

Time-dependent density functional calculation of nuclear response functions

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and



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Collaborators

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Basic equation

- TDHF eq. (TDKS eq.)

$$i\frac{\partial}{\partial t}\varphi_i(t) = h[\rho(t)]\varphi_i(t), \quad h[\rho] \equiv \frac{\delta E[\rho]}{\delta \rho}$$

- TDHFB eq. (TDBdGKS eq.)

$$i\frac{\partial}{\partial t}\begin{pmatrix} U_\mu(t) \\ V_\mu(t) \end{pmatrix} = \begin{pmatrix} h(t) - \lambda & \Delta(t) \\ -\Delta^*(t) & -(h(t) - \lambda)^* \end{pmatrix} \begin{pmatrix} U_\mu(t) \\ V_\mu(t) \end{pmatrix}$$
$$h[\rho, \kappa, \kappa^*] \equiv \frac{\delta E[\rho, \kappa, \kappa^*]}{\delta \rho}, \quad \Delta[\rho, \kappa, \kappa^*] \equiv \frac{\delta E[\rho, \kappa, \kappa^*]}{\delta \kappa^*}$$

Time-dependent DFT (TDDFT)

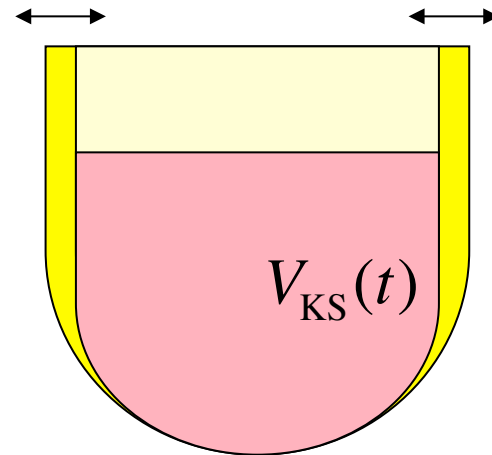
Time-dependent Kohn-Sham equation (1984)

$$i \frac{\partial}{\partial t} \varphi_i(t) = \left\{ -\frac{\hbar^2}{2m} \nabla^2 + V_{\text{KS}}[\rho(t)] \right\} \varphi_i(t)$$

$$V_{\text{KS}}[\rho(t)] = V_0 + \delta V_{\text{KS}}(t)$$

Induced (screening) field

$$\delta V_{\text{KS}}(t) = \frac{\delta V_{\text{KS}}}{\delta \rho} \delta \rho(t)$$



The collective motion is induced by the motion of the potential.

Complete analogue of the unified model by Bohr and Mottelson

$$\delta\rho_n(t) = \rho_n(t) - (\rho_0)_n$$

Time-dep. transition density

Instantaneous weak E1 field

$$\delta\rho > 0$$

$$\delta\rho < 0$$

$$V_{\text{ext}}(t) = \eta M(E1) \delta(t)$$

$$\delta\rho_p(t) = \rho_p(t) - (\rho_0)_p$$

$$\delta\rho_n(t) = \rho_n(t) - (\rho_0)_n$$

Time-dep. transition density

Instantaneous weak E1 field

$$\delta\rho > 0$$

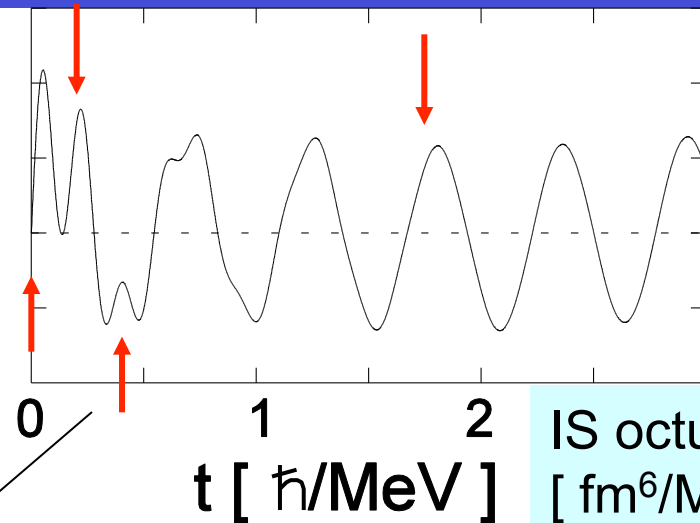
$$\delta\rho < 0$$

$$V_{\text{ext}}(t) = \eta M(E1) \delta(t)$$

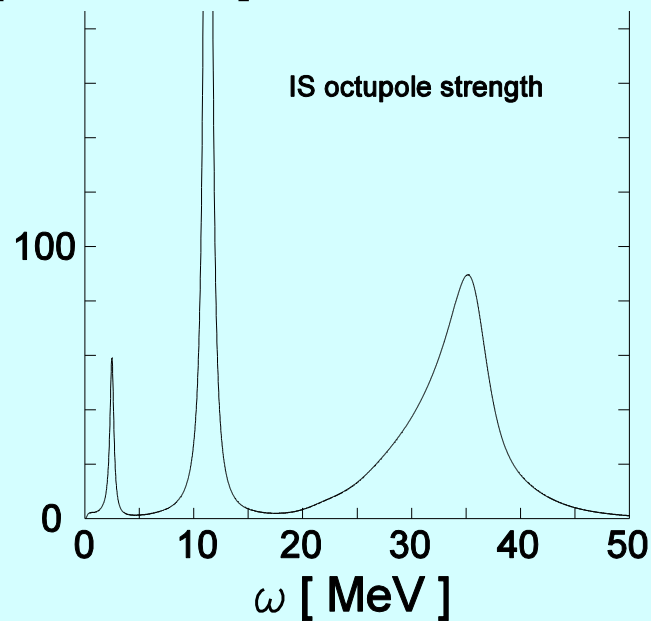
$$\delta\rho_p(t) = \rho_p(t) - (\rho_0)_p$$

IS octupole resonances in ^{16}O

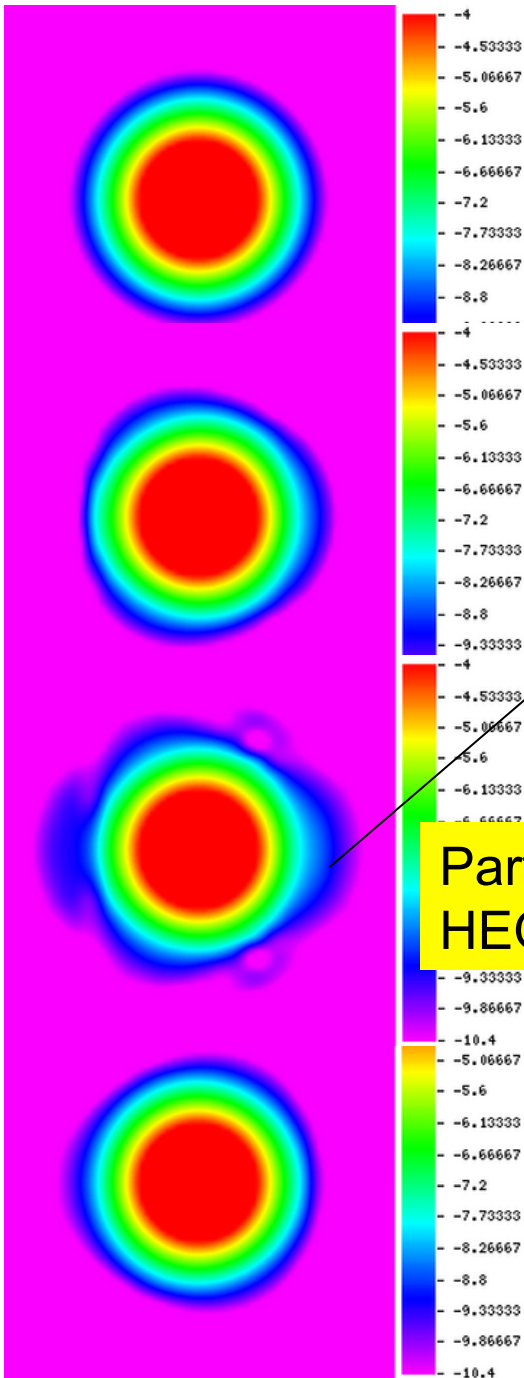
Time evolution of IS octupole moment



IS octupole strength function
[fm^6/MeV]



Particle decay of
HEOR



Canonical-basis TDHFB

Ebata, TN, Inakura, Yoshida, Hashimoto, Yabana, PRC 82 (2010) 034306

$$i \frac{\partial}{\partial t} |k(t)\rangle = (h(t) - \eta_k(t)) |k(t)\rangle$$

$$i \frac{\partial}{\partial t} \rho_k(t) = \Delta_k^*(t) K_k(t) - \text{c.c.}$$

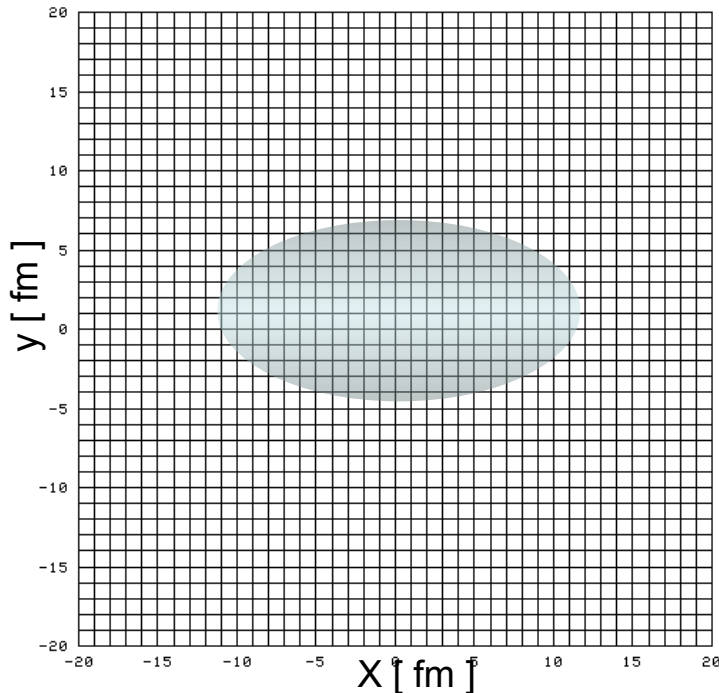
$$i \frac{\partial}{\partial t} K_k(t) = (\eta_k(t) + \eta_{\bar{k}}(t)) K_k(t) + \Delta_k(t) (2\rho_k(t) - 1)$$

- Time-dependent canonical states “k”
- Time-dependent (u,v)-factors

Skyrme Cb-TDHFB in real space & real time

Time evolution is calculated by the predictor-corrector method.

3D mesh representation for canonical states



$$\langle \mathbf{r}, \sigma; t | k \rangle = \left\{ \langle \mathbf{r}_i, \sigma; t_n | k \rangle \right\}_{i=1, \dots, M_r}^{n=1, \dots, M_t}, \quad k = 1, \dots, M \geq N$$

$$u_k(t), v_k(t) \quad k = 1, \dots, M$$

Spatial size is a spherical box of radius of 12 - 15 fm.

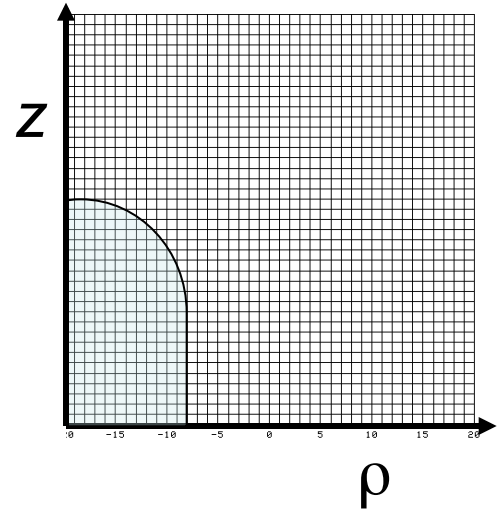
Spatial mesh size is 0.8 fm.

Time step is about 0.2 fm/c

HFB+QRPA for axially deformed nuclei

$$\begin{pmatrix} h - \lambda & \Delta \\ -\Delta^* & -(h - \lambda)^* \end{pmatrix} \begin{pmatrix} U_\mu(\rho, z; \sigma) \\ V_\mu(\rho, z; \sigma) \end{pmatrix} = E_\mu \begin{pmatrix} U_\mu(\rho, z; \sigma) \\ V_\mu(\rho, z; \sigma) \end{pmatrix}$$

$$\left\{ \begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix} - \omega \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right\} \begin{pmatrix} X_{\mu\nu}(\omega) \\ Y_{\mu\nu}(\omega) \end{pmatrix} = 0$$



- HFB equations are solved in the 2D coordinate space, assuming the axial symmetry for the SkM* functional with the cutoff of $E_{\text{qp}} < 60$ MeV.
- The pairing energy functional is the one determined by a global fitting to deformed nuclei (Yamagami, Shimizu, TN, PRC **80**, 064301 (2009))
- QRPA matrix is calculated in the quasiparticle basis ($E_{2\text{qp}} < 60$ MeV).
- All the residual interactions are taken into account, except for the residual Coulomb interaction.

Finite Amplitude Method

T.N., Inakura, Yabana, PRC **76** (2007) 024318

Avogadro and T.N., PRC **84**, 014314 (2011)

To avoid the calculation of “two-body”-like residual interaction

$$\left\{ \begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix} - \omega \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right\} \begin{pmatrix} X_{\mu\nu}(\omega) \\ Y_{\mu\nu}(\omega) \end{pmatrix} = \begin{pmatrix} F_{\mu\nu}^{20} \\ F_{\mu\nu}^{02} \end{pmatrix} \quad \Rightarrow \quad \begin{aligned} (E_\mu + E_\nu - \omega) X_{\mu\nu}(\omega) + \delta H_{\mu\nu}^{20}(\omega) &= F_{\mu\nu}^{20}(\omega) \\ (E_\mu + E_\nu + \omega) Y_{\mu\nu}(\omega) + \delta H_{\mu\nu}^{02}(\omega) &= F_{\mu\nu}^{02}(\omega) \end{aligned}$$

Residual fields can be calculated with small parameter η

$$\delta H(\omega) = \frac{1}{\eta} \left\{ H[\bar{V}_\eta^*, \bar{U}_\eta^*; V_\eta, U_\eta] - H_0 \right\} \quad \begin{aligned} V_\eta &= V + \eta U^* Y, & \bar{V}_\eta^* &= V^* + \eta U X \\ U_\eta &= U + \eta V^* Y, & \bar{U}_\eta^* &= U^* + \eta V X \end{aligned}$$

Trivial programming of the (Q)RPA code

Only need the **single-particle potential**, with **different bras and kets**.

-
- 3D real-space FAM without pairing
 - Inakura, T.N., Yabana, PRC **80**, 044301 (2009); PRC **84**, 021302 (2011)
 - 2D HO-basis FAM with pairing
 - Stoitsov et al, PRC **84**, 041305 (2011)

Nuclear Landscape

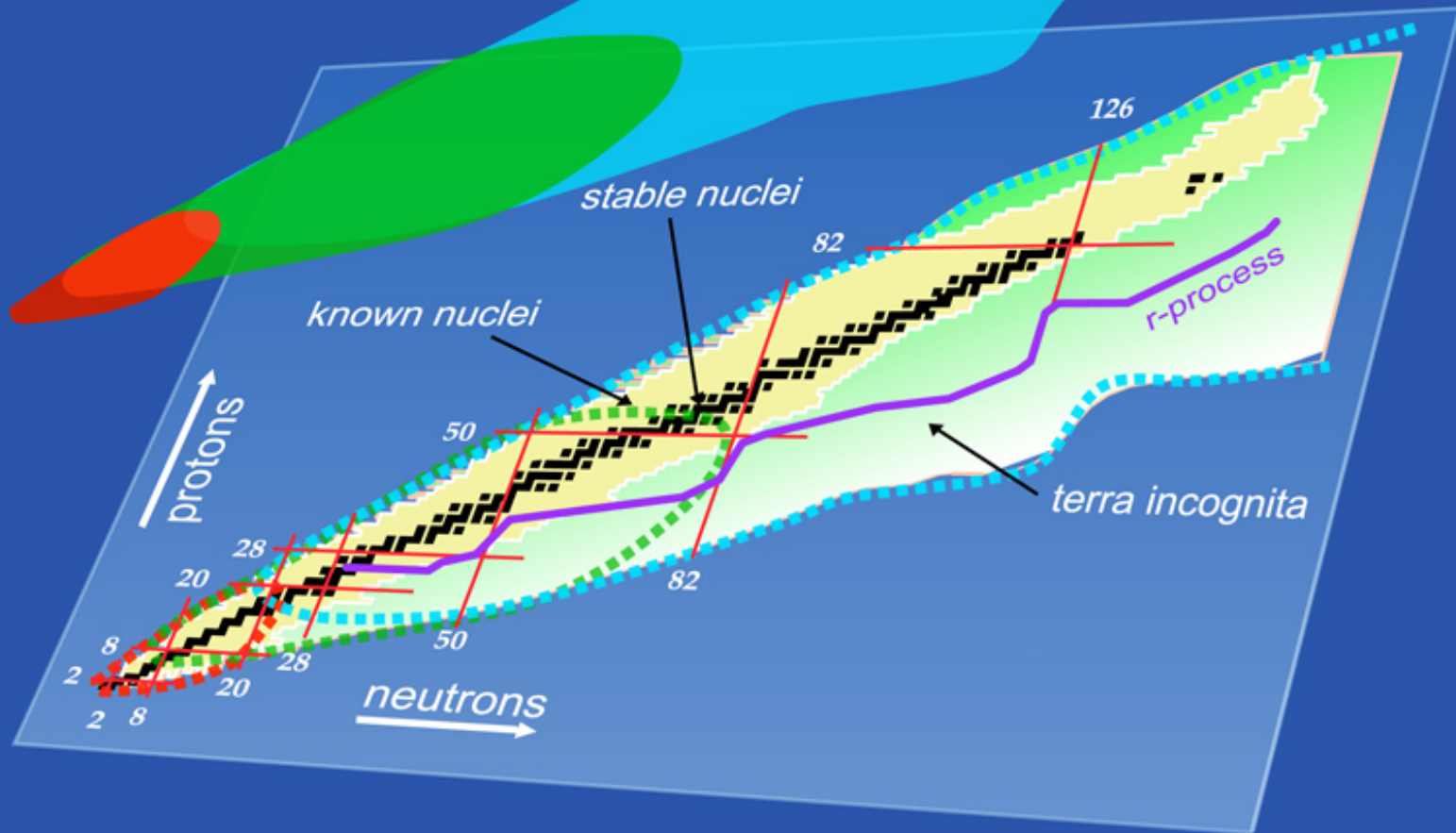


Figure from UNEDF Web Site

Skyrme energy density functional

$$E[\rho_q, \tau_q, \vec{J}_q; \kappa_q]$$

kinetic spin-current pair density

- Kohn-Sham scheme (BdG-KS, HFB)

$$\begin{pmatrix} h - \lambda & \Delta \\ -\Delta^* & -(h - \lambda)^* \end{pmatrix} \begin{pmatrix} U_\mu \\ V_\mu \end{pmatrix} = E_\mu \begin{pmatrix} U_\mu \\ V_\mu \end{pmatrix}$$

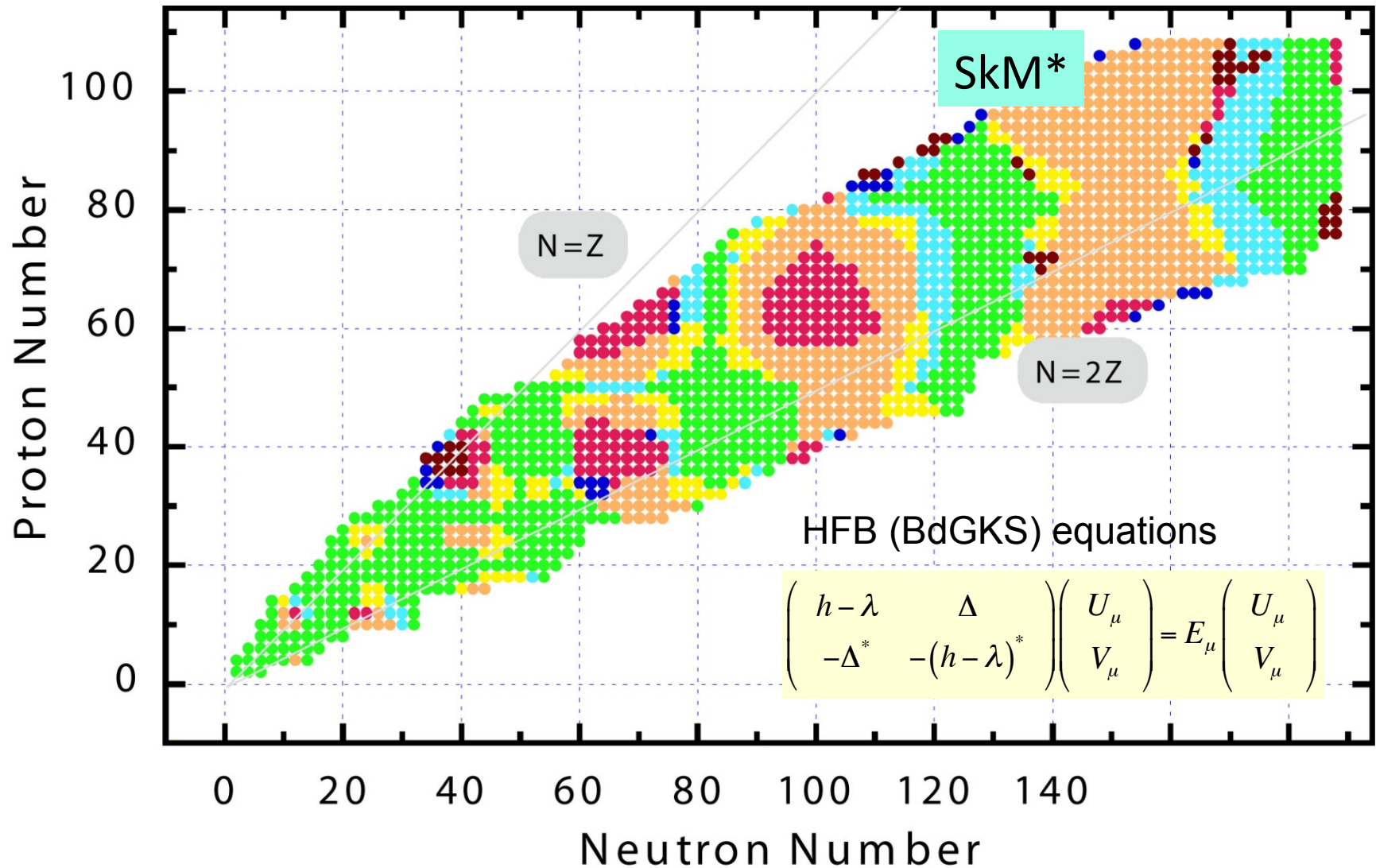
$$\rho = VV^+, \quad \kappa = UV^T$$

$$h[\rho] \equiv \frac{\delta E}{\delta \rho} = -\frac{\hbar^2}{2m} \nabla^2 + V_{\text{KS}}[\rho](\vec{r}), \quad \Delta[\kappa] \equiv \frac{\delta E}{\delta \kappa^*}$$

- Spontaneous symmetry breaking
 - nuclear deformation, pair condensation

In this talk, I present results with the SkM* functional.

