Vertex Functions and Asymptotics of Nuclear Bound-State Wave Functions

L. D. Blokhintsev

Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University, Moscow, 119991, Russia

Abstract

The paper deals with vertex functions representing matrix elements of two-fragment ($a \to 1+2$) or three-fragment ($a \to 1+2+3$) virtual decays of a bound nuclear system a. Much attention is given to the on-shell vertex functions corresponding to the case when all external particles (fragments) are on the mass shell. The relations are established between the on-shell vertex functions and the coefficients multiplying the asymptotic forms of wave functions and overlap integrals in two- or three-fragment channels. It is shown that the on-shell three-fragment vertex functions determine the contributions to the amplitudes of processes described by the Feynman diagrams containing loops. The anomalous asymptotics of the wave functions in the two-fragment channels is discussed.

Keywords: *Vertex function; bound state; asymptotics*

1 Introduction

The vertex function (VF) W for the virtual n-fragment decay of a bound state a is the matrix element of the process

$$a \to 1 + 2 + \ldots + n. \tag{1}$$

If the system $1 + 2 + \ldots + n$ possesses a bound state a, then the matrix element (the amplitude) of the scattering process

$$1 + 2 + \ldots + n \to 1 + 2 + \ldots + n \tag{2}$$

has a pole at the energy corresponding to that bound state, and the VF W is related to the residue of the matrix element at that pole (see Fig. 1).

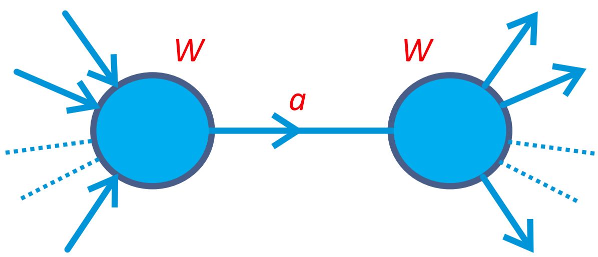

Figure 1:

Proceedings of the International Conference 'Nuclear Theory in the Supercomputing Era — 2014' (NTSE-2014), Khabarovsk, Russia, June 23–27, 2014. Eds. A. M. Shirokov and A. I. Mazur. Pacific National University, Khabarovsk, Russia, 2016, p. 90.

http://www.ntse-2014.khb.ru/Proc/Blokhintsev.pdf.

The matrix element of the process (2) can be written as

$$M = \left\langle \Phi_f | V | P \Psi_i^{(+)} \right\rangle = \langle \Phi_f | V | P (1 + GV) \Phi_i \rangle, \tag{3}$$

P is the (anti)symmetrization operator. Using the spectral decomposition of the Green's function G and the relation $V = H - H_0$ one easily gets

$$W\left(\vec{q}_1, \vec{q}_2, \ldots, \vec{q}_{n-1}\right) = -N^{1/2}\left(T + \epsilon\right)\langle \phi_1 \phi_2 \ldots \phi_n | \phi_a \rangle. \tag{4}$$

Here $\vec{q}_i$ are the Jacobi momenta, ϕ_i is the internal wave function of the fragment i, T is the relative kinetic energy, and $\epsilon > 0$ is the binding energy in the channel $a \to 1 + 2 + \ldots + n$.

The factor $N^{1/2}$ arises due to the identity of constituents. If all fragments consist of nucleons which are considered to be identical, then

$$N = \frac{A_a!}{A_1! A_2! \ldots A_n!}, \tag{5}$$

where A_i denotes the number of nucleons in the fragment i. If all fragments $1, 2, \ldots n$ are treated as structureless, then $\langle \phi_1 \phi_2 \ldots \phi_n | \phi_a \rangle$ turns into the wave function ϕ_a and N equals $n!$.

VFs W for $a \to 1 + 2 + \ldots + n$ are related to the coordinate asymptotics of ϕ_a in the channel $1 + 2 + \ldots + n$. In what follows we discuss this relation for the important cases $n = 2$ and $n = 3$.

The system of units $\hbar = c = 1$ is used throughout the paper.

