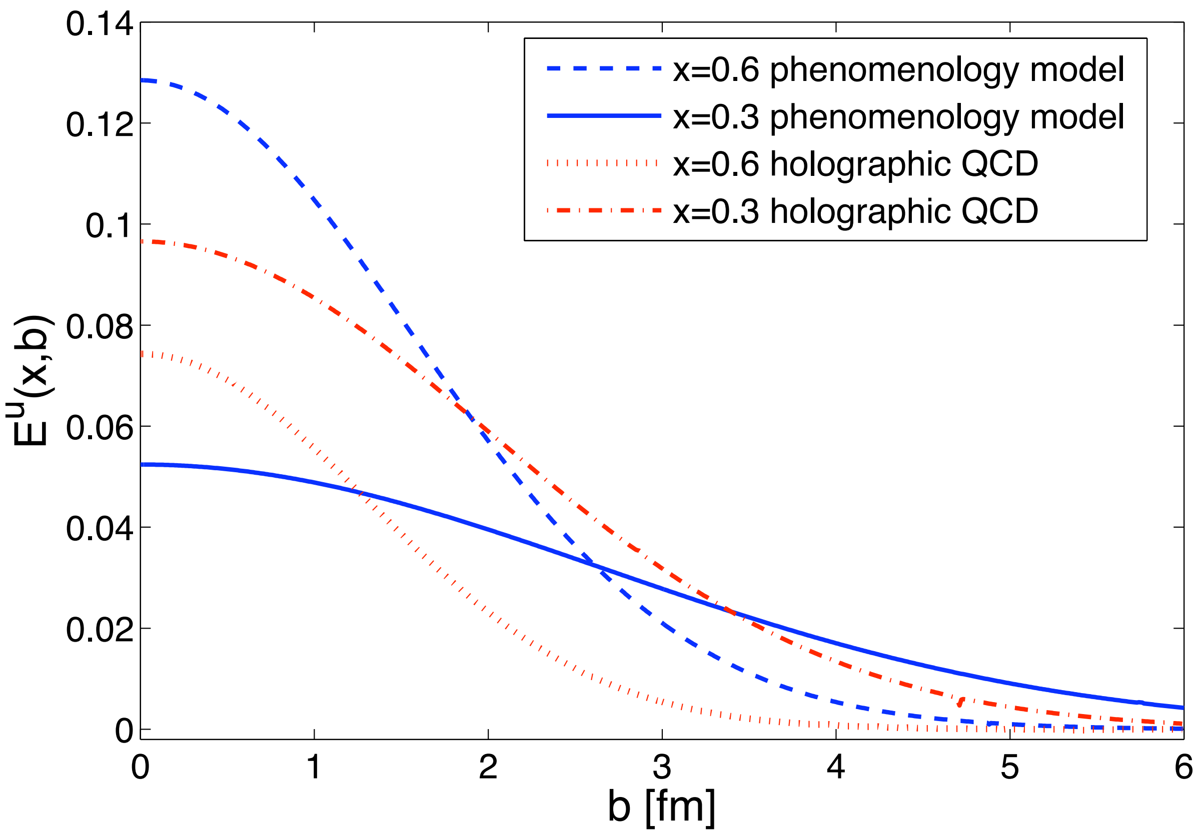
$$q^X(x,b) = q(x,b) - \frac{1}{2M}\frac{\partial e(x,b)}{\partial b_y}$$

- E(x,t) causes a transverse shift(for state polarized in x-dirn, the shift is in y-dirn)

