# Electromagnetic deuteron form factors in point form of RQM


*Nikolay Khokhlov*
Komsomolsk-na-Amure State Technical University

***"Nuclear Theory in the Supercomputing Era"***

# RQM of systems with a fixed number of particles

- Direct generalization of the non-RQM.
- Invariance – Poincare group (PG) instead of Galileo one. The WF of a system must transform according to a UR of the PG in some Hilbert space.
- Cluster separability (in Sokolov packing operators method). It means that symmetries and conservation laws that hold for a system of particles also hold for isolated subsystems.
- Direct interaction. No antiparticles and intermediate particles. But theory directly may be generalized to Lee model (Fuda[1990]) and to a quantum field theory (QFT) (Fubini[1973].

# Forms of RQM

- **Where to insert interaction into the PG generators? Must it come into all of them?**

$$[P^\mu, P^\nu] = 0, \qquad [M^{\mu\nu}, P^\rho] = -i(g^{\mu\rho}P^\nu - g^{\nu\rho}P^\mu),$$

$$[M^{\mu\nu}, M^{\rho\sigma}] = -i(g^{\mu\rho}M^{\nu\sigma} + g^{\nu\sigma}M^{\mu\rho} - g^{\mu\sigma}M^{\nu\rho} - g^{\nu\rho}M^{\mu\sigma})$$

**Dirac[1949]: There are simpler ways.**

**Kinematic generators – subgroup of Poincare group**

**Dynamical ones – Hamiltonians**

- **Instant Form, kinematic subgroup is:**

$$\vec{P}, \quad \vec{M} = (M^{23}, M^{31}, M^{12})$$

**Four Hamiltonians: Energy and Boosts**

- **Light Front, kinematic subgroup is:**

$$P^{+} = \left(P^{0} + P^{Z}\right)\big/\sqrt{2}, P^{j}, M^{12}, M^{+-}, M^{+j}, j = 1,2(x, y)$$

**Three Hamiltonians:** $P^{-}, M^{-j}$

- **Point Form, kinematic subgroup is: The Lorenz Group**

**Four Hamiltonians: 4-momentum**

**All are unitary equivalent, Sokolov[1975,1978]:**

**by some unitary transformation a subgroup of PG may be made free of interaction**

# Why Point Form?

*Orwell [1945]* "All animals are equal, but some animals are more equal than others"

**Why the PF  is more equal:**

- **It seems to be a simplest generalization of non-RQM:  Hamiltonians are the 4-momenta operators, commuting. And states considered usually are their eigenstates.**
- **LG generators a free of interaction:**

1. **Spins and Orbital momenta are summed as in the non-RQM**
2. **Spectator approximation for transition operators is Lorenz covariant.**
3. **It becomes the non-RQM in the non-Rel limit without additional conditions.**

# Only recently it became fashionable (more papers), why?

- I do not know, may be:

1. The quantum field theory taught is in the instant form.
2. Light front is unusual and it was fashionable once. It has only three Hamiltonians. It is convenient for considering parton phenomena.
3. It was not clear how to relate the PF to the QFT.

# How to insert interaction?

- **Bakamjian-Thomas (BT) procedure [1953](two body)**

**in point form:** $M = M_0 + V$

$$P = MG, \qquad \mathbf{M} = \mathbf{l}(\mathbf{G}) + \mathbf{S}, \qquad \mathbf{N} = -\imath G^0 \frac{\partial}{\partial \mathbf{G}} + \frac{\mathbf{S} \times \mathbf{G}}{1 + G^0}$$

$$l(G) = i\, G \times \frac{\partial}{\partial G}$$

- **Sokolov (packing operators) [1978] (many body)**

# Simplest way – direct interaction

$$M\,|\chi\rangle_{\boldsymbol{i}} \equiv \left[\sqrt{\hat{\mathbf{q}}^2 + m_1^2} + \sqrt{\hat{\mathbf{q}}^2 + m_2^2} + V\;\right] |\chi\rangle_{\boldsymbol{i}} = M_{\boldsymbol{i}}\,|\chi\rangle,$$

$$\left[\hat{\mathbf{q}}^2 + m\,\hat{V}\right] |\chi\rangle_{\boldsymbol{i}} = q^2\,|\chi\rangle_{\boldsymbol{i}}$$

$$q^2 = \frac{M_{\boldsymbol{i}}^2}{4} - \frac{m_1^2 + m_2^2}{2} + \frac{(m_1^2 - m_2^2)^2}{4M_{\boldsymbol{i}}^2},$$

# complications in case of not spinless particles

- The BT form of generators is now: $\Gamma^i = U_{12}\tilde{\Gamma}^i U_{12}^{-1}$

where the unitary operator from the internal Hilbert space to the Hilbert rep space of two particle state

$$U_{12} = U_{12}(G, \mathbf{q}) = \prod_{i=1}^{2} D[\mathbf{s}_i; \alpha(p_i/m)^{-1}\alpha(G)\alpha(q_i/m)]$$

With D[s;u] being the rep operator of the SU(2) corresponding to the u in SU(2) and generators s.

$$|P, \chi\rangle = U_{12}|P\rangle \otimes |\chi\rangle$$

- **Momenta of the particles in their c.m.f.**

$$q_i = L[\alpha(G)]^{-1} p_i$$

$$\mathbf{q} = \mathbf{q}_1 = -\mathbf{q}_2$$

- **External part** $$\langle G|P'\rangle \equiv \frac{2}{M'} G'^0 \delta^3(\mathbf{G} - \mathbf{G}')$$

- **Scalar product**

$$\langle P''|P'\rangle = \int \frac{d^3\mathbf{G}}{2G^0} \langle P''|G\rangle\langle G|P'\rangle$$
$$= 2\sqrt{M'^2 + \mathbf{P}'^2}\, \delta^3(\mathbf{P}'' - \mathbf{P}')$$

# As a result

The interaction term is present in all components of total four-momentum. Generators of Lorentz boosts and generators of rotations are free of interaction. In the c.m. frame, the relative orbital angular momentum and spins are coupled together as in the nonrelativistic case. Moreover, most nonrelativistic scattering theory formal results are valid for our case of two particles Keister[1991]

# Can we use the non-RQM NN-potentials in this theory?

## In Schrodinger equation

$$m_1 \approx m_2 = m$$

$$q^2 = \frac{M_{\boldsymbol{i}}^2}{4} - m^2$$

- Coester[1974]:
- In case of NN scattering states there is no corrections:

$$q^2/m = E_{lab}/2$$

- In case of deuteron: $q^2/m = (M-2m)(1+(M-2m)/4m)$

"effective" energy -2.2233 MeV

Instead of the experimental one -2.2246 MeV

# EM current operator (CO):

- Siegert [1937]: It must depend on interaction being 4-vectors (for some generators are)
- Current conservation

Spectator model: EM CO of a system equals to the sum of the EM COs of the constituents.

Point Form: Lev [1994], Klink[1998] – equivalent, but different parameterization.

We use parameterization by F. Lev.

# Lev construction of Spectator Approximation of the EM CO for a two particle system

- **In a special frame:**

$$\mathbf{G}_i + \mathbf{G}_f = 0,$$

$$G_i = P_i/M_i, G_f = P_f/M_f$$

- **A vector-parameter:** $\mathbf{h} = \mathbf{G}_f/G_f^0$

- **Using common properties of the covariant 4-vector operator we come to**

$$\langle P_f, \chi_f|\hat{J}^\mu(x)|P_i, \chi_i\rangle = 4\pi^{3/2}\sqrt{M_i M_f} e^{i(P_f-P_i)x} \langle\chi_f|\hat{j}^\mu(\mathbf{h})|\chi_i\rangle_{\text{n.r.}}$$

- **Reduction to the inner space calculation**

$$j^\mu(\mathbf{h}) = \sum_{i=1,2}(L^i)^\mu_\nu D_1^i D_2^i j_i^\nu(\mathbf{h}) D_3^i K^i I_i(\mathbf{h})$$

# Note:

- **Momentum transferred to Deuteron gives shift in the momentum space WF:**

$$I_i(h)\chi(q)=\chi(q+\Delta q_i)$$

$$\Delta q_i = (-1)^i \frac{2\mathbf{h}}{1-h^2}[w + (-1)^i(\mathbf{h}\cdot\mathbf{q})]$$

$$\frac{h}{1-h^2} = \eta = Q^2/4m_D$$

# One particle COs (s=1/2)

$$j_i^0(\mathbf{h}) = eF_e^i(Q_i^2),$$

$$\mathbf{j}_i(\mathbf{h}) = -\frac{ie}{\sqrt{1-\mathbf{h}_i^2}} F_m^i(Q_i^2)(\mathbf{h}_i \times \mathbf{s}_i)$$

- **And in PF RQM**

$$Q_1^2 = -(q_1' - q_1)^2 = 16\left(m^2 + \mathbf{q}^2 - \frac{(\mathbf{q}\cdot\mathbf{h})^2}{h^2}\right)\frac{h^2}{(1-h^2)^2} \neq Q^2$$

# Details of calculation

- **Helicities of the deuteron states**

$$\xi_i^\Lambda = \begin{cases} (0, \pm 1, -\imath, 0)/\sqrt{2}, & \Lambda = \pm \\ (-Q/2, 0, 0, P_0)/m_d = (-h, 0, 0, 1)/\sqrt{1-h^2} & \Lambda = 0 \end{cases}$$

$$\xi_f^\Lambda = \begin{cases} (0, \mp 1, -\imath, 0)/\sqrt{2}, & \Lambda = \pm \\ (Q/2, 0, 0, P_0)/m_d = (h, 0, 0, 1)/\sqrt{1-h^2} & \Lambda = 0 \end{cases}$$

- **And of the virtual photon**

$$\epsilon^\lambda = \begin{cases} (0, \mp 1, -\imath, 0)/\sqrt{2}, & \lambda = \pm \\ (1, 0, 0, 0) & \lambda = 0 \end{cases}$$

# Details of calculation

- **And we come to helisity amplitudes (matrix elements)**

$$j^{\lambda}_{\Lambda_f \Lambda_i} \equiv \langle \Lambda_f | \left( \epsilon^{\lambda}_{\mu} \cdot j^{\mu}(\mathbf{h}) \right) | \Lambda_i \rangle$$

- **And to form factors:**

$$j^{0}_{00}(Q^2) = G_C + \frac{4}{3}\frac{h^2}{1-h^2} G_Q$$

$$j^{0}_{+-}(Q^2) = j^{0}_{-+}(Q^2) = G_C - \frac{2}{3}\frac{h^2}{1-h^2} G_Q$$

$$j^{+}_{+0}(Q^2) = j^{+}_{0-}(Q^2) = j^{-}_{-0}(Q^2) = j^{-}_{0+}(Q^2) = -\frac{h}{\sqrt{1-h^2}} G_M .$$

$$\eta = Q^2/4m_d^2 = h^2/(1-h^2)$$

# Details of calculation

- **Matrix elements and therefore the deuteron form factors may be expressed in a form:**

$$\sim \sum_{j=e,m} \sum_{i=1,2} \int dq^3 \chi_f(q) f_i(q, Q, \Lambda_i, \Lambda_f, \lambda) F_j^i\left(Q_i^2\right) \chi_i(q + \Delta q_i)$$

# EM deuteron Form Factors: Experimental status

- **In one-photon approximation the FFs may be extracted directly from the elastic ed scattering.**

