Electromagnetic deuteron form factors in point form relativistic quantum mechanics

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Abstract

A study of electromagnetic structure of the deuteron in the framework of relativistic quantum mechanics is presented. The deuteron form factors dependencies on the transferred 4-momentum Q up to 7.5 fm^{-1} are calculated. We compare results obtained with different realistic deuteron wave functions stemming from Nijmegen-I, Nijmegen-II, JISP16, CD-Bonn, Paris and Moscow (with forbidden states) potentials. A nucleon form factor parametrization consistent with modern experimental analysis was used as an input data.

Keywords: *Deuteron; nucleon; electromagnetic form factors*

1 Introduction

Elastic *ed* scattering observables are directly expressed within the Born approximation of one-photon exchange mechanism through electromagnetic (EM) deuteron form factors (FFs) [1–3]. Therefore this process allows to extract the EM FF dependencies on the transferred 4-momentum Q in the space-like region. Relativistic effects may be essential even at low Q [2, 3]. There are different relativistic models of deuteron EM FFs [4–8].

We apply the point-form (PF) relativistic quantum mechanics (RQM) to the elastic electron-deuteron scattering in a Poincaré-invariant way. The RQM concepts and an exhaustive bibliography are presented in the review by Keister and Polyzou [9]. The PF is one of the three forms of RQM proposed by Dirac [10]. The other two are the instant and front forms. These forms are associated with different subgroups of the Poincaré group which may be free of interactions. A general method of allowing for interactions in generators of the Poincaré group was derived in Ref. [11]. It was shown that all the forms are unitary equivalent [12]. Though each form has certain advantages, there are important simplifying features of the PF [13]. In the PF, all generators of the homogeneous Lorentz group (space-time rotations) are free of interactions. Therefore the spectator approximation (SA) preserves its spectator character in any reference frame (r. f.) only in the PF [14–16]. In the case of electromagnetic NN process, it means that the NN interaction does not affect the photon-nucleon interaction and therefore the sum of one-particle EM current operators is invariant under transformations from one r. f. to another. Two equivalent SAs for EM current operator of a composite system in PF RQM were derived in Refs. [13, 15]. The PF SA was applied to calculation of deuteron, pion and nucleon EM FFs [7, 17–21] providing reasonable results.

The present paper is an extension of our previous investigations where we have described the elastic NN scattering up to laboratory energy of 3 GeV [22], bremsstrahlung in pp scattering $pp \to pp\gamma$ [23], deuteron photodisintegration $\gamma D \to np$

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http://www.ntse-2014.khb.ru/Proc/Khokhlov.pdf.

[24–26] and exclusive deuteron electrodisintegration [27]. Here we demonstrate that the developed approach is applicable to the elastic eD scattering.

2 Potential model in PF of RQM

A system of two particles is described within PF RQM by the wave function which is an eigenfunction of the mass operator $\hat{M}$. We may represent this wave function as a product of external and internal parts. The internal wave function $|\chi\rangle$ is also an eigenfunction of the mass operator and for a system of two nucleons with masses $m_1 \approx m_2 \approx m = 2m_1m_2/(m_1+m_2)$ satisfies the equation

$$\hat{M}|\chi\rangle \equiv \left[2\sqrt{\mathbf{q}^2+m^2}+V_{int}\right]|\chi\rangle = M|\chi\rangle, \tag{1}$$

where V_{int} is an operator commuting with the full angular momentum operator and acting on internal variables (spins and relative momentum) only, $\mathbf{q}$ is a momentum operator of one of particles in the center of mass (c. m.) frame (relative momentum), $\hat{M}$ is a system mass operator and M is its eigenvalue. Here we adopt a natural system of units with $\hbar = c = 1$. A rearrangement of Eq. (1) gives

$$\left[\mathbf{q}^2 + mV\right]|\chi\rangle = q^2|\chi\rangle, \tag{2}$$

where the operator

$$mV = \frac{1}{4}\left(2\sqrt{\mathbf{q}^2+m^2}V_{int} + 2V_{int}\sqrt{\mathbf{q}^2+m^2} + V_{int}^2\right), \tag{3}$$

as well as V_{int}, acts on internal variables only, and the eigenvalue of the operator $\mathbf{q}^2 + mV$ is

$$q^2 = \frac{M^2}{4} - m^2. \tag{4}$$

Equation (2) is identical in form to a Schrödinger equation. Relativistic corrections affect the deuteron binding energy ε only and may be easily accounted for by replacing the experimental deuteron binding of 2.2246 MeV by an effective value of 2.2233 MeV. The origin of this relativistic correction is easy to understand [28–30]. Clearly,

$$M = 2m - \varepsilon, \tag{5}$$

and hence Eq. (4) can be rewritten as

$$q^2 = -m\varepsilon\left(1 - \frac{\varepsilon}{4m}\right). \tag{6}$$

Comparing Eq. (6) with the nonrelativistic relation

$$q^2 = -m\varepsilon, \tag{7}$$

one identifies the factor $\left(1 - \frac{\varepsilon}{4m}\right)$ as a relativistic correction to the deuteron binding energy. It is interesting and important to note that there is no similar correction in the scattering domain since $q^2 = mE_{lab}/2$ is a precise relativistic relationship (E_{lab} is the laboratory energy) used in the partial wave analysis [28].

