Evgeny Epelbaum, Ruhr-Universität Bochum

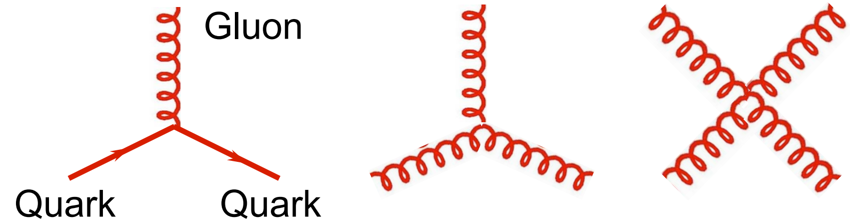
Workshop on Nuclear Theory in the Supercomputing
Era, Khabarovsk, Russia, June 23 - 28, 2014

Chiral nuclear forces:
State of the art and future perspectives

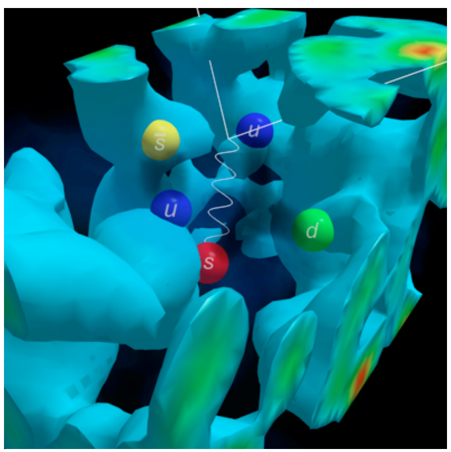
- Introduction
- Chiral EFT and nuclear forces
- Chiral NN potentials
- 3N force
- Summary & outlook

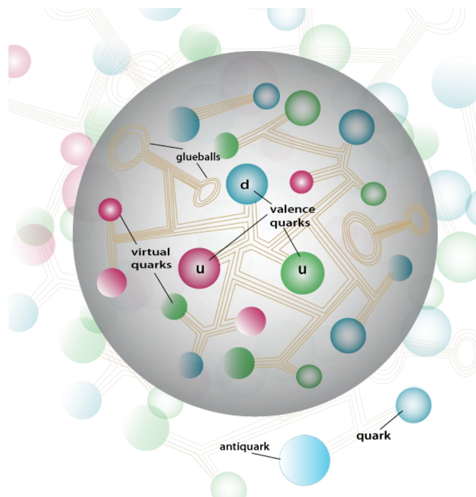
Facets of strong interactions

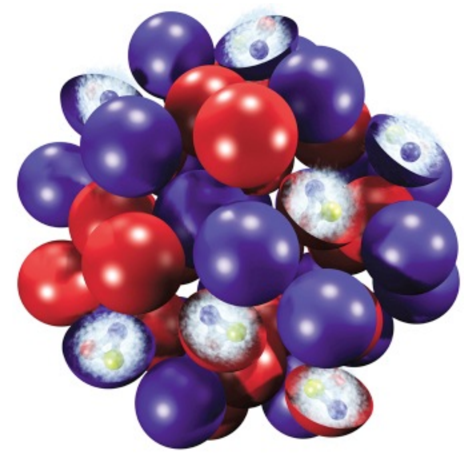
$$\mathcal{L}_{\rm QCD} = \bar{q}(i\not{D} - m)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu}$$



Seemingly very simple formulation is responsible for extremely complex phenomena!

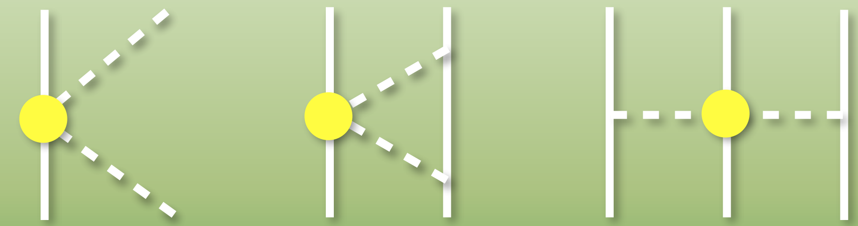






From QCD to nuclear physics

- → **lattice** → **nuclear physics**
- → **effective chiral Lagrangian** → **(low-energy) nuclear physics**



Chiral perturbation theory

- **Ideal world [$m_u = m_d = 0$], zero-energy limit:** non-interacting massless GBs
  (+ strongly interacting massive hadrons)

- **Real world [$m_u,\ m_d \ll \Lambda_{QCD}$], low energy:** weakly interacting light GBs
  (+ strongly interacting massive hadrons)

→ expand about the ideal world (ChPT)

Chiral Perturbation Theory

Chiral Perturbation Theory: expansion of the scattering amplitude in powers of

Weinberg, Gasser, Leutwyler, Meißner, ...

$$Q = \frac{\text{momenta of pions and nucleons or } M_\pi \sim 140 \text{ MeV}}{\text{hard scales [at best } \Lambda_\chi = 4\pi F_\pi \sim 1 \text{ GeV]}}$$

Manohar, Georgi '84

Tool: Feynman calculus using the effective chiral Lagrangian

low-energy constants

$$\mathcal{L}_\pi = \mathcal{L}_\pi^{(2)} + \mathcal{L}_\pi^{(4)} + \ldots$$

$$\mathcal{L}_{\pi N} = \underbrace{\bar{N}\left(i\gamma^\mu D_\mu[\pi] - m + \frac{g_A}{2}\gamma^\mu\gamma_5 u_\mu[\pi]\right)N}_{\mathcal{L}_{\pi N}^{(1)}} + \underbrace{\sum_i \boldsymbol{c_i}\, \bar{N}\hat{O}_i^{(2)}[\pi]N}_{\mathcal{L}_{\pi N}^{(2)}} + \underbrace{\sum_i \boldsymbol{d_i}\, \bar{N}\hat{O}_i^{(3)}[\pi]N}_{\mathcal{L}_{\pi N}^{(3)}} + \ldots$$

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To simplify calculations, expand $\mathcal{L}$ in 1/m [Foldy-Wouthuysen/Heavy-Baryon expansion]

Use NDA [power counting] to estimate the importance of *renormalized* diagrams:

- Vertices with more derivatives are suppressed
- Pion loops are suppressed
- At any order, a finite number of vertices and Feynman diagrams contribute

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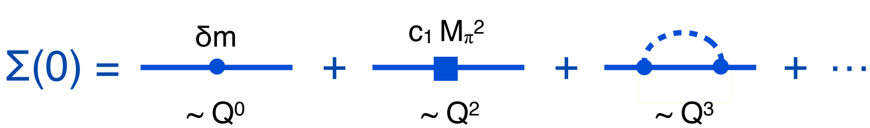
Example: quark mass dependence of m:

$$-i\Sigma_{\text{loop}}(0) \sim \int \frac{d^4 l}{(2\pi)^4}\left[\frac{g_A^2}{F_\pi^2} l\right]\frac{i}{-l_0 + i\epsilon}\,\frac{i}{l^2 - M_\pi^2 + i\epsilon}\left[\frac{g_A^2}{F_\pi^2} l\right]$$



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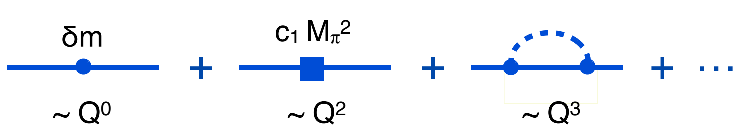
$$\xrightarrow{\Lambda} \frac{g_A^2}{F_\pi^2}\left(\alpha\Lambda^3 + \beta\Lambda M_\pi^2 - \frac{3}{32\pi}M_\pi^3 + \mathcal{O}(\Lambda^{-1})\right)$$

$\Sigma(0) =$



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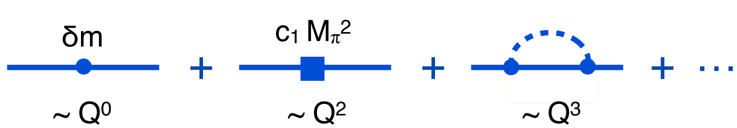

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Pion-nucleon scattering up to Q^4 in heavy-baryon ChPT

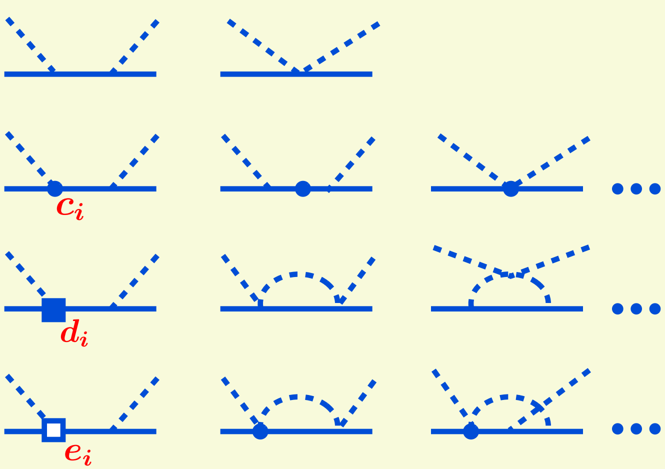
Fettes, Meißner '00; Krebs, Gasparyan, EE '12

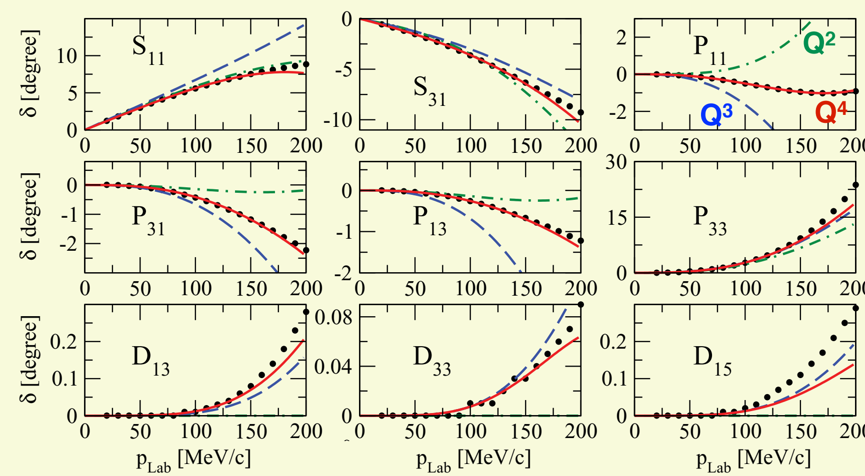
Order Q:

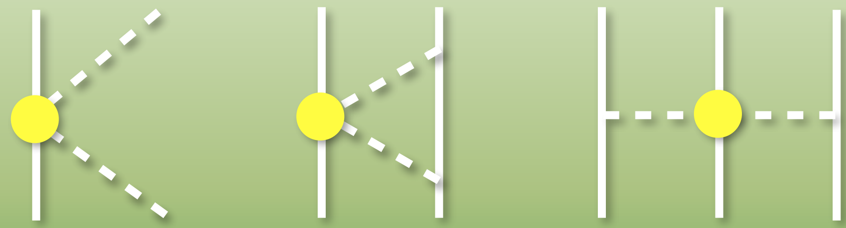
Order Q^2:

Order Q^3:

Order Q^4:

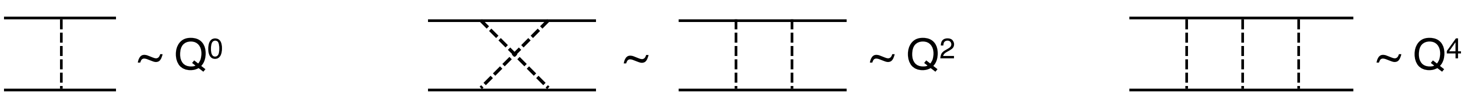


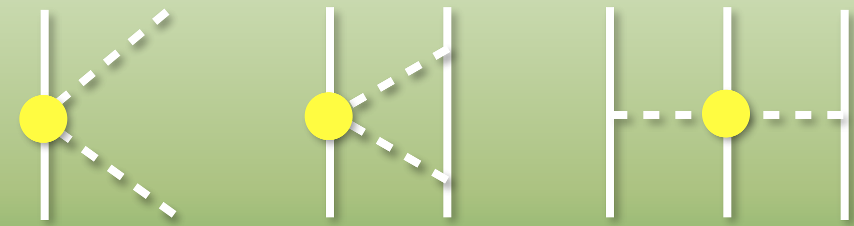




Chiral EFT for nuclear systems

Naive application of power counting to NN scattering seem to suggests perturbativeness (i.e. no bound states…)





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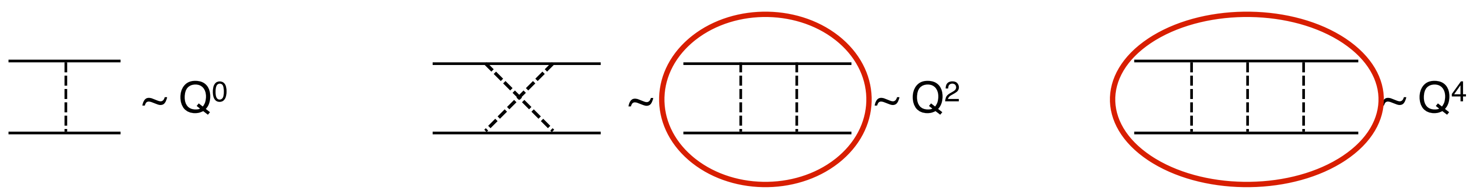


However, diagrams involving NN intermediate states are infrared-divergent in the limit of $m \to \infty$ (due to pinch singularity). For finite m, reducible diagrams are infrared-finite but enhanced and need to be re-summed (e.g. by solving the LS equation).

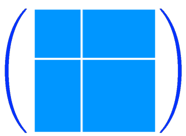
Main steps in the derivation of nuclear forces in the method of UT

EE, Glöckle, Meißner '98

1. Begin with the most general chiral-invariant effective Lagrangian for π, N [+ possibly Δ]

2. Apply standard canonical formalism to switch to πN Hamiltonian

3. Apply unitary transformation in Fock space to decouple purely nucleonic space [i.e. our „model space“] from the rest

$H \to \tilde{H} = U^\dagger$  $U = \begin{pmatrix} \tilde{H}_{\rm nucl} & 0 \\ 0 & \tilde{H}_{\rm rest} \end{pmatrix}$

For U use a minimal Okubo-parametrization in terms of

$$A = \lambda A \eta, \quad \lambda(H - [A,\, H] - AHA)\eta = 0$$

(solved perturbatively in terms of chiral expansion)

4. Apply all possible UTs on the η-subspace consistent with a given chiral order [e.g. static N^3LO nucl.: 6 additional angles α_i, Δ-contributions: 50 additional α^Δ_i …]

5. Evaluate 2-body, 3-body, … momentum-space MEs of the resulting $\eta U^\dagger H U \eta$

6. Demand renormalizability of nuclear potentials. This fixes some of the α_i and α^Δ_i and leads to unique (static) expressions.

7. Calculate the πN system to the same accuracy to determine the relevant LECs, tune NN, NNN, … contact terms to nuclear observables.

Nuclear chiral effective field theory

Weinberg, van Kolck, Kaiser, EE, Glöckle, Meißner, Entem, Machleidt...

- Schrödinger eq. for nucleons interacting via contact forces + long-range potentials (π-exchanges)

$$\left[\left(\sum_{i=1}^{A}\frac{-\vec{\nabla}_i^2}{2m_N}+\mathcal{O}(m_N^{-3})\right)+\underbrace{V_{2N}+V_{3N}+V_{4N}+\ldots}_{\text{derived in ChPT}}\right]|\Psi\rangle = E|\Psi\rangle$$

- access to heavier nuclei (ab initio few-/many-body methods)

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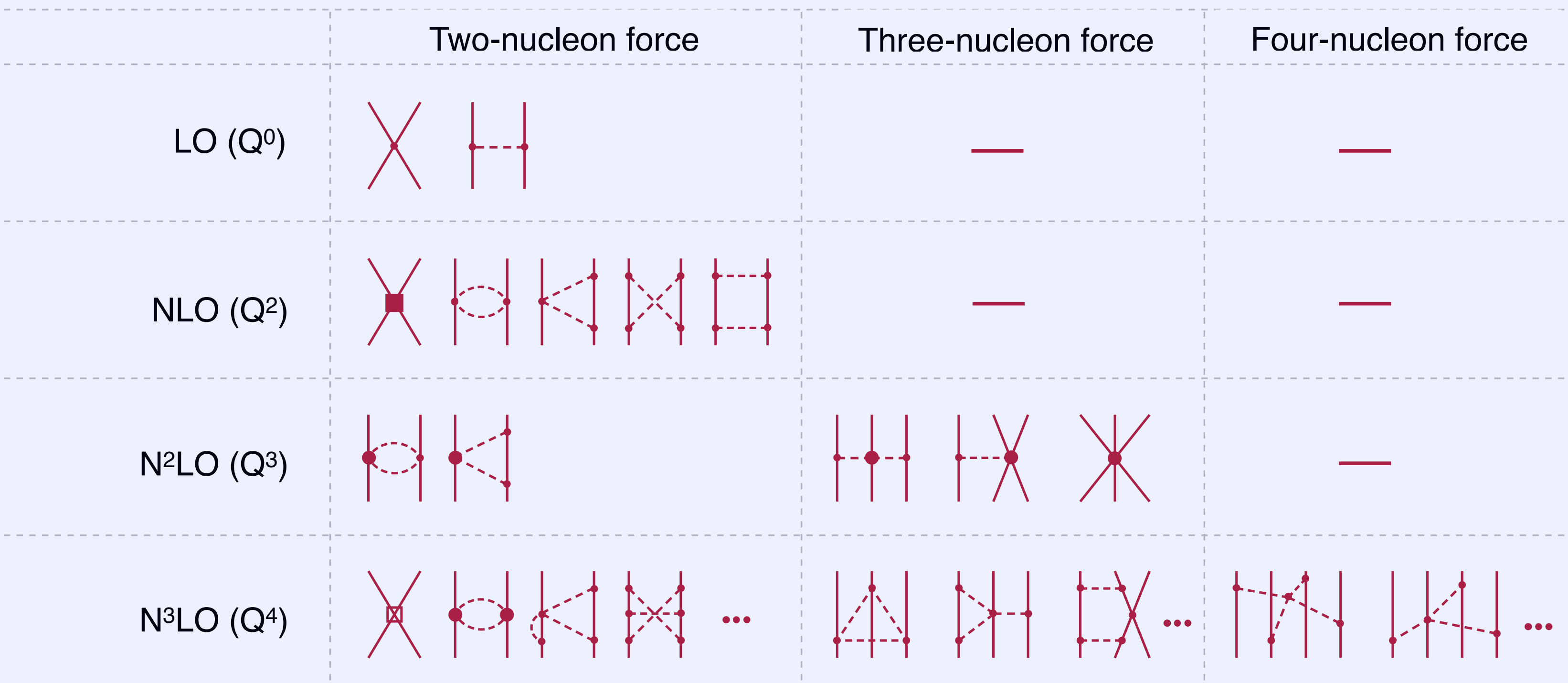
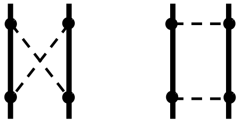


From $\mathcal{L}_{\text{eff}}$ to nuclear forces

Example: chiral 2π-exchange potential proportional to g_A^4 :



$$V_{2\pi}^{(2)}(q) = \; - \eta H_I \frac{\lambda}{E_\pi} H_I \frac{\lambda}{E_\pi} H_I \frac{\lambda}{E_\pi} H_I \eta \; + \; \frac{1}{2} \eta H_I \frac{\lambda}{E_\pi} H_I \eta H_I \frac{\lambda}{E_\pi^2} H_I \eta \; + \; \frac{1}{2} \eta H_I \frac{\lambda}{E_\pi^2} H_I \eta H_I \frac{\lambda}{E_\pi} H_I \eta$$

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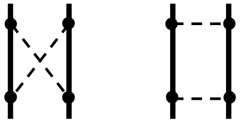

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$$= -\frac{g_A^4}{2(2F_\pi)^4} \int \frac{d^3 l}{(2\pi)^3} \frac{\omega_+^2 + \omega_+\omega_- + \omega_-^2}{\omega_+^3 \omega_-^3 (\omega_+ + \omega_-)} \left\{ \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \left(\vec{l}^{\,2} - \vec{q}^{\,2} \right)^2 + 6 (\vec{\sigma}_2 \cdot [\vec{q} \times \vec{l}\,]) (\vec{\sigma}_1 \cdot [\vec{q} \times \vec{l}\,]) \right\}$$

$$\omega_\pm = \sqrt{(\vec{q} \pm \vec{l}\,) + 4M_\pi^2}$$

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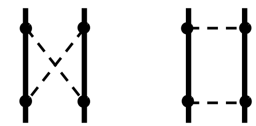

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$$\omega_\pm = \sqrt{(\vec q \pm \vec l\,) + 4M_\pi^2}$$

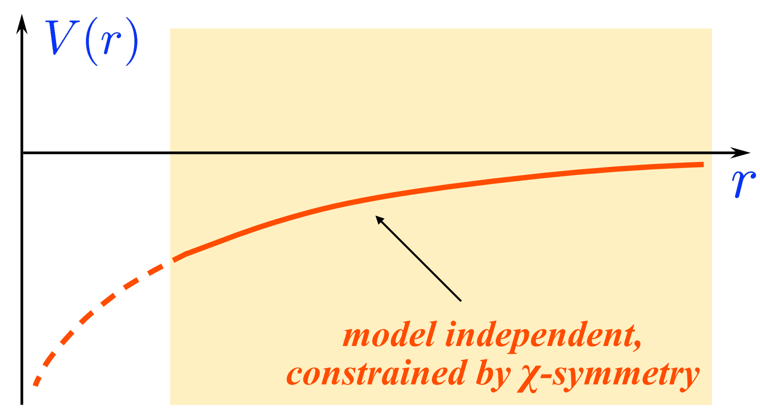
$$= -\frac{g_A^4}{384\pi^2 F_\pi^4} \left[\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \left(20 M_\pi^2 + 23 q^2 + \frac{48 M_\pi^4}{4M_\pi^2 + q^2} \right) - 18 \left(\vec\sigma_1 \cdot \vec q\, \vec\sigma_2 \cdot \vec q - q^2\, \vec\sigma_1 \cdot \vec\sigma_2 \right) \right] L(q) + \ldots$$

where the loop function is given by (in DR):

$$L(q) = \frac{1}{q}\sqrt{4M_\pi^2 + q^2}\, \ln \frac{\sqrt{4M_\pi^2 + q^2} + q}{2M_\pi}$$

