Recent results obtained with JISP16

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- A. Mazur, I. Mazur, E. Mazur (Pacific National University, Khabarovsk)
Plan

- Motivation
- Structure of JISP interaction
- Bindings and spectra of light nuclei with JISP16
- Nuclear matter
- Future prospect
**NN interaction**

- Modern history of realistic NN interaction starts from 1993: Nijmegen NN database and phase shift analysis

- After 1993 various NN interactions describing NN data with $\chi^2/datum \approx 1$ have been suggested, in particular:
  
  - **Meson exchange**: Nijmegen I, II; Reid soft core; Argonne AV$_{18}$; CD-Bonn$_{2000}$; INOY (inside non-local, outside Yukawa)
  
  - **Chiral EFT**: N2LO(next-to-next-to-leading-order), N3LO

- **Inverse scattering**: JISP6, JISP16, JISP16$_{2010}$

  ($\chi^2/datum > 2$ for pre-1993 NN interactions)
Ideal $NN$ interaction from nuclear theorist viewpoint

- Derived from QCD without model approximations and assumptions
- Perfectly describing $NN$ data at low energies
- Perfectly describing bindings, spectra and other observables in light nuclei
- Describing heavier nuclei, nuclear matter, etc.
- Describing other experimental data ($p+d$ scattering, etc.)
- Providing fast convergence of shell model and other calculations
- No need of $NNN$ forces
Ideal $NN$ interaction from nuclear theorist viewpoint

* Derived from QCD without model approximations and assumptions
* Perfectly describing $pn$ data at low energies
* Reasonably describing bindings, spectra and other observables in light nuclei
* Describing heavier nuclei, nuclear matter, etc.
* Describing other experimental data ($p+d$ scattering, etc.)
* Providing fast convergence of shell model and other calculations
* No need of $NNN$ forces
* JISP16
**NNN force**

**NNN** contribution to nuclear observables is small but can be essential.

4\(N\) and higher forces are usually supposed to be inessential for description of nuclei.
Why would be nice to avoid NNN forces?

Approximately 30 times more non-zero Hamiltonian matrix elements when NNN forces are involved; hence much more computer resources are required for calculations.
Role of NNN force?

* W. Polyzou and W. Glöckle theorem (Few-body Syst. 9, 97 (1990)):
  \[ H = T + V_{ij} \Rightarrow H' = T + V'_{ij} + V_{ijk} \]
  where \( V_{ij} \) and \( V'_{ij} \) are phase-equivalent, \( H \) and \( H' \) are isospectral.
  Hope:
  \[ H' = T + V'_{ij} + V_{ijk} \Rightarrow H = T + V_{ij} \]
  with (approximately) isospectral \( H \) and \( H' \).

JISP type interaction seems to be NN interaction minimizing NNN force.

Without NNN force calculations are simpler, calculations are faster, larger model spaces become available; hence predictions are more reliable.
Peter Sauer: ```NNN force is only a baby of theoreticians who would like to work in a restricted Hilbert space’’, i.e., avoiding Δisobar and other excited nucleon degrees of freedom.

From this point of view, JISP NN interaction is an attempt to describe nuclei with nucleon degrees of freedom only.
Modern $NN$ interactions: need of $NNN$

* **Meson exchange**: Nijmegen I, II; Reid soft core; Argonne AV$_{18}$; CD-Bonn$_{2000}$: require $NNN$ potentials (U, IL, TM, ...) which are usually inconsistent with $NN$ interaction

* **Chiral EFT**: N3LO requires $NNN$ potential – N2LO at the moment, consistent with $NN$ at the N2LO level

* **Inverse scattering**: JISP6, JISP16, JISP16$_{2010}$: no need of $NNN$; fitted to light nuclei

+ INOY (inside non-local, outside Yukawa)
**NN interaction: convergence**

* To improve convergence of *ab initio* nuclear structure, usually (exception: GFMC) an effective interaction based on intrinsic NN (and, generally, NNN) interaction is constructed.

