Chiral EFT and nuclear forces: Are we in trouble?

R. Machleidt, University of Idaho

Outline

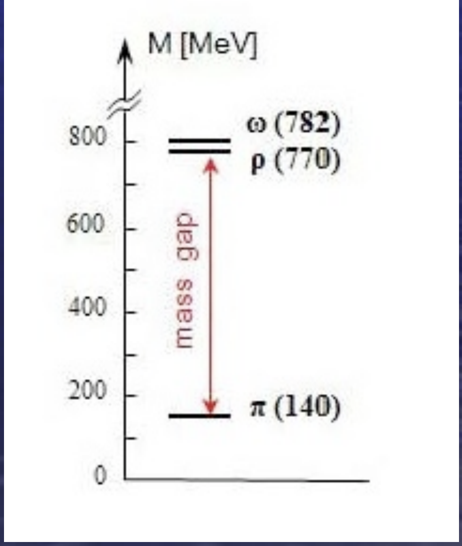
- Nuclear forces from chiral EFT:
  Basic ideas and overview
- Two-nucleon forces (2NF) and renormalization
- Three-nucleon forces (3NF)
- Higher orders: The explosion of contributions
- Conclusions

The ultimate goal of nuclear physics: Understanding nuclei from first principles

- **Forces from first principals (QCD)**
- ***Ab initio* many-body methods**

Forces from first principles, i.e., QCD

- QCD at low energy is strong.
- Quarks and gluons are confined into colorless hadrons.
- Nuclear forces are residual forces (similar to van der Waals forces)
- Separation of scales



- **Calls for an EFT:**

  soft scale: $Q \approx m_\pi$, hard scale: $\Lambda_\chi \approx m_\rho$;
  pions and nucleons are relevant d.o.f.
- **Low-momentum expansion: $(Q/\Lambda_\chi)^\nu$**

  with ν bounded from below.
- **Most general Lagrangian consistent with all symmetries of low-energy QCD, particularly, chiral symmetry which is spontaneously broken.**
- **Weakly interacting Goldstone bosons = pions.**
- **π-π and π-N perturbatively**
- **NN has bound states:**

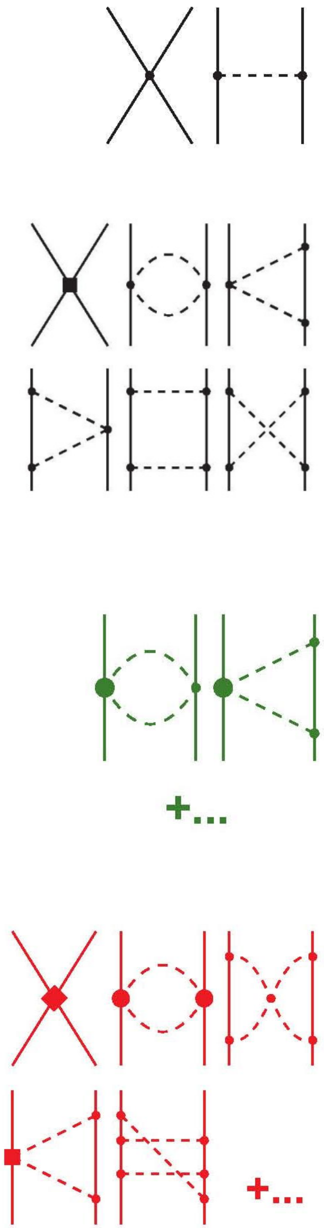
  (i) NN potential perturbatively

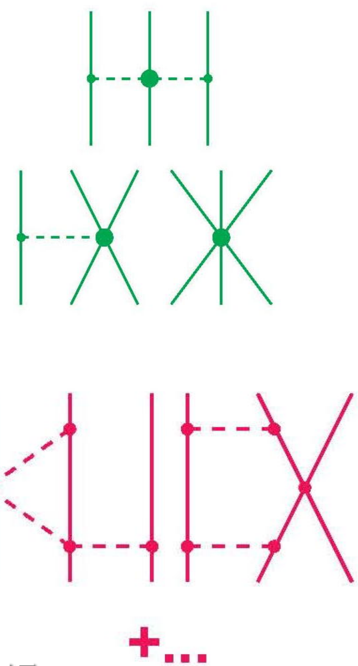
  (ii) apply nonpert. in LS equation.

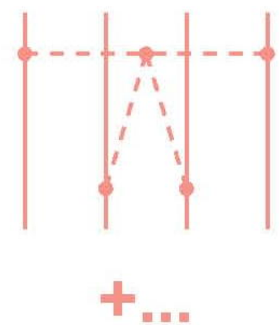
  (Weinberg)

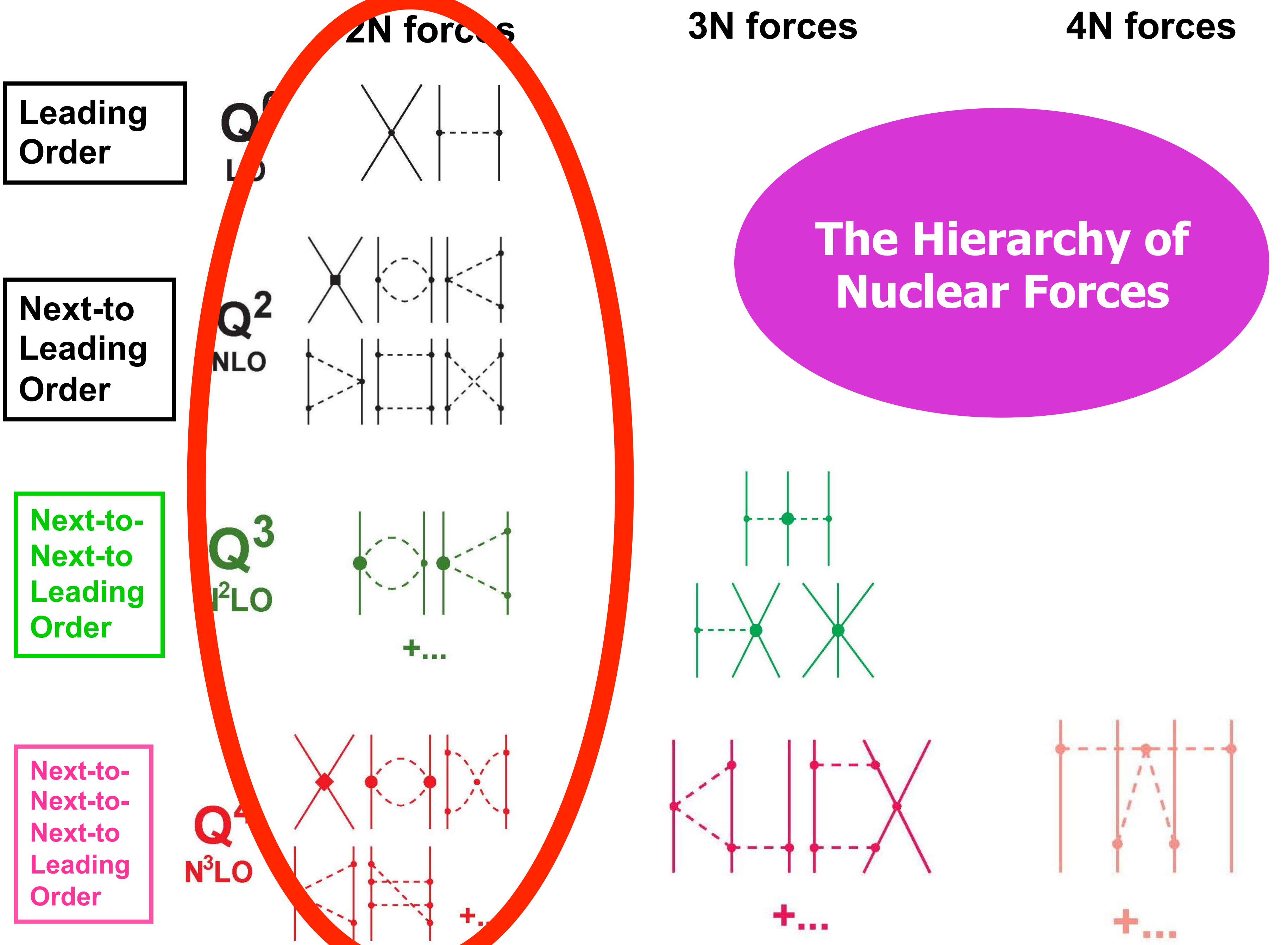
The Hierarchy of Nuclear Forces

		2N forces	3N forces	4N forces
Leading Order	Q^0 LO			
Next-to Leading Order	Q^2 NLO			
Next-to-Next-to Leading Order	Q^3 N^2LO			
Next-to-Next-to-Next-to Leading Order	Q^4 N^3LO			








2N forces
3N forces
4N forces
Leading Order
Q0
LO
Next-to Leading Order
Q2
NLO
Next-to-Next-to Leading Order
Q3
N2LO
+...
Next-to-Next-to-Next-to Leading Order
Q4
N3LO
+...
+...
+...
The Hierarchy of Nuclear Forces

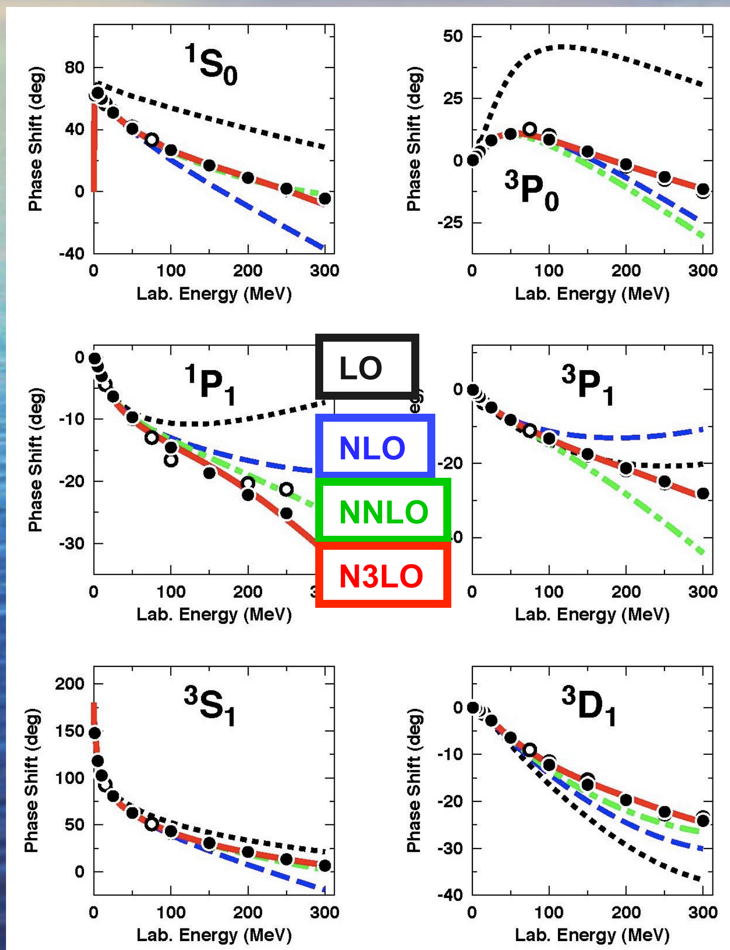
NN phase shifts up to 300 MeV

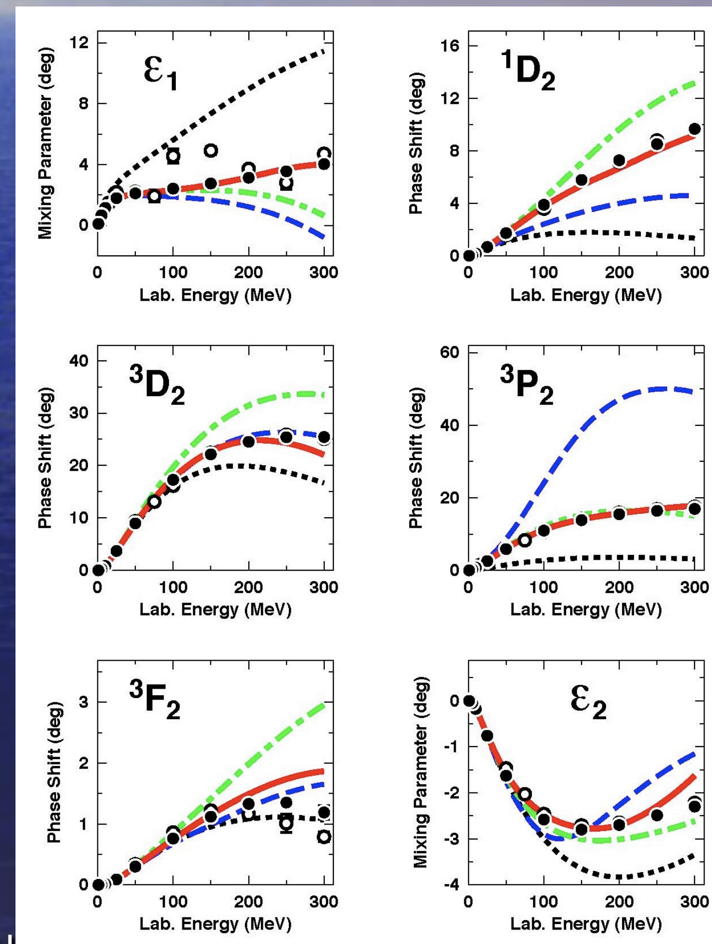
Red Line: N3LO Potential by Entem & Machleidt, PRC 68, 041001 (2003).

Green dash-dotted line: NNLO Potential, and

blue dashed line: NLO Potential

by Epelbaum et al., Eur. Phys. J. A19, 401 (2004).





PRL 110, 192502 (2013) PHYSICAL REVIEW LETTERS week ending 10 MAY 2013

Optimized Chiral Nucleon-Nucleon Interaction at Next-to-Next-to-Leading Order

A. Ekström,[1,2] G. Baardsen,[1] C. Forssén,[3] G. Hagen,[4,5] M. Hjorth-Jensen,[1,2,6] G. R. Jansen,[4,5] R. Machleidt,[7] W. Nazarewicz,[5,4,8] T. Papenbrock,[5,4] J. Sarich,[9] and S. M. Wild[9]

Bin (MeV)	# of data	N^3LO	NNLO	NLO	AV18
0–100	1058	1.05	1.00	4.5	0.95
100–190	501	1.08	1.87	100	1.10
190–290	843	1.15	6.09	180	1.11
0–290	2402	1.10	2.95	86	1.04

N3LO Potential by Entem & Machleidt, PRC 68, 041001 (2003).
NNLO and NLO Potentials by Epelbaum et al., Eur. Phys. J. A19, 401 (2004).

Summary: χ^2/datum

- NLO: ≈ 100
- NNLO: ≈ 10
- N3LO: ≈ 1

Great rate of convergence!

In contrast to older approaches to the nuclear force, like the meson model, chiral EFT wants to be a theory.

How true is that?

If EFT wants to be a theory, it better be renormalizable.

The problem in all field theories are divergent loop integrals.

The method to deal with them in field theories:

1. **Regularize the integral (e.g. apply a "cutoff") to make it finite.**
2. **Remove the cutoff dependence by Renormalization ("counter terms").**

For calculating pi-pi and pi-N reactions no problem.