The integral has logarithmic and quadratic divergences can be absorbed into short-range terms:

$$V_{\rm cont} = (\alpha_1 + \alpha_2\, q^2)\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + \alpha_3 (\vec\sigma_1 \cdot \vec q)(\vec\sigma_2 \cdot \vec q) + \alpha_4 (\vec\sigma_1 \cdot \vec\sigma_2)\, q^2$$



Chiral expansion of the 2π exchange

Kaiser, Brockmann, Weise ’97

$$\mathcal{V}_{NN} = V_C(r) + \vec{\tau}_1 \cdot \vec{\tau}_2 W_C(r) + \left[V_S(r) + \vec{\tau}_1 \cdot \vec{\tau}_2 W_S(r)\right] \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

$$+ \left[V_T(r) + \vec{\tau}_1 \cdot \vec{\tau}_2 W_T(r)\right] (3\vec{\sigma}_1 \cdot \hat{r}\vec{\sigma}_2 \cdot \hat{r} - \vec{\sigma}_1 \cdot \vec{\sigma}_2) + \left[V_{LS}(r) + \vec{\tau}_1 \cdot \vec{\tau}_2 W_{LS}(r)\right] \vec{L} \cdot \vec{S}\,,$$

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The profile functions (in Dimensional Regularization)

$$V_C^{TPE}(r) = \frac{3g^2m^6}{32\pi^2 f^4}\frac{e^{-2x}}{x^6}\left\{\left(2c_1 + \frac{3g^2}{16M}\right)x^2(1+x)^2 + \frac{g^5x^5}{32M} + \left(c_3 + \frac{3g^2}{16M}\right)(6+12x+10x^2+4x^3+x^4)\right\}$$

$$W_T^{TPE}(r) = \frac{g^2m^6}{48\pi^2 f^4}\frac{e^{-2x}}{x^6}\left\{-\left(c_4 + \frac{1}{4M}\right)(1+x)(3+3x+x^2) + \frac{g^2}{32M}(36+72x+52x^2+17x^3+2x^4)\right\},$$

$$V_T^{TPE}(r) = \frac{g^4m^5}{128\pi^3 f^4 x^4}\left\{-12K_0(2x) - (15+4x^2)K_1(2x) + \frac{3\pi m e^{-2x}}{8Mx}(12x^{-1}+24+20x+9x^2+2x^3)\right\},$$

$$W_C^{TPE}(r) = \frac{g^4m^5}{128\pi^3 f^4 x^4}\left\{\left[1+2g^2(5+2x^2) - g^4(23+12x^2)\right]K_1(2x) + x\left[1+10g^2 - g^4(23+4x^2)\right]K_0(2x)\,,\right.$$
$$\left. + \frac{g^2 m\pi e^{-2x}}{4Mx}\left[2(3g^2-2)\left(6x^{-1}+12+10x+4x^2+x^3\right)\right] + g^2x\left(2+4x+2x^2+3x^2\right)\right\},$$

$$V_S^{TPE}(r) = \frac{g^4m^5}{32\pi^3 f^4}\left\{3xK_0(2x) + (3+2x^2)K_1(2x) - \frac{3\pi m e^{-2x}}{16Mx}(6x^{-1}+12+11x+6x^2+2x^3)\right\},$$

$$W_S^{TPE}(r) = \frac{g^2m^6}{48\pi^2 f^4}\frac{e^{-2x}}{x^6}\left\{\left(c_4 + \frac{1}{4M}\right)(1+x)(3+3x+2x^2) - \frac{g^2}{16M}(18+36x+31x^2+14x^3+2x^4)\right\},$$

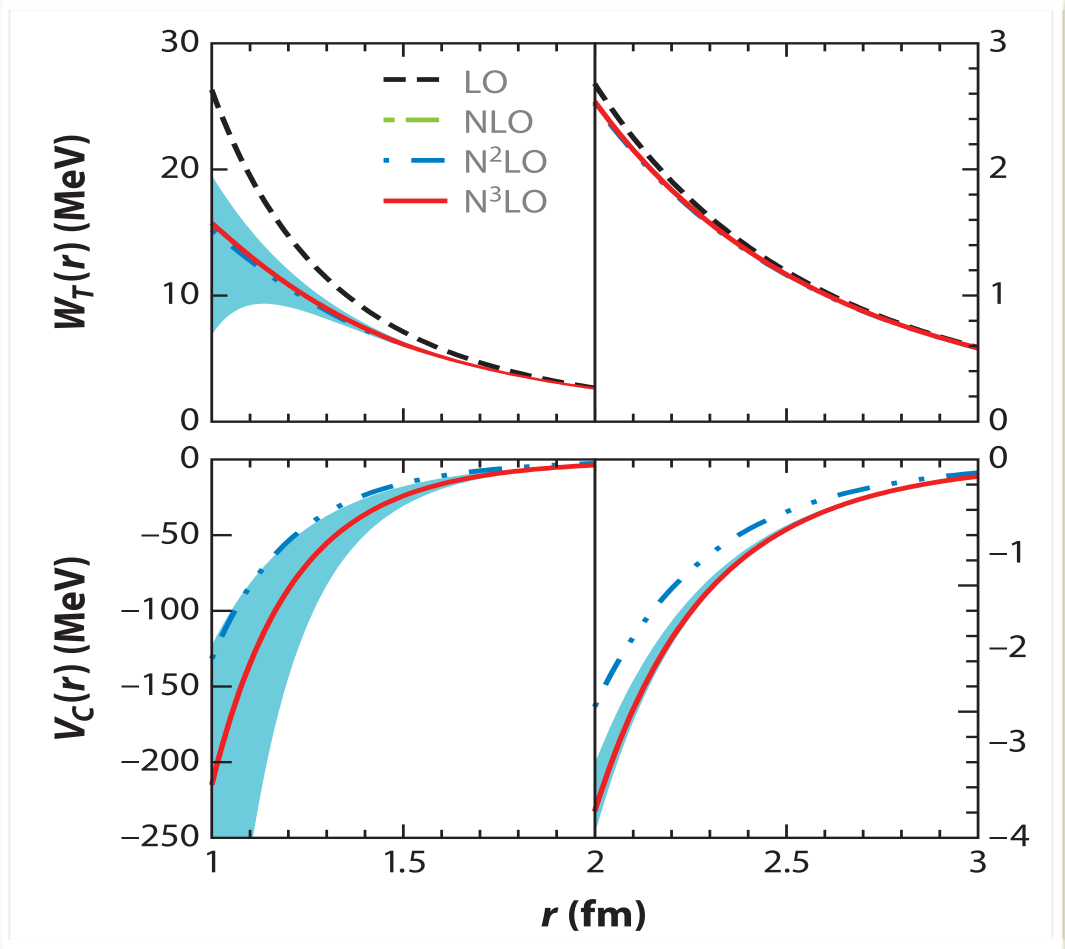
$$V_{LS}^{TPE}(r) = -\frac{3g^4m^6}{64\pi^2 M f^4}\frac{e^{-2x}}{x^6}(1+x)\left(2+2x+x^2\right)\,,$$

$$W_{LS}^{TPE}(r) = \frac{g^2(g^2-1)m^6}{32\pi^2 M f^4}\frac{e^{-2x}}{x^6}(1+x)^2\,,$$

Chiral expansion of the 2π exchange

Kaiser, Brockmann, Weise ’97

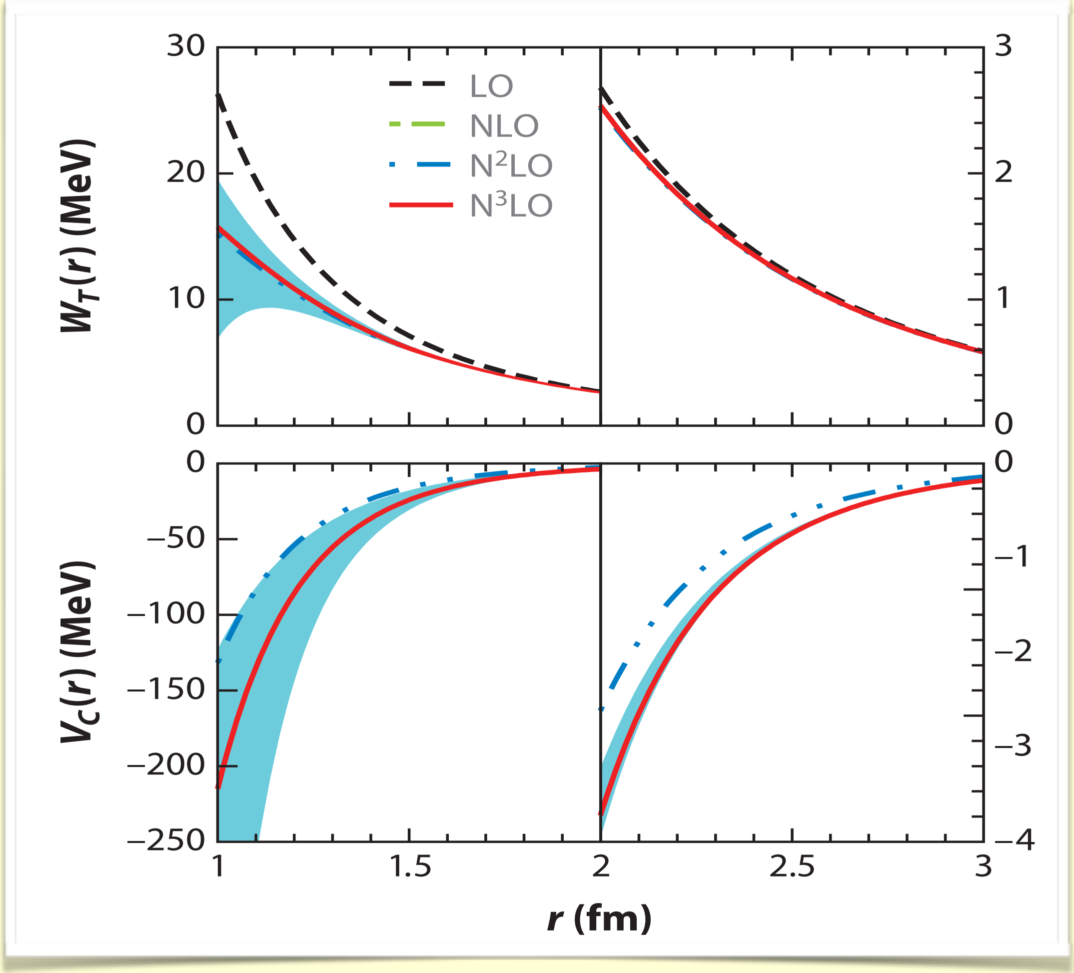
$$\mathcal{V}_{NN} = V_C(r) + \vec{\tau}_1\cdot\vec{\tau}_2 W_C(r) + \left[V_S(r) + \vec{\tau}_1\cdot\vec{\tau}_2 W_S(r)\right]\vec{\sigma}_1\cdot\vec{\sigma}_2$$
$$+ \left[V_T(r) + \vec{\tau}_1\cdot\vec{\tau}_2 W_T(r)\right](3\vec{\sigma}_1\cdot\hat{r}\vec{\sigma}_2\cdot\hat{r} - \vec{\sigma}_1\cdot\vec{\sigma}_2) + \left[V_{LS}(r) + \vec{\tau}_1\cdot\vec{\tau}_2 W_{LS}(r)\right]\vec{L}\cdot\vec{S}\,,$$



Chiral expansion of the 2π exchange

Kaiser, Brockmann, Weise ’97

$$\mathcal{V}_{NN} = V_C(r) + \vec{\tau}_1\cdot\vec{\tau}_2 W_C(r) + [V_S(r) + \vec{\tau}_1\cdot\vec{\tau}_2 W_S(r)]\,\vec{\sigma}_1\cdot\vec{\sigma}_2$$
$$+ [V_T(r) + \vec{\tau}_1\cdot\vec{\tau}_2 W_T(r)]\,(3\vec{\sigma}_1\cdot\hat{r}\vec{\sigma}_2\cdot\hat{r} - \vec{\sigma}_1\cdot\vec{\sigma}_2) + [V_{LS}(r) + \vec{\tau}_1\cdot\vec{\tau}_2 W_{LS}(r)]\,\vec{L}\cdot\vec{S}\,,$$



Is there any evidence from NN data?

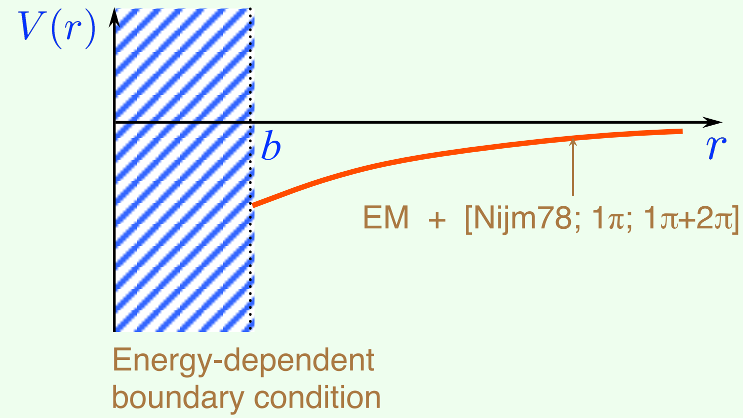
Chiral two-pion exchange and NN data

Nijmegen Partial Wave Analysis

Rentmeester et al.'99,'03

Number of BC parameters needed to achieve $\chi^2_{\text{datum}} \sim 1$ for a given long-range part (input)

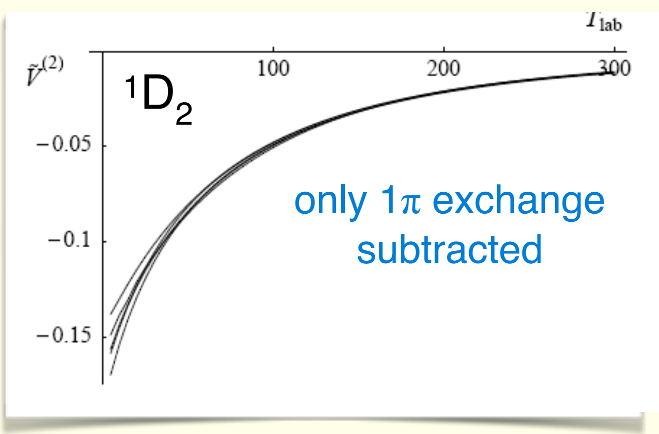
31 (1π) → 28 ($1\pi + 2\pi$ [NLO]) → 23 ($1\pi + 2\pi$ [N^2LO])

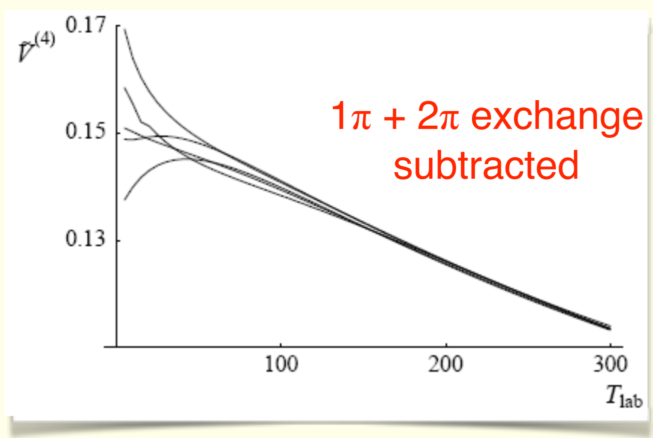


„Deconstructing" neutron-proton phase shufts

Birse, McGovern '06

Idea: Subtract effects of the long-range interaction from phase shifts (DWBA) and look at the residual energy dependence



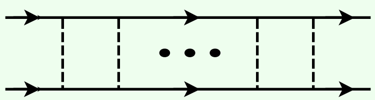


How to renormalize the Schrödinger Eq?

Lowest-order NN potential: $V_{2N}^{(0)} = -\frac{g_A^2}{4F_\pi^2}\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{\vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_2 \cdot \vec{q}}{\vec{q}^{\,2} + M_\pi^2} + C_S + C_T \vec{\sigma}_1 \cdot \vec{\sigma}_2$

Complication: iterations of the LS equation

$$T(\vec{p}\,', \vec{p}) = V_{2N}^{(0)}(\vec{p}\,', \vec{p}) + \int \frac{d^3k}{(2\,\pi)^3} V_{2N}^{(0)}(\vec{p}\,', \vec{k})\, \frac{m_N}{p^2 - k^2 + i\,\epsilon}\, T(\vec{k}, \vec{p})$$



generate divergences whose subtraction requires infinitely many CTs beyond $V_{2N}^{(0)}$

Kaplan, Savage, Wise, Fleming, Mehen, Stewart, Phillips, Beane, Cohen, Frederico, Timoteo, Tomio, Birse, Beane, Bedaque, van Kolck, Pavon Valderrama, Ruiz Arriola, Nogga, Timmermanns, EE, Meißner, Entem, Machleidt, Yang, Elster, Long, Gegelia, ...

$\longrightarrow$ use a **finite** cutoff, self-consistency checks via „Lepage plots“

A new, renormalizable approach (yet to be explored...) EE, Gegelia ’12

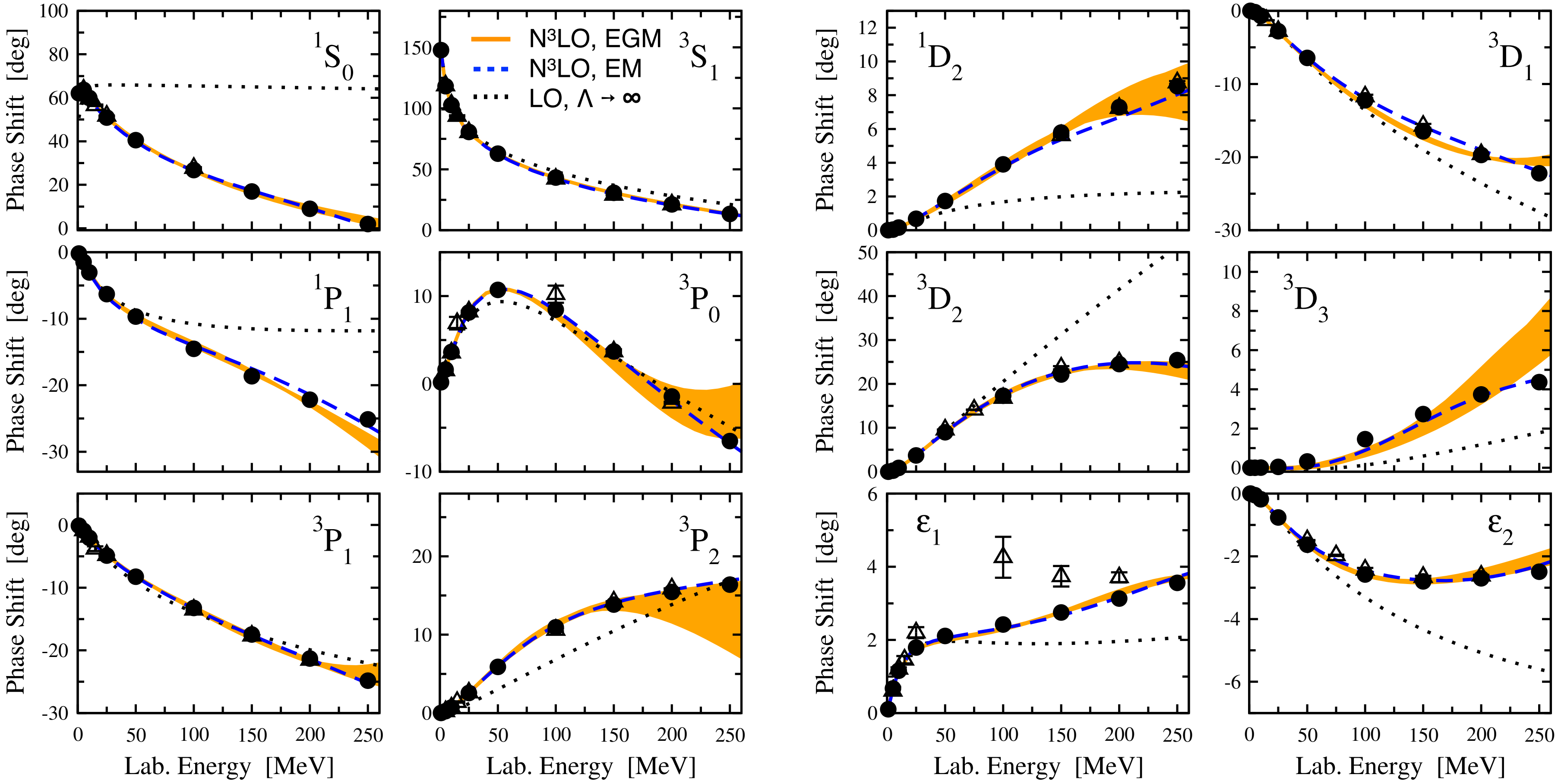
- non-renormalizability of the LO equation is an artifact of the nonrelativistic expansion
- renormalizable LO equation based on manifestly Lorentz-invariant Lagrangian

$$T(\vec{p}\,', \vec{p}) = V_{2N}^{(0)}(\vec{p}\,', \vec{p}) + \frac{m_N^2}{2} \int \frac{d^3k}{(2\,\pi)^3} \frac{V_{2N}^{(0)}(\vec{p}\,', \vec{k})\, T(\vec{k}, \vec{p})}{(k^2 + m_N^2)\,(E - \sqrt{k^2 + m_N^2} + i\,\epsilon)}$$

- higher-order corrections (e.g. two-pion exchange) to be treated perturbatively in progress...

