Recent results obtained with JISP16

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Plan

- Motivation
- Structure of JISP interaction
- Bindings and spectra of light nuclei with JISP16
- Nuclear matter
- Future prospect

NN interaction

- Modern history of realistic *NN* interaction starts from 1993: Nijmegen *NN* database and phase shift analysis
- After 1993 various *NN* interactions describing NN data with χ^2/datum ≈ 1 have been suggested, in particular:

Meson exchange: Nijmegen I, II; Reid soft core; Argonne AV$_{18}$; CD-Bonn$_{2000}$; INOY (inside non-local, outside Yukawa)

Chiral EFT: N2LO(next-to-next-to-leading-order), N3LO

Inverse scattering: JISP6, JISP16, JISP16$_{2010}$

(χ^2/datum > 2 for pre-1993 *NN* interactions)

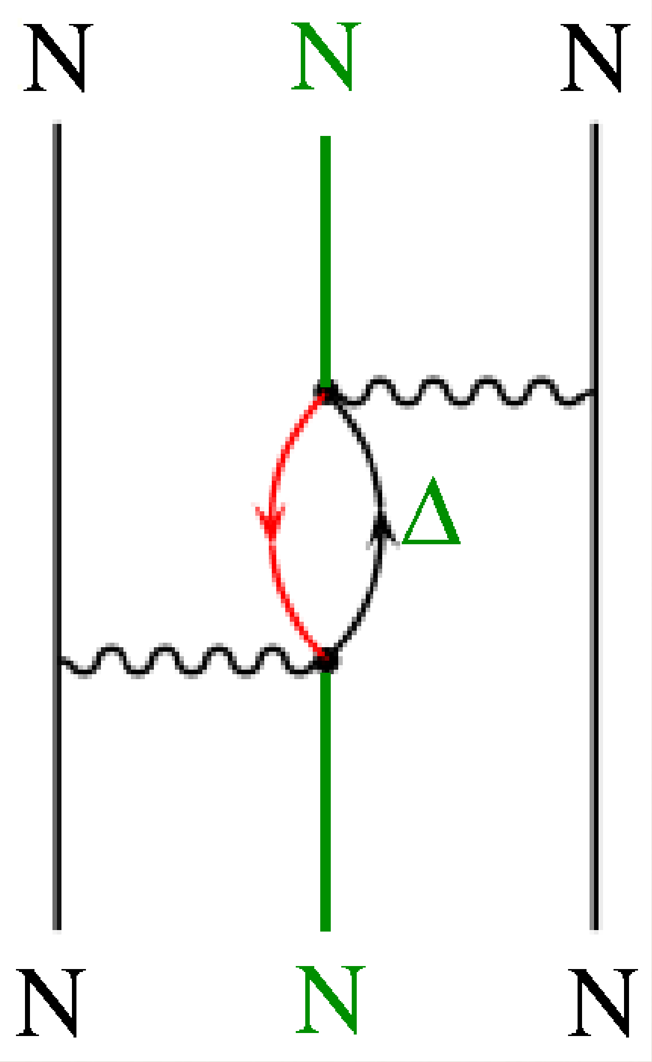
Ideal *NN* interaction from nuclear theorist veiwpoint

- Derived from QCD without model approximations and assumptions
- Perfectly describing *NN* data at low energies
- Perfectly describing bindings, spectra and other observables in light nuclei
- Describing heavier nuclei, nuclear matter, etc.
- Describing other experimental data (*p*+*d* scattering, etc.)
- Providing fast convergence of shell model and other calculations
- No need of *NNN* forces

Ideal *NN* interaction from nuclear theorist veiwpoint

- Derived from QCD without model approximations and assumptions
- Perfectly describing *pn* data at low energies
- Reasonably describing bindings, spectra and other observables in light nuclei
- Describing heavier nuclei, nuclear matter, etc.
- Describing other experimental data (*p*+*d* scattering, etc.)
- Providing fast convergence of shell model and other calculations
- No need of *NNN* forces
- JISP16

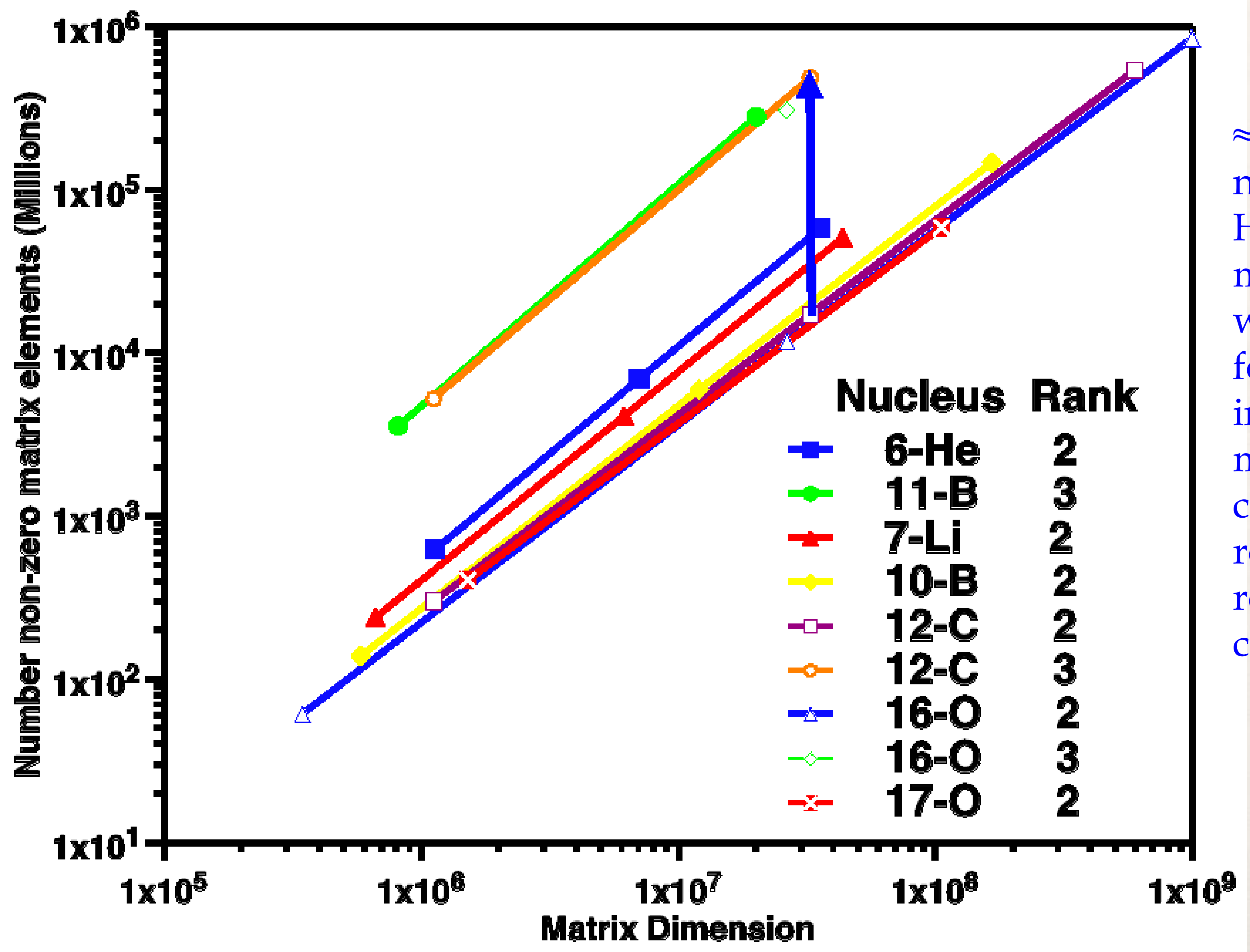
NNN force



NNN contribution to nuclear observables is small but can be essential.

4*N* and higher forces are usually supposed to be inessential for description of nuclei.

Why would be nice to avoid *NNN* forces?



≈30 times more non-zero Hamiltonian matrix elements when *NNN* forces are involved; hence much more computer resources are required for calculations

Role of NNN force?

- W. Polyzou and W. Glöckle theorem (Few-body Syst. **9**, 97 (1990)): $H=T+V_{ij} \Rightarrow H'=T+V'_{ij}+V_{ijk}$, where V_{ij} and V'_{ij} are phase-equivalent, H and H' are isospectral.

  Hope:

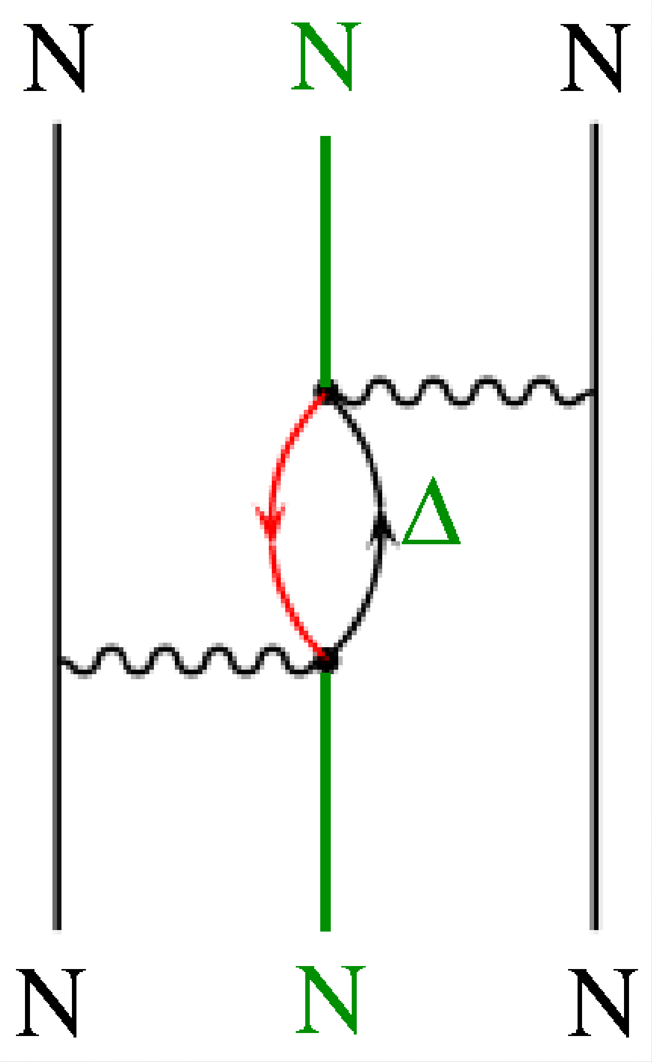
  $$H'=T+V'_{ij}+V_{ijk} \Rightarrow H=T+V_{ij}$$

  with (approximately) isospectral H and H' .

  JISP type interaction seems to be *NN* interaction minimizing *NNN* force.

  Without *NNN* force calculations are simpler, calculations are faster, larger model spaces become available; hence predictions are more reliable.

NNN force



Peter Sauer: ``*NNN* force is only a baby of theoreticians who would like to work in a restricted Hilbert space'', i.e., avoiding Δisobar and other excited nucleon degrees of freedom

From this point of view, JISP *NN* interaction is an attempt to describe nuclei with nucleon degrees of freedom only

Modern *NN* interactions: need of *NNN*

- Meson exchange: Nijmegen I, II; Reid soft core; Argonne AV_{18}; CD-$Bonn_{2000}$: require *NNN* potentials (U, IL, TM, …) which are usually inconsistent with *NN* interaction
- Chiral EFT: N3LO requires *NNN* potential − N2LO at the moment, consistent with *NN* at the N2LO level
- Inverse scattering: JISP6, JISP16, $JISP16_{2010}$: no need of *NNN*; fitted to light nuclei

+ INOY (inside non-local, outside Yukawa)

NN interaction: convergence

- To improve convergence of *ab initio* nuclear structure, usually (exception: GFMC) an effective interaction based on intrinsic *NN* (and, generally, *NNN*) interaction is constructed

- Modern approaches to eff. interaction:

- Lee−Suzuki−Okamoto (LSO): popular up to 2010, less popular now. Idea is to reproduce in a given small model space the results in the infinite model space (of course, approximately). New interaction for each model space. No variational principle, non-monotonic convergence in many-body nuclei, no way to extrapolate results to infinite model space. Induced *NNN* to improve convergence.

- Similarity renormalization group (SRG): a modern trend. Idea is to reduce matrix elements coupling low- and high-momentum components of interaction by unitary transformation. The variational principle works, results can be extrapolated. Induced *NNN* to restore `*bare*' interaction results in many-body nuclei.