2 Two-fragment case ($n = 2$)

2.1 General formalism

From general principles the expression for the vertex function (the matrix element) of the two-body decay (virtual or real) $a \to b + c$ can be written as [1]

$$W_{a \to b+c} = \sqrt{4\pi} \sum_{lsm_l m_s} G_{abc}(ls; \sigma_a, \sigma_b, \sigma_c) \times (J_b M_b J_c M_c | s m_s)(l m_l s m_s | J_a M_a) Y_{lm}(\vec{q}_{bc}/q_{bc}). \tag{6}$$

Here J_i and M_i are the spin of the particle i and its projection, $(a\alpha b\beta|c\gamma)$ are the Clebsh–Gordon coefficients, $\vec{q}_{bc}$ is the relative momentum of b and c, l and m_l are the relative angular momentum of b and c and its projection, s and m_s are the channel spin and its projection, Y_{lm} is the spherical function, $G_{abc}(ls; \sigma_a, \sigma_b, \sigma_c)$ are the invariant vertex form factors (VFF). Generally, when all three particles a, b, c are off-shell, VFFs G_{abc} may depend on three kinematic invariants and the quantities $\sigma_a, \sigma_b, \sigma_c$ are selected as such invariants in Eq. (6). σ_i is defined as $\sigma_i = E_i - \vec{p}_i^{\,2}/2m_i$ where E_i, $\vec{p}_i$ and m_i are the kinetic energy, the momentum, and the mass of the particle i. If the particle i is on-shell, then $\sigma_i = 0$.

However, if one relates the vertex function to the residue of a scattering amplitude and defines it according to Eq. (4), then the VFF $G_{abc}(ls; \sigma_a, \sigma_b, \sigma_c)$ depends on the relative momentum q_{bc} only which is related to σ_i:

$$q_{bc}^2 = -\varkappa^2 - 2\mu_{bc}(\sigma_b + \sigma_c - \sigma_a), \quad \varkappa^2 = 2\mu_{bc}\epsilon, \quad \epsilon = m_b + m_c - m_a, \tag{7}$$

μ_{ij} is the reduced mass of i and j. It follows from Eq. (7) that if all three particles are on shell ($\sigma_a = \sigma_b = \sigma_c = 0$), then $q_{bc} = i\varkappa$.

The on-shell values of VFFs are called vertex constants (VC): $G \equiv G(q)|_{q=i\varkappa}$. They are the analogues of the renormalized coupling constants in quantum field theory. The VCs thus defined are real.

Using Eq. (6) and the analogous expansion of the overlap function (4) in the coordinate representation one obtains

$$G_{abc}(ls;q) = -(\pi N_{bc})^{1/2}\,\frac{q^2+\varkappa^2}{\mu_{bc}}\int\limits_0^\infty j_l(qr)\,I_{abc}(ls;r)\,r^2dr, \tag{8}$$

where $I_{abc}(ls;r)$ is the radial overlap integral of the wave functions of a, b and c, and r is the distance between b and c.

The VC G is directly related to the asymptotic normalization coefficient (ANC) of $I_{abc}(ls;r)$ at $r\to\infty$. In the case of a short-range interaction,

$$I_{abc}(ls;r) \approx C_{abc}(ls)\,\frac{e^{-\varkappa r}}{r}, \quad r\to\infty. \tag{9}$$

Inserting Eq. (9) into Eq. (8) and setting $q=i\varkappa$, it is easy to obtain a relation between the VC $G_{abc}(ls)$ and the ANC $C_{abc}(ls)$ [1]:

$$G_{abc}(ls) = -\frac{(\pi N_{bc})^{1/2}}{\mu_{bc}}\,C_{abc}(ls). \tag{10}$$

Note that the factor $N_{bc}^{1/2}$ is often included into the definition of $C_{abc}(ls)$ and $I_{abc}(ls;r)$.

