Modeling Nuclear Parton Distribution Functions

Heli Honkanen (Penn State University)

With: **Vadim Guzey** (Kurchatov Institute, Petersburg Nuclear Physics Institute)
Mark Strikman (Penn State University)

Introduction

$$f_i^A(x,Q^2) \neq f_i^p(x,Q^2)$$

Nuclear parton distribution functions go well with everything:

$$\sigma_{\mathrm{DIS}}^{\ell+A\to\ell+X} = \sum_{i=q,\overline{q},g} f_i^A \otimes \hat{\sigma}_{\mathrm{DIS}}^{\ell+i\to\ell+X}$$

$$\sigma_{\mathrm{DY}}^{p+A\to l^+l^-+X} = \sum_{i,j=q,\overline{q},g} f_i^p \otimes f_j^A \otimes \hat{\sigma}^{ij\to l^+l^-+X}$$

$$\sigma^{A+B\to\pi+X} = \sum_{i,j,k=q,\overline{q},g} f_i^A \otimes f_j^B \otimes \hat{\sigma}^{ij\to k+X} \otimes D_{k\to\pi}$$

Nuclear Modifications (Phenomenology)

- Decomposition of nPDFs

$$f_i^A(x,Q^2) = R_i^A(x,Q^2)\, f_i^N(x,Q^2)$$

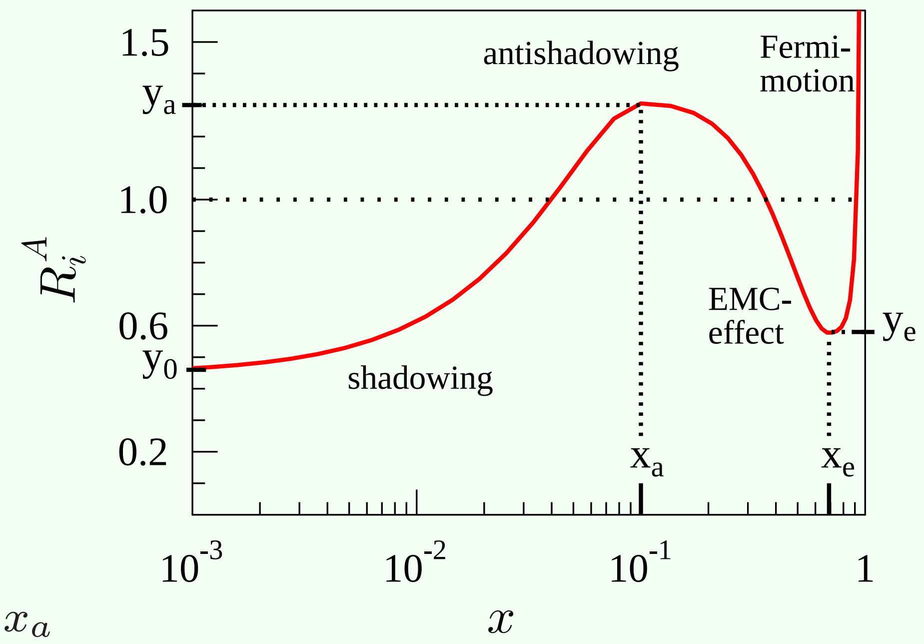
$$f_i^A(x,Q_0) = \int_{x_N}^{A} \frac{dy}{y} W_i^A(y,Q_0) f_i^p\left(\frac{x_N}{y}, Q_0\right)$$

- Initial scale ansatz e.g.

$$R_i(x,A,Z) = 1 + \left(1 - \frac{1}{A^\alpha}\right) \frac{a_i(A,Z) + b_i x + c_i x^2 + d_i x^3}{(1-x)^{\beta_i}}$$

$$R_i^A(x) = \begin{cases} a_0 + (a_1 + a_2 x)[\exp(-x) - \exp(-x_a)] & x \le x_a \\ b_0 + b_1 x + b_2 x^2 + b_3 x^3 & x_a \le x \le x_e \\ c_0 + (c_1 - c_2 x)(1-x)^{-\beta} & x_e \le x \le 1 \end{cases}$$

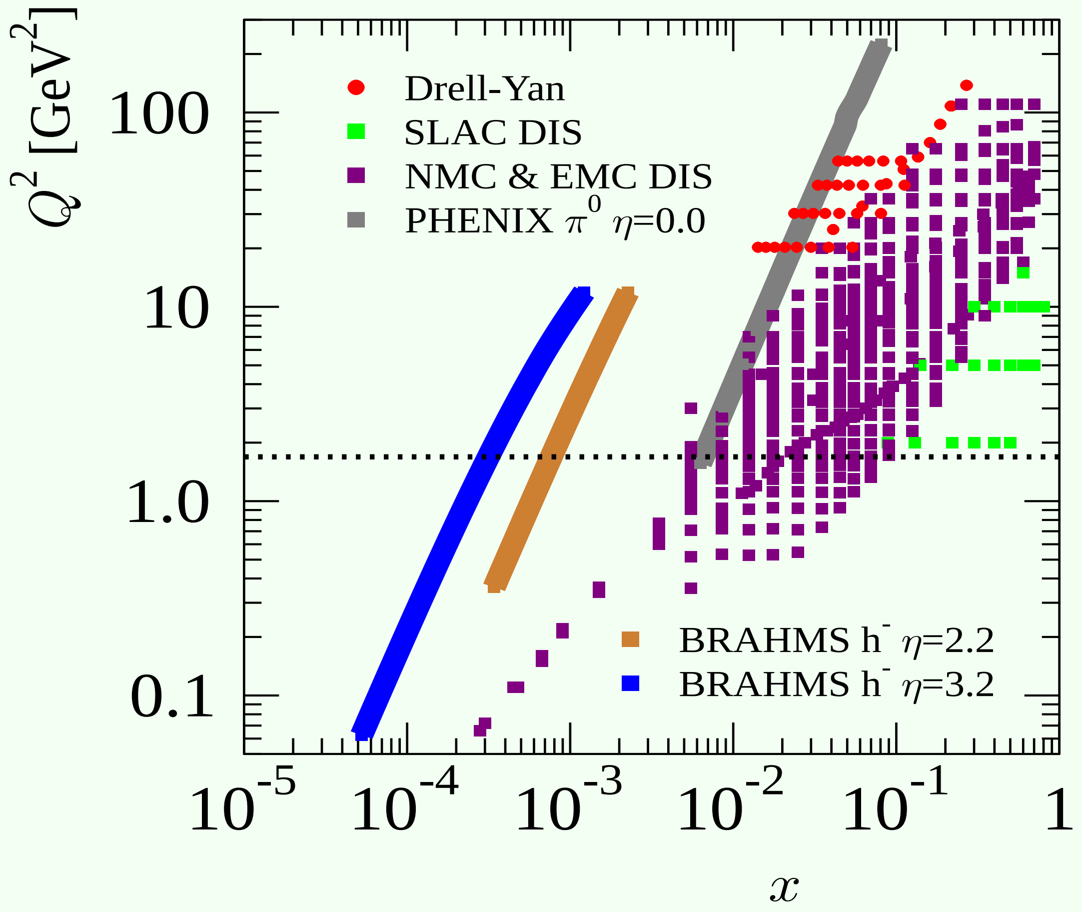
JHEP **0904** (2009) 065



- Built on 'top' of PDF parametrizations $f_i^p(x,Q^2)$ with $0 \le x \le 1$

- Error analysis (Hessian method)