Modern *NN* interactions

- Meson exchange *NN* (Argonne AV_{18}; CD-Bonn_{2000}; INOY, …): high-quality description of *NN* data: χ^2/datum ≈ 1 up to $E_{\text{lab}} = 350$ MeV. Somewhat phenomenological: no ties to QCD, phenomenological terms, inconsistent parameters. Should be combined with (usually inconsistent)(semi)phenomenological *NNN* (IL, U, TM, …). Bad convergence, eff. interaction needed. (*R. Machleidt*: ``If you want more more accuracy, you have to use less theory'')

- Chiral EFT *NN* (N3LO): A modern trend. Less accurate (at the moment) description of *NN* data: *np*:χ^2/datum = 1.10 up to $E_{\text{lab}} = 290$ MeV; *pp*:χ^2/datum = 1.50 up to $E_{\text{lab}} = 290$ MeV. Tied to QCD through expansion in p/p_χ, p_χ is a chiral symmetry breaking momentum. Should be combined with Chiral EFT *NNN* (N2LO now). Bad convergence, eff. interaction needed. (*R. Machleidt*: ``If you want more more accuracy, you have to use more theory'')

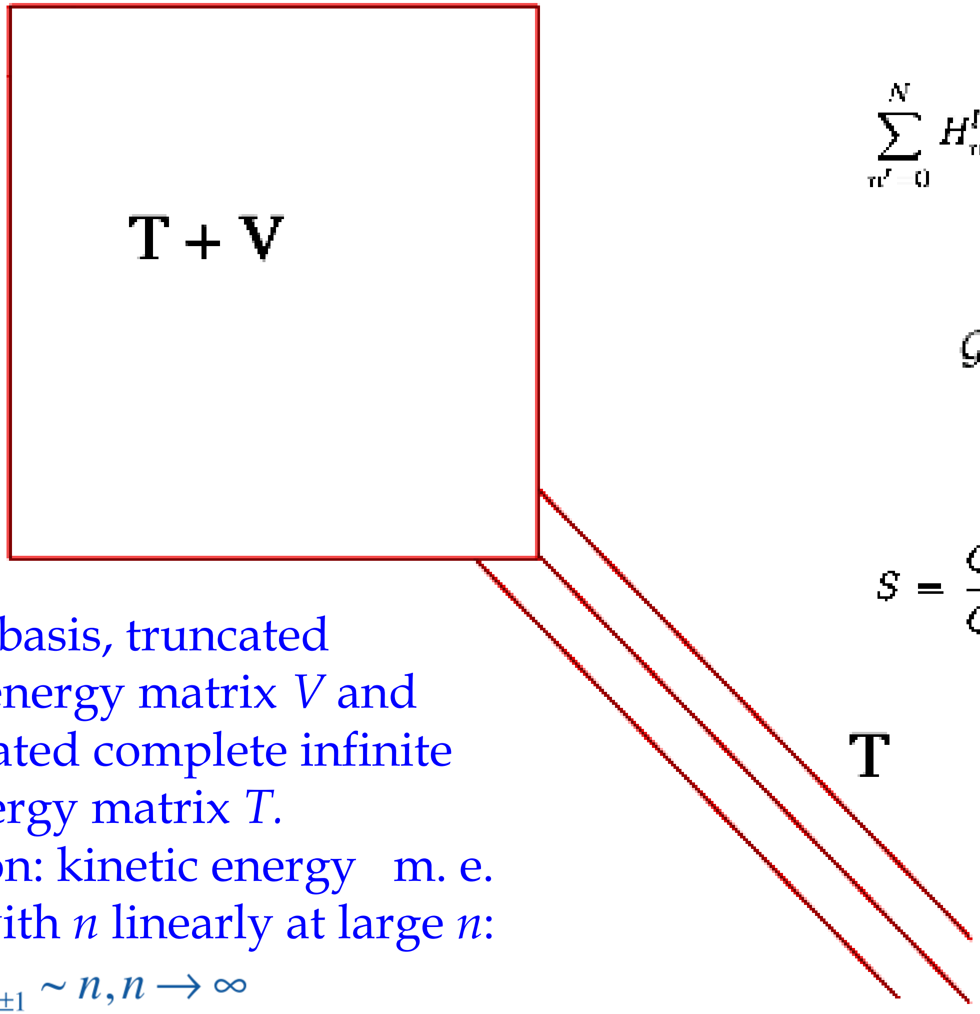
- Inverse scattering *NN* (JISP16, JISP16_{2010}): high-quality description of *np* data: χ^2/datum ≈ 1 up to $E_{\text{lab}} = 350$ MeV. Completely phenomenological. No need of *NNN*

Construction of JISP *NN* interaction

- JISP = *J*-matrix inverse scattering potential

J-matrix formalism:

scattering in the oscillator basis



$$\sum_{n'=0}^{N} H^{l}_{nn'}(n'|\lambda) = E_{\lambda}(n|\lambda), \qquad n \le N.$$

$$\mathcal{G}_{NN}(E) = -\sum_{\lambda=0}^{N} \frac{(N|\lambda)^2}{E_\lambda - E},$$

$$S = \frac{C^{(-)}_{Nl}(q) - \mathcal{G}_{NN}(E)\, T^{l}_{N,\,N+1}\, C^{(-)}_{N+1,\,l}(q)}{C^{(+)}_{Nl}(q) - \mathcal{G}_{NN}(E)\, T^{l}_{N,\,N+1}\, C^{(+)}_{N+1,\,l}(q)},$$

Oscillator basis, truncated potential energy matrix *V* and non-truncated complete infinite kinetic energy matrix *T*. Justification: kinetic energy m. e. increase with *n* linearly at large *n*: $T_{nn} \sim n, T_{n,n\pm1} \sim n, n \to \infty$ while potential energy m. e. V_{nm} decrease with *n* and *m*.

Both direct and inverse scattering *J*-matrix solutions are possible.

JISP *NN* interaction

- *NN* interaction is a small matrix of the in the oscillator basis with $\hbar\Omega$ = 40 MeV:

  9$\hbar\Omega$ truncation, i.e. in each partial wave oscillator quanta $2n+l \leq 9$: 5×5 matrix in *s* (l=0) and *p* (l=1) waves; 4×4 matrix in *d* (l=2) and *f* (l=3) waves; etc.; in coupled waves dimensionalities are summed, e.g., 9×9 matrix in coupled *sd* waves, etc.

- This structure provides a good description of *NN* data and fast convergence of shell model calculations

JISP16 properties

- 1992 *np* data base (2514 data): χ^2/datum = 1.03
- 1999 *np* data base (3058 data): χ^2/datum = 1.05

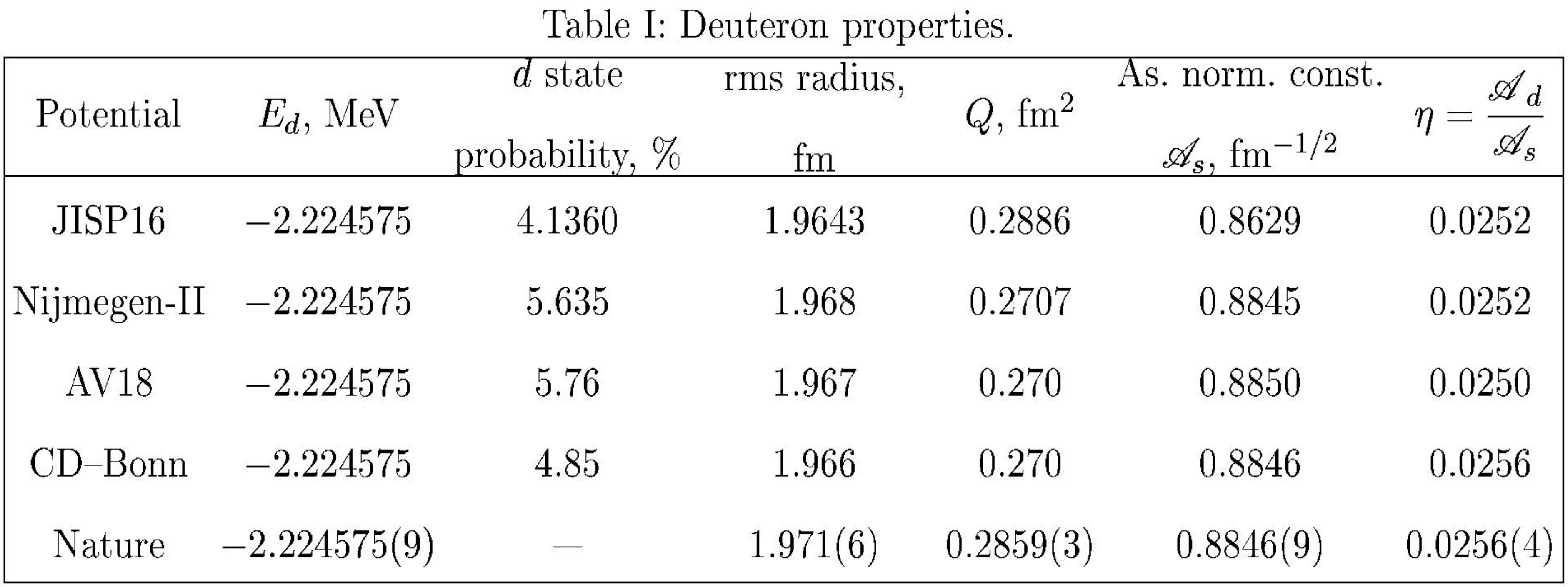
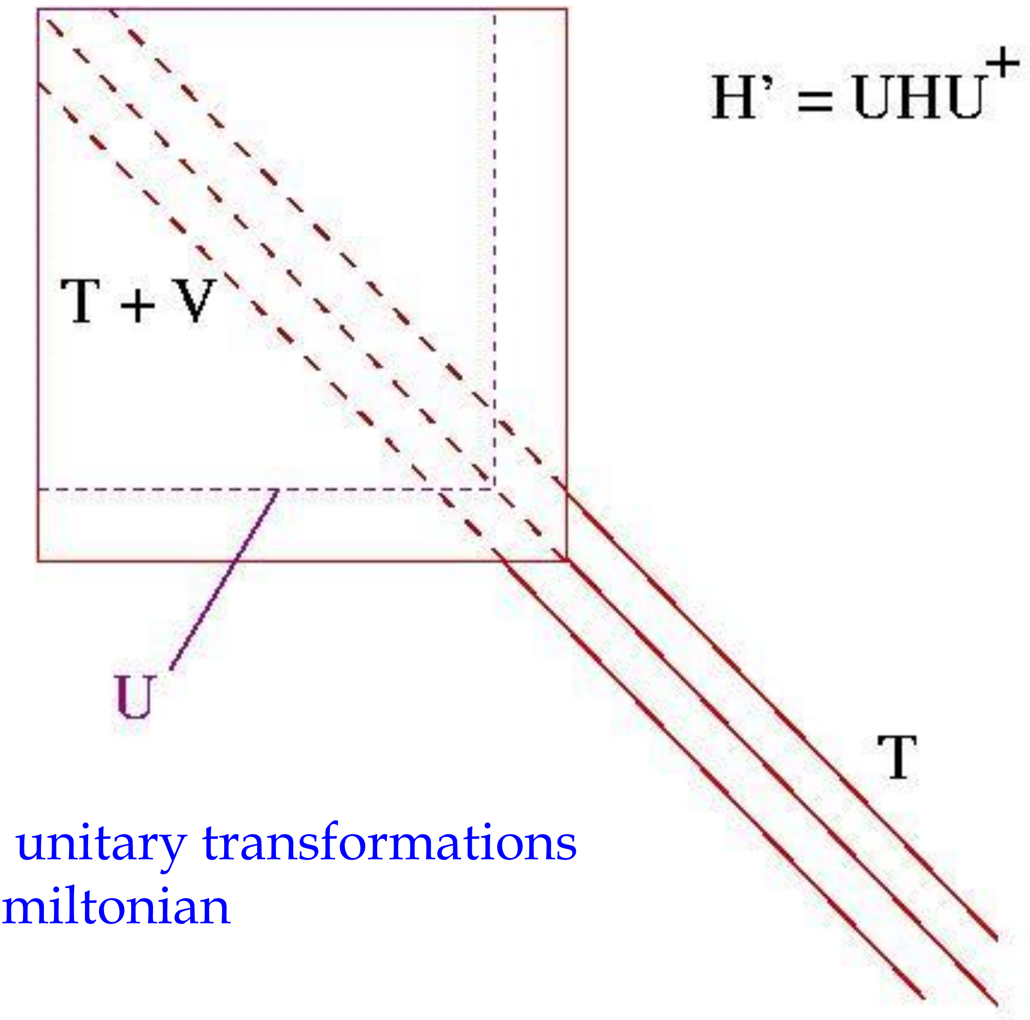


Table I: Deuteron properties.

Potential	E_d, MeV	d state probability, %	rms radius, fm	Q, fm^2	As. norm. const. $\mathcal{A}_s$, fm$^{-1/2}$	$\eta = \frac{\mathcal{A}_d}{\mathcal{A}_s}$
JISP16	−2.224575	4.1360	1.9643	0.2886	0.8629	0.0252
Nijmegen-II	−2.224575	5.635	1.968	0.2707	0.8845	0.0252
AV18	−2.224575	5.76	1.967	0.270	0.8850	0.0250
CD–Bonn	−2.224575	4.85	1.966	0.270	0.8846	0.0256
Nature	−2.224575(9)	—	1.971(6)	0.2859(3)	0.8846(9)	0.0256(4)

Phase-equivalent transformations (PETs)



$H' = UHU^{+}$

PETs are generated by unitary transformations of the two-nucleon Hamiltonian

Ambiguity of JISP interaction

- Any unitary transformation of *NN* Hamiltonian *H* generates a phase-equivalent transformation (PET). Hence the *NN* interaction obtained by *J*-matrix inverse scattering technique is ambiguous.

- This ambiguity is used to fit JISP *NN* interaction to the properties of light nuclei in No-core Shell Model (NCSM) calculations.

- First, the simplest tridiagonal *NN* interaction is constructed fitting *NN* scattering. Next, the simplest PETs with continuous parameters are used in NCSM fit of light nuclei. These PETs are generated by the unitary transformations of the type of rotations mixing the lowest oscillator states in each partial wave:

$$\widetilde{[H]} = [U][H][U^\dagger]$$

$$[U] = [U_0] \oplus [I] = \begin{bmatrix} [U_0] & 0 \\ 0 & [I] \end{bmatrix}$$

$$[U_0] = \begin{bmatrix} \cos\beta & +\sin\beta \\ -\sin\beta & \cos\beta \end{bmatrix}$$