Nuclear deformation predicted by Energy Density Functional Theory (EDFT)

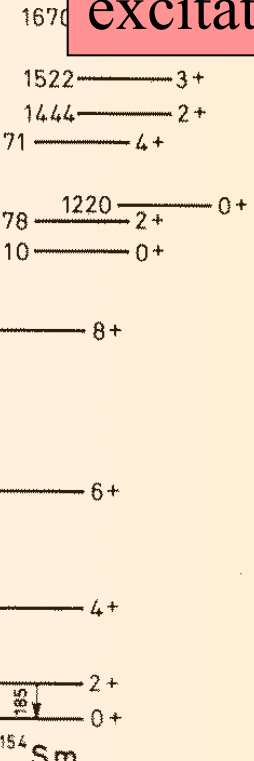
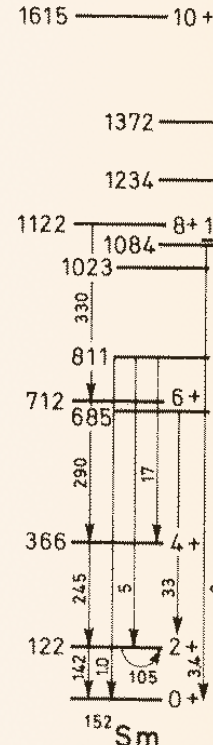
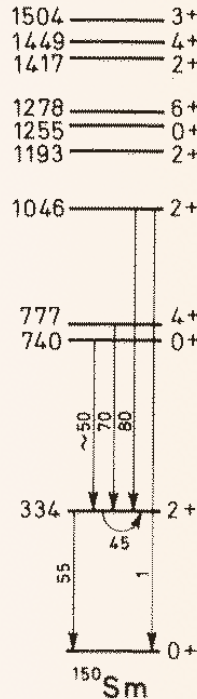
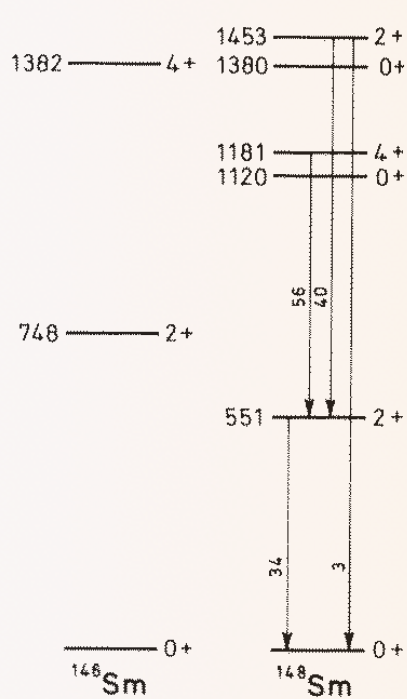
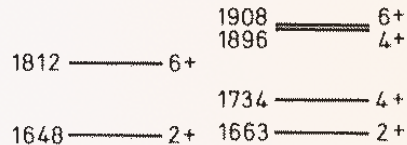
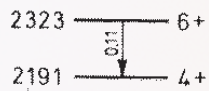


Shape phase transition

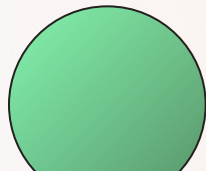
Phonon
excitation

Rotational
excitation

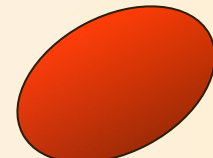
*Nuclear
surface
vibration*



Increasing neutrons



spherical



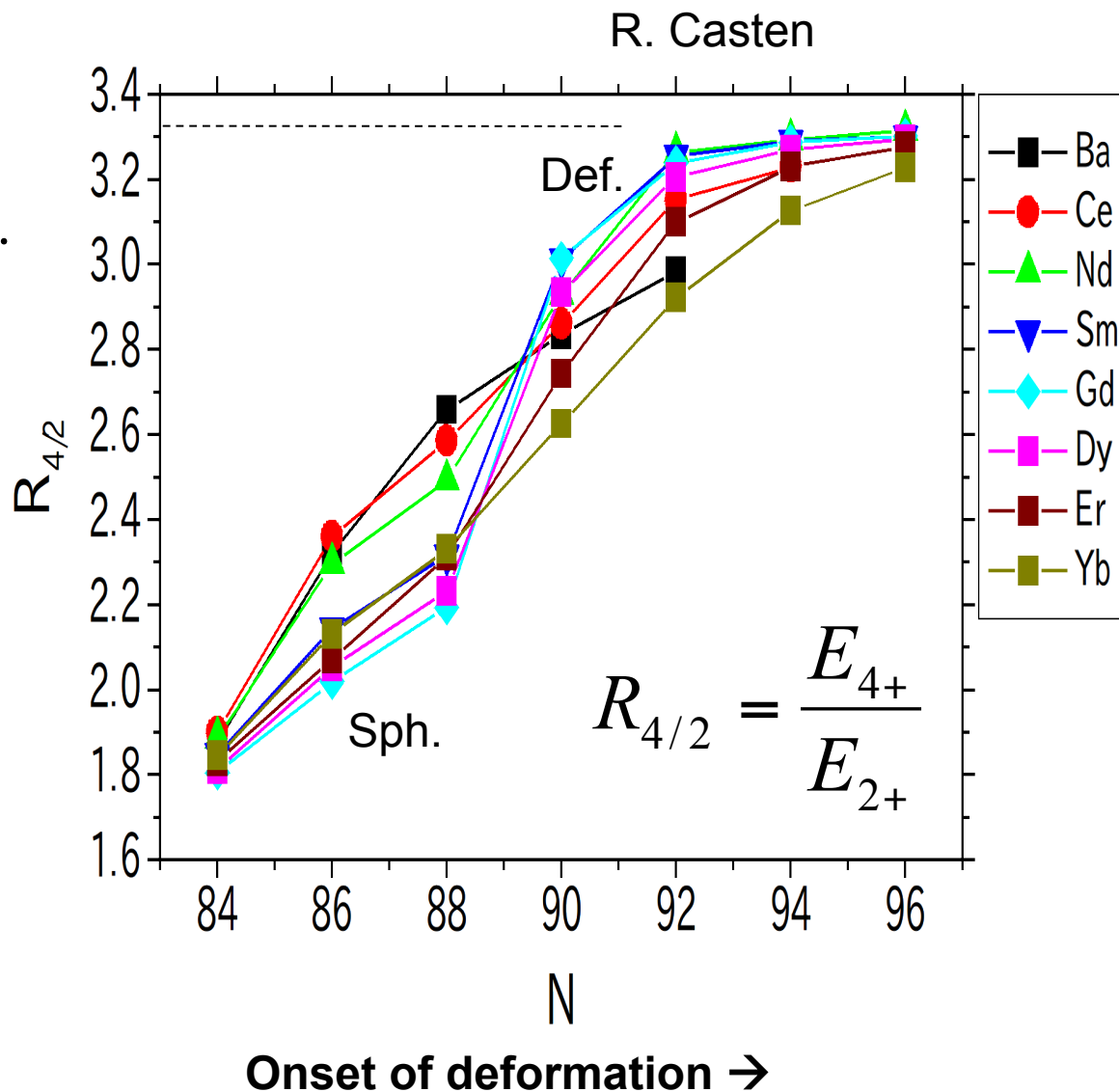
deformed

Evolution of nuclear shapes: $R_{4/2}$

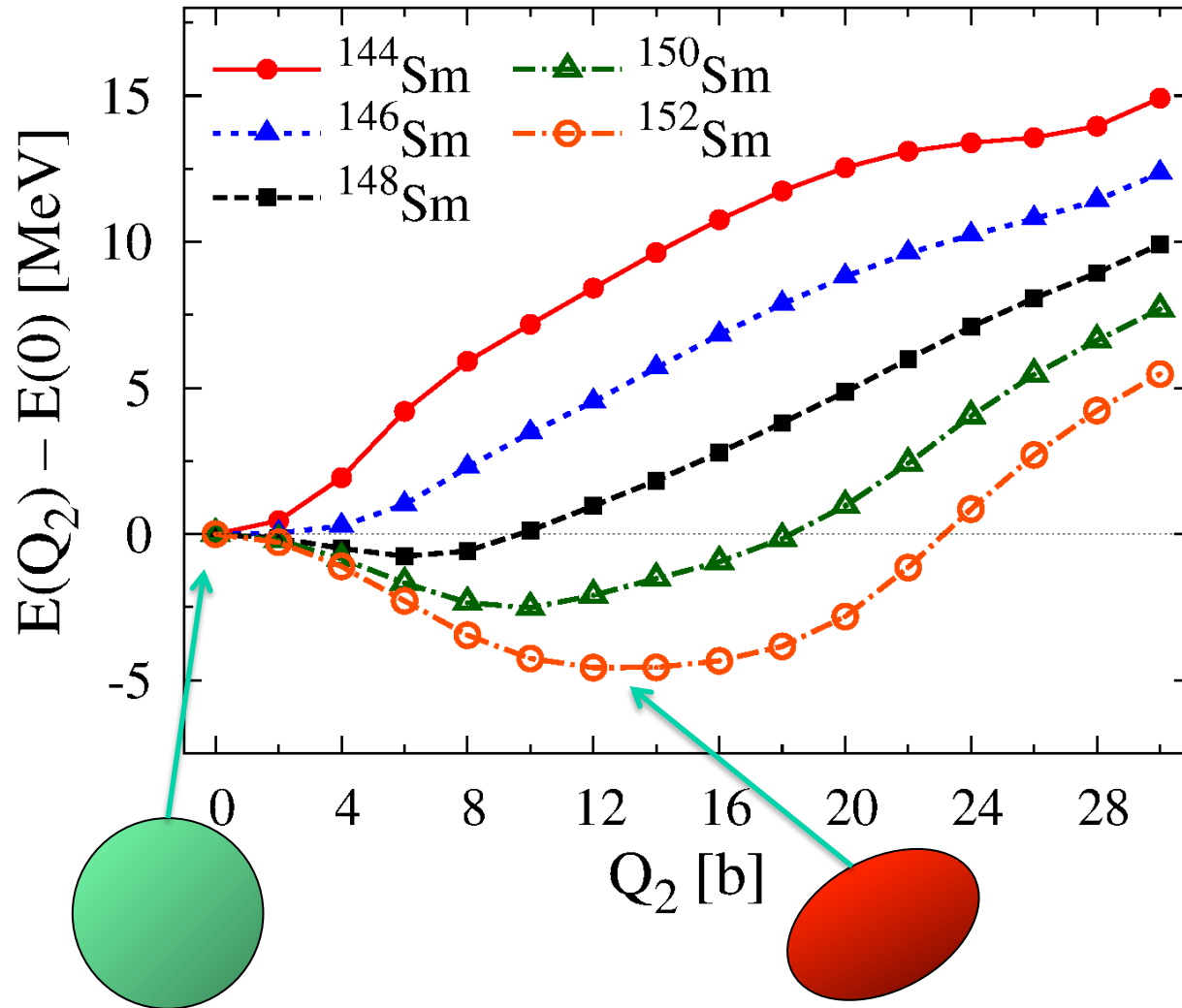
$$E_I = \frac{I(I+1)}{2\mathfrak{I}}$$

↓

$$R_{4/2} \equiv \frac{E_4}{E_2} = \frac{20}{6} = 3.33\dots$$

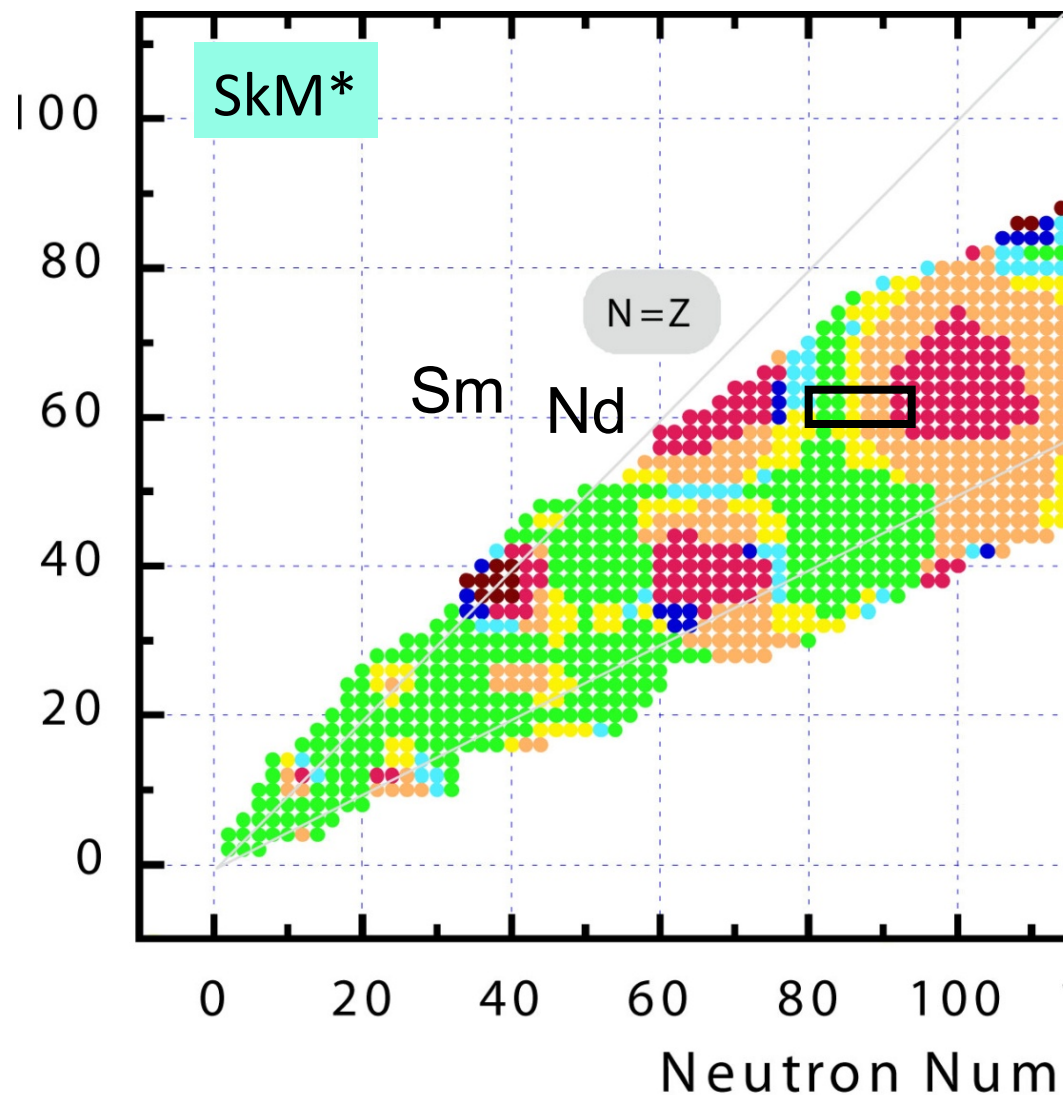
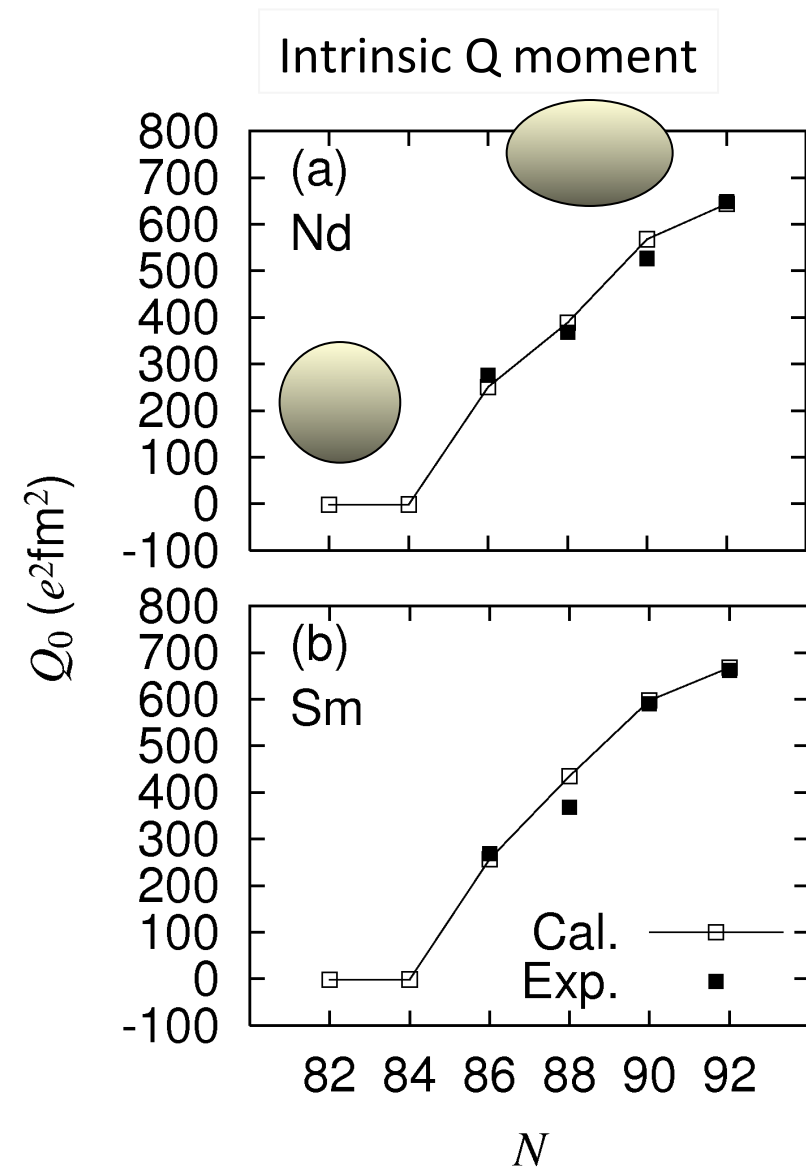


Calculated effective potential



Shape transition produced by EDFT

Yoshida, Nakatsukasa, PRC PRC 83, 021304(R) (2011)



Time-dependent density-functional theory with Skyrme energy density functionals

- Time-odd densities (current density, spin density, etc.)

$$E[\rho_q(t), \tau_q(t), \vec{J}_q(t), \vec{j}_q(t), \vec{s}_q(t), \vec{T}_q(t); \kappa_q(t)]$$

kinetic spin-current current spin spin-kinetic pair density

- Time-dependent BdGKS (HFB) eq.

$$i \frac{\partial}{\partial t} \begin{pmatrix} U_\mu(t) \\ V_\mu(t) \end{pmatrix} = \begin{pmatrix} h(t) - \lambda & \Delta(t) \\ -\Delta^*(t) & -(h(t) - \lambda)^* \end{pmatrix} \begin{pmatrix} U_\mu(t) \\ V_\mu(t) \end{pmatrix}$$