Kinematical range of the data (pre LHC-era)



JHEP **0904** (2009) 065

Gluon constraints

DGLAP (LO):

$$\frac{\partial x q_v(x,t)}{\partial t} = \frac{2}{3}\int_x^1 dy \frac{z}{y}\frac{\left(1+z^2\right) y q_v(y,t) - 2x q_v(x,t)}{1-z} + \left\{1+\frac{4}{3}\ln(1-x)\right\} x q_v(x,t),$$

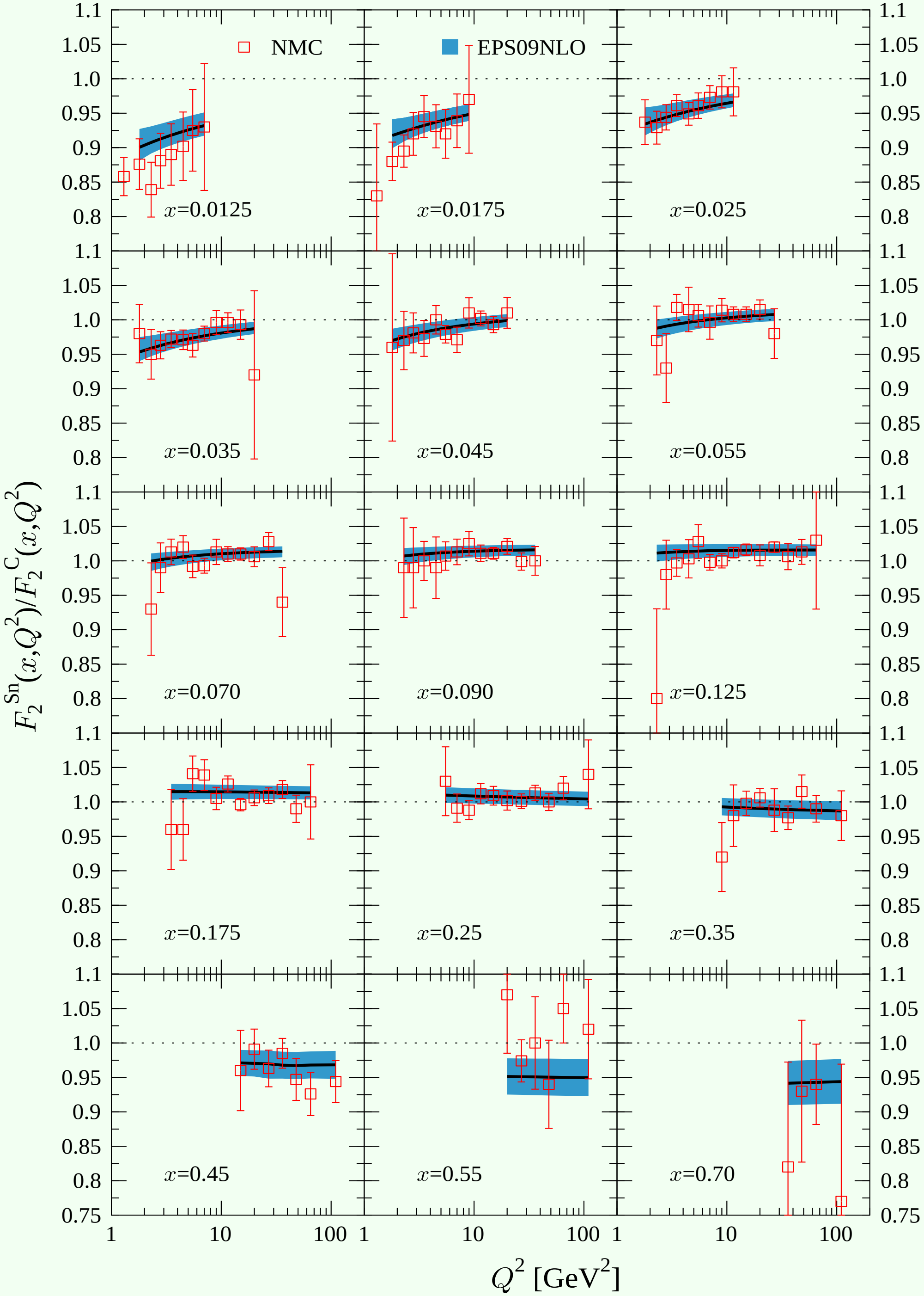
$$\frac{\partial x q_s(x,t)}{\partial t} = \frac{2}{3}\int_x^1 dy \frac{z}{y}\left\{\frac{\left(1+z^2\right) y q_s(y,t) - 2x q_s(x,t)}{1-z} + \frac{3}{8}\left(z^2+(1-z)^2\right) y g(y,t)\right\}$$
$$+\left\{1+\frac{4}{3}\ln(1-x)\right\} x q_s(x,t),$$

$$\frac{\partial x g(x,t)}{\partial t} = \int_x^1 dy \frac{z}{y}\left\{\frac{3\left(z y g(y,t) - x g(x,t)\right)}{1-z} + \frac{3(1-z)(1+z^2)}{z} y g(y,t)\right.$$
$$\left. + \frac{2}{3}\frac{1+(1-z)^2}{z}\sum_{i=q} y\left(q_v(y,t)+2q_s(y,t)\right)\right\}$$
$$+\left\{\frac{11}{4}-\frac{N_f}{6}+3\ln(1-x)\right\} x g(x,t), \text{where } z = x/y.$$

Small-x data on $R^A_{F_2}$ (LO): (Phys. Lett. B **311** (1993) 28, Phys. Lett. B **532** (2002) 222)