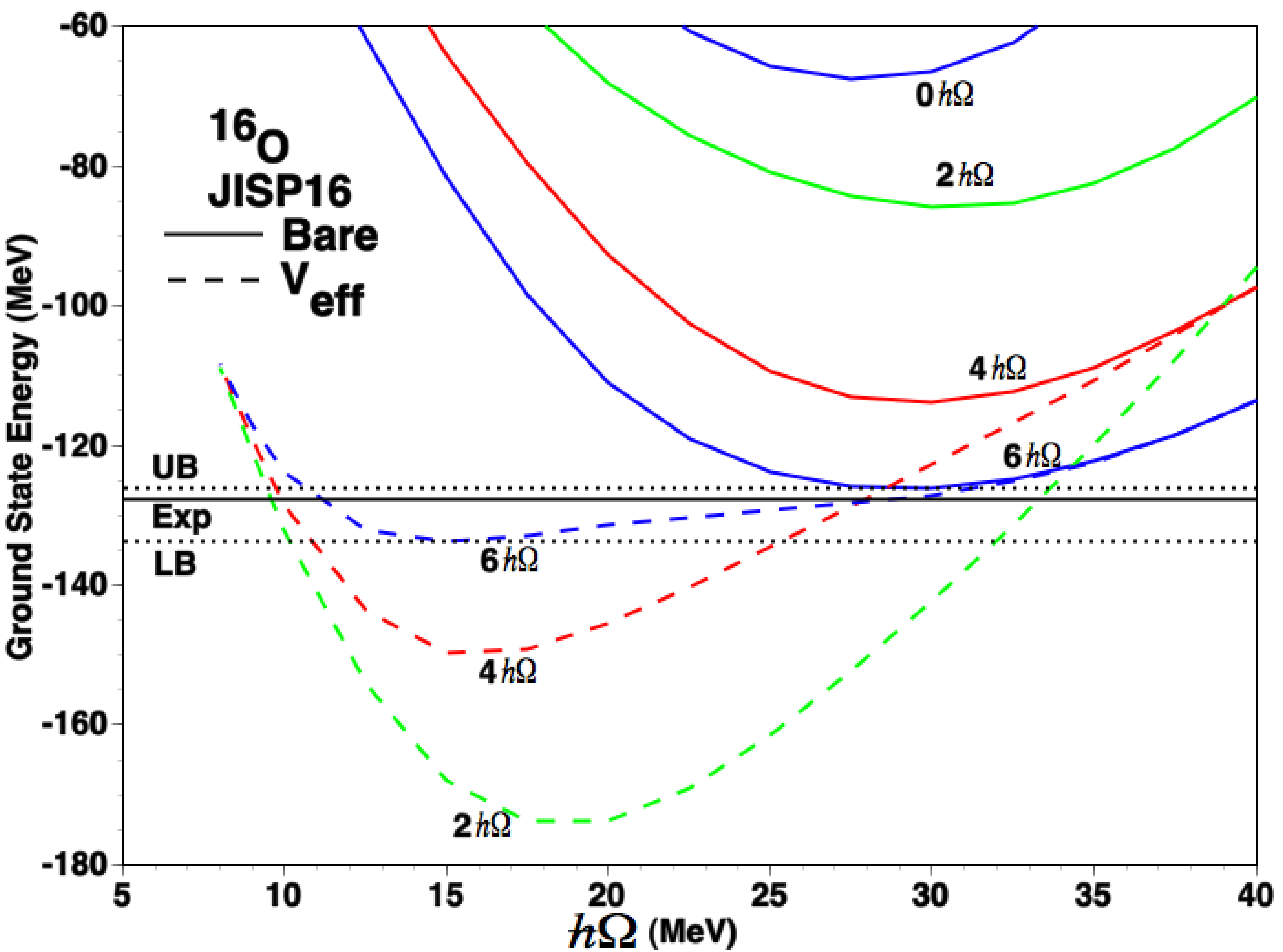
JISP *NN* interactions

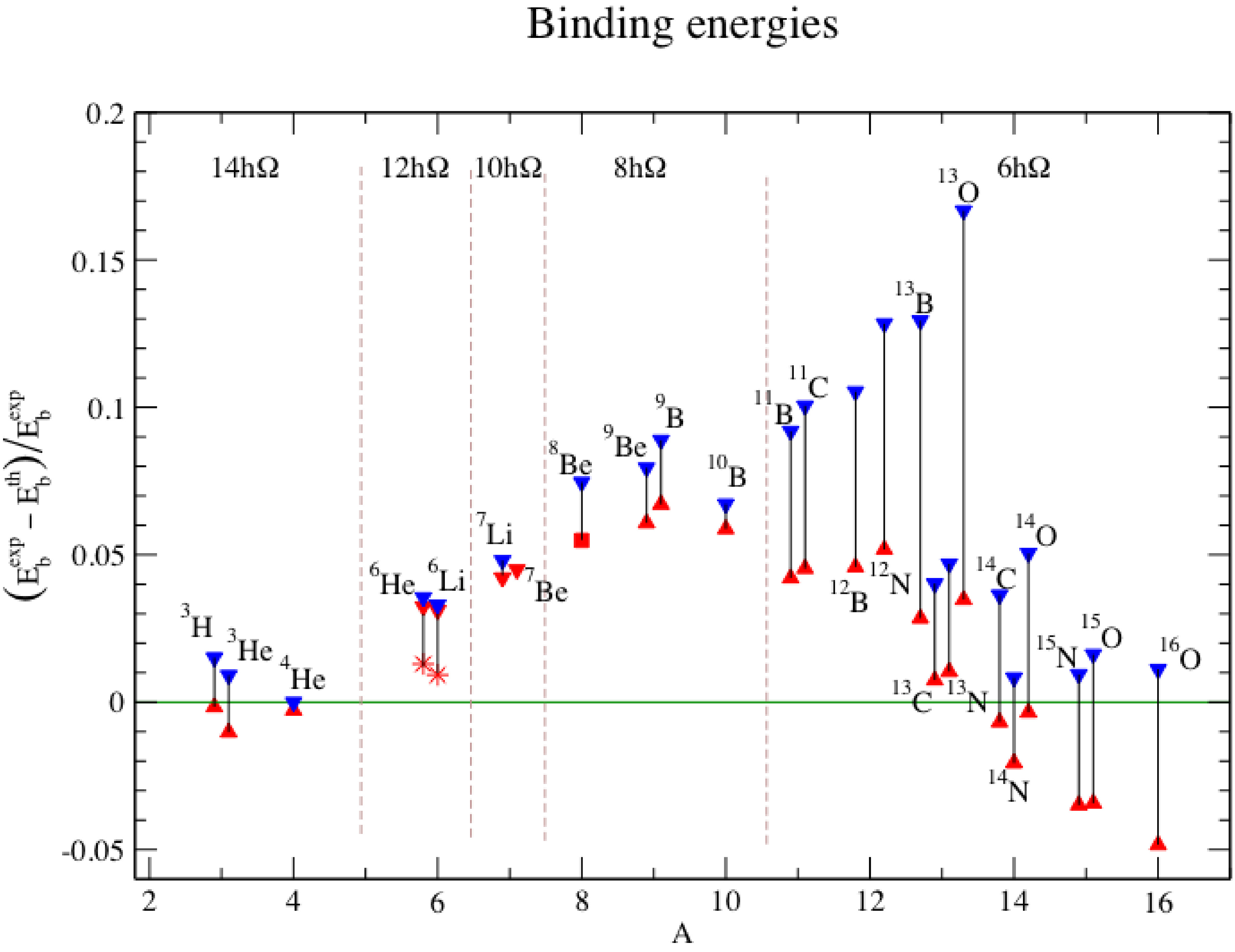
- A. M. Shirokov, A. I. Mazur, S. A. Zaytsev, J. P. Vary, T. A. Weber, Phys. Rev. C **70**, 044005 (2004): $A \leq 4$
- A. M. Shirokov, J. P. Vary, A. I. Mazur, S. A. Zaytsev, T. A. Weber, Phys. Lett. B **621**, 96 (2005): $A \leq 6$ – JISP6
- A. M. Shirokov, J. P. Vary, A. I. Mazur, T. A. Weber, Phys. Lett. B **644**, 33 (2007): $A \leq 16$ – JISP16

JISP16 initial fit

- Fitted manually to binding energies of (^{2}H), ^{3}H, ^{4}He, ^{6}Li, ^{12}C, ^{16}O
- Spectrum: ^{6}Li
- Lee−Suzuki−Okamoto effective interaction was used
- This fit appeared to be surprisingly successful

Typical NCSM results obtained with bare *NN* interaction and Lee−Suzuki−Okamoto effective interaction


16O
JISP16
Bare
Veff
Ground State Energy (MeV)
ħΩ (MeV)
-60
-80
-100
-120
-140
-160
-180
5
10
15
20
25
30
35
40
0 ħΩ
2 ħΩ
4 ħΩ
6 ħΩ
6 ħΩ
4 ħΩ
2 ħΩ
UB
Exp
LB


Binding energies
$(E_b^{exp} - E_b^{th})/E_b^{exp}$
A
0.2
0.15
0.1
0.05
0
-0.05
2
4
6
8
10
12
14
16
14hΩ
12hΩ
10hΩ
8hΩ
6hΩ
^{3}H
^{3}He
^{4}He
^{6}He
^{6}Li
^{7}Li
^{7}Be
^{8}Be
^{9}Be
^{9}B
^{10}B
^{11}B
^{11}C
^{12}B
^{12}N
^{13}B
^{13}C
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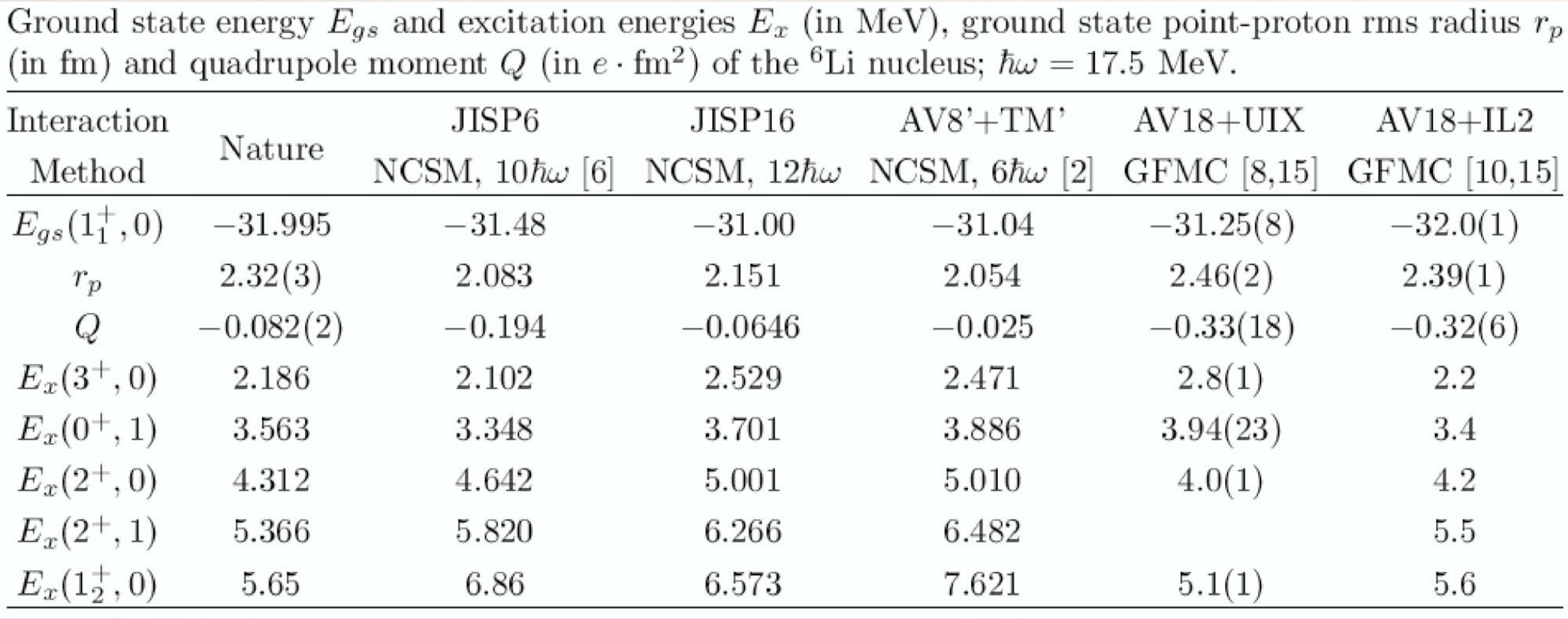
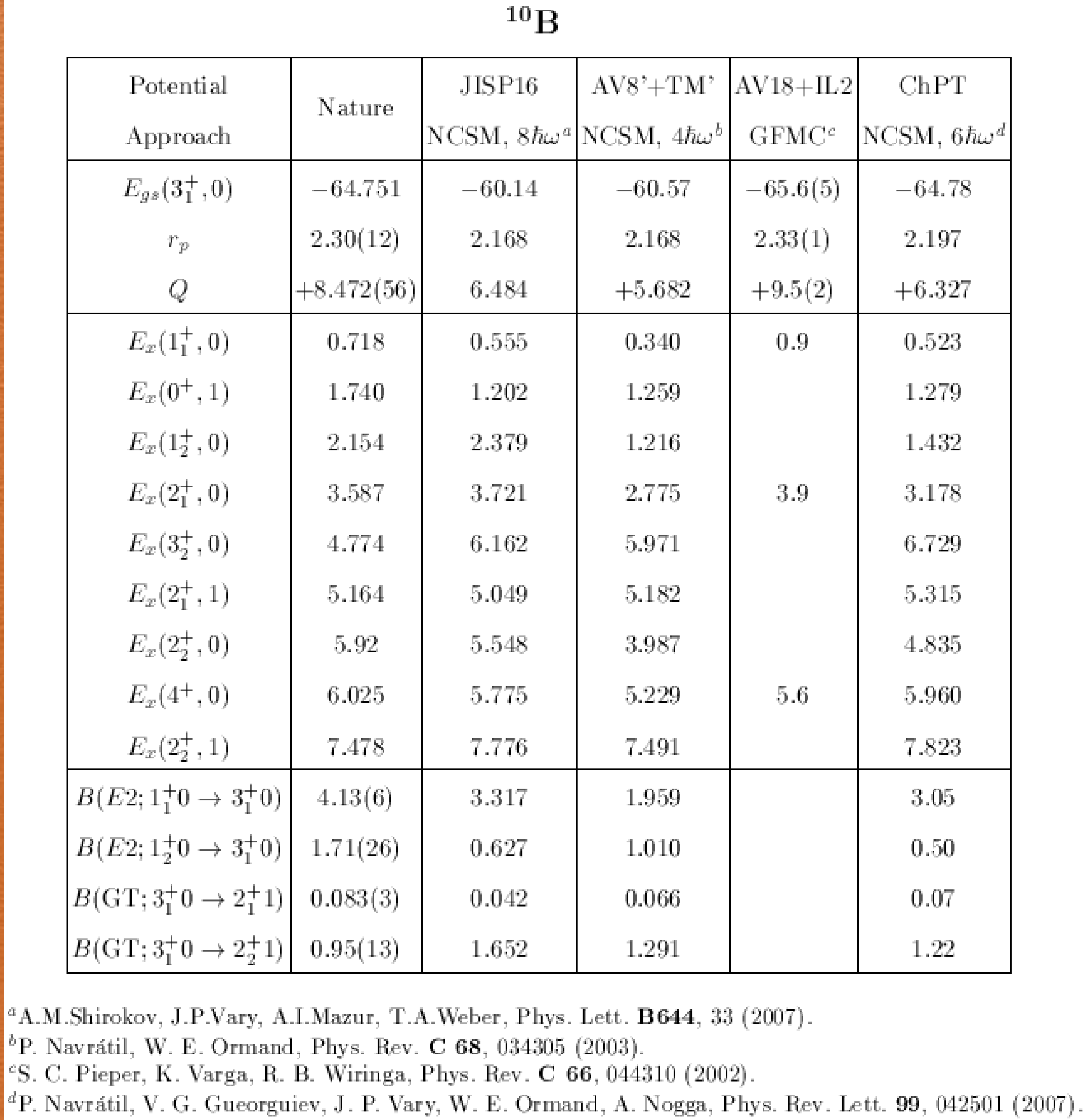
^{15}N
^{15}O
^{16}O

JISP16 results



Ground state energy E_{gs} and excitation energies E_x (in MeV), ground state point-proton rms radius r_p (in fm) and quadrupole moment Q (in $e \cdot \mathrm{fm}^2$) of the ^{6}Li nucleus; $\hbar\omega = 17.5$ MeV.

