Computational approaches to many-body dynamics of unstable nuclear systems

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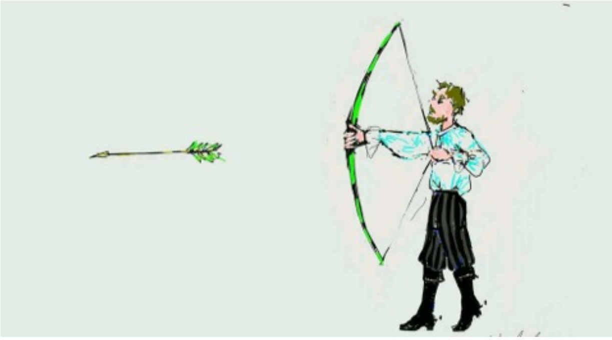
Physics and mathematics of instability and decay

Zeno paradox of arrow (490–430 BC)

The time is composed of moments therefore the flying arrow is motionless. Zeno of Elea , see also Aristotle, *Physics* VI:9, 239b5



Quantum Zeno effect. When a watched pot boils? (1983)

An unstable particle, if observed continuously, will never decay - Maddox, Nature 306, 111-111, 1983 P. T. Greenland, Nature 387, 548-549, 1997

Power-law decay(1961)

A classical source with energy dispersion will exhibit a power-law decay at remote times. R.G Newton. Ann.Phys., 14(1):333, 1961

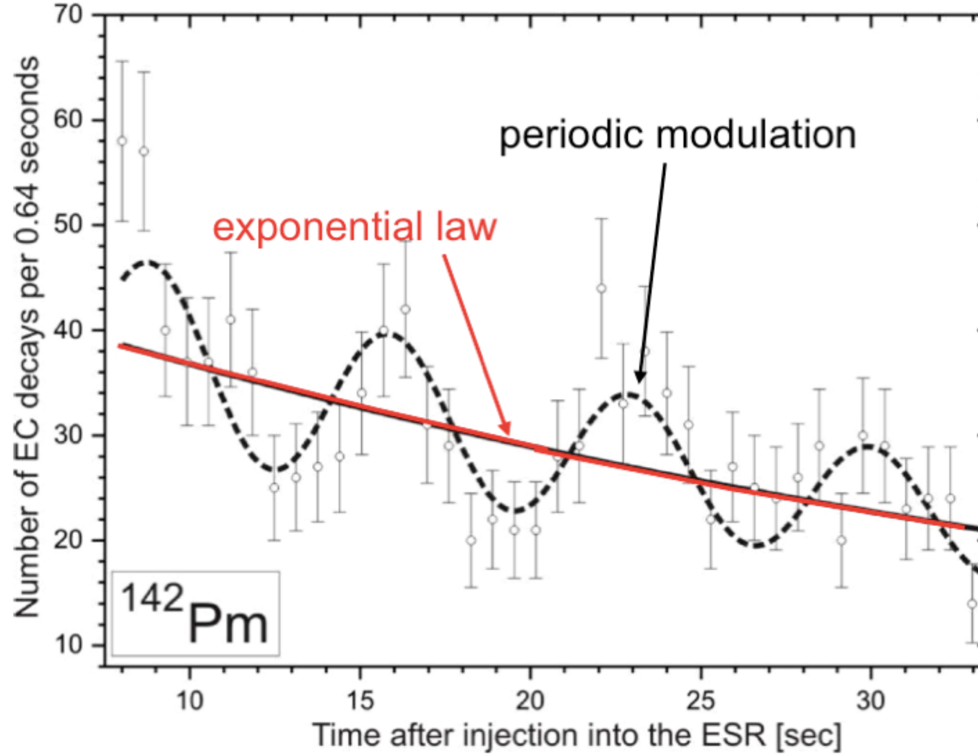
Decays cannot be exponential in the quantum world

L. A. Khalfin, JETP 6 (6), 1958; J. Schwinger Ann.Phys., 9, 169 1960; R. G. Winter Phys Rev, 123,1503 1961.

Discussion continues: Is radioactive decay exponential?

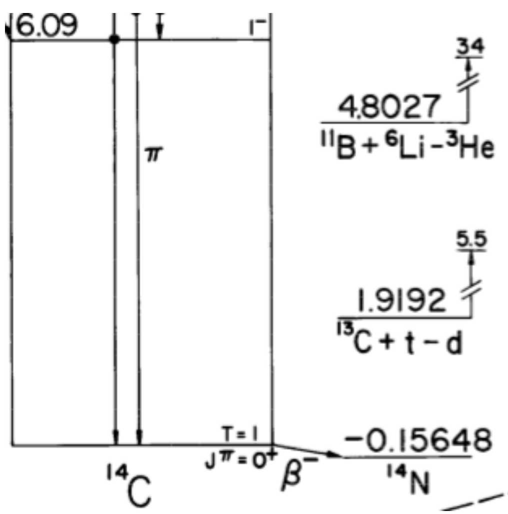
The GSI oscillations Mystery (2008)

Periodic modulation of the expected exponential law in EC-decays of different highly charged ions - Litvinov et al: Phys. Lett. B664, 162 (2008)



Carbon dating and non-exponential decay (2012)

"If the decay of 14C is indeed non-exponential... this would remove a foundation stone of modern dating methods. Aston EPL 97 (2012) 52001.



Half life 5,730 ± 40 years

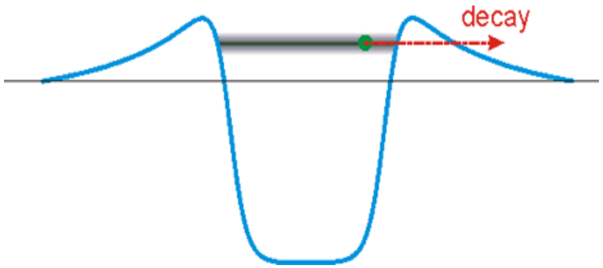
mean-life time 8,033 years

$$ {}^{14}_{6}\mathrm{C} \longrightarrow {}^{14}_{7}\mathrm{N} + e^{-} + \bar{\nu}_e $$

Quantum mechanics of decay

Why exponential decay? $\frac{dN(t)}{dt} = -\Gamma N(t) \quad N(t) = N(0)\, e^{-\Gamma t}$

Time evolution and decay in quantum mechanics

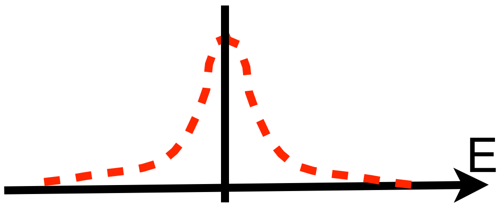


$$\psi(t) = e^{-iHt/\hbar}\psi(0)$$

Survival amplitude and probability

$$A(t) = \langle\psi(0)|e^{-iHt/\hbar}|\psi(0)\rangle \qquad P(t) = |A(t)|^2$$

Resonance wave function



$$\psi_R(t) = \exp\left[-\frac{i}{\hbar}\left(E_0 - i\frac{\Gamma}{2}\right)t\right]\psi_R(0)$$

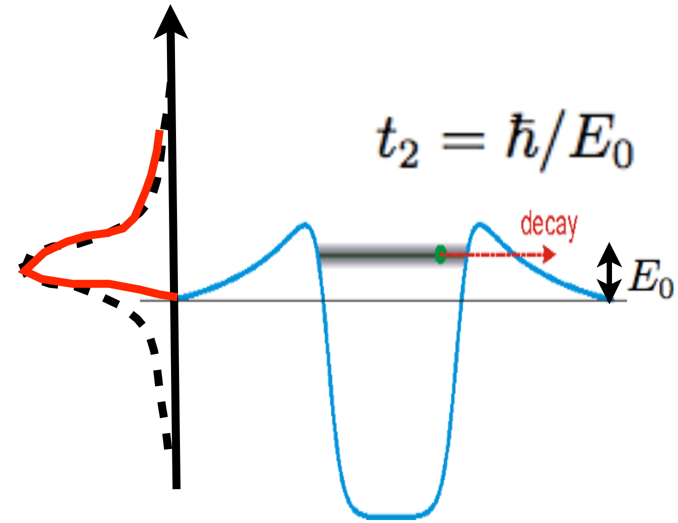
$$|\psi_R(E)|^2 \propto \frac{\Gamma/2}{(E-E_0)^2 - \Gamma^2/4}$$

Why and when decay cannot be exponential

Initial state "memory" time $e^{-iHt/\hbar} \approx 1 - iHt/\hbar \ldots \quad t_1 = \hbar/(\Delta E) \quad t < t_1$

Internal motion in quasi-bound state

$t_2 = \hbar/E_0$



$$|\psi_R(E)|^2 \propto \frac{\Gamma/2}{(E-E_0)^2 - \Gamma^2/4}.$$

$$t < t_2$$

Remote power-law $t > t_3$

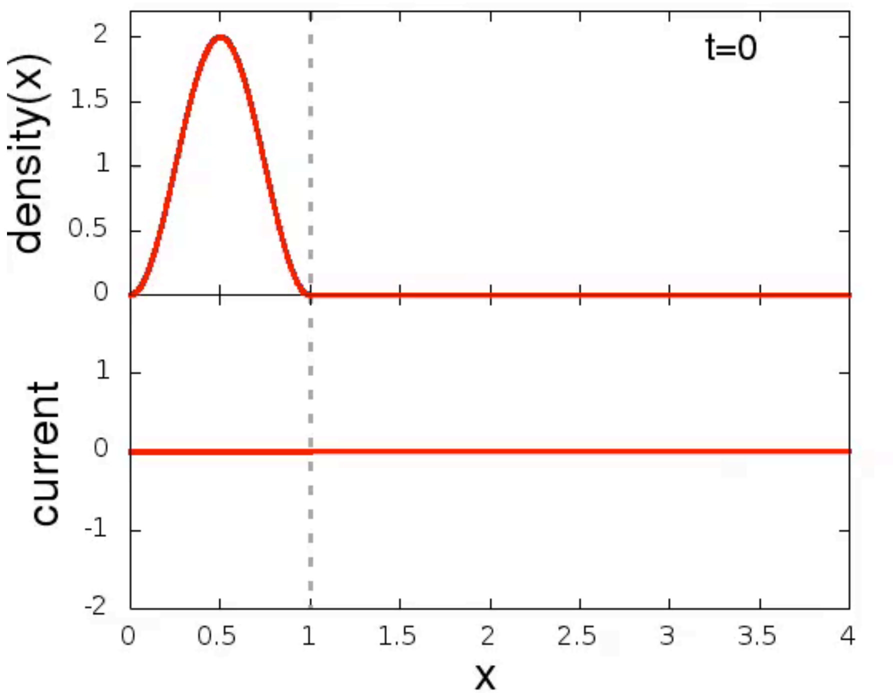
There are "free" slow-moving non-resonant particles, they escape slowly

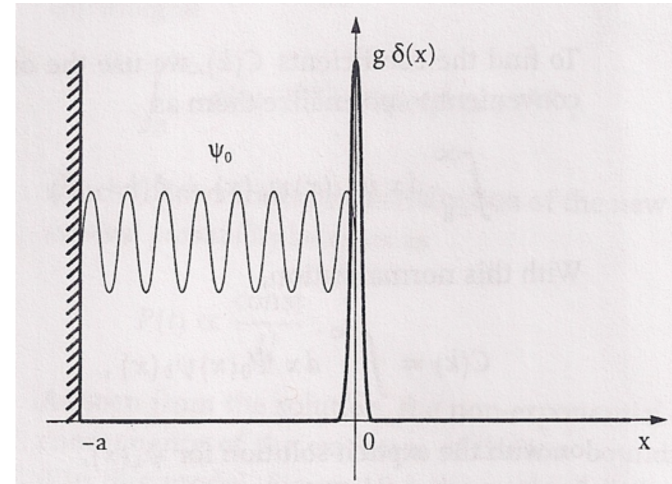
$$N(t) \propto \frac{\Delta x}{vt} = \frac{\hbar}{mv^2 t} = \frac{2\hbar}{E_0 t} \propto |\psi_N(t)|^2 \qquad \Delta x = \frac{\hbar}{mv} \qquad t_3 = \frac{\hbar}{\Gamma} \ln\left(\frac{E_0}{\Gamma}\right)$$

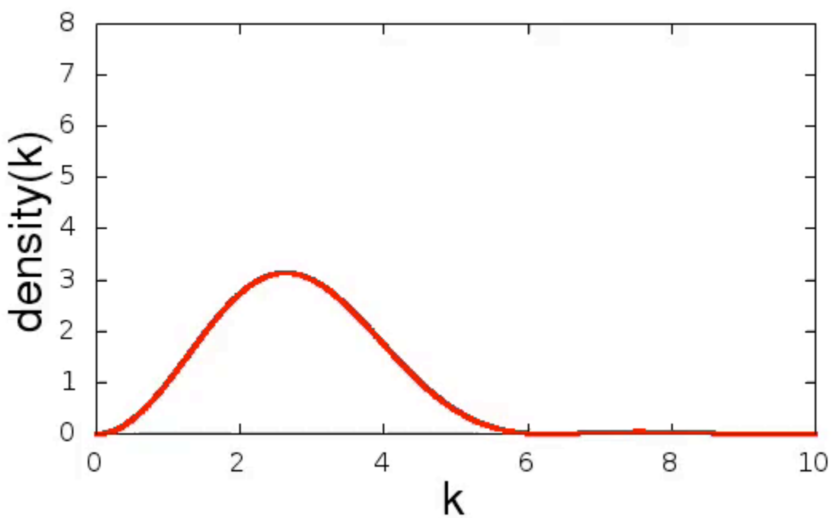
Example ^{14}C decay: E_0=0.157 MeV $t_2=10^{-21}$ s $\quad \ln\left(\frac{E_0}{\Gamma}\right)$=73