* Modern approaches to eff. interaction:

  * **Lee–Suzuki–Okamoto (LSO):** popular up to 2010, less popular now. Idea is to reproduce in a given small model space the results in the infinite model space (of course, approximately). New interaction for each model space. No variational principle, non-monotonic convergence in many-body nuclei, no way to extrapolate results to infinite model space. Induced NNN to improve convergence.

  * **Similarity renormalization group (SRG):** a modern trend. Idea is to reduce matrix elements coupling low- and high-momentum components of interaction by unitary transformation. The variational principle works, results can be extrapolated. Induced NNN to restore `bare` interaction results in many-body nuclei.
Modern NN interactions

- **Meson exchange NN (Argonne AV$_{18}$; CD-Bonn$_{2000}$; INOY, ...):** high-quality description of NN data: $\chi^2$/datum ≈ 1 up to $E_{\text{lab}} = 350$ MeV. Somewhat phenomenological: no ties to QCD, phenomenological terms, inconsistent parameters. Should be combined with (usually inconsistent)(semi)phenomenological NNN (IL, U, TM, ...). Bad convergence, eff. interaction needed. *(R. Machleidt: "If you want more more accuracy, you have to use less theory")*

- **Chiral EFT NN (N3LO):** A modern trend. Less accurate (at the moment) description of NN data: np:$\chi^2$/datum = 1.10 up to $E_{\text{lab}} = 290$ MeV; pp:$\chi^2$/datum = 1.50 up to $E_{\text{lab}} = 290$ MeV. Tied to QCD through expansion in $p/p_\chi$, $p_\chi$ is a chiral symmetry breaking momentum. Should be combined with Chiral EFT NNN (N2LO now). Bad convergence, eff. interaction needed. *(R. Machleidt: "If you want more more accuracy, you have to use more theory")*

- **Inverse scattering NN (JISP16, JISP16$_{2010}$):** high-quality description of np data: $\chi^2$/datum ≈ 1 up to $E_{\text{lab}} = 350$ MeV. Completely phenomenological. No need of NNN
Construction of JISP

NN interaction

- JISP = $J$-matrix inverse scattering potential
J-matrix formalism: scattering in the oscillator basis

Oscillator basis, truncated potential energy matrix \( V \) and non-truncated complete infinite kinetic energy matrix \( T \).

Justification: kinetic energy m. e. increase with \( n \) linearly at large \( n \): 
\[
T_{nn} \sim n, T_{n,n+1} \sim n, n \to \infty
\]
while potential energy m. e. \( V_{nn} \) decrease with \( n \) and \( m \).

Both direct and inverse scattering \( J \)-matrix solutions are possible.

\[
\sum_{n' = 0}^{N} H_{nn'}^l \langle n' \mid \lambda \rangle = E_\lambda \langle n \mid \lambda \rangle, \quad n \leq N.
\]

\[
\mathcal{G}_{NN}(E) = -\sum_{\lambda = 0}^{N} \frac{\langle N \mid \lambda \rangle^2}{E_\lambda - E},
\]

\[
S = \frac{C_{N1}^{(-)}(q)}{C_{N1}^{(+)}(q)} - \mathcal{G}_{NN}(E) T_{N,N+1}^l C_{N+1,l}^{(-)}(q) \frac{C_{N1}^{(-)}(q)}{C_{N1}^{(+)}(q)} - \mathcal{G}_{NN}(E) T_{N,N+1}^l C_{N+1,l}^{(+)}(q).
\]
The JISP NN interaction

- NN interaction is a small matrix of the in the oscillator basis with $\hbar \Omega = 40$ MeV:

  $9\hbar \Omega$ truncation, i.e. in each partial wave oscillator quanta $2n+l \leq 9$:
  - 5×5 matrix in $s$ ($l=0$) and $p$ ($l=1$) waves;
  - 4×4 matrix in $d$ ($l=2$) and $f$ ($l=3$) waves; etc.; in coupled waves dimensionalities are summed, e.g., 9×9 matrix in coupled $sd$ waves, etc.