The long-range Coulomb interaction modifies the asymptotic behavior of the overlap integral $I_{abc}(ls;r)$, namely

$$I_{abc}(ls;r) \approx C_{abc}(ls)\,\frac{W_{-\eta,l+1/2}(\varkappa r)}{r} \approx C_{abc}(ls)\,\frac{e^{-\varkappa r-\eta\ln(2\varkappa r)}}{r}, \qquad r\to\infty. \tag{11}$$

Here $\eta = Z_bZ_ce^2\mu_{bc}/\varkappa$ is the Coulomb (Sommerfeld) parameter for a bound state a, Z_ie is the charge of the fragment i, and W is the Whittaker function.

In the presence of the Coulomb interaction Eq. (8) can not be used for determining the VC since at $q\to i\varkappa$ the right-hand-side of (8) tends to 0 for the repulsive Coulomb potential and to ∞ for the attractive potential.

There are different definitions of VCs in the presence of the Coulomb interaction. The most natural definition relates the VC to the Coulomb-modified scattering amplitude.

The total amplitude of elastic bc scattering in the presence of the Coulomb and short-range interactions is written as

$$f(\vec{k}) = f_C(\vec{k}) + f_{NC}(\vec{k}), \tag{12}$$

$$f_C(\vec{k}) = \sum_{l=0}^{\infty}(2l+1)\,\frac{\exp(2i\sigma_l)-1}{2ik}\,P_l(\cos\theta), \tag{13}$$

$$f_{NC}(\vec{k}) = \sum_{l=0}^{\infty}(2l+1)\,\exp(2i\sigma_l)\,\frac{\exp(2i\delta_l^{NC})-1}{2ik}\,P_l(\cos\theta). \tag{14}$$

Here $\sigma_l = \arg\Gamma(l+1+i\eta_s)$ and δ_l^{NC} are the pure Coulomb and Coulomb-nuclear phase shifts, $\Gamma(z)$ is the Gamma function and $\eta_s = Z_bZ_ce^2\mu/k$ is the Coulomb parameter for a scattering state.

The renormalized Coulomb-nuclear partial-wave amplitude $\tilde{f}_l^N$ in the case of the repulsive Coulomb potential is introduced as follows [2]:

$$\tilde{f}_l^N = \exp(2i\sigma_l)\,\frac{\exp(2i\delta_l^{NC})-1}{2ik}\left(\frac{l!}{\Gamma(l+1+i\eta_s)}\right)^2 e^{\pi\eta_s}. \tag{15}$$

The analytic properties of $\tilde{f}_l^N$ on the physical sheet are analogous to those for the scattering by a short-range potential. In particular, it is regular near zero energy.

If the $b+c$ system possesses a bound state a with the binding energy $\epsilon = \varkappa^2/2\mu$, then the amplitude $\tilde{f}_l^N(k)$ has a pole at $k = i\varkappa$. The residue at that pole is expressed in terms of the Coulomb-renormalized VC $\tilde{G}_l$ and ANC C_l:

$$\operatorname{res} \tilde{f}_l^N(k) = \lim_{k \to i\varkappa} [(k - i\varkappa)\tilde{f}_l^N(k] = i\frac{\mu^2}{2\pi\varkappa}\tilde{G}_l^2, \tag{16}$$

$$C_l = -\frac{\mu}{\sqrt{\pi}}\frac{\Gamma(l+1+\eta)}{l!}\tilde{G}_l. \tag{17}$$

The knowledge of ANCs is essential for an analysis of nuclear reactions between charged particles at low energies. In particular, the value of the ANC $C_{abc}(ls)$ determines essentially the cross section of the radiative capture $b(c,\gamma)a$ reaction at astrophysical energies [3].

2.2 Anomalous asymptotics

In fact, the asymptotic form (9) has been rigorously proved only for the simplest case when the composite system a consists of two elementary constituents. In that case the form (9) follows directly from the Schrödinger equation. It is shown below that the asymptotics of the overlap integral may differ from Eq. (9) if a consists of three or more constituents.