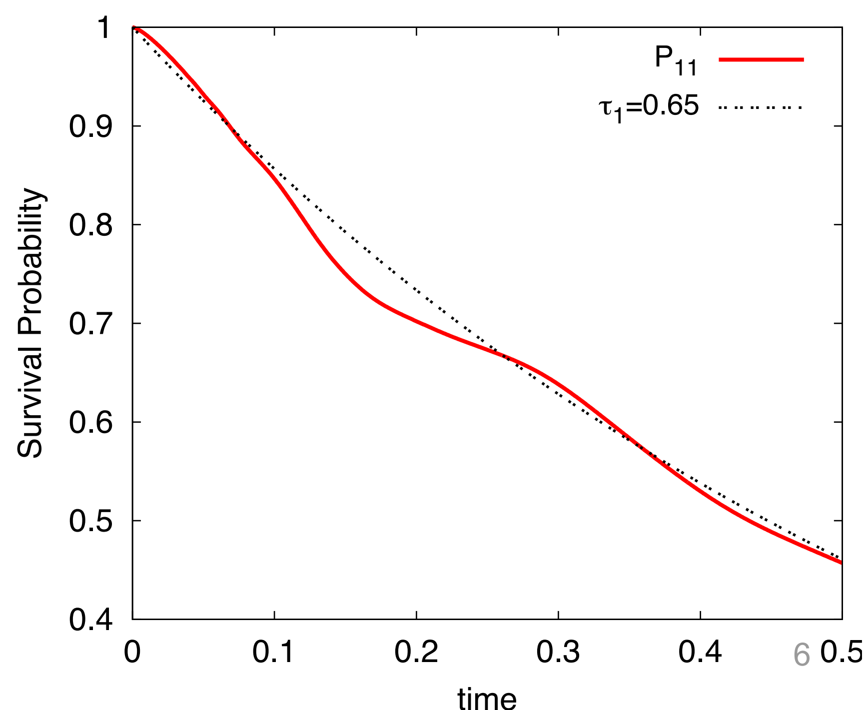
Time dependence of decay, Winter's model