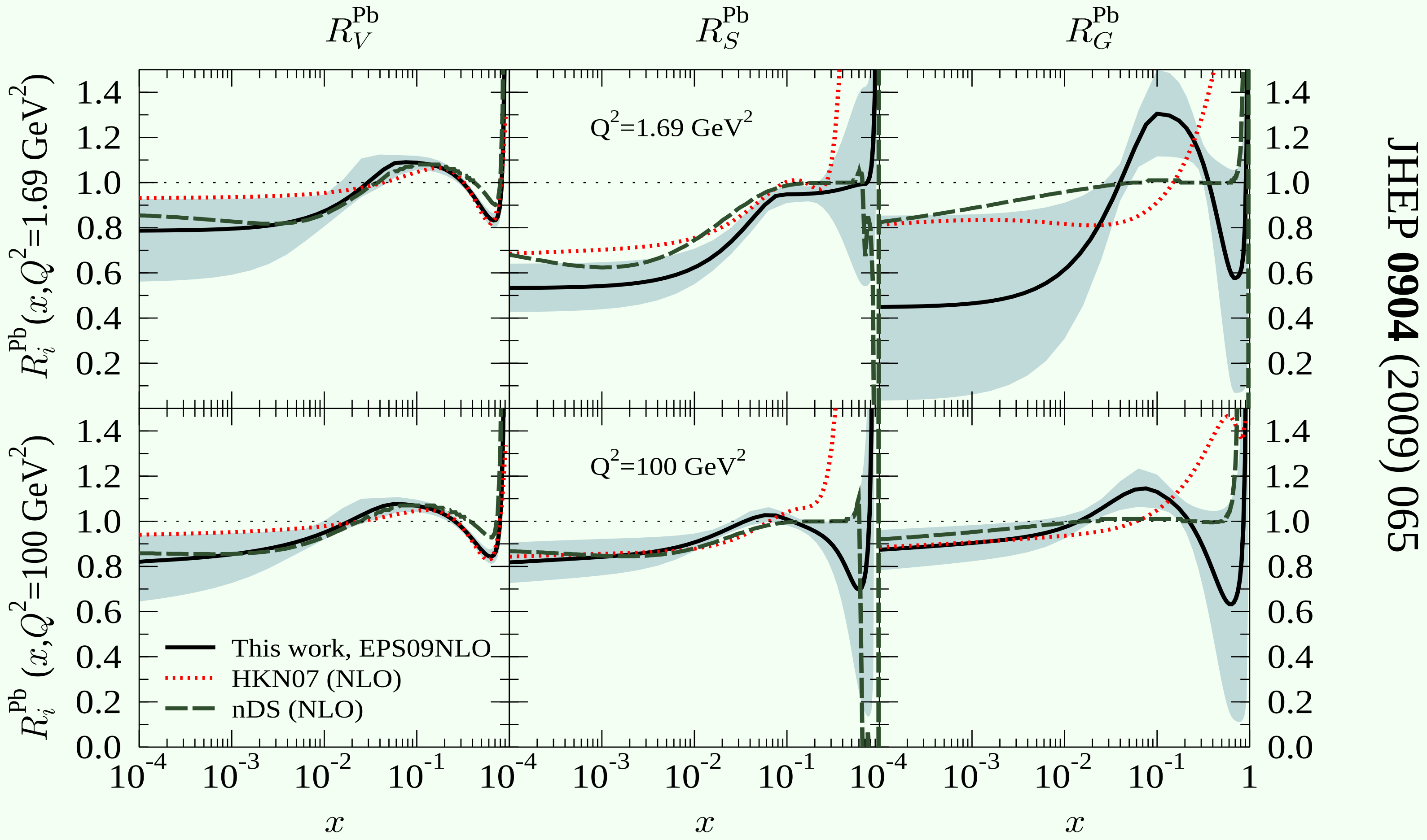
$$\frac{\partial F_2^{p(n)}(x,Q^2)}{\partial \log Q^2} \approx \frac{10\alpha_s}{27\pi} x g(2x,Q^2)$$

$$\Longrightarrow \frac{\partial\left(\frac{1}{117}F_2^{\mathrm{Sn}}/\frac{1}{12}F_2^{\mathrm{C}}\right)}{\partial \log Q^2} \approx \frac{10\alpha_s}{27\pi}\frac{x g(2x,Q^2)}{\frac{1}{2}F_2^{\mathrm{D}}(x,Q^2)}\frac{R^{\mathrm{Sn}}_{F_2}(x,Q^2)}{R^{\mathrm{C}}_{F_2}(x,Q^2)}\left\{\frac{R^{\mathrm{Sn}}_g(2x,Q^2)}{R^{\mathrm{Sn}}_{F_2}(x,Q^2)} - \frac{R^{\mathrm{C}}_g(2x,Q^2)}{R^{\mathrm{C}}_{F_2}(x,Q^2)}\right\}$$



JHEP **0904** (2009) 065

Comparison of different nPDF sets



☞ Recent nPDF sets: EPS09 (JHEP **0904** (2009) 065), HKN07 (Phys. Rev. C **76** (2007) 065207), DSZS (Phys. Rev. D **69** (2004) 074028), nCTEQ (Phys. Rev. Lett. **106** (2011) 122301) ...

nPDFs for pA and AA collisions

- At small (large) values of x gg (qq) processes dominate the hadron production
  - Need more accurate description of small-x sea and gluons at the LHC!
    $\Longrightarrow$ Models for nuclear shadowing: FGS nPDFs
    (Frankfurt, Guzey, Strikman: Phys. Rept. **512** (2012) 255)

- **Definition of x:** (Frankfurt, Strikman: Int. J. Mod. Phys. E **21** (2012) 1230002)
  - DIS data: $x_p = Q^2/(2q_0 m_p)$, independent of the target mass
  - Bjorken-x: $x = A\,Q^2/(2q \cdot p_A)$, with $0 \le x \le A$

$$x_p = x\,(1 + r_x^A), \ \text{with} \ r_x^A = \frac{1}{m_p}(\epsilon_A - (m_n - m_p)N/A)$$

E.g. $\epsilon_{Pb} \approx \epsilon_C \approx 7.7$ MeV, $r_x \approx 0.008$

$\Longrightarrow$ Large effect in the EMC region

- Fraction of nucleus momentum carried by equivalent photons
  - η_γ(Pb) $\sim$0.7%, η_γ(C) $\sim$0.1%

- Impact parameter dependence

Merged Model

FGS+EKS98/EPS09:

- For $x \leq 0.01$ use FGS-model(s), for $x > 0.01$ EKS98 or EPS09 (interpolate smoothly at $x \sim 0.01$) together with a PDF set (CTEQ5)