Consider the Fourier transform $J(q^2)$ of $I(r)$:

$$I(r) = (2\pi)^{-3}\int e^{i\vec{q}\vec{r}}\, J(q^2)\, d^3q. \tag{18}$$

According to Eqs.(2) and (6), $J(q^2)$ can be written in the form:

$$J(q^2) = -N_{bc}^{-1/2}\,\frac{2\mu_{bc}}{q^2+\varkappa^2}\,G(q^2), \tag{19}$$

Inserting Eq. (19) into Eq. (18) and integrating over angular variables, one obtains:

$$I(r) = \text{const}\cdot\frac{1}{ir}\int_{-\infty}^{\infty} e^{iqr}\,\frac{G(q^2)}{q^2+\varkappa^2}\,qdq. \tag{20}$$

In the upper half-plane of the complex variable q the integrand in Eq. (20) has a pole at $q = i\varkappa$ and a cut beginning from the nearest singular point $q = i\varkappa_1$ of the form factor $G(q^2)$. Making use of the Cauchy theorem one gets from Eq. (20)

$$I(r) = \text{const}\cdot\left\{\frac{\pi}{r}e^{-\varkappa r}G(-\varkappa^2) + \frac{1}{ir}\int\limits_{\varkappa_1}^{\infty}\frac{e^{-kr}\,\operatorname{disc} G(-k^2)}{k^2-\varkappa^2}\,kdk\right\} = I_0(r) + I_1(r). \tag{21}$$

An explicit asymptotic form of the second term on the r.h.s. of Eq. (21) depends on the behavior of $\operatorname{disc} G(q^2)$ at $q^2 \to -\varkappa_1^2$, that is, on the type of the singularity at $q = i\varkappa_1$. To investigate the singular behavior of $G(q^2)$, it is convenient to use the formalism of Feynman diagrams. In the vicinity of a proper singularity $z = z_0$, the singular part of the amplitude of a Feynman diagram having n inner lines and v vertices, behaves as [4, 5]

$$\begin{aligned} &M_{nv}\,|_{z\to z_0} \sim (z-z_0)^s && \text{if } s \neq 0,1,2,..., \\ &M_{nv}\,|_{z\to z_0} \sim (z-z_0)^s\,\ln(z-z_0) && \text{if } s = 0,1,2,..., \end{aligned} \tag{22}$$

where $s = (3n - 4v + 3)/2$.

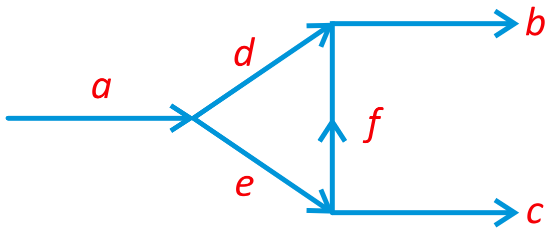

Figure 2: The simplest Feynman diagram for an $a \to b + c$ vertex.

The simplest Feynman diagram for an $a \to b + c$ vertex is a triangle diagram of Fig. 2.

For this diagram, $s = 0$ and it is easy to obtain from Eq. (21) that the contribution of that diagram results in

$$I(r)\,|_{r\to\infty} = c_0 \frac{e^{-\varkappa r}}{r} + c_1 \frac{e^{-\varkappa_1 r}}{r^2}, \tag{23}$$

$$\varkappa_1 = i\frac{m_b}{m_d}(\varkappa_{ade} + \varkappa_{bdf}), \quad \varkappa_{ijk}^2 = 2\mu_{jk}\epsilon_{ijk}, \quad \epsilon_{ijk} = m_j + m_k - m_i. \tag{24}$$

The first term on the r.h.s. of Eq. (23) corresponds to a 'normal' asymptotics. If $\varkappa < \varkappa_1$, then this term is a leading one and the overlap integral $I(r)$ possesses the normal asymptotics. However, in the opposite case, $\varkappa > \varkappa_1$, the asymptotics of $I(r)$ is governed by the second term in Eq. (23) (the 'anomalous' case).