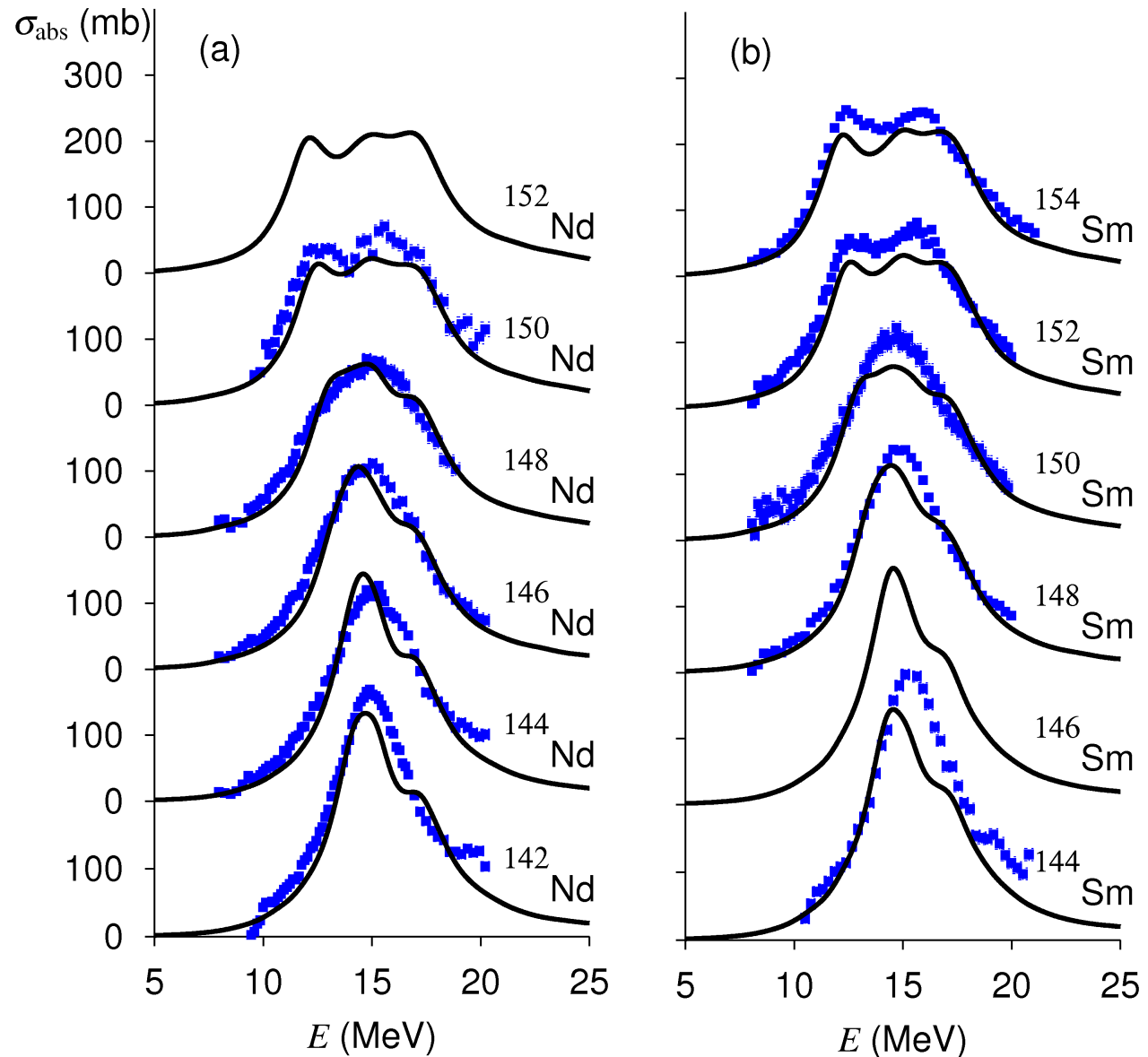
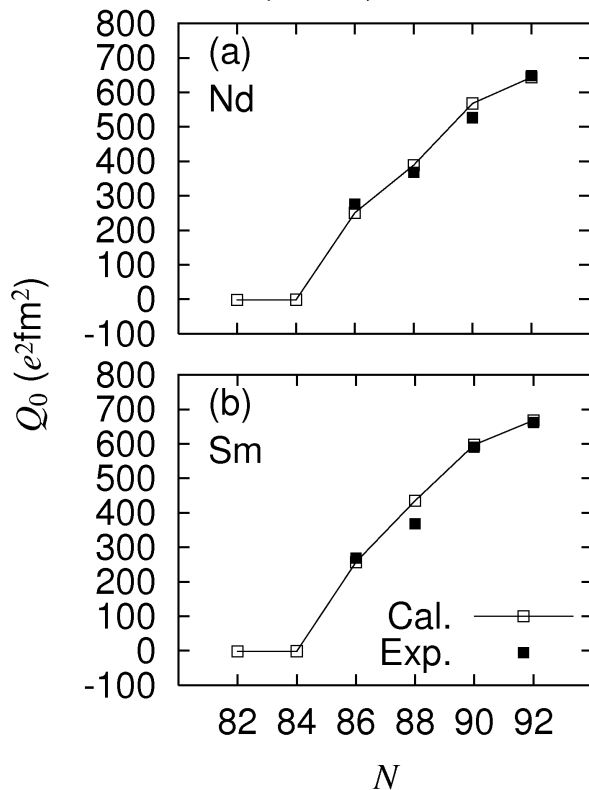
Linear response and photoabsorption cross section

Yoshida, Nakatsukasa, PRC 83, 021304(R) (2011)

SkM* functional

Intrinsic Q moment

$$\langle \hat{Q}_{20} \rangle$$



Giant resonances in Sm isotopes (IS, $L=0\sim 3$)

SkM* functional

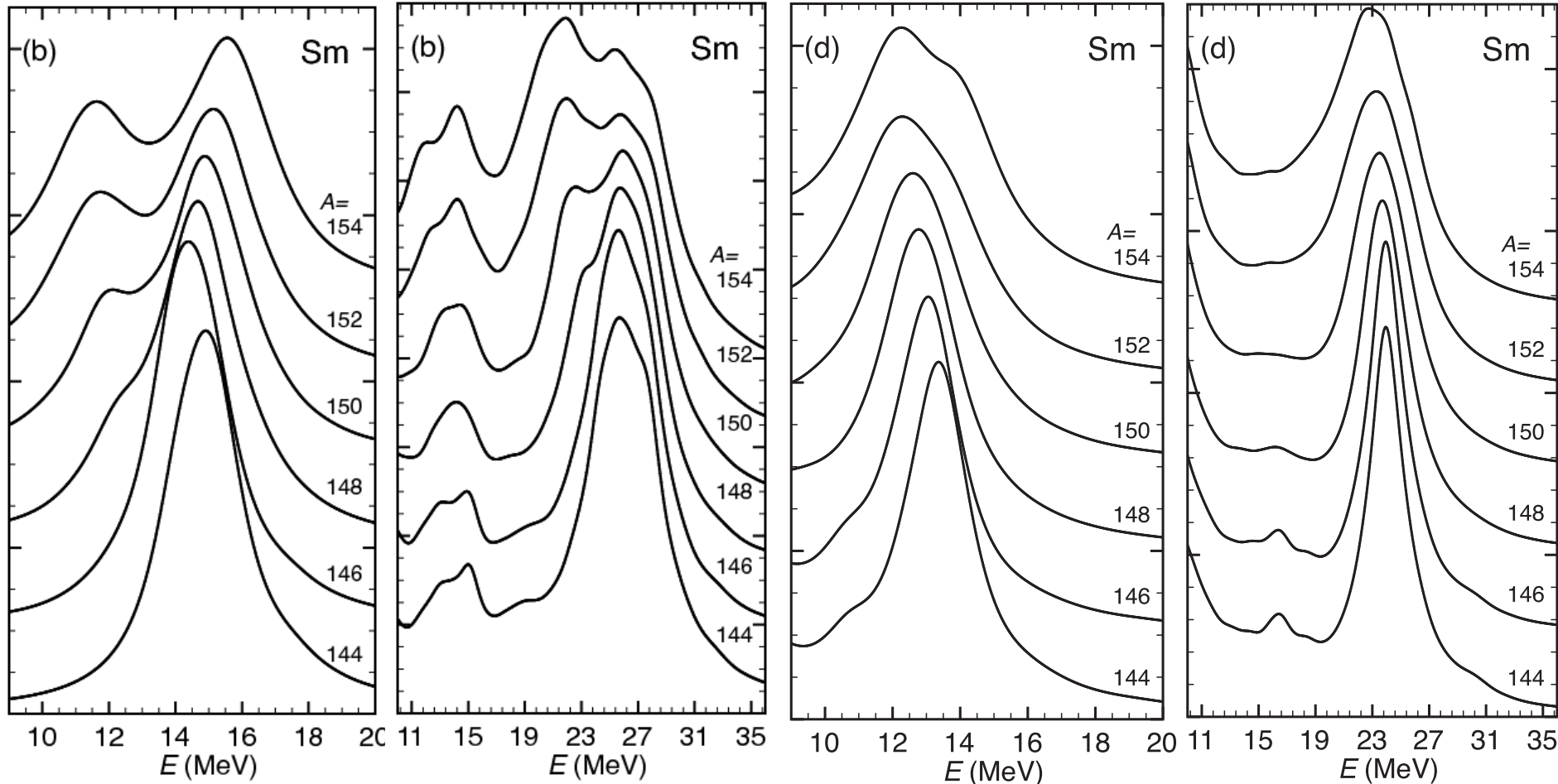
K.Yoshida and TN, PRC 88 034309 (2013)

IS-GOR
(HEOR)

IS-GMR

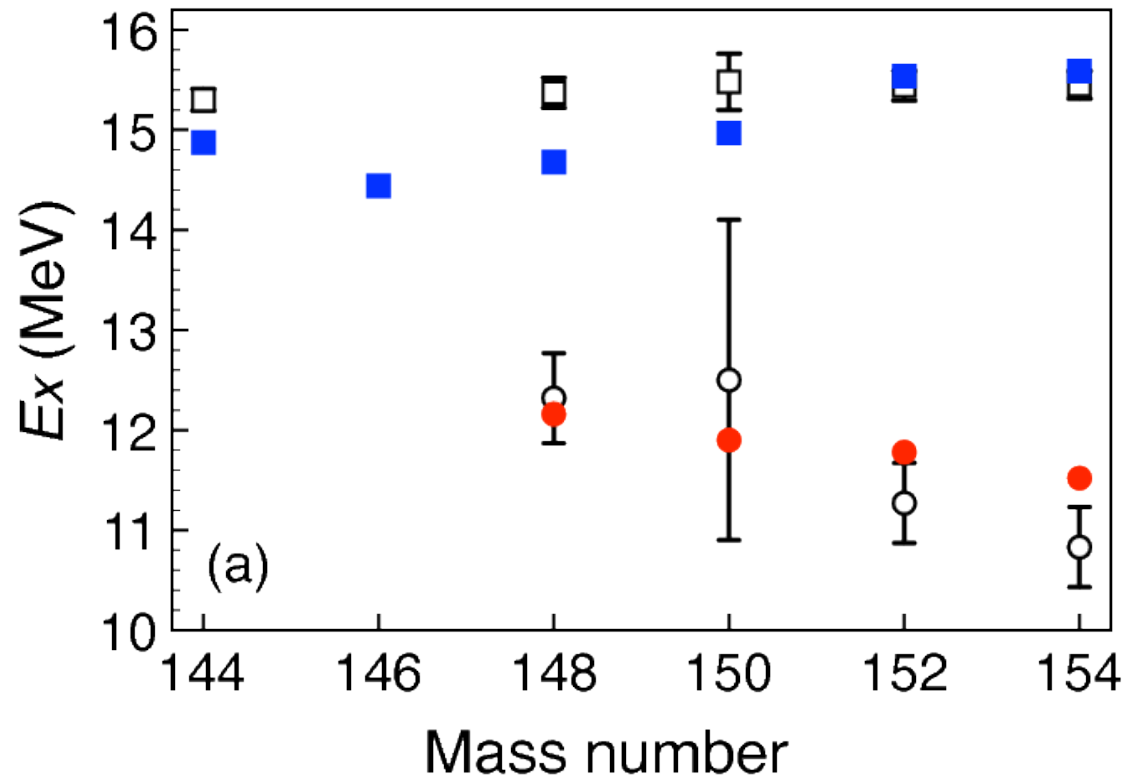
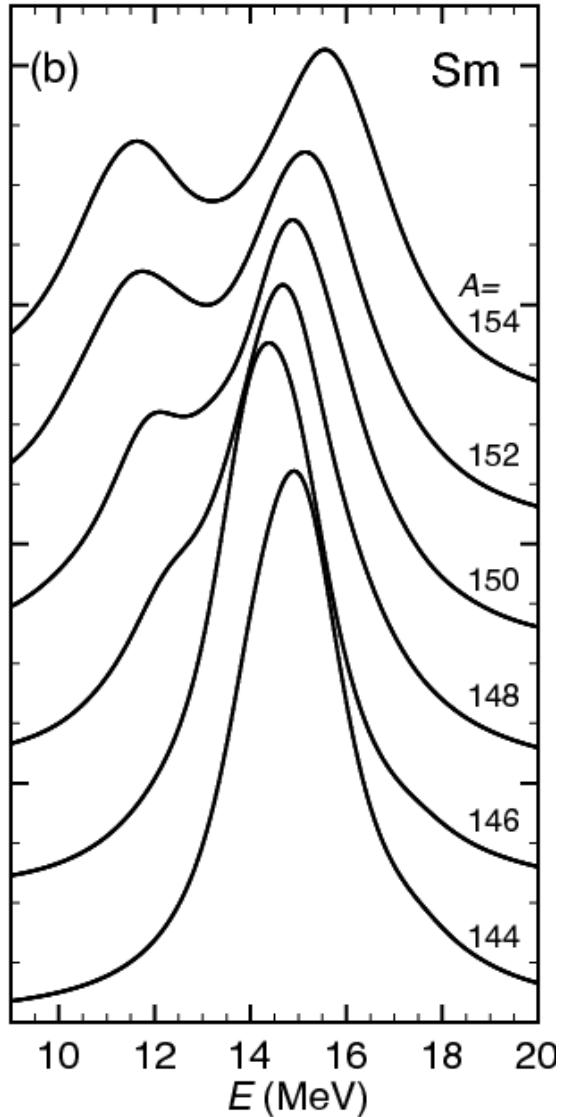
IS-GDR

IS-GQR



Isoscalar giant monopole resonances

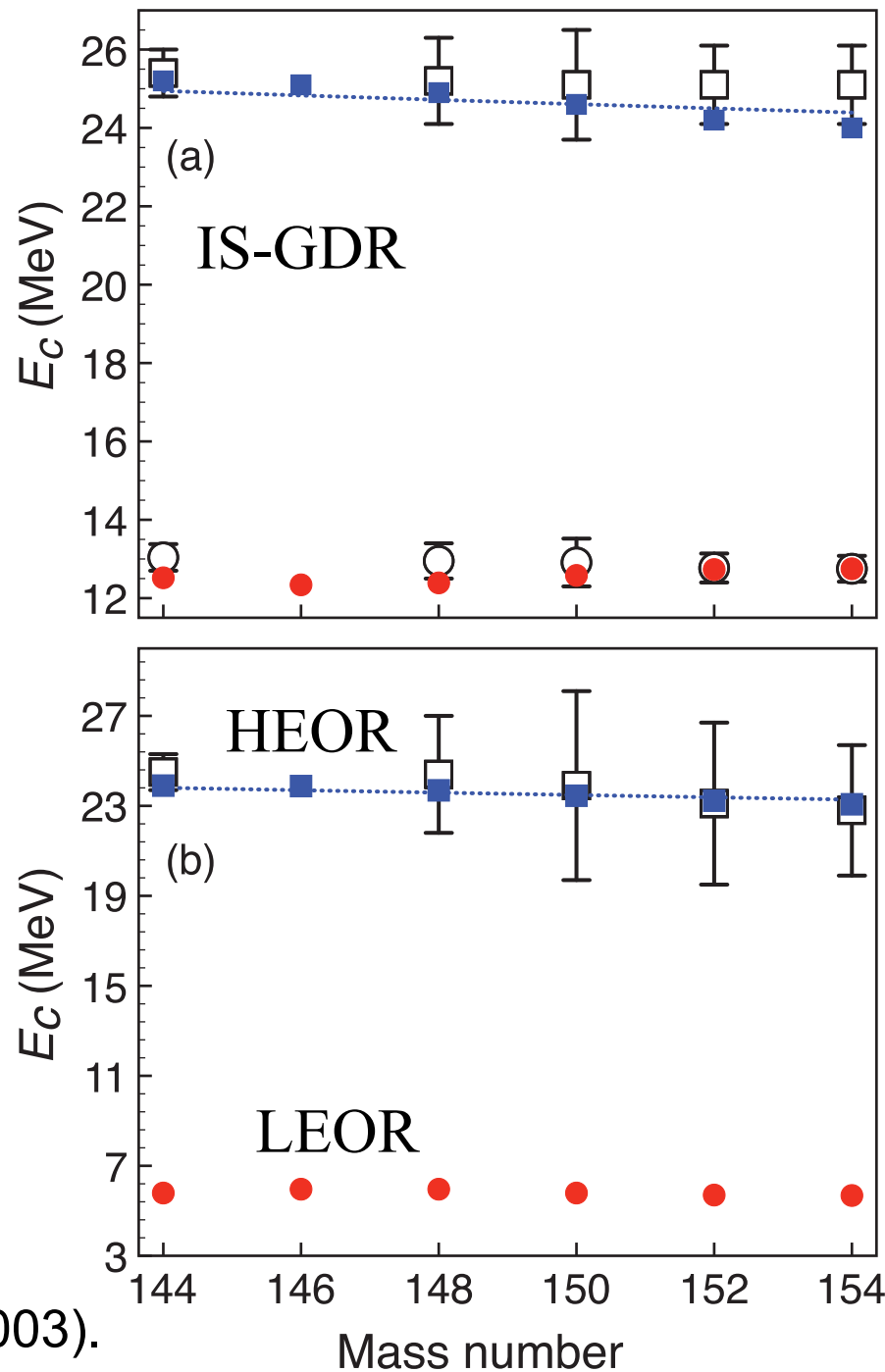
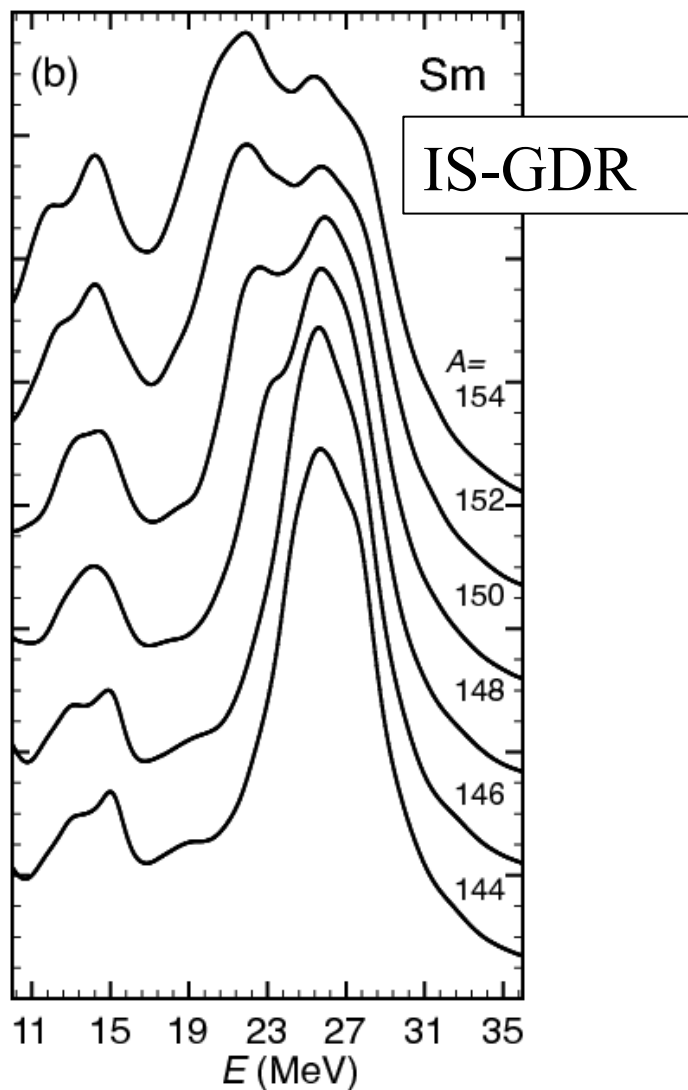
SkM* functional



Exp: M. Itoh et al., PRC 68, 064602 (2003).

Peak splitting is properly described.
Coupling to IS-GQR due to deformation

IS-GDR and IS-GOR



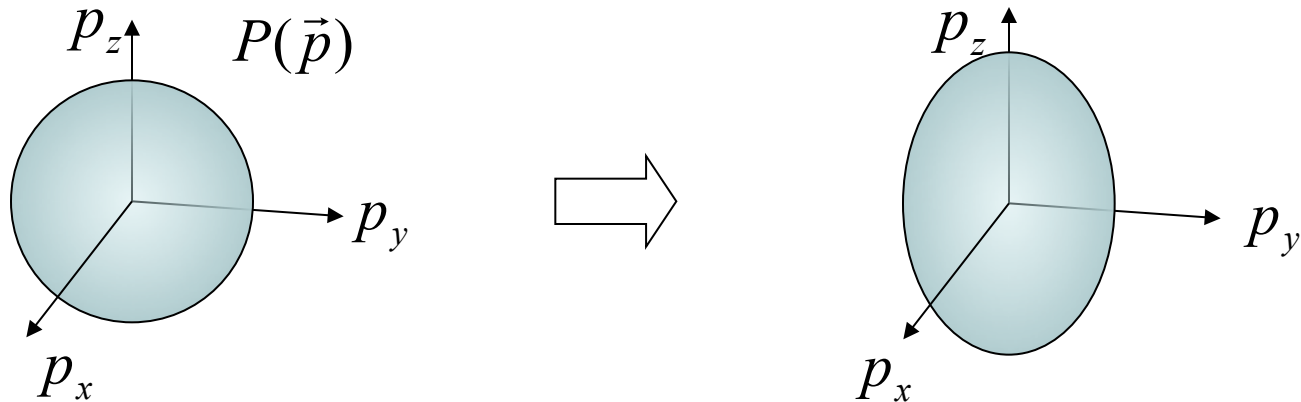
Exp: M. Itoh et al., PRC 68, 064602 (2003).