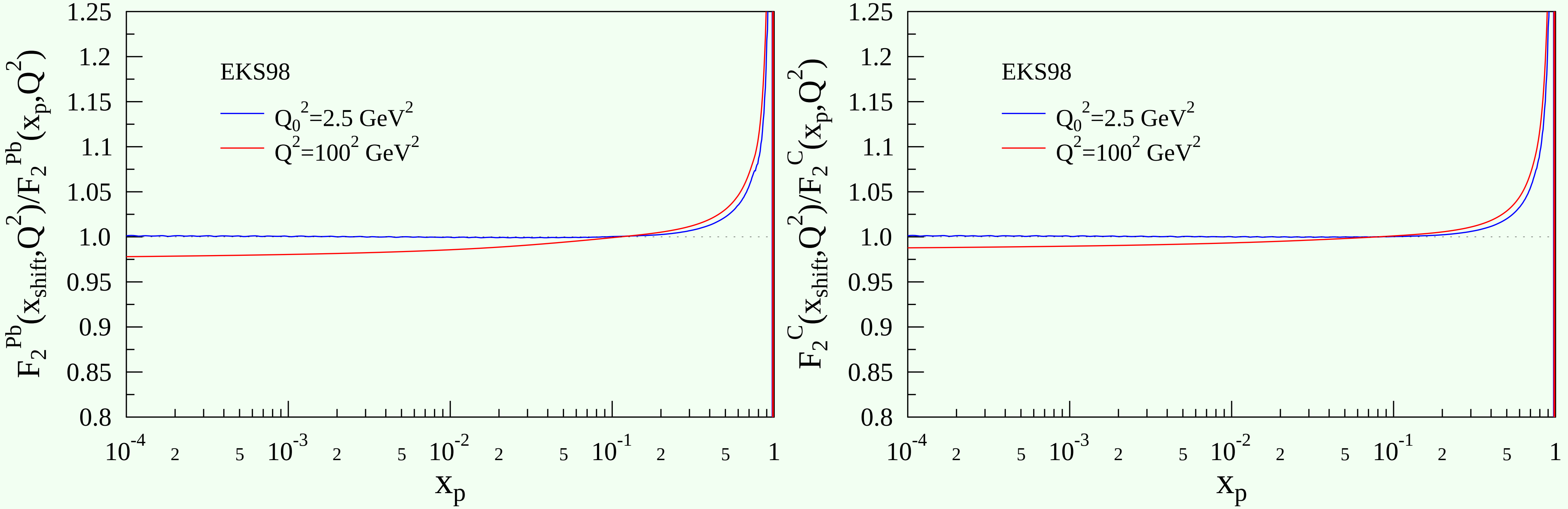
- Rescale gluons (since worst constrained by the data) such that

$$\sum_i \int_0^1 \mathrm{d}x x f_i^A(x, Q^2) = 1 - \eta_\gamma(A)$$

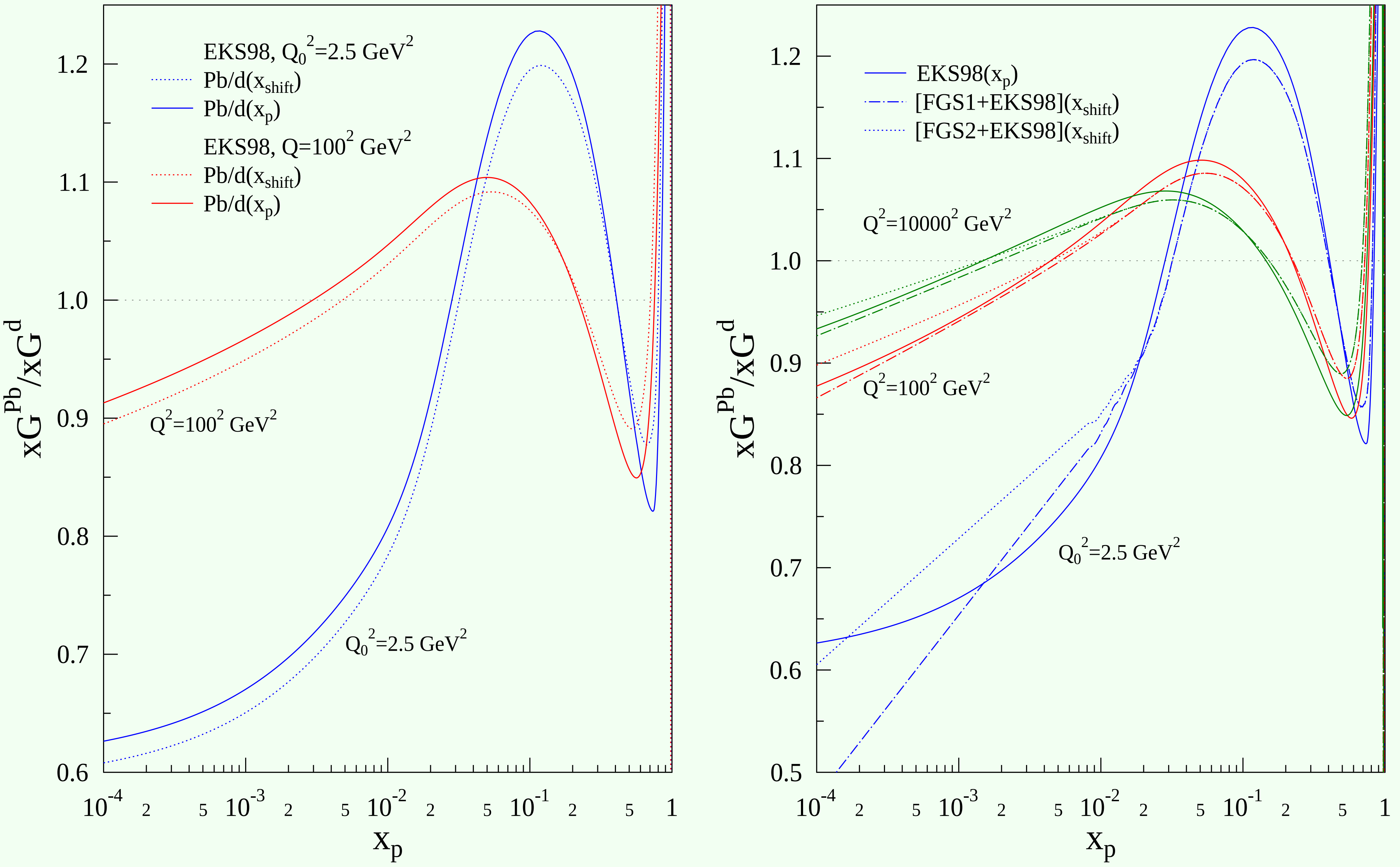
- Prosentage of the momentum carried by the gluons (41.80% for proton):

	EKS98		FGS1+EKS98		FGS2+EKS98	
	Pb	C	Pb	C	Pb	C
x_p	43.98	42.61	44.17	42.55	44.27	42.59
x_{shift}	43.16	42.11	43.35	42.05	43.46	42.09