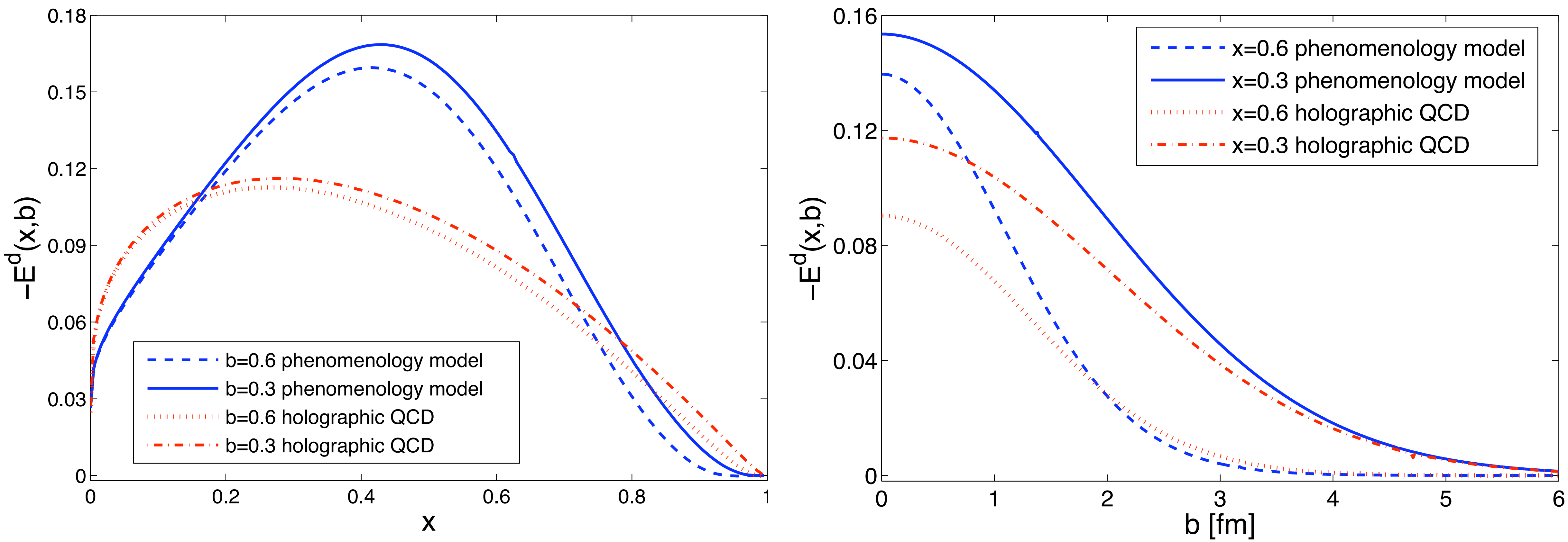
Comparison of the two models

E(x,b) for u quark

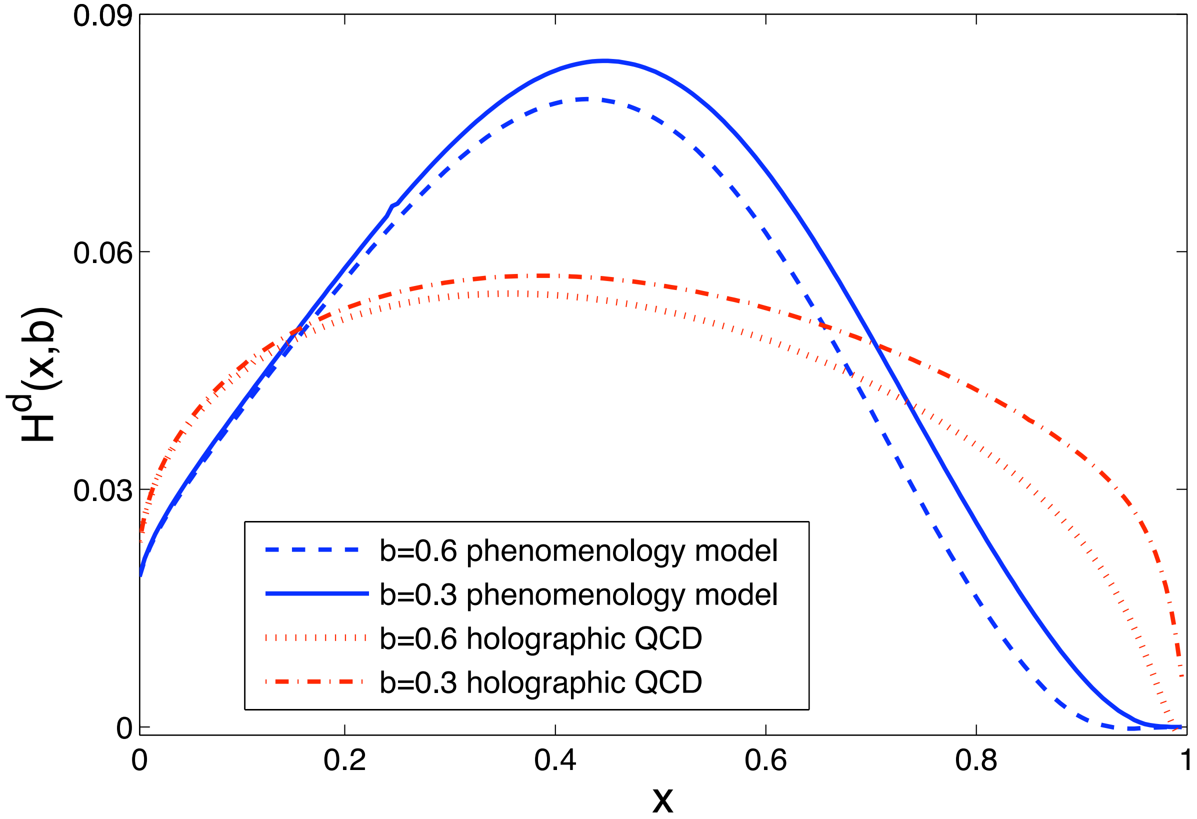


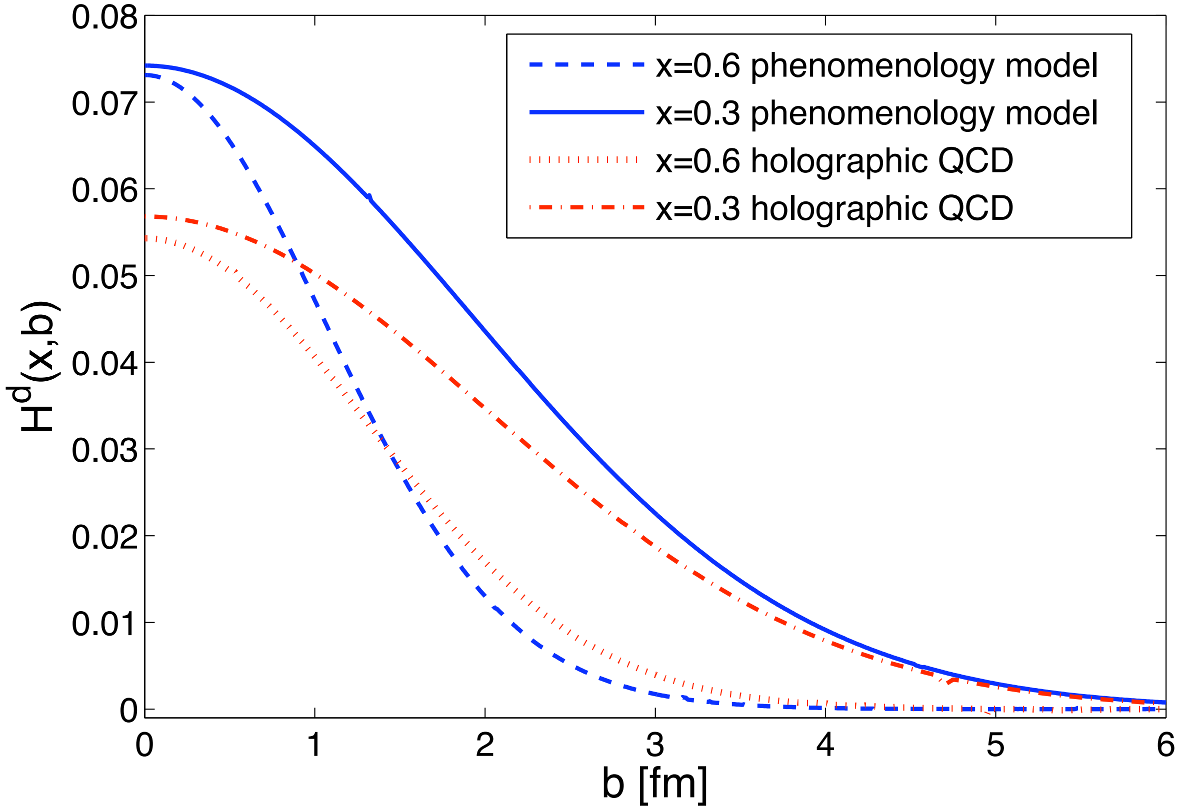


E(x,b) for d-quark



H(x,b) for d-quark





Summary and Conclusions

- GPDs are studied from AdS/QCD and also in a phenomenological model for proton.
- In ADS/QCD, the GPDs are extracted from the nucleon form factors.
- Impact parameter dependent GPDs are studied and compared for the two models.
- Overall behaviors of the GPDs in impact parameter space are similar in both cases.
- For small b, in both models, H(x,b) is larger for u-quark and E(x,b) larger for d-quark.


Wish you many happy returns of the day