However, the NN case is tougher, because it involves two kinds of (divergent) loop integrals.

The NN interaction involves **two kinds** of renormalizations

- **Perturbative:** NN Potential. No problem.
- **Non-perturbative:** NN T-matrix:
  - The potential is inserted into the Schroedinger or Lippmann-Schwinger (LS) equation: non-perturbative re-summation of ladder diagrams (infinite sum):

$$T(\vec{p}\,',\vec{p}) = V(\vec{p}\,',\vec{p}) + \int d^3p''\, V(\vec{p}\,',\vec{p}\,'')\, \frac{M_N}{p^2 - p''^2 + i\epsilon}\, T(\vec{p}\,'',\vec{p}),$$

  - Divergent integral.
  - Regularize it:

$$V(\vec{p}\,',\vec{p}) \longmapsto V(\vec{p}\,',\vec{p})\, e^{-(p'/\Lambda)^{2n}}\, e^{-(p/\Lambda)^{2n}}.$$

  - Cutoff dependent results.
  - Renormalize to get rid of the cutoff dependence:

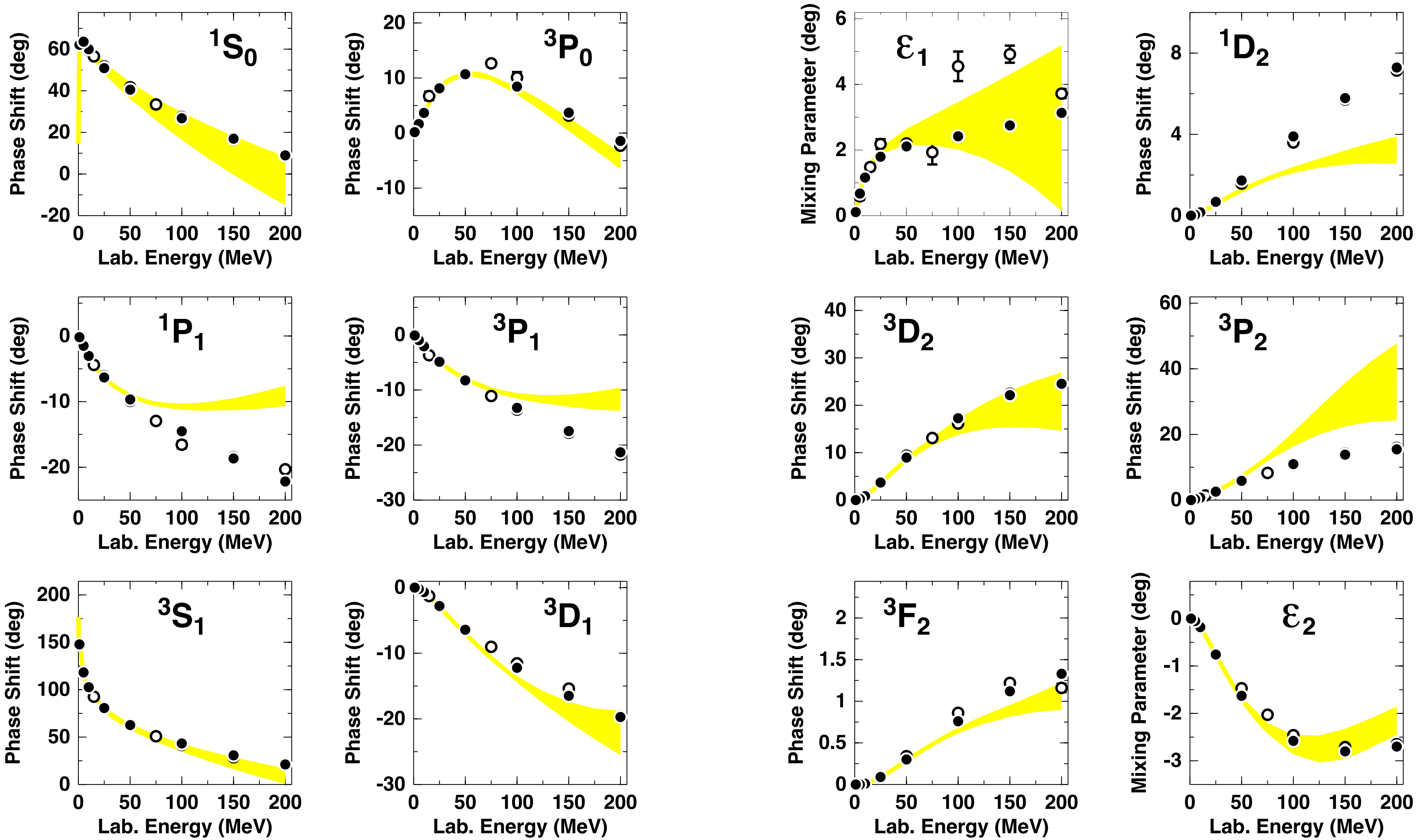
➢ **Non-perturbative renormalization**

Some Results from non-perturbative renormalization

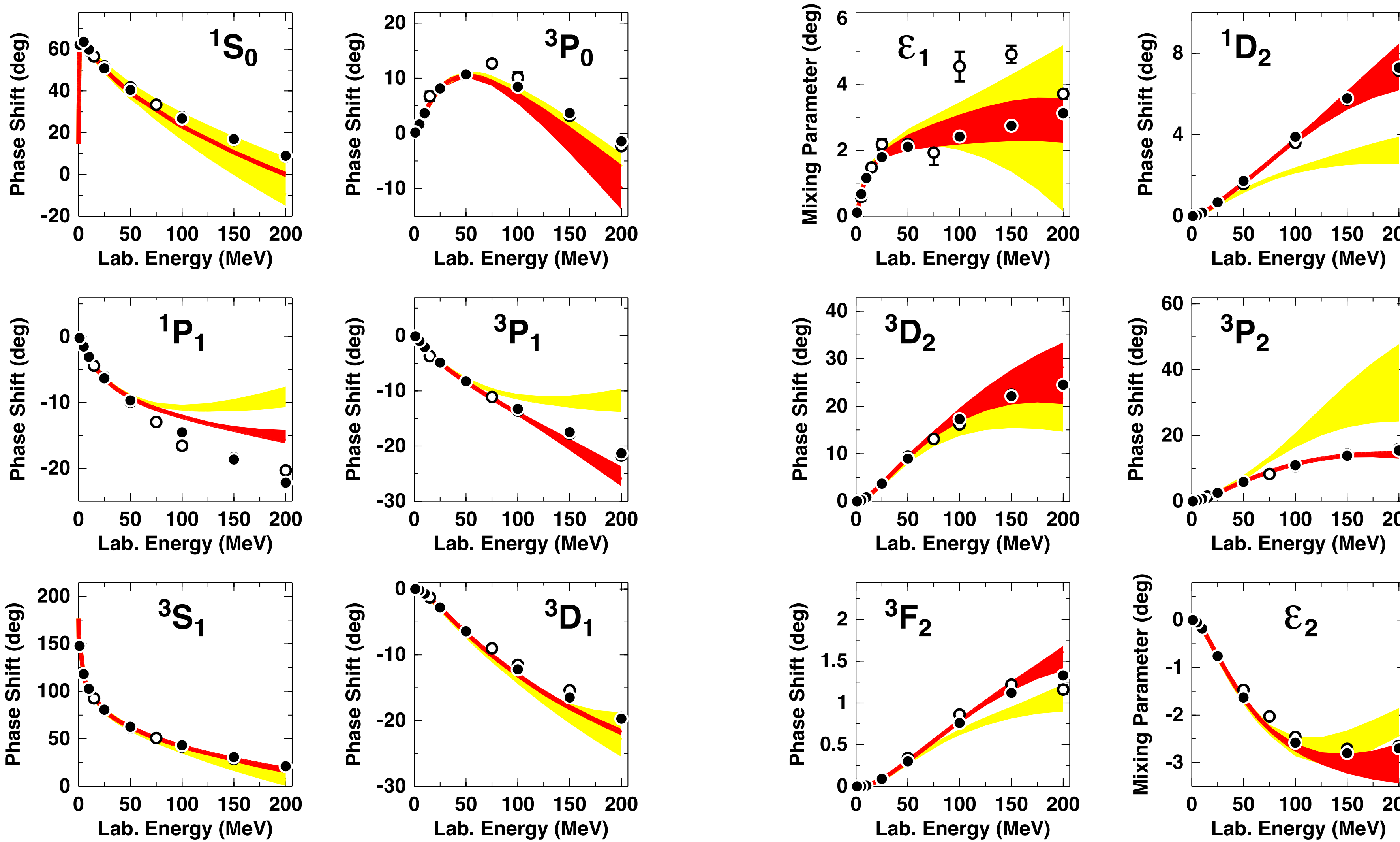
- Infinite cutoff: no reasonable power counting scheme, no order-by-order improvement (Idaho group).
- Infinite cutoff only at LO, higher orders perturbatively (Valderrama; Gegelia): How to implement in nuclear structure calculations? Also: huge tensor force.
- Finite cutoff (below the hard scale): cutoff independence for the range 450-800 MeV, substantial improvements from NLO to NNLO (Idaho group).