Winter, Phys. Rev., ***123****,1503 1961.*

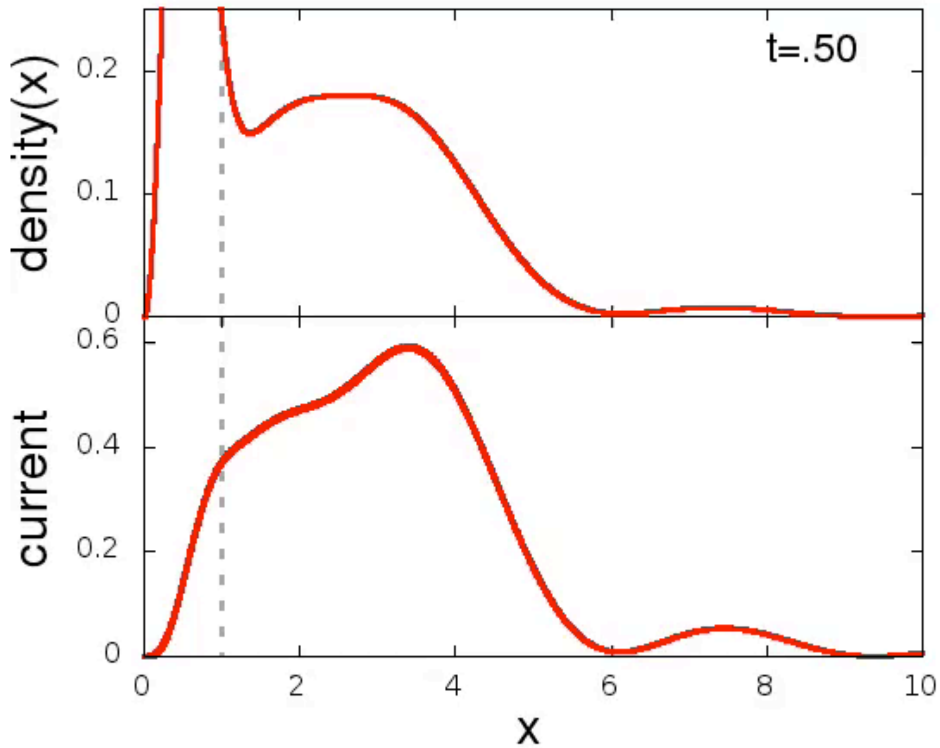


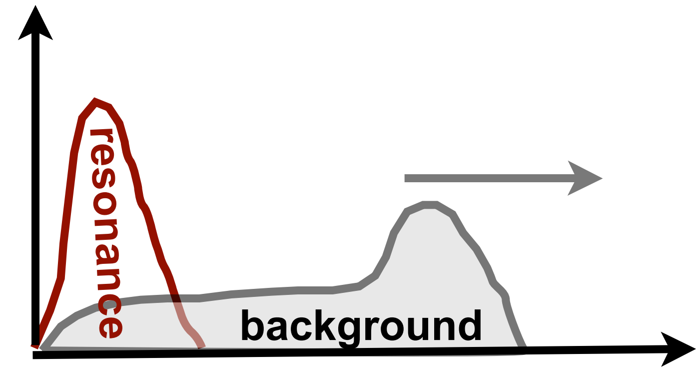


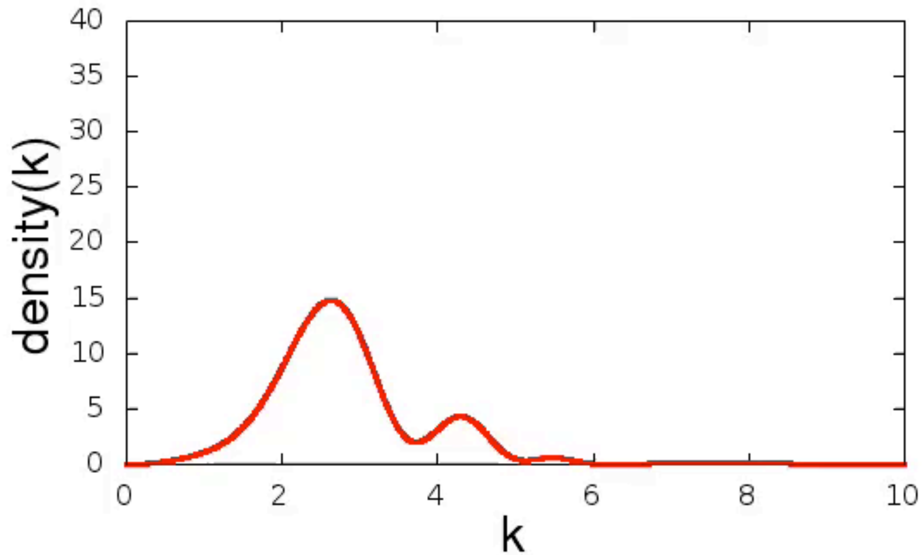


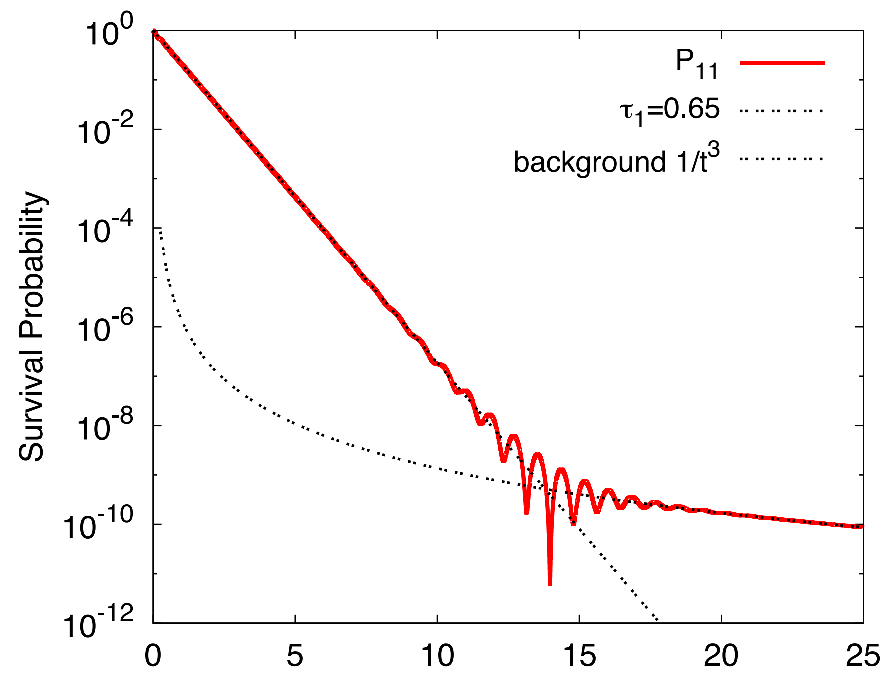


Winter's model: Dynamics at remote times

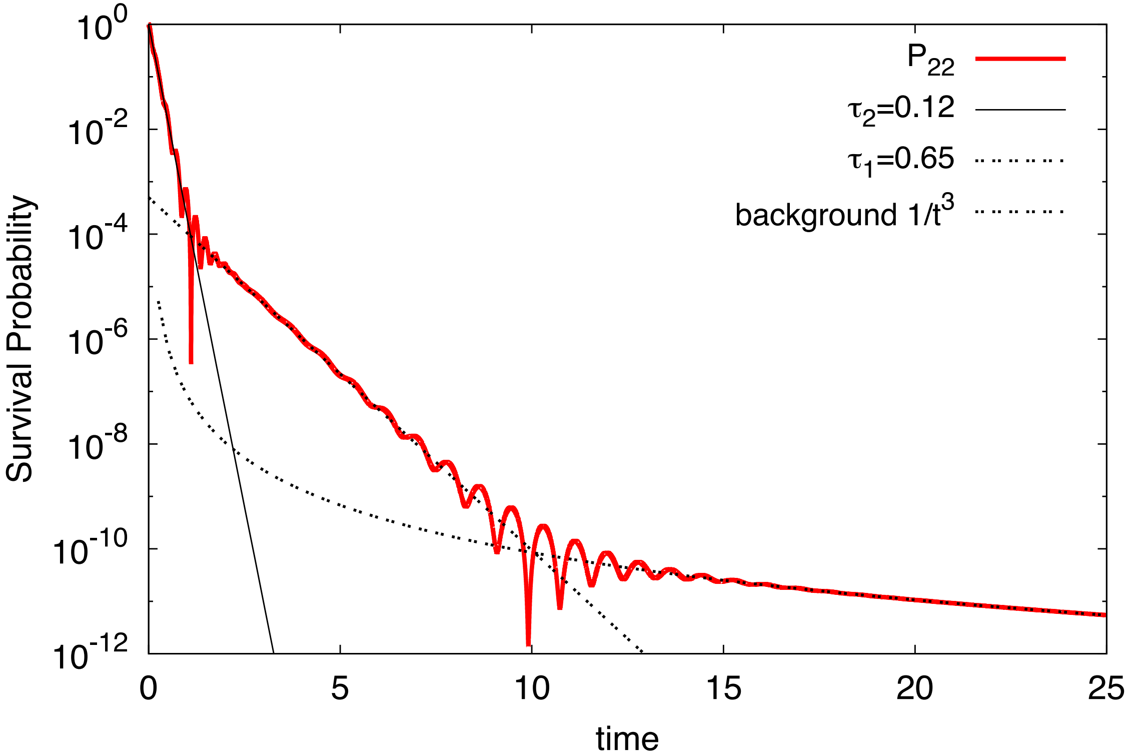


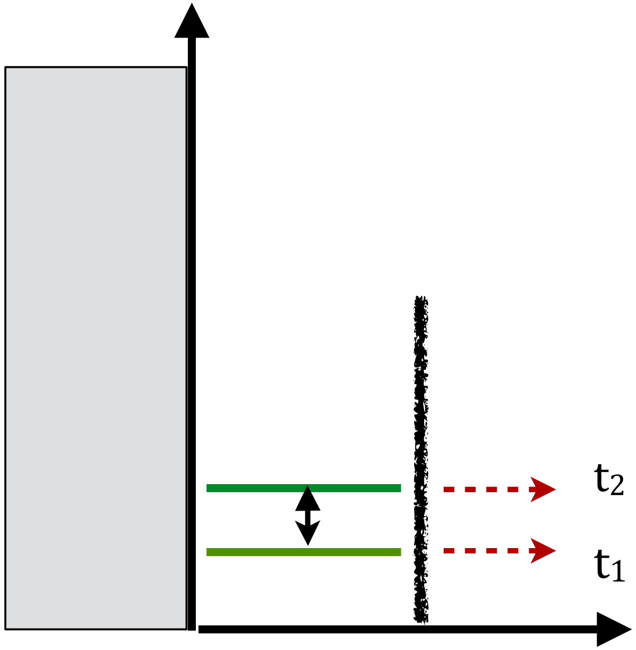






Internal dynamics in decaying system
Winter's model





Internal dynamics in decaying system
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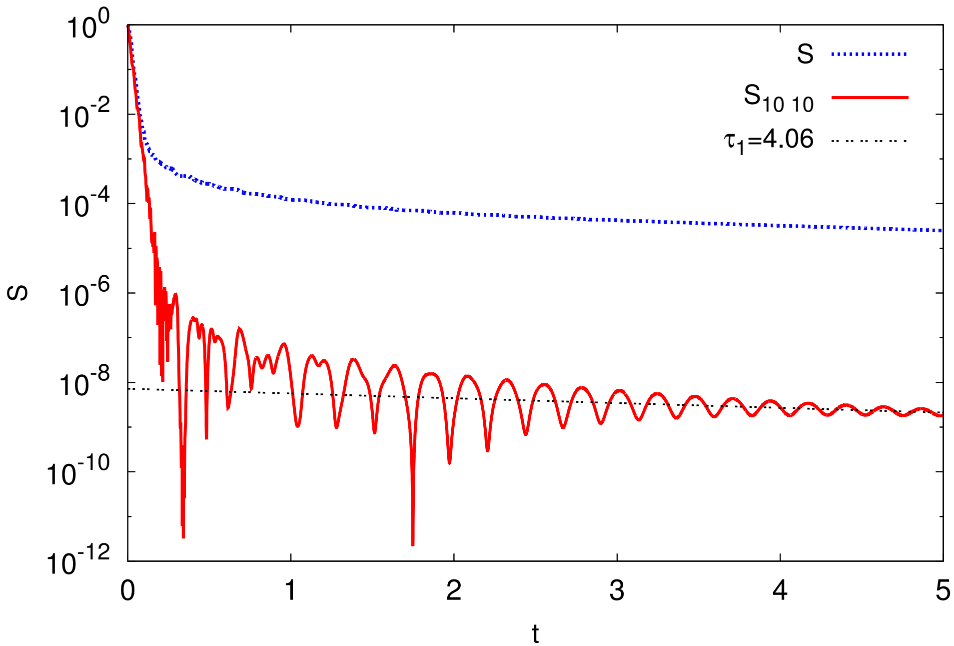
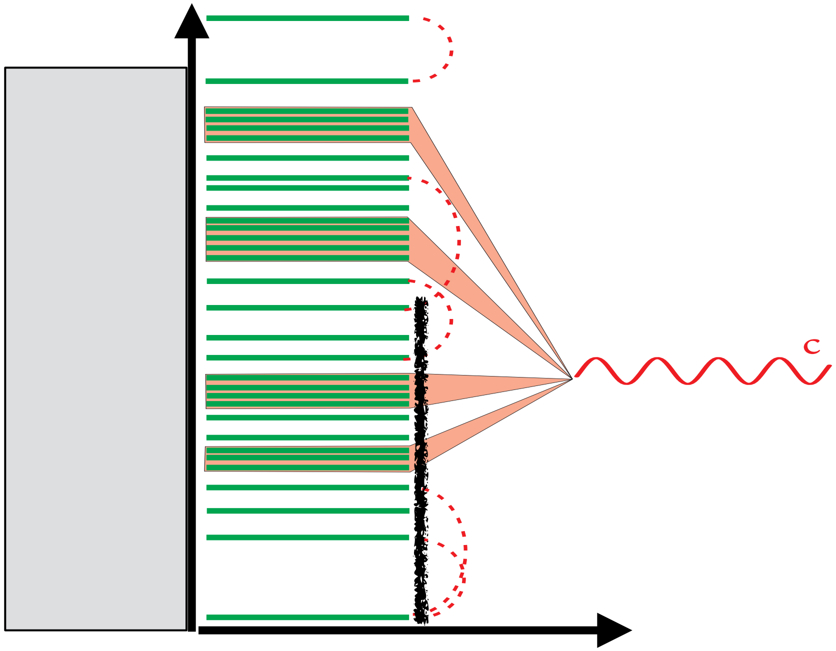
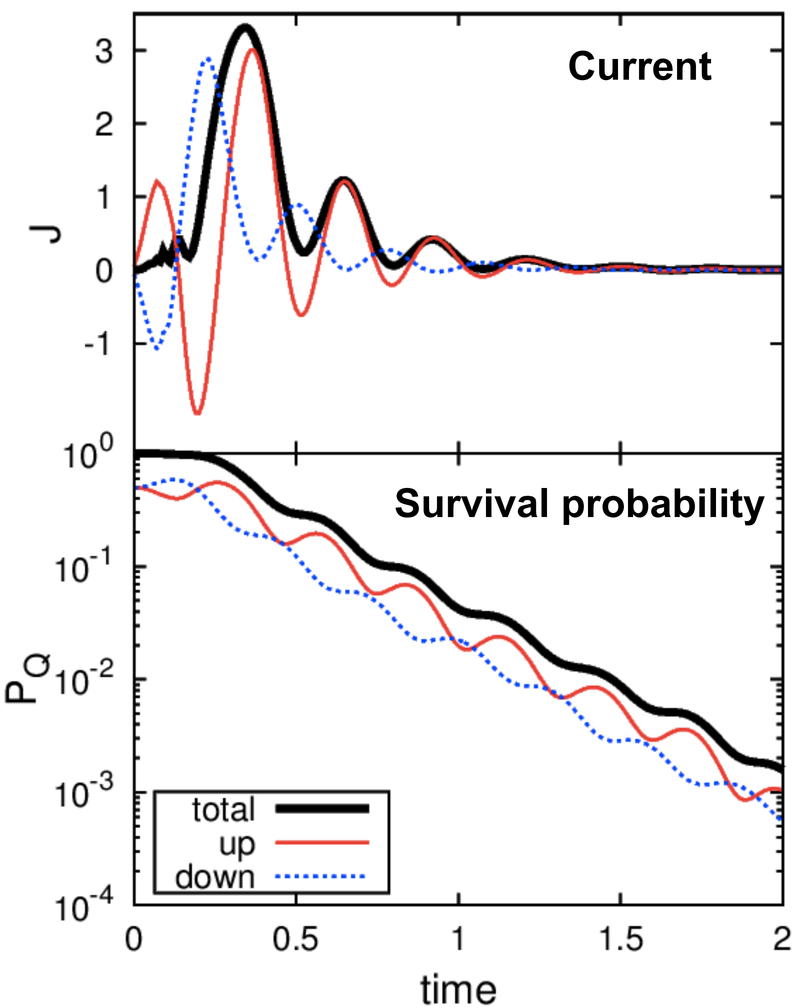




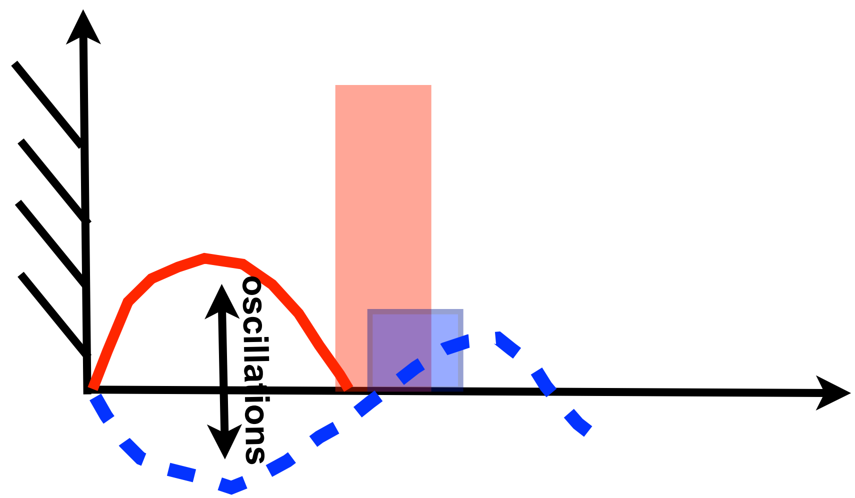
- Internal dynamics can be very complicated and chaotic.
- Transitions are driven by continuum coupling.
- Definitions of survival probability

A note on decay oscillations and neutrino mixing



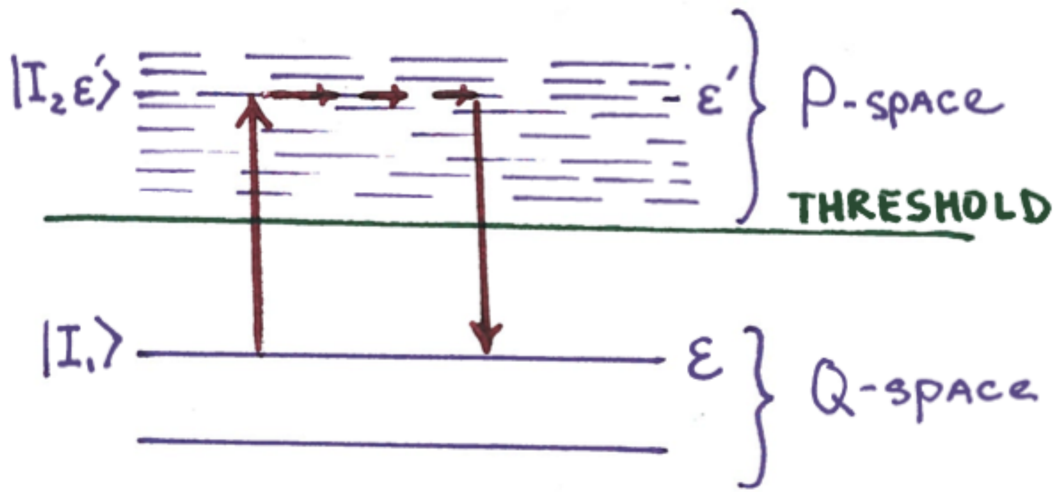
Decay oscillations are possible

- Kinetic energy - mass eigenstates
- Interaction (barrier)- flavor eigenstates
- Fast and slow decaying modes



[1] M. Peshkin, A Volya, and V. Zelevinsky, submitted to EPL

Physics of coupling to continuum



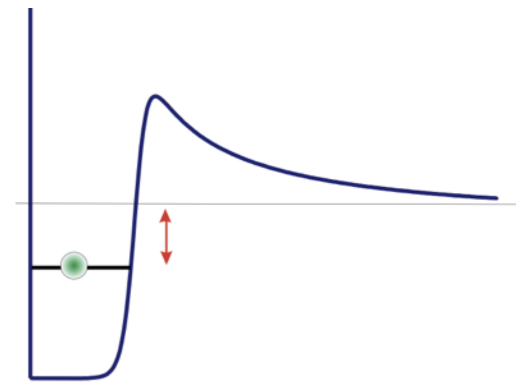
The role of continuum-coupling

$$H'(\epsilon) = \int_0^\infty d\epsilon' \, A^*(\epsilon') \frac{1}{\epsilon - \epsilon' + i0} A(\epsilon') \qquad A(\epsilon') \equiv \langle I_2, \epsilon' | H_{PQ} | I_1 \rangle$$

[1] C. Mahaux and H. Weidenmüller, *Shell-model approach to nuclear reactions*, North-Holland Publishing, Amsterdam 1969

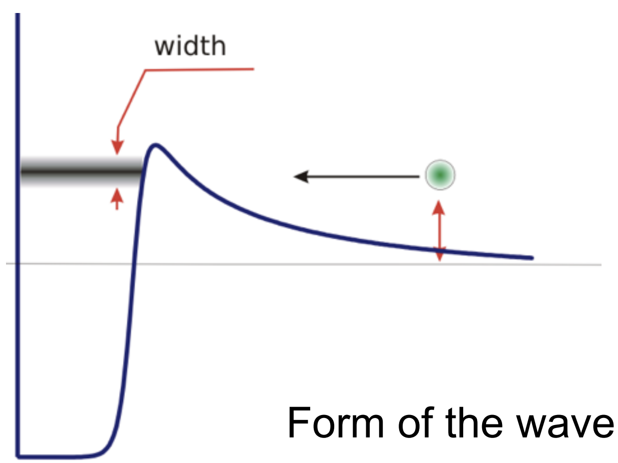
Physics of coupling to continuum

$$H'(\epsilon) = \int_0^\infty d\epsilon' \frac{|A(\epsilon')|^2}{\epsilon - \epsilon' + i0}.$$



Integration region involves no poles

$$H'(\epsilon) = \Delta(\epsilon) \qquad \Delta(\epsilon) = \fint d\epsilon' \frac{|A(\epsilon')|^2}{\epsilon - \epsilon' + i0}$$



State embedded in the continuum

$$\frac{1}{x \pm i0} = \text{p.v.}\frac{1}{x} \mp i\pi\delta(x)$$

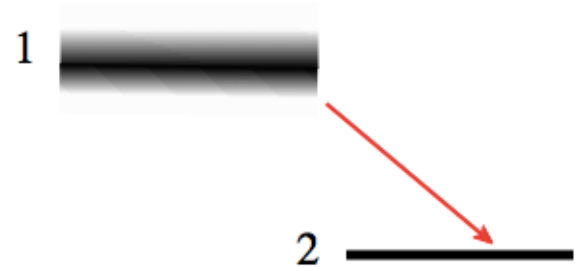
$$H'(\epsilon) = \Delta(\epsilon) - \frac{i}{2}\Gamma(\epsilon) \quad \Gamma(\epsilon) = 2\pi\,|A(\epsilon)|^2$$

Form of the wave function and probability

$$|\exp(-iEt)|^2 = 1 \to |\exp(-iEt - \Gamma t/2)|^2 = \exp(-\Gamma t)$$

One-body decay review

Fermi Golden Rule



$$A_{1,2}(\epsilon) = \langle I_2, \epsilon | H_{QP_1} | I_1 \rangle$$

$$d\Gamma_{1,2}(\epsilon) = 2\pi \left| A_{1,2}(\epsilon) \right|^2 \delta(E_1 - E_2 - \epsilon) dE$$

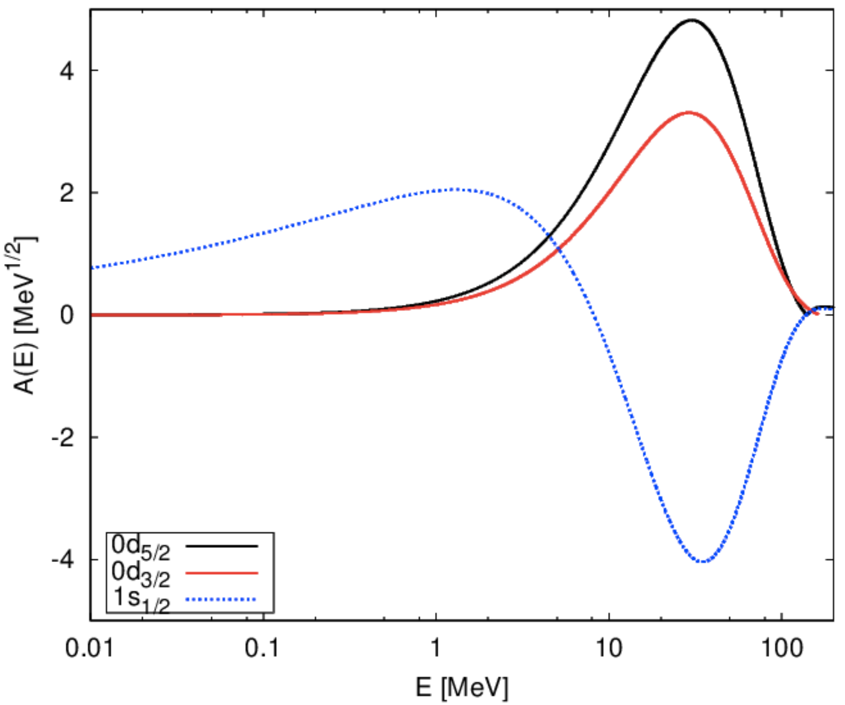
$$\Gamma_{1,2}(\epsilon) = 2\pi \left| A_{1,2}(\epsilon) \right|^2$$

Typical Amplitude

Low energy: phase space

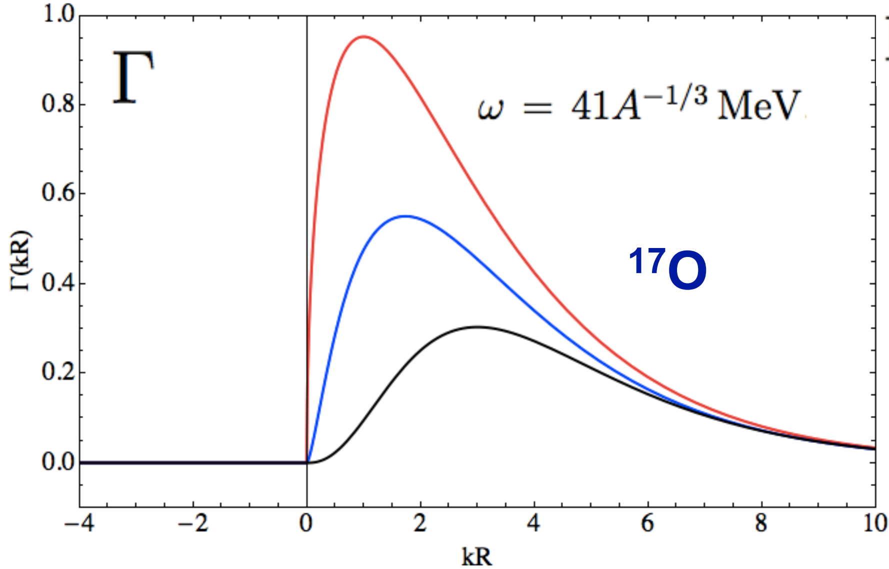
$$A_{1,2}(\epsilon) = \sqrt{\frac{2\mu}{\hbar^2 k \pi}} \int_0^\infty dr\, u_{I_1}(r)\, V(r)\, F_{I_2}(kr)$$