**From the unpolarized differential cross section:**

$$A(Q^2) = G_C^2(Q^2) + \frac{2}{3}\eta G_M^2(Q^2)$$

$$B(Q^2) = \frac{4}{3}\eta(1+\eta)G_M^2(Q^2).$$

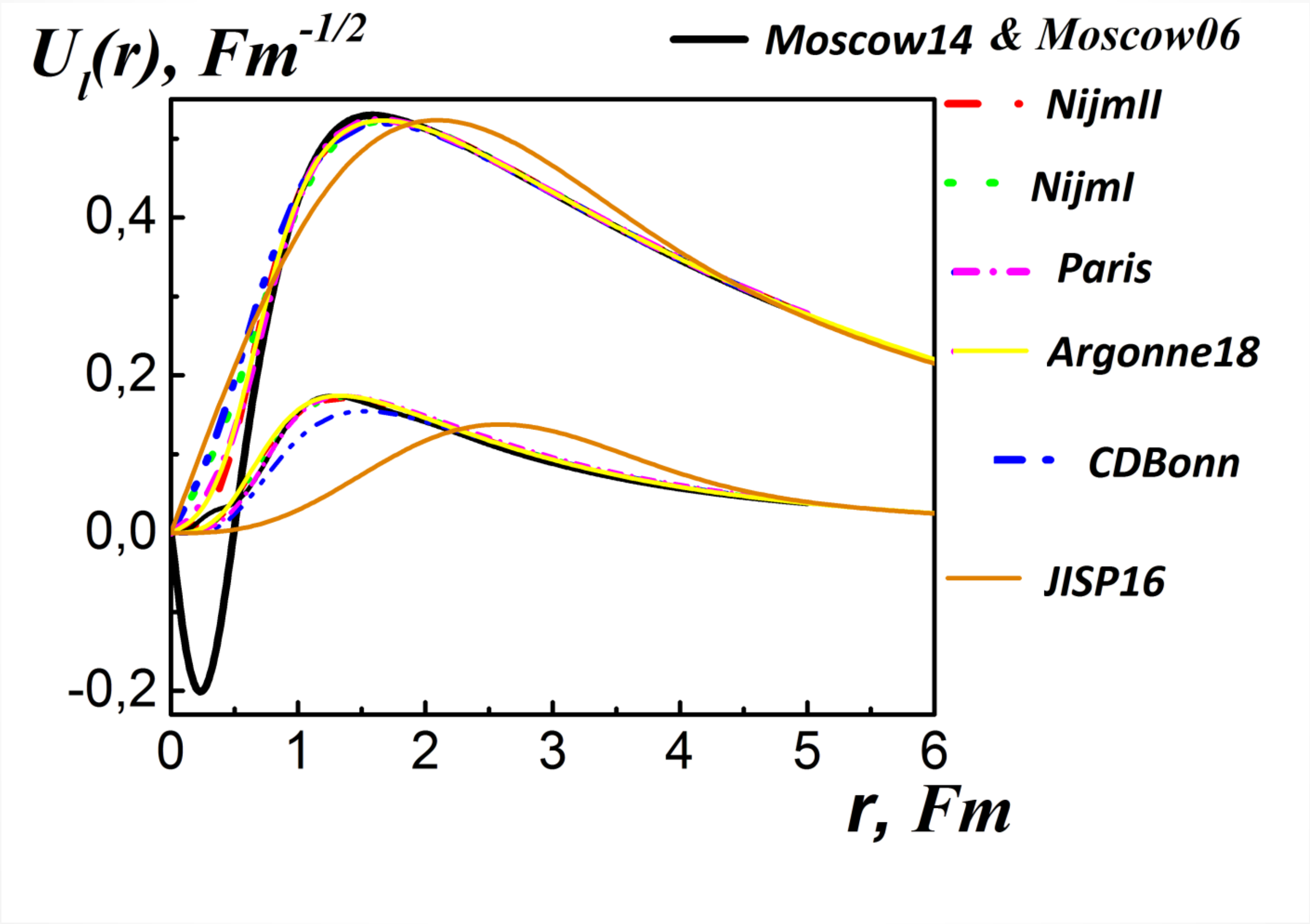
$$\eta = Q^2/4m_d^2 = h^2/(1-h^2)$$

- **Gross[2001] at $\theta$=$70^0$**

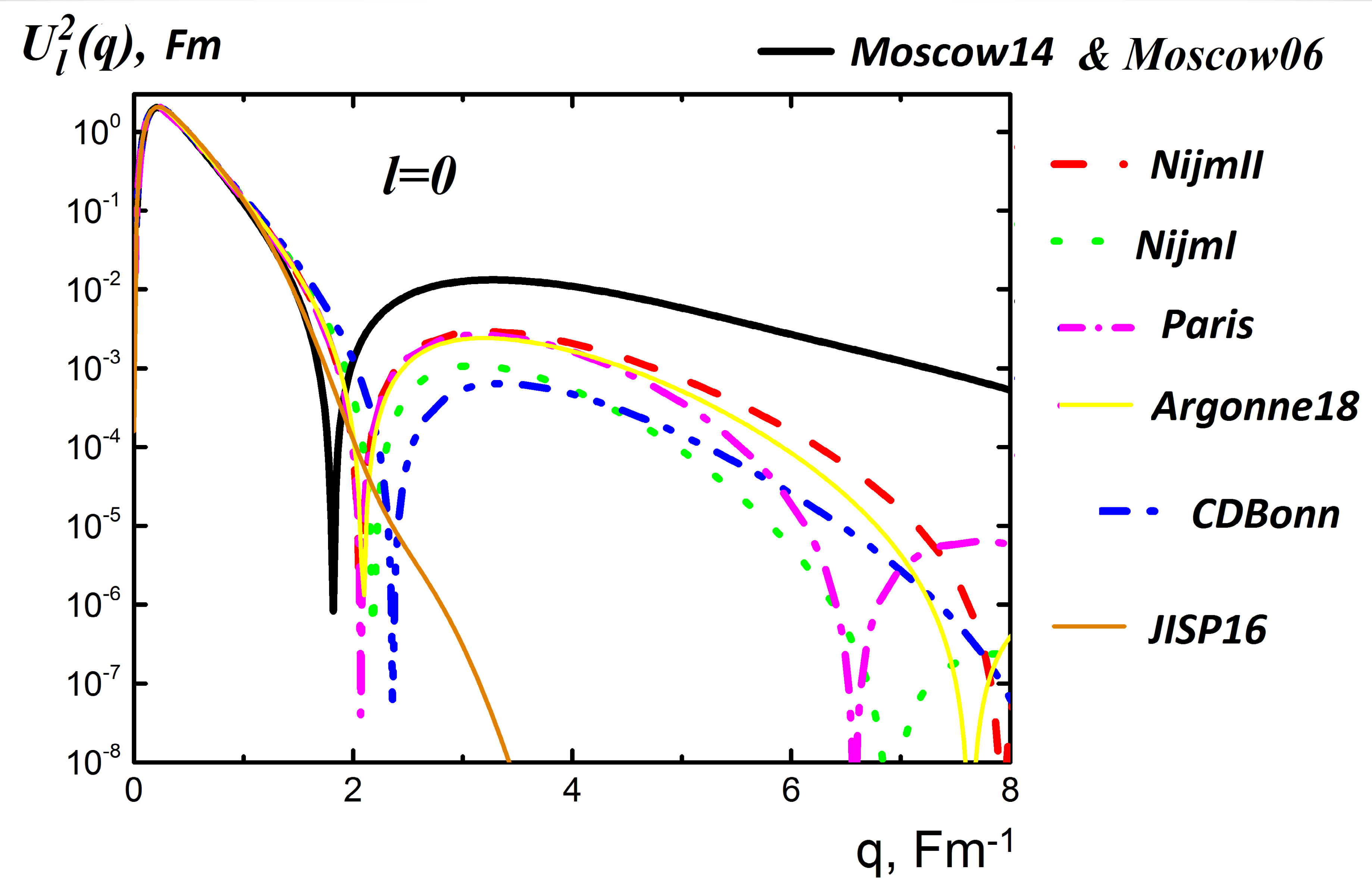
**polarization quantity**

$$t_{20}(Q^2,\theta) \approx \tilde{t}_{20}(Q^2) = -\frac{\frac{8}{3}\eta G_C(Q^2)G_Q(Q^2) + \frac{8}{9}\eta^2 G_Q^2(Q^2)}{\sqrt{2}(G_C^2(Q^2) + \frac{8}{9}\eta^2 G_M^2(Q^2))}$$

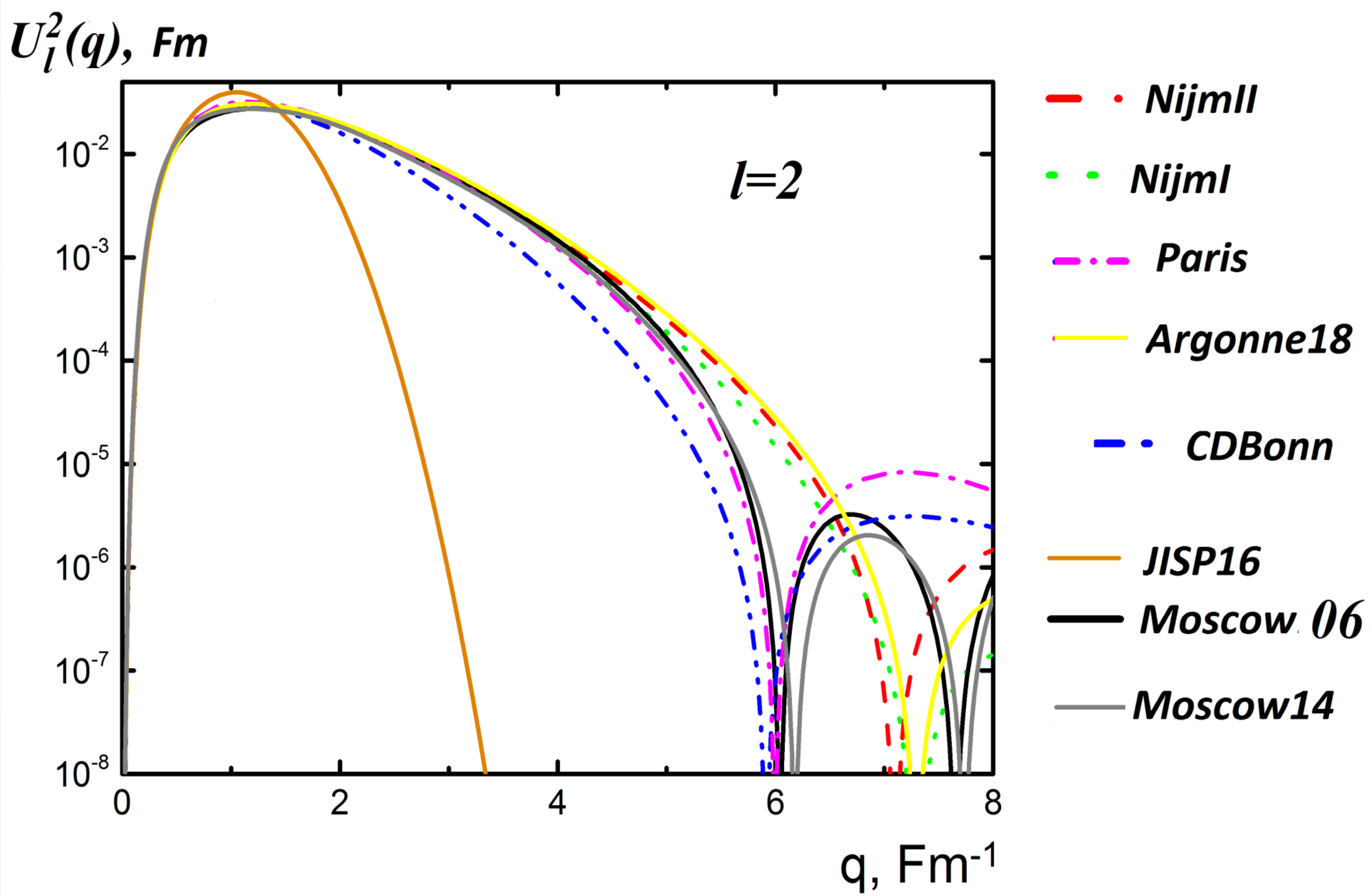
# The deuteron WFs in CS (r)