  - Decrease of 1.86% (1.17%) for Pb (C)



EKS98
Q_0^2=2.5 GeV^2
Q^2=100^2 GeV^2
$F_2^{Pb}(x_{shift},Q^2)/F_2^{Pb}(x_p,Q^2)$
x_p
EKS98
Q_0^2=2.5 GeV^2
Q^2=100^2 GeV^2
$F_2^{C}(x_{shift},Q^2)/F_2^{C}(x_p,Q^2)$
x_p


EKS98, Q_0^2=2.5 GeV^2
Pb/d(x_{shift})
Pb/d(x_p)
EKS98, Q=100^2 GeV^2
Pb/d(x_{shift})
Pb/d(x_p)
Q^2=100^2 GeV^2
Q_0^2=2.5 GeV^2
xG^{Pb}/xG^d
x_p
EKS98(x_p)
[FGS1+EKS98](x_{shift})
[FGS2+EKS98](x_{shift})
Q^2=10000^2 GeV^2
Q^2=100^2 GeV^2
Q_0^2=2.5 GeV^2
xG^{Pb}/xG^d
x_p

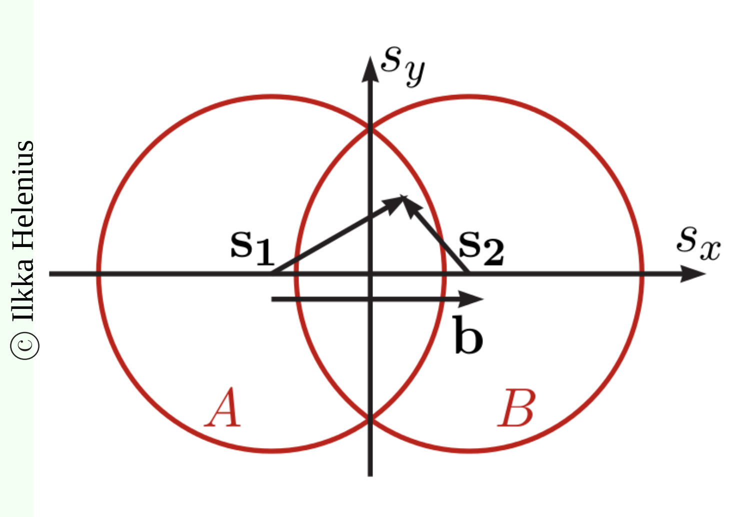
Nuclear Geometry

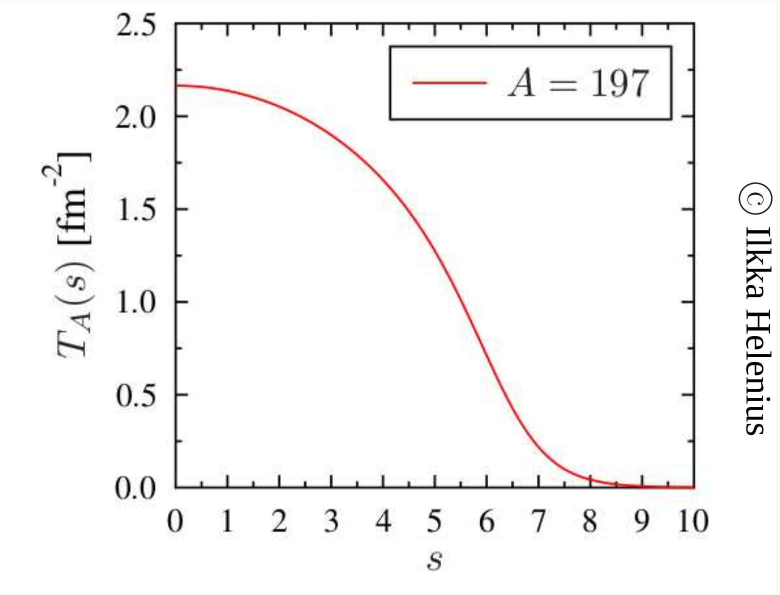
$$d\sigma^{AB\to k+X} = \sum_{i,j,X'} f_i^A \otimes f_j^B \otimes d\hat{\sigma}^{ij\to k+X'} + \mathcal{O}(1/Q^2)$$

$$dN^{AB\to k+X}(\mathbf{b}) = T_{AB}(\mathbf{b})d\sigma^{AB\to k+X}$$

$$T_{AB}(\mathbf{b}) = \int d^2\mathbf{s}\, T_A(\mathbf{s_1})\, T_B(\mathbf{s_2}), \quad \mathbf{s_1} = \mathbf{s} + \mathbf{b}/2, \;\; \mathbf{s_2} = \mathbf{s} - \mathbf{b}/2$$

where $\mathbf{s_1} = \mathbf{s} + \mathbf{b}/2$ and $\mathbf{s_2} = \mathbf{s} - \mathbf{b}/2$. Normalization: $\int d^2\mathbf{b}\, T_{AB}(\mathbf{b}) = AB$.





Impact Parameter Dependence (Phenomenology)

EKS98s/EPS09s: (I. Helenius, K. J. Eskola, HH, C. A. Salgado, JHEP **1207** (2012) 073)

- Define

$$R_i^A(x,Q^2) \equiv \frac{1}{A}\int \mathrm{d}^2\mathbf{s}\, T_A(\mathbf{s})\, r_i^A(x,Q^2,\mathbf{s}),$$

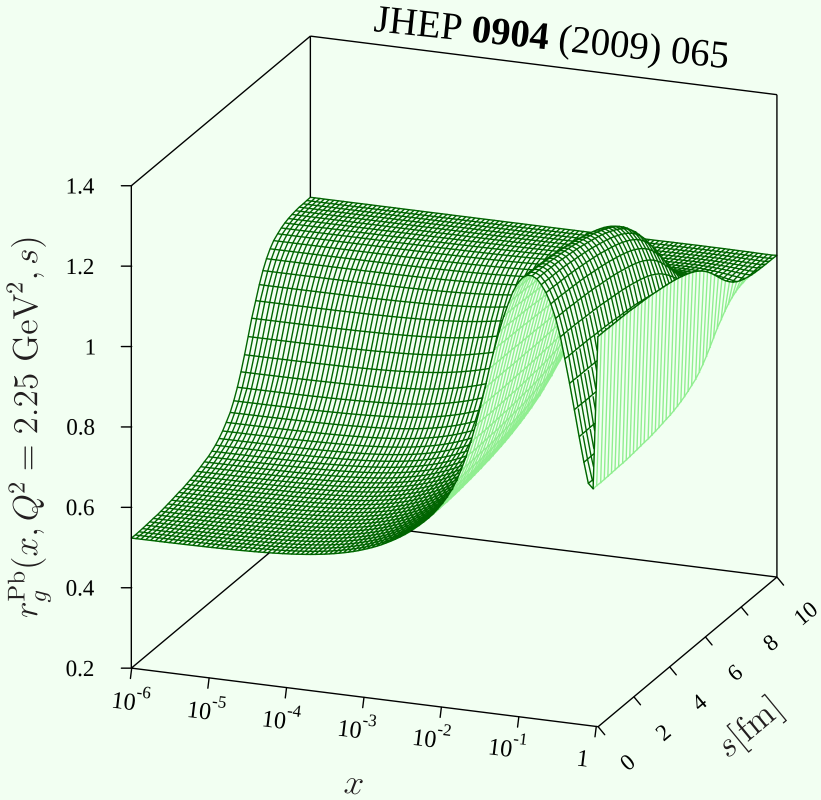
where $\mathbf{s}$ is the transverse position of the nucleon.