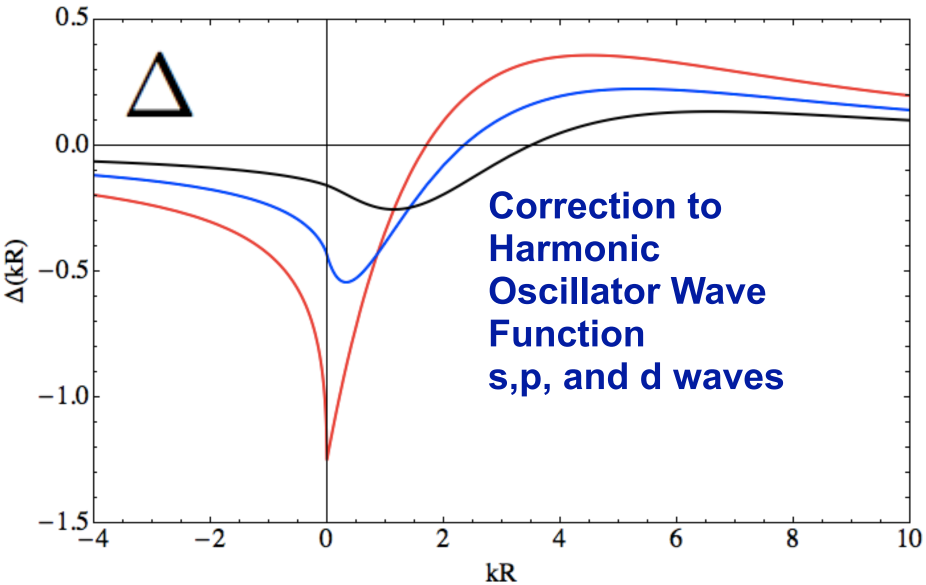
High energy: Born approximation



Self energy, interaction with continuum

$$\Gamma(\epsilon) \propto \epsilon^{l+1/2}$$

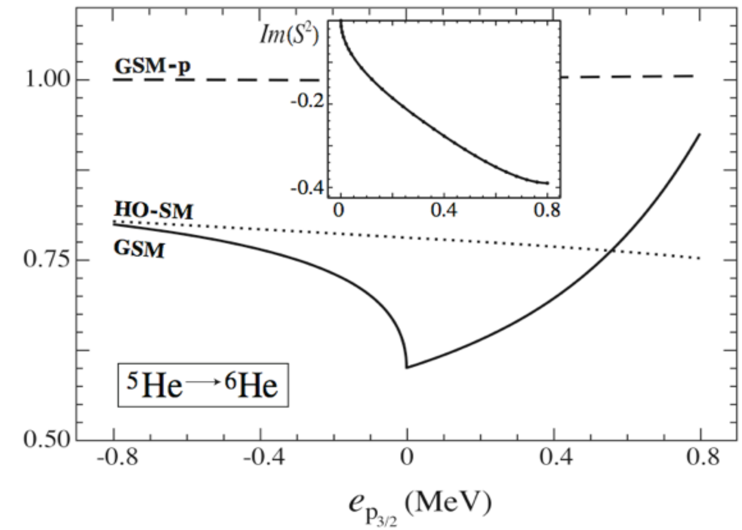




momentum

A. Volya, EPJ Web of Conf. 38, 03003 (2012).

Gamow shell model



N Michel, J. Phys. G: Nucl. Part. Phys. 36 (2009) 013101

Notes:

- Wave functions are not HO
- Phenomenological SM is adjusted to observation
- No corrections for properly solved mean field

Effective Hamiltonian Formulation

The Hamiltonian in P is: $$\mathcal{H}(E) = H + \Delta(E) - \frac{i}{2}W(E)$$

Channel-vector: $$|A^c(E)\rangle = H_{QP}|c;E\rangle$$

Self-energy: $$\Delta(E) = \frac{1}{2\pi}\int dE' \sum_c \frac{|A^c(E')\rangle\langle A^c(E')|}{E-E'}$$

Irreversible decay to the excluded space: $$W(E) = \sum_{c(\text{open})} |A^c(E)\rangle\langle A^c(E)|$$

[1] C. Mahaux and H. Weidenmüller, *Shell-model approach to nuclear reactions*, Amsterdam 1969
[2] A. Volya and V. Zelevinsky, Phys. Rev. Lett. **94**, 052501 (2005).
[3] A. Volya, Phys. Rev. C **79**, 044308 (2009).

Scattering matrix and reactions

$$\mathbf{T}_{cc'}(E) = \langle A^c(E)| \left(\frac{1}{E - \mathcal{H}(E)} \right) |A^{c'}(E)\rangle$$

$$\mathbf{S}_{cc'}(E) = \exp(i\xi_c)\ \{\delta_{cc'} - i\,\mathbf{T}_{cc'}(E)\} \exp(i\xi_{c'})$$

Cross section:

$$\sigma = \frac{\pi}{k'^2} \sum_{cc'} \frac{(2J+1)}{(2s'+1)(2I'+1)} |\mathbf{T}_{cc'}|^2$$

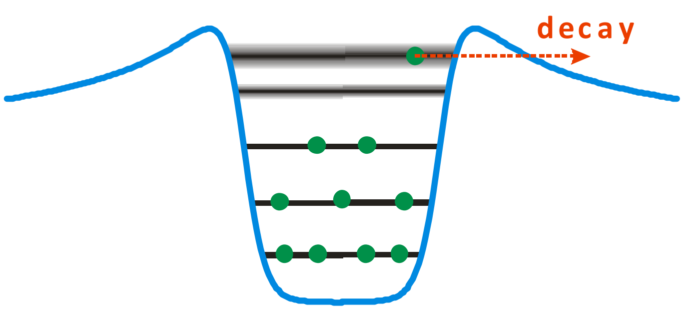
Additional topics:

•Angular (Blatt-Biedenharn) decomposition
•Coulomb cross sections, Coulomb phase shifts, and interference
•Phase shifts from remote resonances.

The nuclear many-body problem

Traditional

- Single-particles state (particle in the well)
- Many-body states (slater determinants)
- Hamiltonian and Hamiltonian matrix
- Matrix diagonalization



Continuum physics

- Effective non-hermitian energy-dependent Hamiltonian
- Channels (parent-daughter structure)
- Bound states and resonances
- Matrix inversion at all energies (time dependent approach)

Formally exact approach
Limit of the traditional shell model
Unitarity of the scattering matrix

Structure of channel vectors and traditional shell model limit

$$|A^c(E)\rangle = a^c(E)\,|c\rangle$$

Channel amplitude

Energy-independent channel vector: structure of spectator components

Perturbative limit in traditional Shell Model: $H|\alpha\rangle = E_\alpha|\alpha\rangle$

$$\Gamma_\alpha = \langle\alpha|W(E_\alpha)|\alpha\rangle \quad \Gamma_\alpha = \sum_c \Gamma_\alpha^c \quad \Gamma_\alpha^c = \gamma_c(E_\alpha)\,|\langle c|\alpha\rangle|^2$$

Single-particle decay width

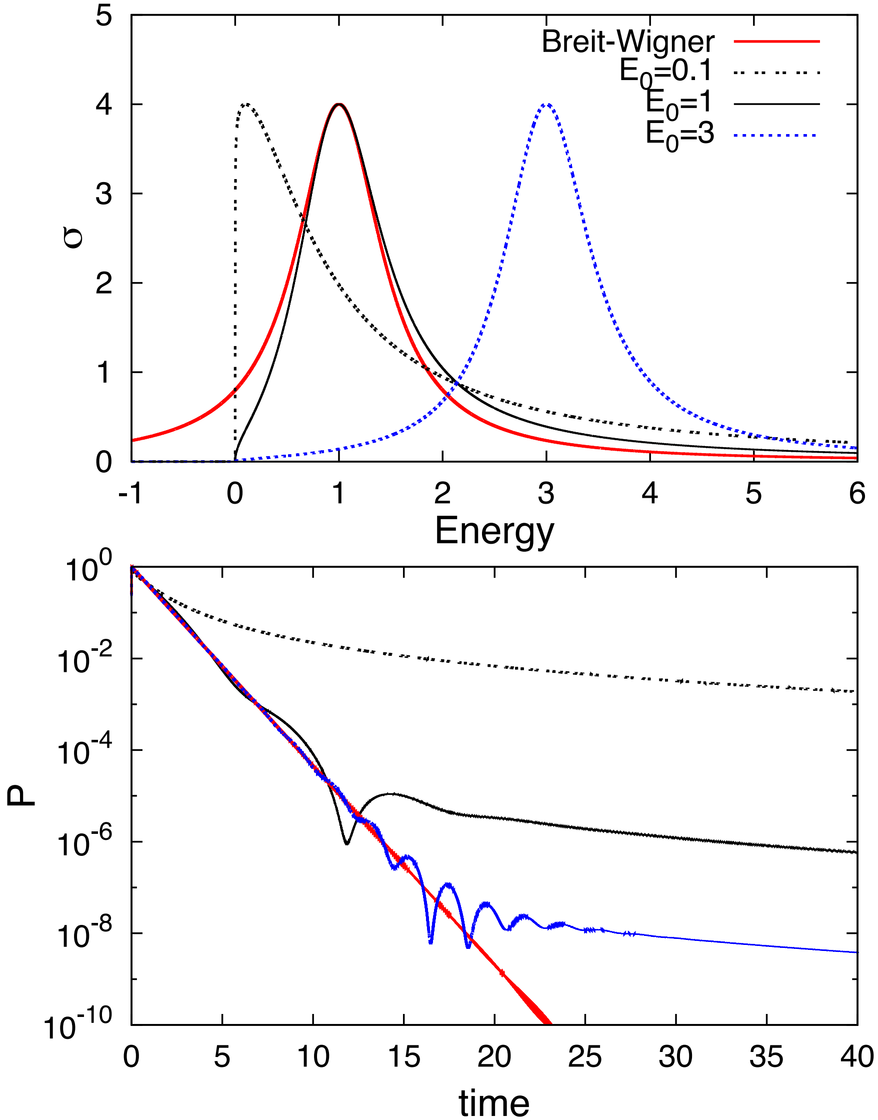
$$\gamma_c(E) = |a^c(E)|^2$$

Spectroscopic factor or transition rate

$$C^2S = |\langle c|\alpha\rangle|^2$$

$$B(\mathrm{EM}) = |\langle c|\alpha\rangle|^2$$

Single s-wave resonance in CSM



$$\mathcal{G} = \frac{1}{E - E_o + i/2\,\Gamma(E)}$$

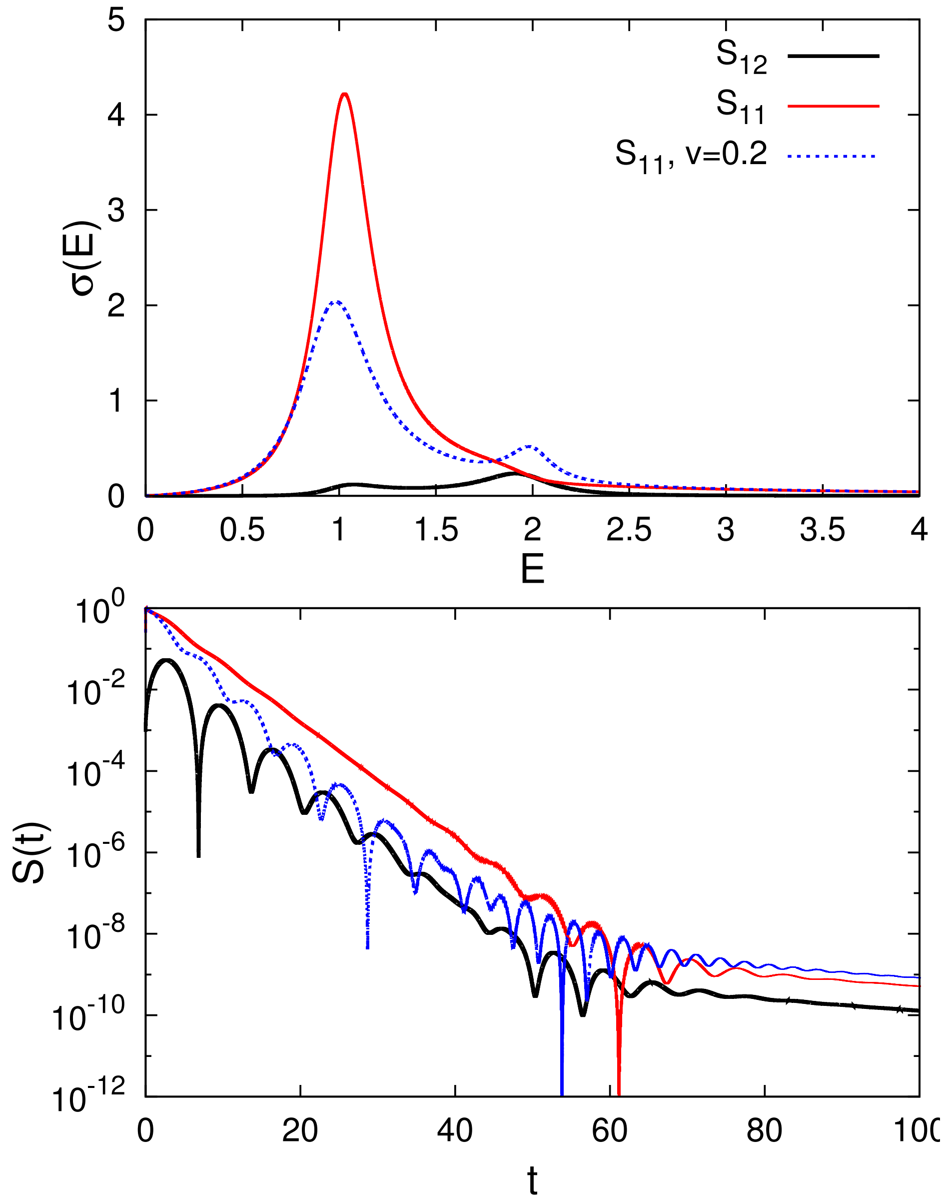
$$\Gamma(E) \propto \sqrt{E}$$

Two-level system

$$\mathcal{H} = \begin{pmatrix} \epsilon_1 - (i/2)\Gamma_1 & v - (i/2)A_1A_2 \\ v - (i/2)A_1A_2 & \epsilon_2 - (i/2)\Gamma_2 \end{pmatrix}$$

$$\Gamma_1 = A_1^2, \quad \Gamma_2 = A_2^2,$$

$$S(t) = |\langle\Psi(0)|\Psi(t)\rangle|^2$$



Time-dependent
Continuum Shell Model Approach

- Reflects time-dependent physics of unstable systems
- Direct relation to observables
- Linearity of QM equations maintained
- No matrix diagonalization
- Powerful many-body numerical techniques
- Stability for broad and narrow resonances
- Ability to work with experimental data