Neutron-proton phase shifts at N^3LO

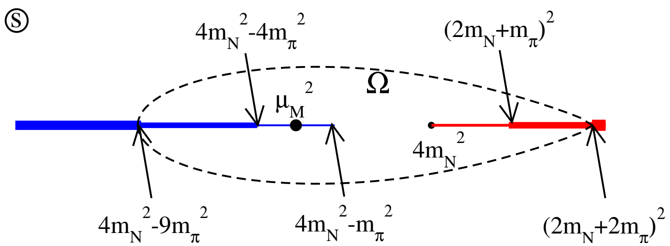
Entem, Machleidt '04; E.E., Glöckle, Meißner '05



Current topics & ongoing developments

Renormalitazion and power counting
van Kolck, Pavon Valderrama, Brise, Gegelia, EE, Machleidt, …

Merging chiral EFT with dispersion relations
Albaladejo, Oller ’11,’12; Gasparyan, EE, Lutz ’12; Guo, Oller, Rios ’13



- Calculate the discontinuity of the amplitude along the left-hand cut using ChPT
- Reconstruct the amplitude in the physical region using dispersion relations + analytic cont. (conformal mapping)

Generalization to the SU(3) sector
Haidenbauer, Meißner, Kaiser, Petschauer, Nogga, …

Nuclear parity violation
Schindler, Viviani, Kievski, Girlanda, de Vries, van Kolck, Kaiser, Meißner, EE, …

Partial wave analysis
Rentmeester et al., Birse, McGovern, Navarro Perez, Ruiz Arriola et al.

- Role of 2π-exchange
- Error propagation in nuclear observables

New generation of chiral NN potentials

Current topics & ongoing developments

Renormalitazion and power counting

van Kolck, Pavon Valderrama, Brise, Gegelia, EE, Machleidt, …

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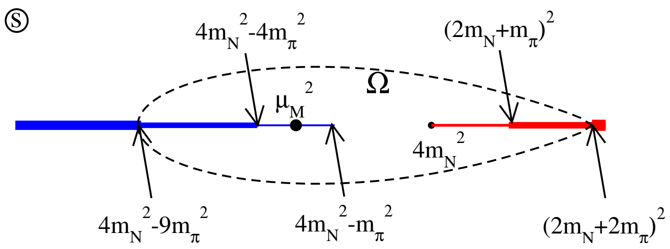

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Partial wave analysis

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- Role of 2π-exchange
- Error propagation in nuclear observables

New generation of chiral NN potentials

New chiral NN interactions

Already available:

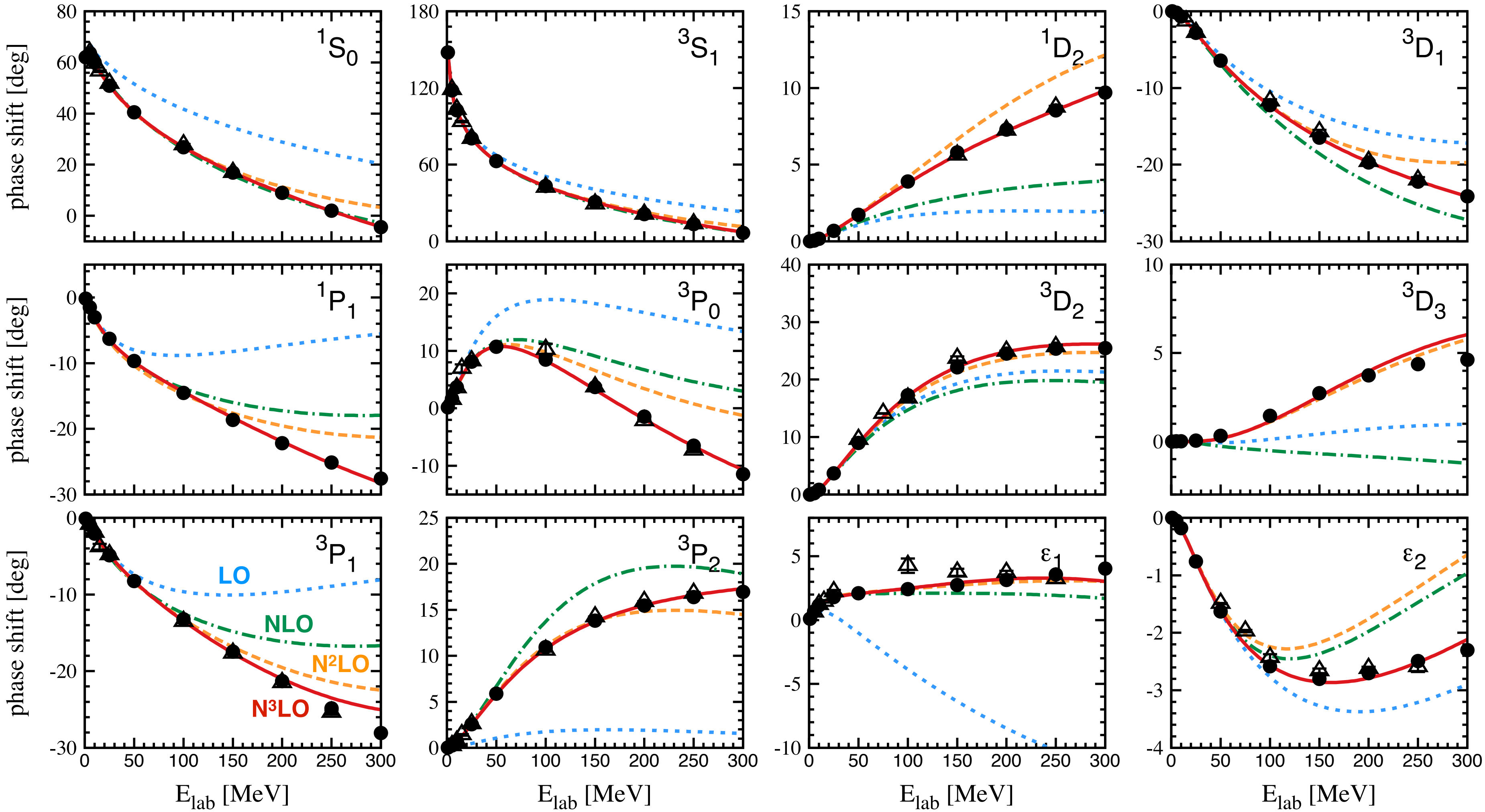
- Optimized N^{2}LO chiral nuclear force (tune LECs to reduce the impact of 3NF in the > 2N systems) Ekström, Baardsen, et al. '13. Justified from EFT point of view?
- Fully local potentials @ LO, NLO, N^{2}LO [R_0 = 1.0, 1.1 and 1.2 fm and Λ_{SFR} = 0.8…1.4 GeV] Gezerlis, Tews, EE, Gandolfi, Hebeler, Nogga, Schwenk, PRL 111 (13) 032501; Gezerlis, Tews, EE, Freunek, Gandolfi, Hebeler, Nogga, Schwenk, arXiv:1406.0454; Lynn, Carlson, EE, Gandolfi, Gezerlis, Schwenk, arXiv:1406.2787

In development/testing [in collaboration with: Krebs, Nogga, Meißner, Golak, Skibinski, Witala, Kamada]

- New version of **l**ocal**-chiral** potentials @ LO, NLO, N^{2}LO [Λ_{SFR} up to Infinity, PWD MEs and operator form both in r-space and p-space]
- New **i**mproved**-chiral** potentials up to N^{3}LO [Λ_{SFR} up to Infinity, PWD MEs and operator form in p-space]
  - Local regulator preserves the analytic structure of the amplitude and allows to minimize cutoff artifacts → better performance at high energies!
  - No need for SFR cutoff, can accommodate for LECs from πN

i-chiral 2NF: Order-by-order improvement

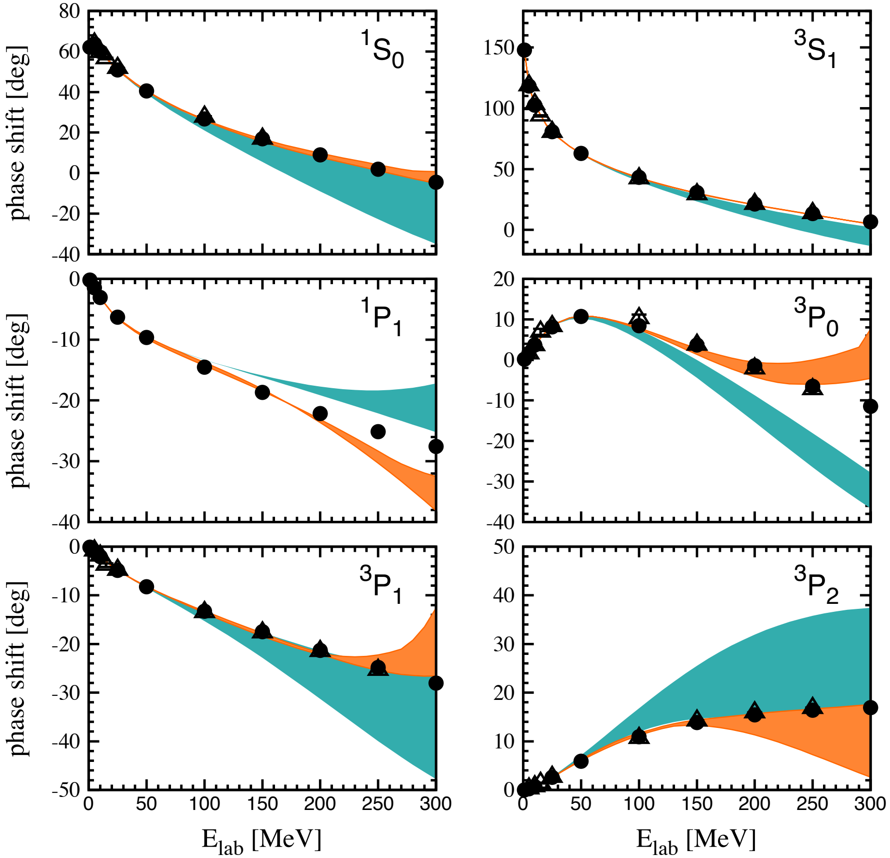
neutron-proton phase shifts on i-chiral 2NF at LO, NLO, N^2LO and N^3LO (w.o. 1/m)



R_0 = 0.9 fm, Λ_{SFR} = Infinity [i.e. DR]

Cutoff dependence: i-chiral vs old EGM'04

np phase shifts based on EGM'04 N²LO/N³LO 2NF

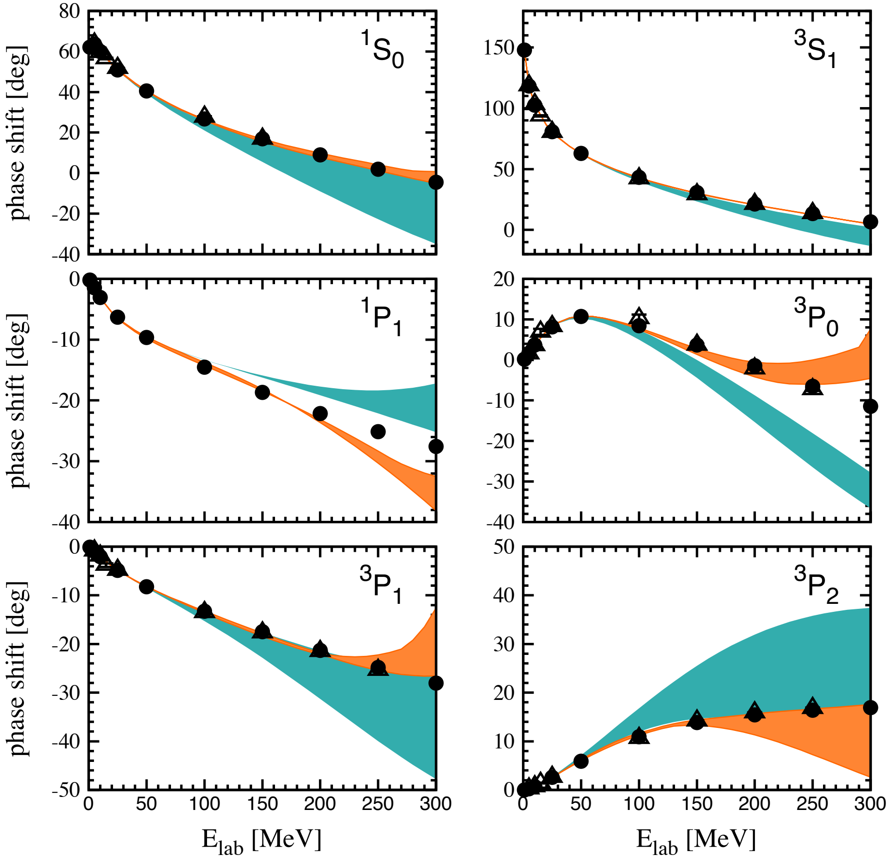


N²LO: Λ = 450…600 MeV, Λ_{SFR} =500…700 MeV

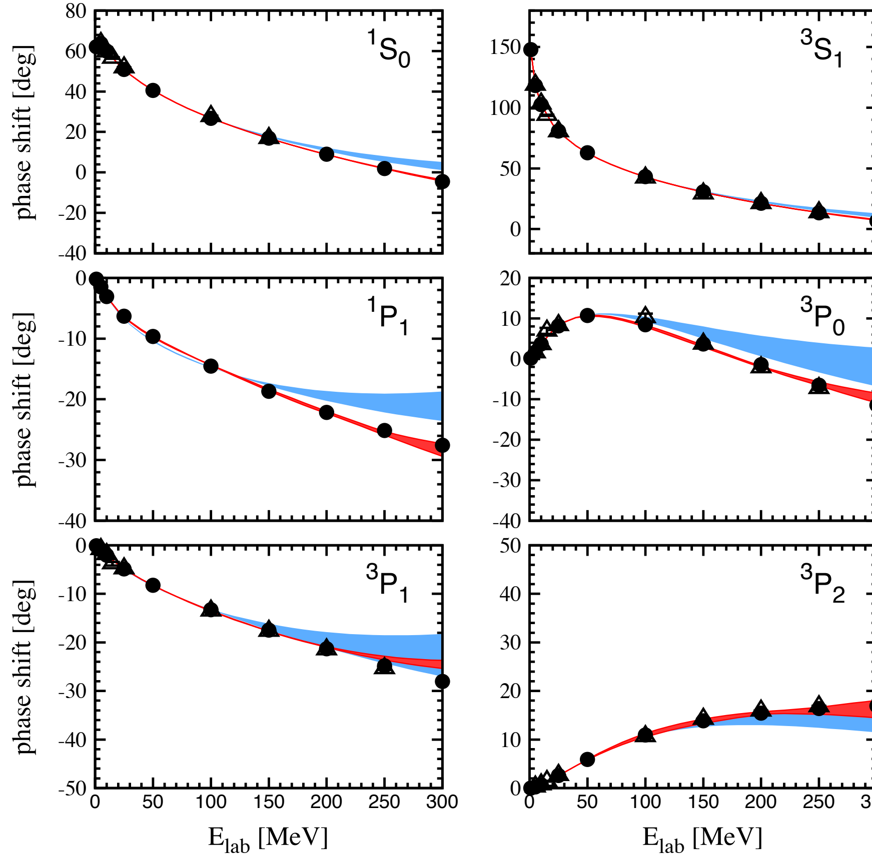
N³LO: Λ = 450…600 MeV, Λ_{SFR} =500…700 MeV

Cutoff dependence: i-chiral vs old EGM'04

np phase shifts based on EGM'04 N²LO/N³LO 2NF



np phase shifts based on i-chiral N²LO/N³LO 2NF



N²LO: Λ = 450…600 MeV, Λ_{SFR} =500…700 MeV

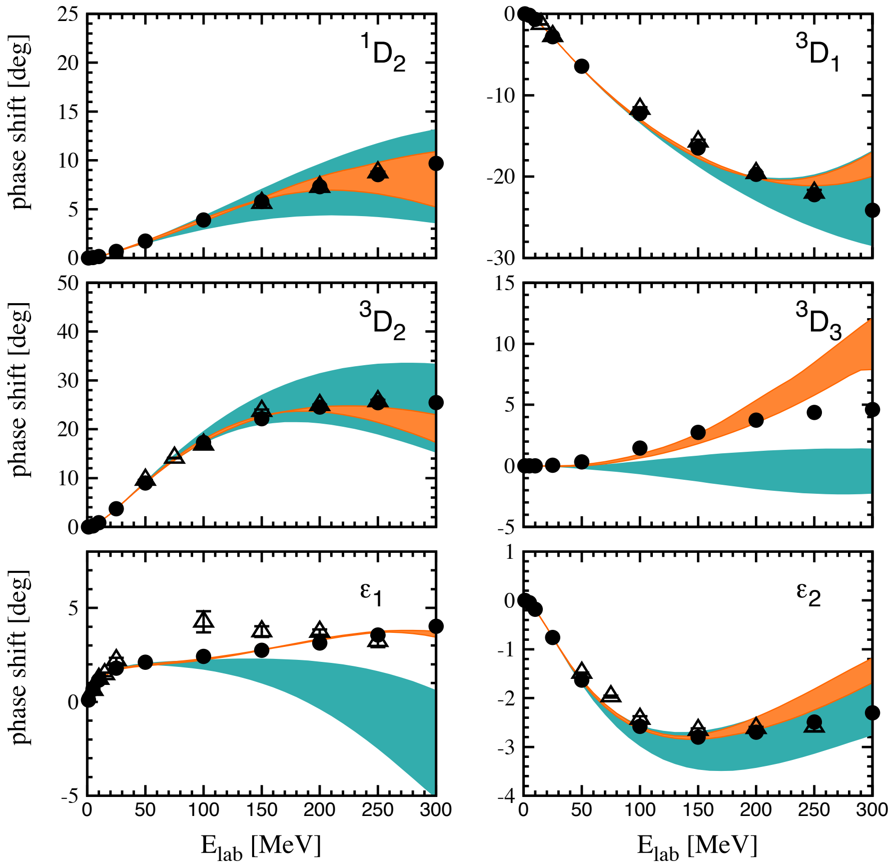
N³LO: Λ = 450…600 MeV, Λ_{SFR} =500…700 MeV

N²LO: R_0 = 0.8…1.0 fm, Λ_{SFR} =1GeV…Infinity

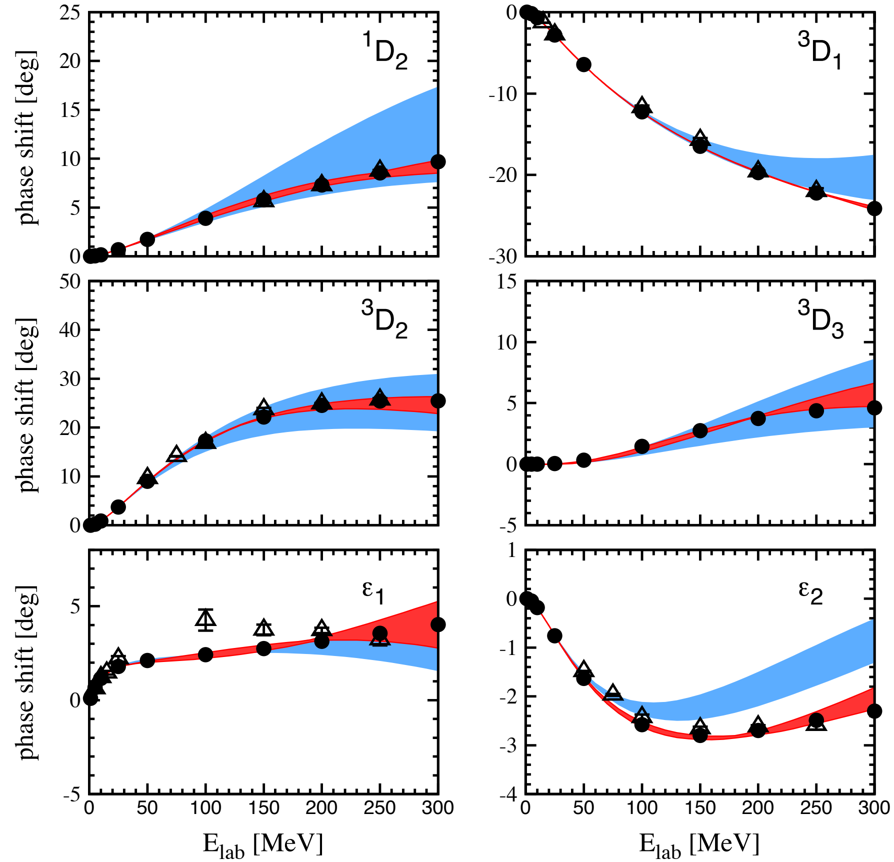
N³LO: R_0 = 0.8…1.1 fm, Λ_{SFR} =1GeV…Infinity
LECs from Q⁴ KH πN