Interaction	Nature	JISP6	JISP16	AV8'+TM'	AV18+UIX	AV18+IL2
Method		NCSM, $10\hbar\omega$ [6]	NCSM, $12\hbar\omega$	NCSM, $6\hbar\omega$ [2]	GFMC [8,15]	GFMC [10,15]
$E_{gs}(1_1^+, 0)$	-31.995	-31.48	-31.00	-31.04	$-31.25(8)$	$-32.0(1)$
r_p	2.32(3)	2.083	2.151	2.054	2.46(2)	2.39(1)
Q	$-0.082(2)$	-0.194	-0.0646	-0.025	$-0.33(18)$	$-0.32(6)$
$E_x(3^+, 0)$	2.186	2.102	2.529	2.471	2.8(1)	2.2
$E_x(0^+, 1)$	3.563	3.348	3.701	3.886	3.94(23)	3.4
$E_x(2^+, 0)$	4.312	4.642	5.001	5.010	4.0(1)	4.2
$E_x(2^+, 1)$	5.366	5.820	6.266	6.482		5.5
$E_x(1_2^+, 0)$	5.65	6.86	6.573	7.621	5.1(1)	5.6



^{10}B

Potential Approach	Nature	JISP16 NCSM, $8\hbar\omega^a$	AV8'+TM' NCSM, $4\hbar\omega^b$	AV18+IL2 GFMC^c	ChPT NCSM, $6\hbar\omega^d$
$E_{gs}(3_1^+,0)$	-64.751	-60.14	-60.57	$-65.6(5)$	-64.78
r_p	2.30(12)	2.168	2.168	2.33(1)	2.197
Q	+8.472(56)	6.484	+5.682	+9.5(2)	+6.327
$E_x(1_1^+,0)$	0.718	0.555	0.340	0.9	0.523
$E_x(0^+,1)$	1.740	1.202	1.259		1.279
$E_x(1_2^+,0)$	2.154	2.379	1.216		1.432
$E_x(2_1^+,0)$	3.587	3.721	2.775	3.9	3.178
$E_x(3_2^+,0)$	4.774	6.162	5.971		6.729
$E_x(2_1^+,1)$	5.164	5.049	5.182		5.315
$E_x(2_2^+,0)$	5.92	5.548	3.987		4.835
$E_x(4^+,0)$	6.025	5.775	5.229	5.6	5.960
$E_x(2_2^+,1)$	7.478	7.776	7.491		7.823
$B(E2;1_1^+0\rightarrow 3_1^+0)$	4.13(6)	3.317	1.959		3.05
$B(E2;1_2^+0\rightarrow 3_1^+0)$	1.71(26)	0.627	1.010		0.50
$B(\mathrm{GT};3_1^+0\rightarrow 2_1^+1)$	0.083(3)	0.042	0.066		0.07
$B(\mathrm{GT};3_1^+0\rightarrow 2_2^+1)$	0.95(13)	1.652	1.291		1.22

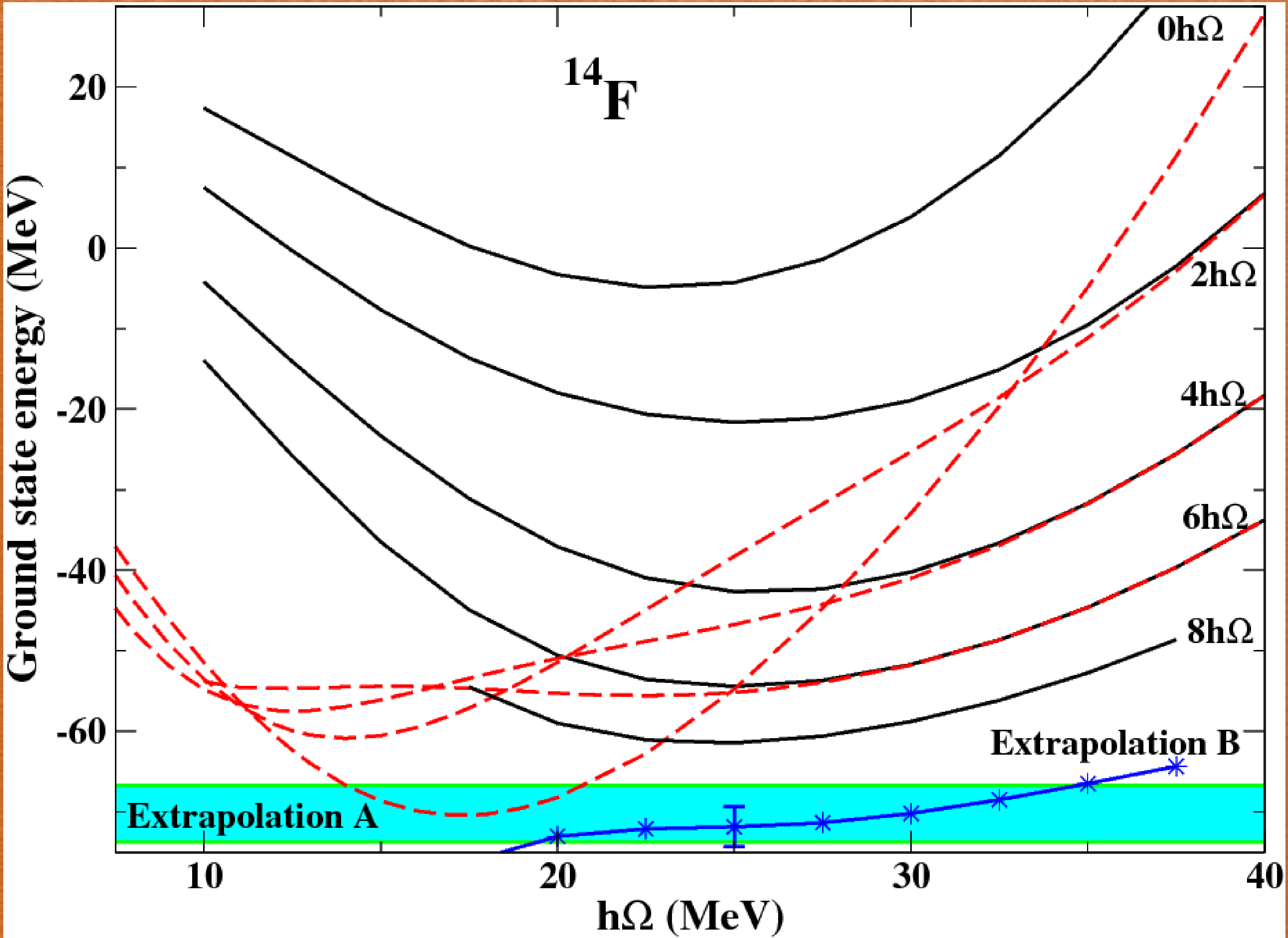
[a] A.M.Shirokov, J.P.Vary, A.I.Mazur, T.A.Weber, Phys. Lett. **B644**, 33 (2007).

[b] P. Navrátil, W. E. Ormand, Phys. Rev. **C 68**, 034305 (2003).

[c] S. C. Pieper, K. Varga, R. B. Wiringa, Phys. Rev. **C 66**, 044310 (2002).

[d] P. Navrátil, V. G. Gueorguiev, J. P. Vary, W. E. Ormand, A. Nogga, Phys. Rev. Lett. **99**, 042501 (2007).

Is LSO effective interaction reliable?

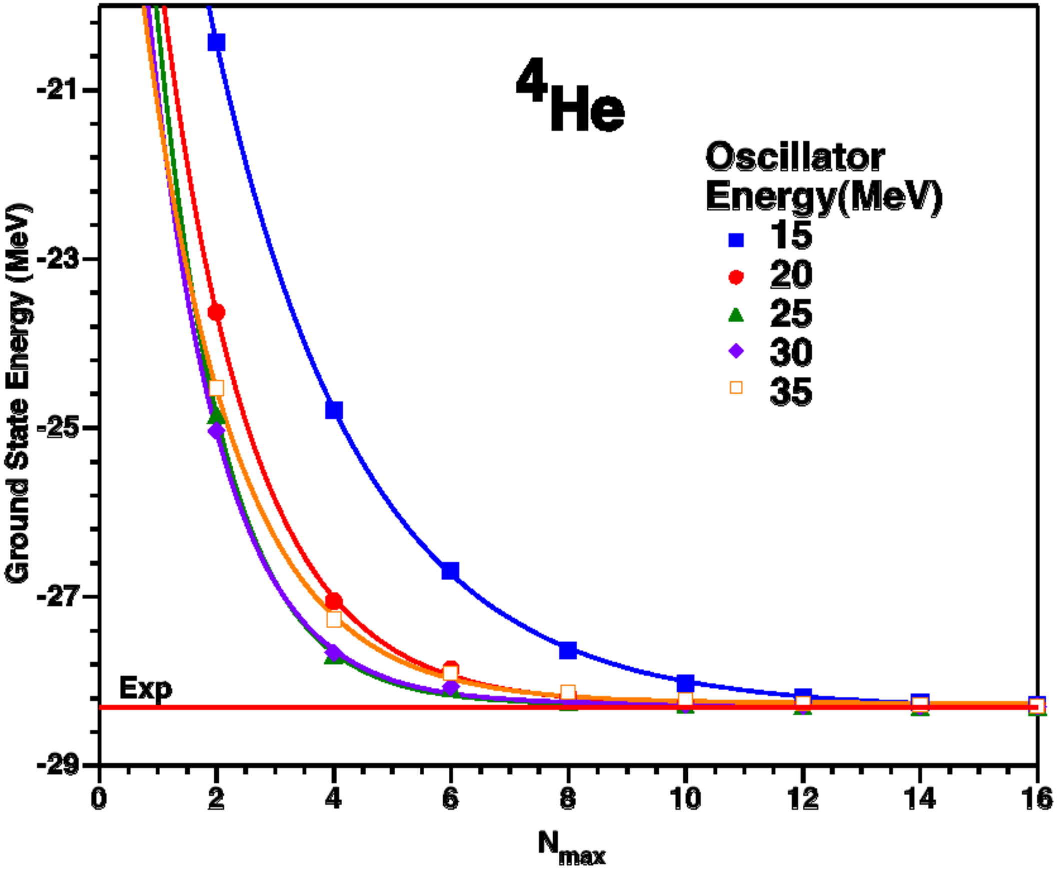


From effective interactions to no-core full configuration (NCFC) calculations

- Extrapolation:

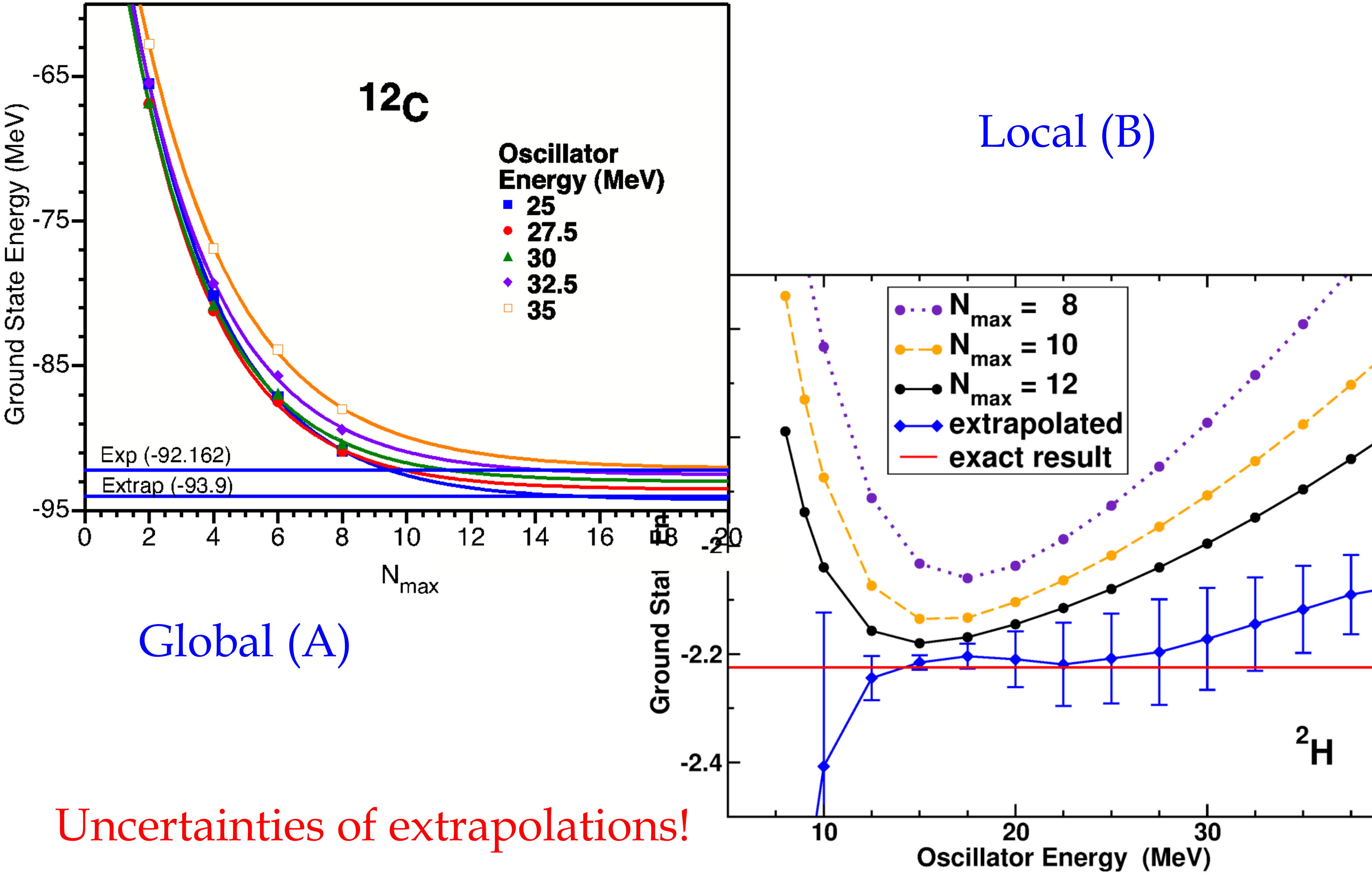
$$E_{gs}(N_{max}) = a e^{-bN_{max}} + E_{gs}(\infty)$$

- Works with bare interaction only (e.g., JISP16)
- Example:



P. Maris, J. P. Vary, A. M. Shirokov, Phys. Rev. C <u>79</u>, 014308 (2009)

2 types of extrapolations



Local (B)

Global (A)

Uncertainties of extrapolations!