- This structure provides a good description of NN data and fast convergence of shell model calculations.
JISP16 properties

- 1992 np data base (2514 data): $\chi^2$/datum = 1.03
- 1999 np data base (3058 data): $\chi^2$/datum = 1.05

<table>
<thead>
<tr>
<th>Potential</th>
<th>$E_d$, MeV</th>
<th>$d$ state probability, %</th>
<th>rms radius, fm</th>
<th>$Q$, fm$^2$</th>
<th>As. norm. const. $A_s$, fm$^{-1/2}$</th>
<th>$\eta = \frac{A_d}{A_s}$</th>
</tr>
</thead>
<tbody>
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<td>JISP16</td>
<td>-2.224575</td>
<td>4.1360</td>
<td>1.9643</td>
<td>0.2886</td>
<td>0.8629</td>
<td>0.0252</td>
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<td>5.635</td>
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<td>5.76</td>
<td>1.967</td>
<td>0.270</td>
<td>0.8850</td>
<td>0.0250</td>
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<td>CD–Bonn</td>
<td>-2.224575</td>
<td>4.85</td>
<td>1.966</td>
<td>0.270</td>
<td>0.8846</td>
<td>0.0256</td>
</tr>
<tr>
<td>Nature</td>
<td>-2.224575(9)</td>
<td>—</td>
<td>1.971(6)</td>
<td>0.2859(3)</td>
<td>0.8846(9)</td>
<td>0.0256(4)</td>
</tr>
</tbody>
</table>
Phase-equivalent transformations (PETs)

PETs are generated by unitary transformations of the two-nucleon Hamiltonian

\[ H' = UHU^+ \]
Ambiguity of JISP interaction

* Any unitary transformation of $NN$ Hamiltonian $H$ generates a phase-equivalent transformation (PET). Hence the $NN$ interaction obtained by $J$-matrix inverse scattering technique is ambiguous.

* This ambiguity is used to fit JISP $NN$ interaction to the properties of light nuclei in No-core Shell Model (NCSM) calculations.

* First, the simplest tridiagonal $NN$ interaction is constructed fitting $NN$ scattering. Next, the simplest PETs with continuous parameters are used in NCSM fit of light nuclei. These PETs are generated by the unitary transformations of the type of rotations mixing the lowest oscillator states in each partial wave:

$$\tilde{H} = U H U^\dagger$$

$$[U] = [U_0] \oplus [I] = \begin{bmatrix} [U_0] & 0 \\ 0 & [I] \end{bmatrix}$$

$$[U_0] = \begin{bmatrix} \cos \beta & +\sin \beta \\ -\sin \beta & \cos \beta \end{bmatrix}$$
JISP $NN$ interactions


JISP16 initial fit

* Fitted manually to binding energies of (2H), 3H, 4He, 6Li, 12C, 16O

* Spectrum: 6Li

* Lee–Suzuki–Okamoto effective interaction was used

* This fit appeared to be surprisingly successful
Typical NCSM results obtained with bare NN interaction and Lee–Suzuki–Okamoto effective interaction
Ground state energy $E_{gs}$ and excitation energies $E_x$ (in MeV), ground state point-proton rms radius $r_p$ (in fm) and quadrupole moment $Q$ (in $e \cdot fm^2$) of the $^6$Li nucleus; $\hbar \omega = 17.5$ MeV.