NLO

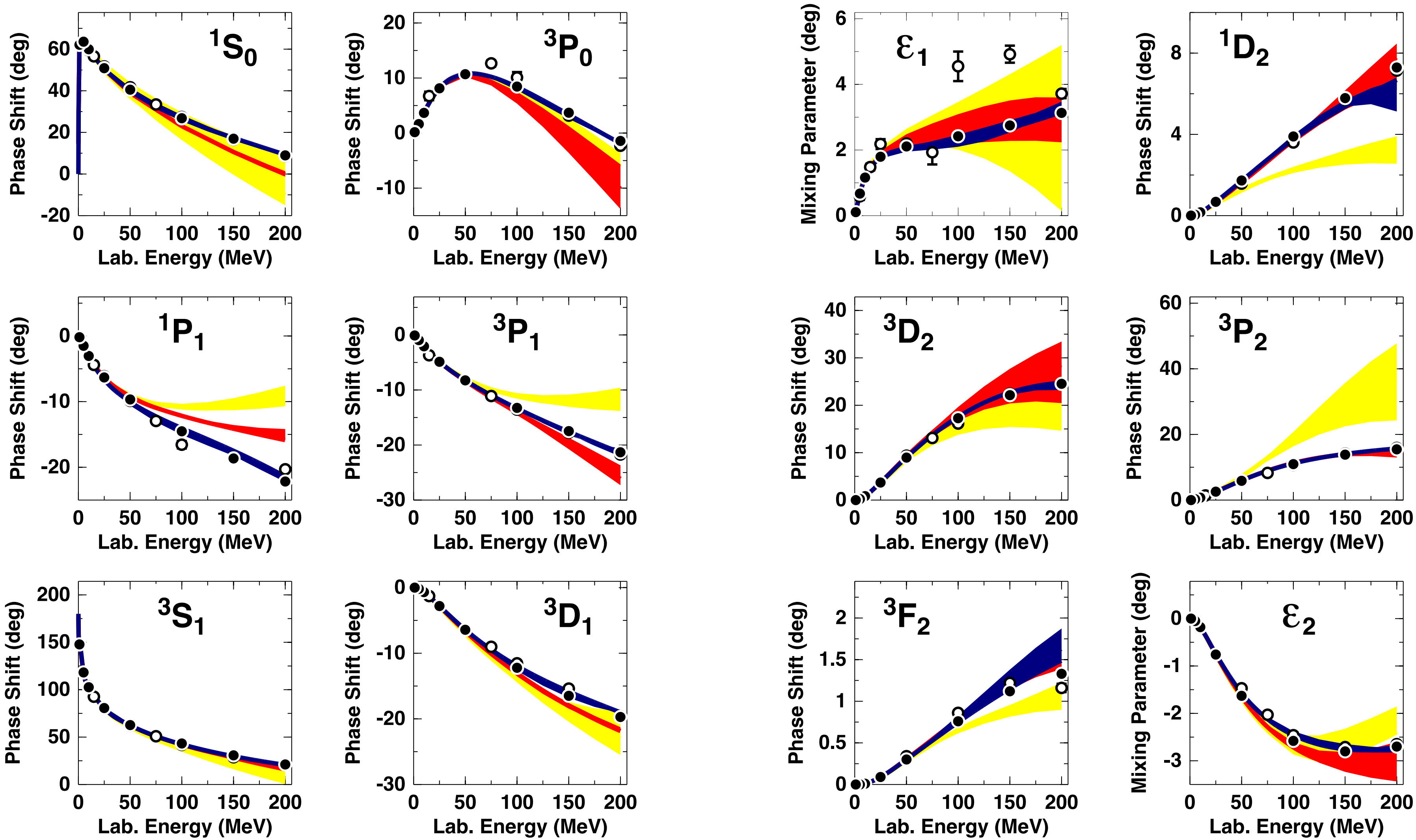
Cutoff = 450-800 MeV

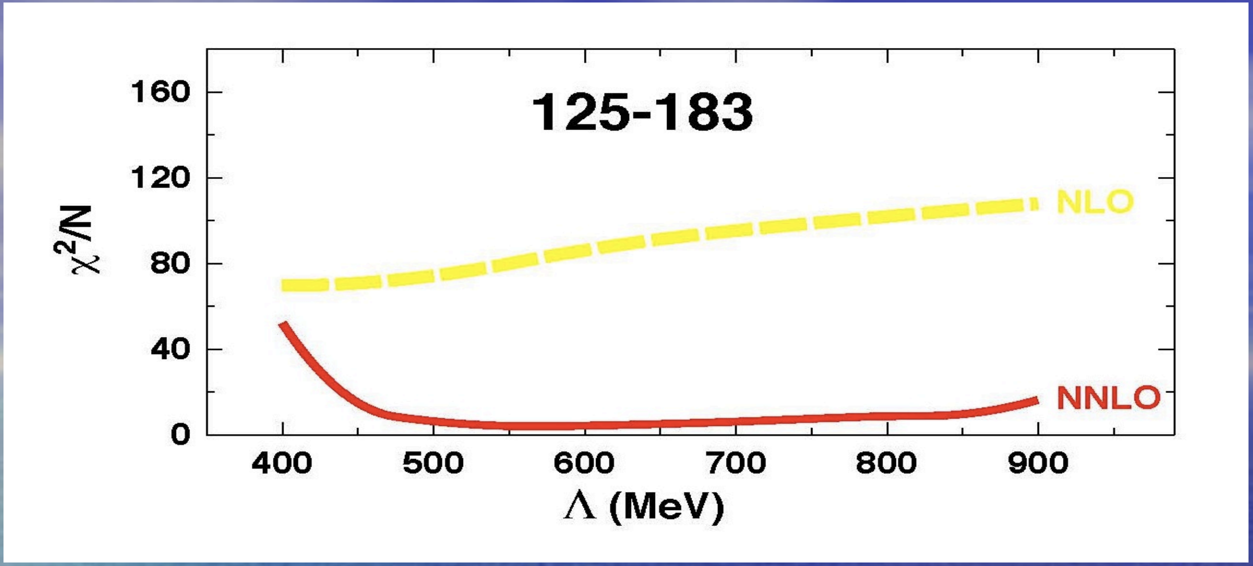


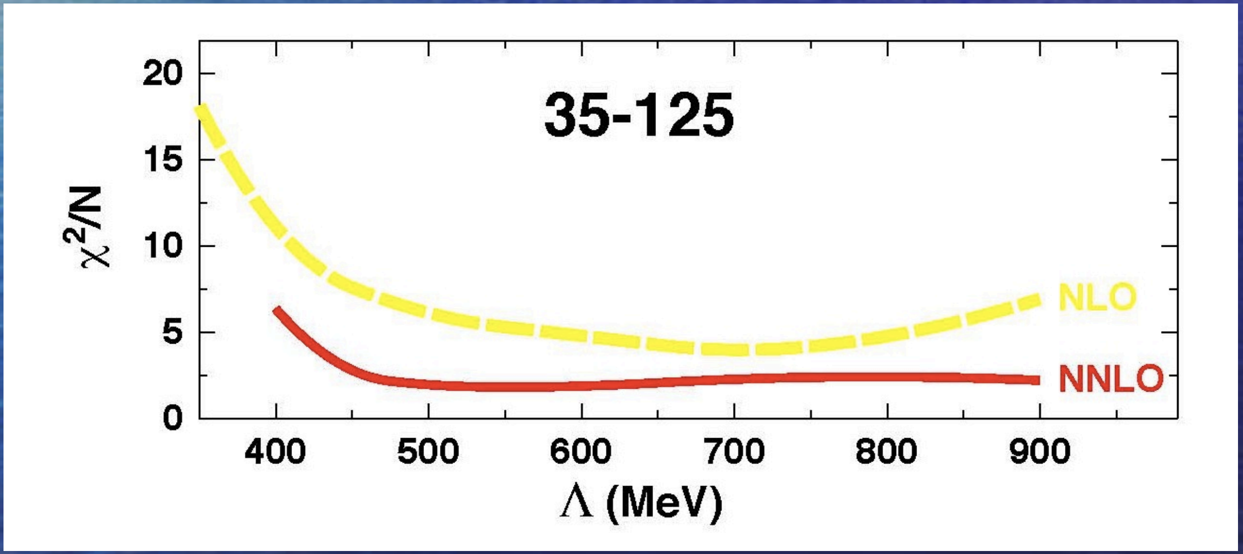
NLO NNLO Cutoff = 450-800 MeV



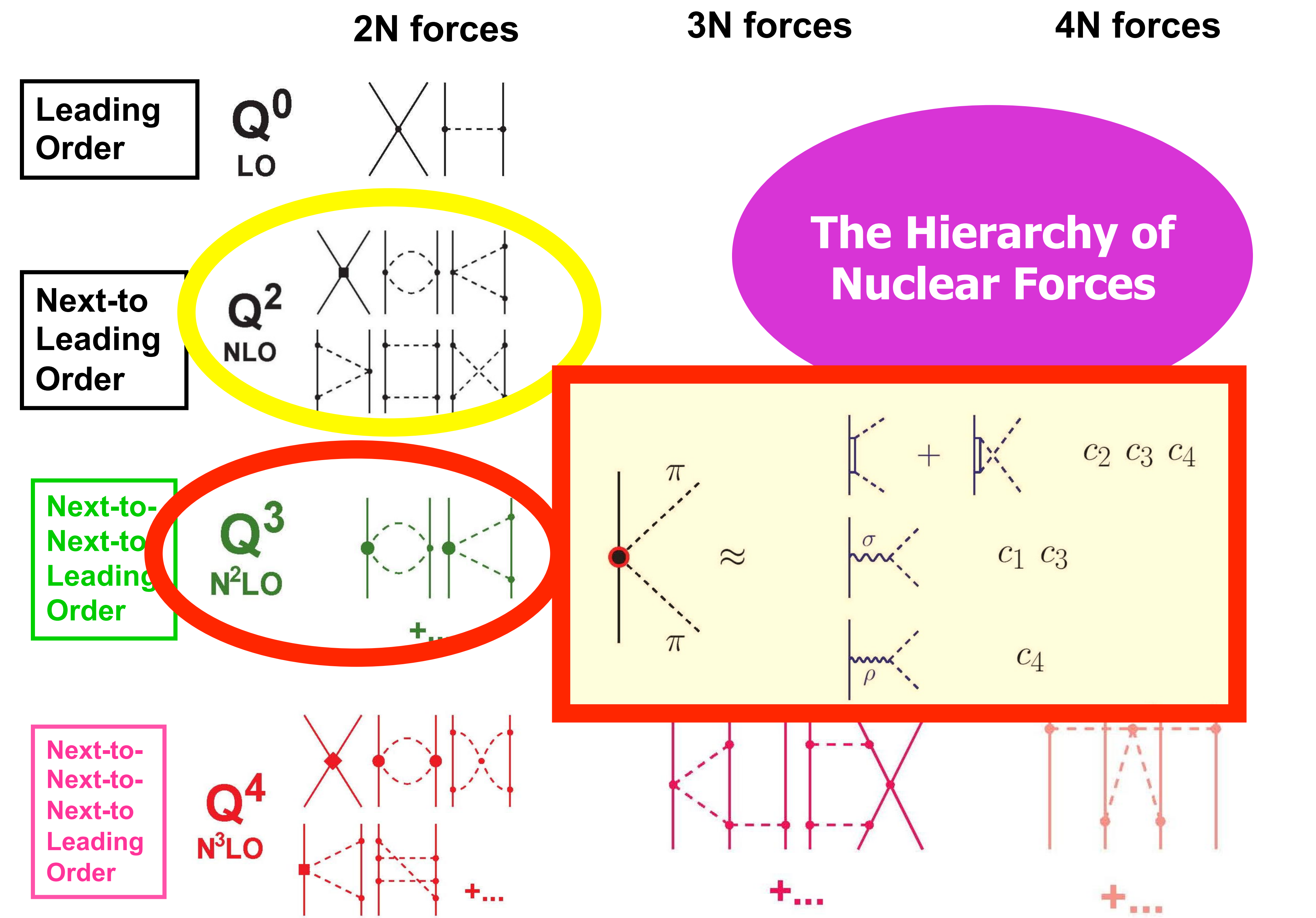
NLO NNLO N3LO Cutoff = 450-600 MeV

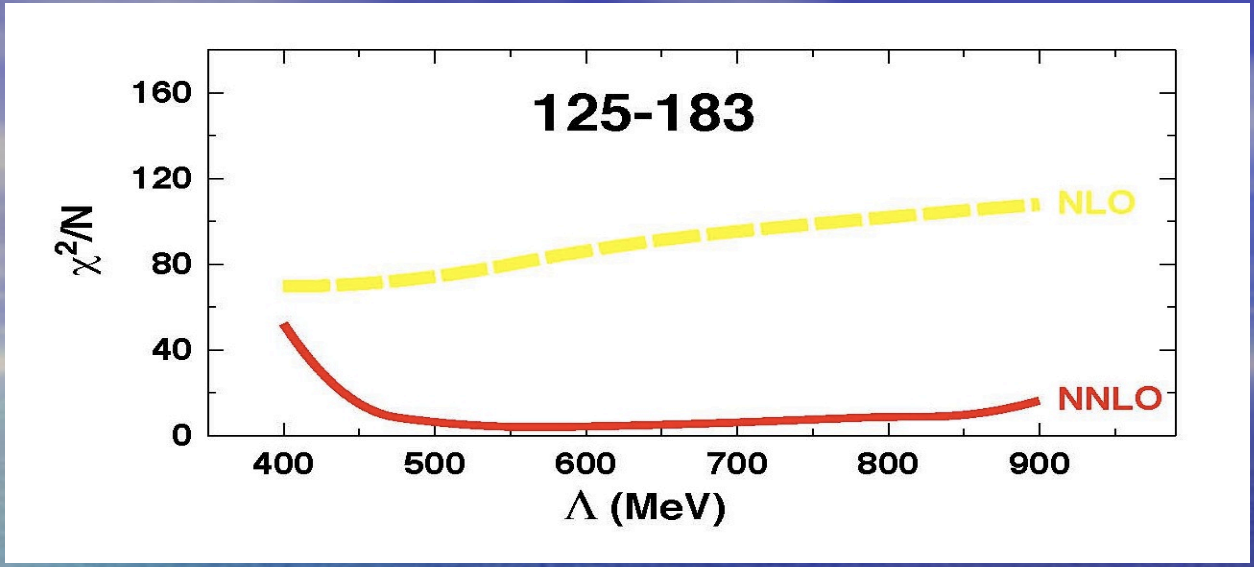


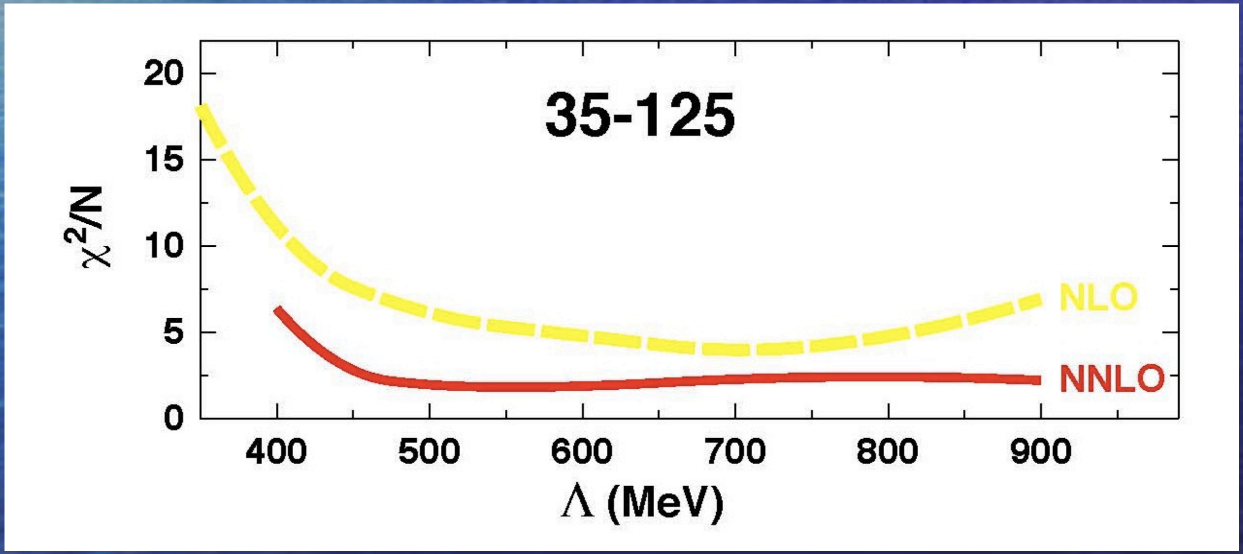




The plateaus improve with increasing order.


2N forces
3N forces
4N forces
Leading Order
Q^0
LO
Next-to Leading Order
Q^2
NLO
Next-to-Next-to Leading Order
Q^3
N^2LO
+...
The Hierarchy of Nuclear Forces
π
π
$\approx$
$+$
c_2 c_3 c_4
σ
c_1 c_3
ρ
c_4
Next-to-Next-to-Next-to Leading Order
Q^4
N^3LO
+...
+...
+...





The plateaus improve with increasing order.

Renormalization Summary

Non-perturbative reno using finite cutoffs ≤ $\Lambda_\chi \approx 1$ GeV.
For this, we have shown:

Cutoff independence for a certain finite range below 1 GeV (shown for NLO and NNLO).

Order-by-order improvement of the predictions.

This is what you want to see in an EFT!

So much about two-body forces;
there isn't much more to say, because …

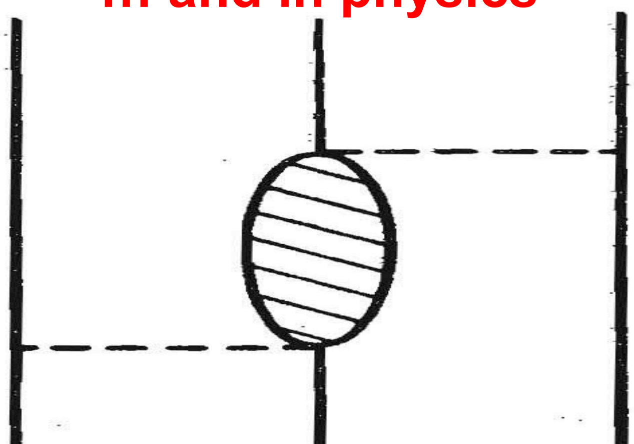
Two-body interactions are easy – in physics

and in human life:

But, now let's turn to three-body forces …

Three-body interactions are difficult –
in human life …

… and in physics



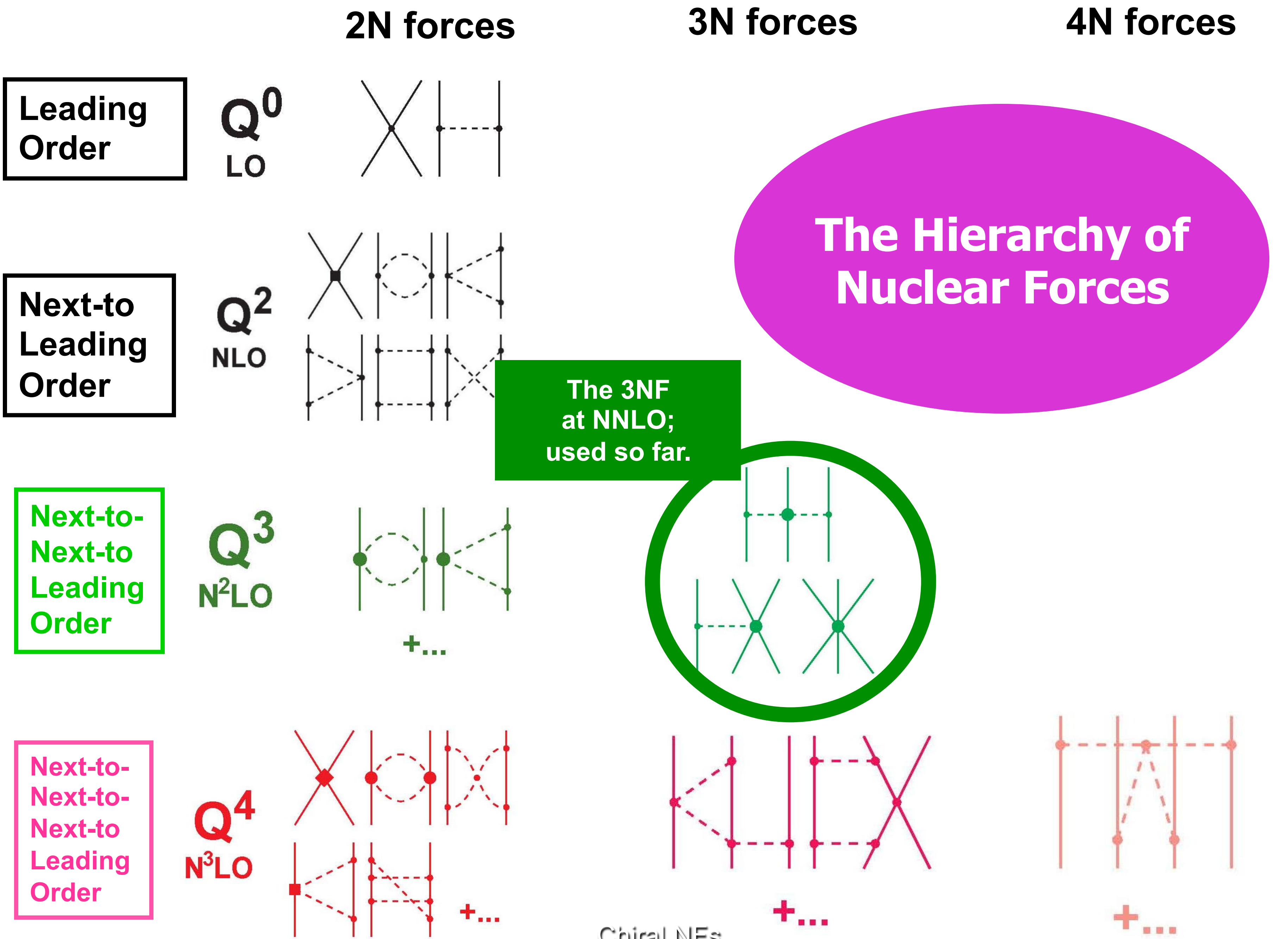
Fujita-Miyazawa, 1957

Status of
Three-body forces
50+ years ago

What progress did we have in the past 50 years on the topic of three-body forces?