Other extrapolations

- Other extrapolation techniques were suggested recently:
- S. A. Coon, M. I. Avetian, M. K. G. Kruse, U. van Kolck, P. Maris, and J. P. Vary, Phys. Rev. C 86, 054002 (2012)
- R. J. Furnstahl, G. Hagen, and T. Papenbrock, Phys. Rev. C 86, 031301(R) (2012)
- These extrapolations are better theoretically grounded. However, from our resent analysis of large number of nuclei, they seem to be less accurate than our phenomenological extrapolations

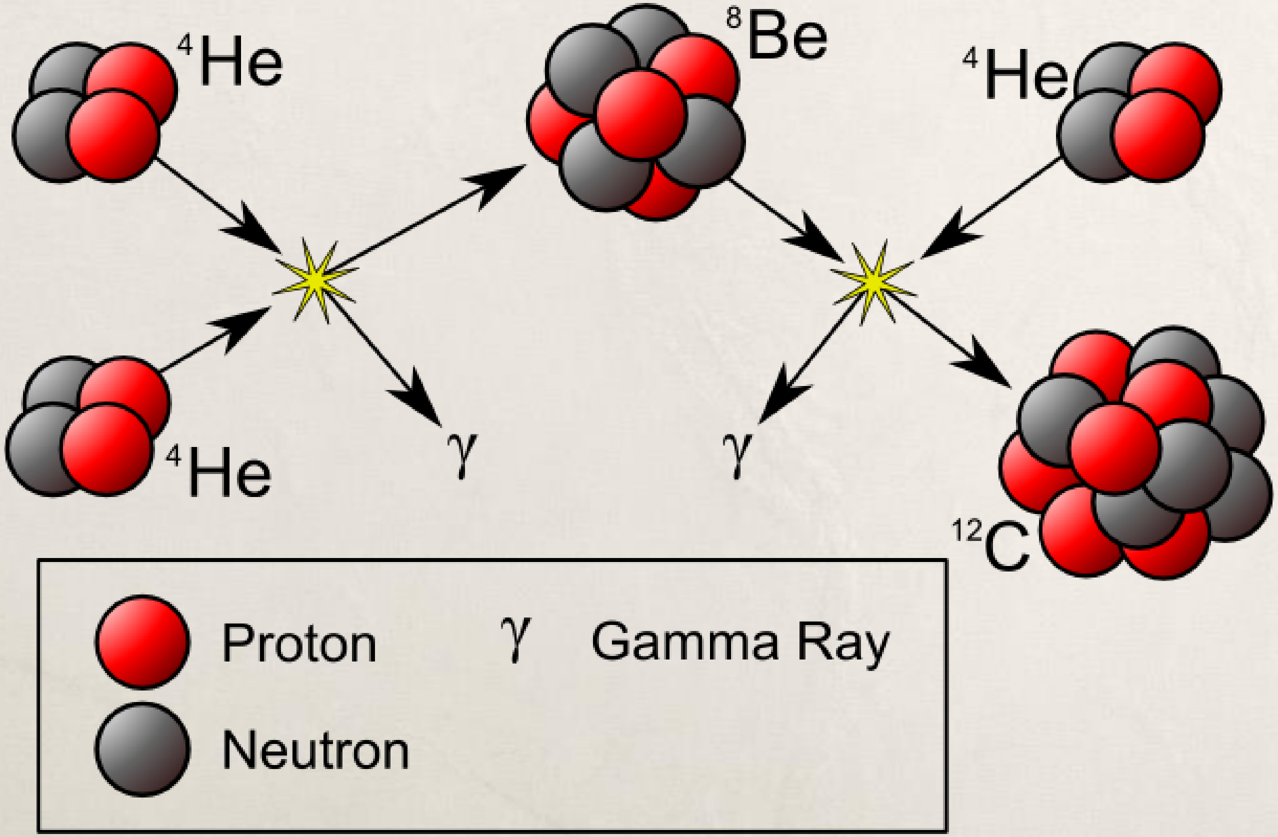
NCSM-NCFC approach: Some problems

Temporary problems

- We can extrapolate energies but still cannot extrapolate other observables: rms radii, EM moments, EM transition probabilities, etc.
- We calculate these observables but they have an $\hbar\Omega$ dependence, so, we cannot estimate the uncertainties that are large.
- Note: extrapolation technique for rms radii was suggested: R. J. Furnstahl, G. Hagen, and T. Papenbrock, Phys. Rev. C 86, 031301(R) (2012). However it has not been carefully tested yet

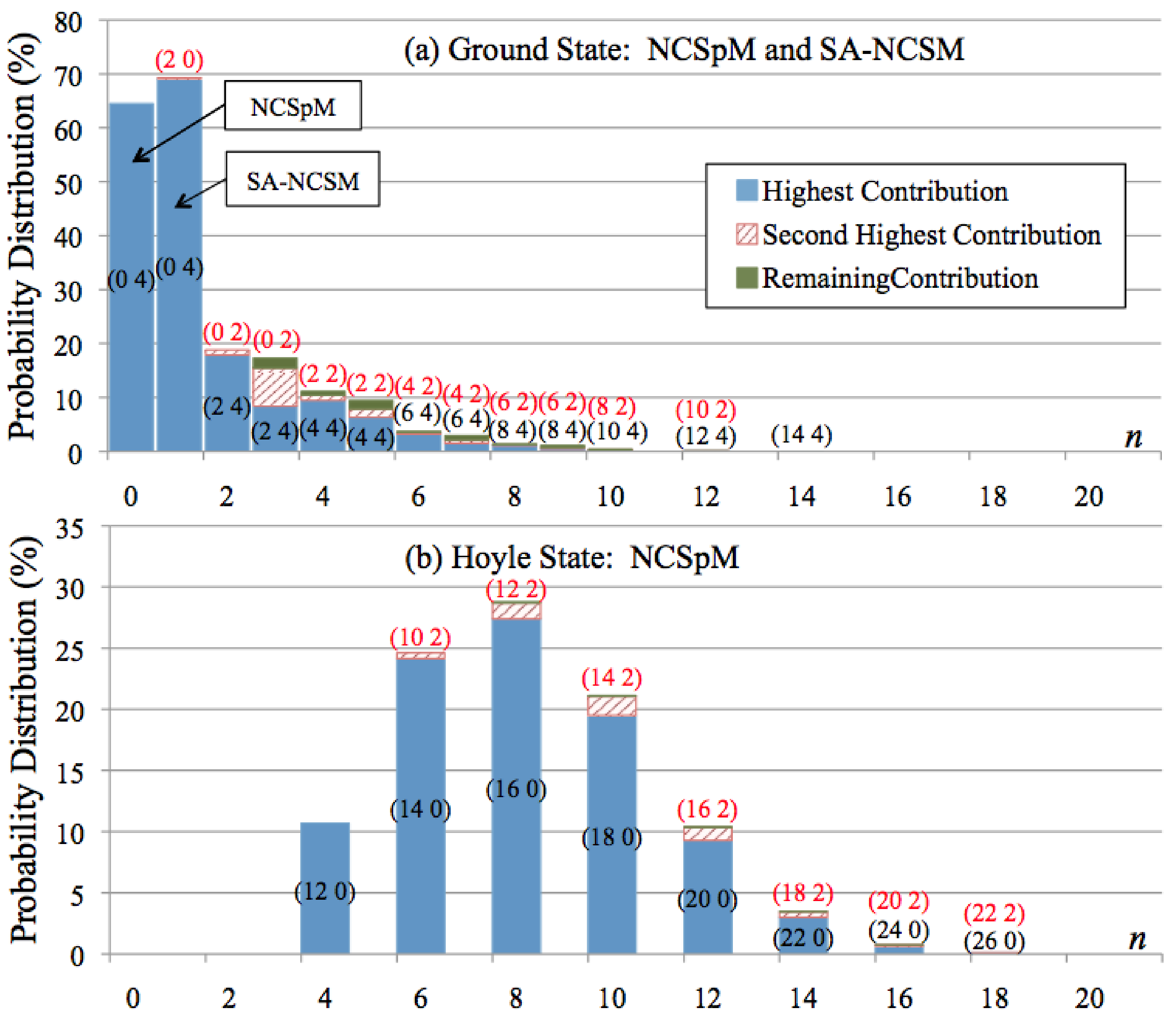
Problems

- We cannot obtain some levels, e.g., the Hoyle state predicted by Fred Hoyle in the 1950s and later confirmed experimentally. This state is essential for the production of the ^{12}C isotope in stars via the triple-α process, i.e., for the origin of life on Earth.

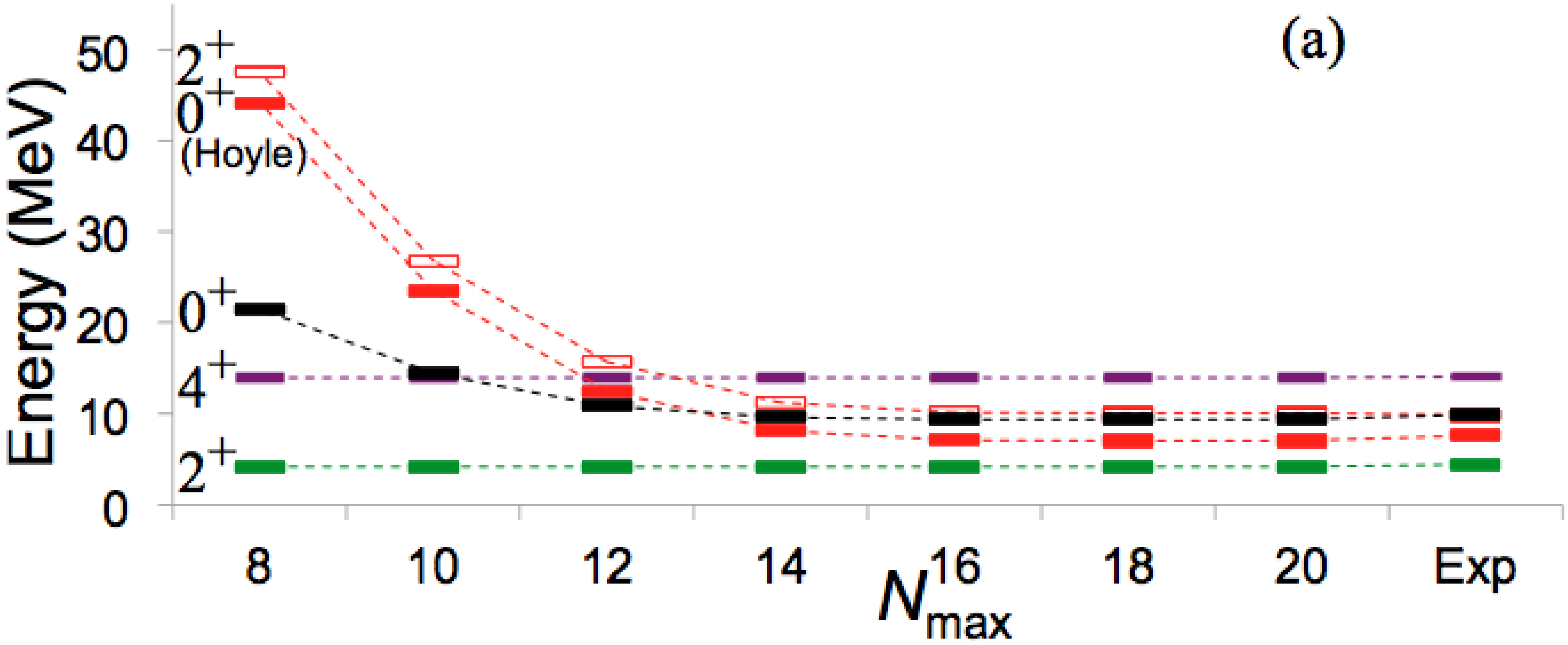


Hoyle state

Too large N_{max} required for Hoyle



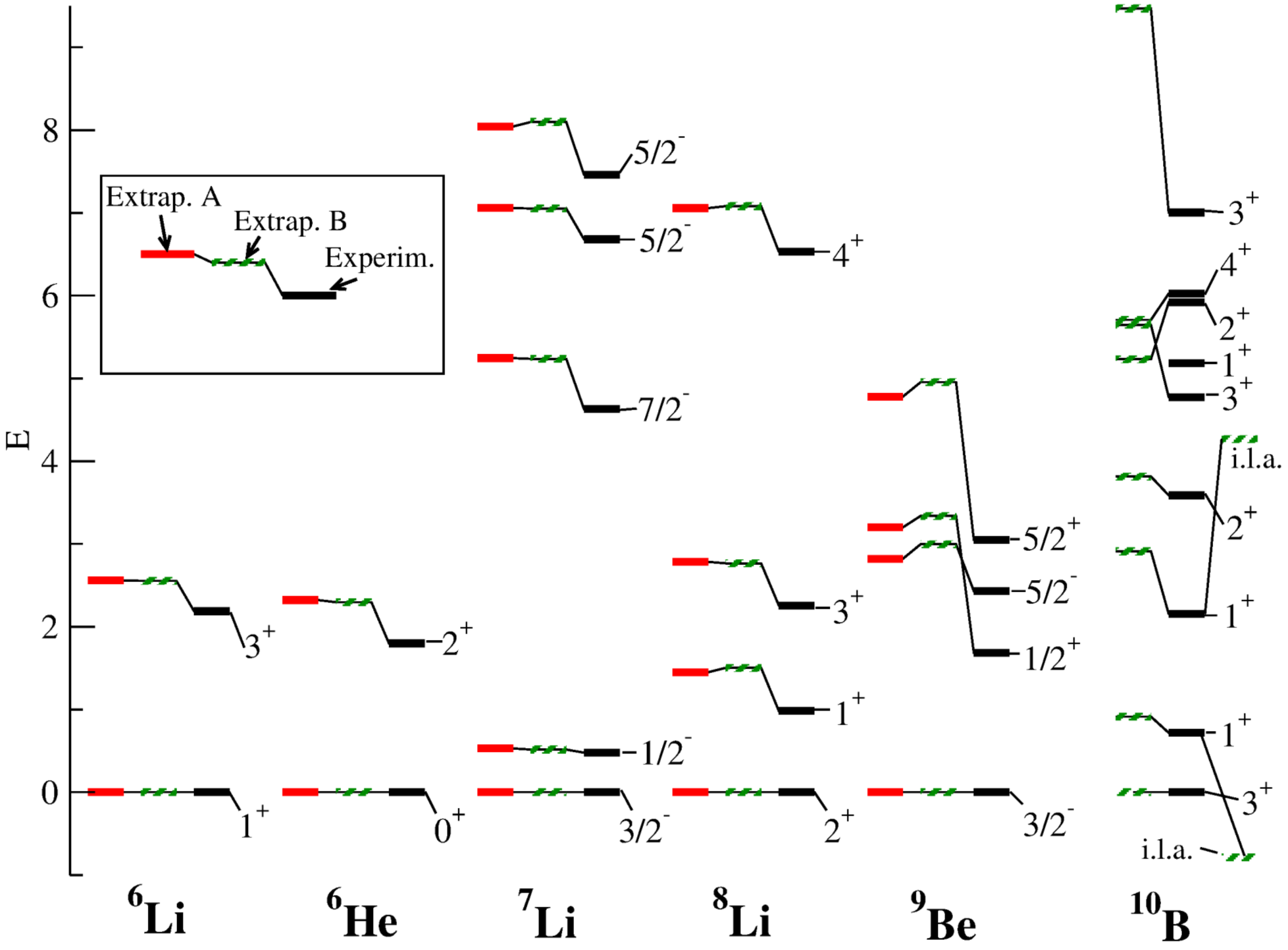
Hoyle state



Taken from A. C. Dreyfuss, K. D. Launey, T. Dytrych, J. P. Draayer, C. Bahri, arXiv:212.2255 (2012).