Cutoff dependence: i-chiral vs old EGM'04

np phase shifts based on EGM'04 N^2LO/N^3LO 2NF



np phase shifts based on <u>i-chiral</u> N^2LO/N^3LO 2NF



N^2LO: $\Lambda = 450 \ldots 600$ MeV, $\Lambda_{SFR} = 500 \ldots 700$ MeV

N^3LO: $\Lambda = 450 \ldots 600$ MeV, $\Lambda_{SFR} = 500 \ldots 700$ MeV

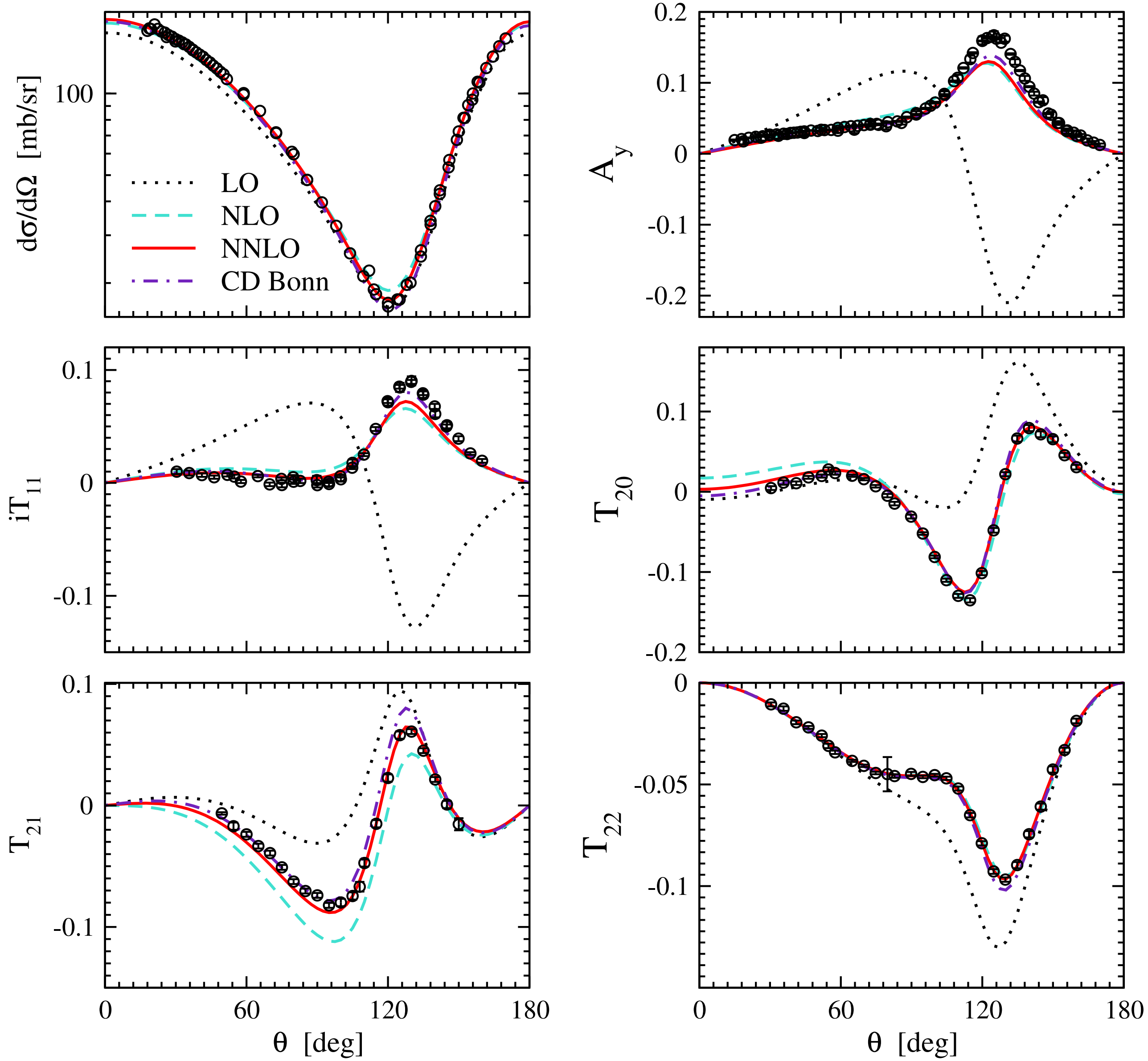
N^2LO: $R_0 = 0.8 \ldots 1.0$ fm, $\Lambda_{SFR} = 1$GeV…Infinity

N^3LO: $R_0 = 0.8 \ldots 1.1$ fm, $\Lambda_{SFR} = 1$GeV…Infinity
LECs from Q^4 KH πN

I-chiral 2NF: elastic nd scattering order-by-order

EE, Golak, Kamada, Krebs, Meißner, Nogga, Skibinski, Witala, in preparation

3 MeV:



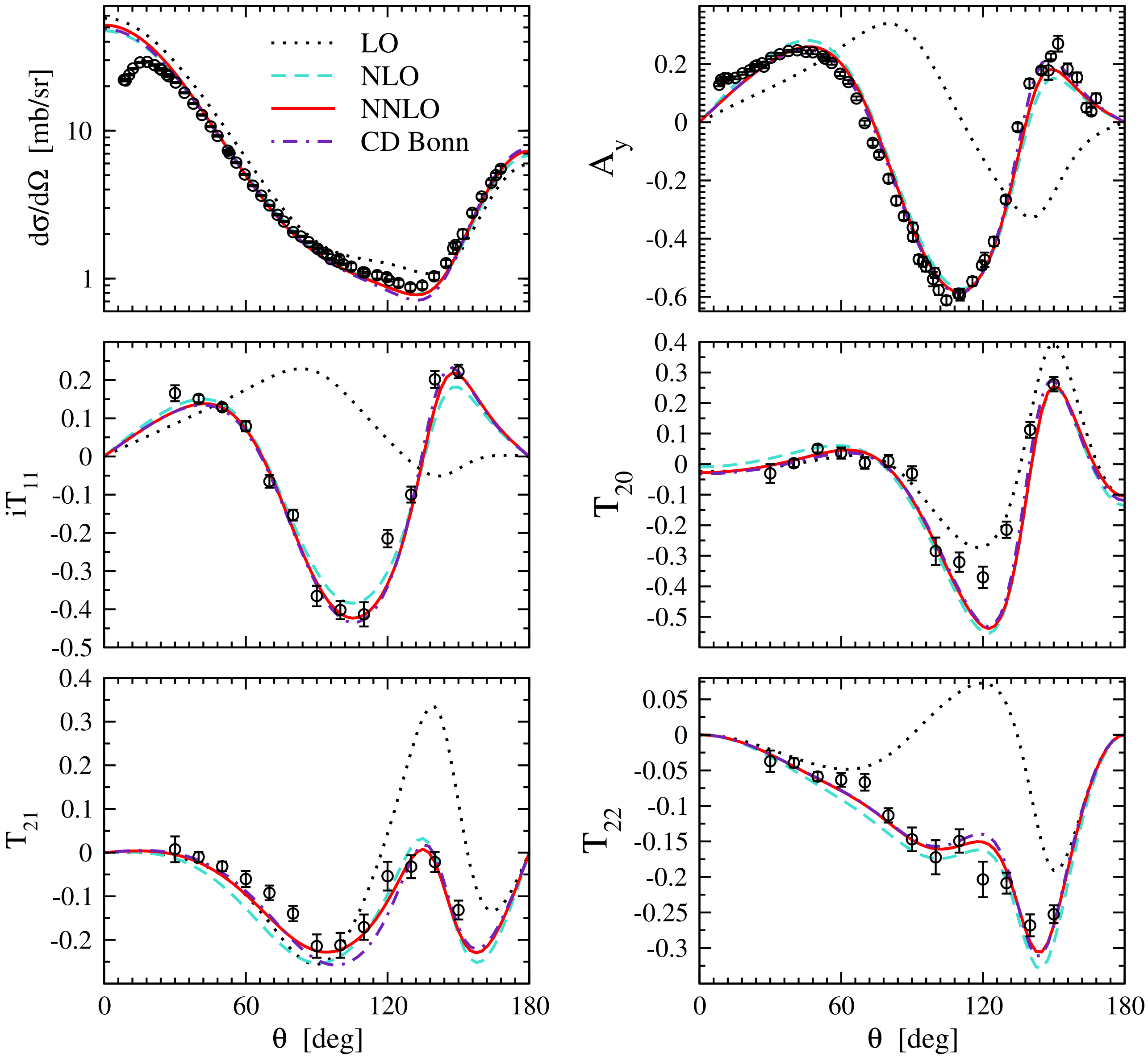
$R_0 = 1$ fm

$\Lambda_{SFR} = 2$ GeV

I-chiral 2NF: elastic nd scattering order-by-order

EE, Golak, Kamada, Krebs, Meißner, Nogga, Skibinski, Witala, in preparation

65 MeV:

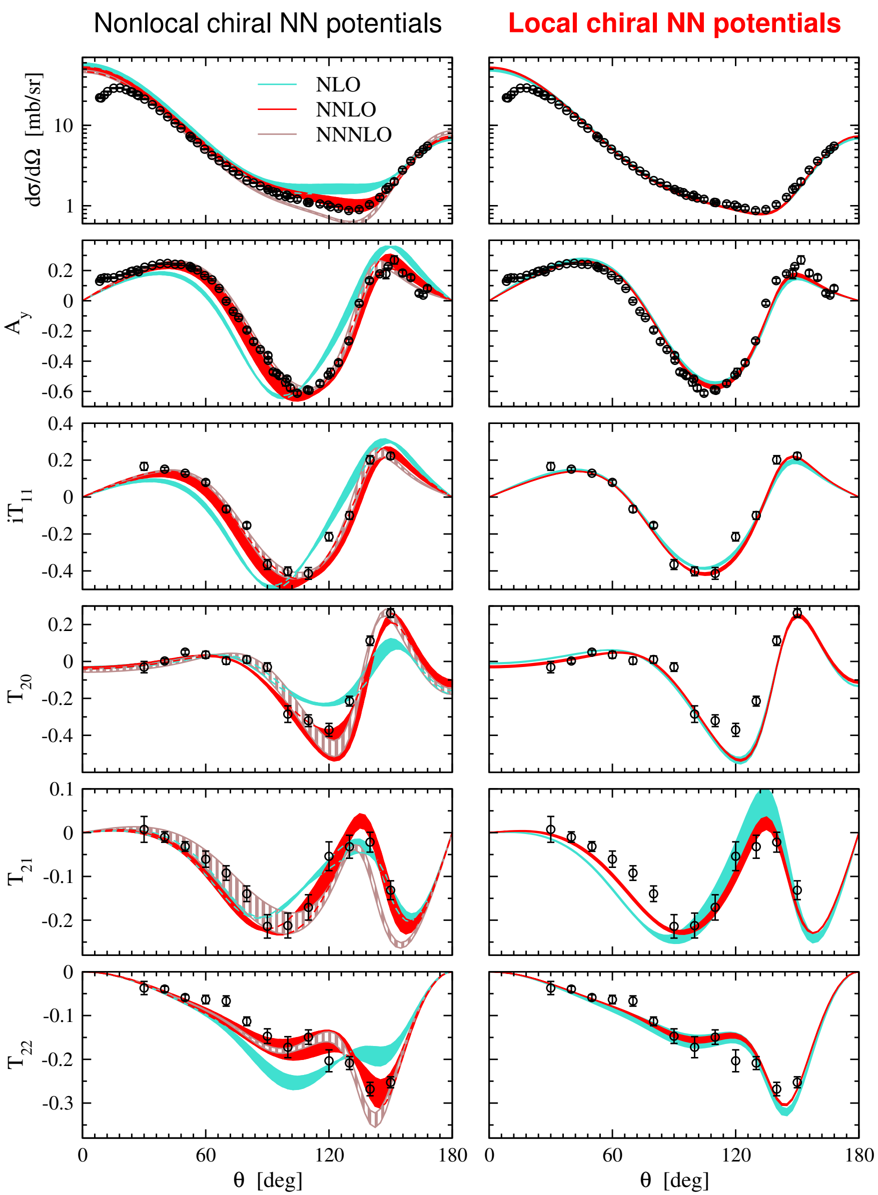


$R_0 = 1$ fm
$\Lambda_{SFR} = 2$ GeV

nd scattering with l-chiral 2NF: Cutoff dependence

EE, Golak, Kamada, Krebs, Meißner, Nogga, Skibinski, Witala, in preparation

65 MeV:



nonlocal NLO/N²LO/N³LO:

Λ = 450…600 MeV,

Λ_{SFR} = 500…700 MeV

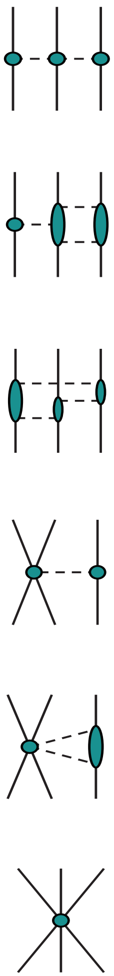
local NLO/N²LO:

R_0 = 1…1.2 fm,

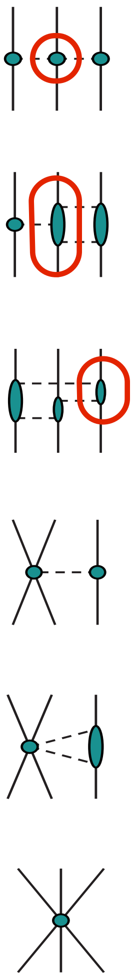
Λ_{SFR} = 1…2 GeV

Three-nucleon force:
Status and ongoing developments

Chiral expansion of the 3NF (Δ-less EFT)

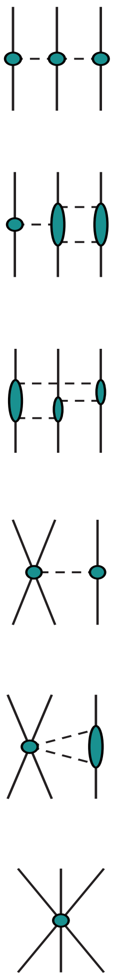


Chiral expansion of the 3NF (Δ-less EFT)



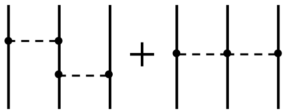
3NF structure functions at large distance are
model-independent and parameter-free predictions
based on χ symmetry of QCD + exp. information on πN system

Chiral expansion of the 3NF (Δ-less EFT)



Chiral expansion of the 3NF (Δ-less EFT)

NLO (Q^2)



Weinberg ’91, van Kolck ’94

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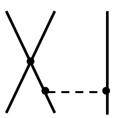

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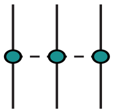
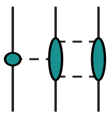
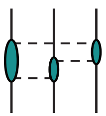
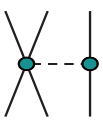
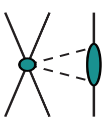
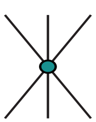
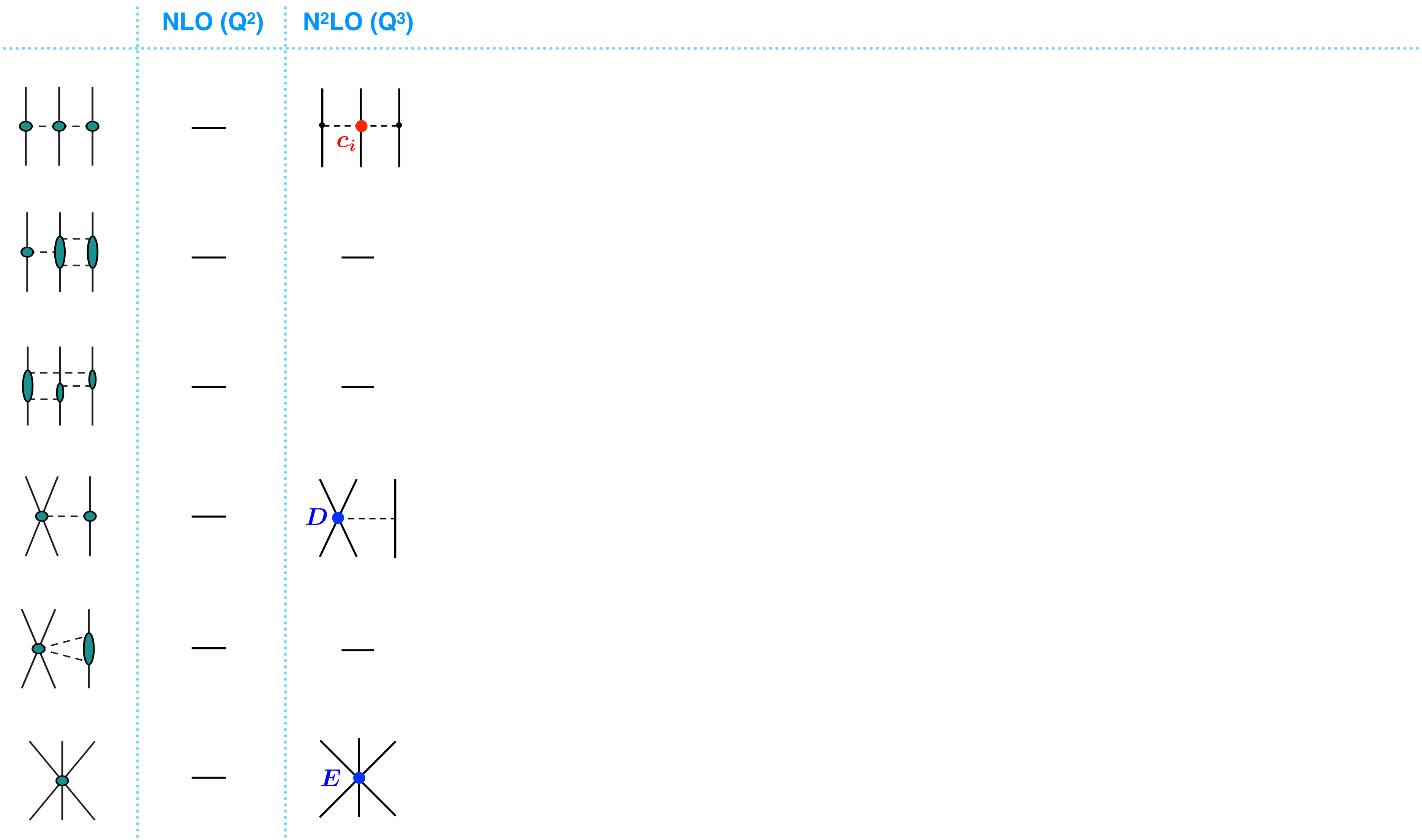
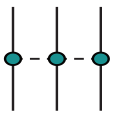
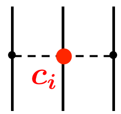
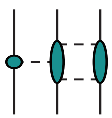
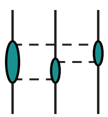
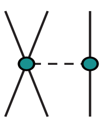
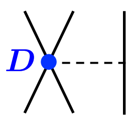
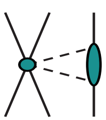
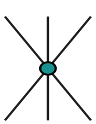
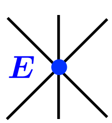
Chiral expansion of the 3NF (Δ-less EFT)

	NLO (Q^2)
	—
	—
	—
	—
	—
	—

Chiral expansion of the 3NF (Δ-less EFT)

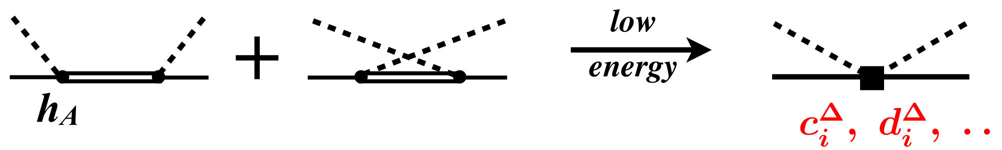


Chiral expansion of the 3NF (Δ-less EFT)

	NLO (Q^2)	N^2LO (Q^3)
	—	
	—	—
	—	—
	—	
	—	—
	—	

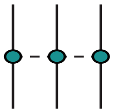
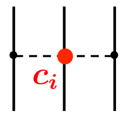
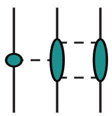
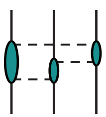
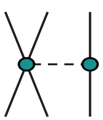
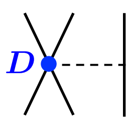
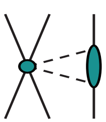
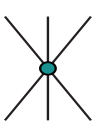
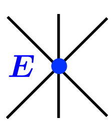
Notice: c_i receive large Δ(1232) contributions

Bernard, Kaiser, Meißner '97

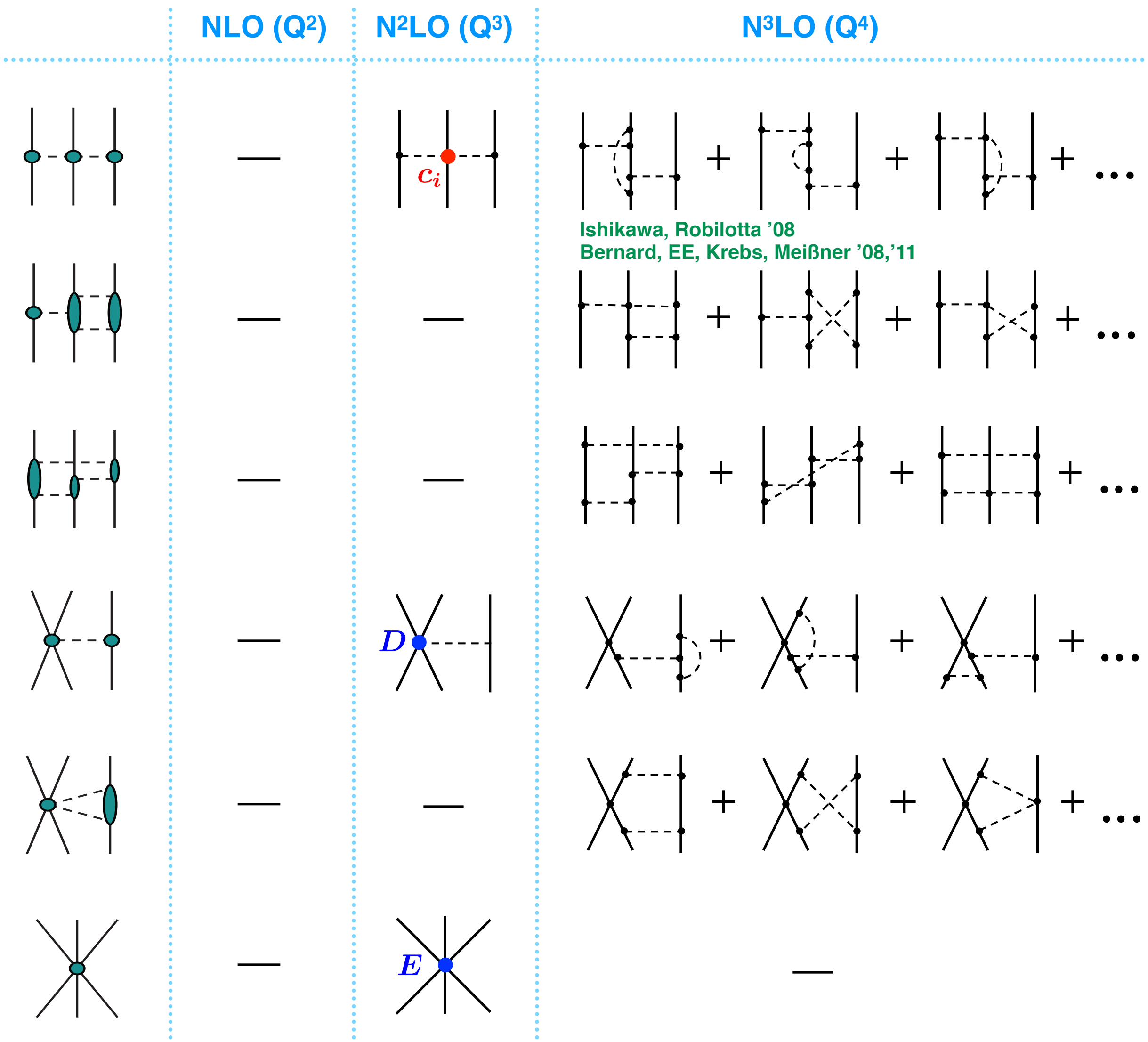


$$c_2^\Delta = -c_3^\Delta = 2c_4^\Delta = \frac{4h_A^2}{9(m_\Delta - m_N)} \simeq 2.8\,\mathrm{GeV}^{-1}$$