Fermi Liquid Properties

Sum-rule analysis for nuclear IS Giant Quadrupole Resonance

$$m_3 \propto \left. \frac{1}{2} \frac{\partial^2}{\partial \eta^2} E(\eta) \right|_{\eta=0} \propto \langle T \rangle$$

The restoring force originates from the kinetic energy.



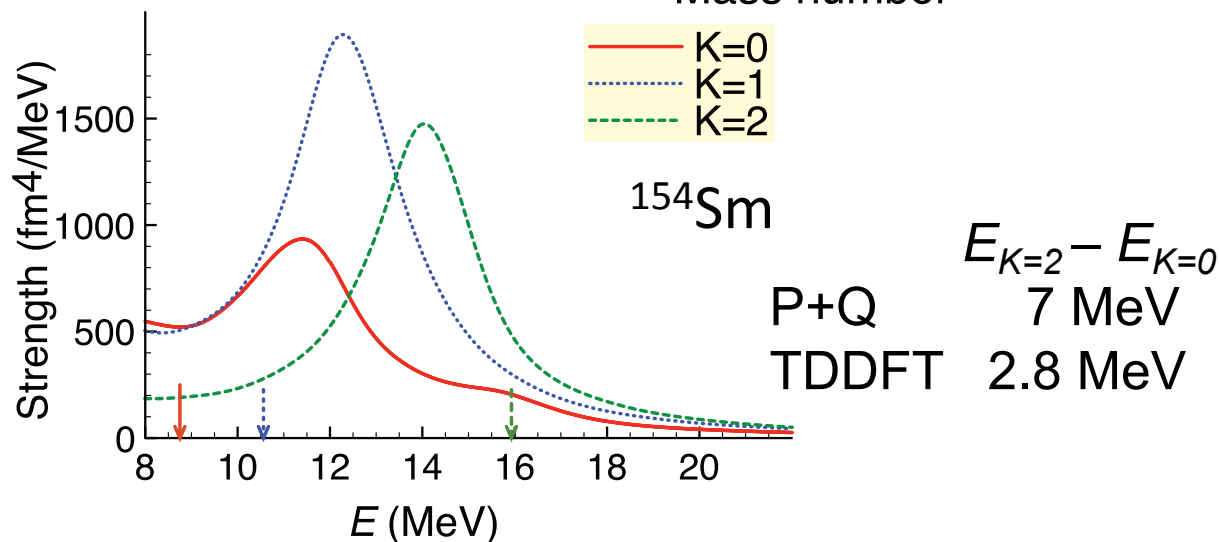
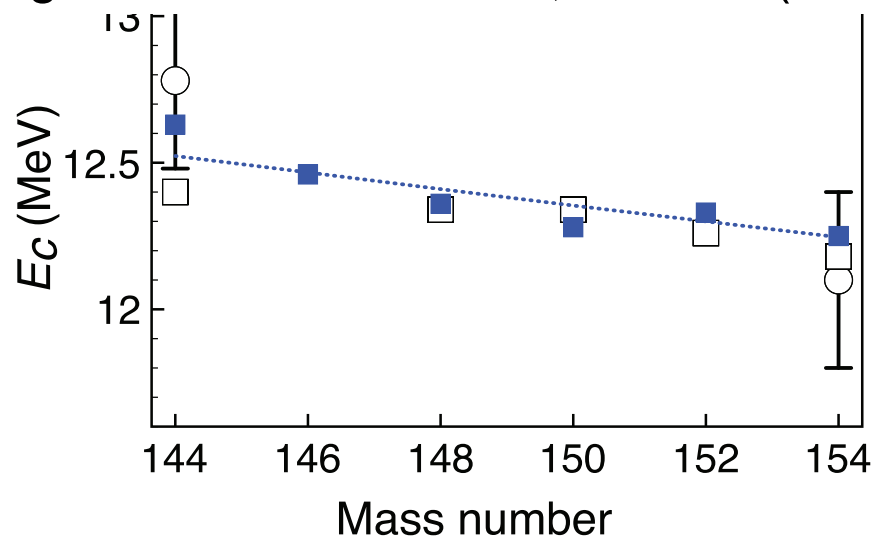
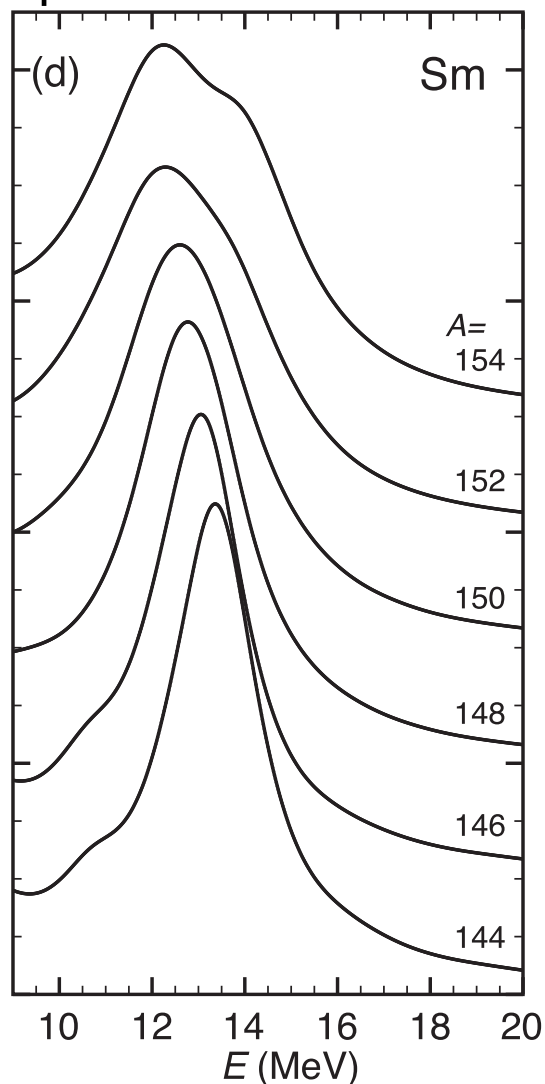
The spatial deformation leads to the deformation of the Fermi sphere.

This is different from the classical (incompressible) liquid model.

Isoscalar giant quadrupole resonances

Exp: Youngblood, et al., PRC **69**, 034315 (2004)

No prominent deformation splitting



Self-consistency between potential and density

Nuclear matter properties

$$\omega_{ISGMR} = \Omega \sqrt{5K / 3m \langle r^2 \rangle}$$

$$\omega_{ISGDR} = \Omega \sqrt{\frac{7}{3} \left[\frac{5K}{3m \langle r^2 \rangle} + \frac{8}{5} \left(\frac{m}{m^*} \right) \right]}$$

$$\Omega = 41 A^{-1/3} \text{ MeV}$$

$$\omega_{ISGQR} = \Omega \sqrt{2m / 3m^*}$$

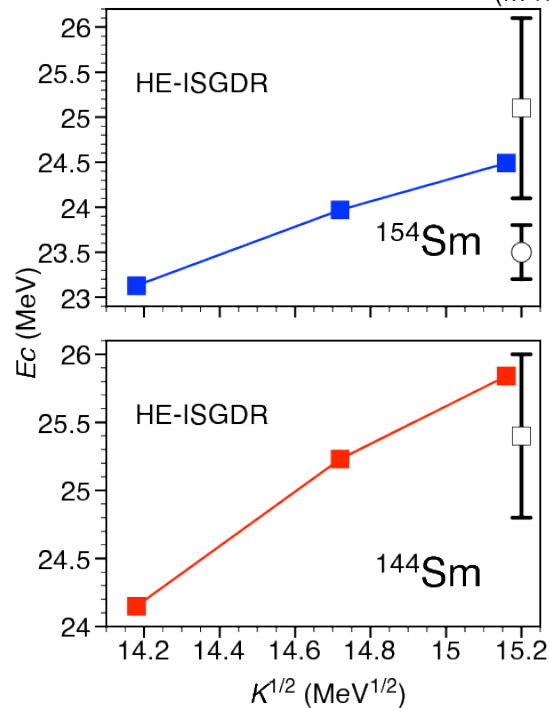
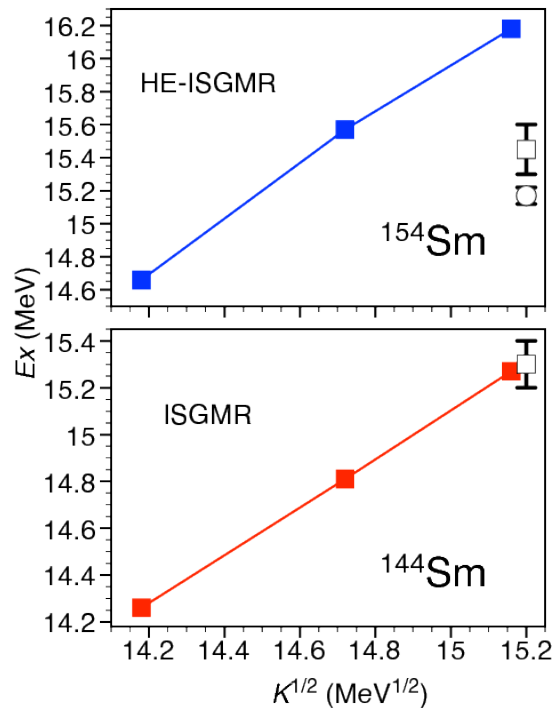
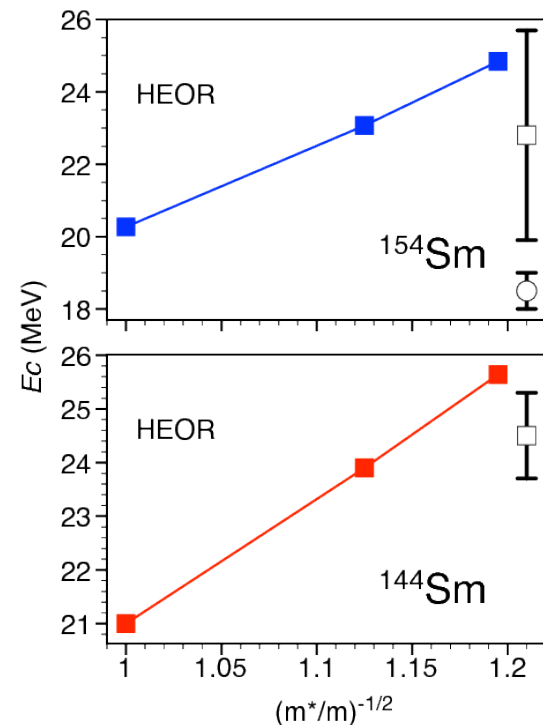
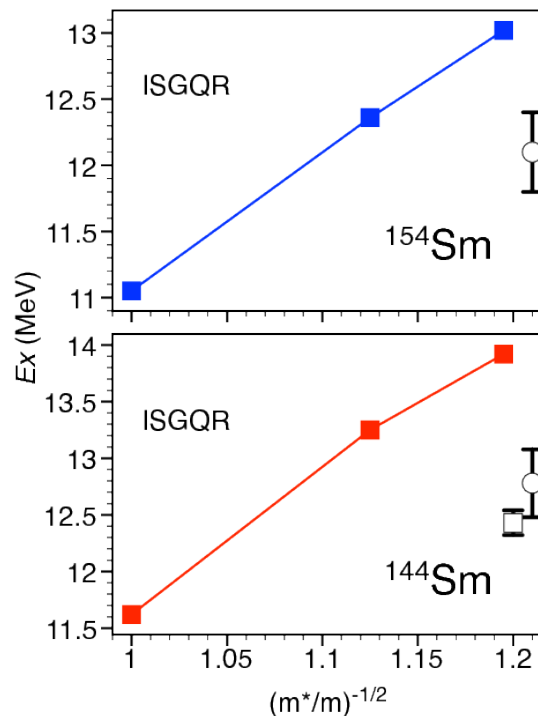
$$\omega_{HEQR} = \Omega \sqrt{28m / 5m^*}$$

Effective mass
and
Incompressibility

SkM*, SLy4, SkP

Effective mass

$$m^*/m = 0.8 \sim 0.9$$



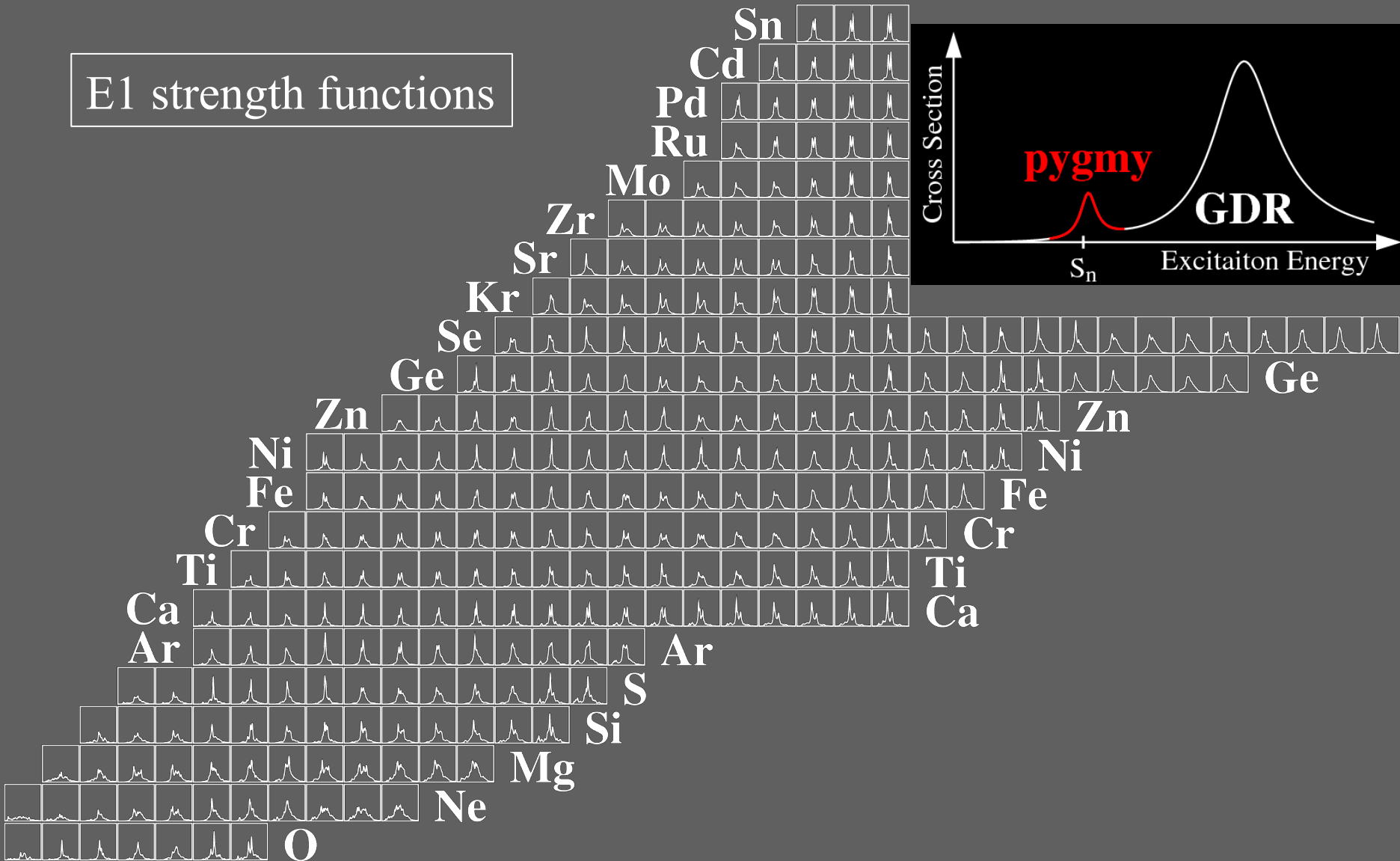
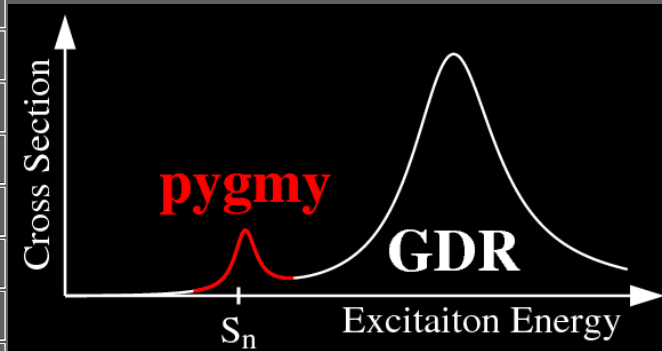
Incompressibility

$$K = 210 \sim 230 \text{ MeV}$$

Photoabsorption cross sections

Inakura, T.N., Yabana, PRC 84, 021302 (R) (2011); PRC 88, 051305(R) (2013)