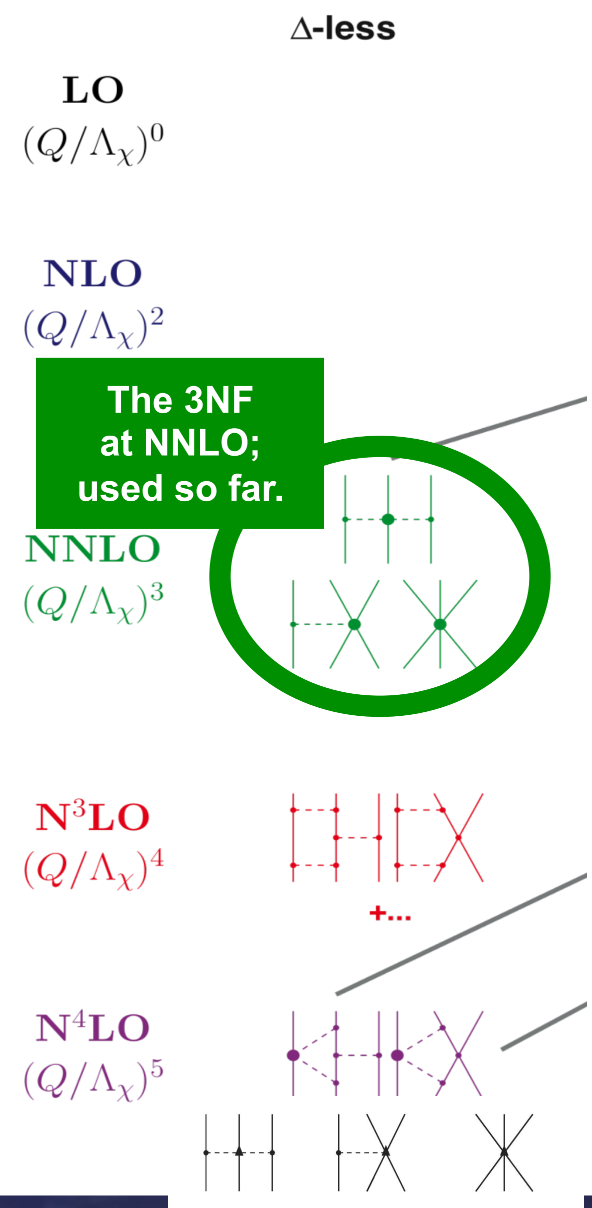
Three-body forces in physics

- **Phenomenological three-nucleon forces (3NFs):**
  - Fujita-Miyazawa (1957)
  - Tucson-Melbourne (1975-1999)
  - Urbana (1995)
  - Illinois (2001-2010)
  - CD-Bonn + Δ (Deltuva, Sauer, 2003)
- **Chiral three-nucleon forces (3NFs)**


2N forces
3N forces
4N forces
Leading Order
Q0
LO
The Hierarchy of Nuclear Forces
Next-to Leading Order
Q2
NLO
The 3NF at NNLO; used so far.
Next-to-Next-to Leading Order
Q3
N2LO
+...
Next-to-Next-to-Next-to Leading Order
Q4
N3LO
+...
+...
+...

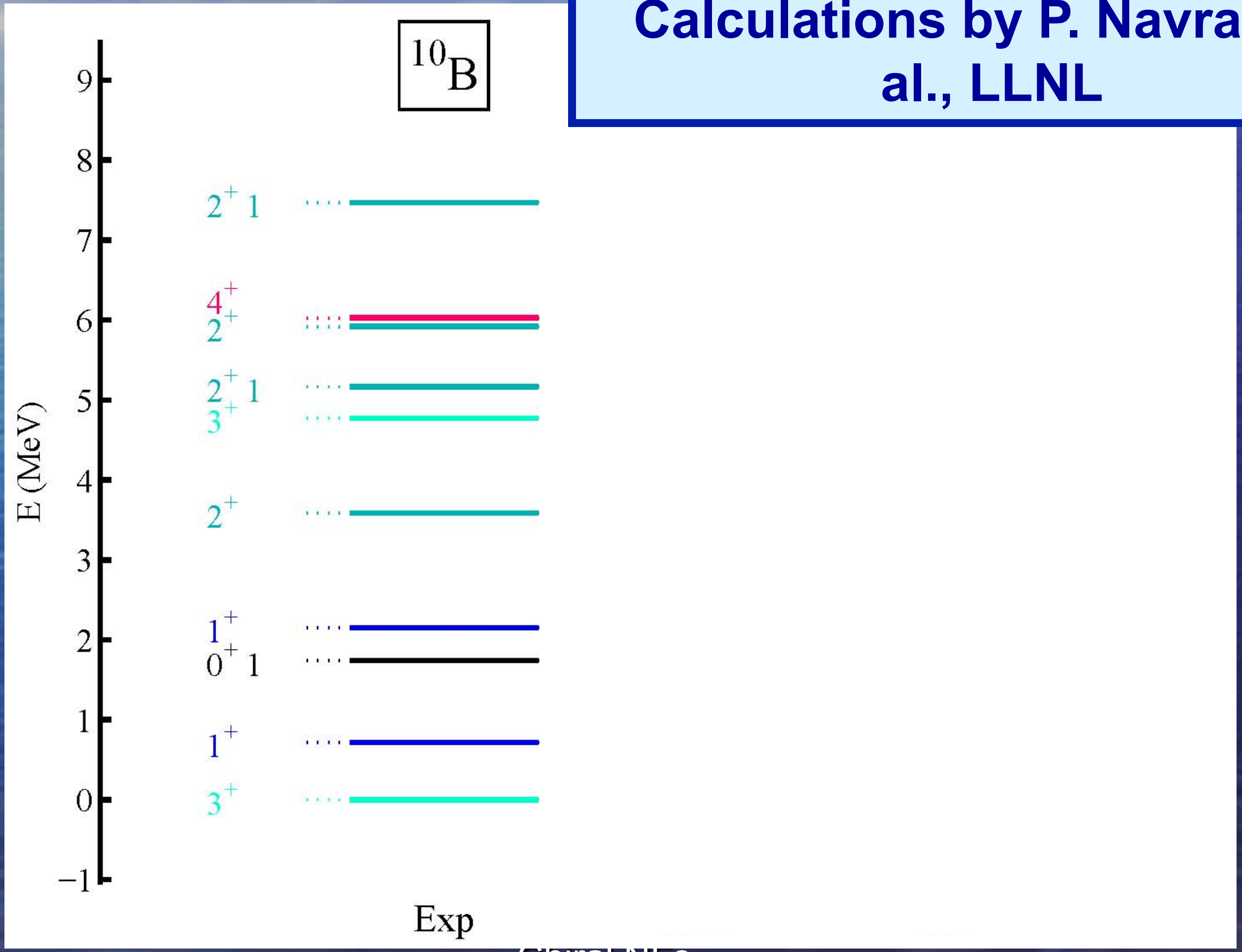
Chiral 3N Force

Now, showing only 3NF diagrams.



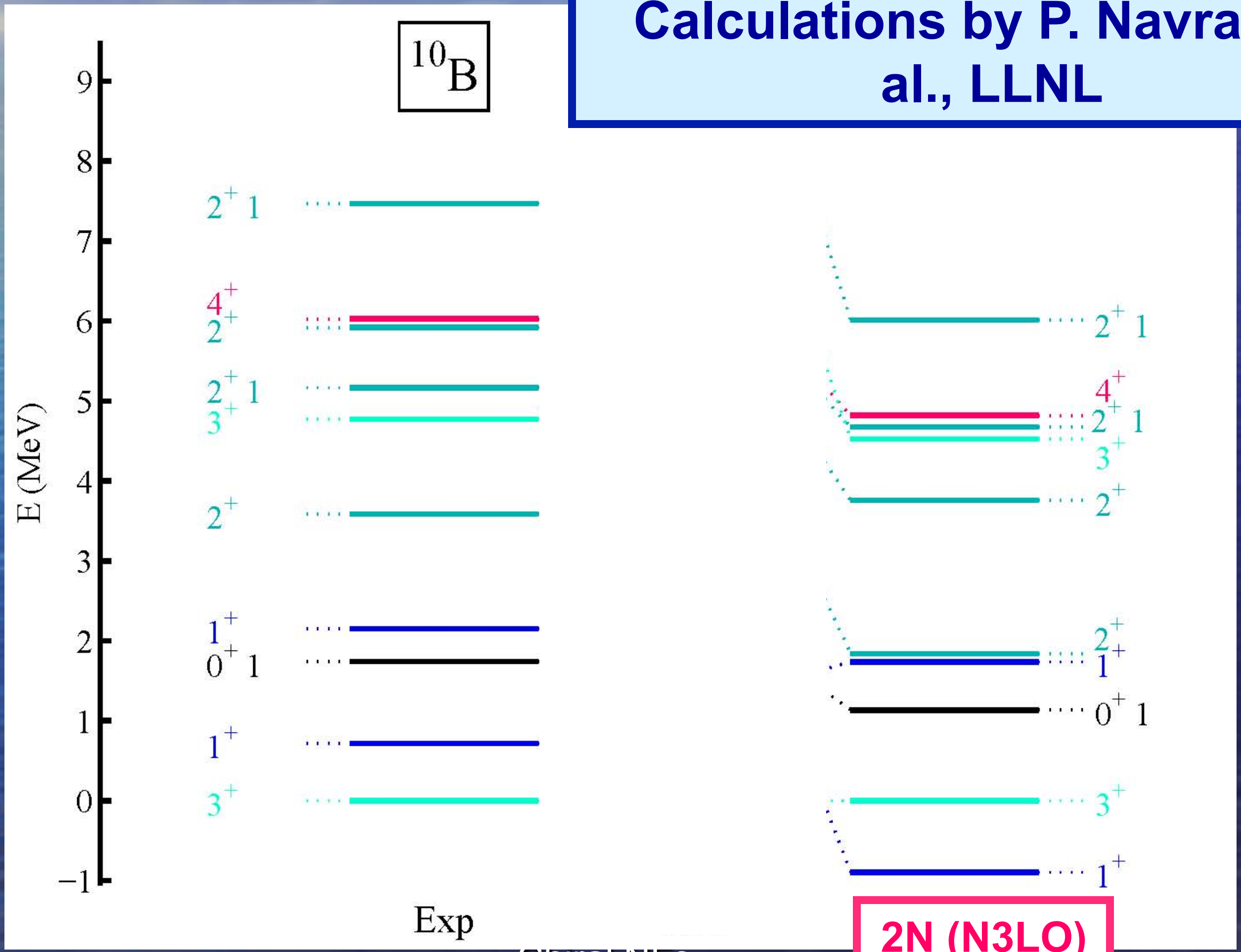
Calculating the properties of light nuclei using chiral 2N and 3N forces

"No-Core Shell Model " Calculations by P. Navratil et al., LLNL



Calculating the properties of light nuclei using chiral 2N and 3N forces

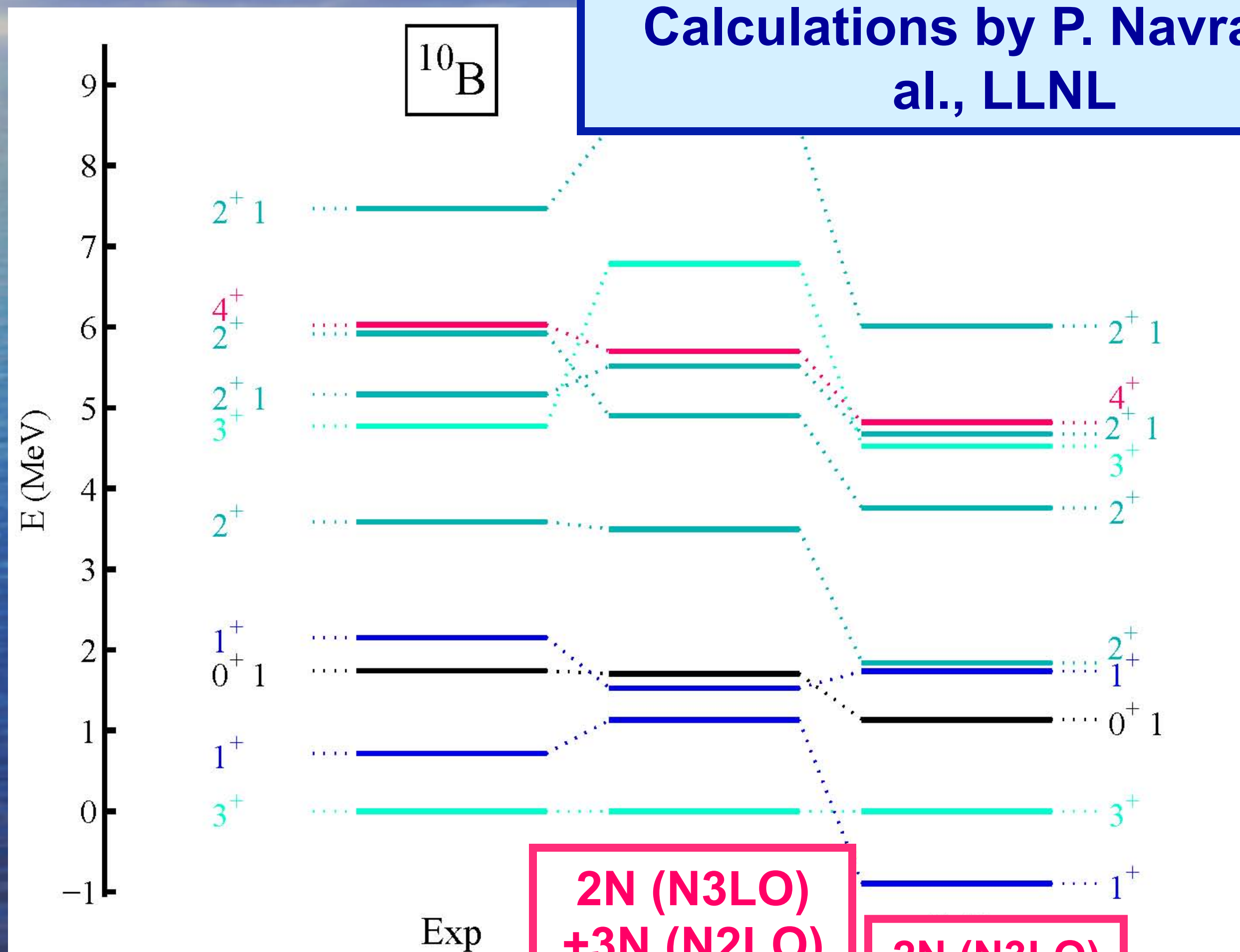
“No-Core Shell Model “ Calculations by P. Navratil et al., LLNL