Propagator and Strength Function

$$G(E) = \frac{1}{E - H} = -i \int_0^\infty dt\ \exp(iEt)\exp(-iHt)$$

- Scale Hamiltonian so that eigenvalues are in [-1 1]
- Expand Using evolution operator in Chebyshev polynomials

$$\exp(-iHt) = \sum_{n=0}^{\infty} (-i)^n (2 - \delta_{n0}) J_n(t) T_n(H)$$

- Chebyshev polynomial $T_n[\cos(\theta)] = \cos(n\theta)$
- Use iterative relation and matrix-vector multiplication to generate

$$|\lambda_n\rangle = T_n(H)|\lambda\rangle$$

$$|\lambda_0\rangle = |\lambda\rangle, \quad |\lambda_1\rangle = H|\lambda\rangle \quad |\lambda_{n+1}\rangle = 2H|\lambda_n\rangle - |\lambda_{n-1}\rangle$$

$$\langle\lambda'|T_{n+m}(H)|\lambda\rangle = 2\langle\lambda'_m|\lambda_n\rangle - \langle\lambda'|\lambda_{n-m}\rangle, \quad n \geqslant m$$

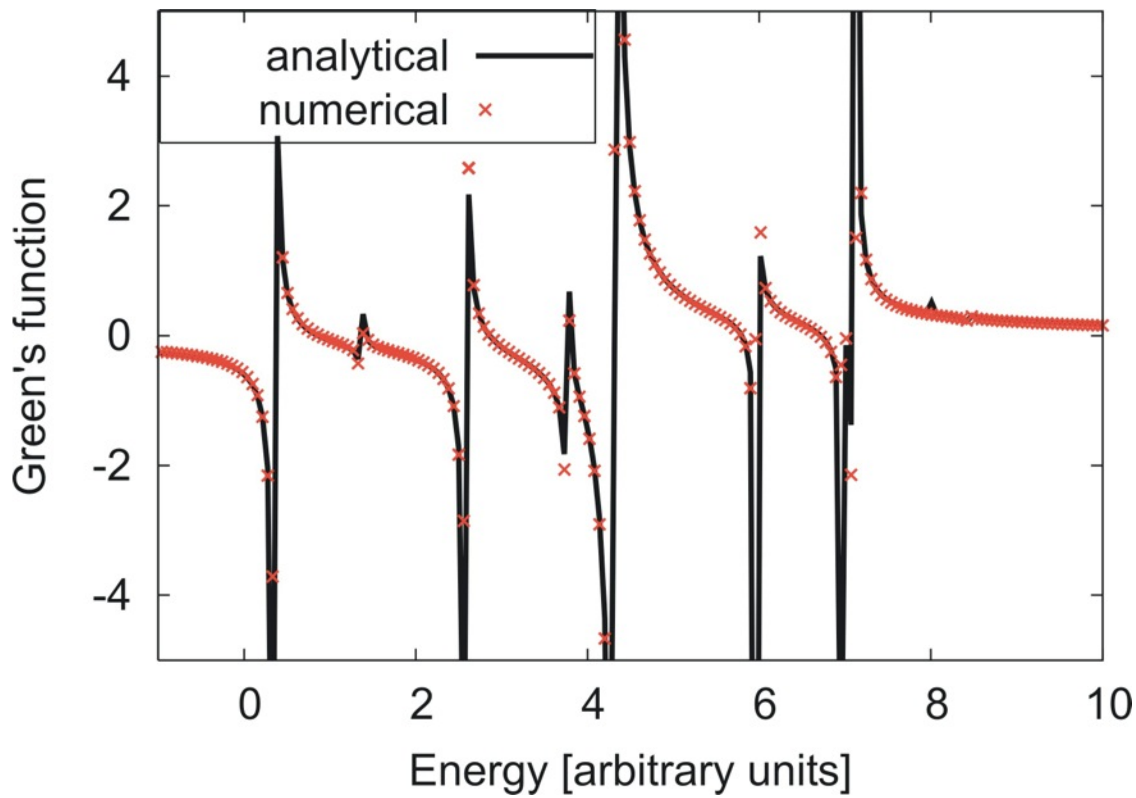
- Use FFT to find return to energy representation

T. Ikegami and S. Iwata, J. of Comp. Chem. **23** (2002) 310-318
A. Volya, Phys. Rev. C 79, 044308 (2009).

Chebyshev expansion
Green's function calculation

Advantages of the method

- No need for full diagonalization or inversion at different E
- Only matrix-vector multiplications
- Numerical stability
- Controlled energy resolution



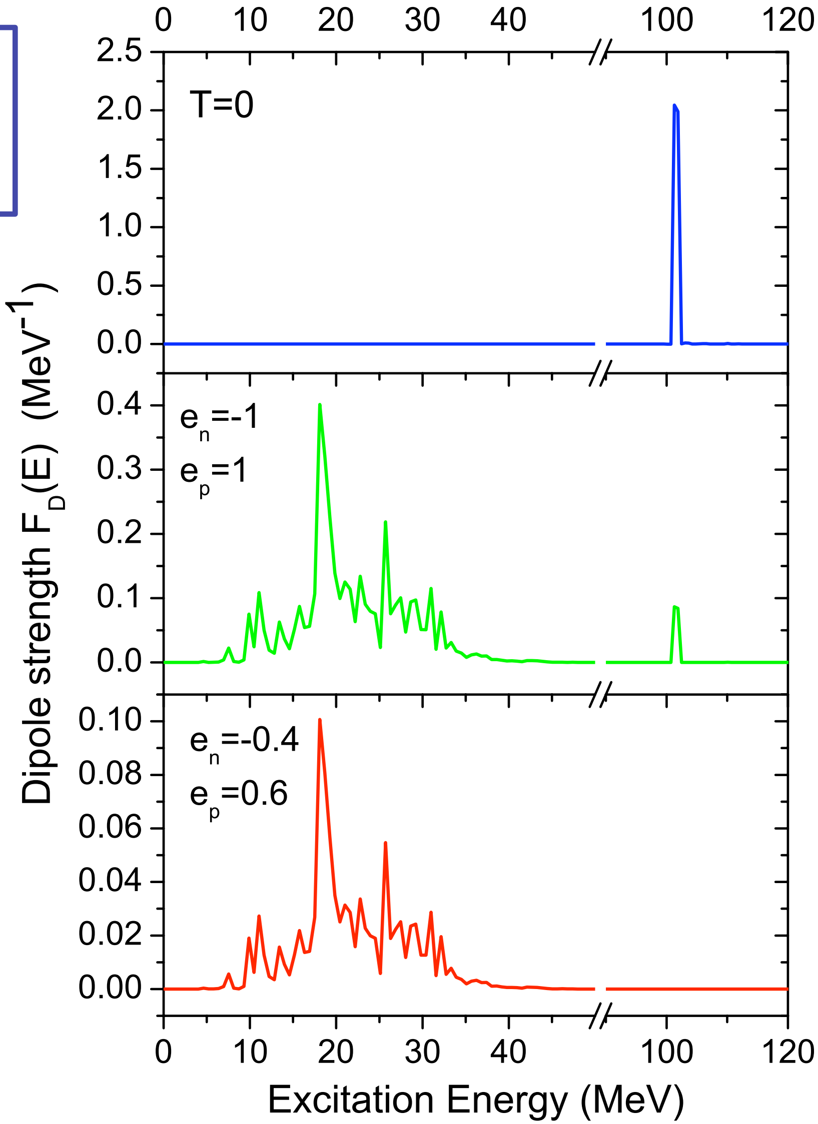
Center-of-mass problem
The strength-function example

Figure: Strength function for E1 and CM excitation in ^{20}O example, spsdfp –shell model WBP interaction.

CM spurious states are moved to high energy
- Top plot-isoscalar dipole E1 T=0 excitation
- Center- E1 excitation with incorrect effective charges
- Bottom-E1 with e_p=0.6 and e_n=-0.4

$$F_\lambda(E) = \langle\lambda|\delta(E-H)|\lambda\rangle = -\frac{1}{\pi}\,\mathrm{Im}\,\langle\lambda|G(E)|\lambda\rangle$$

$$|D\rangle = D|0^+_{\mathrm{g.s.}}\rangle \qquad \vec{D} = \sum_a e_a \vec{r}_a$$



Dysion's equation,
including other interaction terms

$$\mathcal{H}(E) = H + \Delta(E) - \frac{i}{2}W(E)$$

$$\mathcal{H}(E) = H + V(E) \qquad V(E) = \sum_{ab} |a\rangle \mathbf{V}_{ab}(E) \langle b|$$

$$G(E) = \frac{1}{E - H} \qquad \mathcal{G}(E) = \frac{1}{E - \mathcal{H}(E)}$$

Propagators in channel space

$$\mathbf{G}_{ab} = \langle a|G(E)|b\rangle \qquad \mathbb{G}_{ab} = \langle a|\mathcal{G}(E)|b\rangle$$

Include non-Hermitian terms with Dyson's equation

$$\mathcal{G}(E) = G(E) + G(E)V(E)\mathcal{G}(E)$$

$$\mathbb{G} = \mathbf{G}\left[\mathbf{1} - \mathbf{V}\mathbf{G}\right]^{-1} = \left[\mathbf{1} - \mathbf{G}\mathbf{V}\right]^{-1}\mathbf{G}$$

Strength function and decay in ^{22}O

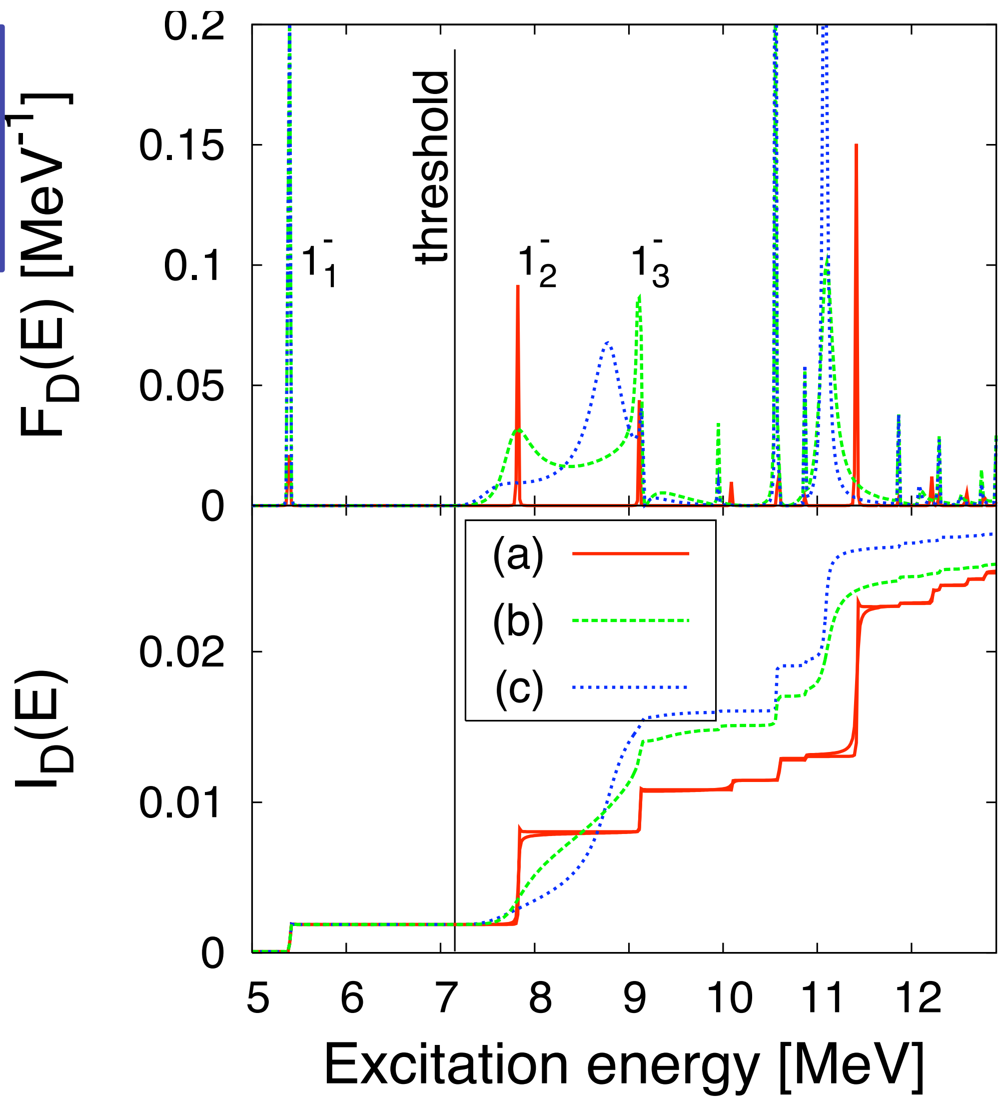
Upper panel: Isovector dipole strength in ^{22}O low-energy region.
Lower panel: Integrated strength

$$F_\lambda(E) = -\frac{1}{\pi}\,\mathrm{Im}\,\langle\lambda|\mathcal{G}(E)|\lambda\rangle$$

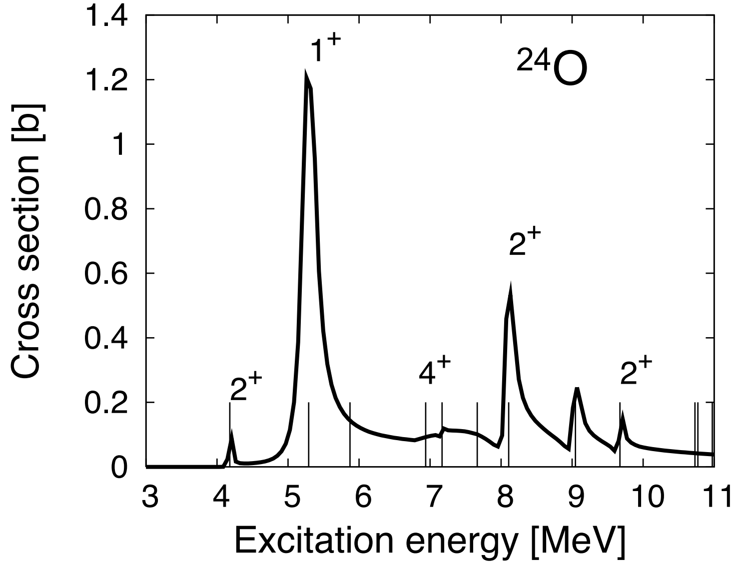
$$I_\lambda(E) = \int_{-\infty}^{E} F_\lambda(E')dE'$$