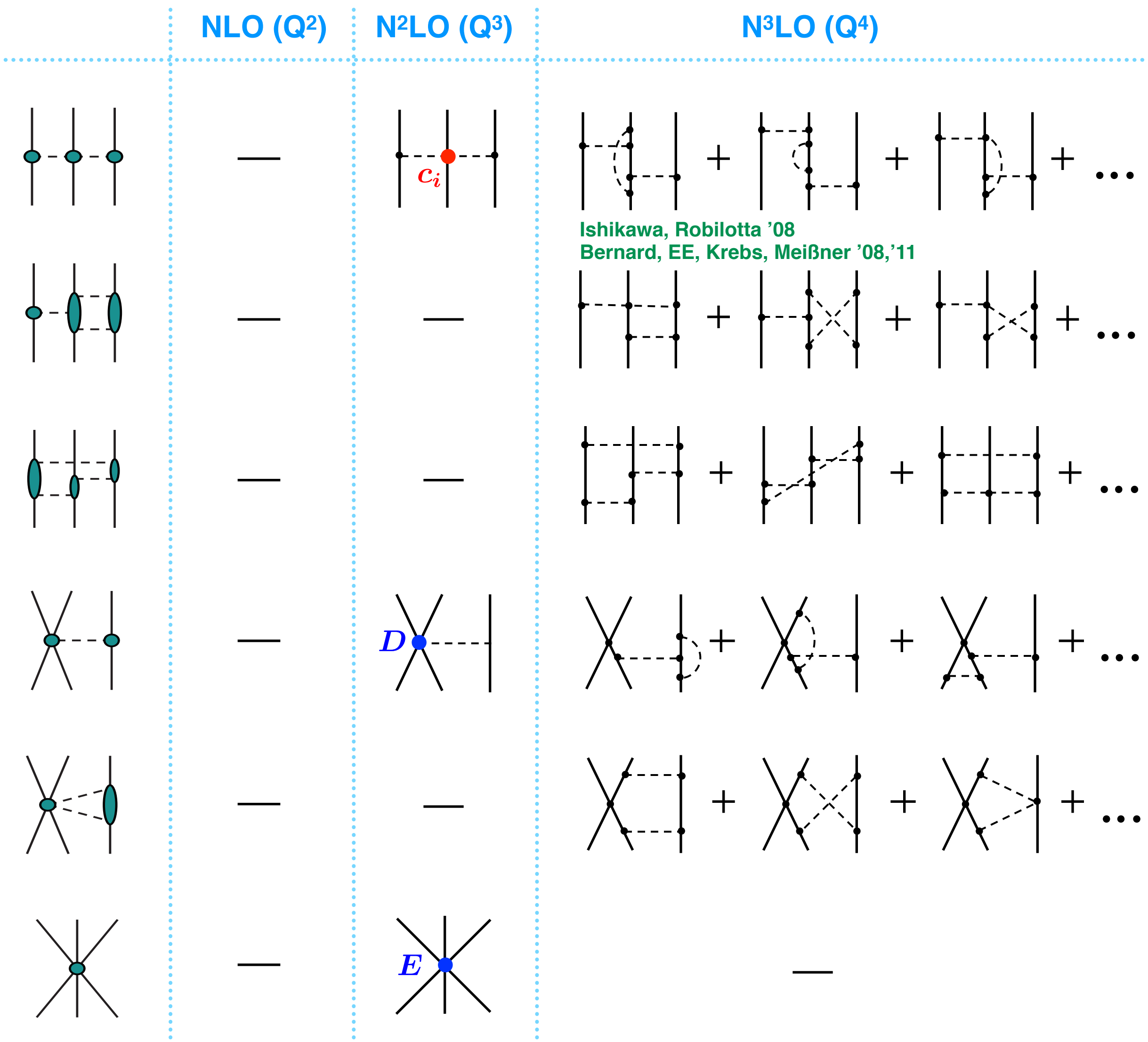
Chiral expansion of the 3NF (Δ-less EFT)

	NLO (Q^2)	N^2LO (Q^3)
	—	
	—	—
	—	—
	—	
	—	—
	—	

Chiral expansion of the 3NF (Δ-less EFT)

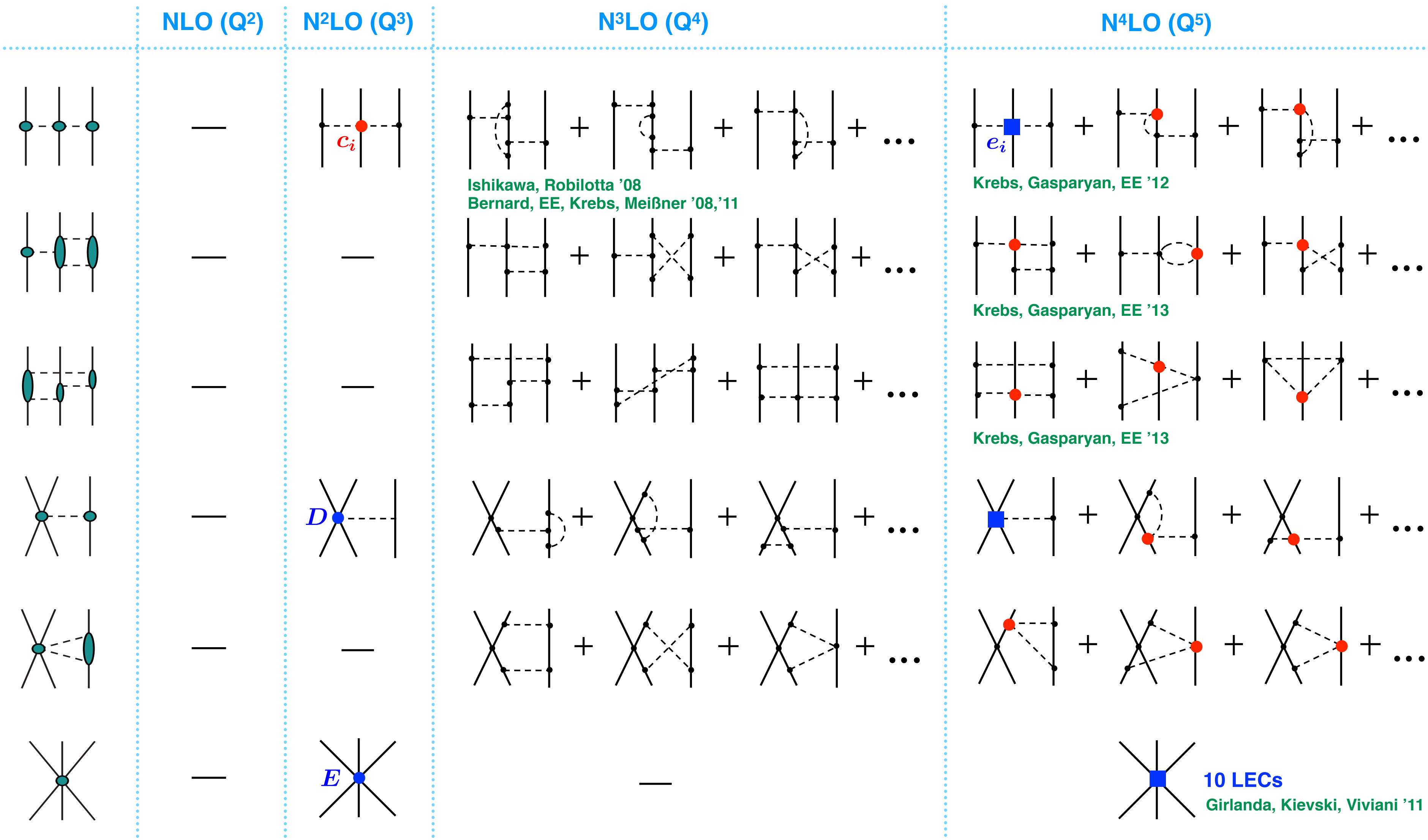


Chiral expansion of the 3NF (Δ-less EFT)



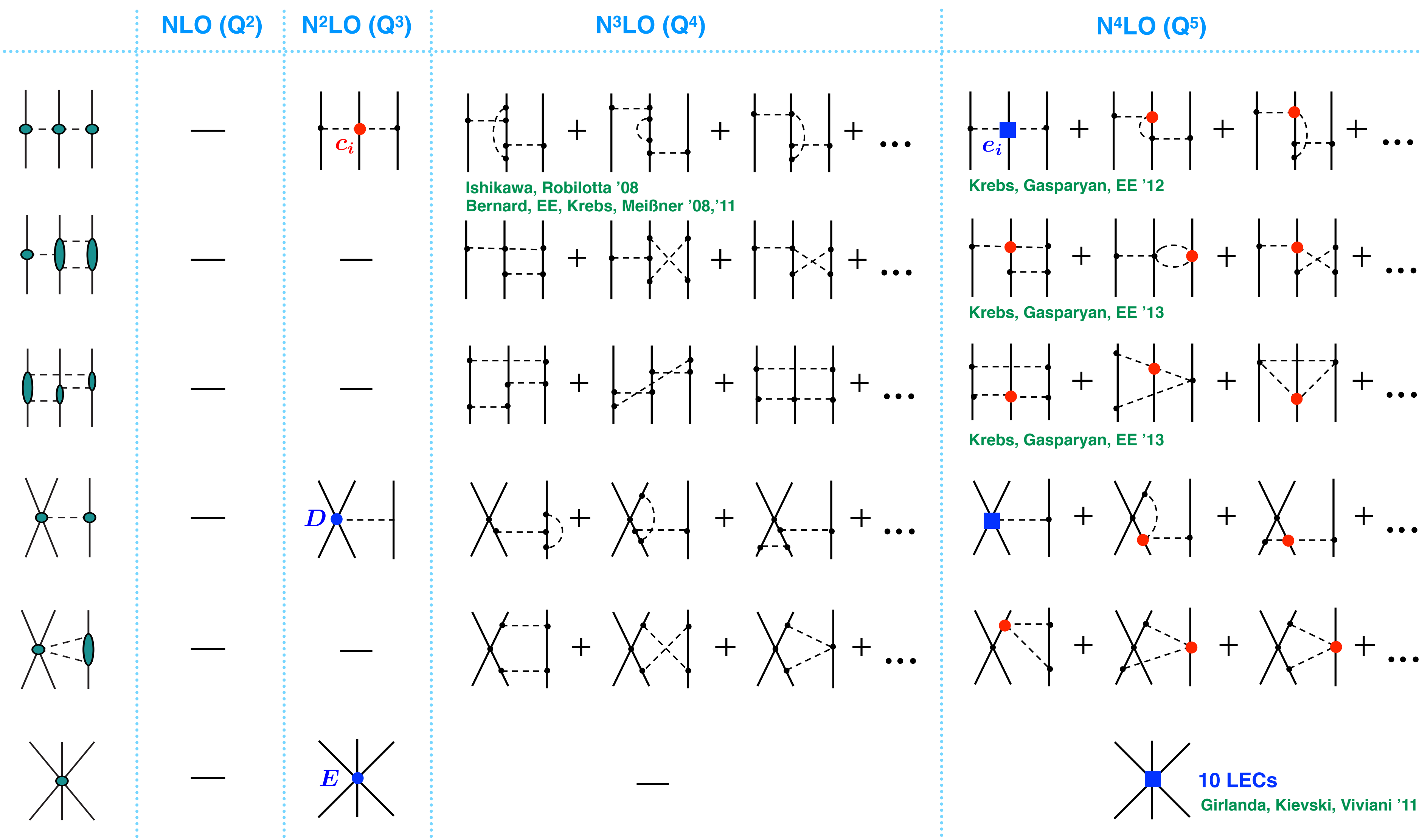
- parameter-free!
- Δ-effects are missing (except for the 2π 3NF) → expect large N⁴LO corrections

Chiral expansion of the 3NF (Δ-less EFT)



- parameter-free!
- Δ-effects are missing (except for the 2π 3NF) → expect large N^{4}LO corrections

Chiral expansion of the 3NF (Δ-less EFT)

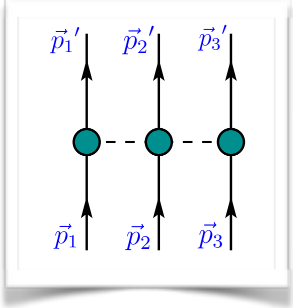


- parameter-free!
- Δ-effects are missing (except for the 2π 3NF) → expect large N⁴LO corrections

- long range parameter-free (after determination of LECs in πN)
- converged?? (graphs ~ c_i^2, c_i^3…)

Longest-range 3NF up to N^{4}LO

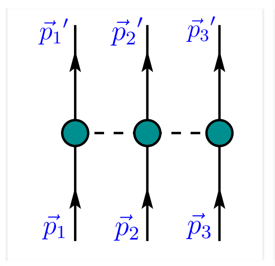
The TPE 3NF has the form (modulo 1/m-terms):



$$V_{2\pi} = \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \, \vec{\sigma}_3 \cdot \vec{q}_3}{[q_1^2 + M_\pi^2]\,[q_3^2 + M_\pi^2]} \Big(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \mathcal{A}(q_2) + \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_2 \, \vec{q}_1 \times \vec{q}_3 \cdot \vec{\sigma}_2 \, \mathcal{B}(q_2) \Big)$$

Longest-range 3NF up to N⁴LO

The TPE 3NF has the form (modulo 1/m-terms):



$$V_{2\pi} = \frac{\vec{\sigma}_1 \cdot \vec{q}_1 \, \vec{\sigma}_3 \cdot \vec{q}_3}{[q_1^2 + M_\pi^2]\,[q_3^2 + M_\pi^2]} \Big(\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \mathcal{A}(q_2) + \boldsymbol{\tau}_1 \times \boldsymbol{\tau}_3 \cdot \boldsymbol{\tau}_2 \, \vec{q}_1 \times \vec{q}_3 \cdot \vec{\sigma}_2 \, \mathcal{B}(q_2) \Big)$$

- N²LO [Q³]:
  van Kolck '94

$$\mathcal{A}^{(3)}(q_2) = \frac{g_A^2}{8F_\pi^4}\Big((2c_3 - 4c_1)M_\pi^2 + c_3 q_2^2\Big)\,, \qquad \mathcal{B}^{(3)}(q_2) = \frac{g_A^2 c_4}{8F_\pi^4}$$

- N³LO [Q⁴]:

$$\mathcal{A}^{(4)}(q_2) = \frac{g_A^4}{256\pi F_\pi^6}\Big[A(q_2)\left(2M_\pi^4 + 5M_\pi^2 q_2^2 + 2q_2^4\right) + \left(4g_A^2 + 1\right)M_\pi^3 + 2\left(g_A^2 + 1\right)M_\pi q_2^2\Big]\,,$$

$$\mathcal{B}^{(4)}(q_2) = -\frac{g_A^4}{256\pi F_\pi^6}\Big[A(q_2)\left(4M_\pi^2 + q_2^2\right) + (2g_A^2 + 1)M_\pi\Big]$$

Ishikawa, Robilotta '07
Bernard, EE, Krebs, Meißner '08

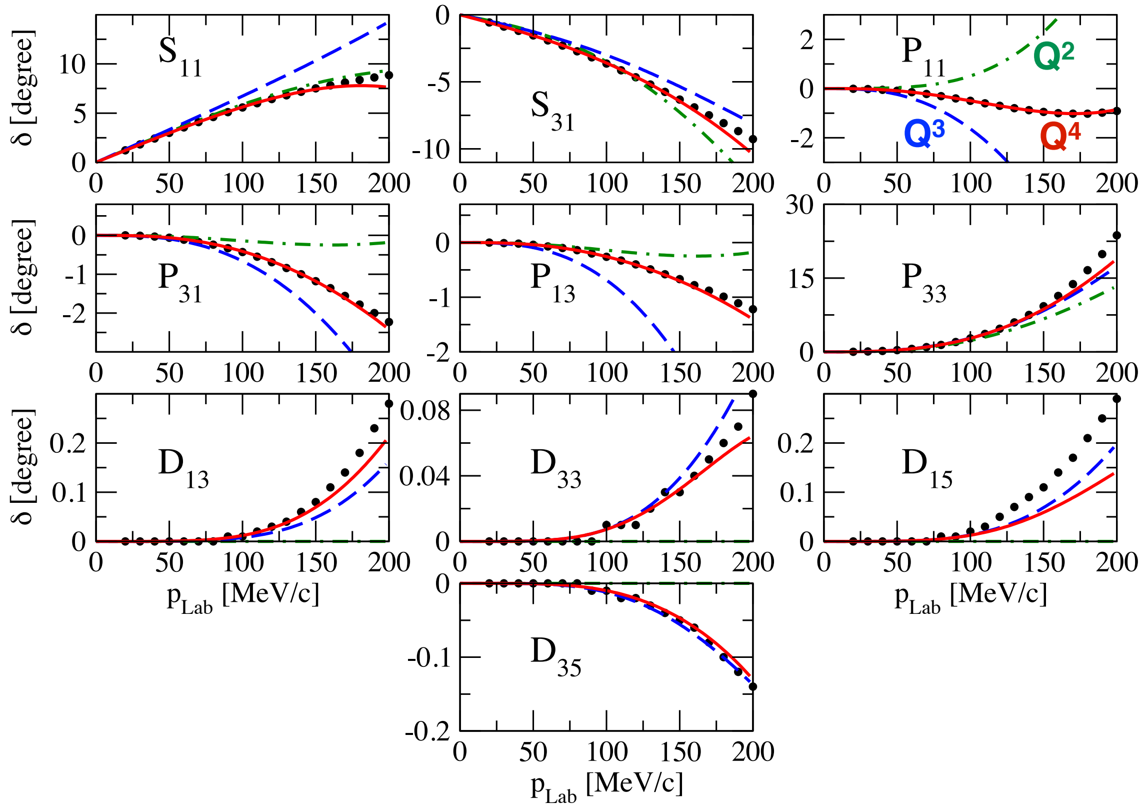
- N⁴LO [Q⁵]:
  Krebs, Gasparyan, EE '12

$$\begin{aligned}
\mathcal{A}^{(5)}(q_2) = {} & \frac{g_A}{4608\pi^2 F_\pi^6}\Big[M_\pi^2 q_2^2 (F_\pi^2\left(2304\pi^2 g_A(4\bar{e}_{14} + 2\bar{e}_{19} - \bar{e}_{22} - \bar{e}_{36}) - 2304\pi^2 \bar{d}_{18} c_3\right) \\
& + g_A(144c_1 - 53c_2 - 90c_3)) + M_\pi^4\left(F_\pi^2\left(4608\pi^2 \bar{d}_{18}(2c_1 - c_3) + 4608\pi^2 g_A(2\bar{e}_{14} + 2\bar{e}_{19} - \bar{e}_{36} - 4\bar{e}_{38})\right)\right. \\
& + g_A\left(72\left(64\pi^2 \bar{l}_3 + 1\right)c_1 - 24c_2 - 36c_3\right)\Big) + q_2^4\left(2304\pi^2 \bar{e}_{14} F_\pi^2 g_A - 2g_A(5c_2 + 18c_3)\right)\Big] \\
& - \frac{g_A^2}{768\pi^2 F_\pi^6} L(q_2)\left(M_\pi^2 + 2q_2^2\right)\left(4M_\pi^2(6c_1 - c_2 - 3c_3) + q_2^2(-c_2 - 6c_3)\right)\,, \\
\mathcal{B}^{(5)}(q_2) = {} & -\frac{g_A}{2304\pi^2 F_\pi^6}\Big[M_\pi^2\left(F_\pi^2\left(1152\pi^2 \bar{d}_{18} c_4 - 1152\pi^2 g_A(2\bar{e}_{17} + 2\bar{e}_{21} - \bar{e}_{37})\right) + 108 g_A^3 c_4 + 24 g_A c_4\right) \\
& + q_2^2\left(5g_A c_4 - 1152\pi^2 \bar{e}_{17} F_\pi^2 g_A\right)\Big] + \frac{g_A^2 c_4}{384\pi^2 F_\pi^6} L(q_2)\left(4M_\pi^2 + q_2^2\right)
\end{aligned}$$

Longest-range 3NF up to N^{4}LO

Krebs, Gasparyan, EE '12

πN phase shifts in HB ChPT up to Q^4 (KH PWA)