2N (N3LO) force only

Calculating the properties of light nuclei using chiral 2N and 3N forces

“No-Core Shell Model “ Calculations by P. Navratil et al., LLNL



2N (N3LO) +3N (N2LO) forces

2N (N3LO) force only

PRL 108, 242501 (2012) PHYSICAL REVIEW LETTERS week ending 15 JUNE 2012

Continuum Effects and Three-Nucleon Forces in Neutron-Rich Oxygen Isotopes

G. Hagen,[1,2] M. Hjorth-Jensen,[3,4,5] G. R. Jansen,[3] R. Machleidt,[6] and T. Papenbrock[2,1]

PRL 109, 032502 (2012) PHYSICAL REVIEW LETTERS week ending 20 JULY 2012

Evolution of Shell Structure in Neutron-Rich Calcium Isotopes

G. Hagen,[1,2] M. Hjorth-Jensen,[3,4] G. R. Jansen,[3] R. Machleidt,[5] and T. Papenbrock[1,2]

Oxygen

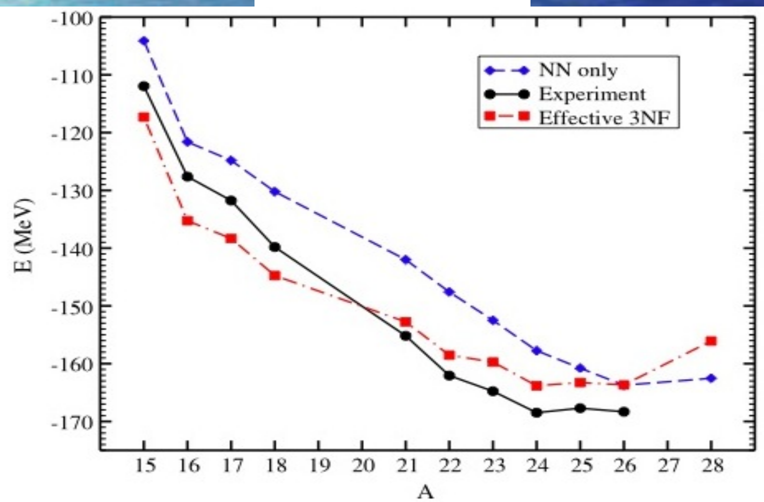


FIG. 1 (color online). Ground-state energy of the oxygen isotope ^{A}O as a function of the mass number A. Black circles: experimental data; blue diamonds: results from nucleon-nucleon interactions; red squares: results including the effects of three-nucleon forces.

Calcium

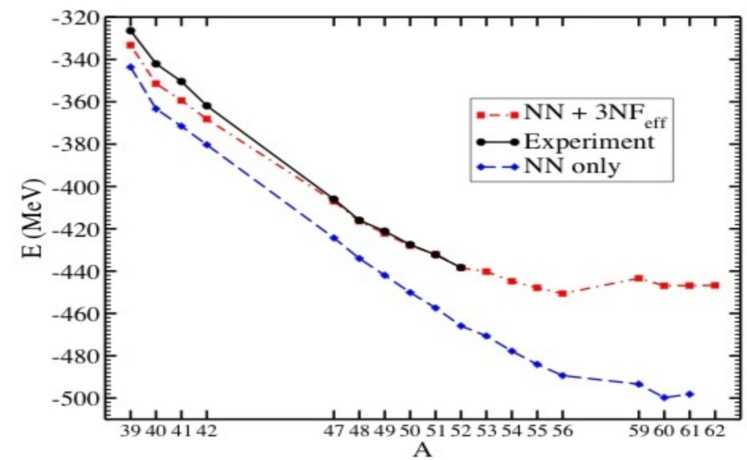
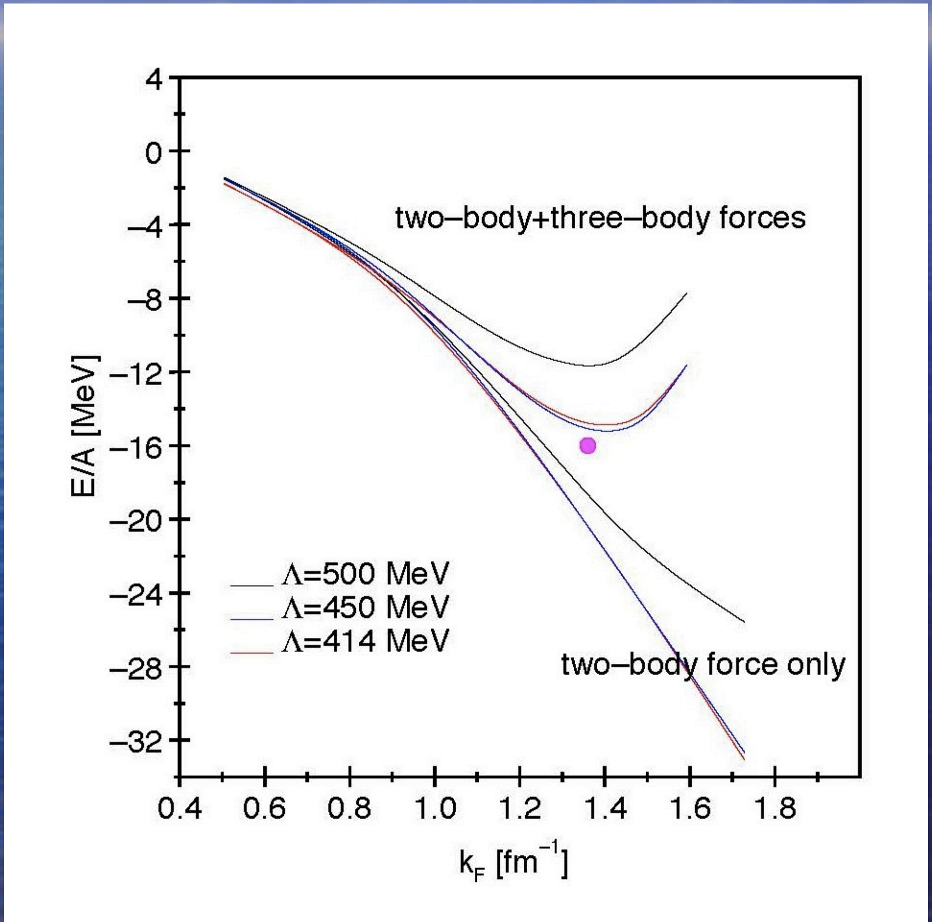


FIG. 1: (Color online) Ground-state energy of the calcium isotopes as a function of the mass number A. Black circles: experimental data; red squares: theoretical results including the effects of three-nucleon forces; blue diamonds: predictions from chiral NN forces alone. The experimental results for ^{51,52}Ca are from Ref. [34].

The nuclear matter equation of state with consistent two- and three-body perturbative chiral interactions

L. Coraggio,[1] J. W. Holt,[2] N. Itaco,[1,3] R. Machleidt,[4] L. E. Marcucci,[5,6] and F. Sammarruca[4]



Analyzing Power Ay

Calculations by the Pisa Group

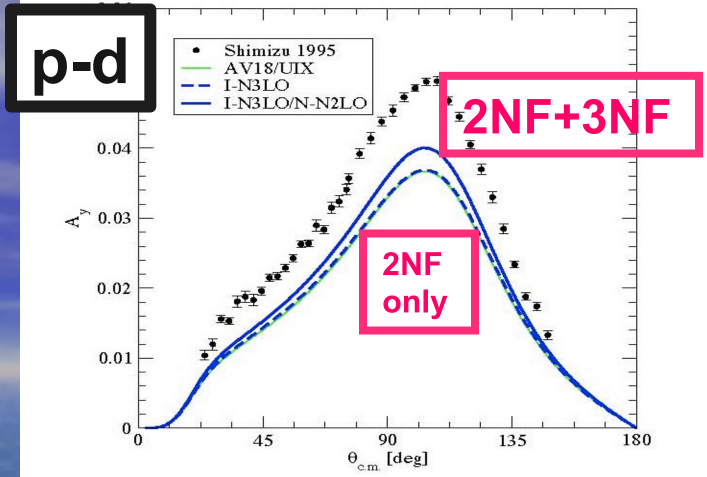


Fig. 9. $p-d$ observable A_y at $E_p = 3$ MeV calculated with the I-N3LO (blue dashed line), I-N3LO/N-N2LO (blue solid line) AV18/UIX (thin green solid line) interaction mod MeV. The experimental data are from Ref

The Ay puzzle is NOT solved by the 3NF at NNLO.

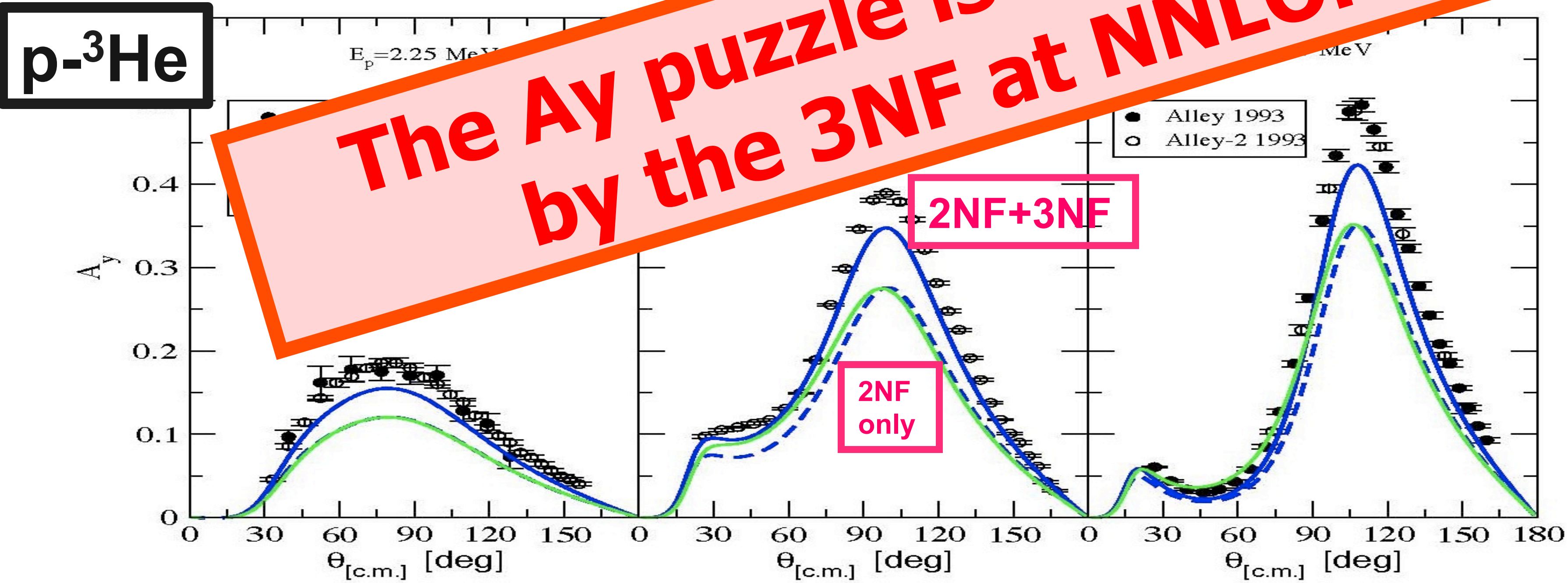


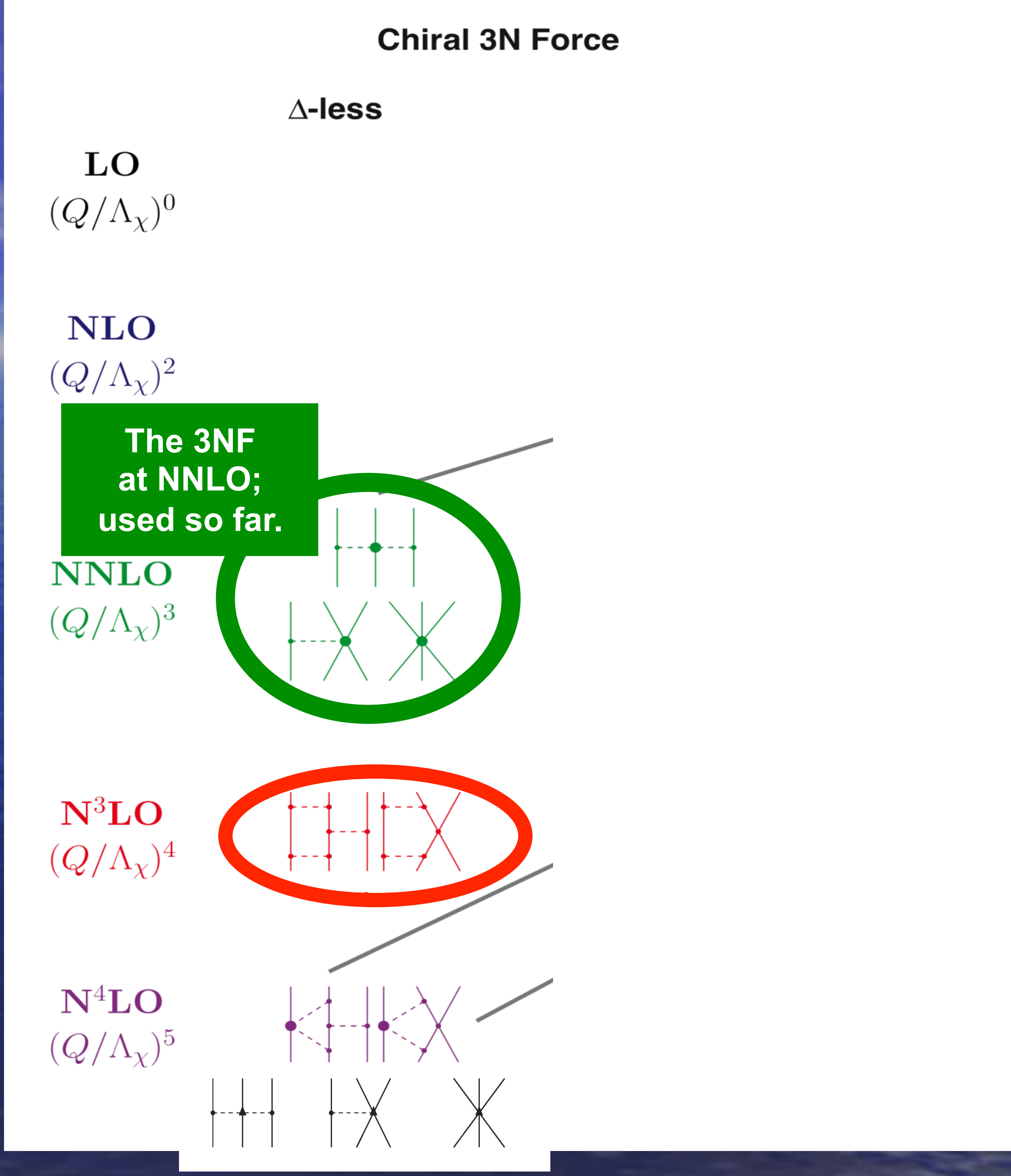
Fig. 6. $p - {}^3$He A_y observable calculated with the I-N3LO (blue dashed line), the I-N3LO/N-N2LO (blue solid line), and the AV18/UIX (thin green solid line) interaction models for three different incident proton energies. The experimental data are from Refs. [37,22,36].