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<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>$E_{gs}(1^+_1, 0)$</td>
<td>-31.995</td>
<td>-31.48</td>
<td>-31.00</td>
<td>-31.04</td>
<td>-31.25(8)</td>
<td>-32.0(1)</td>
</tr>
<tr>
<td>$r_p$</td>
<td>2.32(3)</td>
<td>2.083</td>
<td>2.151</td>
<td>2.054</td>
<td>2.46(2)</td>
<td>2.39(1)</td>
</tr>
<tr>
<td>$Q$</td>
<td>-0.082(2)</td>
<td>-0.194</td>
<td>-0.0646</td>
<td>-0.025</td>
<td>-0.33(18)</td>
<td>-0.32(6)</td>
</tr>
<tr>
<td>$E_x(3^+_0)$</td>
<td>2.186</td>
<td>2.102</td>
<td>2.529</td>
<td>2.471</td>
<td>2.8(1)</td>
<td>2.2</td>
</tr>
<tr>
<td>$E_x(0^+_1)$</td>
<td>3.563</td>
<td>3.348</td>
<td>3.701</td>
<td>3.886</td>
<td>3.94(23)</td>
<td>3.4</td>
</tr>
<tr>
<td>$E_x(2^+_0)$</td>
<td>4.312</td>
<td>4.642</td>
<td>5.001</td>
<td>5.010</td>
<td>4.0(1)</td>
<td>4.2</td>
</tr>
<tr>
<td>$E_x(2^+_1)$</td>
<td>5.366</td>
<td>5.820</td>
<td>6.266</td>
<td>6.482</td>
<td>5.1(1)</td>
<td>5.5</td>
</tr>
<tr>
<td>$E_x(1^+_2, 0)$</td>
<td>5.65</td>
<td>6.86</td>
<td>6.573</td>
<td>7.621</td>
<td>5.6</td>
<td></td>
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<tr>
<td>Potential Approach</td>
<td>Nature</td>
<td>JISP16 NCSM, $8h\omega^a$</td>
<td>AV8'*+TM' NCSM, $4h\omega^b$</td>
<td>AV18+IL2 GFMC$^c$</td>
<td>ChPT NCSM, $6h\omega^d$</td>
<td></td>
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</tr>
<tr>
<td>$E_g^s(3^+_1,0)$</td>
<td>-64.751</td>
<td>-60.14</td>
<td>-60.57</td>
<td>-65.6(5)</td>
<td>-64.78</td>
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<tr>
<td>$r_p$</td>
<td>2.30(12)</td>
<td>2.168</td>
<td>2.168</td>
<td>2.33(1)</td>
<td>2.197</td>
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</tr>
<tr>
<td>$Q$</td>
<td>+8.472(56)</td>
<td>6.484</td>
<td>+5.682</td>
<td>+9.5(2)</td>
<td>+6.327</td>
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<tr>
<td>$E_x(1^+_1,0)$</td>
<td>0.718</td>
<td>0.555</td>
<td>0.340</td>
<td>0.9</td>
<td>0.523</td>
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<tr>
<td>$E_x(0^+,1)$</td>
<td>1.740</td>
<td>1.202</td>
<td>1.259</td>
<td>1.279</td>
<td></td>
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<tr>
<td>$E_x(1^+_2,0)$</td>
<td>2.154</td>
<td>2.379</td>
<td>1.216</td>
<td>1.432</td>
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<tr>
<td>$E_x(2^+_1,0)$</td>
<td>3.587</td>
<td>3.721</td>
<td>2.775</td>
<td>3.9</td>
<td>3.178</td>
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<tr>
<td>$E_x(3^+_2,0)$</td>
<td>4.774</td>
<td>6.162</td>
<td>5.971</td>
<td></td>
<td>6.729</td>
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<tr>
<td>$E_x(2^+_1,1)$</td>
<td>5.164</td>
<td>5.049</td>
<td>5.182</td>
<td>5.315</td>
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<tr>
<td>$E_x(2^+_2,0)$</td>
<td>5.92</td>
<td>5.548</td>
<td>3.987</td>
<td>4.835</td>
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<tr>
<td>$E_x(4^+,0)$</td>
<td>6.025</td>
<td>5.775</td>
<td>5.229</td>
<td>5.6</td>
<td>5.960</td>
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<tr>
<td>$E_x(2^+_2,1)$</td>
<td>7.478</td>
<td>7.776</td>
<td>7.491</td>
<td>7.823</td>
<td></td>
<td></td>
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<tr>
<td>$B(E2;1^+_10 \rightarrow 3^+_10)$</td>
<td>4.13(6)</td>
<td>3.317</td>
<td>1.959</td>
<td>3.05</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B(E2;1^+_20 \rightarrow 3^+_10)$</td>
<td>1.71(26)</td>
<td>0.627</td>
<td>1.010</td>
<td>0.50</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B(GT;3^+_10 \rightarrow 2^+_11)$</td>
<td>0.083(3)</td>
<td>0.042</td>
<td>0.066</td>
<td>0.07</td>
<td></td>
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<tr>
<td>$B(GT;3^+_10 \rightarrow 2^+_21)$</td>
<td>0.95(13)</td>
<td>1.652</td>
<td>1.291</td>
<td>1.22</td>
<td></td>
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</tr>
</tbody>
</table>


Is LSO effective interaction reliable?
From effective interactions to no-core full configuration (NCFC) calculations

* Extrapolation:

\[ E_{gs}(N_{\text{max}}) = ae^{-bN_{\text{max}}} + E_{gs}(\infty) \]

* Works with bare interaction only (e.g., JISP16)

* Example:

2 types of extrapolations

Global (A)

Uncertainties of extrapolations!