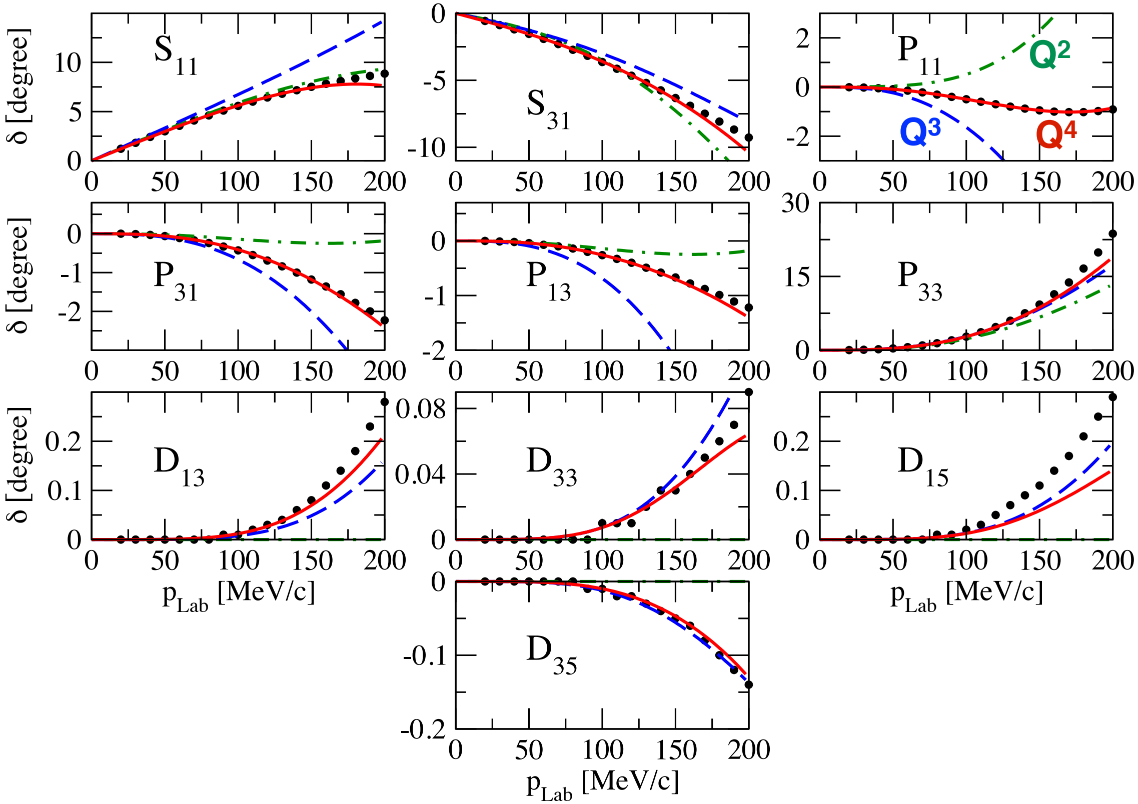
The determined values of LECs

	c_1	c_2	c_3	c_4	$\bar{d}_1+\bar{d}_2$	$\bar{d}_3$	$\bar{d}_5$	$\bar{d}_{14}-\bar{d}_{15}$	$\bar{e}_{14}$	$\bar{e}_{15}$	$\bar{e}_{16}$	$\bar{e}_{17}$	$\bar{e}_{18}$
Q^4 fit to GW	-1.13	3.69	-5.51	3.71	5.57	-5.35	0.02	-10.26	1.75	-5.80	1.76	-0.58	0.96
Q^4 fit to KH	-0.75	3.49	-4.77	3.34	6.21	-6.83	0.78	-12.02	1.52	-10.41	6.08	-0.37	3.26

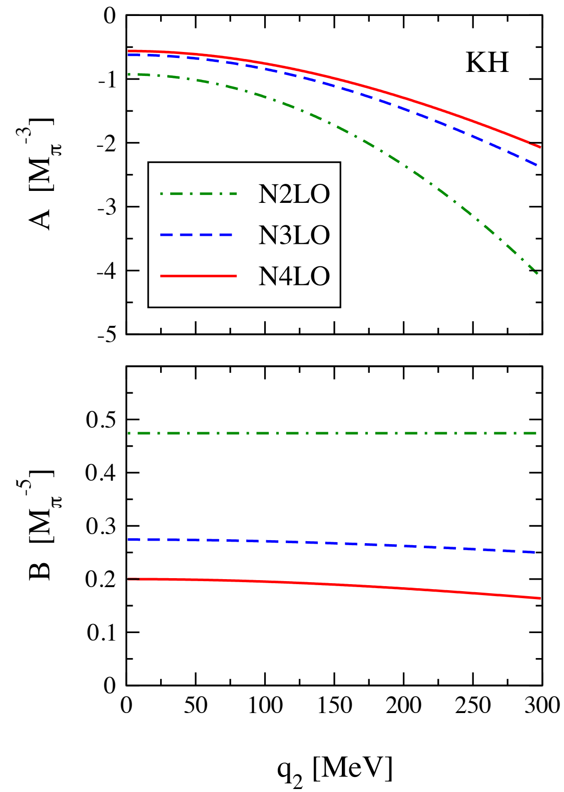
Longest-range 3NF up to N^4LO

Krebs, Gasparyan, EE '12

πN phase shifts in HB ChPT up to Q^4 (KH PWA)



3NF „structure functions“



The determined values of LECs

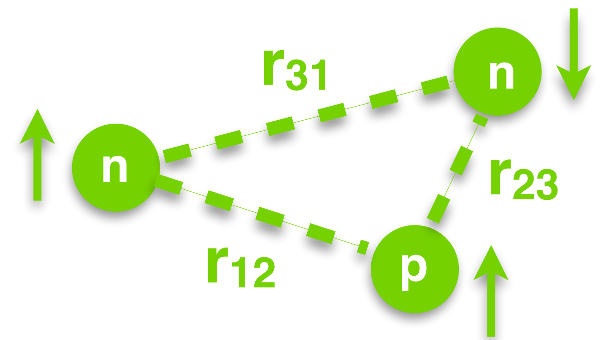
	c_1	c_2	c_3	c_4	$\bar{d}_1 + \bar{d}_2$	$\bar{d}_3$	$\bar{d}_5$	$\bar{d}_{14} - \bar{d}_{15}$	$\bar{e}_{14}$	$\bar{e}_{15}$	$\bar{e}_{16}$	$\bar{e}_{17}$	$\bar{e}_{18}$
Q^4 fit to GW	−1.13	3.69	−5.51	3.71	5.57	−5.35	0.02	−10.26	1.75	−5.80	1.76	−0.58	0.96
Q^4 fit to KH	−0.75	3.49	−4.77	3.34	6.21	−6.83	0.78	−12.02	1.52	−10.41	6.08	−0.37	3.26

Most general structure of a IC local 3NF

Most general local isospin-conserving 3NF can be written via

$$V(q_1, q_2, q_3) = \sum_{i=1}^{20} \mathcal{G}_i \, F_i(q_1, q_2, q_3) + \text{permutations}$$

$$V(r_{12}, r_{23}, r_{31}) = \sum_{i=1}^{20} \tilde{\mathcal{G}}_i \, F_i(r_{12}, r_{23}, r_{31}) + \text{permutations}$$



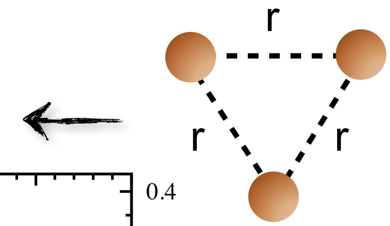
(2 operators out of the 22 given in **Krebs, Gasparyan, EE, PRC87 (2013)** are redundant **EE, Gasparyan, Krebs, Schat, to appear**)

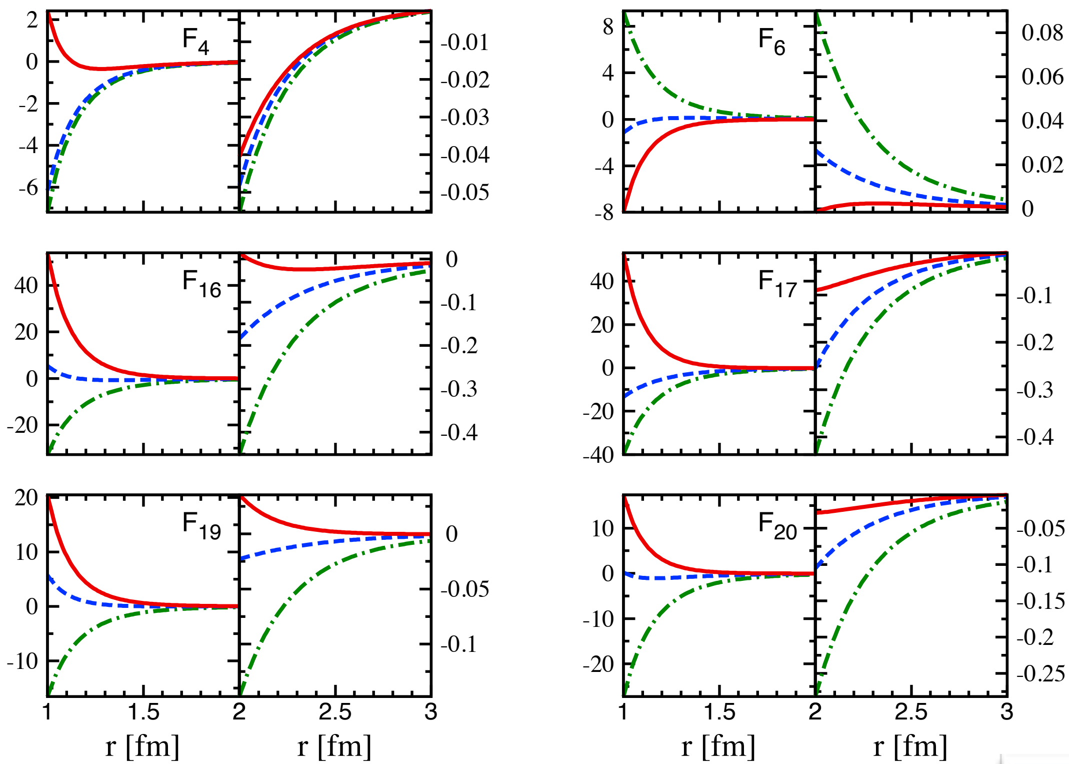
Generators $\mathcal{G}$ in momentum space	Generators $\tilde{\mathcal{G}}$ in coordinate space
$\mathcal{G}_1 = 1$	$\tilde{\mathcal{G}}_1 = 1$
$\mathcal{G}_2 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3$	$\tilde{\mathcal{G}}_2 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3$
$\mathcal{G}_3 = \vec{\sigma}_1 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_3 = \vec{\sigma}_1 \cdot \vec{\sigma}_3$
$\mathcal{G}_4 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{\sigma}_1 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_4 = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \vec{\sigma}_1 \cdot \vec{\sigma}_3$
$\mathcal{G}_5 = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{\sigma}_1 \cdot \vec{\sigma}_2$	$\tilde{\mathcal{G}}_5 = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \, \vec{\sigma}_1 \cdot \vec{\sigma}_2$
$\mathcal{G}_6 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot (\vec{\sigma}_2 \times \vec{\sigma}_3)$	$\tilde{\mathcal{G}}_6 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \, \vec{\sigma}_1 \cdot (\vec{\sigma}_2 \times \vec{\sigma}_3)$
$\mathcal{G}_7 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_2 \cdot (\vec{q}_1 \times \vec{q}_3)$	$\tilde{\mathcal{G}}_7 = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})$
$\mathcal{G}_8 = \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_1 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_8 = \hat{r}_{23} \cdot \vec{\sigma}_1 \, \hat{r}_{23} \cdot \vec{\sigma}_3$
$\mathcal{G}_9 = \vec{q}_1 \cdot \vec{\sigma}_3 \vec{q}_3 \cdot \vec{\sigma}_1$	$\tilde{\mathcal{G}}_9 = \hat{r}_{23} \cdot \vec{\sigma}_3 \, \hat{r}_{12} \cdot \vec{\sigma}_1$
$\mathcal{G}_{10} = \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_{10} = \hat{r}_{23} \cdot \vec{\sigma}_1 \, \hat{r}_{12} \cdot \vec{\sigma}_3$
$\mathcal{G}_{11} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_1 \cdot \vec{\sigma}_2$	$\tilde{\mathcal{G}}_{11} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \, \hat{r}_{23} \cdot \vec{\sigma}_1 \, \hat{r}_{23} \cdot \vec{\sigma}_2$
$\mathcal{G}_{12} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_2$	$\tilde{\mathcal{G}}_{12} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \, \hat{r}_{23} \cdot \vec{\sigma}_1 \, \hat{r}_{12} \cdot \vec{\sigma}_2$
$\mathcal{G}_{13} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_3 \cdot \vec{\sigma}_1 \vec{q}_1 \cdot \vec{\sigma}_2$	$\tilde{\mathcal{G}}_{13} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \, \hat{r}_{12} \cdot \vec{\sigma}_1 \, \hat{r}_{23} \cdot \vec{\sigma}_2$
$\mathcal{G}_{14} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_3 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_2$	$\tilde{\mathcal{G}}_{14} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \, \hat{r}_{12} \cdot \vec{\sigma}_1 \, \hat{r}_{12} \cdot \vec{\sigma}_2$
$\mathcal{G}_{15} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{q}_2 \cdot \vec{\sigma}_1 \vec{q}_2 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_{15} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \hat{r}_{13} \cdot \vec{\sigma}_1 \, \hat{r}_{13} \cdot \vec{\sigma}_3$
$\mathcal{G}_{16} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \vec{q}_3 \cdot \vec{\sigma}_2 \vec{q}_3 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_{16} = \boldsymbol{\tau}_2 \cdot \boldsymbol{\tau}_3 \, \hat{r}_{12} \cdot \vec{\sigma}_2 \, \hat{r}_{12} \cdot \vec{\sigma}_3$
$\mathcal{G}_{17} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \vec{q}_1 \cdot \vec{\sigma}_1 \vec{q}_3 \cdot \vec{\sigma}_3$	$\tilde{\mathcal{G}}_{17} = \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_3 \, \hat{r}_{23} \cdot \vec{\sigma}_1 \, \hat{r}_{12} \cdot \vec{\sigma}_3$
$\mathcal{G}_{18} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot \vec{\sigma}_3 \vec{\sigma}_2 \cdot (\vec{q}_1 \times \vec{q}_3)$	$\tilde{\mathcal{G}}_{18} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \, \vec{\sigma}_1 \cdot \vec{\sigma}_3 \, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})$
$\mathcal{G}_{19} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_3 \cdot \vec{q}_1 \vec{q}_1 \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)$	$\tilde{\mathcal{G}}_{19} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \, \vec{\sigma}_3 \cdot \hat{r}_{23} \, \hat{r}_{23} \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)$
$\mathcal{G}_{20} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \vec{\sigma}_1 \cdot \vec{q}_1 \vec{\sigma}_3 \cdot \vec{q}_3 \vec{\sigma}_2 \cdot (\vec{q}_1 \times \vec{q}_3)$	$\tilde{\mathcal{G}}_{20} = \boldsymbol{\tau}_1 \cdot (\boldsymbol{\tau}_2 \times \boldsymbol{\tau}_3) \, \vec{\sigma}_1 \cdot \hat{r}_{23} \, \vec{\sigma}_3 \cdot \hat{r}_{12} \, \vec{\sigma}_2 \cdot (\hat{r}_{12} \times \hat{r}_{23})$

Long-range 3NF up to N^4LO (preliminary)

EE, Gasparyan, Krebs, Schat, to appear

Chiral expansion of TPE „structure functions“ F_i (in MeV) in the equilateral-triangle configuration





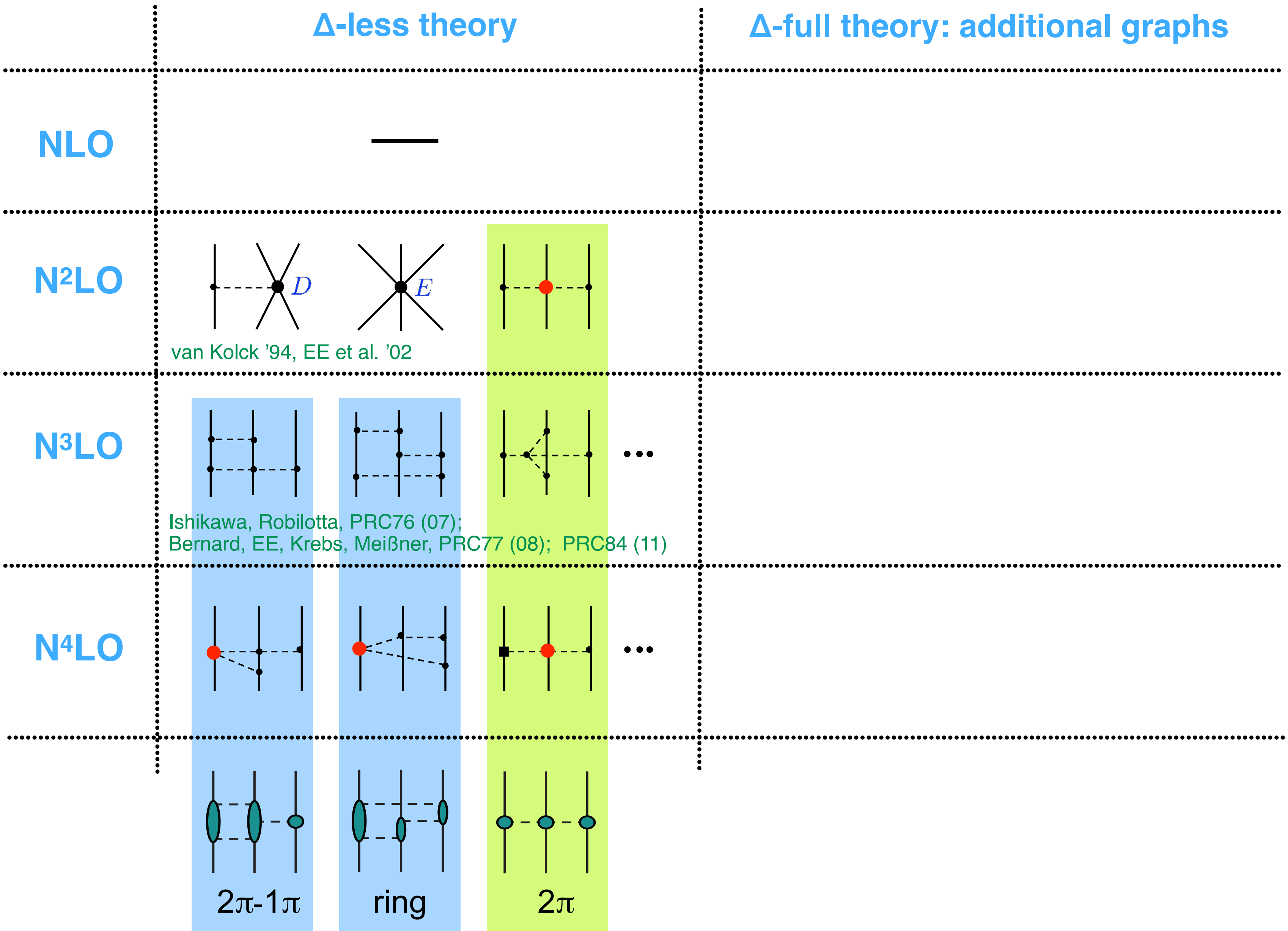
tree level (N^2LO)

+ N^3LO

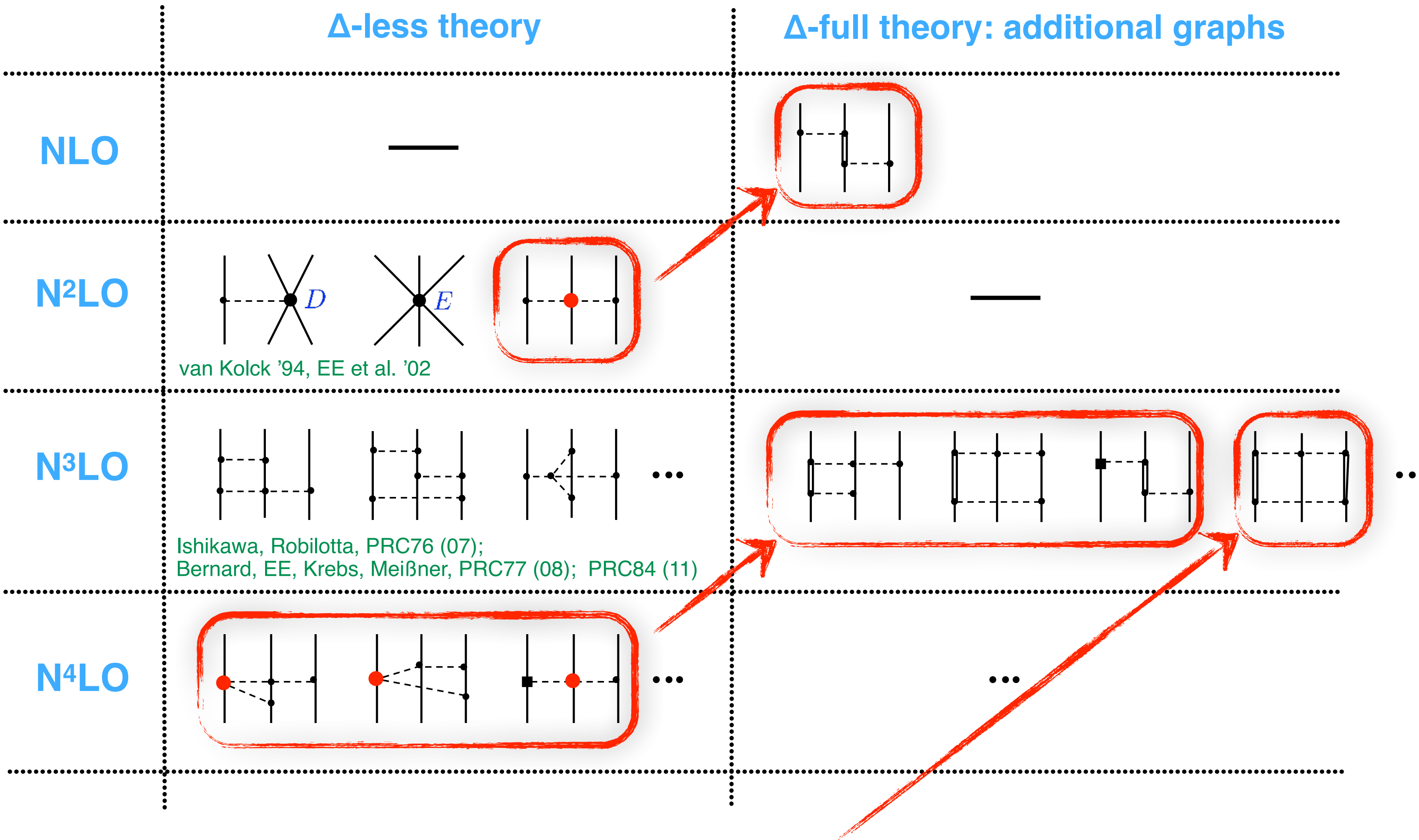
+ N^4LO

- 8 structures out of 20
- N^4LO corrections are large, seem to converge only at very large r