And so,
we need 3NFs beyond NNLO, because …

- The 2NF is N3LO;
  consistency requires that all contributions are included up to the same order.

- There are unresolved problems in 3N and 4N scattering, and nuclear structure.

Back to the drawing board.



The explosion is starting.

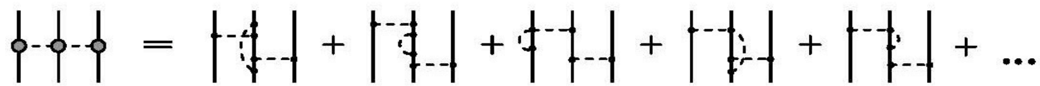
Chiral 3N Force

Δ-less

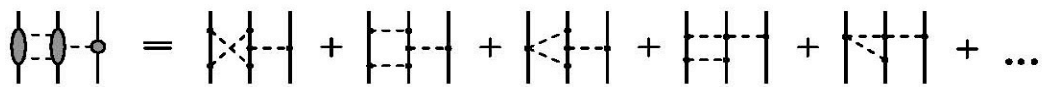
LO
$(Q/\Lambda_\chi)^0$

NLO
$(Q/\Lambda_\chi)^2$

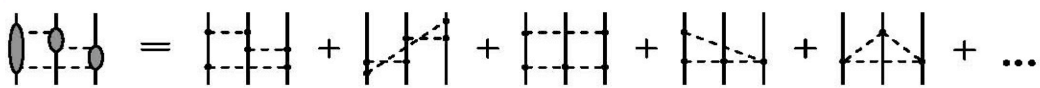
2π–exchange



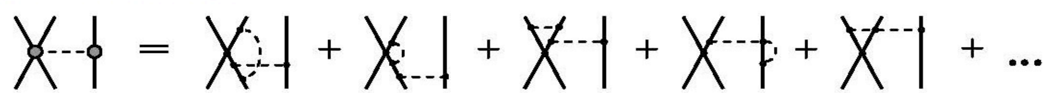
2π-1π–exchange



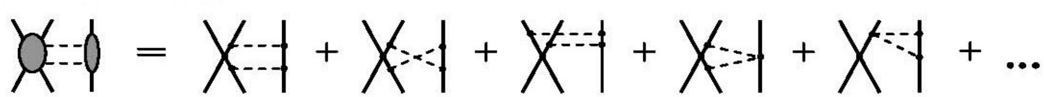
ring diagrams



contact-1π–exchange



contact-2π–exchange



R. Machleidt

$$
\begin{aligned}
S_5 &= -\frac{A(q_1)\, g_A^4 q_1 \left((1+z^2)\, q_1 + 2 z q_3\right)}{128 F^6 \pi \left(-1+z^2\right)^2 q_3^2} - \frac{A(q_3)\, g_A^4 \left(2 z q_1 + (1+z^2)\, q_3\right)}{128 F^6 \pi \left(-1+z^2\right)^2 q_3} + \\
&\quad \frac{I(4:0,-q_1,q_3;0)\, g_A^4 q_1 \left(-4z\left(-1+z^2\right) M_\pi^2 + 2 z q_1^2 + \left(1+3z^2\right) q_1 q_3 + 2 z q_3^2\right)}{32 F^6 \left(-1+z^2\right)^2 q_3} + \\
&\quad \frac{A(q_2)\, g_A^4 \left(\left(1+z^2\right) q_1^2 + z\left(3+z^2\right) q_1 q_3 + \left(1+z^2\right) q_3^2\right)}{128 F^6 \pi \left(-1+z^2\right)^2 q_3^2}, \\
S_6 &= -\frac{A(q_3)\, g_A^4 q_3 \left(z q_1 + q_3\right)}{128 F^6 \pi \left(-1+z^2\right)} + \frac{A(q_2)\, g_A^4 \left(q_1^2 - z\left(-3+z^2\right) q_1 q_3 + q_3^2\right)}{128 F^6 \pi \left(-1+z^2\right)} - \frac{A(q_1)\, g_A^4 q_1 \left(q_1 + z q_3\right)}{128 F^6 \pi \left(-1+z^2\right)} + \\
&\quad \frac{I(4:0,-q_1,q_3;0)\, g_A^4 q_1 q_3 \left(z q_1 + q_3\right)\left(q_1 + z q_3\right)}{32 F^6 \left(-1+z^2\right)}, \\
S_7 &= \frac{A(q_1)\, g_A^4 \left(2M_\pi^2 + q_3^2\right)}{256 F^6 \pi \left(-1+z^2\right) q_3^2} - \frac{A(q_2)\, g_A^4 \left(z q_3^2 \left(z q_1 + q_3\right) + 2 M_\pi^2 \left(q_1 + z q_3\right)\right)}{256 F^6 \pi \left(-1+z^2\right) q_1 q_3^2} + \frac{z A(q_3)\, g_A^4 \left(2M_\pi^2 + q_3^2\right)}{256 F^6 \pi \left(-1+z^2\right) q_1 q_3} - \\
&\quad \frac{I(4:0,-q_1,q_3;0)\, g_A^4 \left(z q_1 + q_3\right)\left(2M_\pi^2 + q_3^2\right)}{64 F^6 \left(-1+z^2\right) q_3}. \qquad \text{(A.7)}
\end{aligned}
$$

Examining the above results one observes that the individual terms in the expressions for R_i and S_i are singular for $z = \pm 1$, $q_1 = 0$ and/or $q_3 = 0$. These singularities, however, cancel in such a way that the resulting terms in Eqs. (A.1) and (A.6) are finite. In principle, it is possible to obtain a representation for functions R_i and S_i which is free of at least some of the singularities. In particular, the singularities at $z = \pm 1$ can be avoided if one expresses the results in terms of the functions J_1 and J_2 defined as

$$
\begin{aligned}
J_1(d, \vec q_1, \vec q_3) &= \frac{1}{1-z^2}\Big\{ J(d:\vec 0, -\vec q_1, \vec q_3) - \frac{1}{2}(1+z)\Big[\frac{J(d:\vec 0,\vec q_1)}{q_3^2+q_1 q_3} + \frac{J(d:\vec 0,\vec q_3)}{q_1^2+q_1 q_3} - \frac{J(d:\vec 0,\vec q_1+\vec q_3)}{q_1 q_3}\Big] \\
&\quad - \frac{1}{2}(1-z)\Big[\frac{J(d:\vec 0,\vec q_1)}{q_3^2-q_1 q_3} + \frac{J(d:\vec 0,\vec q_3)}{q_1^2-q_1 q_3} + \frac{J(d:\vec 0,\vec q_1+\vec q_3)}{q_1 q_3}\Big]\Big\}, \qquad \text{(A.8)} \\
J_2(d, \vec q_1, \vec q_3) &= \frac{1}{(1-z^2)^2}\Big\{ J(d:\vec 0, -\vec q_1, \vec q_3) - \frac{1}{4}(1-z)^2\Big[\frac{J(d:\vec 0,\vec q_1)}{q_3^2-q_1 q_3} + \frac{J(d:\vec 0,\vec q_3)}{q_1^2-q_1 q_3} + \frac{J(d:\vec 0,\vec q_1+\vec q_3)}{q_1 q_3} \qquad \text{(A.9)} \\
&\quad + (1+z)\Big[-\frac{8M^2-2q_1^2+(d-1)(q_1-q_3)(2q_1-q_3)}{(d-1)q_3(q_1-q_3)^3} J(d:\vec 0,\vec q_1) \\
&\quad + \frac{8M^2-2q_3^2+(d-1)(q_1-q_3)(q_1-2q_3)}{(d-1)q_1(q_1-q_3)^3} J(d:\vec 0,\vec q_3) + \frac{2(4M^2+(d-2)(q_1-q_3)^2)}{(d-1)q_1 q_3 (q_1-q_3)^2} J(d:\vec 0,\vec q_1+\vec q_3)\Big]\Big] \\
&\quad - \frac{1}{4}(1+z)^2\Big[\frac{J(d:\vec 0,\vec q_1)}{q_3^2+q_1 q_3} + \frac{J(d:\vec 0,\vec q_3)}{q_1^2+q_1 q_3} - \frac{J(d:\vec 0,\vec q_1+\vec q_3)}{q_1 q_3} \\
&\quad + (1-z)\Big[\frac{8M^2-2q_1^2+(d-1)(q_1+q_3)(2q_1+q_3)}{(d-1)q_3(q_1+q_3)^3} J(d:\vec 0,\vec q_1) \\
&\quad + \frac{8M^2-2q_3^2+(d-1)(q_1+q_3)(q_1+2q_3)}{(d-1)q_1(q_1+q_3)^3} J(d:\vec 0,\vec q_3) - \frac{2(4M^2+(d-2)(q_1+q_3)^2)}{(d-1)q_1 q_3 (q_1+q_3)^2} J(d:\vec 0,\vec q_1+\vec q_3)\Big]\Big]\Big\},
\end{aligned}
$$

rather than the three-point function $J(d:\vec 0, -\vec q_1, \vec q_3)$ and uses certain linear combinations of two-point functions and tadpole integrals. In the above expressions, the two-point function is defined as

$$
J(d:\vec p_1, \vec p_2) = \int \frac{d^d l}{(2\pi)^d} \frac{1}{(\vec l + \vec p_1)^2 + M_\pi^2} \frac{1}{(\vec l + \vec p_2)^2 + M_\pi^2}. \qquad \text{(A.10)}
$$

[1] N. Kalantar-Nayestanaki and E. Epelbaum, Nucl. Phys. News **17**, 22 (2007) [arXiv:nucl-th/0703089].

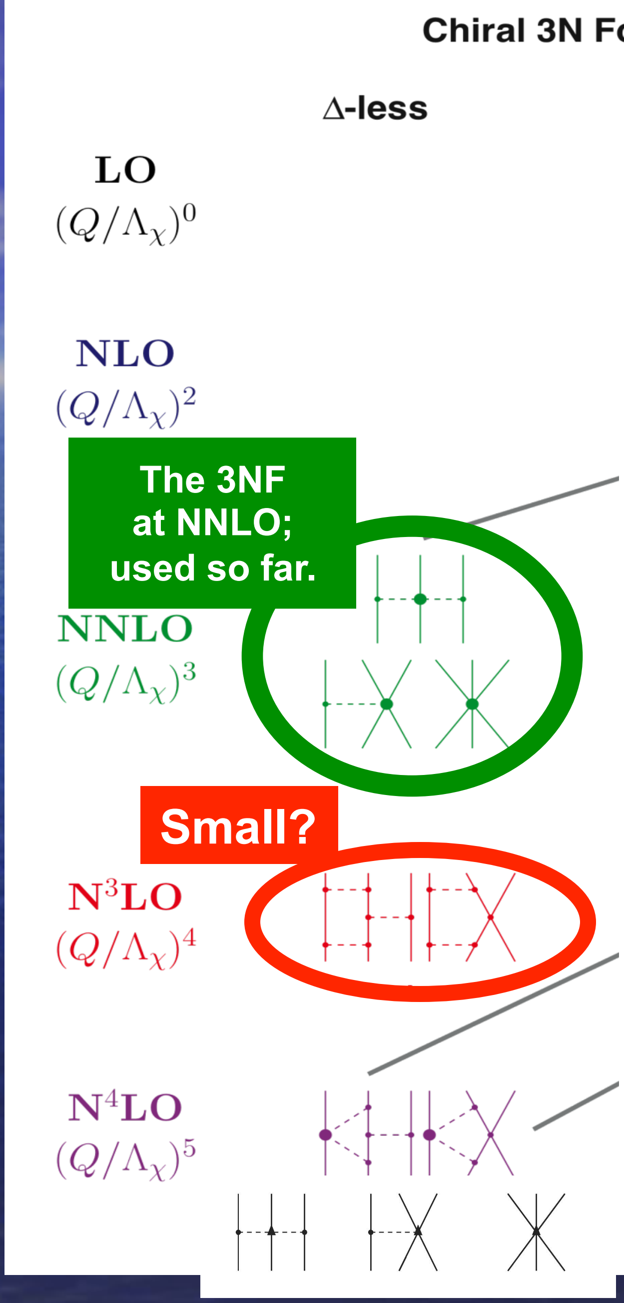
[2] D. R. Entem and R. Machleidt, Phys. Rev. C **68**, 041001 (2003) [arXiv:nucl-th/0304018].

Apps of N3LO 3NF:

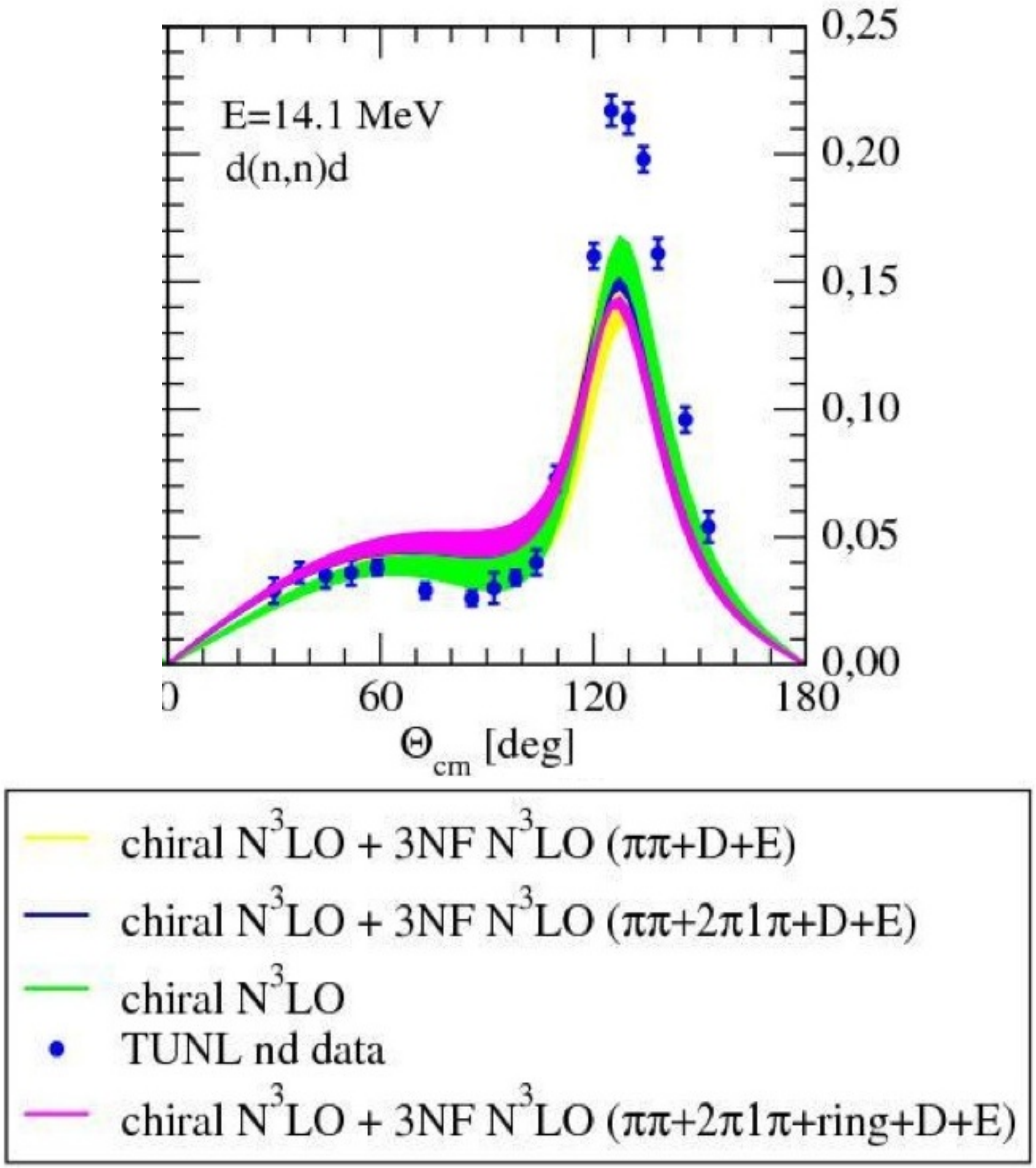
Triton: Skibinski et al., PRC 84, 054005 (2011). **Not conclusive.**

Neutron matter: Hebeler, Schwenk and co-workers, PRL 110, 032504 (2013). **Not small!(?)**

N-d scattering (Ay): Witala et al. **Small!**



N-d A_y calculations by Witala et al.