Obtained in symplectic NCSM

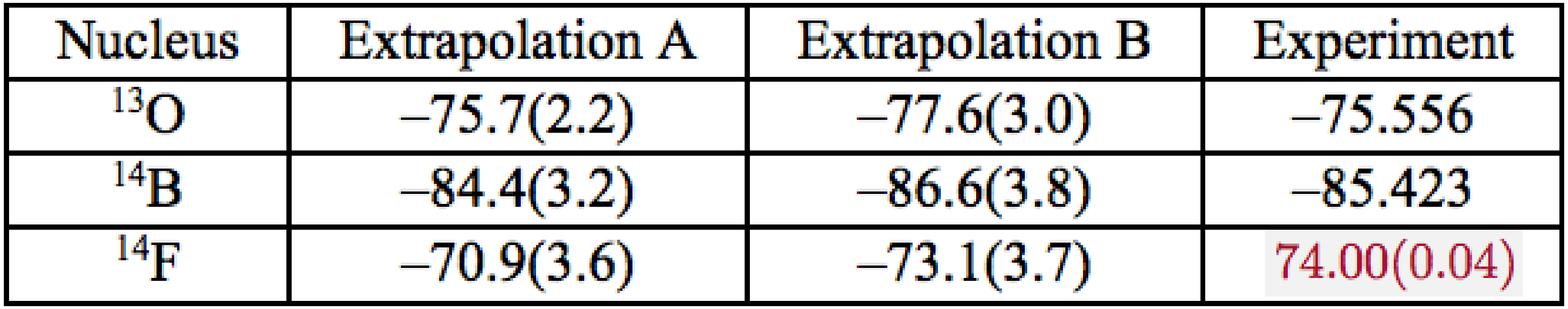
NCFC & JISP16: levels with Γ< 300 kev



Success of NCSM calculations with JISP16 interaction and NCFC extrapolations: Predictions of ^{14}F properties (2009)

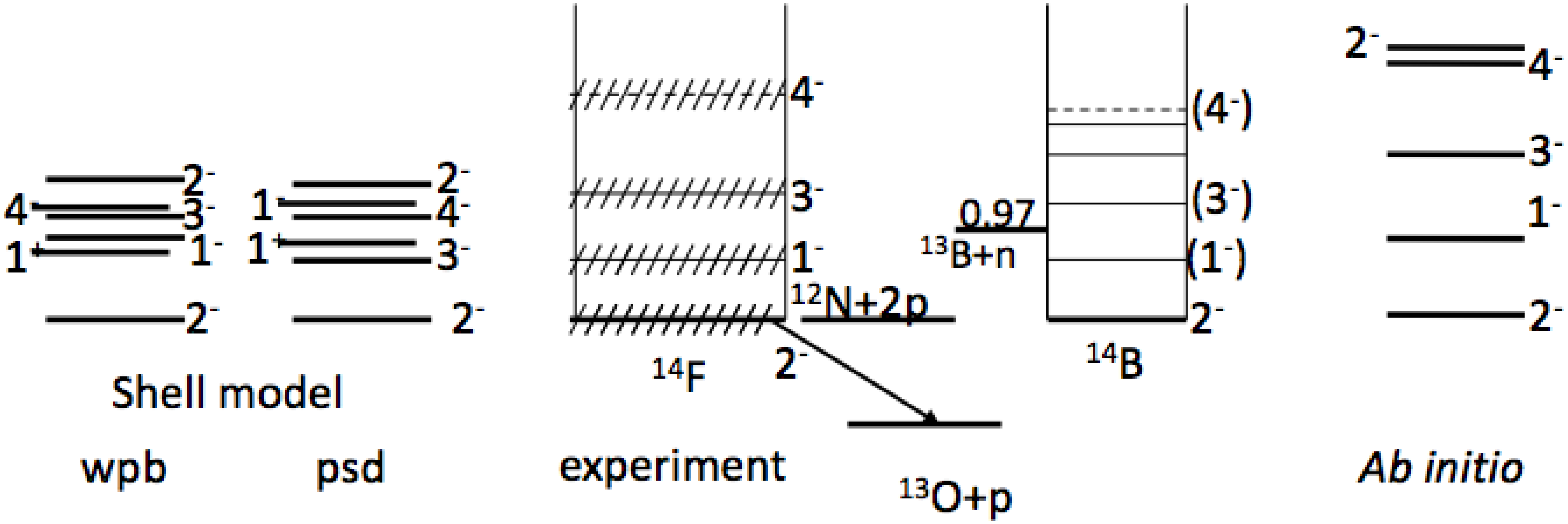
^{14}F

- 1,990,061,078 basis states in $N_{\text{max}} = 8$ model space
- each $\hbar\Omega$ point requires 2 to 3 hours on 7,626 quad-core compute nodes (30,504 processors in total) at the Jaguar supercomputer at ORNL



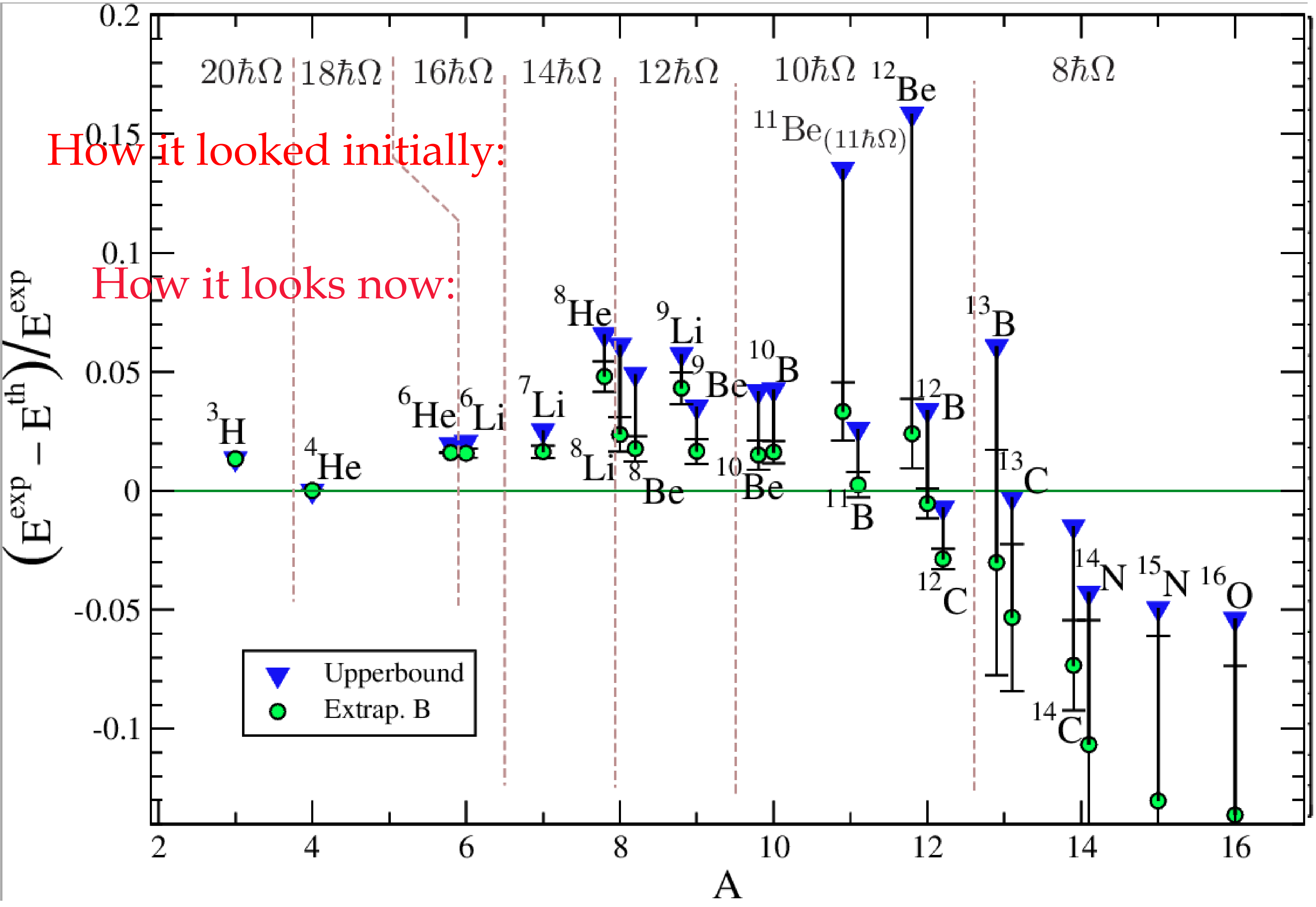
Nucleus	Extrapolation A	Extrapolation B	Experiment
^{13}O	–75.7(2.2)	–77.6(3.0)	–75.556
^{14}B	–84.4(3.2)	–86.6(3.8)	–85.423
^{14}F	–70.9(3.6)	–73.1(3.7)	74.00(0.04)

^{14}F spectrum



Back to JISP16: drawbacks

- Deficiency of JISP16 revealed by NCFC extrapolations and by the use of larger model spaces attainable due to new supercomputers



How it looked initially:

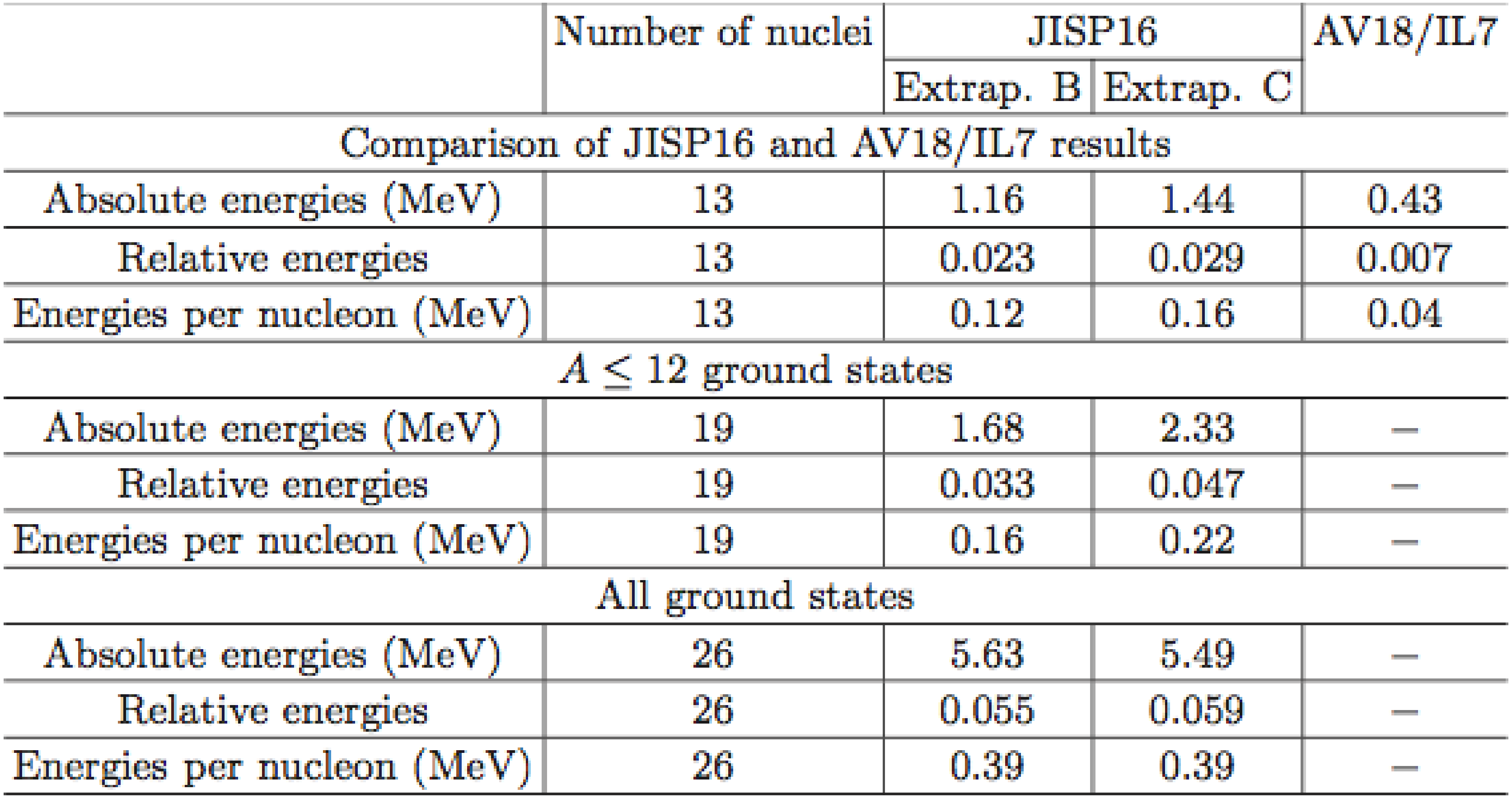
How it looks now:

Light nuclei with JISP16: comprehensive analysis

- 26 nuclei, 135 natural and unnatural parity states
- Analyzed rms deviations from experiment for absolute energies E_i^{th}, for energies per nucleon E_i^{th}/A, and rms for relative energies $(E_i^{th}-E_i^{exp})/E_i^{exp}$.