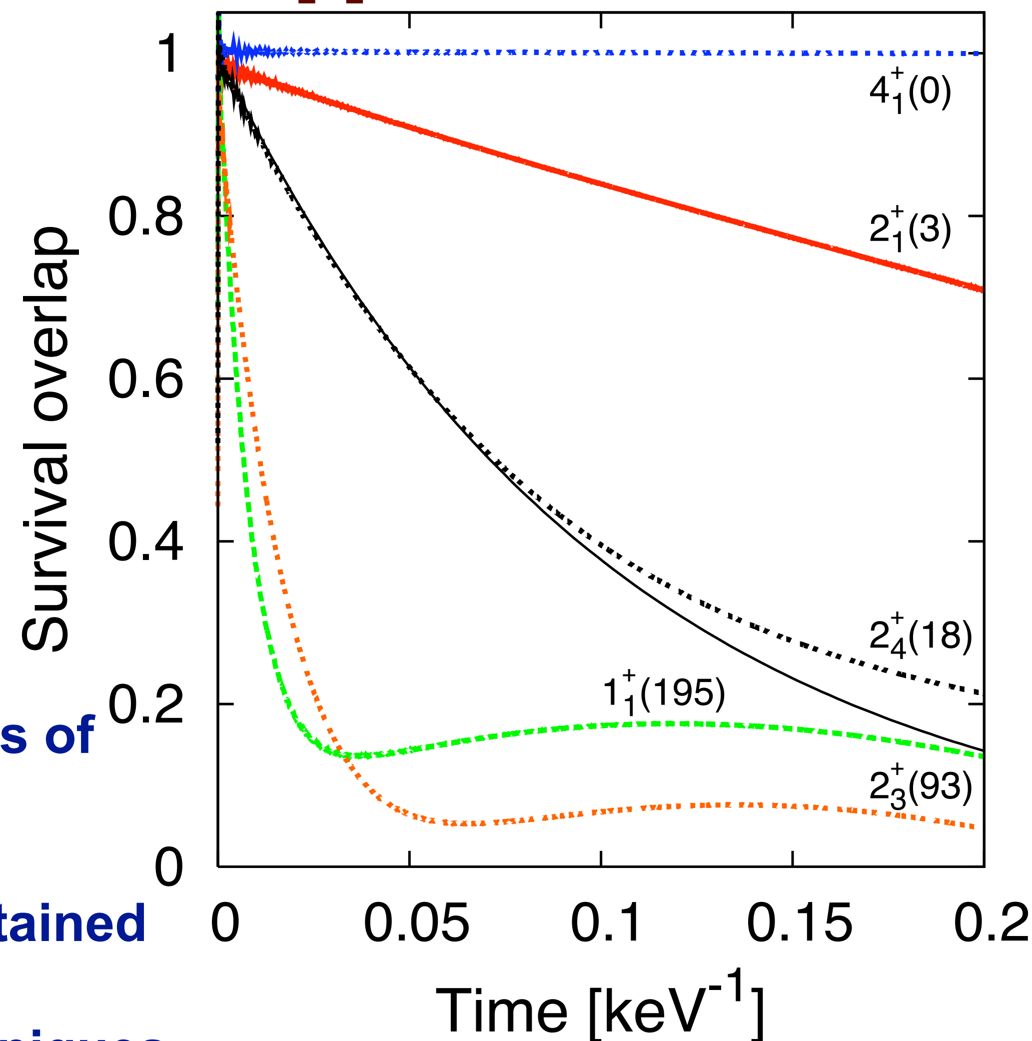
In the limit of weak decay

$$I_D(E) = \sum_{\alpha}^{E_\alpha<E} B(\mathrm{E1};\alpha\to 0^+_{\mathrm{g.s.}})$$



Time-dependent approach

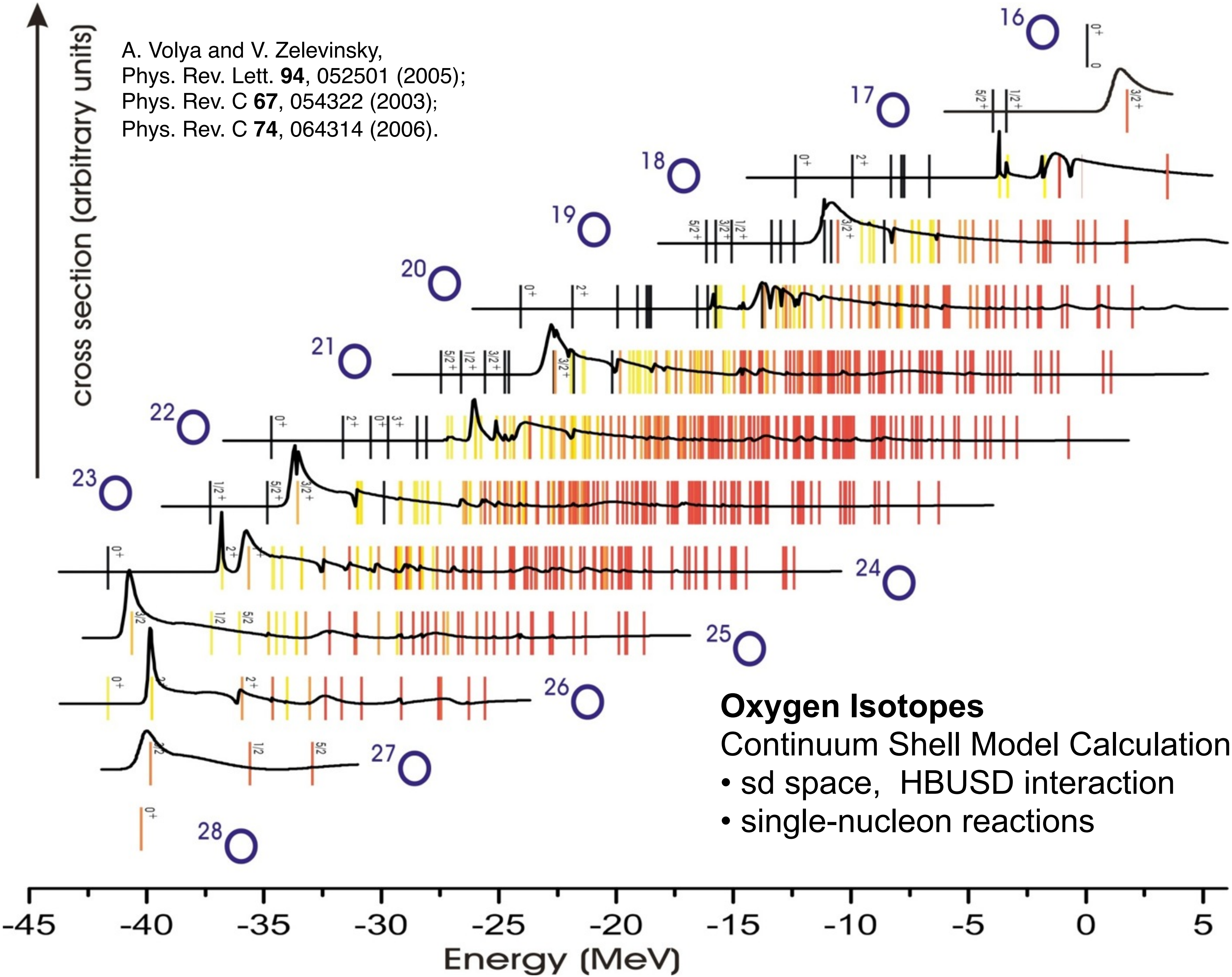




Time evolution of several SM states in ^{24}O. The absolute value of the survival overlap is shown

- **Reflects time-dependent physics of unstable systems**
- **Direct relation to observables**
- **Linearity of QM equations maintained**
- **No matrix diagonalization**
- **New many-body numerical techniques**
- **Stability for broad and narrow resonances**
- **Ability to work with experimental data**

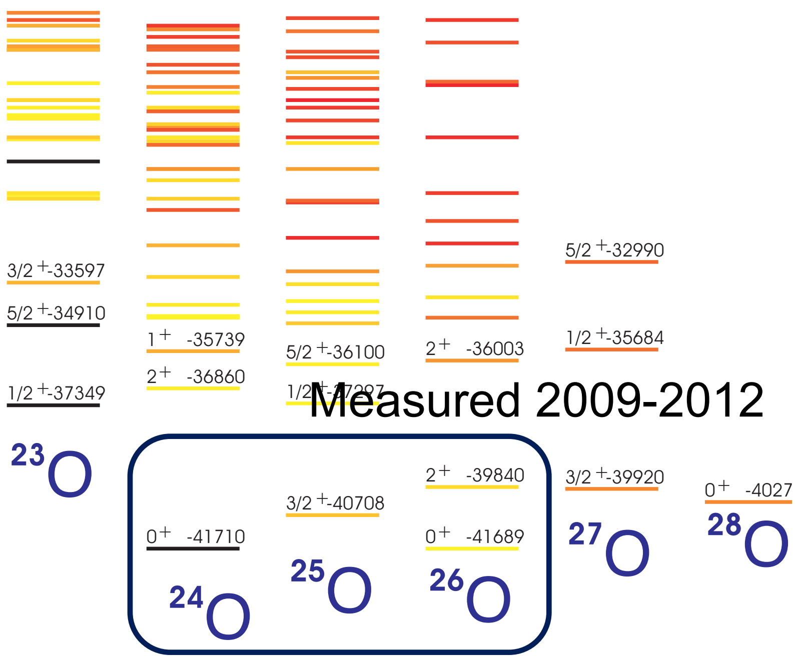
A. Volya, *Time-dependent approach to the continuum shell model,* Phys. Rev. C **79**, 044308 (2009).

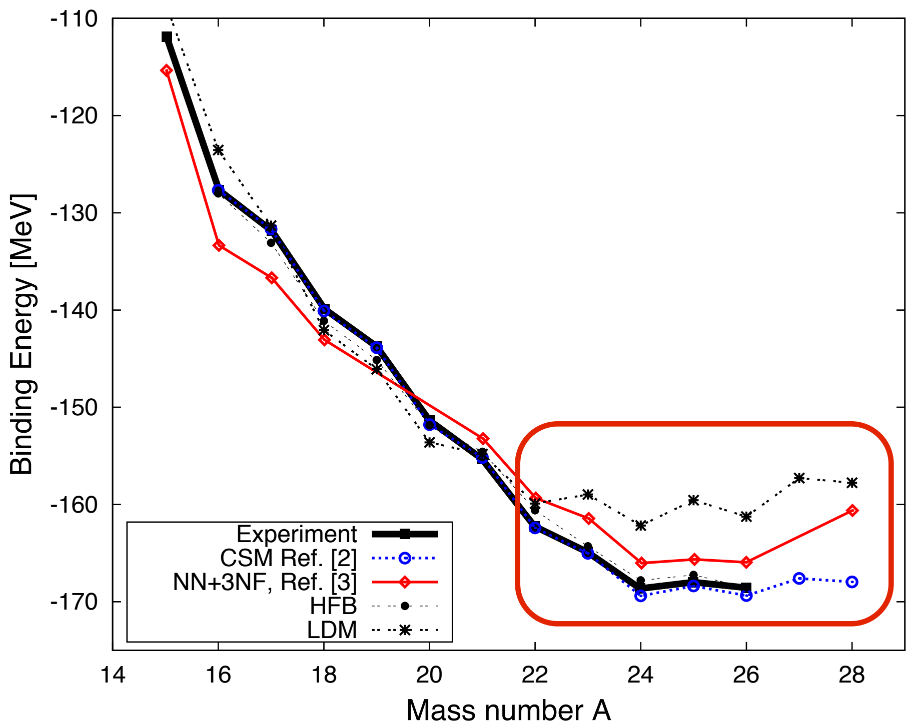


A. Volya and V. Zelevinsky,
Phys. Rev. Lett. **94**, 052501 (2005);
Phys. Rev. C **67**, 054322 (2003);
Phys. Rev. C **74**, 064314 (2006).
cross section (arbitrary units)
16 O
17 O
18 O
19 O
20 O
21 O
22 O
23 O
24 O
25 O
26 O
27 O
28 O
0+
5/2+
1/2+
3/2+
2+
3+
Oxygen Isotopes
Continuum Shell Model Calculation
• sd space, HBUSD interaction
• single-nucleon reactions
-45
-40
-35
-30
-25
-20
-15
-10
-5
0
5
Energy (MeV)

Predictive power of theory

Continuum Shell Model prediction 2003-2006





[1] C. R. Hoffman et al., Phys. Lett. B **672**, 17 (2009); Phys.Rev.Lett.**102**,152501(2009); Phys.Rev.C **83**,031303(R)(2011); E. Lunderberg et al., Phys. Rev. Lett. **108**, 142503 (2012).

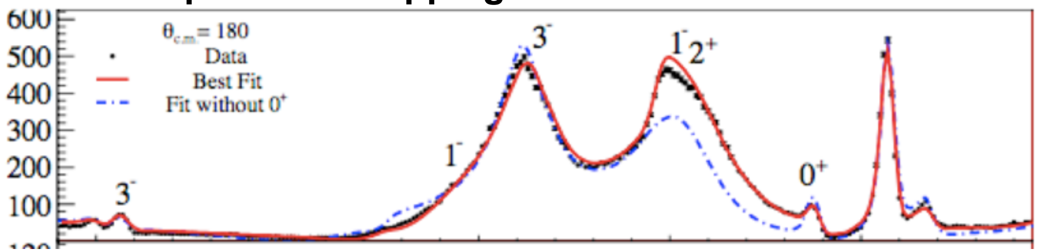
[2] A. Volya and V. Zelevinsky, Phys. Rev. Lett. **94**, 052501 (2005); Phys. Rev. C **67**, 054322 (2003); **74**, 064314 (2006).