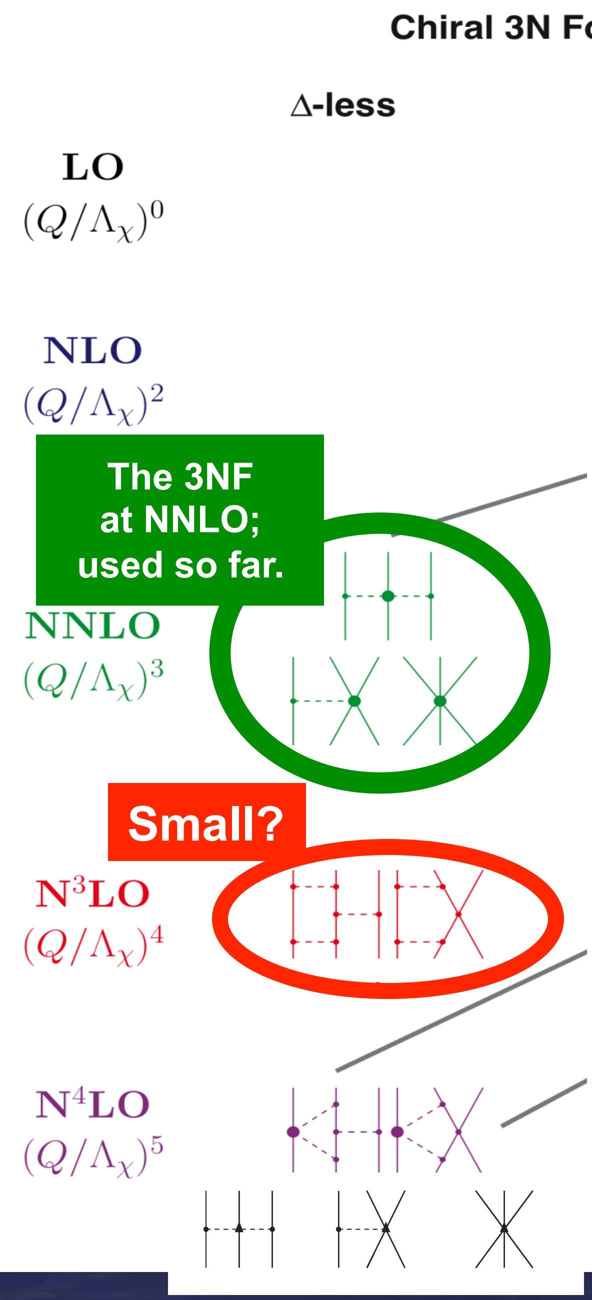


Apps of N3LO 3NF:

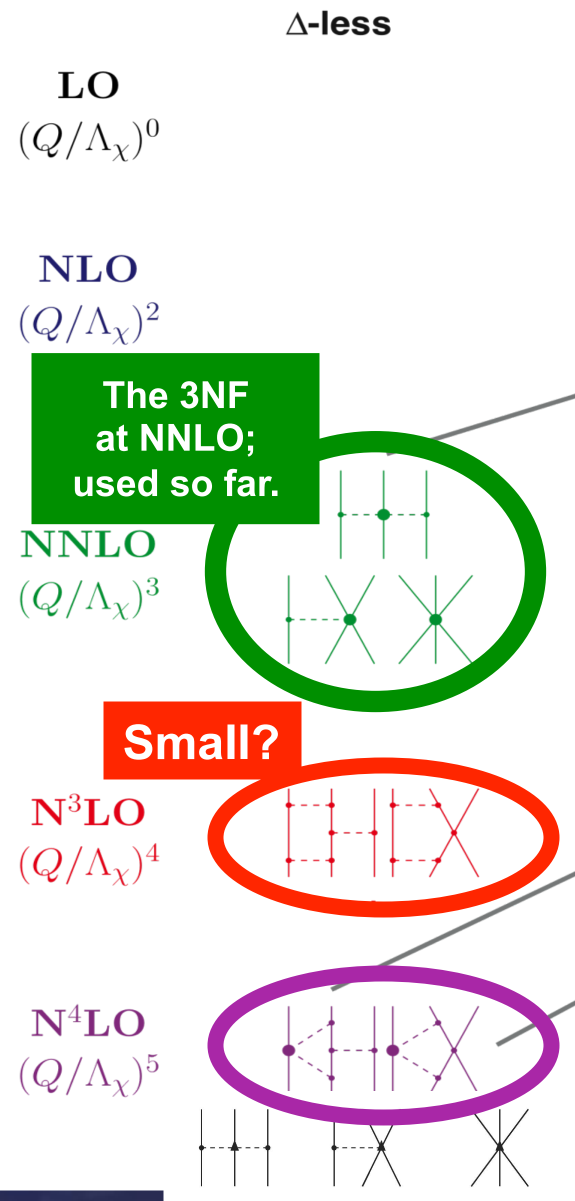
Triton: Skibinski et al., PRC 84, 054005 (2011).
Not conclusive.

Neutron matter:
Hebeler, Schwenk and co-workers, PRL 110, 032504 (2013).
Not small!(?)

N-d scattering (Ay):
Witala et al.
Small!



Chiral 3N Force



Chiral 3N Force

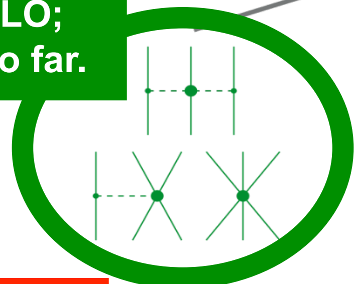
Δ-less

LO
$(Q/\Lambda_\chi)^0$

NLO
$(Q/\Lambda_\chi)^2$

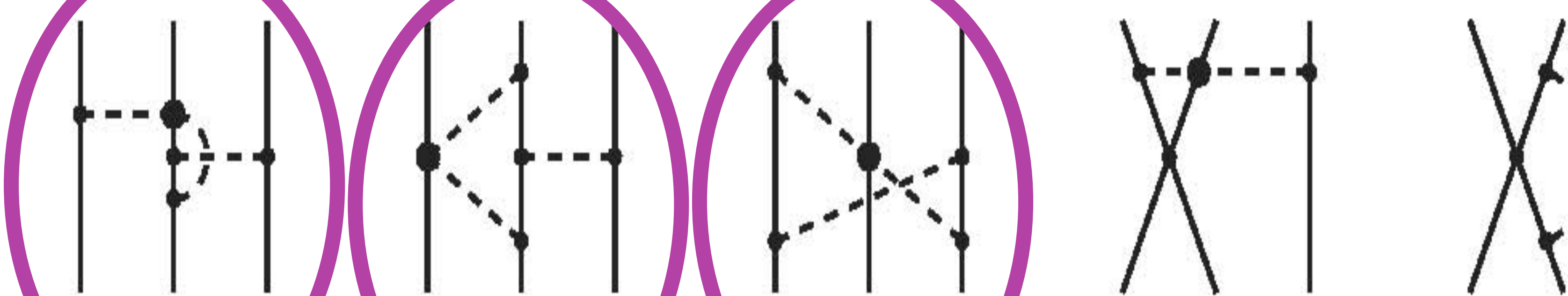
The 3NF
at NNLO;
used so far.

NNLO
$(Q/\Lambda_\chi)^3$

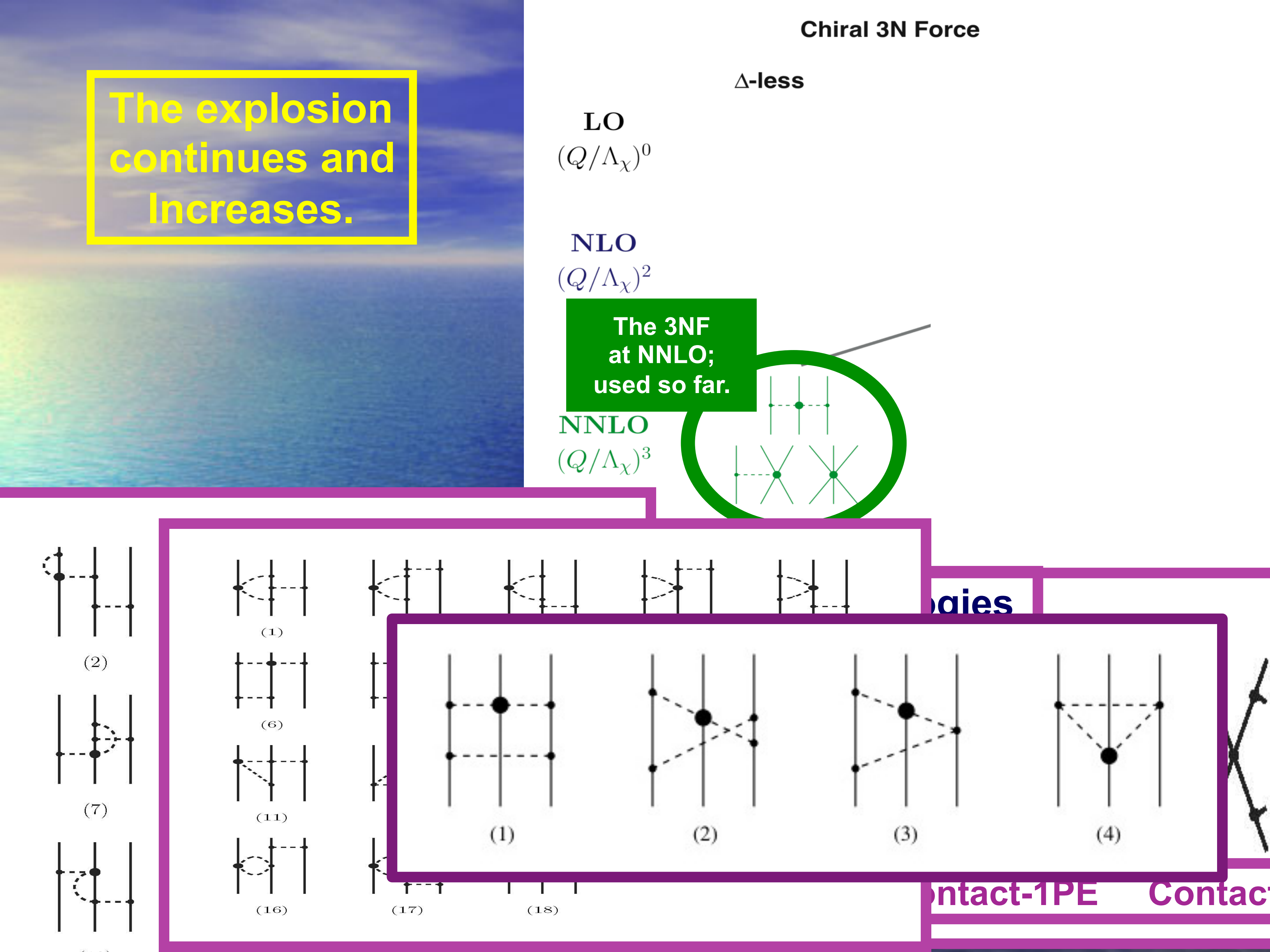


Small?

1-loop graphs: 5 topologies



2PE | 2PE-1PE | Ring | Contact-1PE | Contact



The explosion continues and Increases.
Chiral 3N Force
Δ-less
LO
(Q/Λχ)^0
NLO
(Q/Λχ)^2
The 3NF at NNLO; used so far.
NNLO
(Q/Λχ)^3
(2)
(7)
(1)
(6)
(11)
(16)
(17)
(18)
(1)
(2)
(3)
(4)

Chiral 3N Force

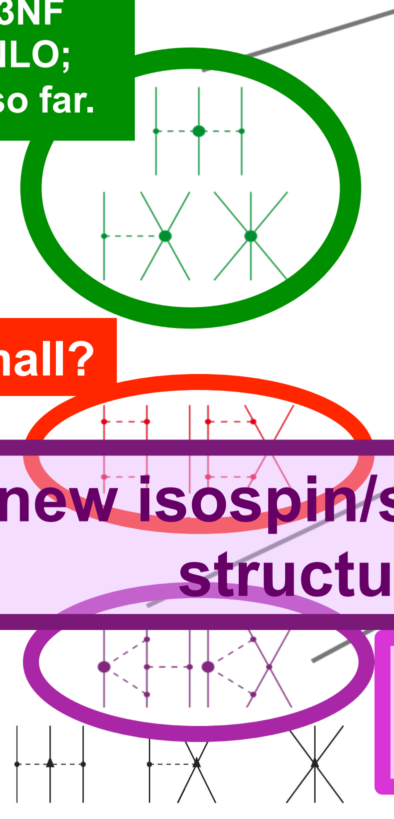
Δ-less

LO
$(Q/\Lambda_\chi)^0$

NLO
$(Q/\Lambda_\chi)^2$

The 3NF at NNLO; used so far.

NNLO
$(Q/\Lambda_\chi)^3$



Small?