E1 strength functions

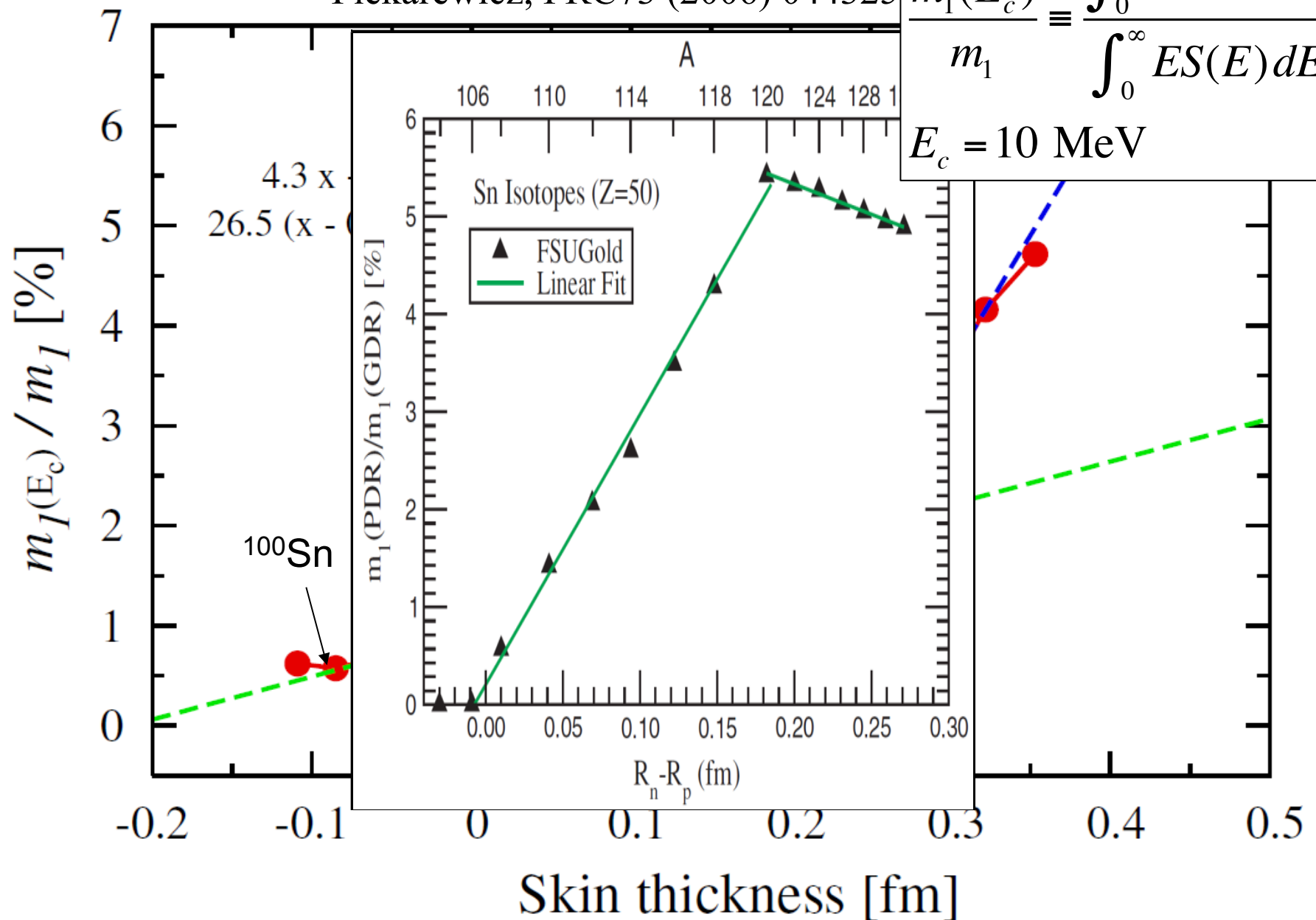


Pygmy dipole states

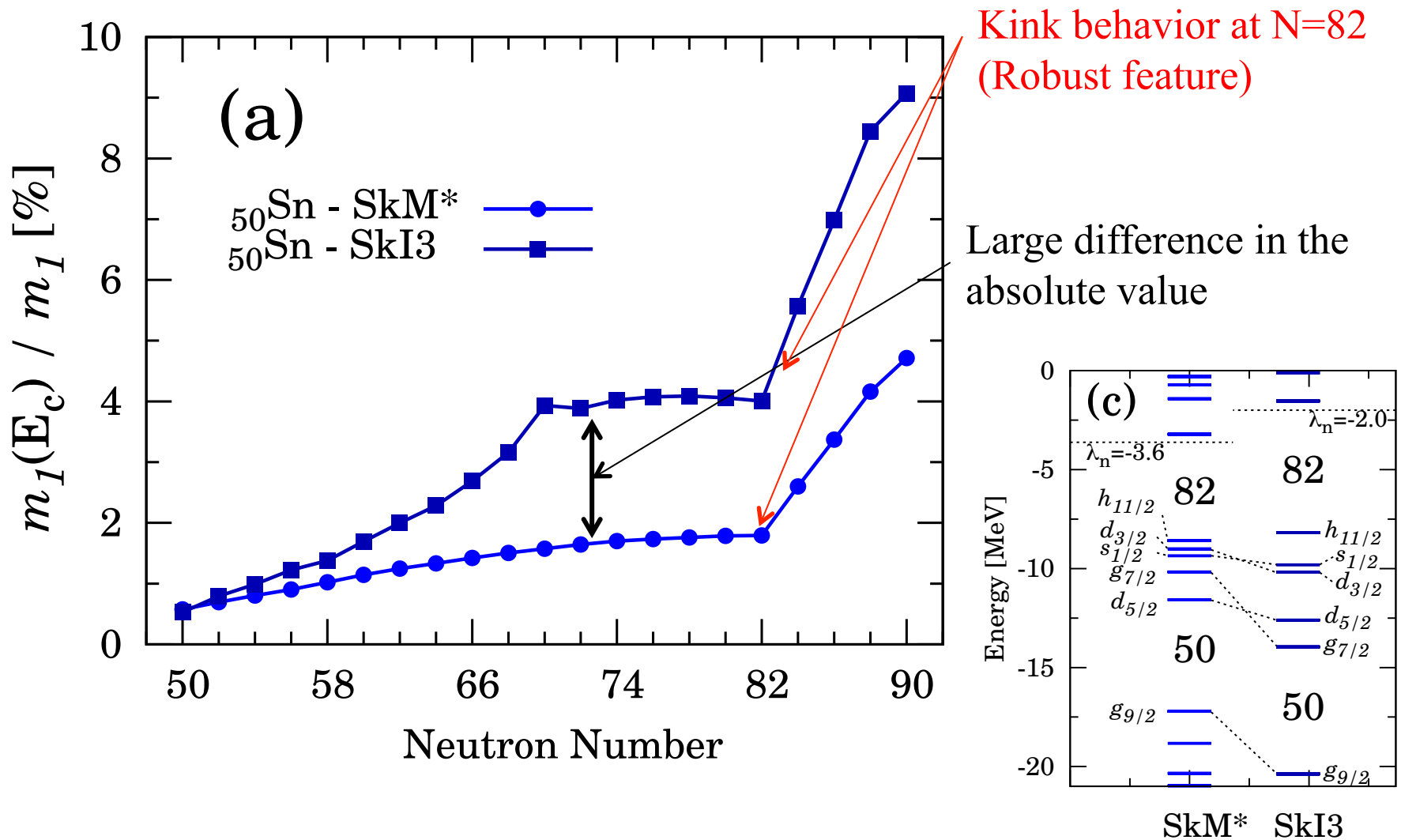
- Robust features
 - Strong shell effects for $E1$ strengths $S(E1)$
 - “Magic numbers”
 - Correlation between $S(E1)$ and the skin thickness [L parameter] in select nuclei
- Model dependent features
 - Absolute values of $S(E1)$
 - Collective nature
 - Correlation between $S(E1)$ and skin thickness in general

$$\frac{m_1(E_c)}{m_1} \equiv \frac{\int_0^{E_c} ES(E) dE}{\int_0^{\infty} ES(E) dE}$$

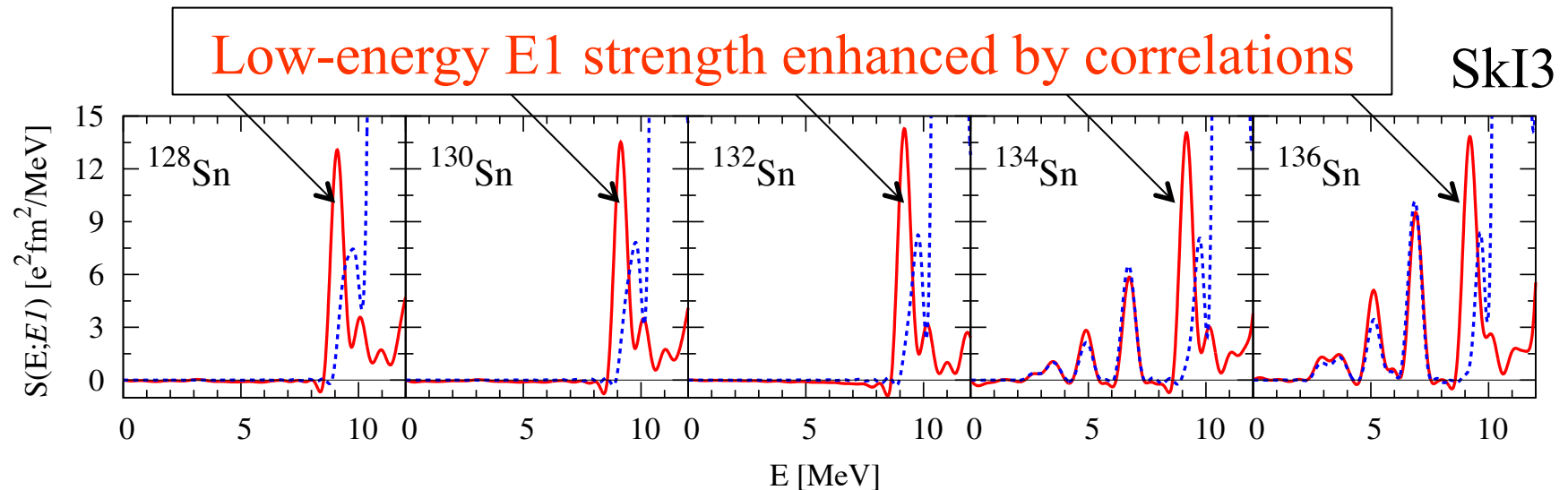
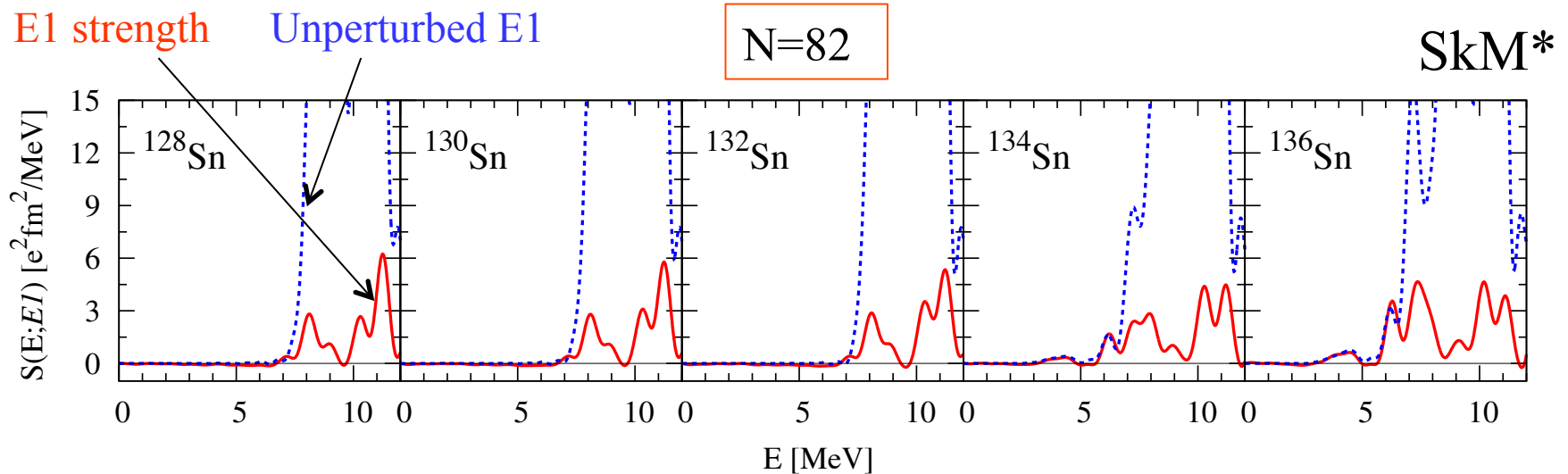
$$E_c = 10 \text{ MeV}$$



Model dependence & independence

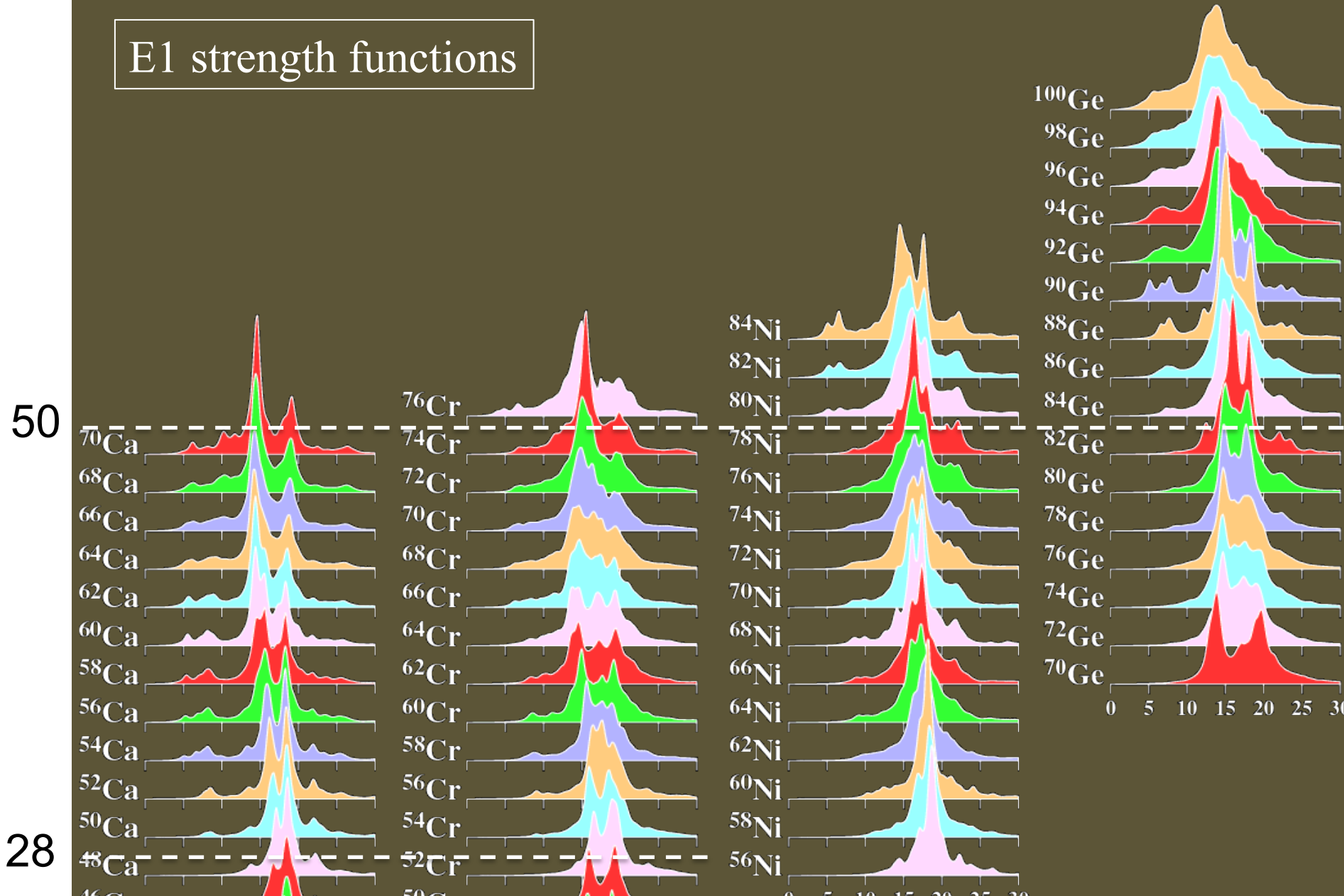


E1 hindrance or enhancement?

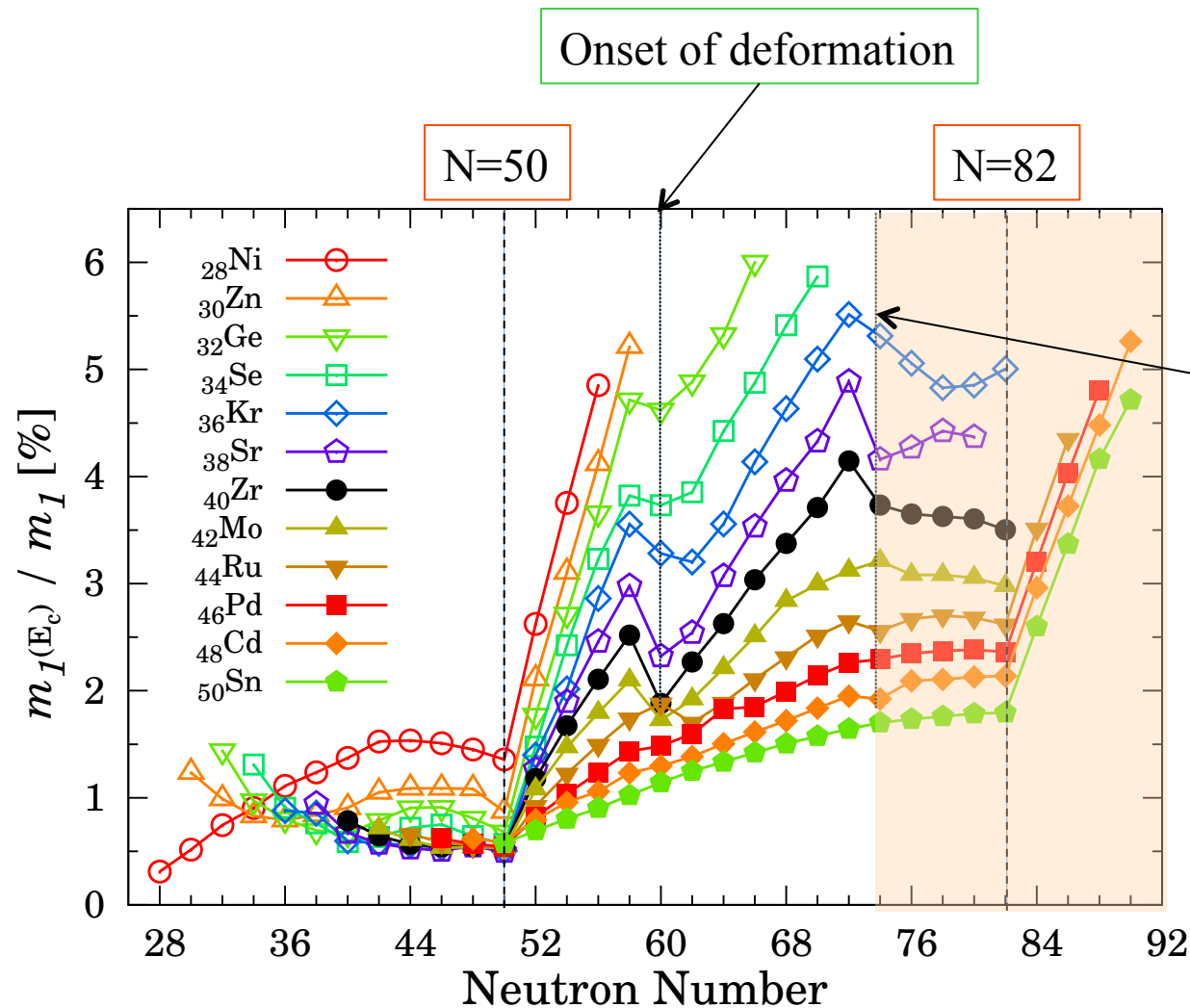


“Magic” numbers

E1 strength functions



Shell effect



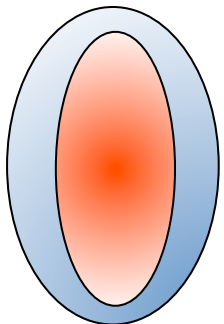
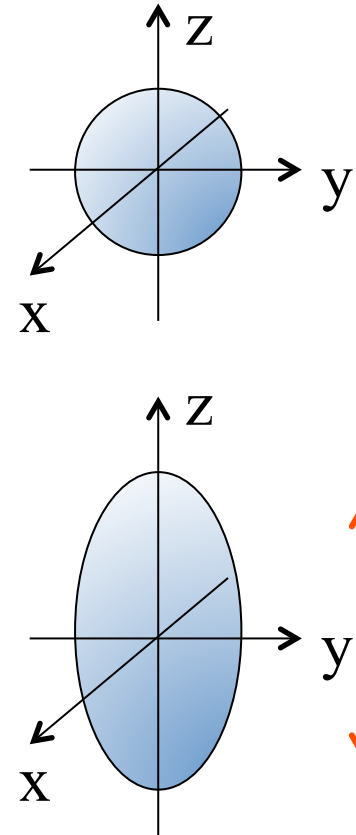
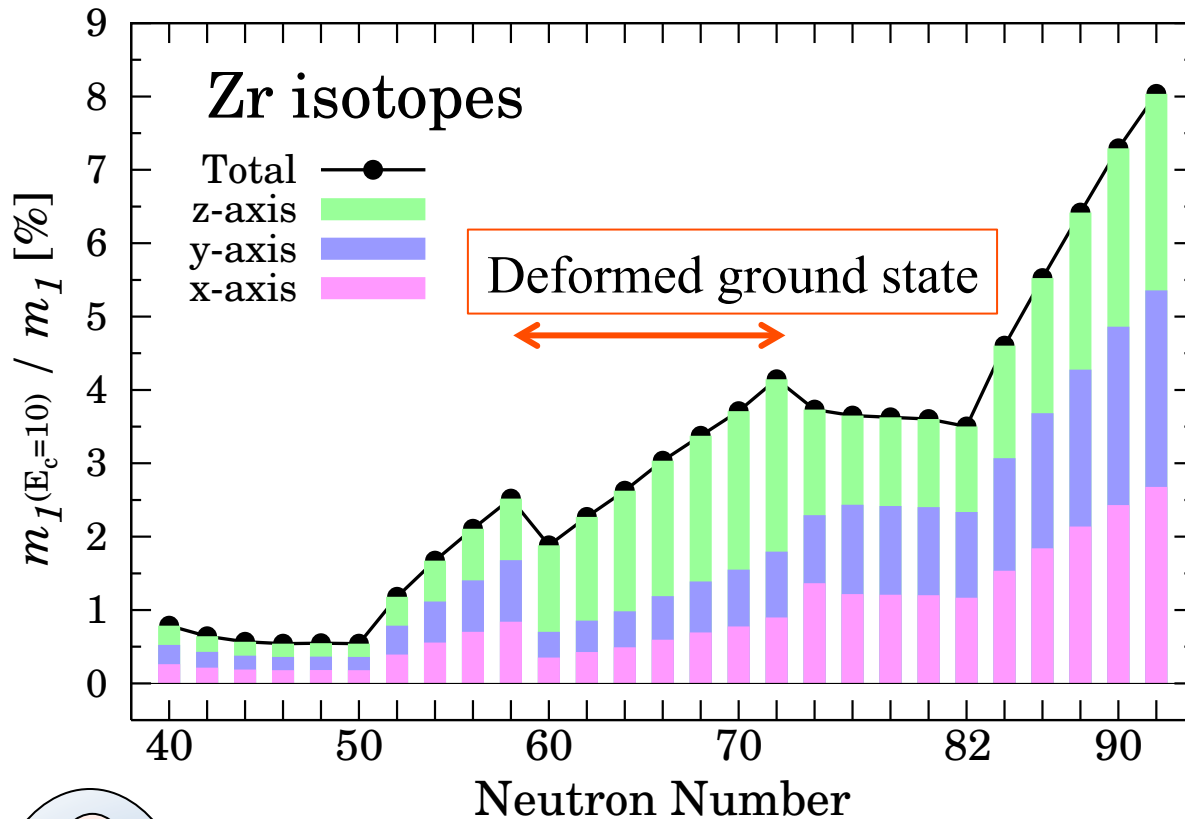
$$\frac{m_1(E_c)}{m_1} \equiv \frac{\int_0^{E_c} ES(E) dE}{\int_0^{\infty} ES(E) dE}$$

$$E_c = 10 \text{ MeV}$$

Back to spherical shape
(Occ. of $h_{11/2}$ orbitals)

Calculated with
SkM* functional

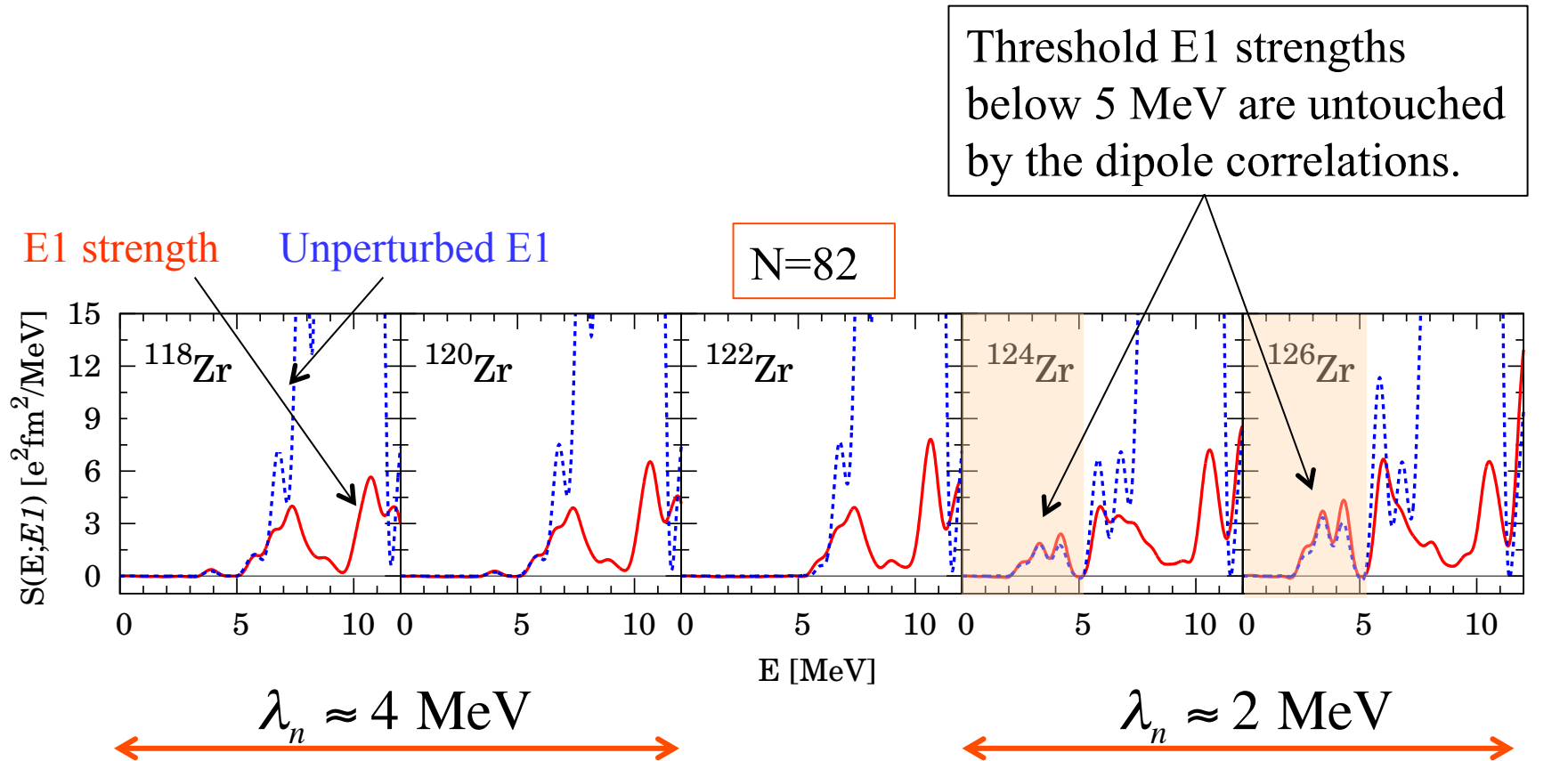
Deformation effect



The skin thickness is larger for (x,y) directions than z direction.
Inconsistent with the skin-mode picture?