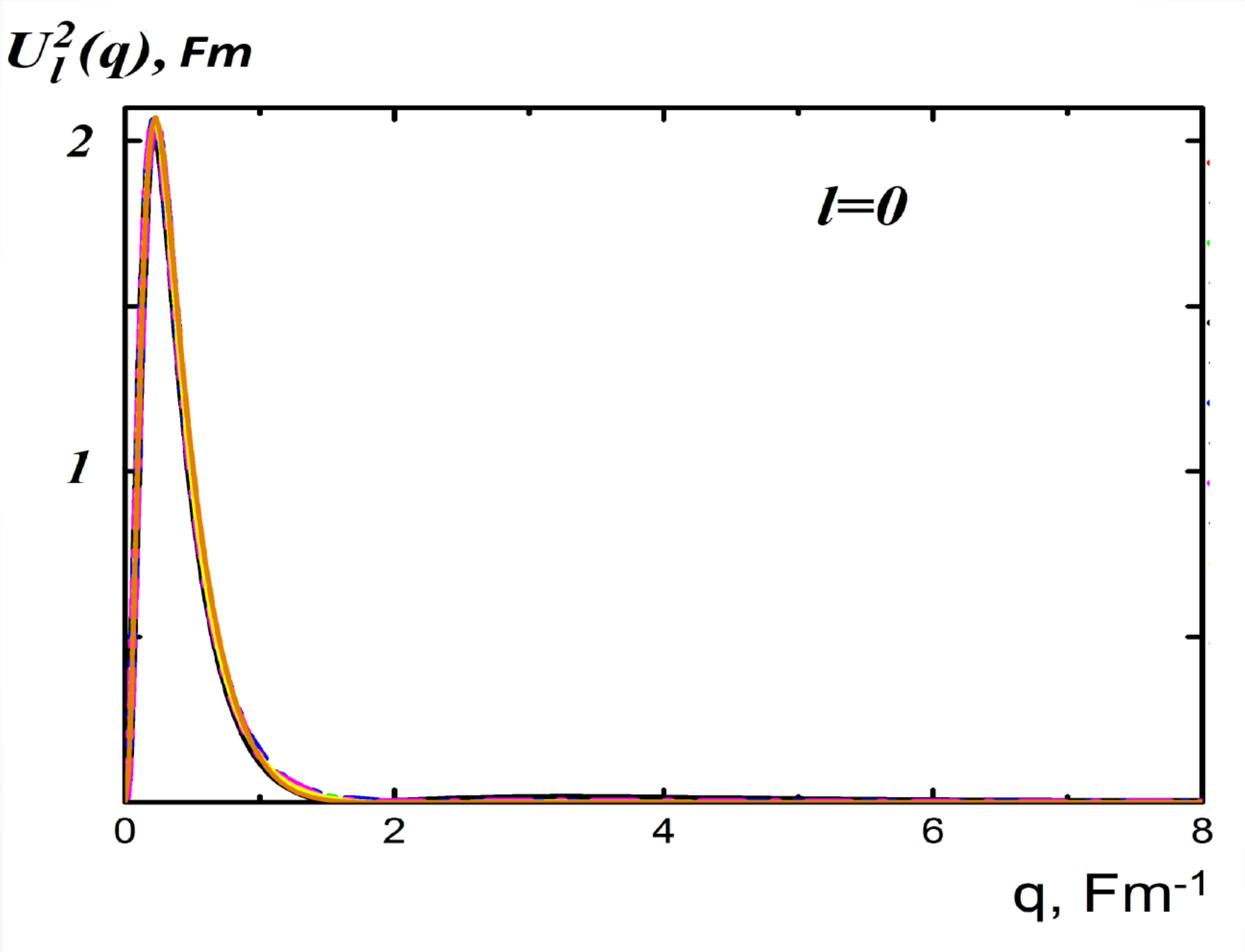
# The deuteron WFs in MS (q)



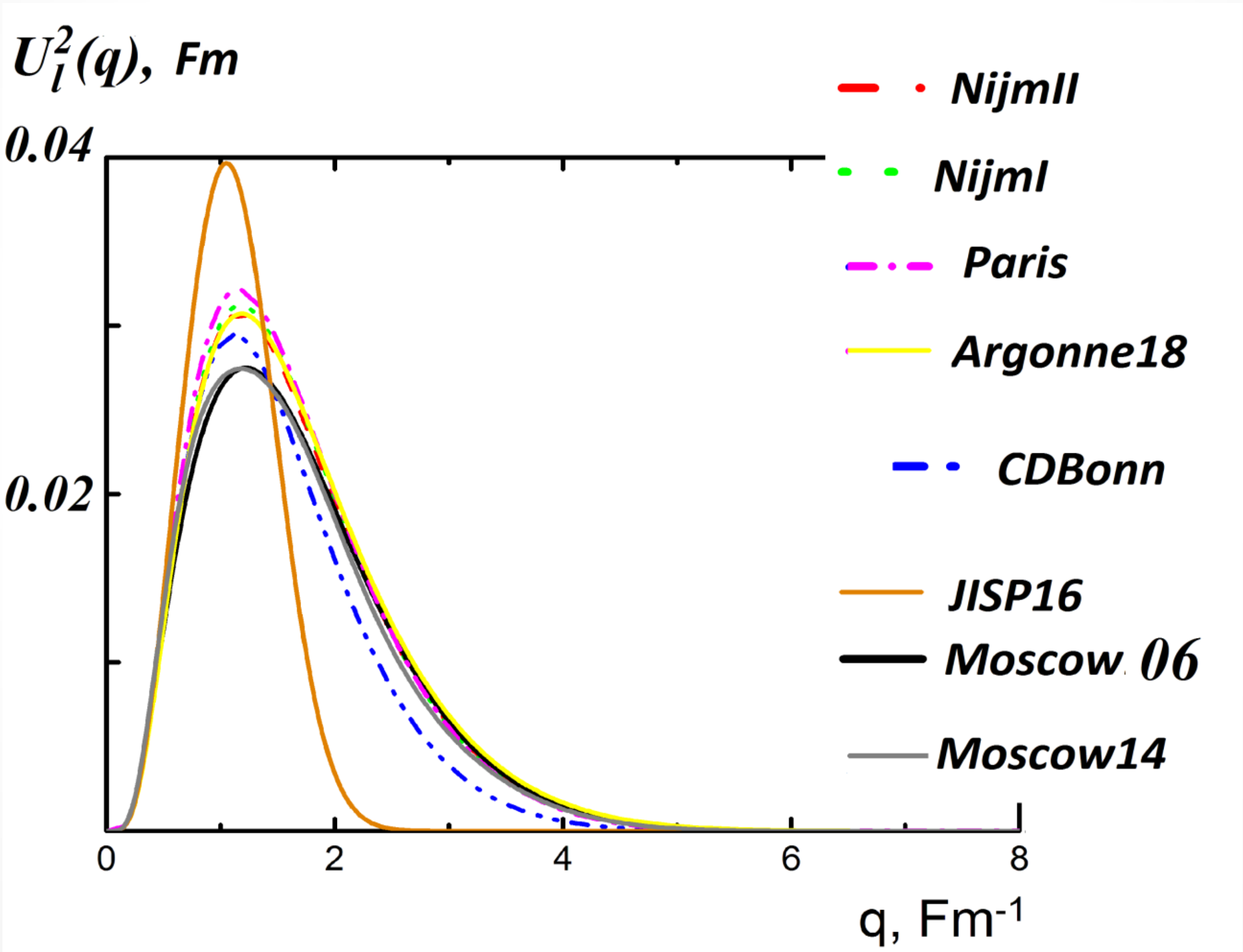
# The deuteron WFs in MS (q)



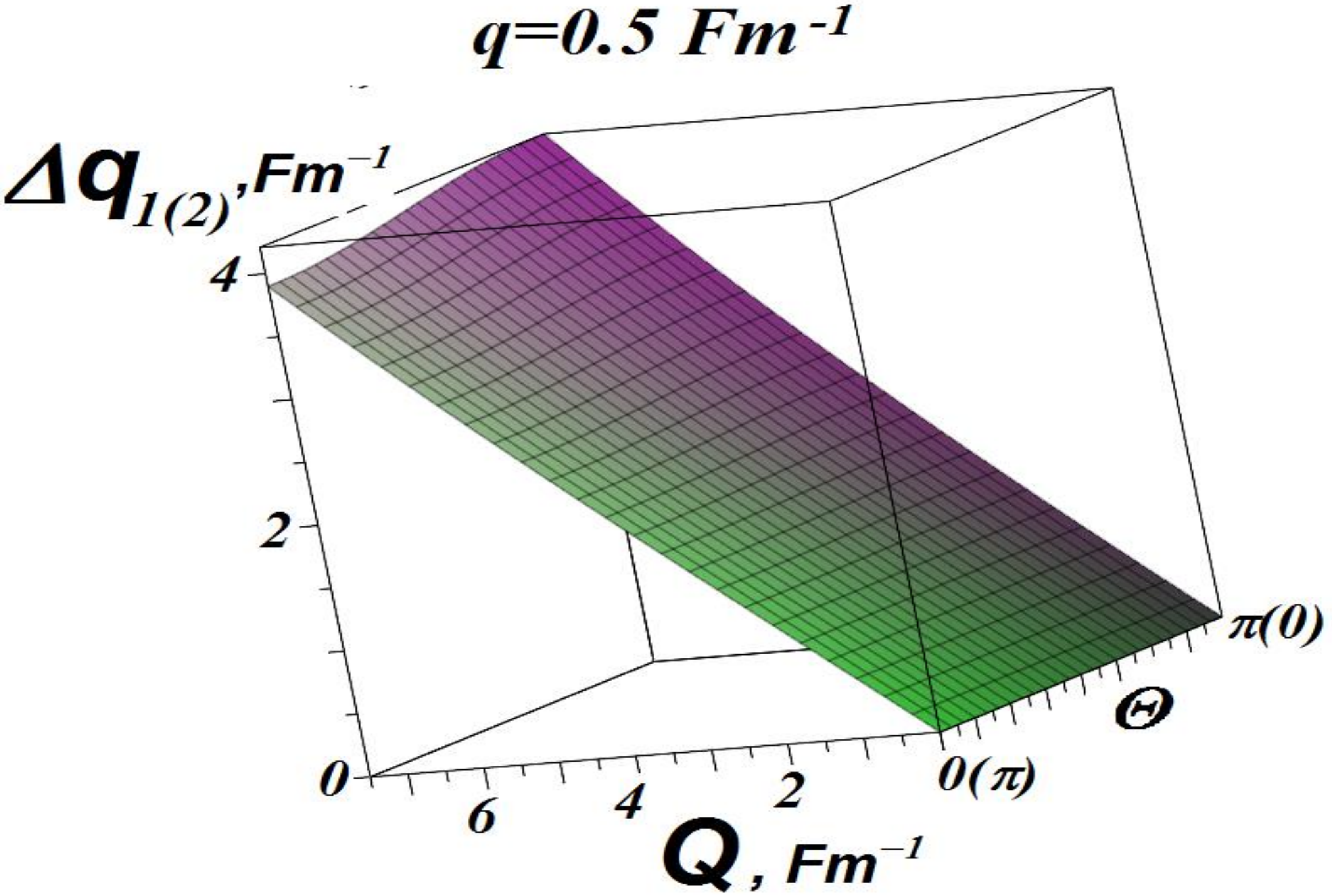
# The deuteron WFs in MS (q)



# The deuteron WFs in MS (q)



$$\sim \sum_{j=e,m} \sum_{i=1,2} \int dq^3 \chi_f(q) f_i(q, Q, \Lambda_i, \Lambda_f, \lambda) F_j^i\left(Q_i^2\right) \chi_i(q + \Delta q_i)$$



# Nucleon FFs

- Proton FFs:

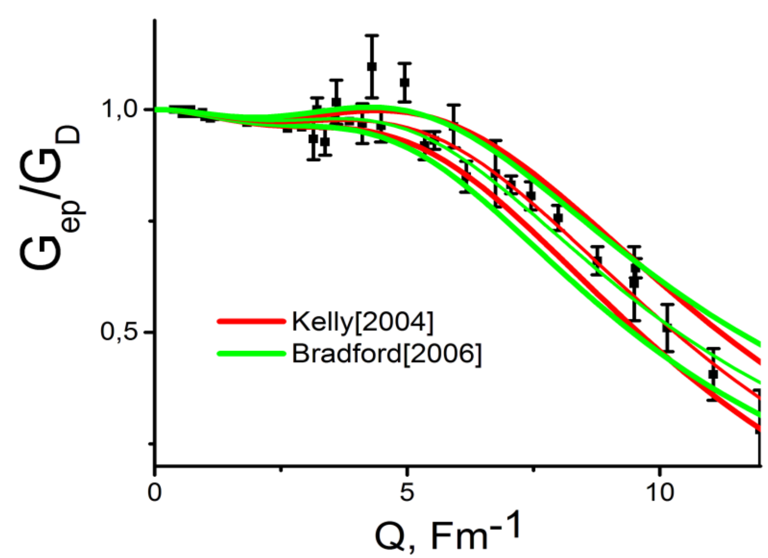
Rosenbluth separation from ep elastic cs at different scat angles: Large uncertainties at large Q for Ge and at small Q for Gm: Ge/Gm is almost a constant

- Akhiezer[1968]: Recoil polarization to improve the *FF accuracy*: Proton Ge/Gm is decreasing with Q