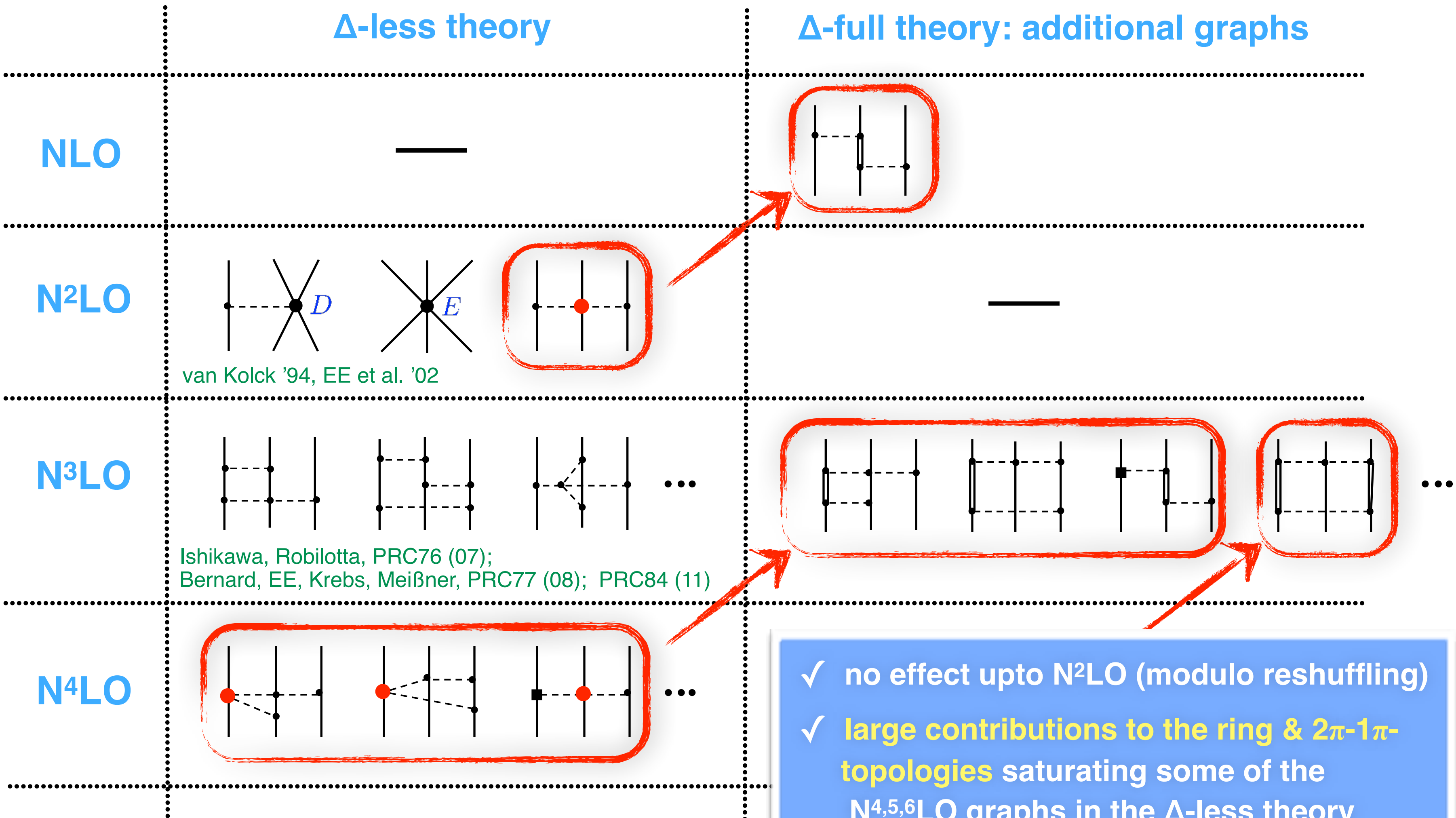
Chiral expansion of the 3NF



Chiral expansion of the 3NF



Chiral expansion of the 3NF

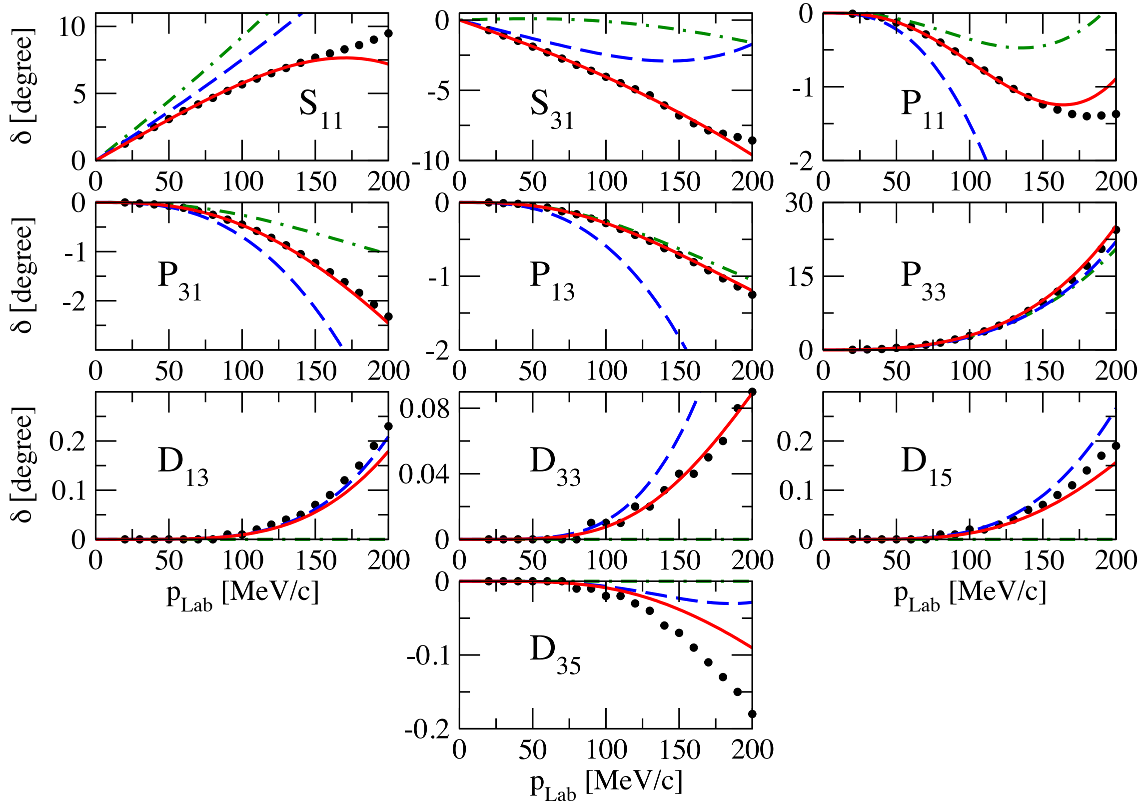


- √ no effect upto N²LO (modulo reshuffling)
- √ large contributions to the ring & 2π-1π-topologies saturating some of the N^{4,5,6}LO graphs in the Δ-less theory
- √ What is more efficient: Δ-less N⁴LO (and beyond?) vs Δ-full N³LO ??

Pion-nucleon system in Δ-full EFT up to Q^4

Krebs, Gasparyan, EE, to appear

πN phase shifts in HB ChPT up to Q^4 (KH PWA)

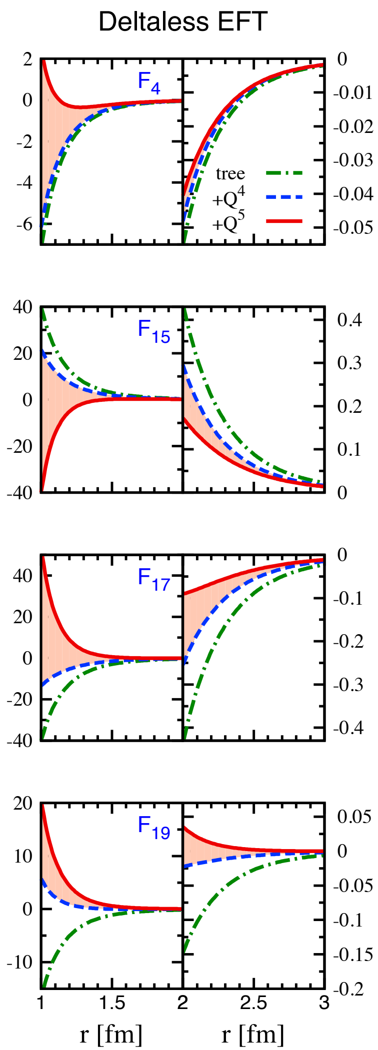


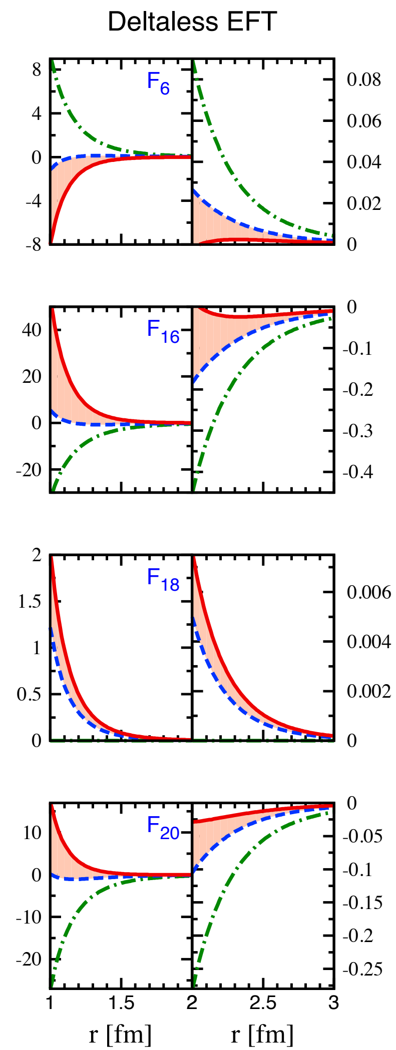
LECs from pion-nucleon scattering (HB ChPT) in units of GeV^{-n} (fit to KH PWA)

	c_1	c_2	c_3	c_4	$\bar{d}_1 + \bar{d}_2$	$\bar{d}_3$	$\bar{d}_5$	$\bar{d}_{14} - \bar{d}_{15}$	$\bar{e}_{14}$	$\bar{e}_{15}$	$\bar{e}_{16}$	$\bar{e}_{17}$	$\bar{e}_{18}$
Δ-less approach	−0.75	3.49	−4.77	3.34	6.21	−6.83	0.78	−12.02	1.52	−10.41	6.08	−0.37	3.26
Δ-full approach	−0.95	1.90	−1.78	1.50	2.40	−3.87	1.21	−5.25	−0.24	−6.35	2.34	−0.39	2.81
Δ-contribution	0	2.81	−2.81	1.40	2.39	−2.39	0	−4.77	1.87	−4.15	4.15	−0.17	1.32

2π-exchange 3NF: Δ-full vs Δ-less EFT

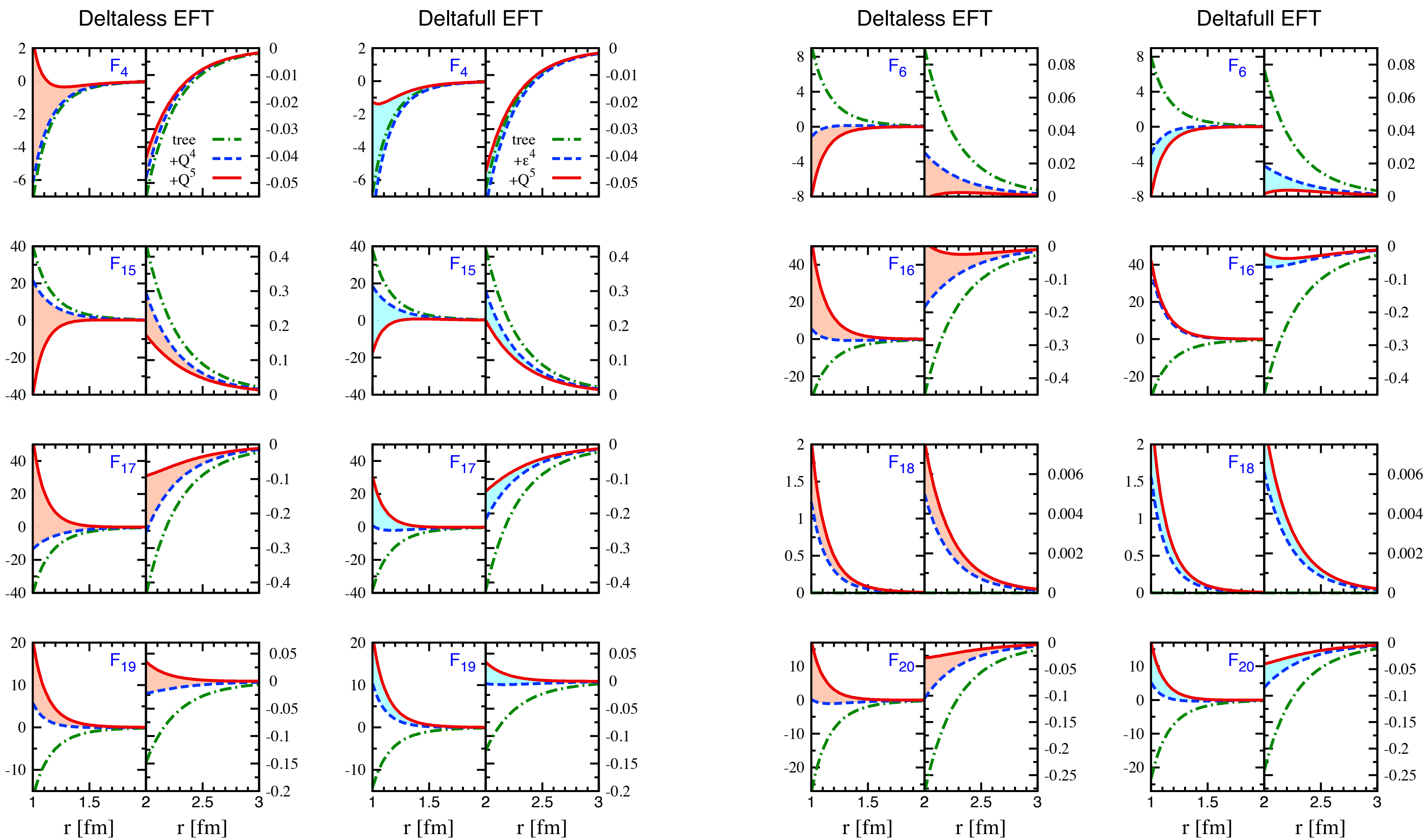
Krebs, Gasparyan, EE, to appear





2π-exchange 3NF: Δ-full vs Δ-less EFT

Krebs, Gasparyan, EE, to appear



- Δ-full and Δ-less EFT predictions agree well with each other
- Δ-full approach shows clearly a superior convergence
- remarkably, the final 2π 3NF turns out to be rather weak at large distances…

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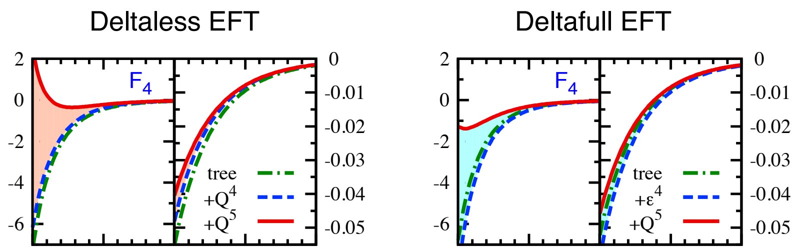
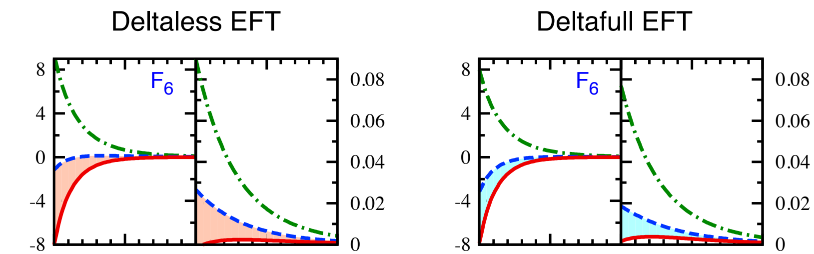
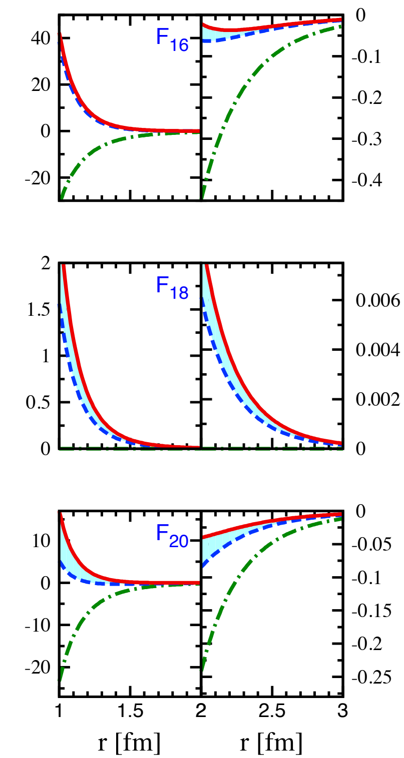






Numerical implementation of the 3NF at N3LO and applications to few-/many-N systems are is being carried out by the

Low **E**nergy **N**uclear **P**hysics **I**nternational **C**ollaboration **(LENPIC)**

- J.Golak, R.Skibinski, K.Topolnicki, H.Witala (Cracow)
- EE, H.Krebs (Bochum)
- S.Binder, A.Calci, K.Hebeler, J.Langhammer, R.Roth (Darmstadt)
- P.Maris, H.Potter, James Vary (Iowa State)
- R.J.Furnstahl (Ohio State)
- A.Nogga (Jülich)
- U.-G. Meißner (Bonn)
- V. Bernard (Orsay)
- H.Kamada (Kyushu)

e distances…

Summary and outlook

- **Nonperturbative renormalization with nonperturbative 1π-exchange**
  - It is possible to completely eliminate Λ using relativistic equations (e.g. Kadyshevsky) assuming that 2π exchange can be treated in perturbation theory
  - Promising results for phase shifts, deuteron FFs and χ-extrapolations at LO

  Future plans: higher orders (TPE), generalization to SU(3)

- **New NN chiral potentials about to emerge**

  A new generation of chiral NN potentials up to N^{3}LO is being developed:

  $\mathbf{l}_{\text{ocal}}$-**chiral** (up to N^{2}LO): local interactions, can be used in QMC

  $\mathbf{i}_{\text{mproved}}$-**chiral** (up to N^{3}LO): nonlocal potentials

  Common features: better performance at higher energies, less sensitivity to cutoffs, no need for SFR, can use c_i's from πN.