Enhancement of the low-energy E1 strength along z axis

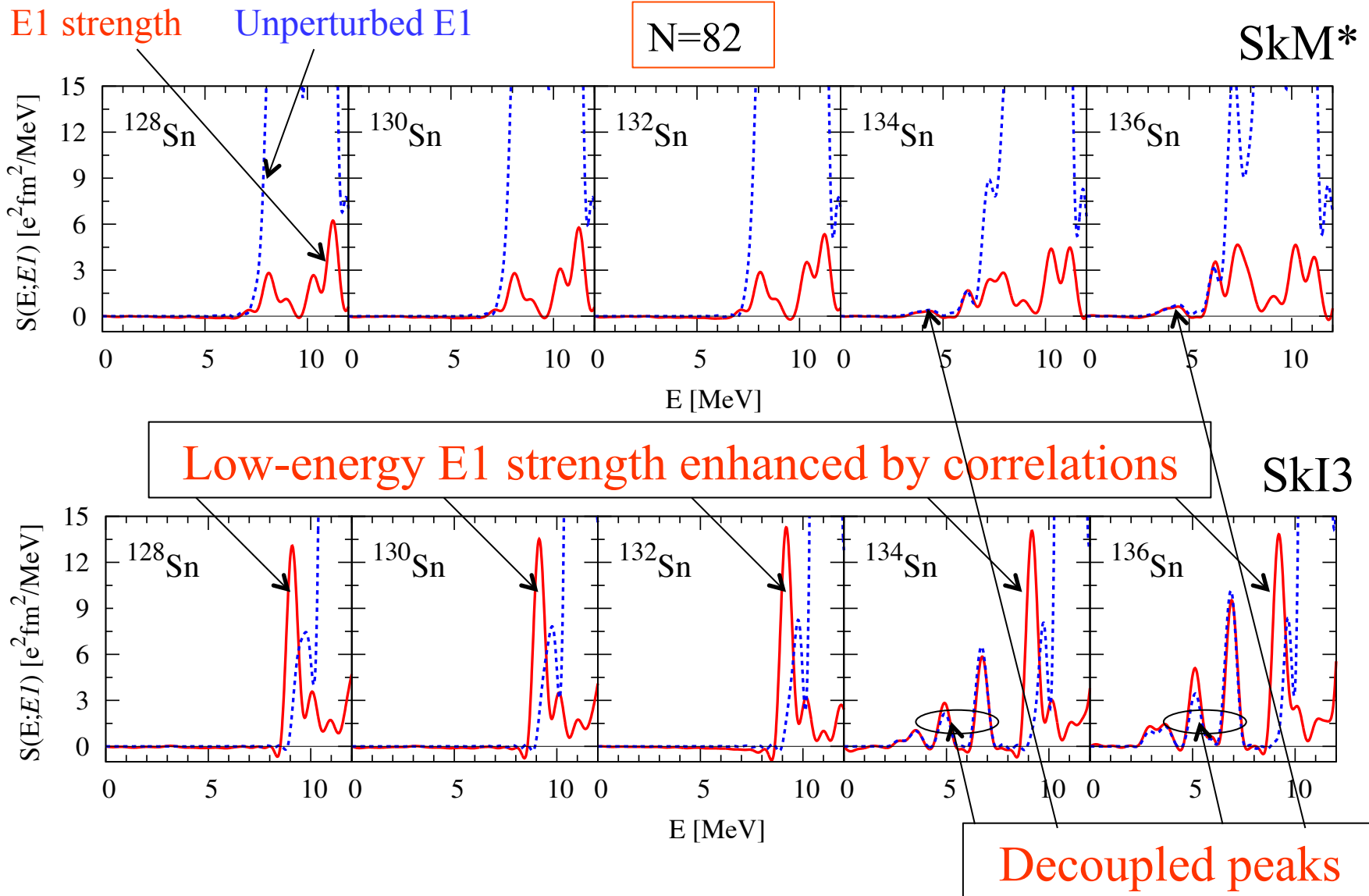
E1 Hindrance and decoupling



- Hindrance of low-energy E1
- “Destructive coherence”
- Strength goes to GDR

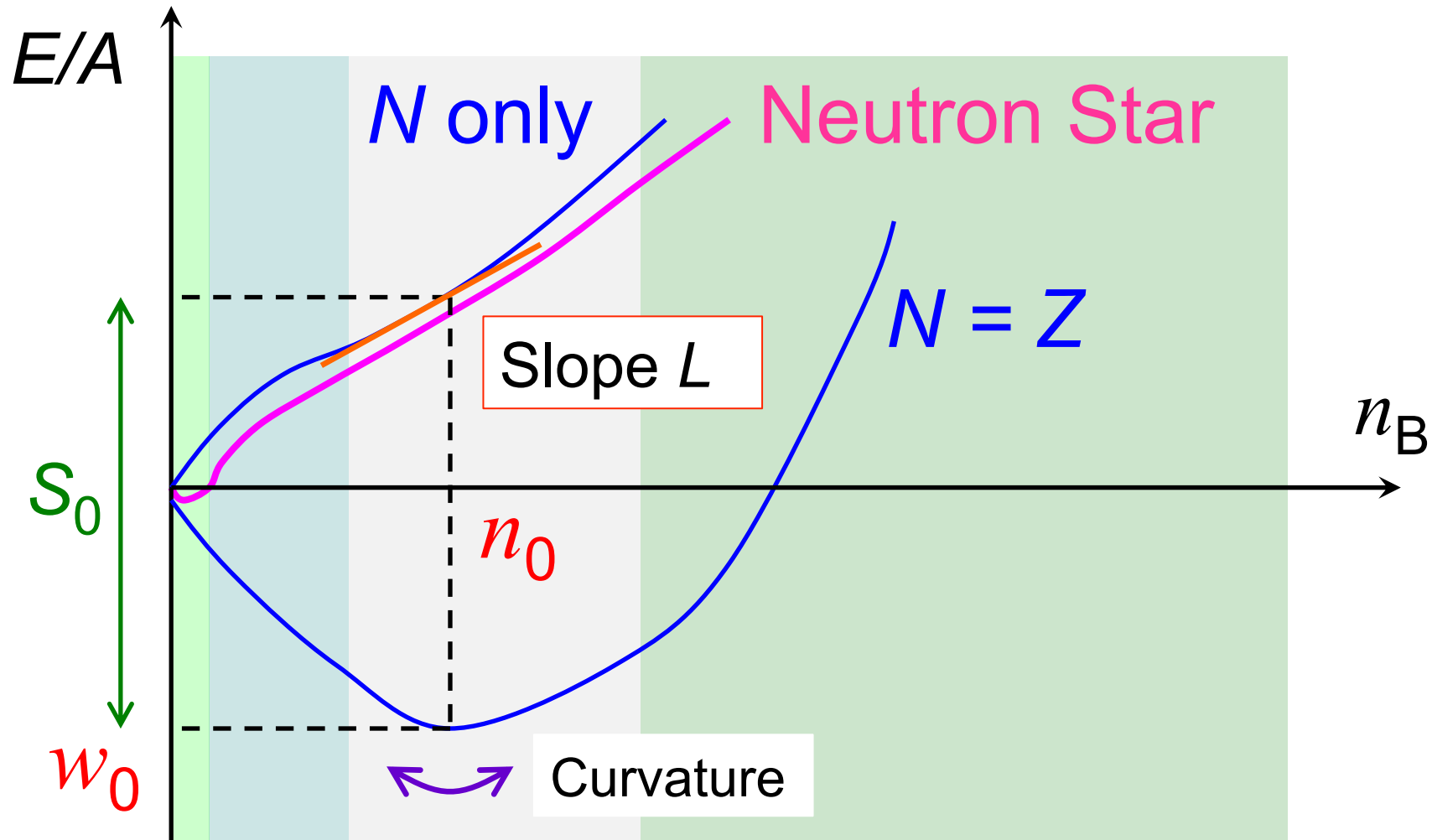
- Decoupling of low-lying dipole modes
- “Single-particle” excitations

Decoupling of low-E $E1$



Slope parameter of symmetry energy

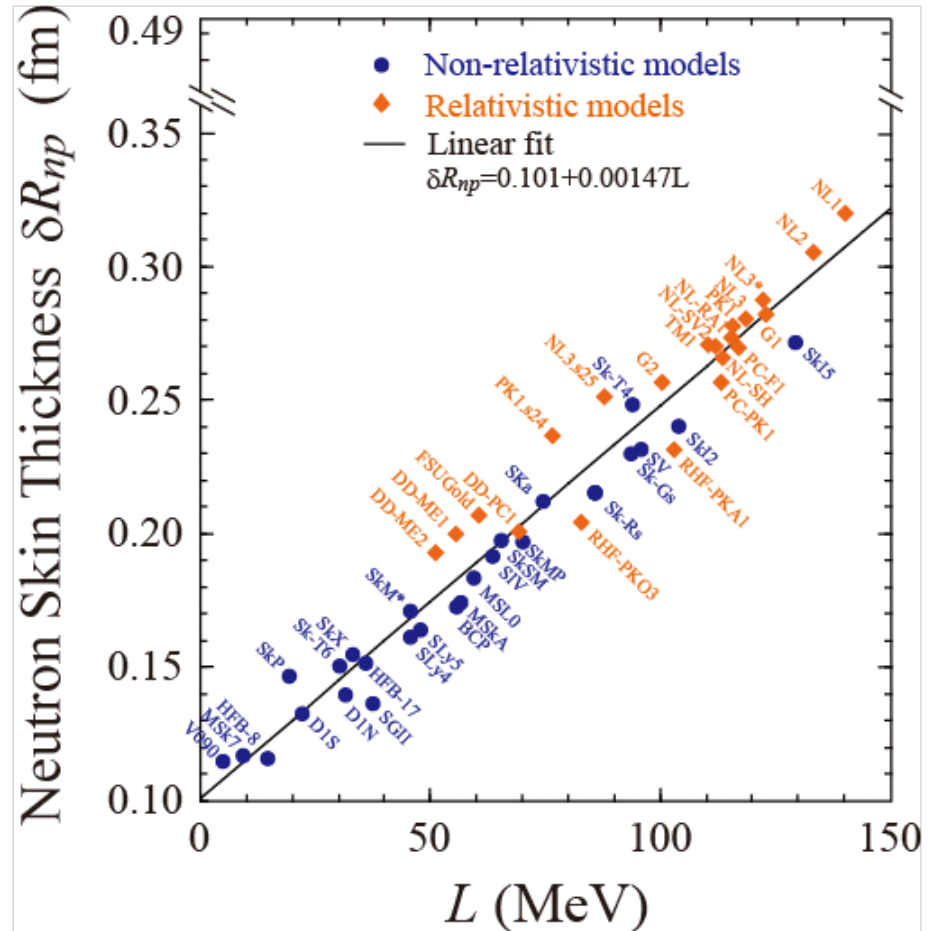
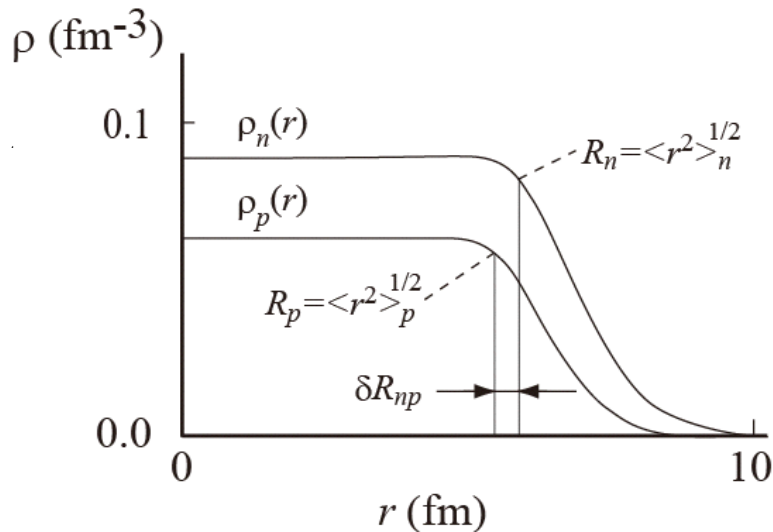
- Symmetry energy



Neutron skin thickness

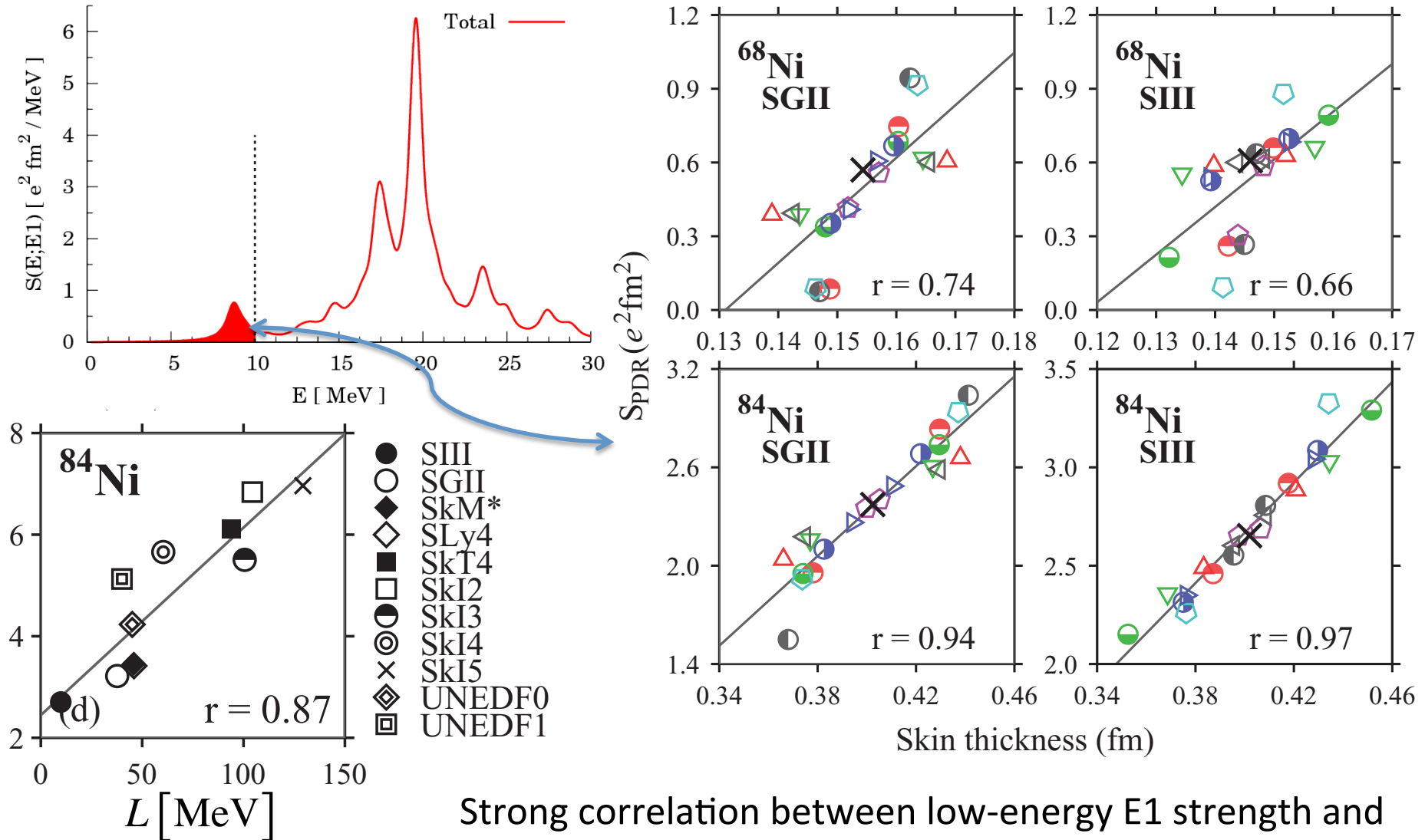
Roca-Maza et al., PRL106, 252501 (2011)

Neutron skin thickness
(Difference in radius between
neutrons and protons)



Selected isotopes

Inakura, Nakatsukasa, Yabana, PRC 88, 051305 (2013)



Strong correlation between low-energy E1 strength and skin thickness (slope parameter L) in selected nuclei

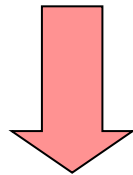
- Nuclear TDDFT
 - Calculation of response functions
 - Parallel computing
 - Orbital parallelization (real-time cal.)
 - Matrix element parallelization (QRPA cal.)
- Evolution of IS giant resonances
 - Peak splitting due to deformation
 - Prominent in GMR, “invisible” in GQR/GOR
 - $m^*/m = 0.8 \sim 0.9$; $K = 210 \sim 230$ MeV
- Pygmy dipole states in neutron-rich nuclei
 - Strong neutron shell effects
 - Decoupling from GDR
 - Skin thickness (L parameter) in select nuclei

Phonon condensation

$|\Phi_0\rangle$ Ground state

$B_{2^+}^+ |\Phi_0\rangle$ Excited state (2^+)

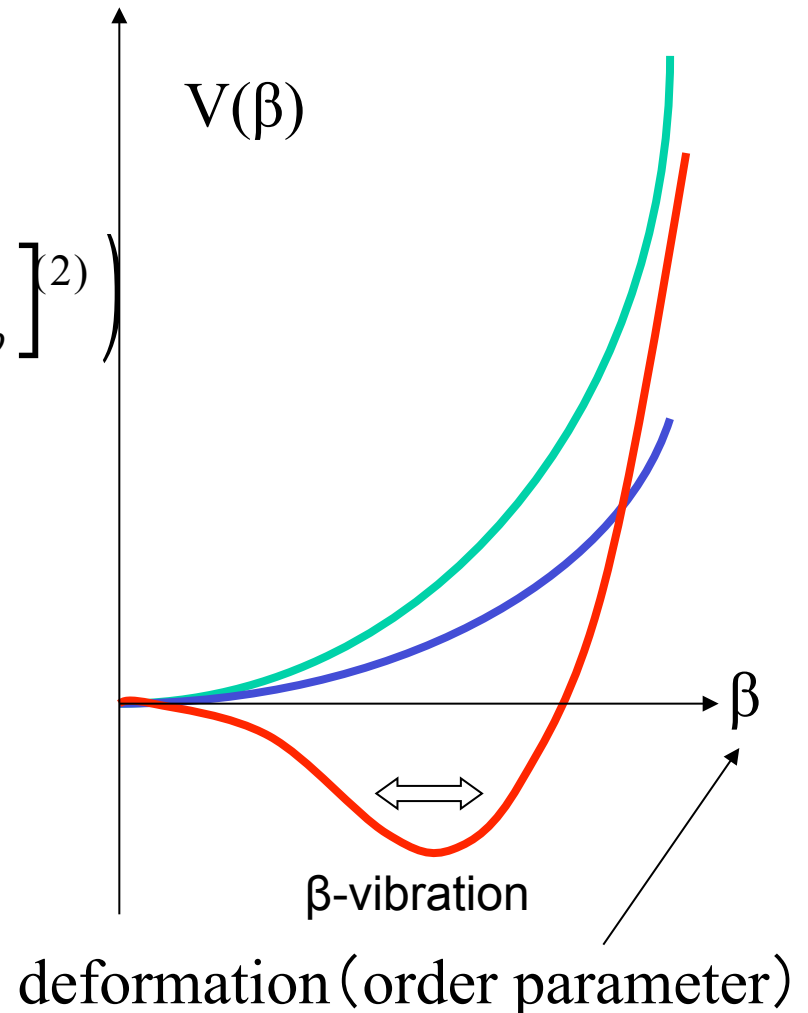
$$B_{2^+}^+ \approx \sum_{p,h} \left(X_{ph} [c_p^+ c_h]^{(2)} + Y_{ph} [c_h^+ c_p]^{(2)} \right)$$