Local (B)
Other extrapolations

- Other extrapolation techniques were suggested recently:

- These extrapolations are better theoretically grounded. However, from our recent analysis of large number of nuclei, they seem to be less accurate than our phenomenological extrapolations
NCSM-NCFC approach: Some problems
Temporary problems

- We can extrapolate energies but still cannot extrapolate other observables: rms radii, EM moments, EM transition probabilities, etc.

- We calculate these observables but they have an $\hbar \Omega$ dependence, so, we cannot estimate the uncertainties that are large.

- Note: extrapolation technique for rms radii was suggested: R. J. Furnstahl, G. Hagen, and T. Papenbrock, Phys. Rev. C 86, 031301(R) (2012). However it has not been carefully tested yet
Problems

We cannot obtain some levels, e.g., the Hoyle state predicted by Fred Hoyle in the 1950s and later confirmed experimentally. This state is essential for the production of the $^{12}\text{C}$ isotope in stars via the triple-$\alpha$ process, i.e., for the origin of life on Earth.
Hoyle state

Too large $N_{\text{max}}$ required for Hoyle
Hoyle state


Obtained in symplectic NCSM
NCFC & JISP16: levels with $\Gamma < 300$ kev
Success of NCSM calculations with JISP16 interaction and NCFC extrapolations:
Predictions of $^{14}$F properties (2009)
14F

* 1,990,061,078 basis states in $N_{\text{max}} = 8$ model space

* Each $\hbar \Omega$ point requires 2 to 3 hours on 7,626 quad-core compute nodes (30,504 processors in total) at the Jaguar supercomputer at ORNL

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Extrapolation A</th>
<th>Extrapolation B</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{13}\text{O}$</td>
<td>$-75.7(2.2)$</td>
<td>$-77.6(3.0)$</td>
<td>$-75.556$</td>
</tr>
<tr>
<td>$^{14}\text{B}$</td>
<td>$-84.4(3.2)$</td>
<td>$-86.6(3.8)$</td>
<td>$-85.423$</td>
</tr>
<tr>
<td>$^{14}\text{F}$</td>
<td>$-70.9(3.6)$</td>
<td>$-73.1(3.7)$</td>
<td>$74.00(0.04)$</td>
</tr>
</tbody>
</table>
$^{14}\text{F}$ spectrum
Back to JISP16: drawbacks

- Deficiency of JISP16 revealed by NCFC extrapolations and by the use of larger model spaces attainable due to new supercomputers
How it looked initially:

How it looks now:
Light nuclei with JISP16: comprehensive analysis

- 26 nuclei, 135 natural and unnatural parity states
- Analyzed rms deviations from experiment for absolute energies $E_{i}^{th}$, for energies per nucleon $E_{i}^{th}/A$, and rms for relative energies $(E_{i}^{th} - E_{i}^{exp})/E_{i}^{exp}$. 


Light nuclei with JISP16: comprehensive analysis

rms for binding energies

<table>
<thead>
<tr>
<th></th>
<th>Number of nuclei</th>
<th>JISP16 Extrap. B</th>
<th>JISP16 Extrap. C</th>
<th>AV18/IL7</th>
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<tbody>
<tr>
<td><strong>Comparison of JISP16 and AV18/IL7 results</strong></td>
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<tr>
<td>Absolute energies (MeV)</td>
<td>13</td>
<td>1.16</td>
<td>1.44</td>
<td>0.43</td>
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<tr>
<td>Relative energies</td>
<td>13</td>
<td>0.023</td>
<td>0.029</td>
<td>0.007</td>
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<tr>
<td>Energies per nucleon (MeV)</td>
<td>13</td>
<td>0.12</td>
<td>0.16</td>
<td>0.04</td>
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</table>