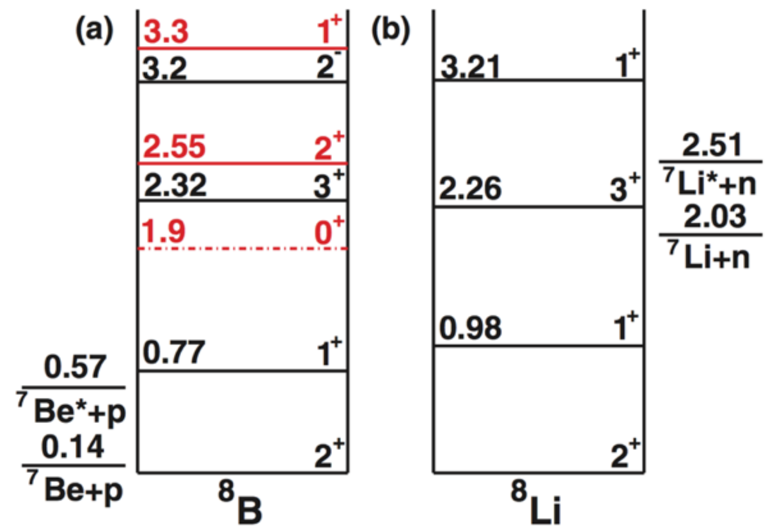
[3] G. Hagen et.al Phys. Rev. Lett. **108**, 242501 (2012)

http://www.nscl.msu.edu/general-public/news/2012/O26

Interference of resonances, ^{8}B study

Example of overlapping resonances





Physics of overlapping resonances: ^{8}B example

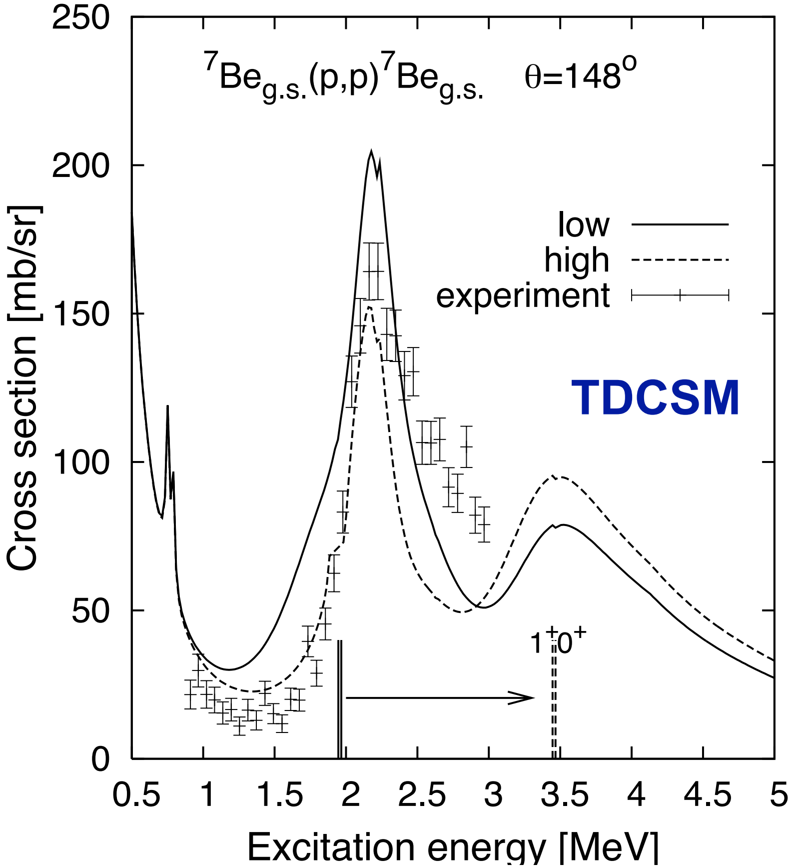
Trying to find missing states

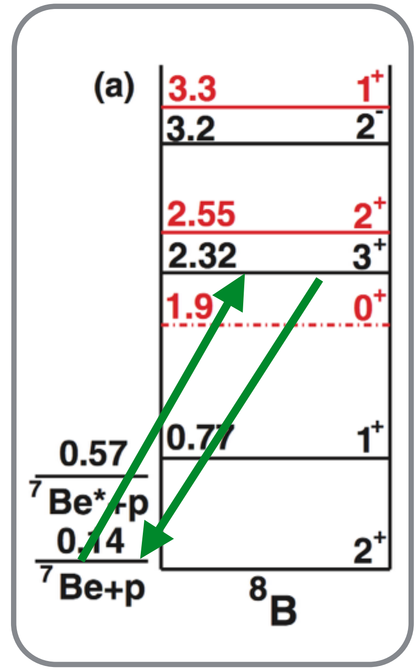
- Ab-initio and no core theoretical models predict low-lying 2^{+}, 0^{+}, and 1^{+} states
- Recoil-Corrected CSM suggests low-lying states
- Traditional SM mixed results
- These states were not seen in ^{8}B and in ^{8}Li

J. P. Mitchell, G. V. Rogachev, E. D. Johnson, L. T. Baby, K. W. Kemper, A. M. Moro, P. N. Peplowski, A. Volya, and I. Wiedenhoever, Phys. Rev. C **82**, 011601 (2010); **87**, 054617 (2013).

Observation of 2⁺, 0⁺, and 1⁺ states

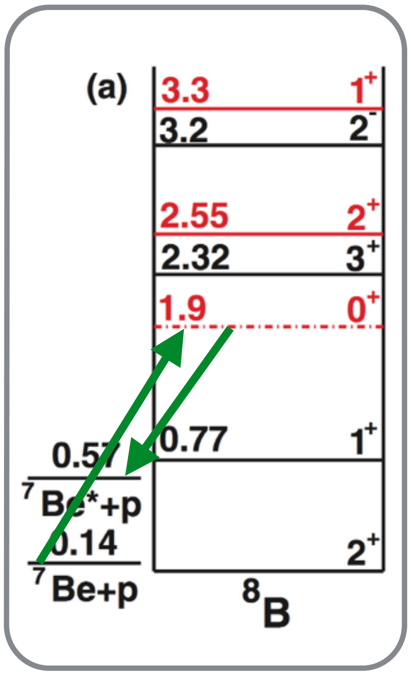
Elastic cross section



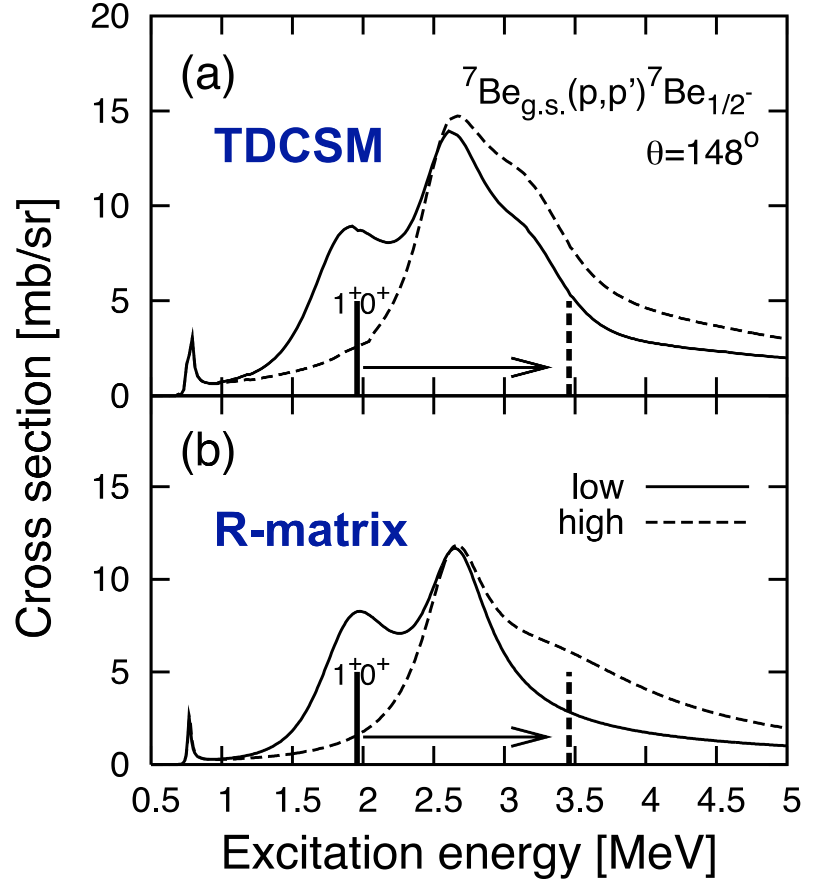


TDCSM: WBP interaction +WS potential, threshold energy adjustment.
R-Matrix: WBP spectroscopic factors, R_c=4.5 fm, only 1^+ 1^+ 0^+ 3^+ and 2^+ l=1 channels

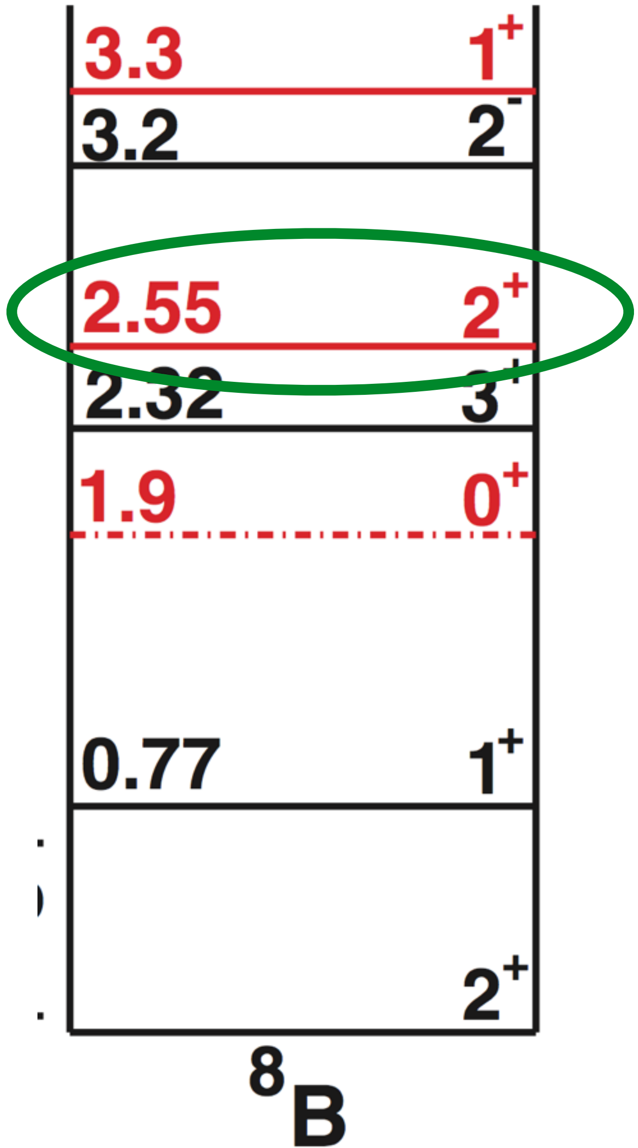
Observation of 2⁺, 0⁺, and 1⁺ states



Inelastic cross section



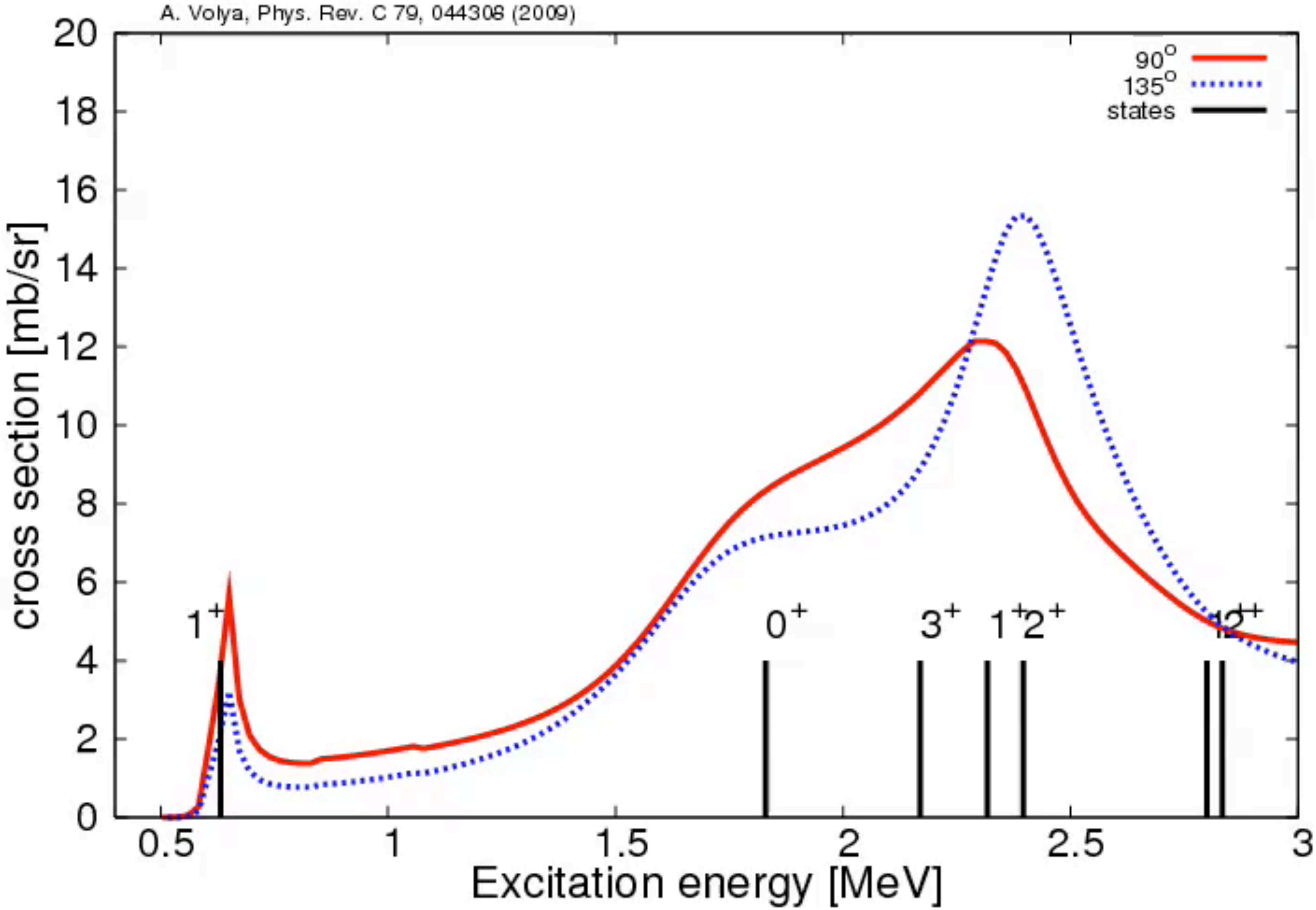
Position of the 2+ and its role in $^7Be(p,p')^7Be$



- Theoretical calculations predict two 2^+ states
- Predictions are higher in energy
- Only one state is observed