Though there is no a general rules preventing the 'anomalous' condition $\varkappa > \varkappa_1$ from being satisfied, it appears that for real nuclear systems this condition is satisfied not very often. The nuclear vertices $^{16}O \to {}^{13}N({}^{13}C) + {}^{3}H({}^{3}He)$ and $^{20}Ne \to {}^{17}F({}^{17}O) + {}^{3}H({}^{3}He)$ can serve as examples of the anomalous asymptotics of the overlap integrals due to the triangle diagram of Fig. 2.

3 Three-fragment case ($n = 3$)

Consider a 3-body bound system $a = \{123\}$ with the wave function

$$\psi_a(\vec\rho, \vec r), \quad \vec\rho = \vec r_1 - \vec r_2, \quad \vec r = \vec r_3 - \frac{m_1\vec r_1 + m_2\vec r_2}{m_1 + m_2}. \tag{25}$$

The constituents 1, 2, and 3 might be composite, then ψ_a turns into an overlap integral.

Introduce the Fourier transform $\varphi_a(\vec k, \vec p)$ of $\psi_a(\vec\rho, \vec r)$ and the vertex function (VF) $W(\vec k, \vec p)$:

$$\vec k = (m_2\vec k_1 - m_1\vec k_2)/m_{12}, \quad \vec p = \left[m_{12}\vec k_3 - m_3(\vec k_1 + \vec k_2)\right]/M,$$

$$m_{ij} = m_i + m_j, \quad M = m_1 + m_2 + m_3. \tag{26}$$

$$\psi_a(\vec\rho, \vec r) = \int \exp\left[i(\vec k\vec\rho + \vec p\vec r)\right]\varphi_a(\vec k, \vec p)\,\frac{d\vec k}{(2\pi)^3}\frac{d\vec p}{(2\pi)^3}, \tag{27}$$

$$\varphi_a(\vec k, \vec p) = -W(\vec k, \vec p)/L(k, p), \quad L(k, p) = -(\epsilon + k^2/2\mu_1 + p^2/2\mu_2),$$

$$\epsilon = m_1 + m_2 + m_3 - m_a, \quad \mu_1 = m_1 m_2/m_{12}, \quad \mu_2 = m_1 m_{12}/M. \tag{28}$$

ψ_a and φ_a are normalized:

$$\int |\psi_a(\vec\rho, \vec r)|^2\, d\vec\rho\, d\vec r = 1, \quad \int |\varphi_a(\vec k, \vec p)|^2\, d\vec k\, d\vec p/(2\pi)^6 = 1. \tag{29}$$

Introduce modified Jacobi variables:

$$\vec{x}_1 = \sqrt{2\mu_1}\vec{\rho}, \quad \vec{x}_2 = \sqrt{2\mu_2}\vec{r}, \quad \vec{k}_1 = \vec{k}/\sqrt{2\mu_1}, \quad \vec{k}_2 = \vec{p}/\sqrt{2\mu_2},$$
$$x_1^2 + x_2^2 = R^2, \quad k_1^2 + k_2^2 = P^2. \tag{30}$$

$\psi_a(\vec{x}_1, \vec{x}_2)$ and $W(\vec{k}_1, \vec{k}_2)$ can be expanded through their partial-wave components $\psi_a^{(l,\lambda,L)}(x_1, x_2)$ and $W^{(l,\lambda,L)}(k_1, k_2)$ corresponding to Jacobi angular momenta l and λ ($\vec{l} + \vec{\lambda} = \vec{L}$). Spin variables could be taken into account as well. Strictly speaking, the following text applies to these partial-wave components. However, to simplify the presentation, we suppose that the $l = \lambda = 0$ components contribute only to ψ_a and W. Then after integrating over the angular variables Eq. (27) assumes the form

$$\psi_a(x_1, x_2) = \frac{(\mu_1\mu_2)^{3/2}}{2\pi^4}\frac{1}{x_1x_2}\int_0^\infty dk_1 \int_0^\infty dk_2 k_1 k_2 \frac{W(k_1, k_2)}{\epsilon + P^2}$$
$$\times (e^{ik_1x_1} - e^{-ik_1x_1})(e^{ik_2x_2} - e^{-ik_2x_2}). \tag{31}$$