\( A \leq 12 \) ground states

<table>
<thead>
<tr>
<th></th>
<th>Number of nuclei</th>
<th>JISP16 Extrap. B</th>
<th>JISP16 Extrap. C</th>
<th>AV18/IL7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absolute energies (MeV)</td>
<td>19</td>
<td>1.68</td>
<td>2.33</td>
<td>–</td>
</tr>
<tr>
<td>Relative energies</td>
<td>19</td>
<td>0.033</td>
<td>0.047</td>
<td>–</td>
</tr>
<tr>
<td>Energies per nucleon (MeV)</td>
<td>19</td>
<td>0.16</td>
<td>0.22</td>
<td>–</td>
</tr>
</tbody>
</table>

All ground states

<table>
<thead>
<tr>
<th></th>
<th>Number of nuclei</th>
<th>JISP16 Extrap. B</th>
<th>JISP16 Extrap. C</th>
<th>AV18/IL7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absolute energies (MeV)</td>
<td>26</td>
<td>5.63</td>
<td>5.49</td>
<td>–</td>
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<tr>
<td>Relative energies</td>
<td>26</td>
<td>0.055</td>
<td>0.059</td>
<td>–</td>
</tr>
<tr>
<td>Energies per nucleon (MeV)</td>
<td>26</td>
<td>0.39</td>
<td>0.39</td>
<td>–</td>
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</table>
Light nuclei with JISP16: comprehensive analysis

Rms for level energies

<table>
<thead>
<tr>
<th></th>
<th>Number of levels</th>
<th>JISP16</th>
<th>AV18/IL7</th>
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<tr>
<td></td>
<td></td>
<td>Extrap. B</td>
<td>Extrap. C</td>
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<tr>
<td><strong>Comparison of JISP16 and AV18/IL7 results</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Absolute energies (MeV)</td>
<td>38</td>
<td>1.38</td>
<td>1.8</td>
</tr>
<tr>
<td>Relative energies</td>
<td>38</td>
<td>0.03</td>
<td>0.04</td>
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<tr>
<td>Energies per nucleon (MeV)</td>
<td>38</td>
<td>0.16</td>
<td>0.21</td>
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<td><strong>Natural parity states</strong></td>
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<tr>
<td>Absolute energies (MeV)</td>
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<td>3.71</td>
<td>3.97</td>
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<tr>
<td>Relative energies</td>
<td>96</td>
<td>0.043</td>
<td>0.05</td>
</tr>
<tr>
<td>Energies per nucleon (MeV)</td>
<td>96</td>
<td>0.28</td>
<td>0.31</td>
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<tr>
<td><strong>Natural and unnatural parity states</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Absolute energies (MeV)</td>
<td>135</td>
<td>3.54</td>
<td>4.04</td>
</tr>
<tr>
<td>Relative energies</td>
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<td>0.056</td>
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<tr>
<td>Energies per nucleon (MeV)</td>
<td>135</td>
<td>0.28</td>
<td>0.34</td>
</tr>
</tbody>
</table>
Light nuclei with JISP16: comprehensive analysis

rms for excitation energies

<table>
<thead>
<tr>
<th></th>
<th>Number of levels</th>
<th>JISP16</th>
<th>AV18/IL7</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Extrap. B</td>
<td>Extrap. C</td>
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<tr>
<td>Natural parity states</td>
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<tr>
<td>Excitation energies (MeV)</td>
<td>25</td>
<td>0.61</td>
<td>0.74</td>
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<td>Excitation energies (MeV)</td>
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<td>1.62</td>
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<tr>
<td>Unnatural parity states</td>
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<tr>
<td>Excitation energy (MeV)</td>
<td>31</td>
<td>2.25</td>
<td>1.57</td>
</tr>
</tbody>
</table>
Nuclear Matter from JISP16 with various J truncations

Nuclear matter with JISP16
Nuclear matter with JISP16

- Nuclear matter is a model that somehow simulates bulk properties of heavy nuclei.