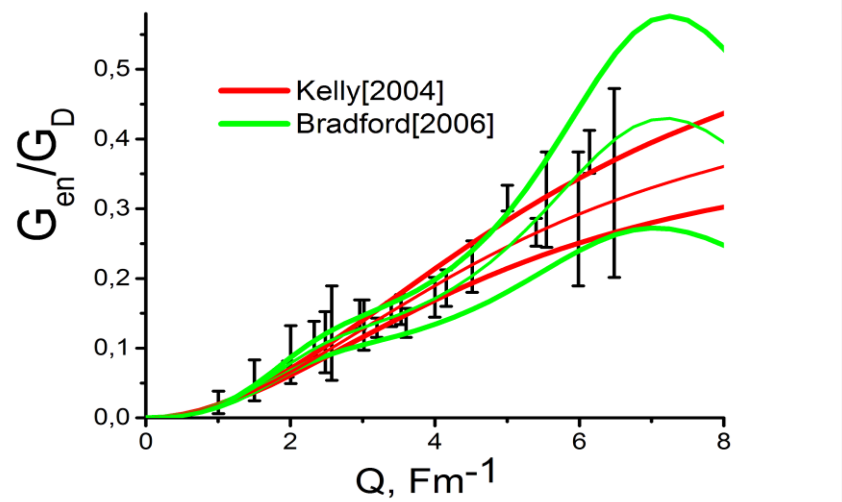
Two photon exchange

- Neutron GE and GM are extracted from reactions with deuteron and helium-3 these extractions may be affected by large nuclear structure correlations

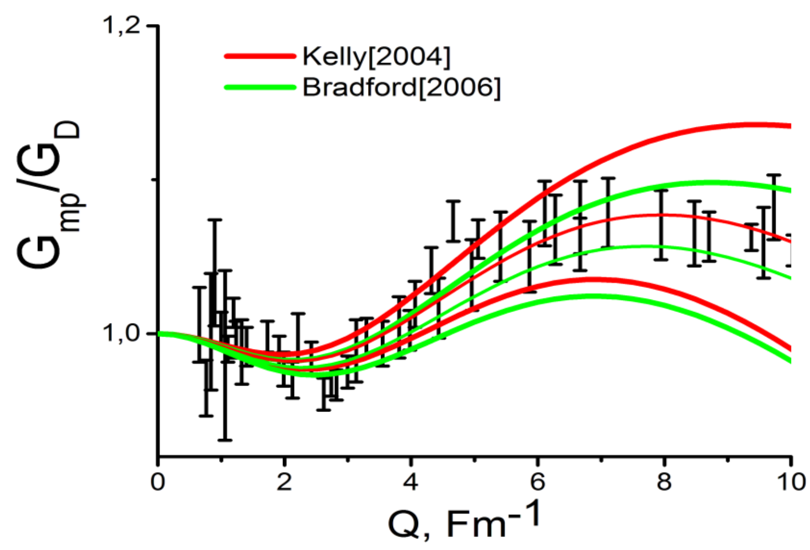
# Nucleon FFs



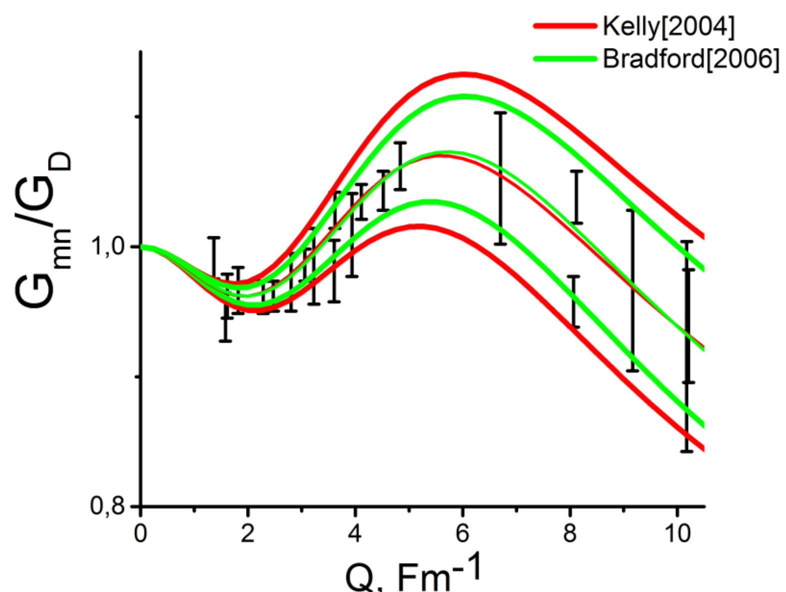
$G_{ep}/G_D$
1,0
0,5
0
5
10
Q, Fm$^{-1}$
Kelly[2004]
Bradford[2006]

$G_{en}/G_D$
0,0
0,1
0,2
0,3
0,4
0,5
0
2
4
6
8
Q, Fm$^{-1}$
Kelly[2004]
Bradford[2006]

$G_{mp}/G_D$
1,0
1,2
0
2
4
6
8
10
Q, Fm$^{-1}$
Kelly[2004]
Bradford[2006]

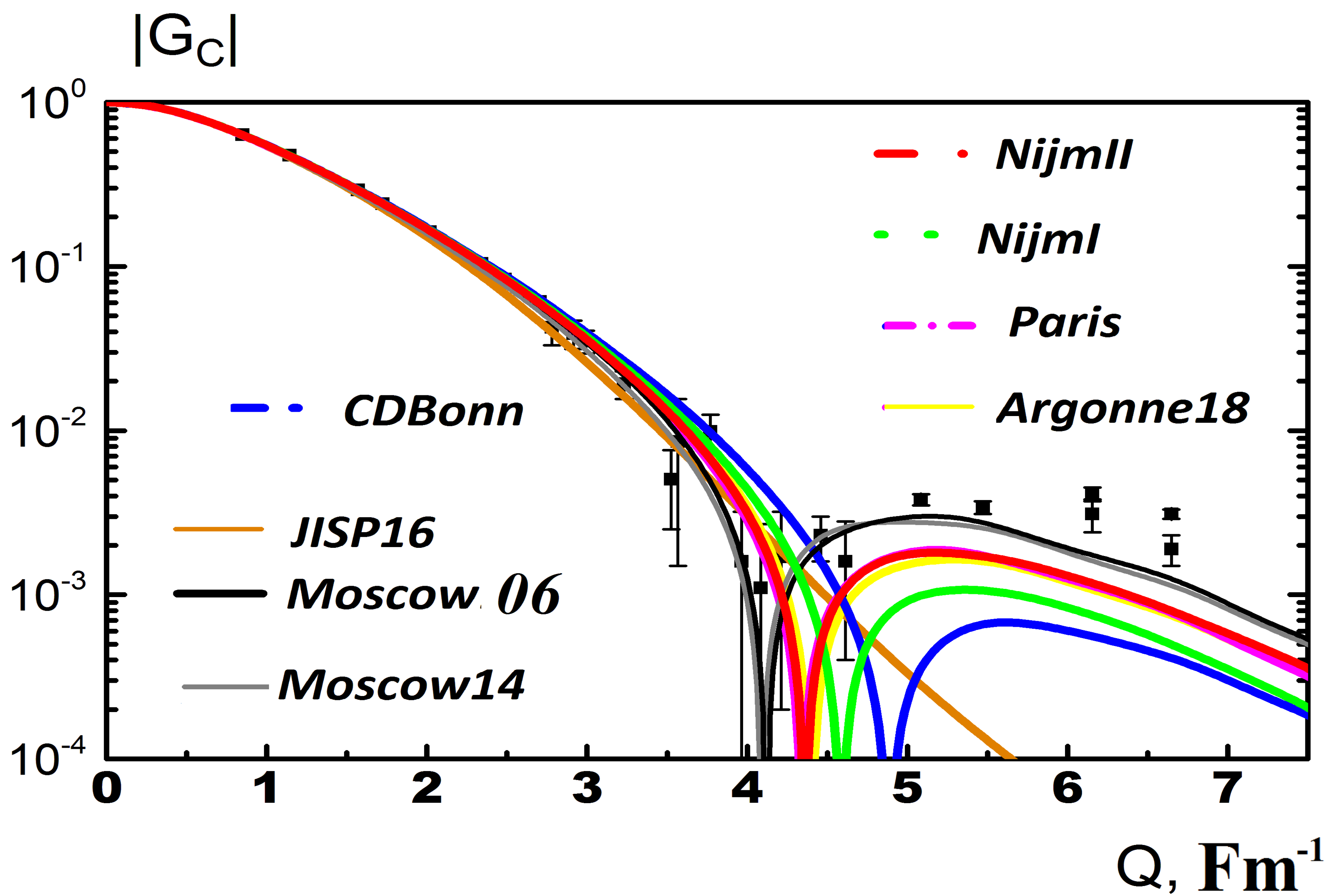



$G_{mn}/G_D$
0,8
1,0
0
2
4
6
8
10
Q, Fm$^{-1}$
Kelly[2004]
Bradford[2006]