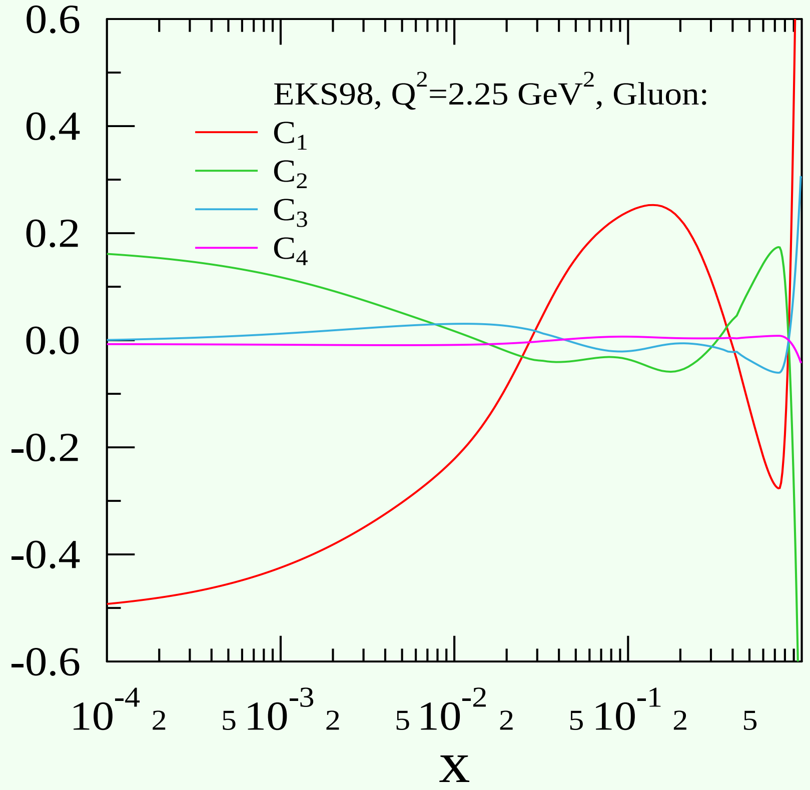
  - $R_i^A(x,Q^2)$ taken from EKS98 or EPS09

- Spatial dependence related to $T_A(\mathbf{s})$

$$r_i^A(x,Q^2,\mathbf{s}) = 1 + \sum_{j=1}^{4} c_j^i(x,Q^2)\,[T_A(\mathbf{s})]^j\,.$$

  - Fit parameters $c_j^i(x,Q^2)$ A-independent!



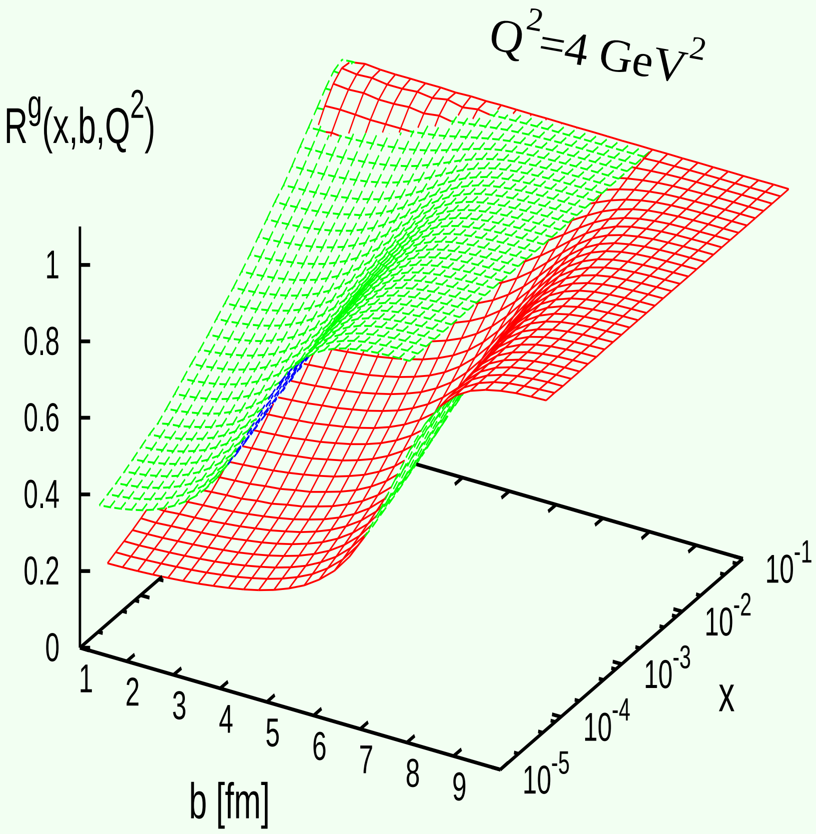


☞ At small **s**: $|1 - r_i^A(x, Q^2, \mathbf{s})| > |1 - R_i^A(x, Q^2)|$

☞ At large **s**: $r_i^A(x, Q^2, \mathbf{s}) \approx 1$

Impact Parameter Dependence (Theory)

FGS: (L. Frankfurt, V. Guzey, M. Strikman, Phys. Rept. **512** (2012) 255)



- Gribov-Glauber multiple scattering formalism modeling for nuclear shadowing
  - input rescattering cross section $\sigma^{j}_{\rm soft}$
- Qualitatively similar to EKS98s/EPS09s
- Allows for more detailed modeling of **b**-dependence
  - EMC effect $\Longleftrightarrow$ SRC

Calculation Of Observables

Spatially averaged case:

$$\mathrm{d}N^{AB\to k+X}(\mathbf{b}) = T_{AB}(\mathbf{b}) \sum_{i,j,X'} R_i^A f_i^N \otimes R_j^B f_j^N \otimes \mathrm{d}\hat{\sigma}^{ij\to k+X'}$$