- Surprisingly strong $J$ dependence of nuclear matter equation of state even for high $J$ in case of JISP16 NN interaction.

- Light nuclei are insensitive to high-$J$ components of the NN interaction. Hence it will be possible to fit JISP16 to nuclear matter properties by PETs in high-$J$ partial waves. This will be interesting!
Improved interaction

Obtained by a more accurate fit to nuclear data using NCFC extrapolations
JISP16\textsubscript{2010} is still somewhat preliminary version of the interaction: it is needed to calculate more nuclei and to check it in more detail. However it is clear that it improves JISP16 essentially.
Tetraneutron

* Interest from experimentalists
* How to evaluate the energy of unbound 4n system?
* Increase JISP16 to obtain bound 4 neutron state
* Use NCSM to obtain ground state energy
* Extrapolate to $NN$ interaction without enhancement
Tetraneutron

4 neutrons with enhanced JISP16

variational min. (if there is a local var. min...) on available hw grid

N_{max} = 6
N_{max} = 8
N_{max} = 10
N_{max} = 12
N_{max} = 14
N_{max} = 16
N_{max} = 18

quadratic fit

twice N^2 at N_{max} = 20
Tetraneutron

* How to evaluate the energy of unbound 4n system?
* Why not to try to do the same with PETs?
* I. e. to bind 4n by PETs
* and to leave unchanged ⁴He binding…
Tetraneutron: The power of PETs

* It is possible!
* Fits in small model spaces, hence not very accurate.
* The PETted JISP16 provides the following NCFC extrapolated results:
  * $^4$He energy: $-29.634$ MeV (exp. $-28.296$ MeV) + low-lying state $(J,T)=(3,1)$
  * $^3$H energy: $-8.231$ MeV (exp. $-8.482$ MeV)
  * 3n unbound
  * 4n: two bound states: $(J,T)=(2,2)$ at $-14.1$ MeV and $(J,T)=(0,2)$ at $-6$ MeV
**J-matrix formalism:**

scattering in the oscillator basis

Oscillator basis, truncated potential energy matrix $V$ and non-truncated complete infinite kinetic energy matrix $T$. Justification: kinetic energy m. e. increase with $n$ linearly at large $n$: $T_{nn} \sim n, T_{n,n\pm 1} \sim n, n \to \infty$ while potential energy m. e. $V_{nm}$ decrease with $n$ and $m$.

$$\sum_{n' \leq n} H_{nn'}^{\lambda}(n') = E_{\lambda}(n) \lambda \geq n.$$  

$$\mathcal{G}_{NN}(E) = -\sum_{\lambda=0}^{N} \frac{(N|\lambda)^2}{E_{\lambda} - E}.$$  

$$S = \frac{c_{N1}^{-}(q) - \mathcal{G}_{NN}(E)T_{N,N+1}^{I} c_{N+1,1}^{-}(q)}{c_{N1}^{+}(q) - \mathcal{G}_{NN}(E)T_{N,N+1}^{I} c_{N+1,1}^{+}(q)}.$$  

Both direct and inverse scattering $J$-matrix solutions are possible.
Tetraneutron: 

*Impossible to calculate all $E_\lambda$

At $E = E_\lambda$:

$$S(E_\lambda) = \frac{C_{N+1,K}^{(-)}(E_\lambda)}{C_{N+1,K}^{(+)}(E_\lambda)}$$

4-body decay, hyperspherical formalism, lowest possible hypermomentum $K$

In the vicinity of the resonance

$$S(E) = e^{2i\delta}$$

$$\delta(E) = \frac{\pi}{2} + \arctan \frac{E-b}{a\sqrt{E}} + c\sqrt{E}$$

$a, b, c$ – fitting parameters
Tetraneutron: 
\textit{J}-matrix formalism

\begin{align*}
E_r &= 0.56 \text{ MeV} \\
\Gamma_r &= 2.98 \text{ MeV}
\end{align*}
Future

* Involving more observables including resonance energies and widths
* Improved JISP16$_{2010}$ fitted also to nuclear matter
* PETted N3LO to avoid use $NNN$ forces
* ...

Present

Thank you!