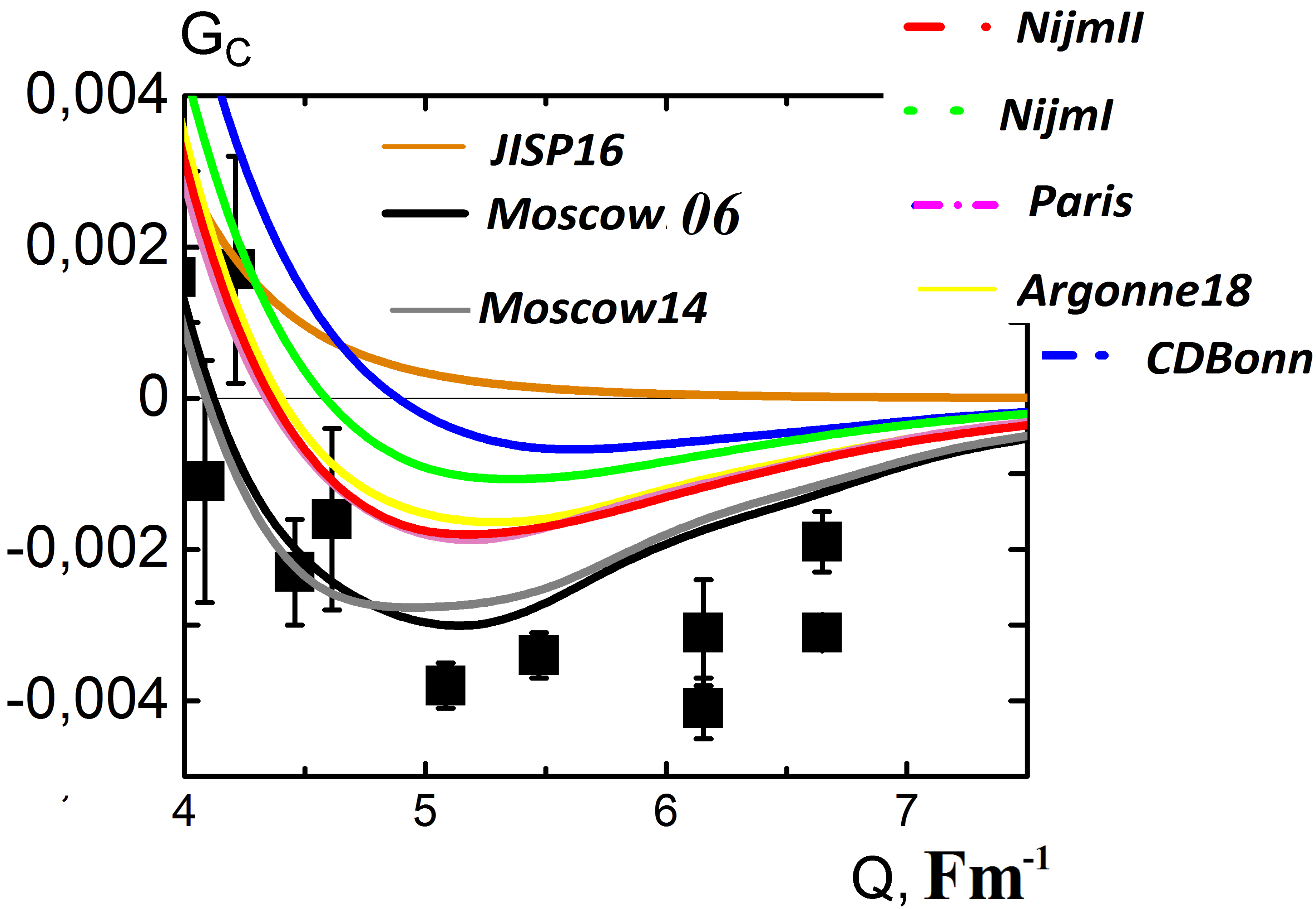
# Results: static FFs

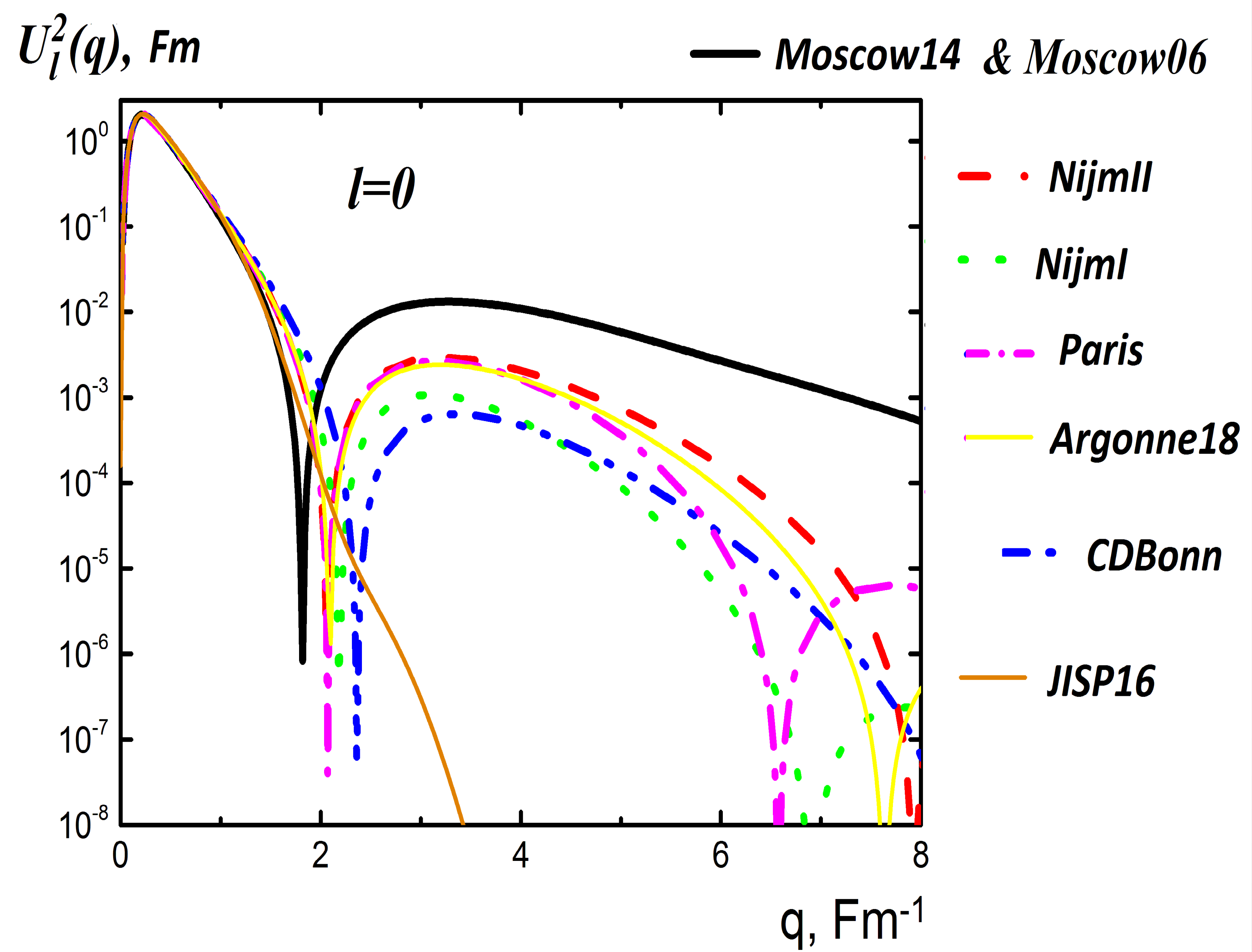
| Rel/nonrel | $G_M(0) = \frac{M_d}{m_p}\mu_d$ | $G_Q(0) = M_d^2 Q_d$ |
|---|---|---|
| Exp | 1.7148 | 25.83 |
| NijmI | 1.697/1.695 | 24.8/24.6 |
| NijmII | 1.700/1.695 | 24.7/24.5 |
| Paris | 1.696/1.694 | 25.6/25.2 |
| CD-Bonn | 1.708/1.704 | 24.8/24.4 |
| Argonne18 | 1.696/1.694 | 24.7/24.4 |
| JISP16 | 1.720/1.714 | 26.3/26.1 |
| Moscow06 | 1.711/1.699 | 24.5/24.2 |
| Moscow14 (preliminary) | 1.716/1.700 | 26.0/25.8 |

# Results

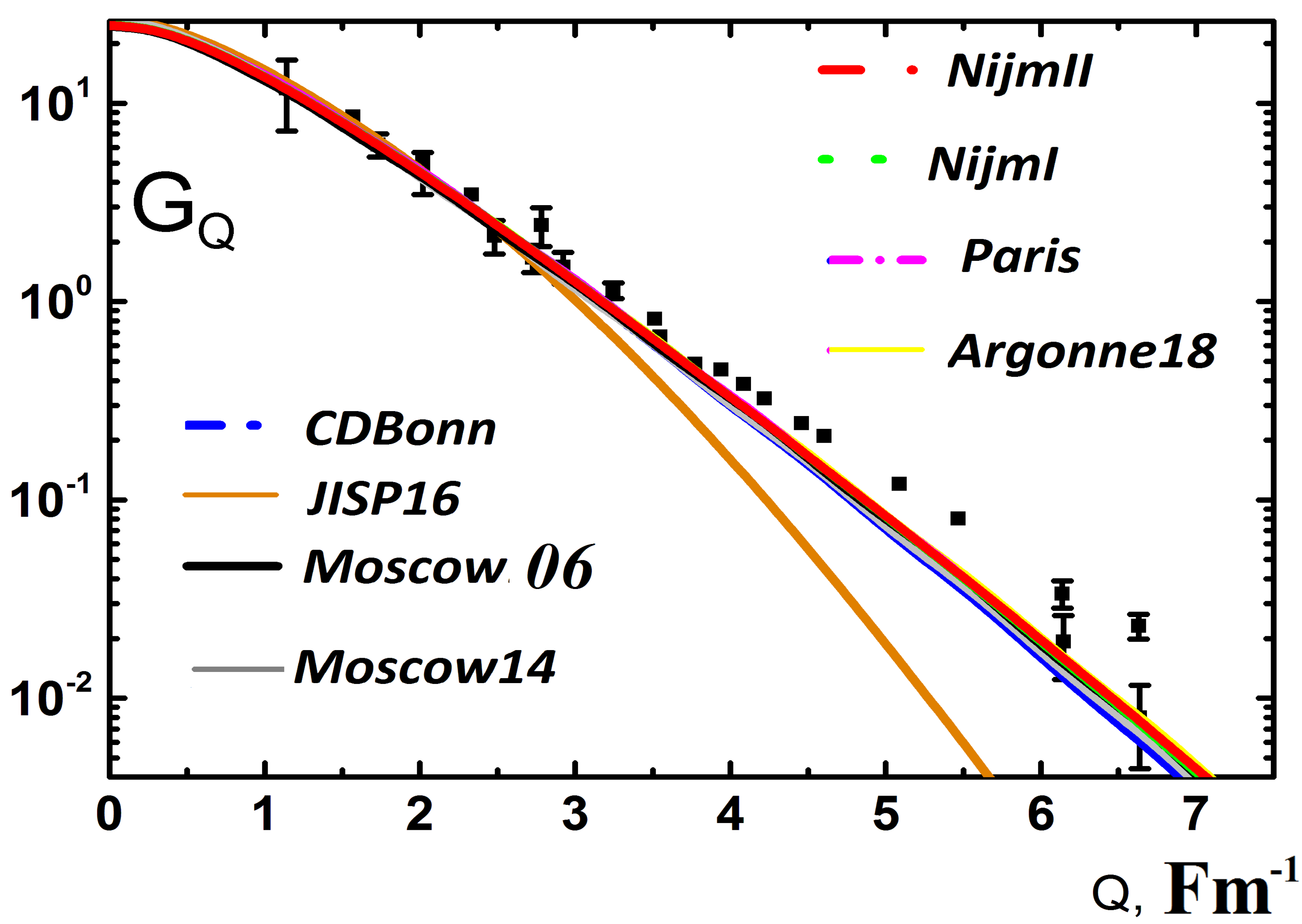


|GC|
10^0
10^-1
10^-2
10^-3
10^-4
NijmII
NijmI
Paris
Argonne18
CDBonn
JISP16
Moscow 06
Moscow14
0
1
2
3
4
5
6
7
Q, Fm^-1

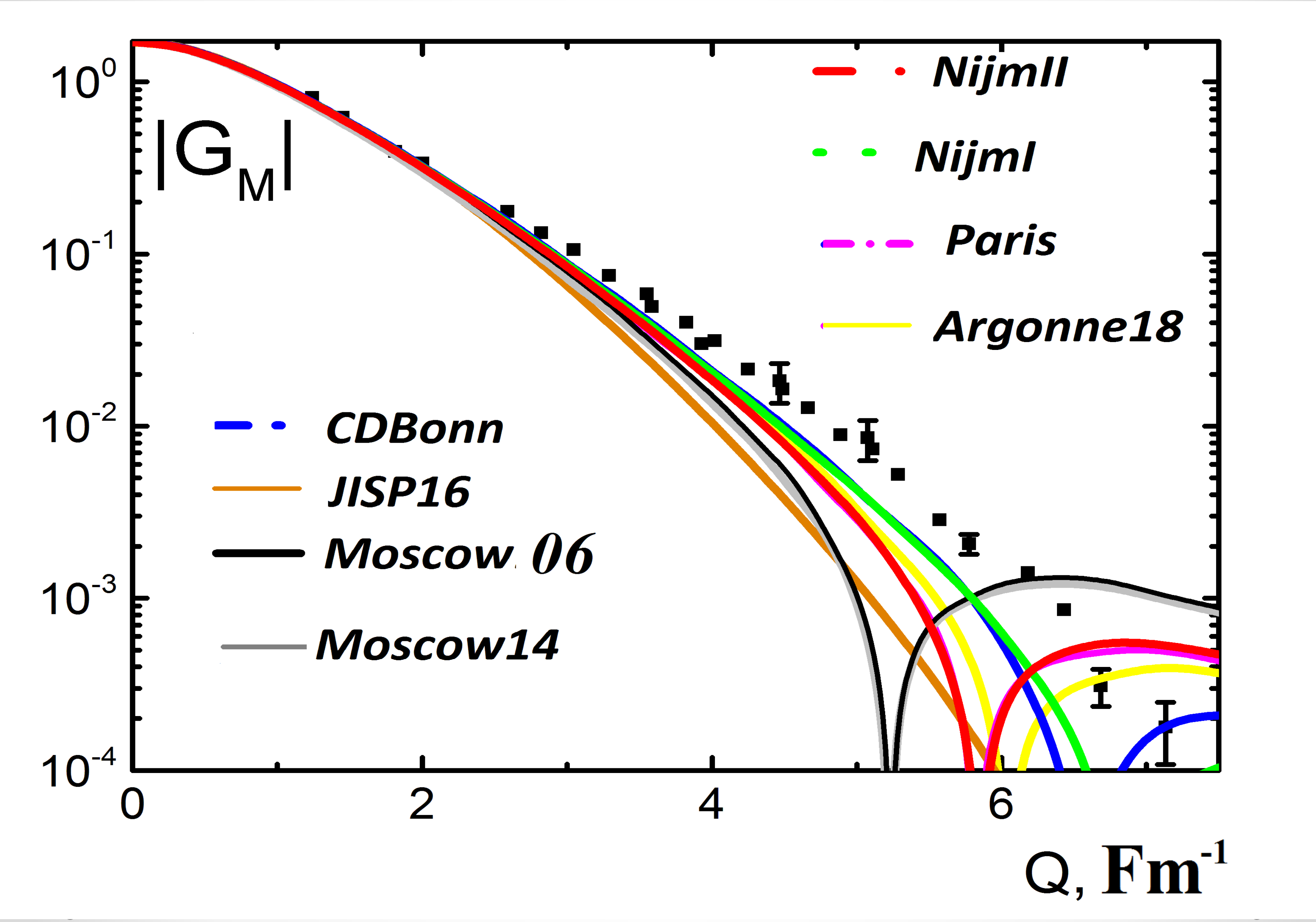

G_C
0,004
0,002
0
-0,002
-0,004
4
5
6
7
Q, Fm^-1
JISP16
Moscow 06
Moscow14
NijmII
NijmI
Paris
Argonne18
CDBonn