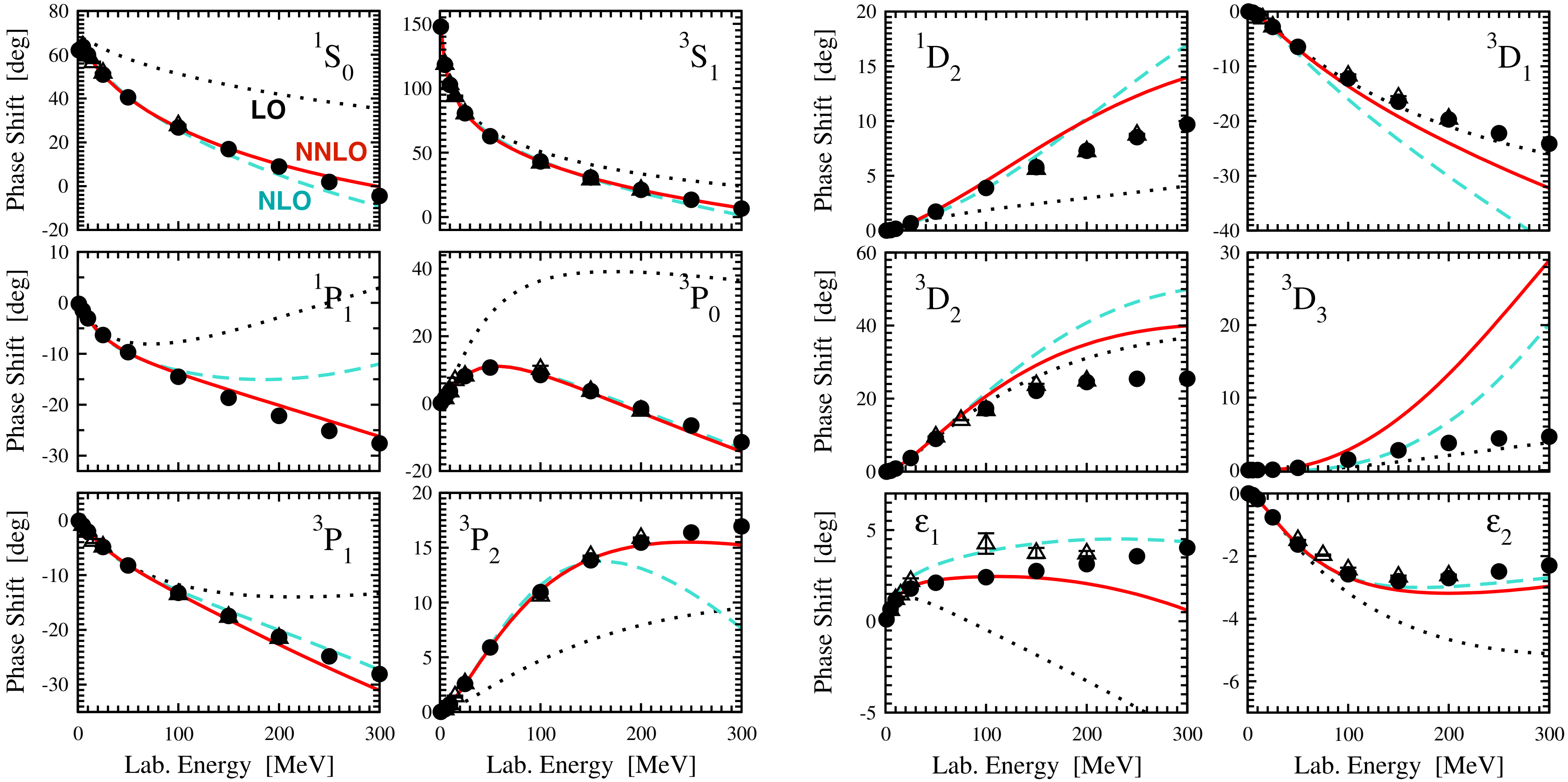
  Future plans: sensitivity to c_i's, extension to Δ-full theory

- **3N force**
  - A complicated object: 20 independent structures even in local case
  - Worked out up to N^{3}LO level (parameter-free), first results are emerging
  - Still not converged at this order (certain Δ effects are missing)
  - Long-range terms worked out at N^{4}LO and N^{3}LO-Δ: signs of convergence…

  Future plans: Nd scattering & nuclear structure at N^{3}LO and beyond

I-chiral 2NF: Order-by-order improvement

neutron-proton phase shifts on <u>I-chiral</u> 2NF at LO, NLO and N²LO



R_0 = 1 fm, Λ_{SFR} = 2 GeV

Regularization of the chiral NN potentials

order of the chiral expansion

$$\langle \vec{p}\,'| V^{\rm NNLO} |\vec{p}\rangle = \left[V_{1\pi}^{(0)} + V_{2\pi}^{(2)} + V_{2\pi}^{(3)} + V_{\rm cont}^{(0)} + V_{\rm cont}^{(2)} \right] e^{\frac{-p'^4 - p^4}{\Lambda^4}}$$

The cutoff Λ should not be chosen too large (spurious bound states, nonlinearities, nonrenormalizable theory) Lepage'97, EE., Meißner '06, EE, Gegelia '09. On the other hand, smaller values of Λ introduce unnecessary errors.

Typical choice: Λ = 450...600 MeV [N³LO potentials by EGM, EM]

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Claim: while the above nonlocal regulator simplifies the determination of the LECs, it cuts off some model-independent long-range physics one would like to keep and leaves some model-dependent short-range physics one would like to cut off...

Given that $V_{1\pi}^{(0)} + V_{2\pi}^{(2)} + V_{2\pi}^{(3)}$ **is local, local regulator will do a better job!**

Reminder:

$$V_{\rm local}(\vec{p}\,', \vec{p}\,) \equiv \langle \vec{p}\,'|V_{\rm local}|\vec{p}\,\rangle = V(\vec{p}\,' - \vec{p}\,) \quad \longrightarrow \quad V(\vec{r}\,', \vec{r}\,) \equiv \langle \vec{r}\,'|V|\vec{r}\,\rangle = \delta^3(\vec{r}\,' - \vec{r}\,)V(\vec{r}\,)$$

Regularization of the chiral NN potentials

Peripheral NN scattering as a long-range filter: insensitive to short-range physics and determined by the model-independent long-range interaction ($V_{1\pi}$). Can be computed using Born approximation: $T_{\alpha'\alpha}(p) \equiv \langle p, \alpha' | T | p, \alpha \rangle = \langle p, \alpha' | V_{1\pi} | p, \alpha \rangle$

where $V_{1\pi}(\vec{q}) = -\frac{g_A^2}{4F_\pi^2} \frac{(\vec{q} \cdot \vec{\sigma}_1)(\vec{q} \cdot \vec{\sigma}_2)}{\vec{q}^{\,2} + M_\pi^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$

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- **Standard, nonlocal regularization** $V_{1\pi}^{\rm reg}(\vec{q}) = V_{1\pi}(\vec{q}) \; F\left(\frac{p'}{\Lambda}, \frac{p}{\Lambda}\right)$

  Partial-wave decomposition: $\langle p', \alpha' | V_{1\pi}^{\rm reg} | p, \alpha \rangle = \langle p', \alpha' | V_{1\pi} | p, \alpha \rangle \; F\left(\frac{p'}{\Lambda}, \frac{p}{\Lambda}\right)$

  Regulator affects all partial waves at high momenta independently on $\boldsymbol{\alpha}, \; \boldsymbol{\alpha}'$

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- **Local regularization**

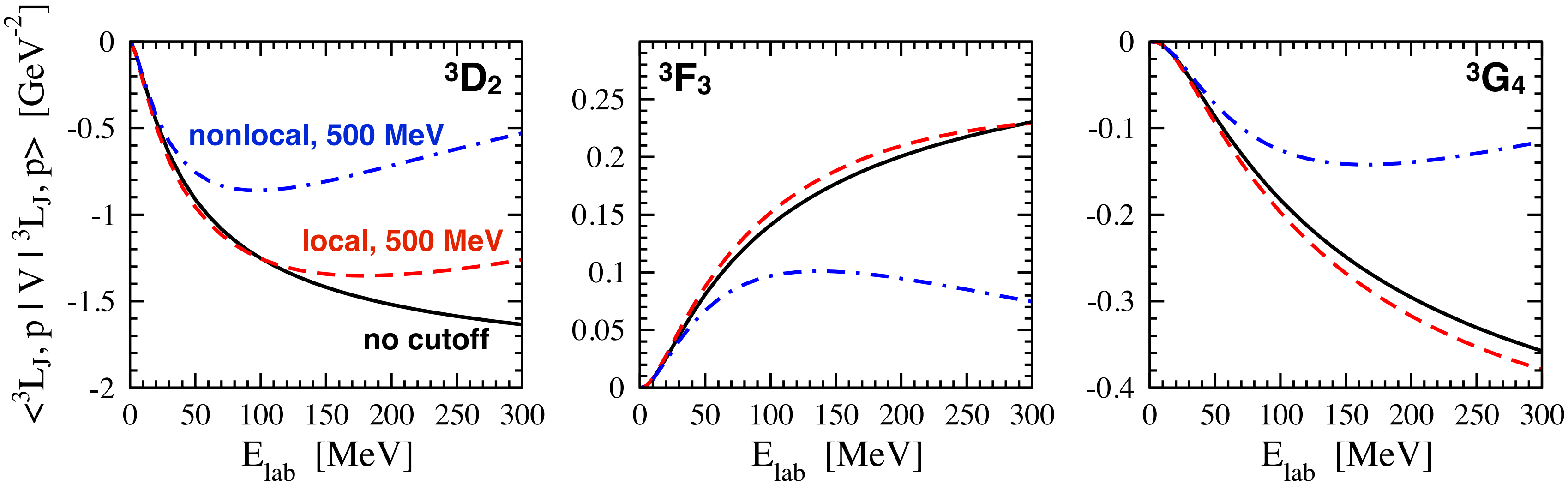
  $V_{1\pi}^{\rm reg}(\vec{q}) = V_{1\pi}(\vec{q})\, F\left(\frac{q}{\Lambda}\right)$ or, alternatively, $V_{1\pi}^{\rm reg}(\vec{r}) = V_{1\pi}(\vec{r})\, F(r/R_0)$

  Partial-wave matrix elements in momentum space:

$$\langle p', \alpha' | V_{1\pi}^{\rm reg} | p, \alpha \rangle \sim \underbrace{\int r^2 dr\, j_{l'}(p'r) \left[V_{1\pi}^{\alpha'\alpha}(r) F(r/R_0) \right] j_l(pr)}_{\text{becomes insensitive to } F \text{ for high } \boldsymbol{l}, \; \boldsymbol{l'}}$$

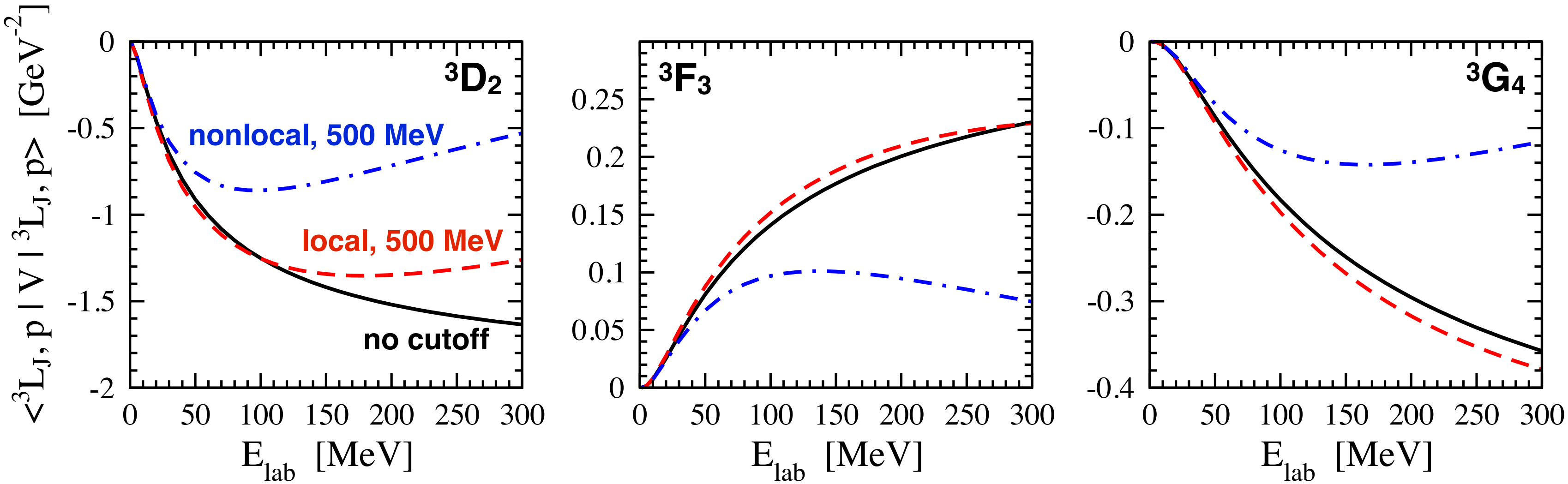
Regularization of the chiral NN potentials

PW projected MEs of the OPEP: $\exp[-(p'^2+p^2)/\Lambda^2]$ versus $\exp[-q^2/\Lambda^2]$ for Λ = 500 MeV

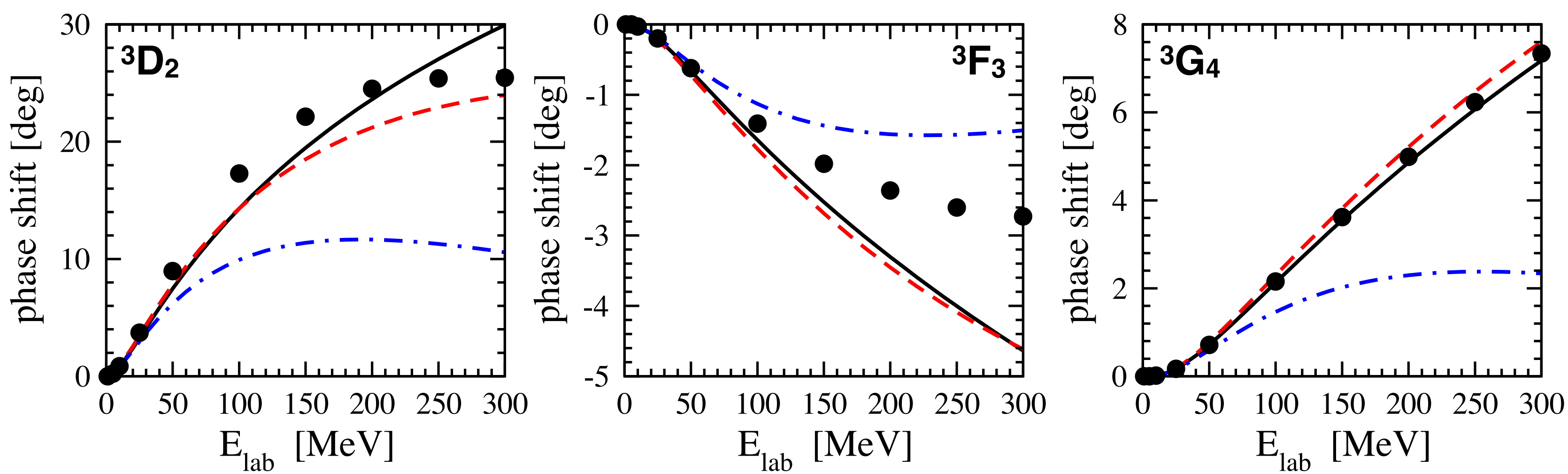


Regularization of the chiral NN potentials

PW projected MEs of the OPEP: exp[-(p'²+p²)/Λ²] versus exp[-q²/Λ²] for Λ = 500 MeV



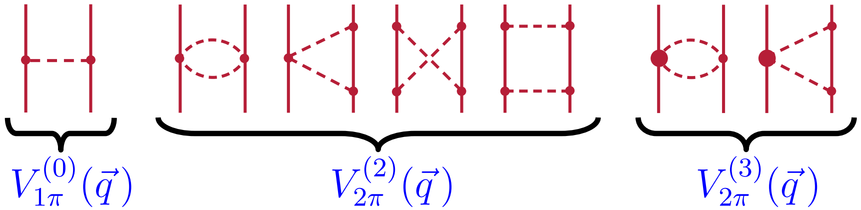
Peripheral partial waves based on the OPE potential (Born approx.)



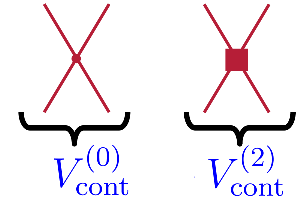
Construction of the potential (published local version)

Gezerlis, Tews, EE, Gandolfi, Hebeler, Nogga, Schwenk, PRL 111 (2013) 032501; more details in the talk by Ingo

Long-range:



Short-range:



There are 9 isospin-concerving contact terms whose choice is not unique. Standard:

$$V_{\rm cont}^{(0)} = C_S + C_T(\vec{\sigma}_1 \cdot \vec{\sigma}_2)$$

$$V_{\rm cont}^{(2)} = C_1 q^2 + C_2 k^2 + C_3 q^2 (\vec{\sigma}_1 \cdot \vec{\sigma}_2) + C_4 k^2 (\vec{\sigma}_1 \cdot \vec{\sigma}_2) + \frac{iC_5}{2}(\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \vec{q} \times \vec{k} + C_6 (\vec{\sigma}_1 \cdot \vec{q})(\vec{\sigma}_2 \cdot \vec{q}) + C_7 (\vec{\sigma}_1 \cdot \vec{k})(\vec{\sigma}_2 \cdot \vec{k})$$

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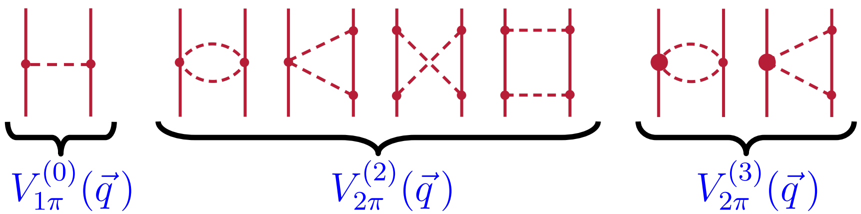

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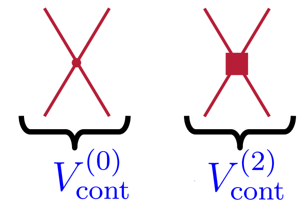

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One can choose instead a local basis:

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Construction of the potential (published local version)

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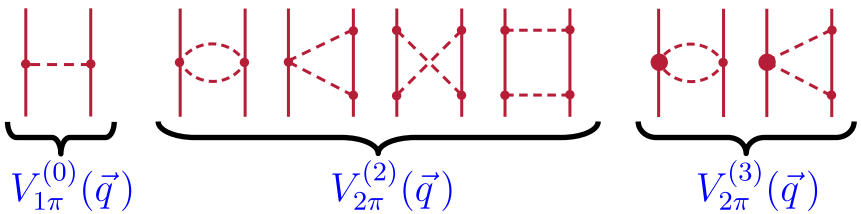

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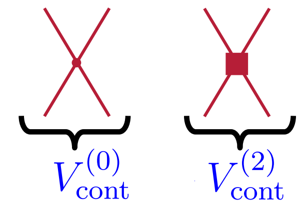

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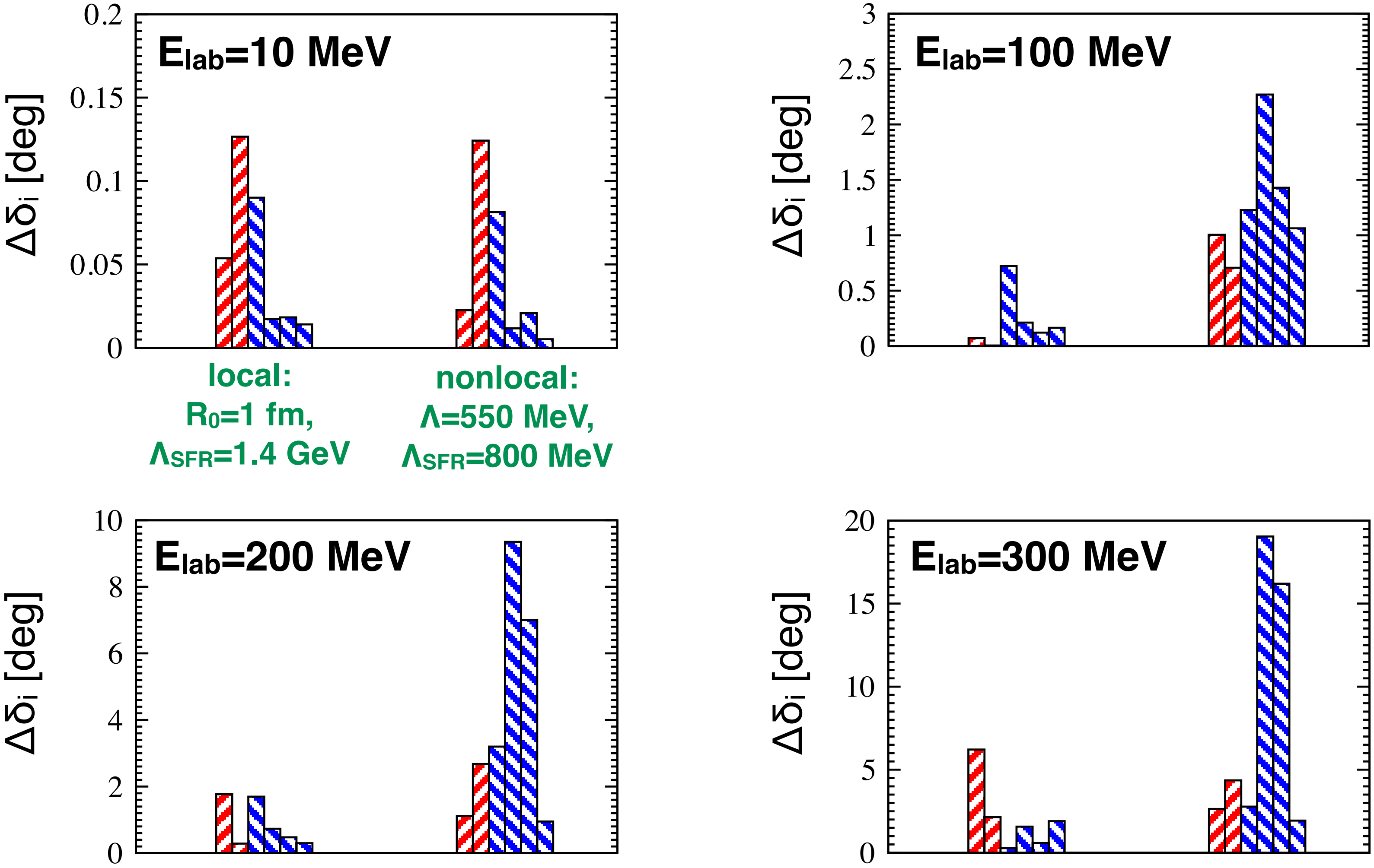
Make Fourier Transform and regularize in configuration space, e.g.:

$$V_{\rm long}(\vec{r}) \;\to\; V_{\rm long}(\vec{r})\left[1 - e^{-r^4/R_0^4}\right]$$ and $$\delta^3(\vec{r}) \;\to\; \alpha e^{-r^4/R_0^4}$$ where $$\alpha = \frac{1}{\pi \Gamma(3/4) R_0^3}$$

The LECs are determined from NN S-, P-waves and the mixing angle ε_1

Error budget: local vs nonlocal regulators

Absolute errors in S- and P-wave phase shifts at N^2LO



Ordering of partial waves: $^1S_0$, $^3S_1$, $^1P_1$, $^3P_0$, $^3P_1$, $^3P_2$