$W(k_1, k_2)$ should depend on k_1^2, k_2^2; that is $W(k_1, k_2)$ is an even function of k_1, k_2. Hence Eq. (31) can be written as

$$\psi_a(x_1, x_2) = \frac{(\mu_1\mu_2)^{3/2}}{2\pi^4}\frac{1}{x_1x_2}\int_{-\infty}^\infty dk_1 \int_{-\infty}^\infty dk_2\, k_1 k_2 e^{i(k_1x_1 + k_2x_2)}\,\frac{W(k_1, k_2)}{\epsilon + P^2}. \tag{32}$$

We neglect the Coulomb interaction in what follows though the results could be easily generalized to the case when two of the particles 1, 2 and 3 are charged.

If a pair subsystem ij ($ij = 12, 23, 31$) can form a bound state with the binding energy ϵ_{ij}, then the VF $W(k_1, k_2)$ has a two-body pole at the relative kinetic energy $E_{ij} = -\epsilon_{ij}$. Such poles lead to the two-body asymptotics analogous to those considered in Section 2. In the present Section we will consider the true three-body asymptotics generated by the pole $P^2 = -\epsilon$ in Eq. (32). Denoting its contribution by ψ_3 and integrating over k_2 in the integral (32) by taking the residue at $k_2^2 = -\epsilon - k_1^2$, one obtains

$$\psi_3(x_1, x_2) = i\frac{(m_1m_2m_3/M)^{3/2}}{2\pi^3}\,\frac{1}{x_1x_2}\,J(x_1, x_2),$$
$$J(x_1, x_2) = \int_{-\infty}^\infty dk_1\, k_1 \exp\Big(ik_1x_1 - \sqrt{\epsilon + k_1^2}\,x_2\Big)\, W\Big(k_1, i\sqrt{\epsilon + k_1^2}\Big). \tag{33}$$

Denoting $x_1 = R\cos\alpha$, $x_2 = R\sin\alpha$ one can evaluate $J(x_1, x_2)$ at $R \to \infty$ by the saddle-point method (the saddle-point is $k_1 = i\epsilon^{1/2}\cos\alpha$). As a result, one obtains the following expression for the leading contribution to the asymptotic form of $\psi_3(x_1, x_2)$:

$$\psi_{3as}^{(0)}(R, \alpha) = C_3\,\frac{e^{-\sqrt{\epsilon}R}}{R^{5/2}},$$
$$C_3 = -\frac{(m_1m_2m_3/M)^{3/2}}{\sqrt{2}\pi^{5/2}}\,W(i\sqrt{\epsilon}\cos\alpha, i\sqrt{\epsilon}\sin\alpha). \tag{34}$$

The R dependence of the asymptotic form (34) agrees with that presented in [6].

C_3 is the three-body asymptotic normalization factor. It is expressed in terms of the on-shell three-body vertex function $W(\alpha) \equiv W(i\sqrt{\epsilon}\cos\alpha, i\sqrt{\epsilon}\sin\alpha)$ corresponding to $P^2 = -\epsilon$. Eq. (34) is the three-body analogue of the two-body relation (9).

The saddle-point method allows one to calculate corrections to the leading term (34). In the present work, the expressions for the correction terms of the order $(\sqrt{\epsilon}R)^{-1}$

and $(\sqrt{\epsilon}R)^{-2}$ are obtained. These corrections are expressed in terms of $W(\alpha)$ and its derivatives. The explicit expression for $\psi^{(2)}_{3as}$ including the corrections of the order $(\sqrt{\epsilon}R)^{-1}$ and $(\epsilon R^2)^{-1})$ is of the form:

$$\psi^{(2)}_{3as} = \psi^{(0)}_{3as}\left[1 + (\sqrt{\epsilon}R)^{-1}\chi_\alpha + (\epsilon R^2)^{-1}\xi_\alpha\right],$$
$$\chi_\alpha = \frac{15}{8} - 2\cot(2\alpha)\,\gamma_1(\alpha) - \frac{1}{2}\gamma_2(\alpha),$$
$$\xi_\alpha = \frac{105}{128} - \frac{11}{4}\cot(2\alpha)\,\gamma_1(\alpha) - \frac{43}{16}\gamma_2(\alpha) + \cot(2\alpha)\,\gamma_3(\alpha) + \frac{1}{8}\gamma_4(\alpha),$$
$$\gamma_n(\alpha) \equiv \frac{1}{W(\alpha)}\frac{d^nW(\alpha)}{d\alpha^n}. \quad (35)$$