See animation at www.volya.net

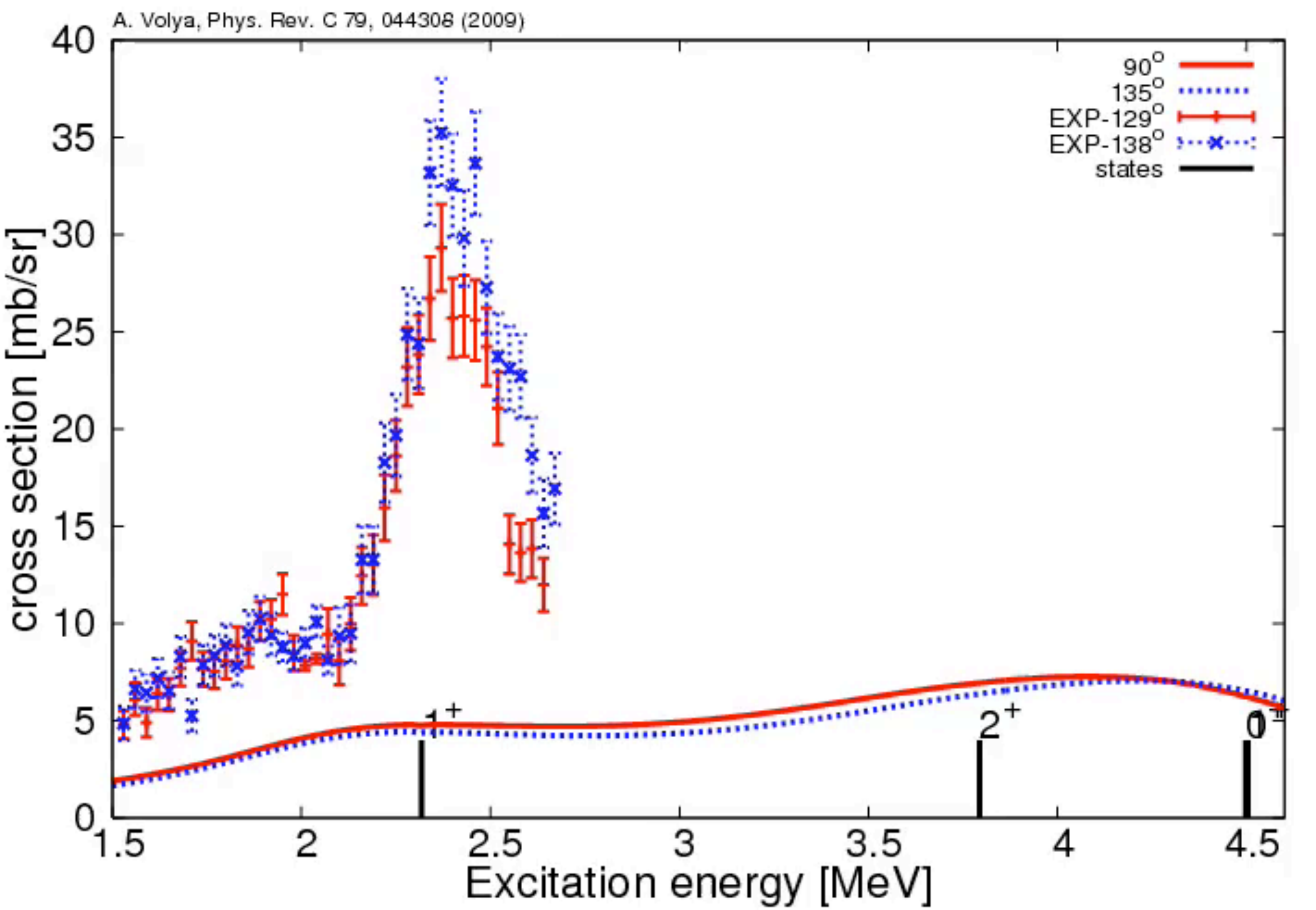
Position of the 2+ and its role in $^7Be(p,p')^7Be$



See animation at www.volya.net

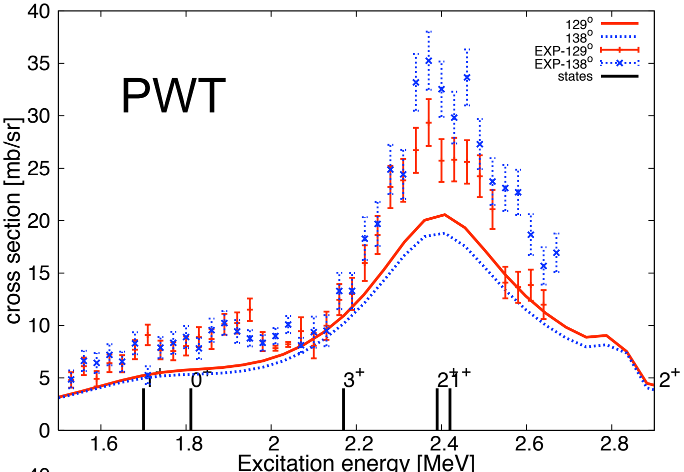
Resonances and their positions inelastic $^7Be(p,p')^7Be$ reaction in TDCSM

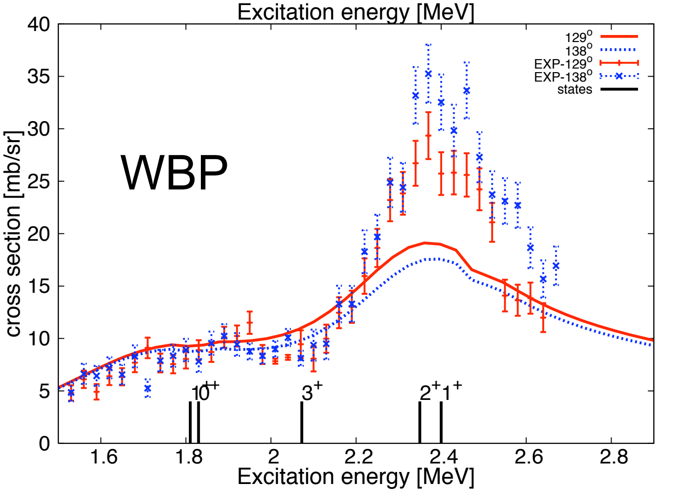
CKI+WS Hamiltonian

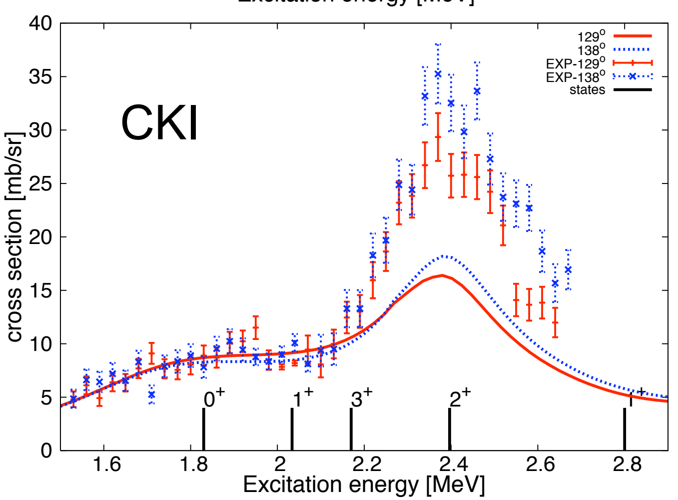


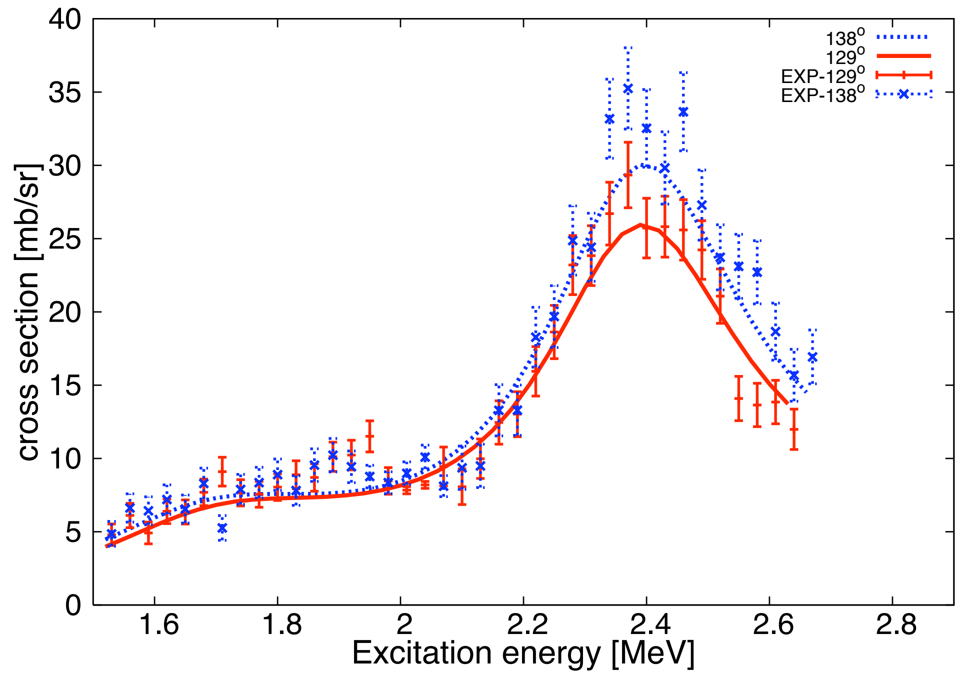
See animation at www.volya.net

R-matrix fit and TDCSM for $^7Be(p,p)^7Be^*$









Chanel Amplitudes from TDCM and final best fit

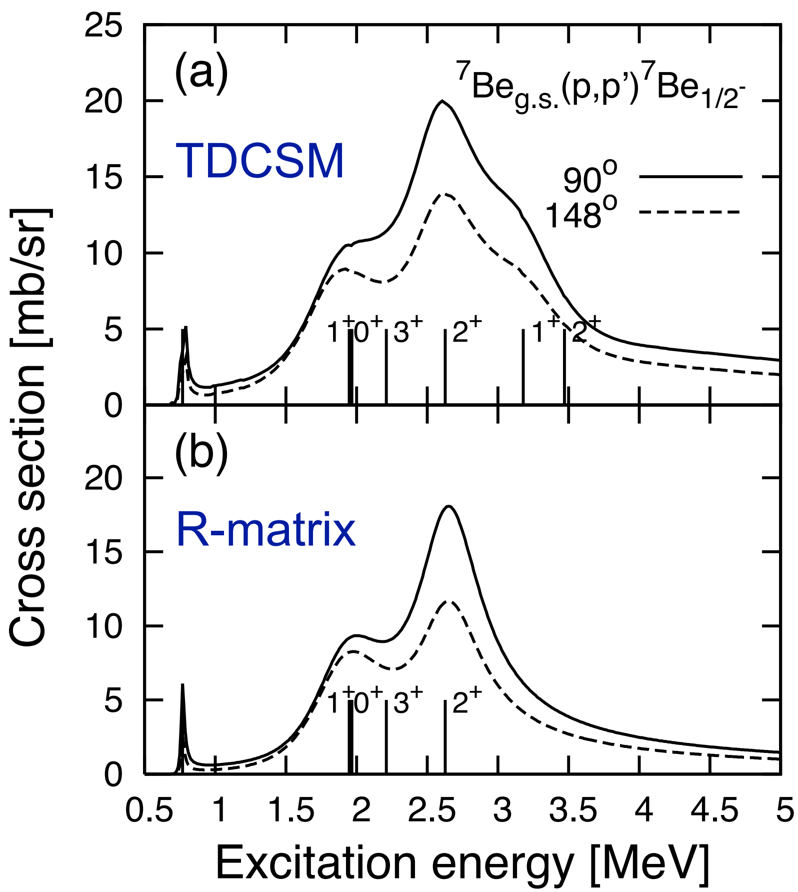
$\langle c|\alpha\rangle$

	J	p I=3/2	p I=3/2	p I=1/2	p I=1/2
FIT	2	-0.293	0.293		0.534
CKI	2	-0.168	0.164		0.521
FIT	1	-0.821	-0.612	0.375	0.175
CKI	1	-0.840	-0.617	0.332	0.178

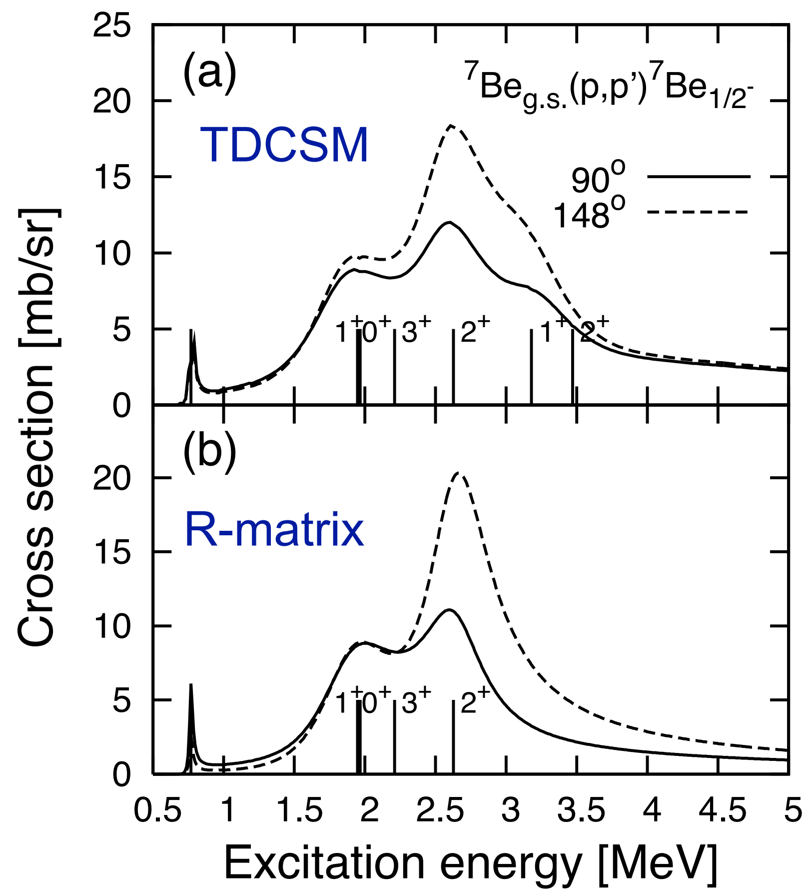
From cross section to many-body structure $^7Be(p,p)^7Be$

J^π	E(MeV)	$p_{3/2}$(g.s.)	$p_{1/2}$(gs)	$p_{3/2}$	$p_{1/2}$
1_1^+	0.7693	-0.563	0.303	0.867	-0.138
1_2^+	1.947	0.597	0.826	0.284	0.240
0_1^+	1.967	0.693	0	0	-0.918
3_1^+	2.2098	0.612	0	0	0
2_2^+	2.628	0.149	0.326	-0.632	0

identical energies, identical widths, identical spectroscopic factors but different cross section



Phase changed



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