Away from the closed-shell

$$|\Phi_{\text{def}}\rangle \approx \exp(c B_{2^+}^+) |\Phi_0\rangle$$

$$\beta \propto \langle \Phi_{\text{def}} | r^2 Y_{20}(\hat{r}) | \Phi_{\text{def}} \rangle$$

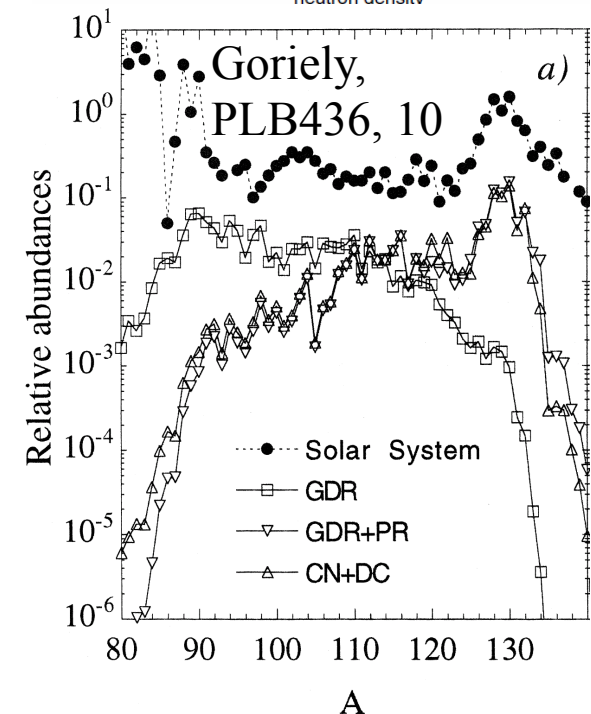
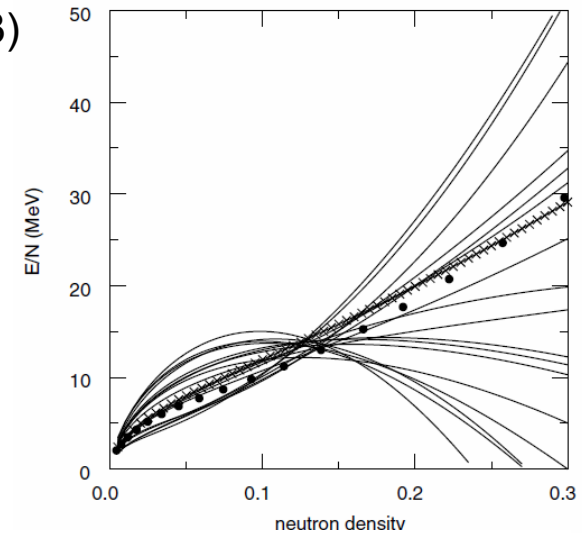


Low-energy $E1$ strength in exotic nuclei

Inakura, Nakatsukasa, Yabana, PRC 84, PRC 88, 051305(R) (2013)

Ebata, Nakatsukasa, Inakura, in preparation.

- Constrain the neutron skin thickness and the neutron matter EOS?
 - Yes, but better in very neutron rich!
 - Data on ^{84}Ni are better than ^{68}Ni
- Influence the r-process?
 - Significantly influence the direct neutron capture process near the neutron drip line
 - We need calculation with a proper treatment of the continuum.



Beyond the linear regime

Future subjects

- Nuclear reaction involving a large shape change, such as fission, fusion, etc.
- Problems
 - Numerical cost for solution of TDHFB eq.

$$i \frac{\partial}{\partial t} \begin{pmatrix} U_{\mu}(t) \\ V_{\mu}(t) \end{pmatrix} = \begin{pmatrix} h(t) - \lambda & \Delta(t) \\ -\Delta^*(t) & -(h(t) - \lambda)^* \end{pmatrix} \begin{pmatrix} U_{\mu}(t) \\ V_{\mu}(t) \end{pmatrix}$$

- How to obtain quantum spectra?

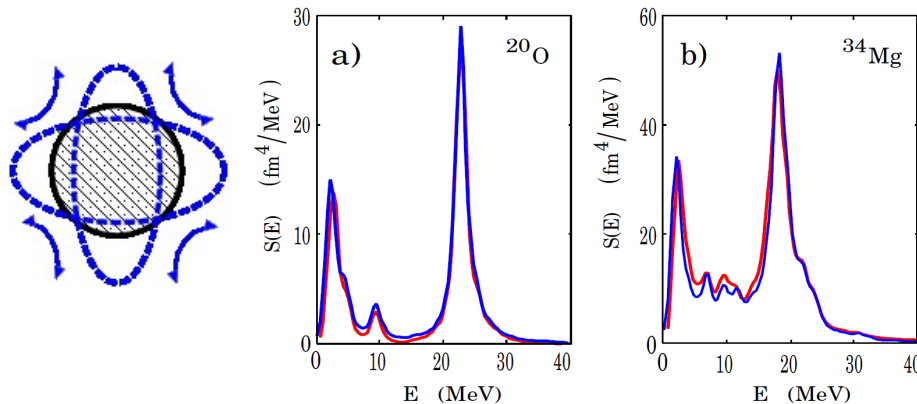
Time-dependent Hartree-Fock-Bogoliubov (TDHFB) calculations with Gogny interaction (Y. Hashimoto)

1. Aim:

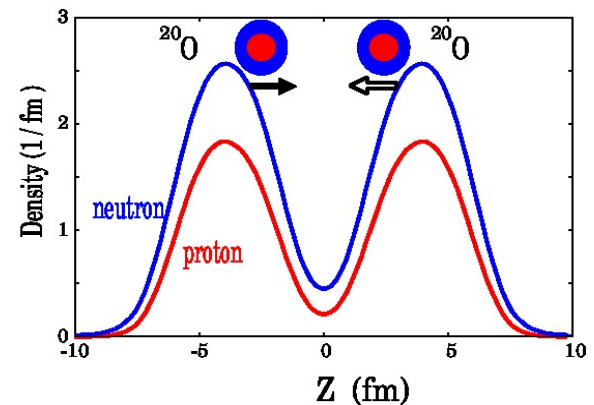
The aim of the TDHFB calculations is to understand the dynamical role of the pairing correlation in the large-amplitude collective motions including the reaction processes.

2. Method :

- i) A new method of carrying out the **Gogny-TDHFB calculations** was proposed with the three-dimensional harmonic oscillator basis (**3DHO**).
- ii) The program codes were extended to make use of the **spatial grids** (**Lagrange mesh**) instead of the 3DHO. At present, two-dimensional harmonic oscillator + one-dimensional Lagrange mesh (**2DHO+LM**) is used.

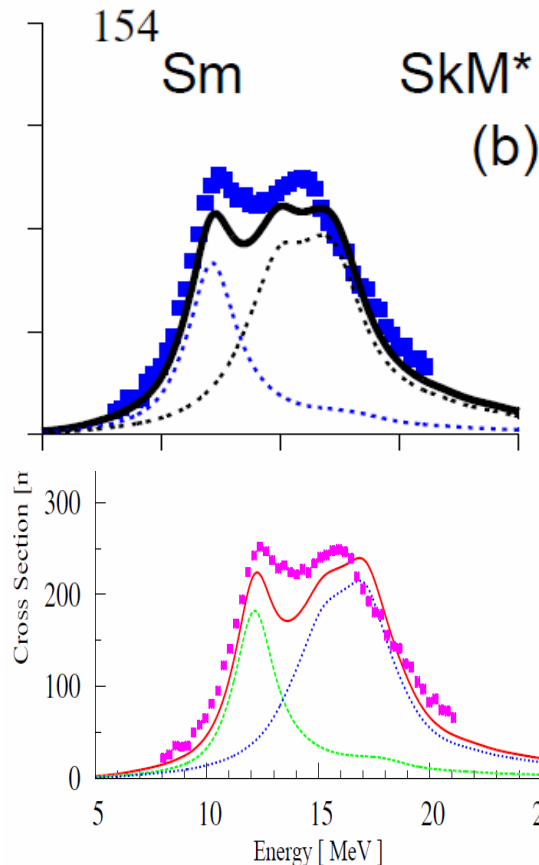


Strength functions of quadrupole vibrations in superconducting nuclei ^{20}O and ^{34}Mg .



Colliding superconducting nuclei $^{20}\text{O} + ^{20}\text{O}$.

Synopsis: Time-saving steps



$$i \frac{\partial}{\partial t} |k(t)\rangle = (h(t) - \eta_k(t)) |k(t)\rangle$$

$$i \frac{\partial}{\partial t} \rho_k(t) = \Delta_k^*(t) K_k(t) - \text{c.c.}$$

$$i \frac{\partial}{\partial t} K_k(t) = (\eta_k(t) + \eta_{\bar{k}}(t)) K_k(t) + \Delta_k(t) (2\rho_k(t) - 1)$$

Canonical-basis time-dependent Hartree-Fock-Bogoliubov theory and linear-response calculations

Shuichiro Ebata, Takashi Nakatsukasa, Tsunenori Inakura, Kenichi Yoshida, Yukio Hashimoto, and Kazuhiro Yabana

Phys. Rev. C **82**, 034306 (2010)

Published September 9, 2010

2D cal. w/o Coulomb
~1000 CPU h



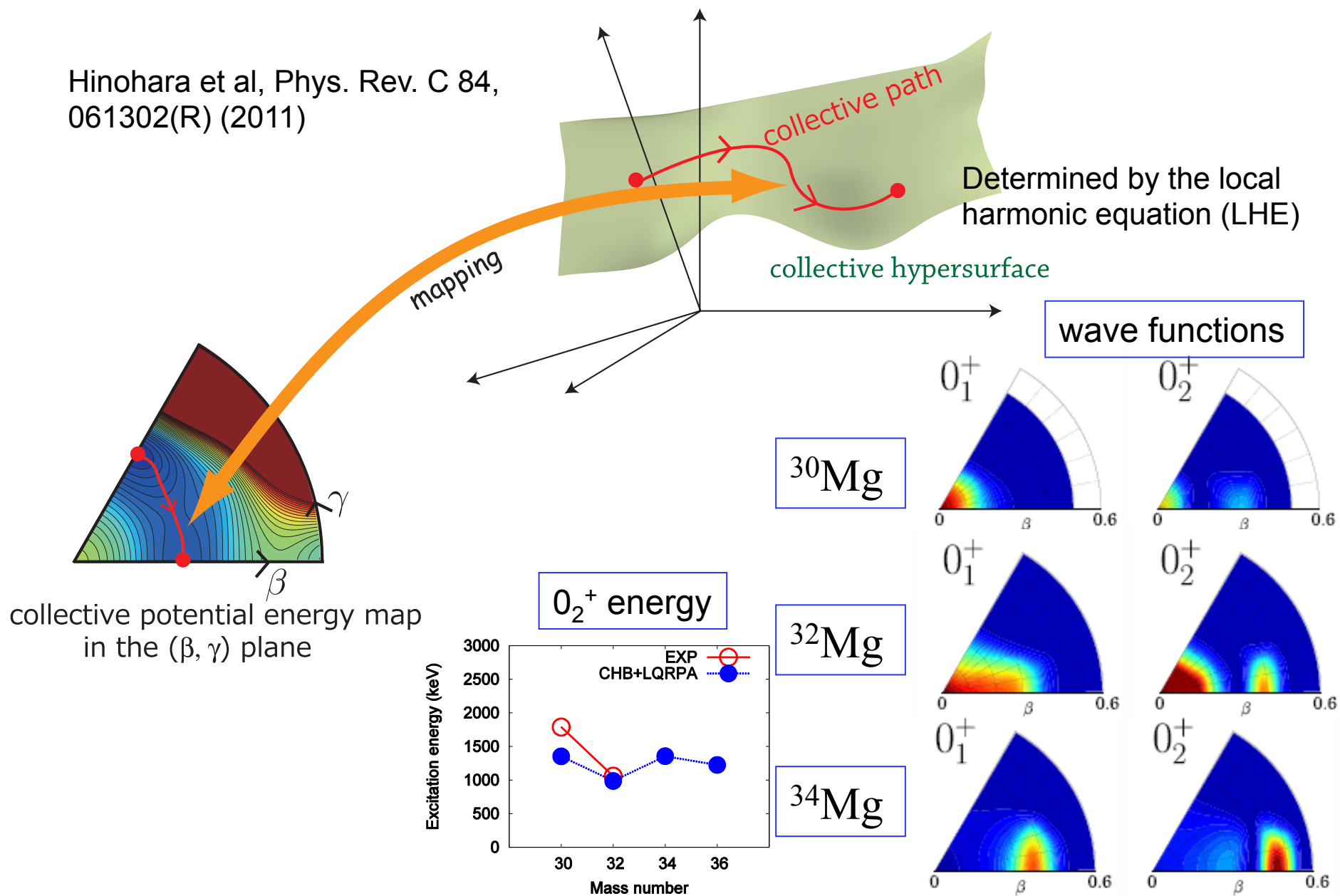
3D cal. w Coulomb
~ 20 CPU hours

Canonical-basis real-time method may significantly reduces computational task.

Applicable to nuclear dynamics beyond the liner regime: fusion and fission reactions.

Collective subspace

Hinohara et al, Phys. Rev. C 84,
061302(R) (2011)



Next stage: Applications to fission problems

- Constrained Hamiltonian

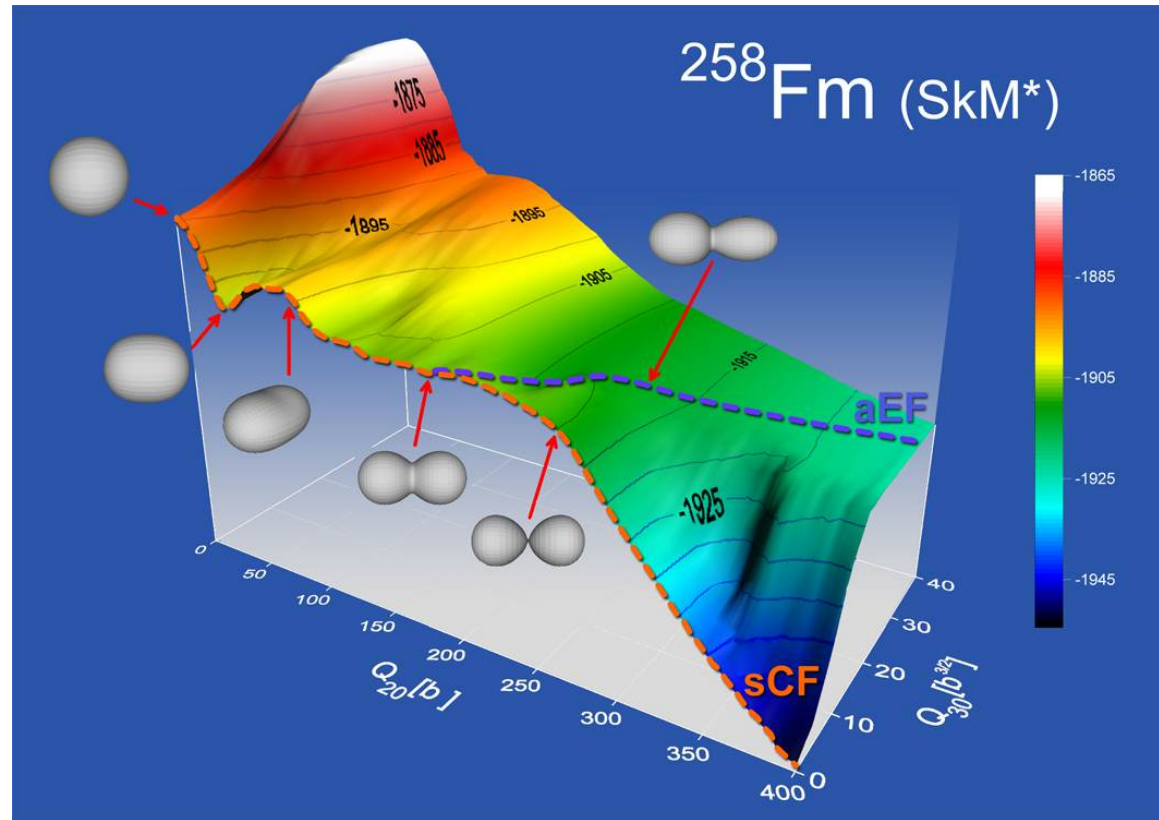
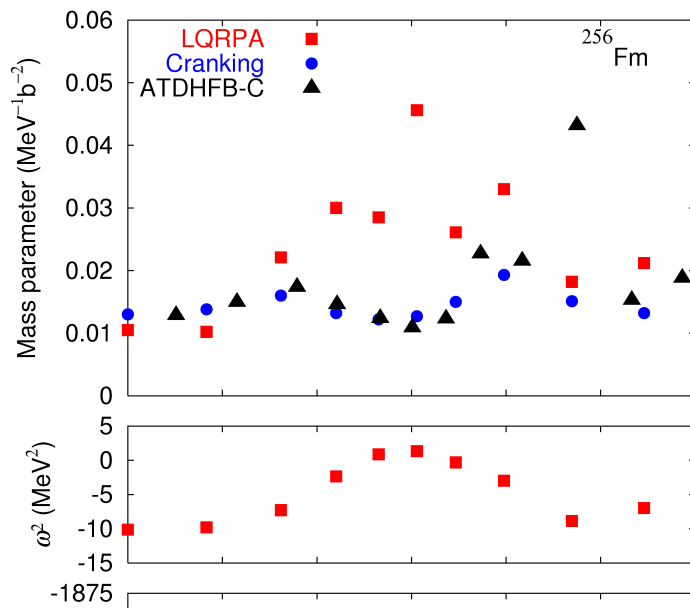
$$H = H - \lambda q$$

- Constrained operator

q Solution of LHE

- Collective mass parameters

Test calculation

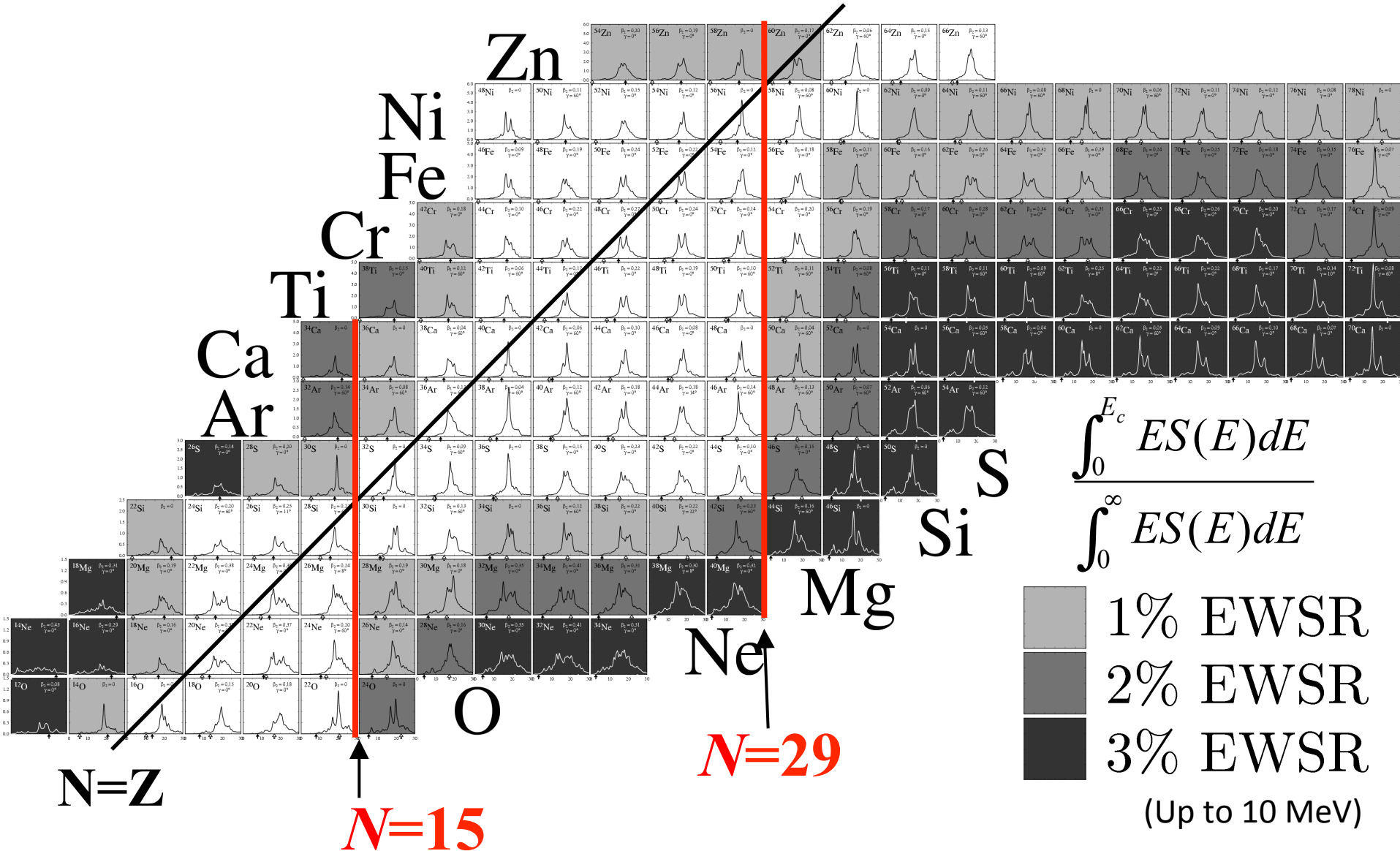


Local Harmonic Equation (LHE) is able to calculate the collective mass parameters including dynamical effect by the time-odd mean fields.