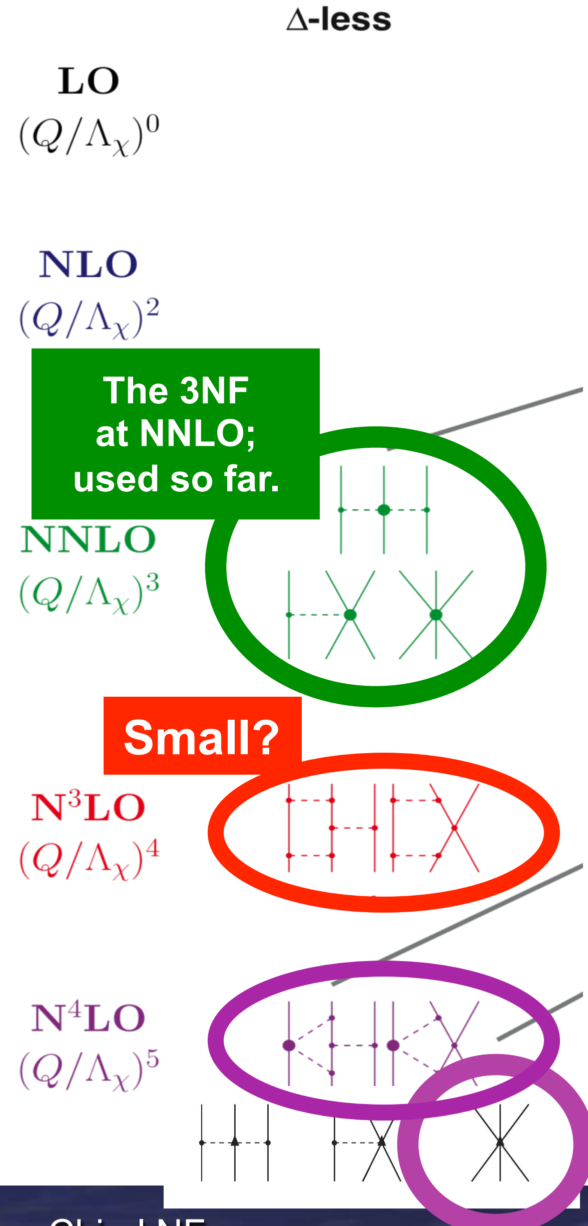
N^3LO
$(Q/\Lambda_\chi)^4$

Many new isospin/spin/momentum structures.

N^4LO
$(Q/\Lambda_\chi)^5$

Large!

Chiral 3N Force



Chiral 3N Force

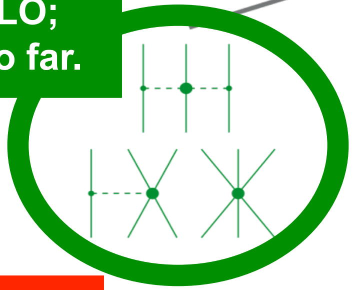
Δ-less

LO
$(Q/\Lambda_\chi)^0$

NLO
$(Q/\Lambda_\chi)^2$

NNLO
$(Q/\Lambda_\chi)^3$

The 3NF at NNLO; used so far.



Small?

3NF contacts at N4LO

Girlanda, Kievsky, Viviani, PRC 84, 014001 (2011)

$\mathbf{k}_i = \mathbf{p}_i - \mathbf{p}'_i$ and $\mathbf{Q}_i = \mathbf{p}_i + \mathbf{p}'_i$, $\mathbf{p}_i$ and $\mathbf{p}'_i$ being the initial and final momenta of nucleon i, the potential in mome
space is found to be

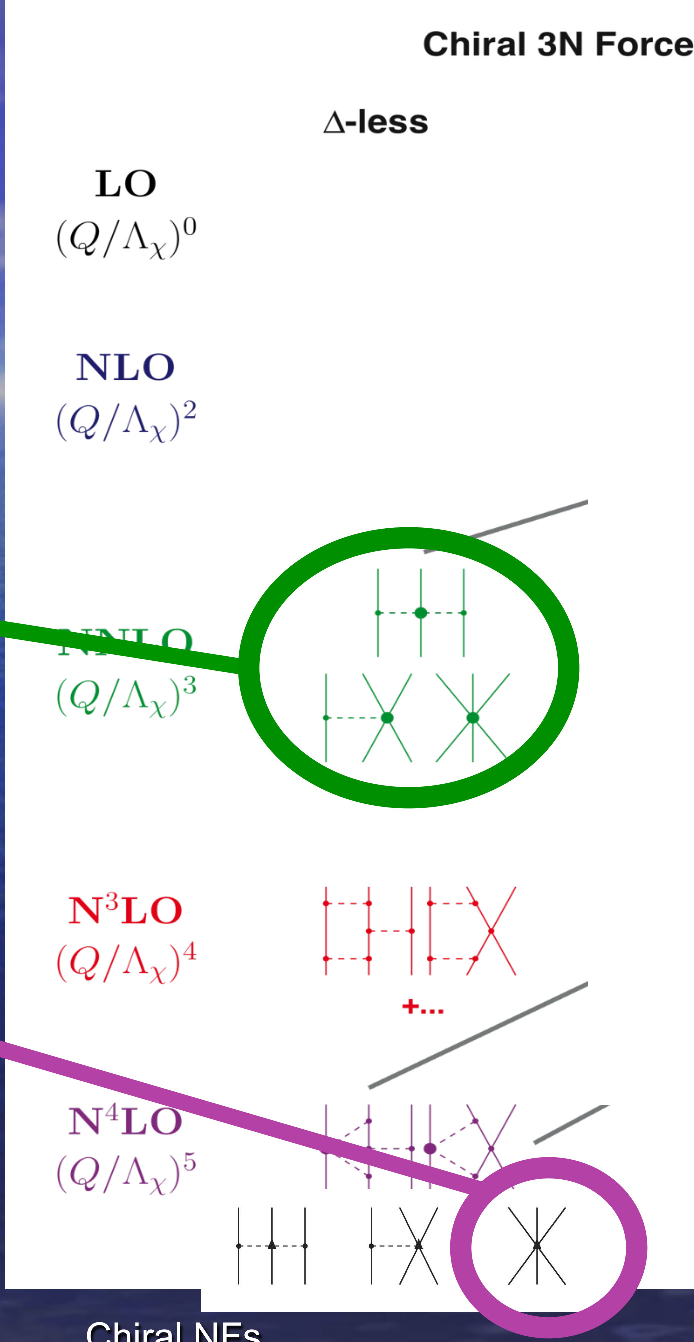
$$V = \sum_{i\neq j\neq k} \Big[-E_1\mathbf{k}_i^2 - E_2\mathbf{k}_i^2\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j - E_3\mathbf{k}_i^2\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j - E_4\mathbf{k}_i^2\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j$$
$$-E_5(3\mathbf{k}_i\cdot\boldsymbol{\sigma}_i\mathbf{k}_i\cdot\boldsymbol{\sigma}_j - \mathbf{k}_i^2) - E_6(3\mathbf{k}_i\cdot\boldsymbol{\sigma}_i\mathbf{k}_i\cdot\boldsymbol{\sigma}_j - \mathbf{k}_i^2))\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j$$
$$+\frac{i}{2}E_7\mathbf{k}_i\times(\mathbf{Q}_i-\mathbf{Q}_j)\cdot(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j) + \frac{i}{2}E_8\mathbf{k}_i\times(\mathbf{Q}_i-\mathbf{Q}_j)\cdot(\boldsymbol{\sigma}_i+\boldsymbol{\sigma}_j)\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k$$

Spin-Orbit Force!

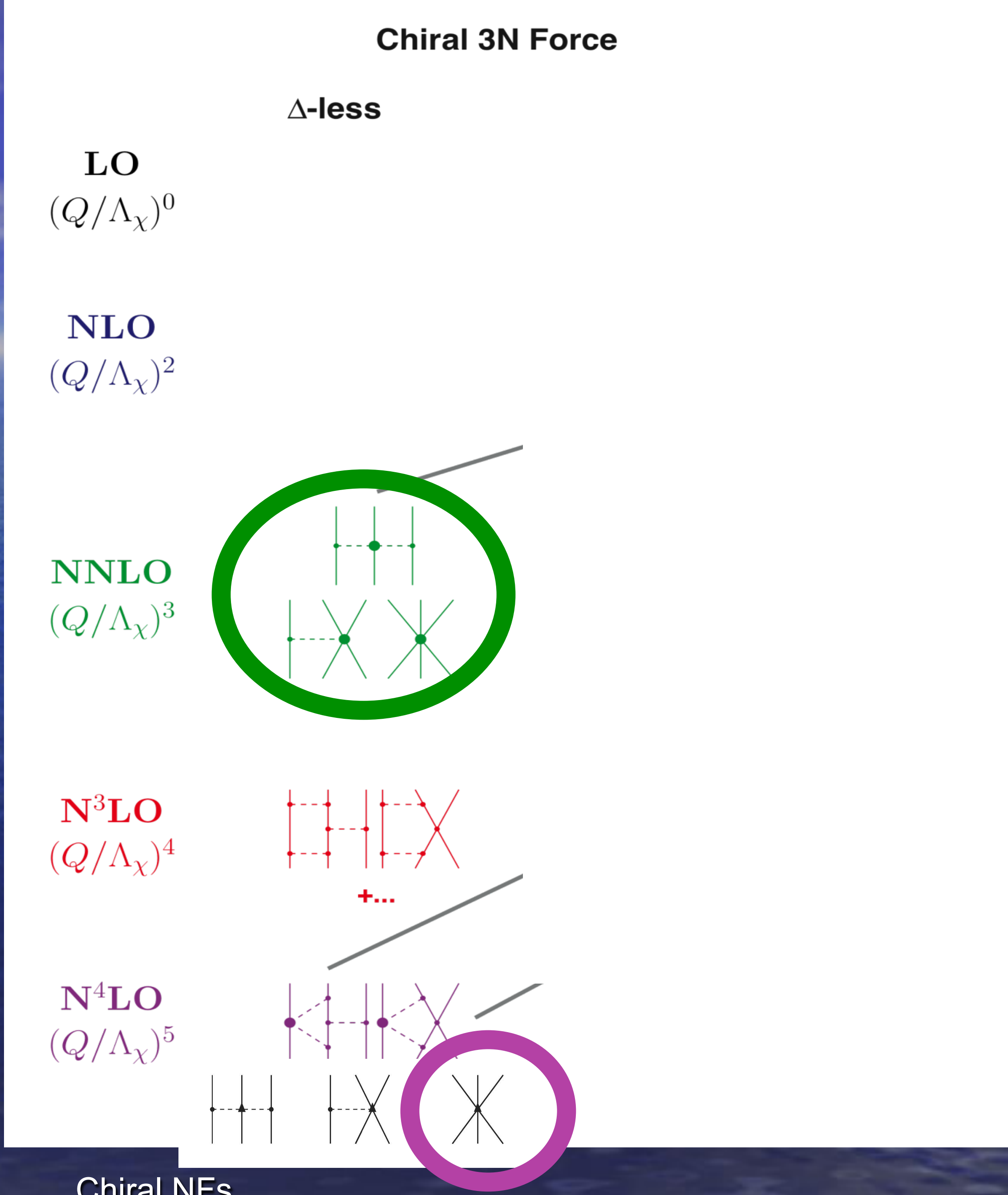
R. Machleidt

How to deal with the explosion?

- use Δ-less
- include NNLO 3NF
- skip N3LO 3NF
- at N4LO start with contact 3NF, use one term at a time, e.g. spin-orbit
- that may already solve some of your problems.

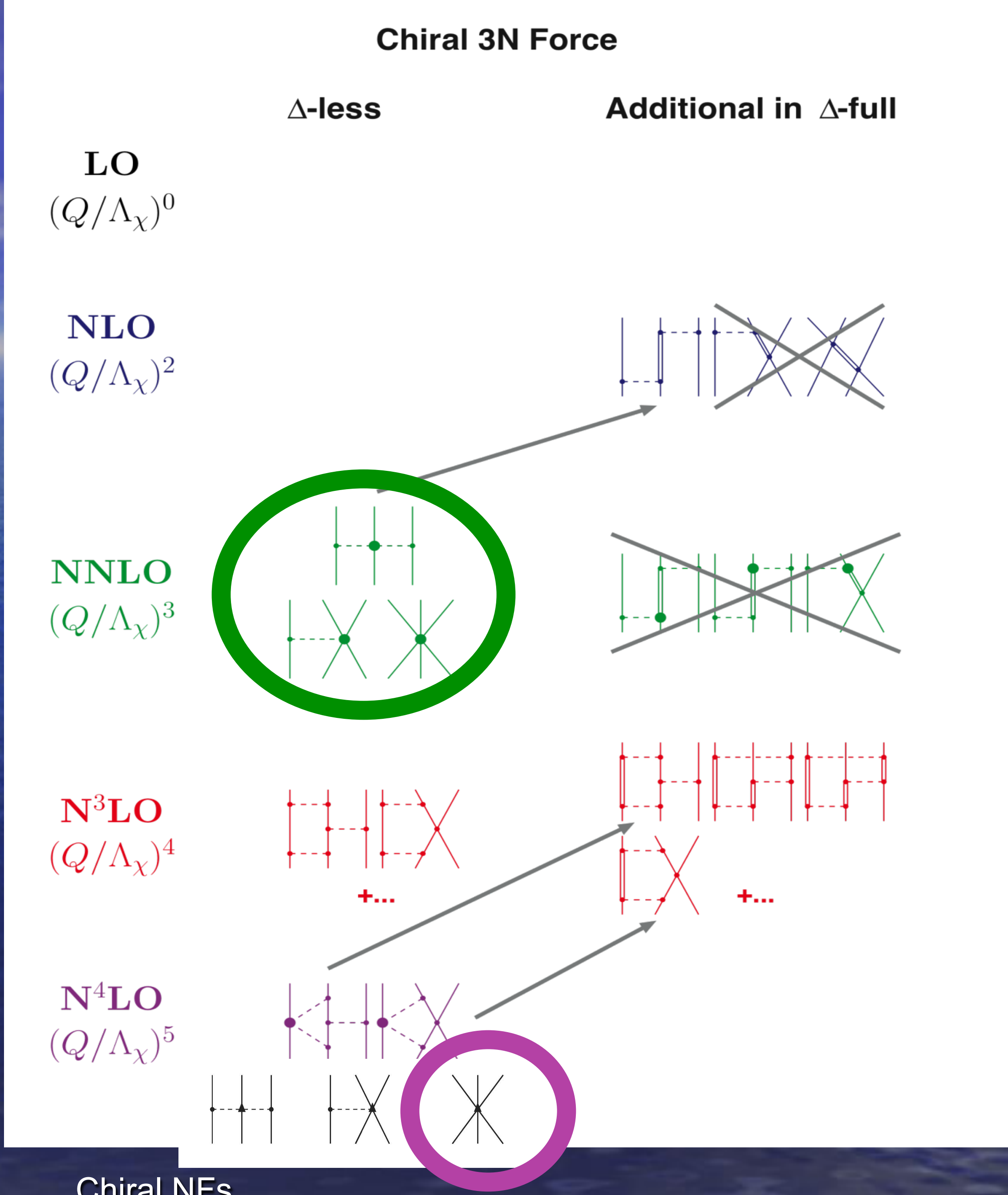


… and then there Is also the Δ-full theory …



… and then there Is also the Δ-full theory …

More explosions!



Conclusions

- **The chiral EFT approach has substantially advanced our understanding of nuclear forces.**
- **Two-, three, and four-nucleon forces have been derived up to N3LO.**
- **The chiral forces are a perfect starting point for *ab initio* nuclear structure calculations.**
- **But there are still some not so subtle "subtleties" to be taken care of:**
- **The non-perturbative renormalization of the chiral forces**
- **Sub-leading 3NFs**
- **The role of 4NFs**

The 3NF issue

- The 3NF at NNLO is good,
  but not good enough.
- The 3NF at N3LO (in the Δ-less theory) may be weak (and useless?).
- However, there is a burst of (potentially large) 3NF contributions at N4LO (including a new set of contact 3NFs!).
- Order by order convergence of the chiral 3NF may be questionable.
- There will be many new 3NFs in the near future. Too many?
- But, practitioners, NO PANIC! For a while you have to pick and choose, and not go for "complete" calculations.

And so,
we are not yet completely done with the nuclear force problem,
but …