$U_l^2(q)$, Fm
Moscow14 & Moscow06
NijmII
NijmI
Paris
Argonne18
CDBonn
JISP16
l=0
$10^0$
$10^{-1}$
$10^{-2}$
$10^{-3}$
$10^{-4}$
$10^{-5}$
$10^{-6}$
$10^{-7}$
$10^{-8}$
0
2
4
6
8
q, Fm$^{-1}$

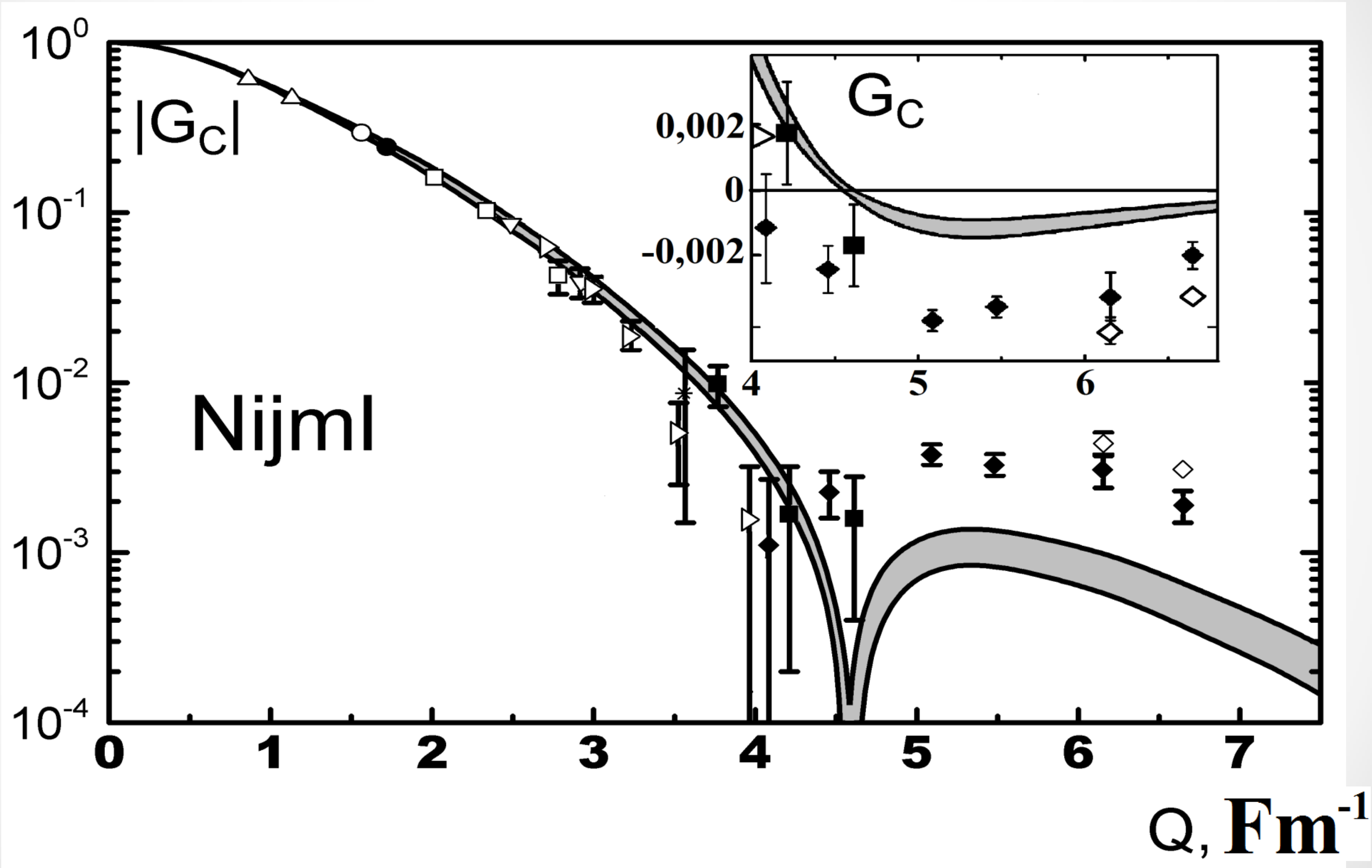


G_Q
NijmII
NijmI
Paris
Argonne18
CDBonn
JISP16
Moscow 06
Moscow14
10^1
10^0
10^-1
10^-2
0
1
2
3
4
5
6
7
Q, Fm^-1



$|G_M|$
NijmII
NijmI
Paris
Argonne18
CDBonn
JISP16
Moscow 06
Moscow14
$10^0$
$10^{-1}$
$10^{-2}$
$10^{-3}$
$10^{-4}$
0
2
4
6
Q, $Fm^{-1}$

# Uncertainties in nucleon FFs lead to



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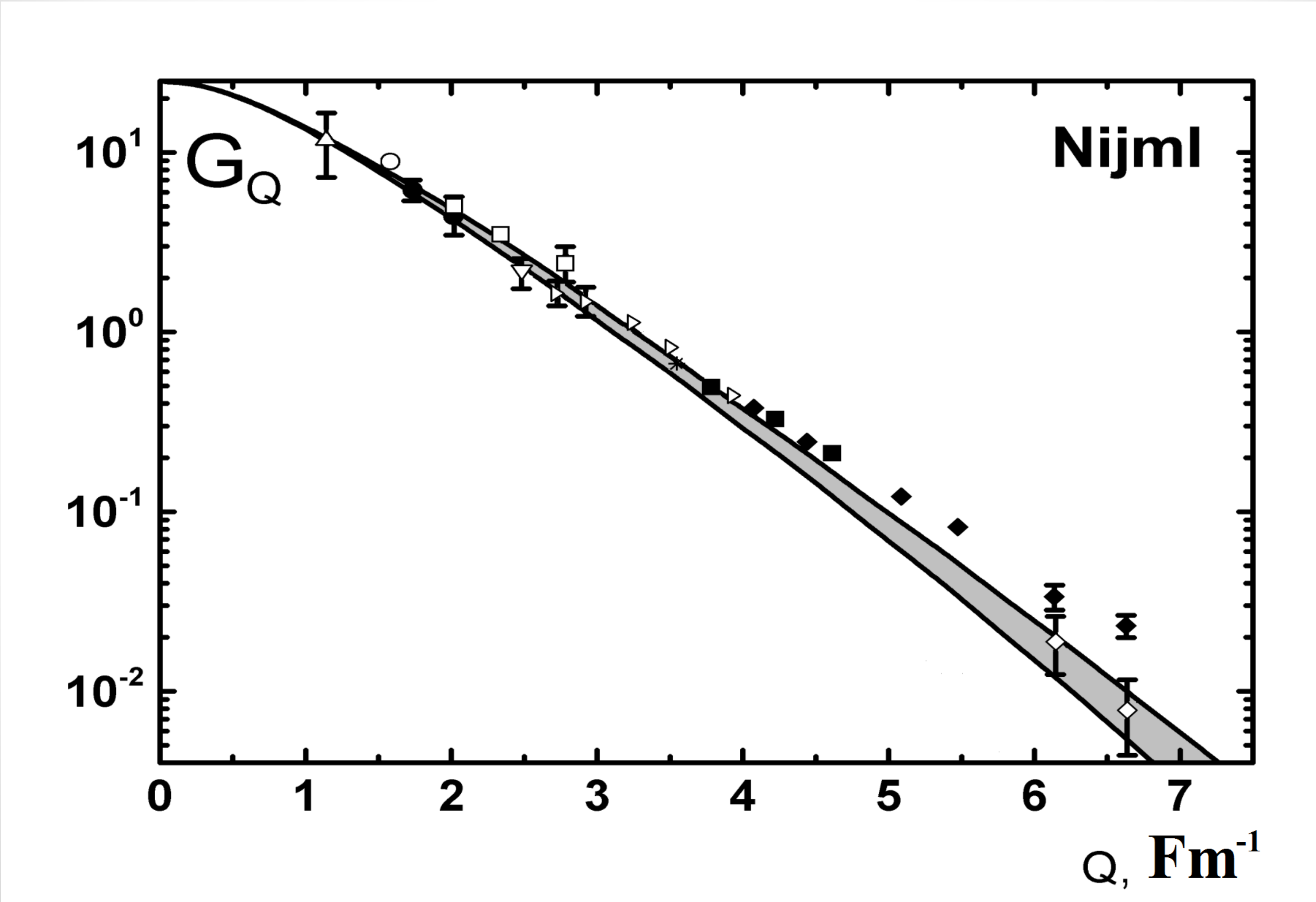
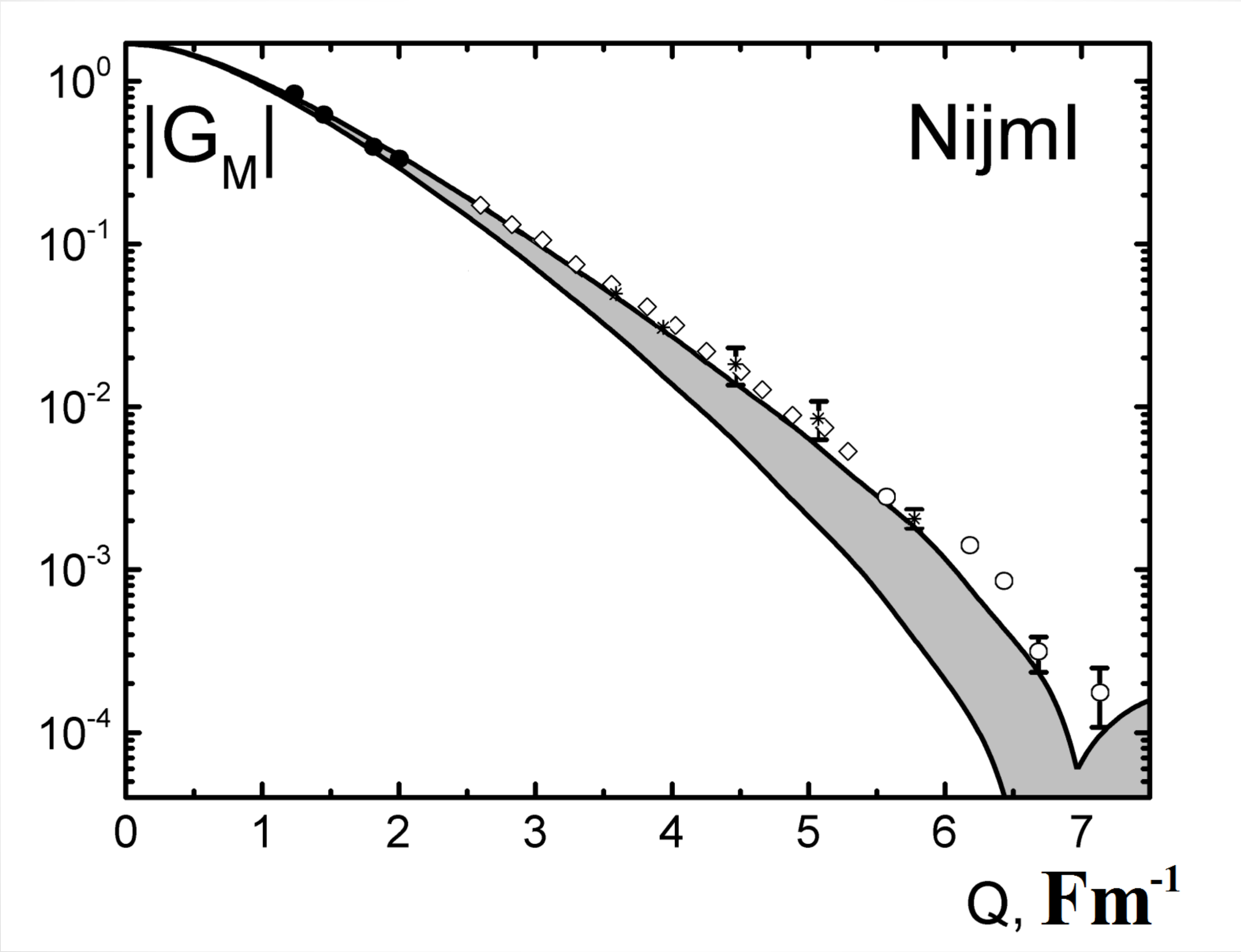
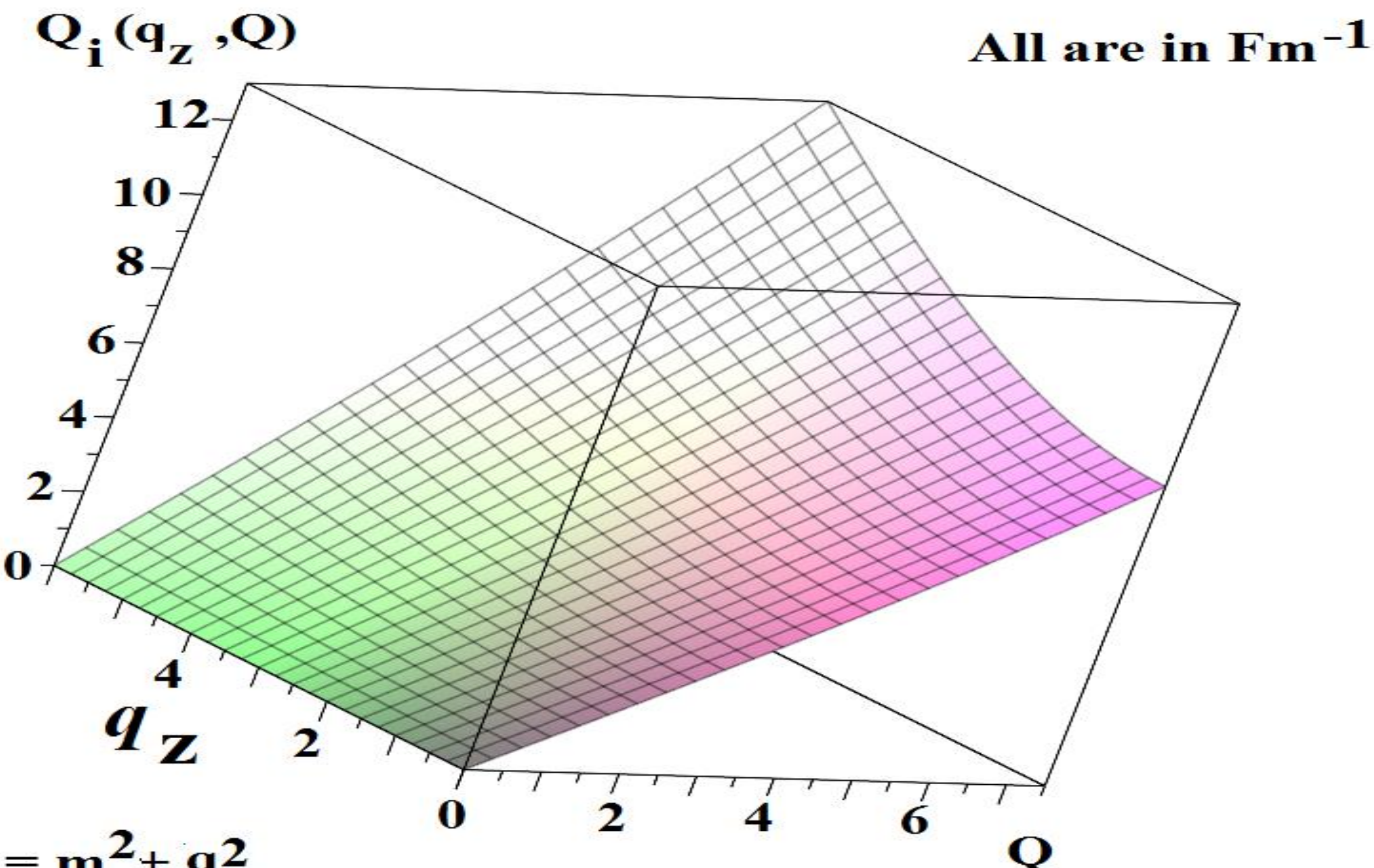