Spatially dependent case (EKS98/EPS09S):

$$\int_{b_1}^{b_2} \mathrm{d}^2\mathbf{b}\, \mathrm{d}N_{AB}^k(\mathbf{b}) = \sum_{n,m=0}^{4} T_{AB}^{nm}(b_1, b_2) \sum_{i,j,X'} \frac{1}{AB} \sum_{N_A,N_B} c_n^i\, f_i^{N_A} \otimes c_m^j\, f_j^{N_B} \otimes \mathrm{d}\hat{\sigma}^{ij\to k+X'},$$

where $c_0^{i,j} \equiv 1$

$$T_{AB}^{nm}(b_1, b_2) \equiv \int_{b_1}^{b_2} \mathrm{d}^2\mathbf{b} \int \mathrm{d}^2\mathbf{s}\, [T_A(\mathbf{s}-\mathbf{b}/2)]^{n+1}\, [T_B(\mathbf{s}+\mathbf{b}/2)]^{m+1}$$

- For $p(d)+A$ collisions replace $T_{AB}(\mathbf{b}) \to T_A(\mathbf{b})$
- For final state particle, add fragmentation function, e.g.

$$\mathrm{d}N_{\mathrm{d}A}^{\pi^0}(\mathbf{b}) = \sum_k \mathrm{d}N_{\mathrm{d}A}^k(\mathbf{b}) \otimes D_{\pi^0/k}(z, Q_F^2)$$

Centrality classes: Nuclear modification factor for $|\mathbf{b}| \in [b_1, b_2]$

$$R_{\mathrm{d}A}^{\pi^0}(p_T, y; b_1, b_2) \equiv \frac{\left\langle \frac{\mathrm{d}^2 N_{\mathrm{d}A}^{\pi^0}}{\mathrm{d}p_T \mathrm{d}y} \right\rangle_{b_1, b_2}}{\langle N_{bin}^{\mathrm{d}A} \rangle_{b_1, b_2} \frac{1}{\sigma_{inel}^{NN}} \frac{\mathrm{d}^2 \sigma_{\mathrm{pp}}^{\pi^0}}{\mathrm{d}p_T \mathrm{d}y}} = \frac{\int_{b_1}^{b_2} \mathrm{d}^2\mathbf{b} \frac{\mathrm{d}^2 N_{\mathrm{d}A}^{\pi^0}(\mathbf{b})}{\mathrm{d}p_T \mathrm{d}y}}{\frac{\mathrm{d}^2 \sigma_{\mathrm{pp}}^{\pi^0}}{\mathrm{d}p_T \mathrm{d}y} \int_{b_1}^{b_2} \mathrm{d}^2\mathbf{b}\, T_{\mathrm{d}A}(\mathbf{b})}$$

☞ Impact parameters $b1$ and $b2$ for given centrality class from optical Glauber model:

$$p_{inel}^{AB}(\mathbf{b}) \approx 1 - \mathrm{e}^{-T_{AB}(\mathbf{b})\sigma_{inel}^{NN}},$$

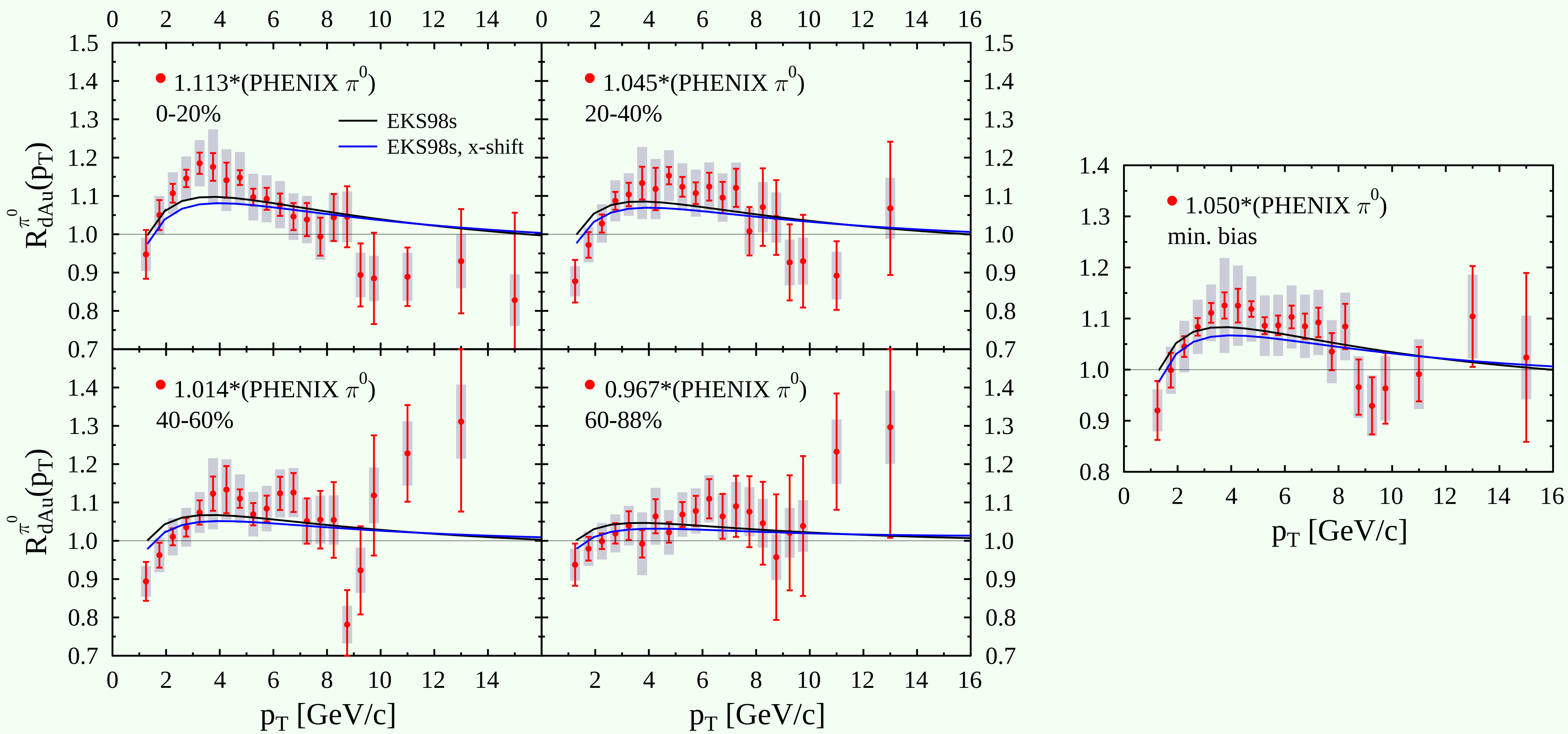
$$\sigma_{inel}^{AB} = \int \mathrm{d}^2\mathbf{b}\, p_{inel}^{AB}(\mathbf{b}) = \int \mathrm{d}^2\mathbf{b} \left(1 - \mathrm{e}^{-T_{AB}(\mathbf{b})\sigma_{inel}^{NN}}\right).$$

$$(c_2 - c_1)\,\% = \frac{1}{\sigma_{inel}^{AB}} \int_{b_1}^{b_2} \mathrm{d}^2\mathbf{b}\, p_{inel}^{AB}(\mathbf{b}) = \frac{\sigma_{inel}^{AB}(b_1, b_2)}{\sigma_{inel}^{AB}}$$