Light nuclei with JISP16: comprehensive analysis

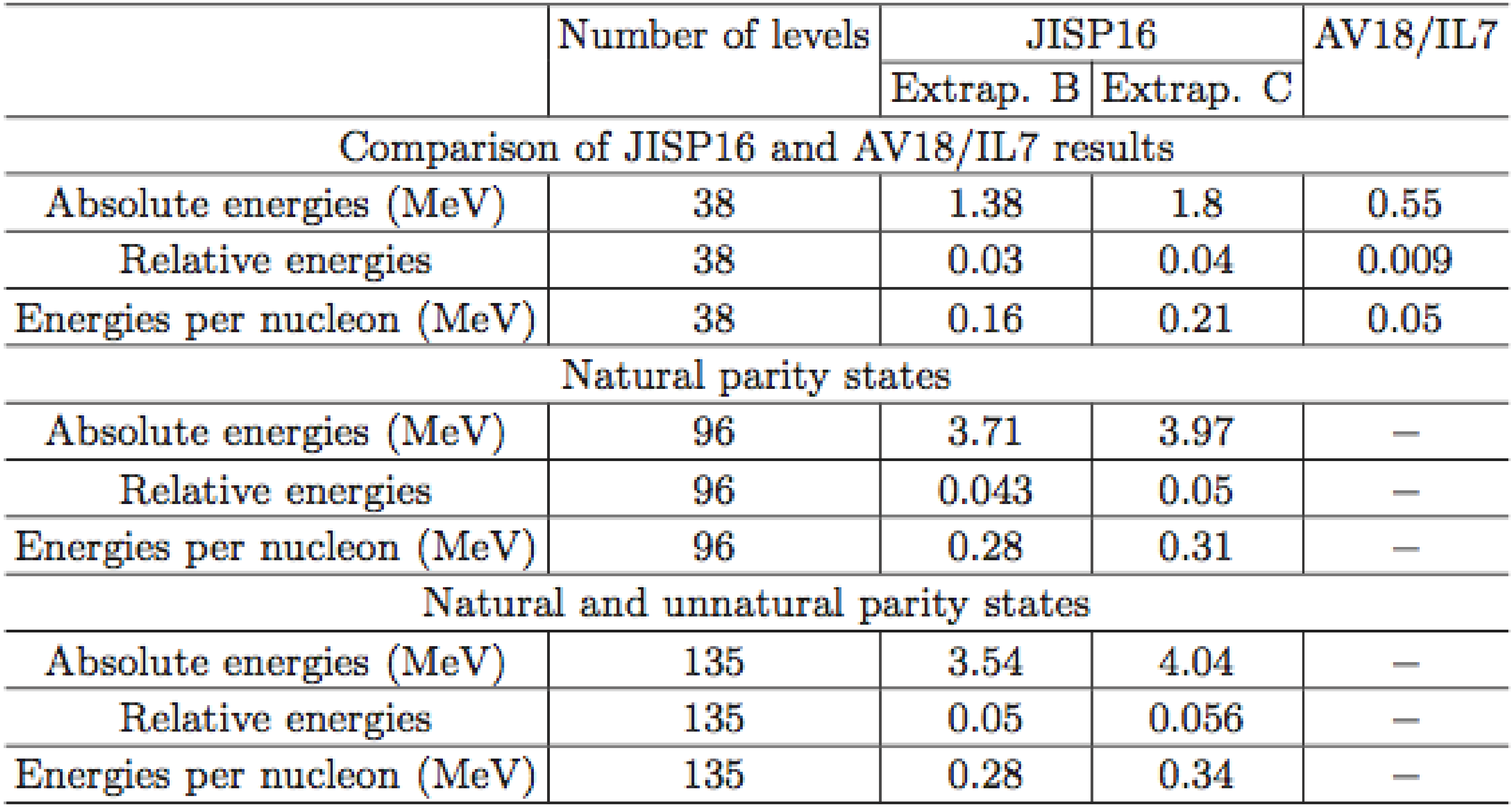
rms for binding energies



	Number of nuclei	JISP16		AV18/IL7
		Extrap. B	Extrap. C	
Comparison of JISP16 and AV18/IL7 results				
Absolute energies (MeV)	13	1.16	1.44	0.43
Relative energies	13	0.023	0.029	0.007
Energies per nucleon (MeV)	13	0.12	0.16	0.04
$A \leq 12$ ground states				
Absolute energies (MeV)	19	1.68	2.33	–
Relative energies	19	0.033	0.047	–
Energies per nucleon (MeV)	19	0.16	0.22	–
All ground states				
Absolute energies (MeV)	26	5.63	5.49	–
Relative energies	26	0.055	0.059	–
Energies per nucleon (MeV)	26	0.39	0.39	–

Light nuclei with JISP16: comprehensive analysis

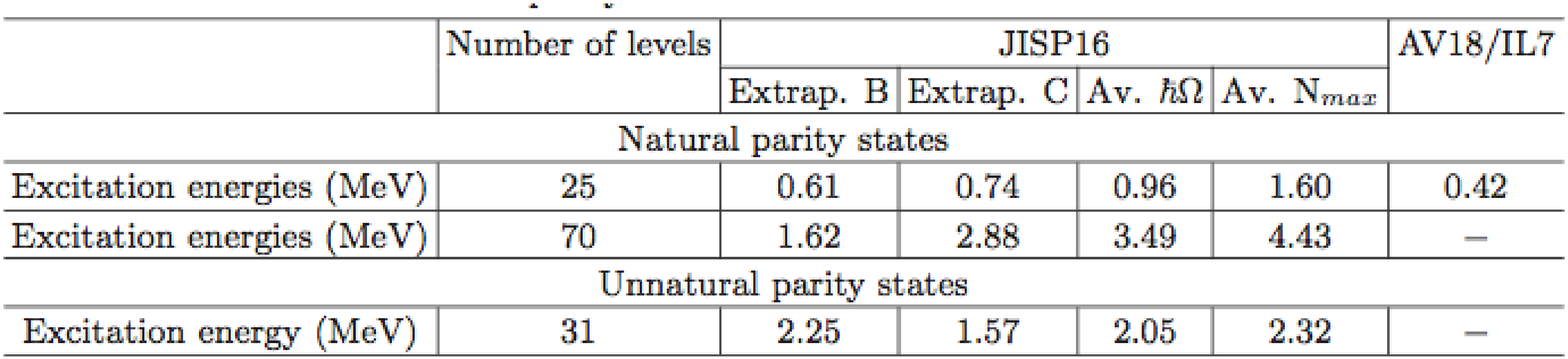
rms for level energies



	Number of levels	JISP16		AV18/IL7
		Extrap. B	Extrap. C	
Comparison of JISP16 and AV18/IL7 results				
Absolute energies (MeV)	38	1.38	1.8	0.55
Relative energies	38	0.03	0.04	0.009
Energies per nucleon (MeV)	38	0.16	0.21	0.05
Natural parity states				
Absolute energies (MeV)	96	3.71	3.97	–
Relative energies	96	0.043	0.05	–
Energies per nucleon (MeV)	96	0.28	0.31	–
Natural and unnatural parity states				
Absolute energies (MeV)	135	3.54	4.04	–
Relative energies	135	0.05	0.056	–
Energies per nucleon (MeV)	135	0.28	0.34	–

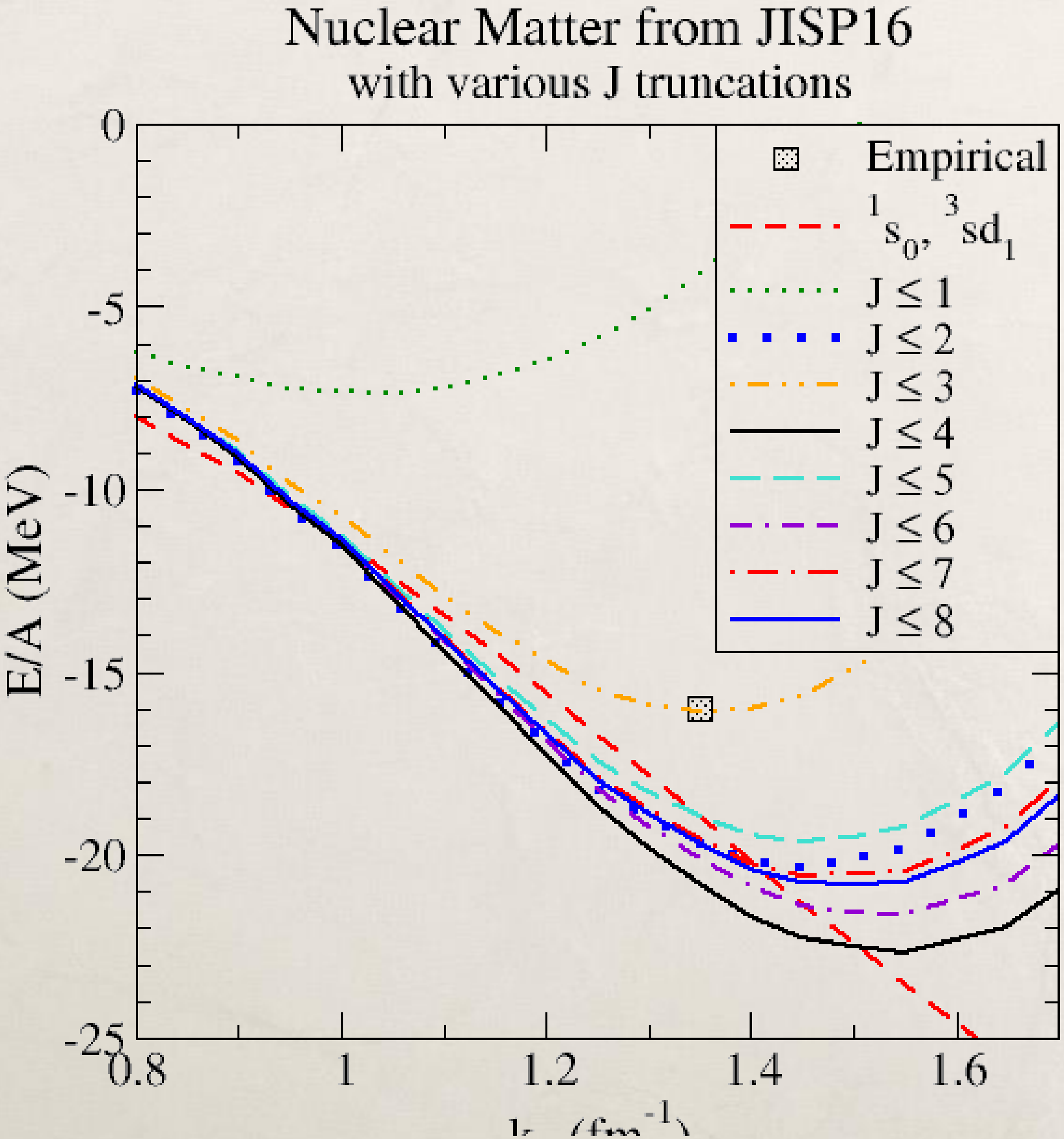
Light nuclei with JISP16: comprehensive analysis

rms for excitation energies



	Number of levels	JISP16				AV18/IL7
		Extrap. B	Extrap. C	Av. $\hbar\Omega$	Av. N_{max}	
Natural parity states						
Excitation energies (MeV)	25	0.61	0.74	0.96	1.60	0.42
Excitation energies (MeV)	70	1.62	2.88	3.49	4.43	–
Unnatural parity states						
Excitation energy (MeV)	31	2.25	1.57	2.05	2.32	–

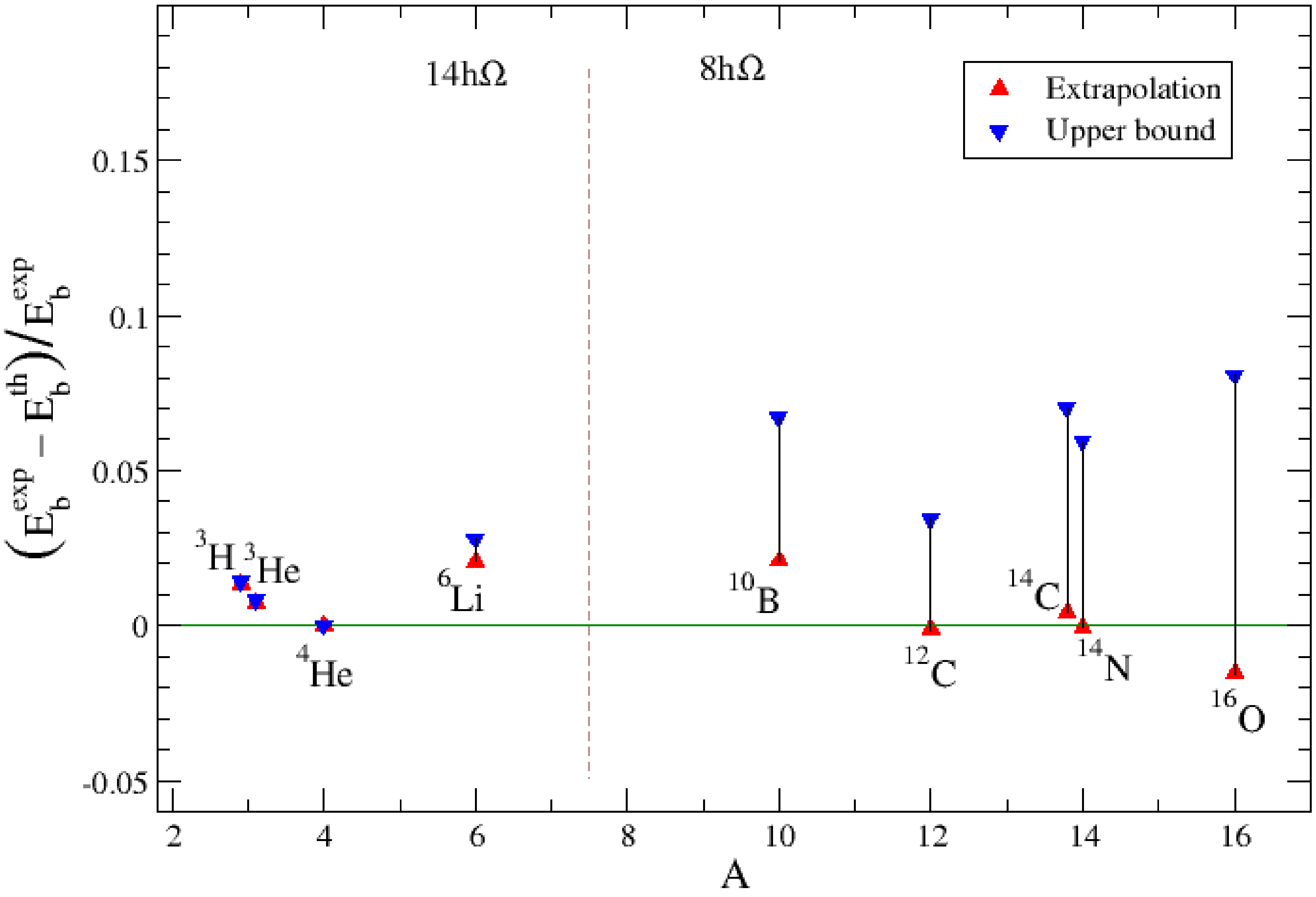
Nuclear matter with JISP16



Nuclear matter with JISP16

- Nuclear matter is a model that somehow simulates bulk properties of heavy nuclei.
- Surprisingly strong J dependence of nuclear matter equation of state even for high J in case of JISP16 NN interaction.
- Light nuclei are insensitive to high-J components of the NN interaction. Hence it will be possible to fit JISP16 to nuclear matter properties by PETs in high-J partial waves. This will be interesting!