# Uncertainties in nucleon FFs lead to



$$\sim \sum_{j=e,m} \sum_{i=1,2} \int dq^3 \chi_f(q) f_i(q, Q, \Lambda_i, \Lambda_f, \lambda) F_j^i(Q_i^2) \chi_i(q + \Delta q_i)$$



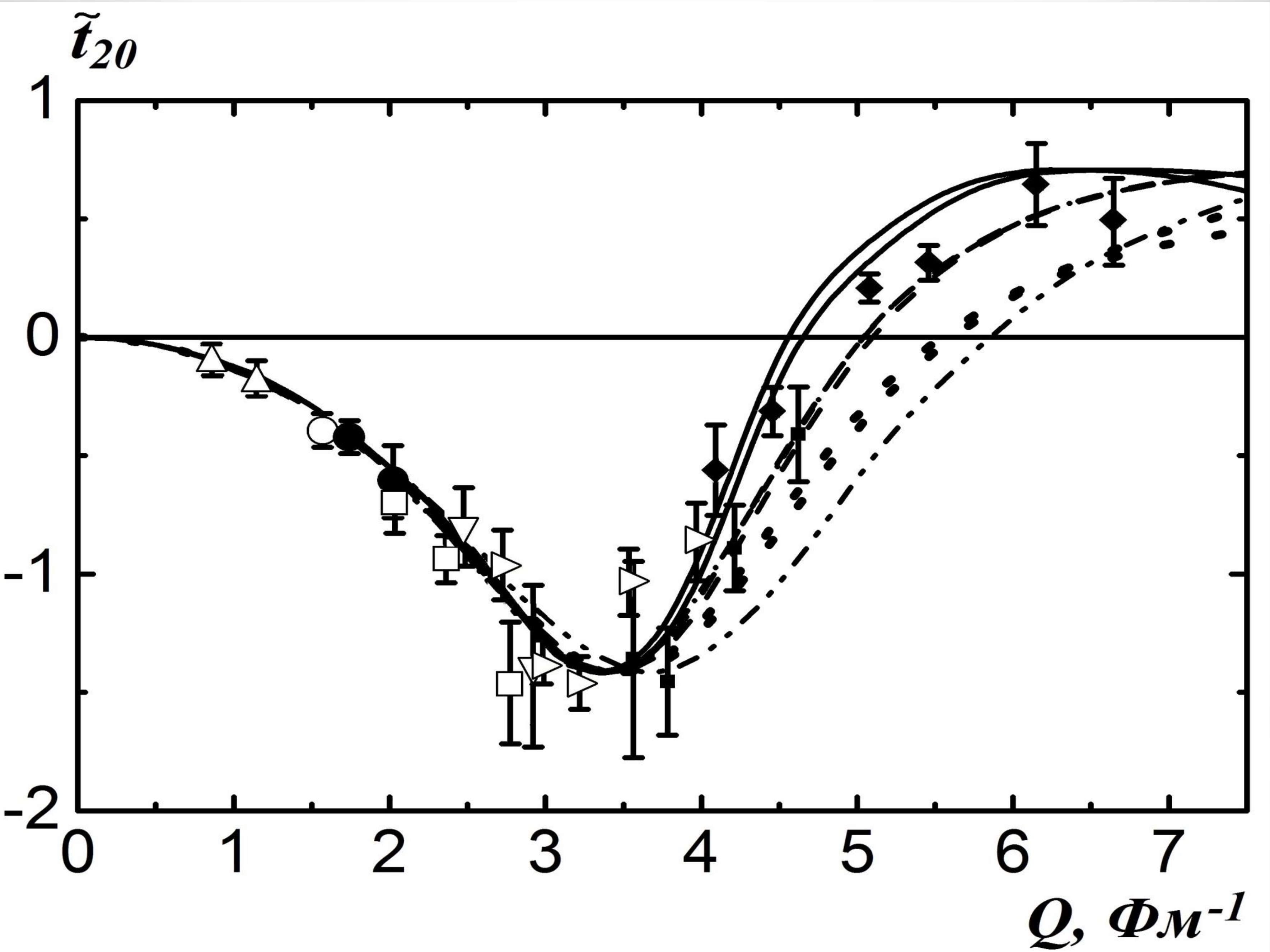

$$w^2 = m^2 + q^2$$

$$Q_i^2 = 4(w^2 - q_z^2)\frac{Q^2}{m_D^2}\left(1 + \frac{Q^2}{4m_D^2}\right)$$

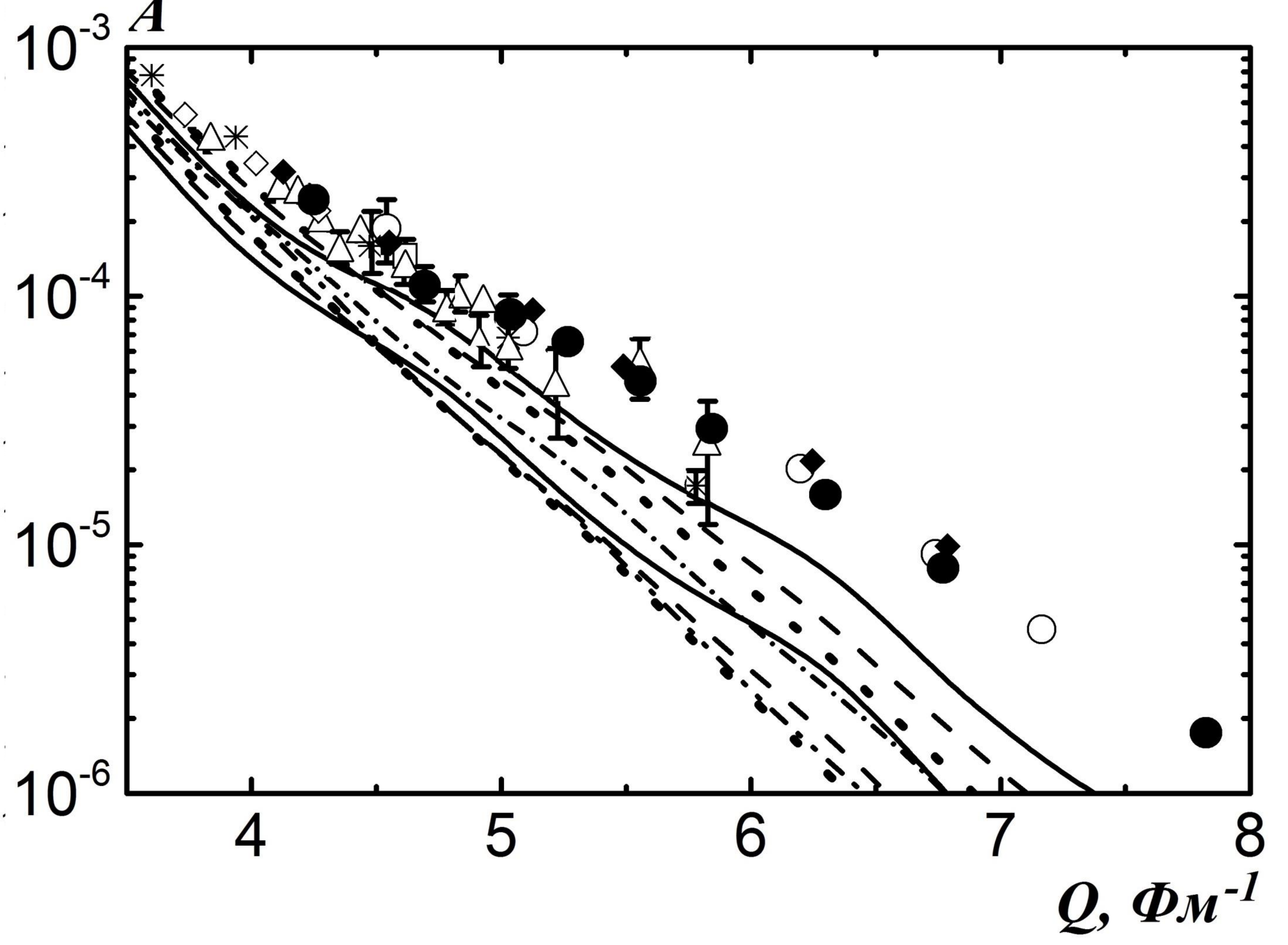
**Allen et al [2001]**

# Comparing with other results

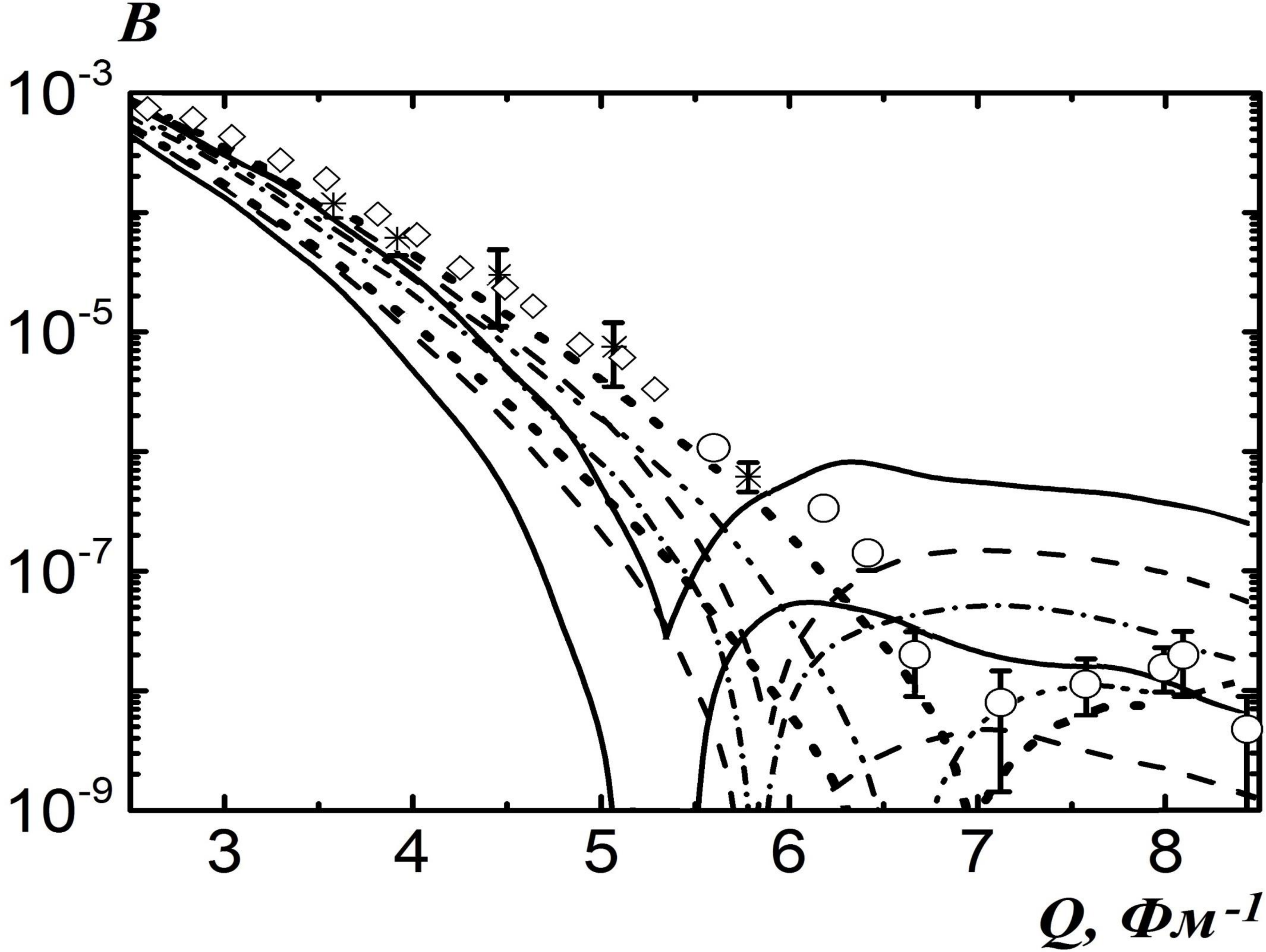
- Allen, Klink & Polyzou[2001:
- Point form
- ECO parameterization of Klink[1998]
- Argonne18 and Reid93 potentials
- Gari-Krumpelmann[1992] and Mergell-Meissner-Drechsel[1996]



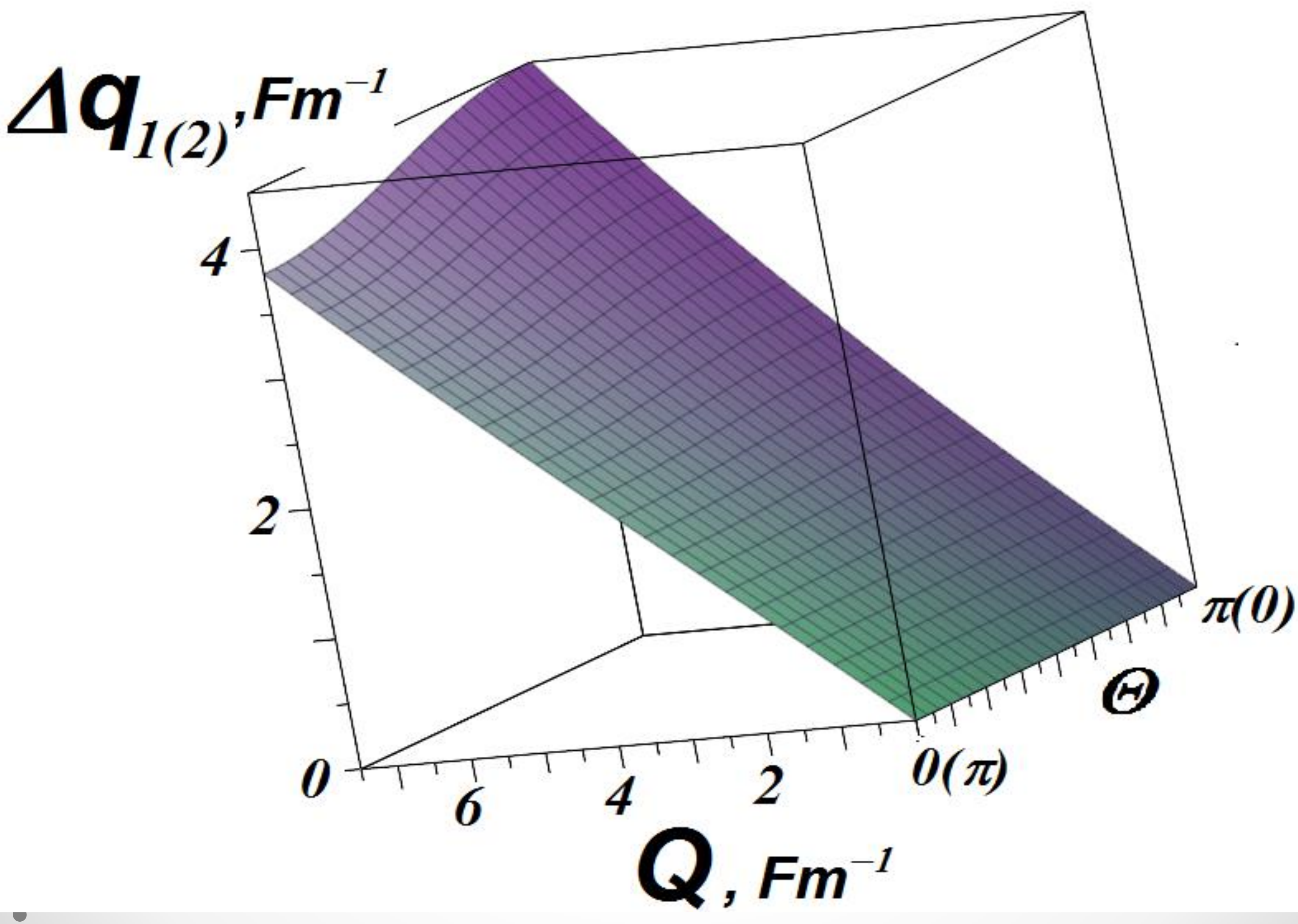
$\tilde{t}_{20}$
1
0
-1
-2
0
1
2
3
4
5
6
7
$Q$, Фм$^{-1}$

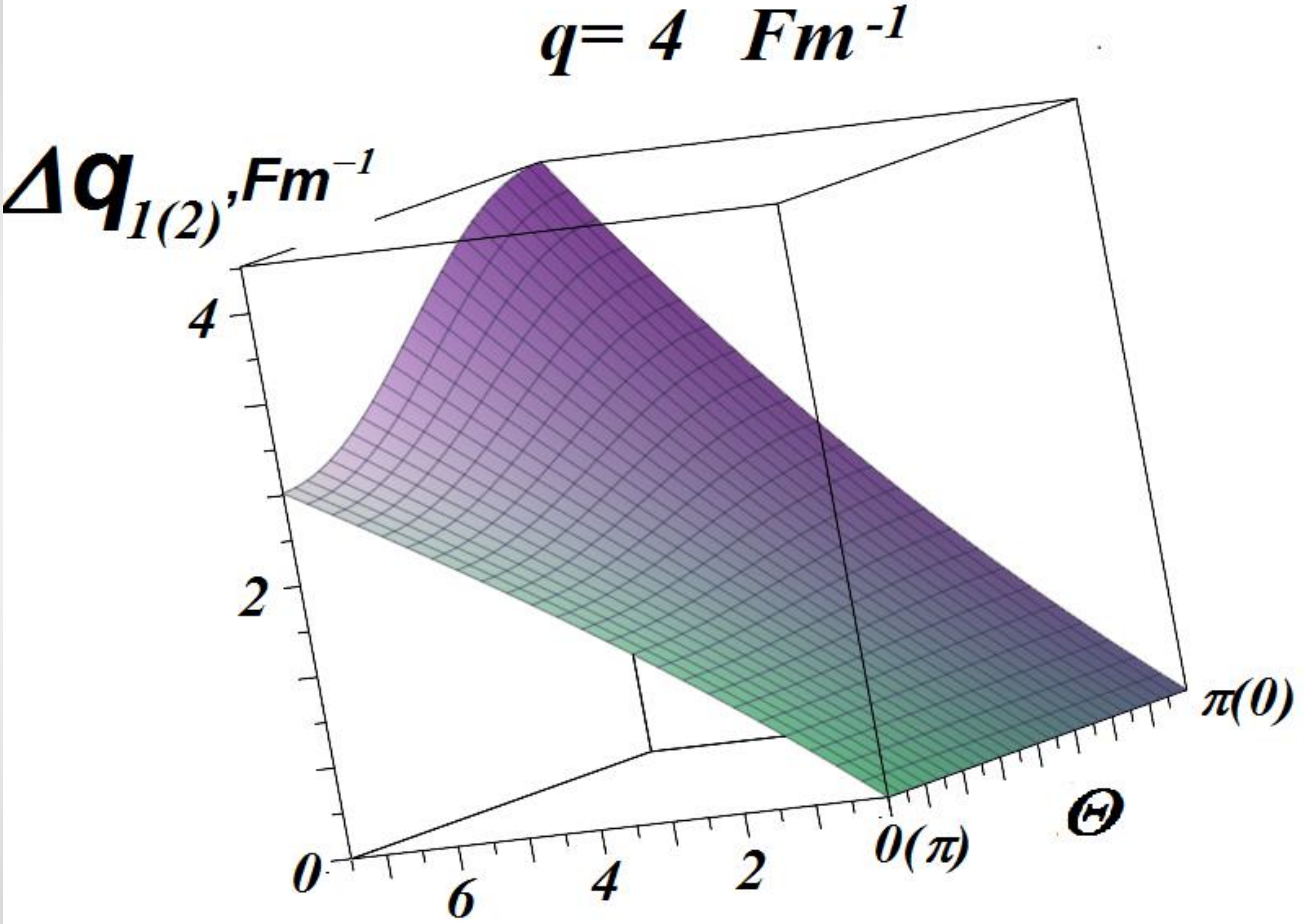


A
$10^{-3}$
$10^{-4}$
$10^{-5}$
$10^{-6}$
4
5
6
7
8
Q, Фм$^{-1}$


B
10-3
10-5
10-7
10-9
3
4
5
6
7
8
Q, Фм-1

$q = 1\ Fm^{-1}$





q= 4 Fm-1
Δq1(2),Fm−1
4
2
0
6
4
2
0(π)
π(0)
Θ