The asymptotics of the three-body wave function was considered in [7]. The results of that work include the corrections due to non-zero values of the angular momenta l and λ. However, these corrections do not include the terms of the same order due to using the saddle-point method. Making use of the results of the present work, one can calculate the reliable correction terms for $l + \lambda \leq 2$.

4 Discussion and Conclusions

The on-shell VFs $W(\alpha)$ are important three-body characteristics determining the asymptotics of three-body wave functions. Of a special interest are the quantities $W_0 = W(\alpha = \pi/2)$ corresponding to $k_1 = 0$ what means that the particles 1 and 2 move as a single body with the mass $m_{12} = m_1 + m_2$. W_0 is a constant which is an analog of the two-body vertex constant G_{abc}. It could be called the generalized vertex constant (GVC).

It follows from Landau equations [4] that the GVCs determine the contributions of proper singularities of Feynman diagrams containing the loops consisting of two particles (as in Fig. 3). Thus $W_0(a \to 1+2+3)$ and $W_0(1+2+4 \to c)$ in Fig. 3a determine a possible anomalous asymptotics of the overlap integral I_{abc}. In particular, the vertices $W_0(^9\text{Be} \to n+\alpha+\alpha)$ and $W_0(n+\alpha+p \to {}^6\text{Li})$ $(W_0(n+\alpha+n \to {}^6\text{He}))$ in the diagrams of the Fig. 3a type were used in Ref. [8] to analyze the anomalous asymptotics of the overlap integrals for the vertices $^9\text{Be} \to {}^6\text{Li} + {}^3\text{H}$ $(^9\text{Be} \to {}^6\text{He} + {}^3\text{He})$. $W_0(a \to 1+2+3)$ and $W_0(x+1+2 \to y)$ in Fig. 3b determine the contribution of the t-channel normal threshold to the amplitude of the process $a + x \to 3 + y$.

The concept of the GVC could be directly extended to the loops containing more than two particles.

In conclusion it is worthwhile to note that the GVC W_0 for the vertex $a \to 1+2+3$ could in principle be determined by the analytic continuation of the differential cross section of the $a + x \to 1 + 2 + y$ reaction to the pole of the diagram of Fig. 4.

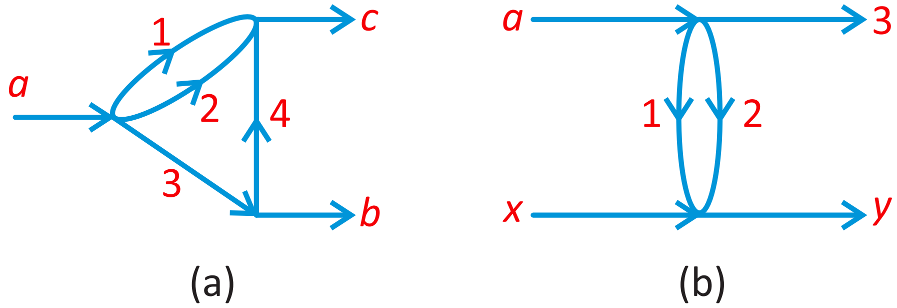

Figure 3:

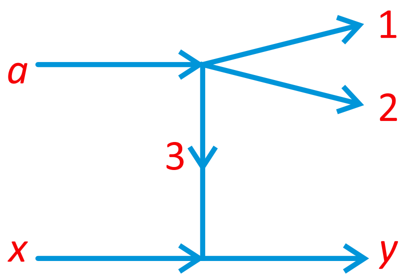

Figure 4:

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