Binding energies



JISP16$_{2010}$

- JISP16$_{2010}$ is still somewhat preliminary version of the interaction: it is needed to calculate more nuclei and to check it in more detail. However it is clear that it improves JISP16 essentially

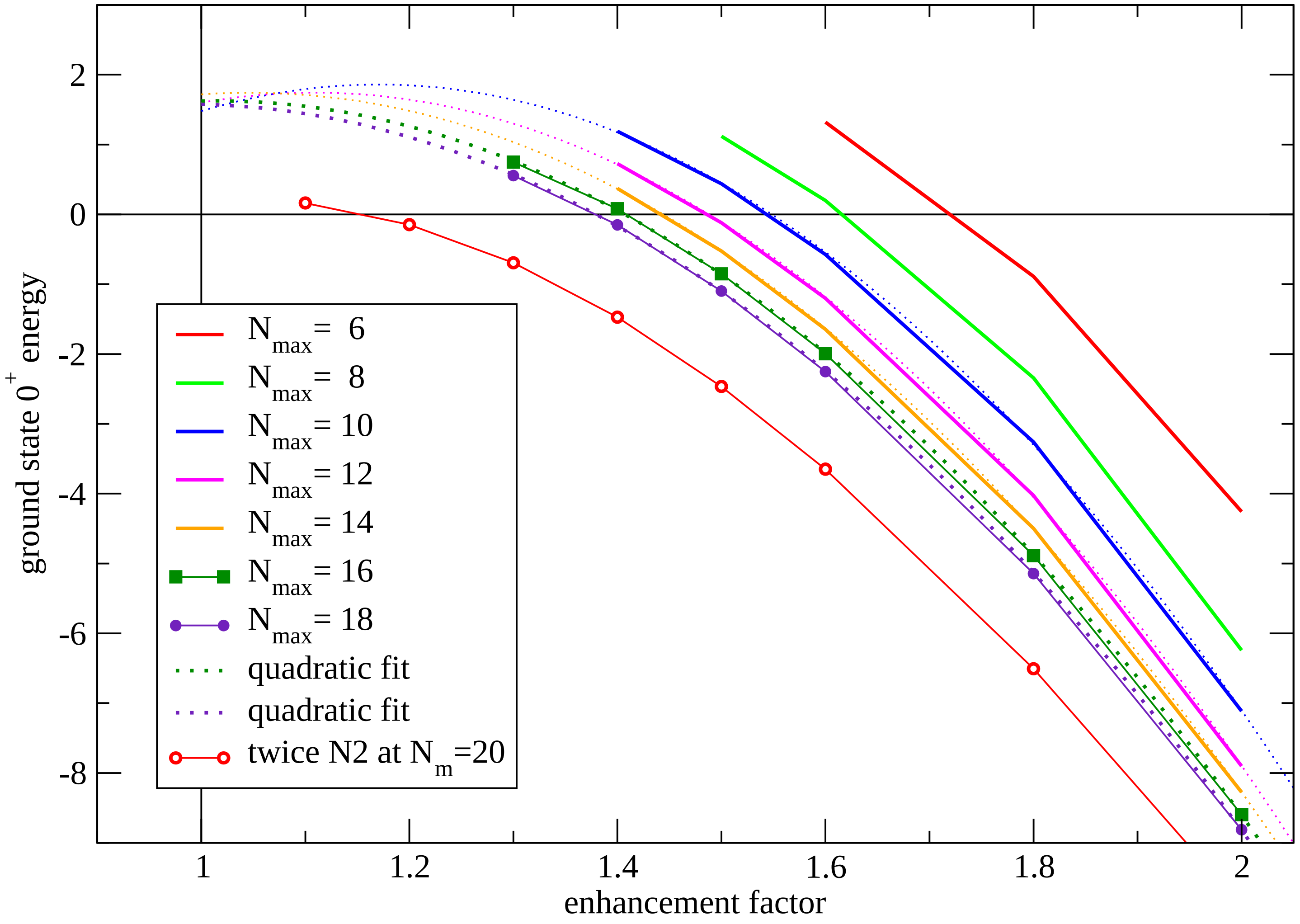
Tetraneutron

- Interest from experimentalists
- How to evaluate the energy of unbound 4n system?
- Increase JISP16 to obtain bound 4 neutron state
- Use NCSM to obtain ground state energy
- Extrapolate to *NN* interaction without enhancement

Tetraneutron

4 neutrons with enhanced JISP16

variational min. (if there is a local var. min...) on available hw grid



Tetraneutron

- How to evaluate the energy of unbound 4n system?
- Why not to try to do the same with PETs?
- I. e. to bind 4n by PETs
- and to leave unchanged ^{4}He binding…

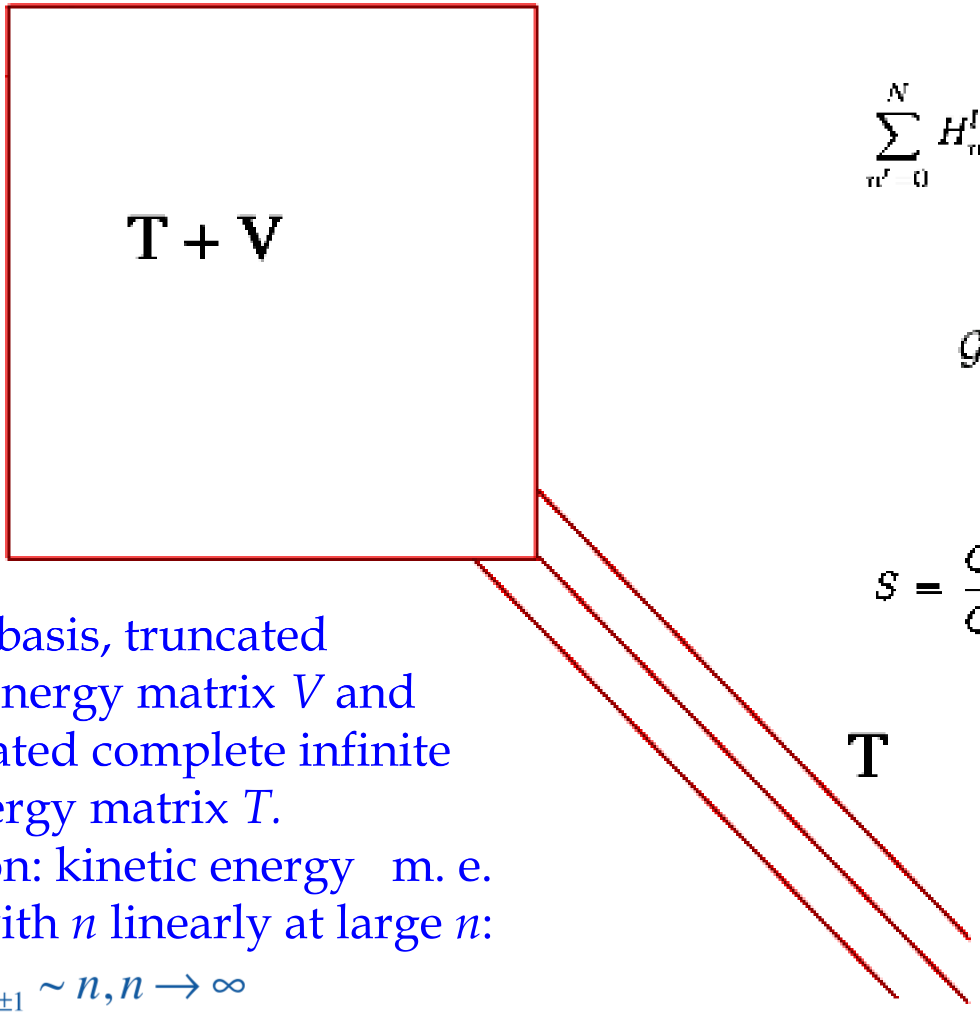
Tetraneutron: The power of PETs

- It is possible!
- Fits in small model spaces, hence not very accurate.
- The PETted JISP16 provides the following NCFC extrapolated results:
- ^{4}He energy: -29.634 MeV (exp. -28.296 MeV) + low-lying state (J,T)=(3,1)
- ^{3}H energy: -8.231 MeV (exp. -8.482 MeV)
- 3n unbound
- 4n: two bound states: (J,T)=(2,2) at -14.1 MeV and

  (J,T)=(0,2) at -6 MeV

J-matrix formalism:

scattering in the oscillator basis



$$\sum_{n'=0}^{N} H^{l}_{nn'}(n'|\lambda) = E_{\lambda}(n|\lambda), \qquad n \leq N.$$

$$\mathcal{G}_{NN}(E) = -\sum_{\lambda=0}^{N} \frac{(N|\lambda)^2}{E_\lambda - E},$$

$$S = \frac{C^{(-)}_{Nl}(q) - \mathcal{G}_{NN}(E)\, T^{l}_{N,\,N+1}\, C^{(-)}_{N+1,\,l}(q)}{C^{(+)}_{Nl}(q) - \mathcal{G}_{NN}(E)\, T^{l}_{N,\,N+1}\, C^{(+)}_{N+1,\,l}(q)},$$

Oscillator basis, truncated potential energy matrix *V* and non-truncated complete infinite kinetic energy matrix *T*. Justification: kinetic energy m. e. increase with *n* linearly at large *n*: $T_{nn} \sim n, T_{n,n\pm1} \sim n, n \to \infty$ while potential energy m. e. V_{nm} decrease with *n* and *m*.

Both direct and inverse scattering *J*-matrix solutions are possible.

Tetraneutron: *J*-matrix formalism

- Impossible to calculate all E_λ

At $E = E_\lambda$: $\quad S(E_\lambda) = \dfrac{C^{(-)}_{N+1,K}(E_\lambda)}{C^{(+)}_{N+1,K}(E_\lambda)}$

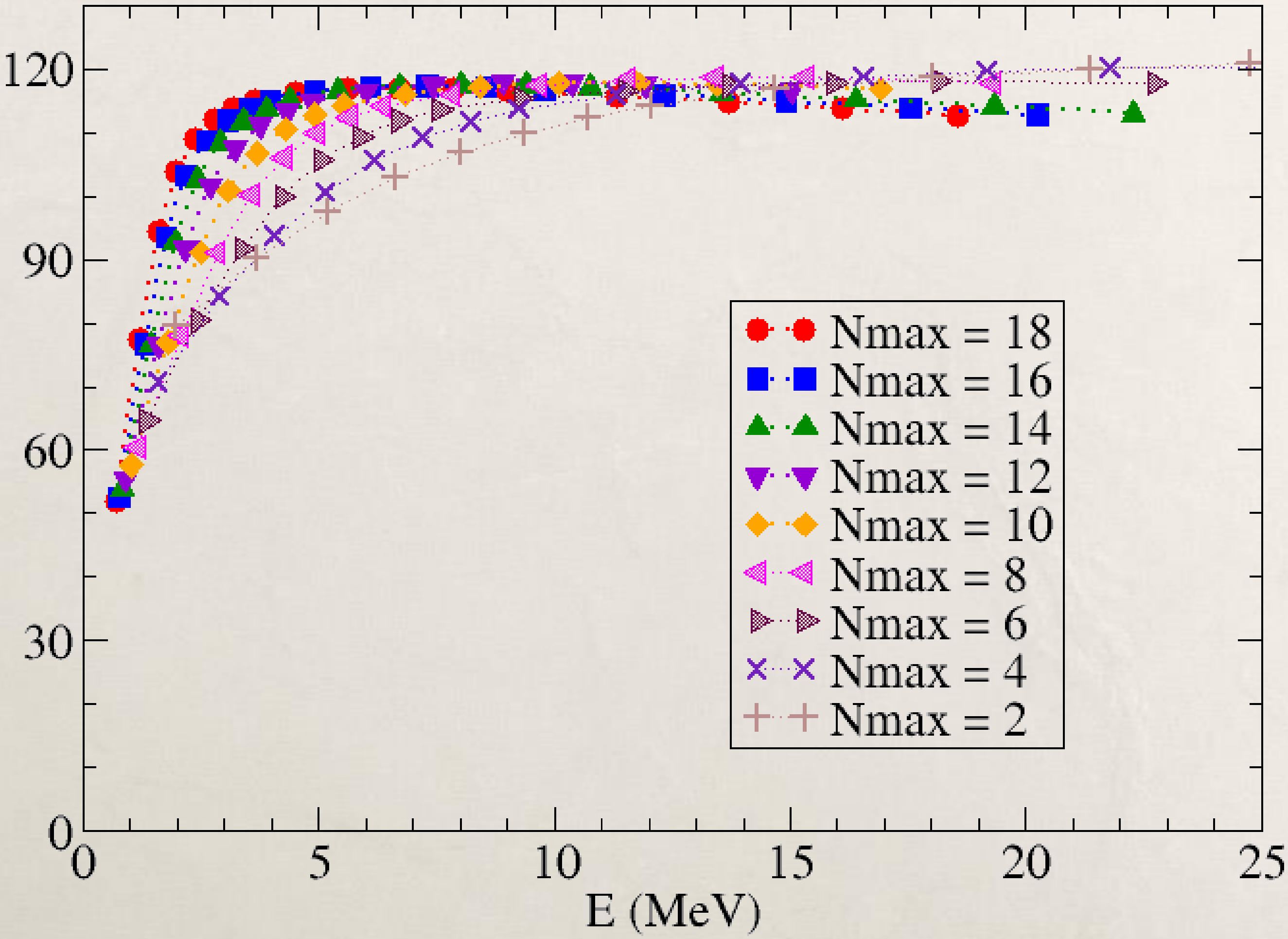
4-body decay, hyperspherical formalism, lowest possible hypermomentum *K*

In the vicinity of the resonance

$$S(E) = e^{2i\delta}$$
$$\delta(E) = \tfrac{\pi}{2} + \arctan\tfrac{E-b}{a\sqrt{E}} + c\sqrt{E}$$

a, *b*, *c* – fitting parameters

Tetraneutron:
J-matrix formalism



$E_r = 0.56$ MeV

$\Gamma_r = 2.98$ MeV

Future

- Involving more observables including resonance energies and widths
- Improved JISP16$_{2010}$ fitted also to nuclear matter
- PETted N3LO to avoid use *NNN* forces
- …

Present

Thank you!