Magic numbers for low-energy E1 strength

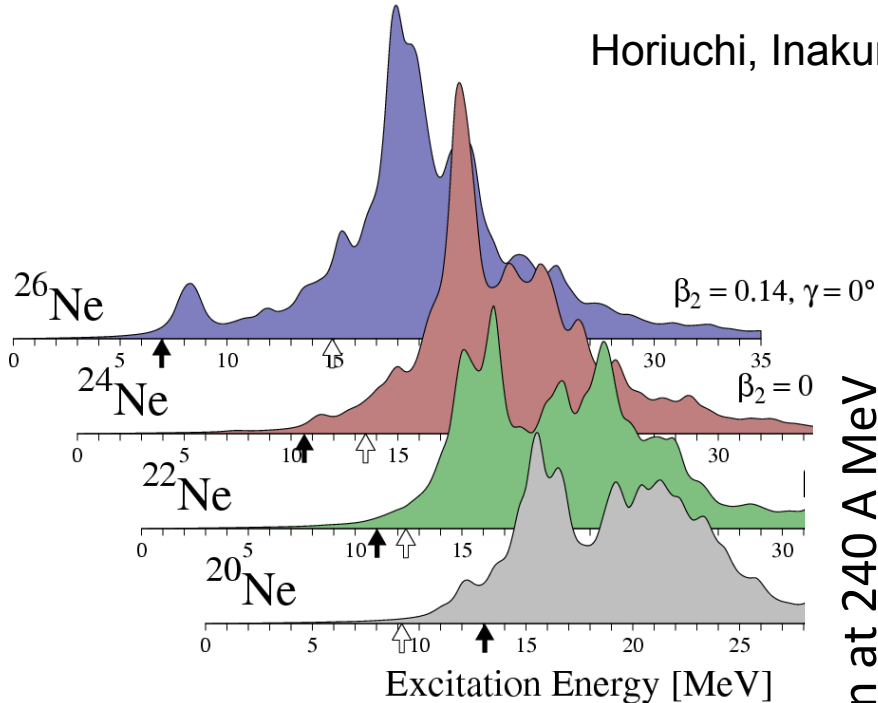
Pygmy dipole resonance (PDR)

Cal. by Inakura



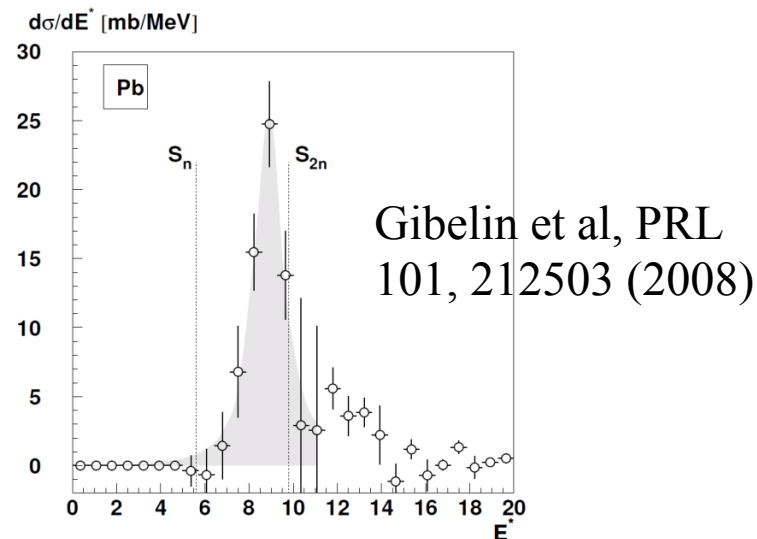
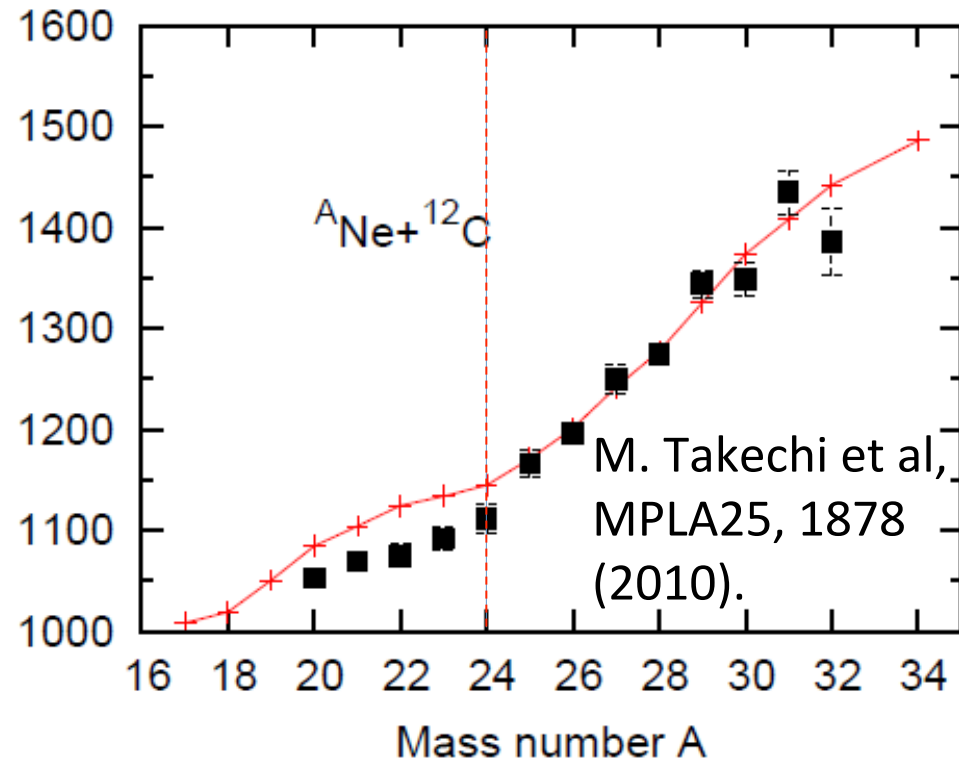
Development of neutron radius

Horiuchi, Inakura, Nakatsukasa, Suzuki, PRC 86, 024614 (2012)



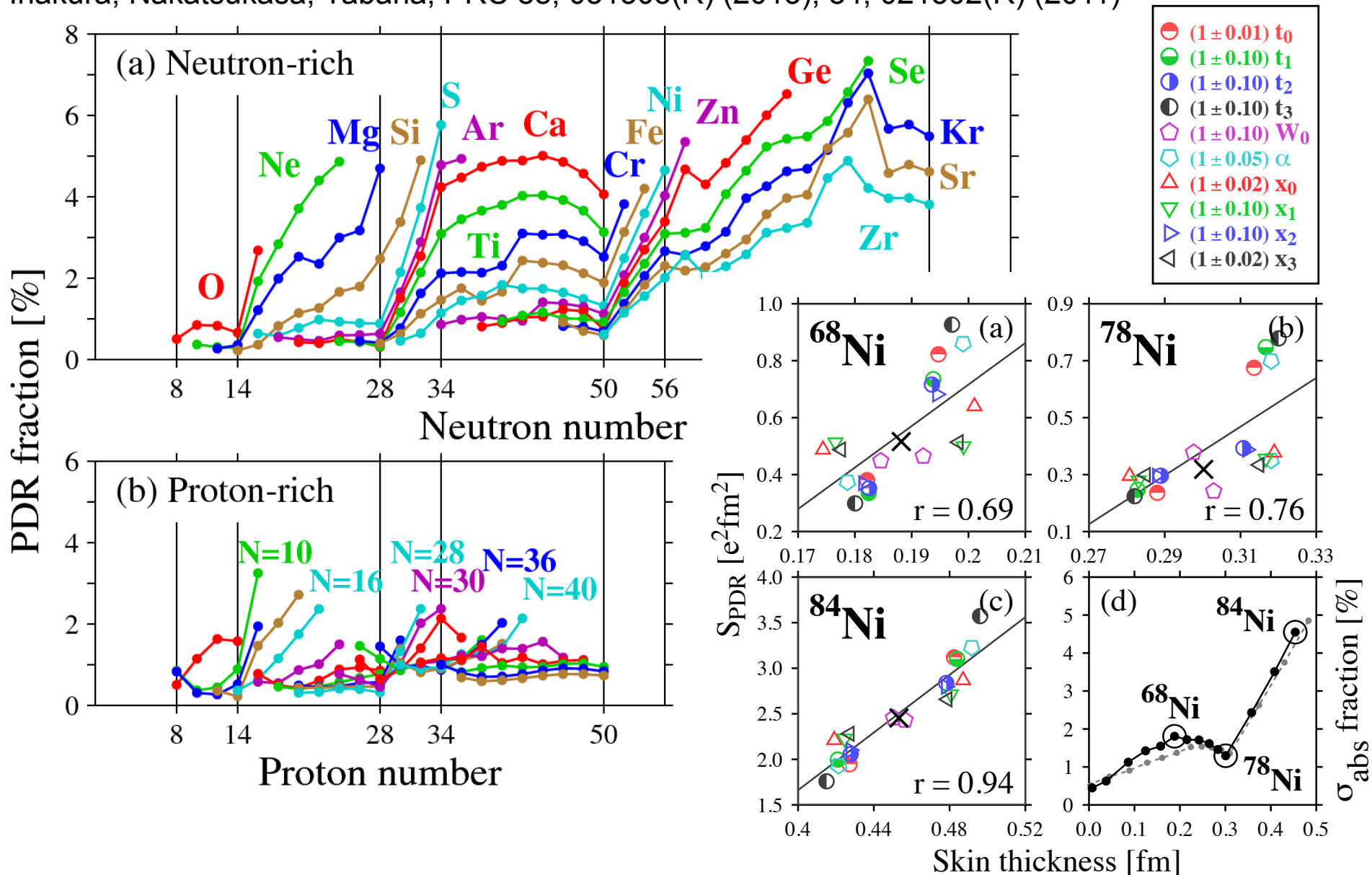
Glauber calculation using the density distribution obtained with the DFT.

Reaction cross section at 240 A MeV



Pygmy dipoles & neutron skins

Inakura, Nakatsukasa, Yabana, PRC 88, 051305(R) (2013); 84, 021302(R) (2011)

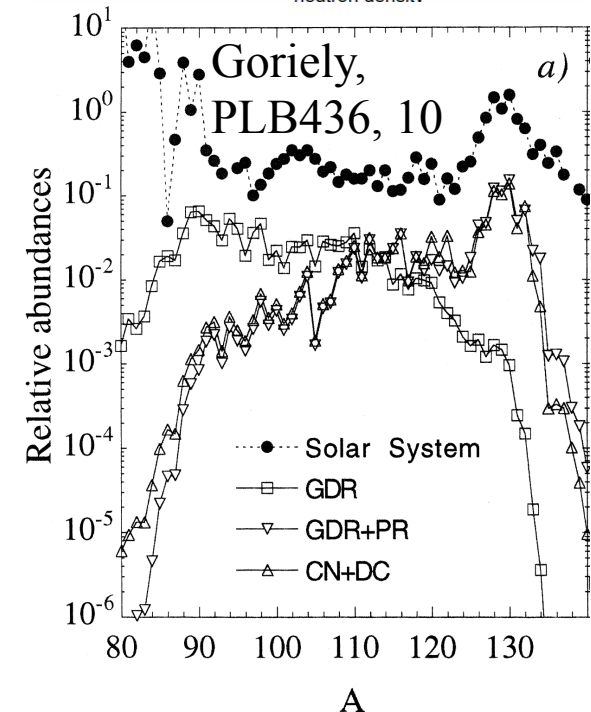
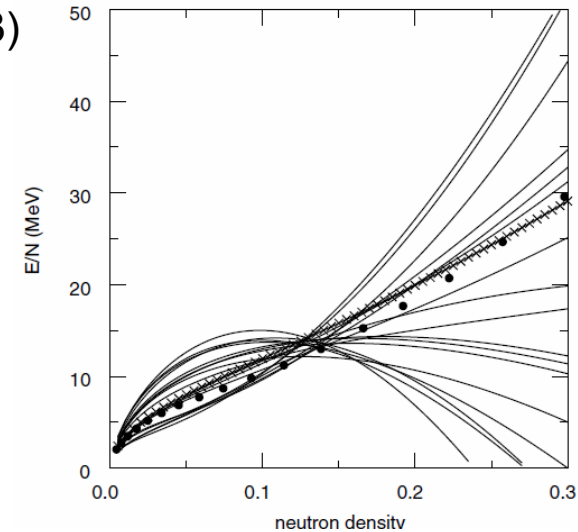


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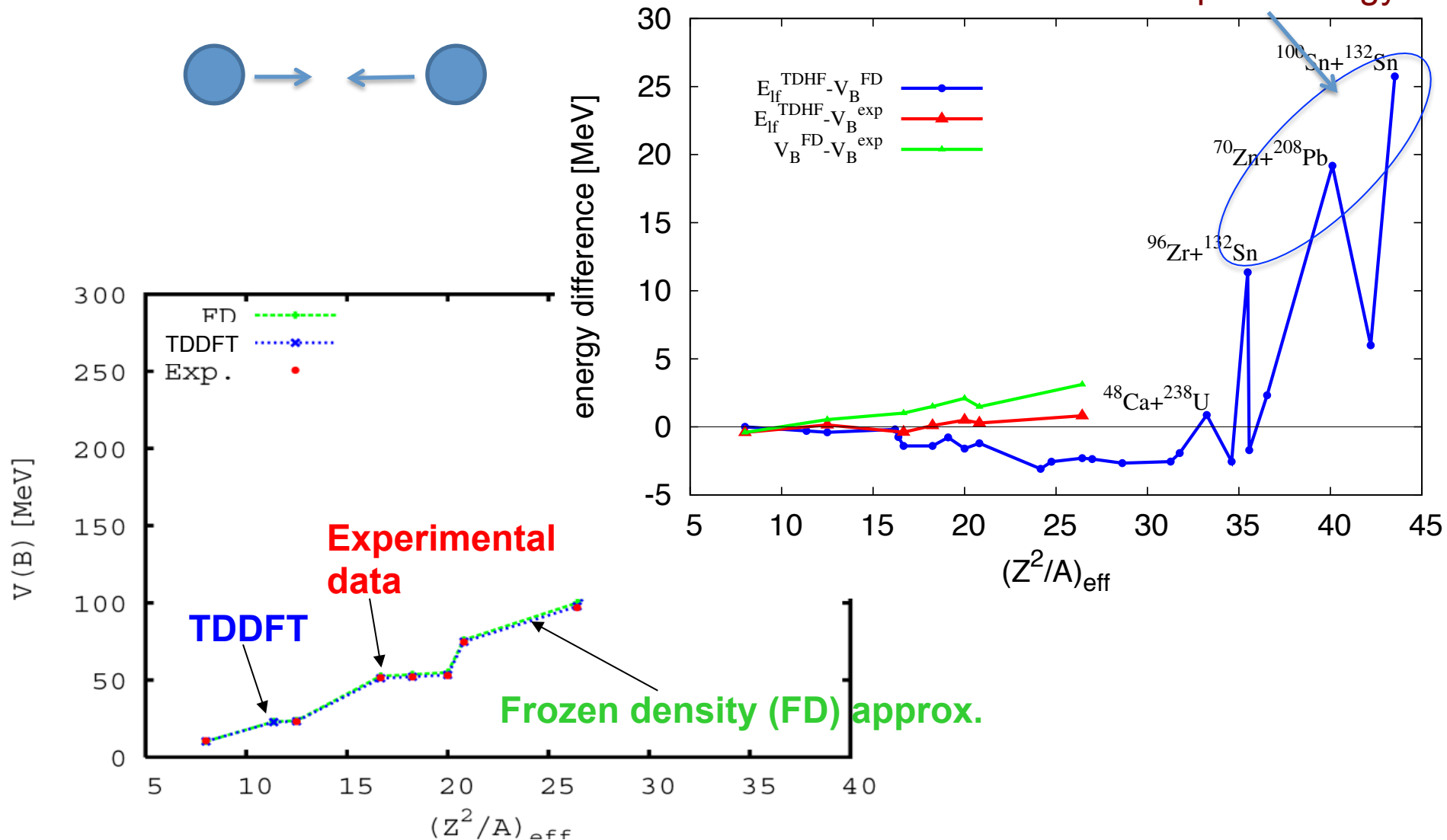
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 - We need calculation with a proper treatment of the continuum.



TDDFT simulation of nuclear fusion reaction

Guo, Nakatsukasa, EPJWC 38, 09003 (2012)

Fusion barrier threshold



Computational efforts

- Ground state: 2D solution of BdGKS equations
 - Use of 64 processors in parallel (MPI)
- Linear response and E1 strength functions

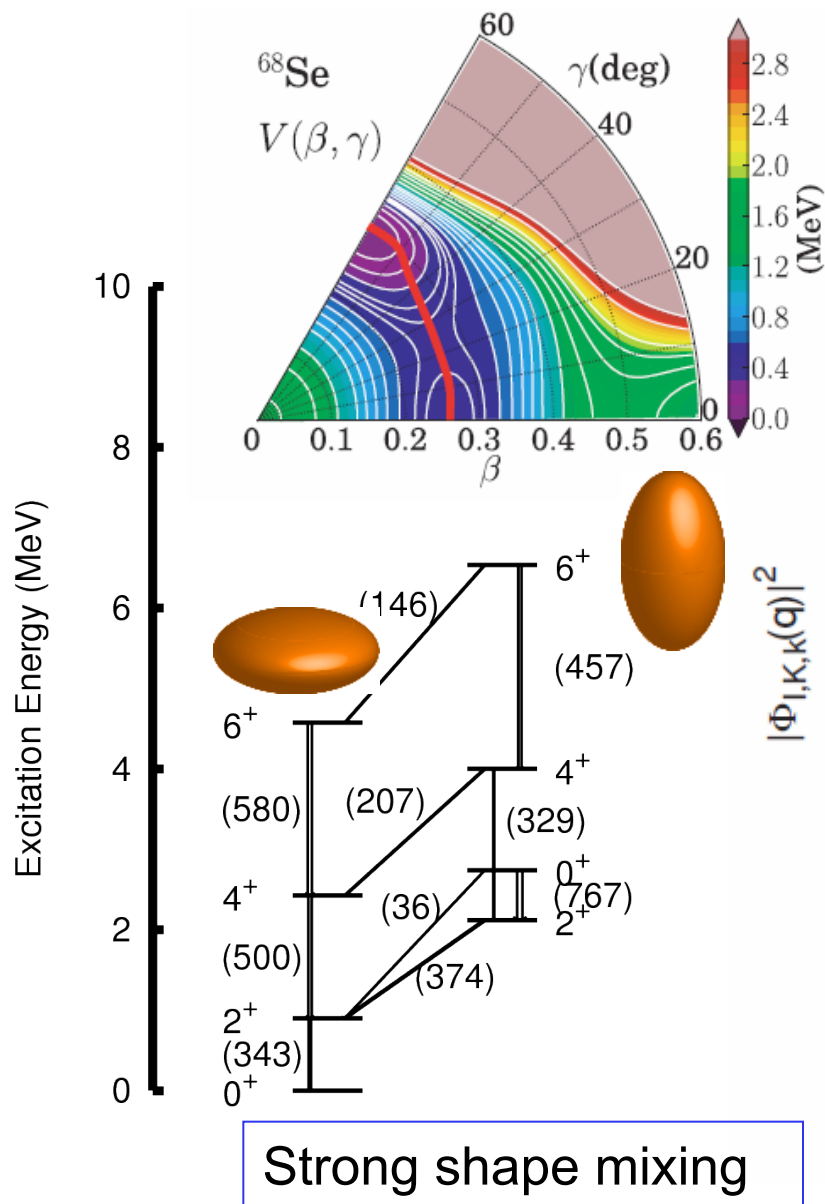
$$\left\{ \begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix} - \omega \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right\} \begin{pmatrix} X_{mi}(\omega) \\ Y_{mi}(\omega) \end{pmatrix} = 0$$

- Use of 500-1000 processors in parallel (MPI)
- Use of BLACS (Basic Linear Algebra Communication Subprograms) for load balancing and ScaLAPACK for matrix diagonalization
- Truncate the space into about 50,000 2qp excitations
- Neglect the residual Coulomb terms
- Several hours to obtain the total strengths

Including residual Coulomb in a larger model space (canonical basis), the project is currently running on use of “K” computer. [4.4 M node*h]

Low-lying spectra in ^{68}Se

Hinojara, et al



Rotation-induced localization