Minimum bias:

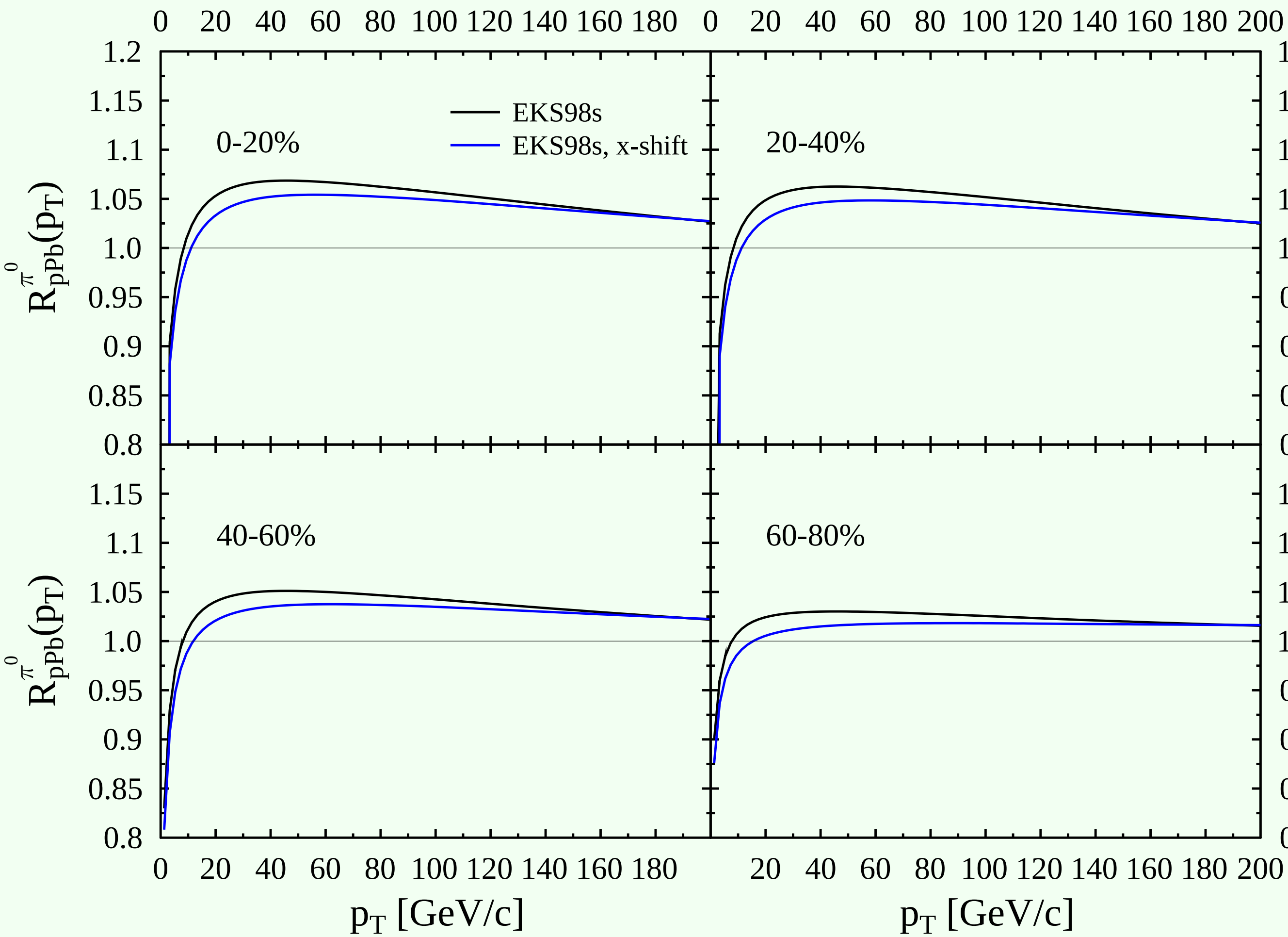
$$\left\langle R_{\mathrm{d}A}^{\pi^0}(p_T, y) \right\rangle = \frac{1}{2A} \frac{\mathrm{d}^2 \sigma_{\mathrm{d}A,\mathrm{MB}}^{\pi^0}}{\mathrm{d}p_T \mathrm{d}y} \Big/ \frac{\mathrm{d}^2 \sigma_{\mathrm{pp}}^{\pi^0}}{\mathrm{d}p_T \mathrm{d}y},$$

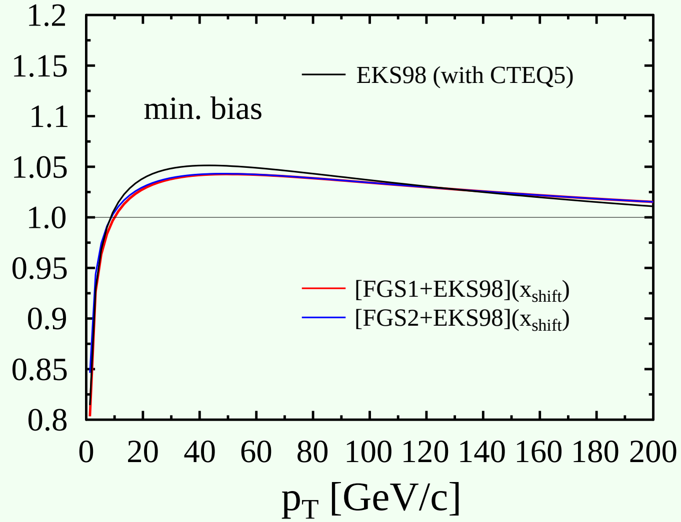
Nuclear Effects At RHIC



(EKS98s results from JHEP **1207** (2012) 073)

Nuclear Effects At The LHC





☞ Centrality dependent FGS+EKS98/EPS09 results soon!

Hyvää Syntymäpäivää James!