The difference between the experimental and effective deuteron binding energies is negligible for our problem. Therefore, due to the formal identity between Eq. (2) and Schrödinger equation, we can use non-relativistic deuteron wave functions in our calculations.

3 *eD* elastic scattering

We sketch here some PF RQM results needed for our calculations. We use formalism and notations of Ref. [15].

We consider the pn system and neglect the difference between neutron and proton masses. Let p_i be the 4-momentum of nucleon i, $P \equiv (P^0, \mathbf{P}) = p_1 + p_2$ is the system 4-momentum, M is the system mass and $G = P/M$ is the system 4-velocity. The wave function of two particles with 4-momentum P is expressed through a tensor product of external and internal parts,

$$|P, \chi\rangle = U_{12}\, |P\rangle \otimes |\chi\rangle, \tag{8}$$

where the internal wave function $|\chi\rangle$ fits Eqs. (1)–(2). The unitary operator

$$U_{12} = U_{12}(G, \mathbf{q}) = \prod_{i=1}^{2} D[\mathbf{s}_i; \alpha(p_i/m)^{-1}\alpha(G)\alpha(q_i/m)] \tag{9}$$

relates the "internal" Hilbert space with the Hilbert space of two-particle states [15]. $D[\mathbf{s}; u]$ is a SU(2) operator corresponding to the element $u \in \mathrm{SU}(2)$, $\mathbf{s}$ are the SU(2) generators. In our case of spin $s = 1/2$ particles, we deal with the fundamental representation, i. e., $\mathbf{s}_i \equiv \frac{1}{2}\boldsymbol{\sigma}_i$ [$\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli matrices] and $D[\mathbf{s}; u] \equiv u$. Matrix $\alpha(g) = (g^0 + 1 + \boldsymbol{\sigma}\cdot\mathbf{g})/\sqrt{2(g^0+1)}$ corresponds to a 4-velocity g. The momenta of particles in their c. m. frame are

$$q_i = L[\alpha(G)]^{-1} p_i, \tag{10}$$

where $L[\alpha(G)]$ is the Lorentz boost to the frame moving with 4-velocity G.

The "external" part of the wave function is defined as

$$\langle G|P'\rangle \equiv \frac{2}{M'}\, G'^0\, \delta^3(\mathbf{G} - \mathbf{G}'). \tag{11}$$

Its scalar product is

$$\langle P''|P'\rangle = \int \frac{d^3\mathbf{G}}{2G^0}\, \langle P''|G\rangle\langle G|P'\rangle = 2\sqrt{M'^2 + \mathbf{P}'^2}\, \delta^3(\mathbf{P}'' - \mathbf{P}'), \tag{12}$$

where $G^0(\mathbf{G}) \equiv \sqrt{1+\mathbf{G}^2}$. The internal part $|\chi\rangle$ is characterized by momentum $\mathbf{q} = \mathbf{q}_1 = -\mathbf{q}_2$ of one of the particles in the c. m. frame.

According to the Bakamjian—Thomas procedure [11], the 4-momentum $\hat{P} = \hat{G}\hat{M}$ incorporates the interaction V_{int}, where $\hat{M}$ is the sum of the free mass operator M_{free} and interaction, $\hat{M} = M_{free} + V_{int}$ [see Eq. (1)]. The interaction operator acts only on internal variables. The operators V_{int} and V (and therefore $\hat{M}$ and M_{free}) commute with S, the spin (full angular momentum) operator, and $\hat{G}$, the 4-velocity operator. The generators of space-time rotations are interaction-free. Most of formal non-relativistic scattering theory results are valid in the case of two relativistic particles [9]. For example, the relative orbital angular momentum and spins are coupled in the c. m. frame in the same manner as in the non-relativistic case.

The deuteron wave function $|P_i, \chi_i\rangle$ is normalized,

$$\langle P_f, \chi_f|P_i, \chi_i\rangle = 2P_i^0\, \delta^3(\mathbf{P}_i - \mathbf{P}_f)\langle\chi_f|\chi_i\rangle. \tag{13}$$

There is a convenient r. f. for calculation of current operator matrix elements [15] (it coincides with the Breit r. f. in the case of elastic ed scattering). This r. f. is defined by the following condition for all EM reactions with two nucleons:

$$\mathbf{G}_f + \mathbf{G}_i = 0, \tag{14}$$

where $\mathbf{G}_f = \mathbf{P}_f/M_D$, $\mathbf{G}_i = \mathbf{P}_i/M_D$ are the final and initial 4-velocities of the deuteron and M_D is its mass. The matrix element of the current operator is [15]

$$\langle P_f, \chi_f | \hat{J}^\mu(x) | P_i, \chi_i \rangle = 2(M_f M_i)^{1/2} \exp(\imath(P_f - P_i)x) \langle \chi_f | \hat{j}^\mu(\mathbf{h}) | \chi_i \rangle, \qquad (15)$$

where $\hat{j}^\mu(\mathbf{h})$ defines action of current operator in the internal space of the NN system,

$$\mathbf{h} = \frac{2(M_i M_f)^{1/2}}{(M_i + M_f)^2}\,\mathbf{k} = \frac{\mathbf{k}}{2M_D} \qquad (16)$$

is the vector-parameter [15] ($0 \le h \le 1$), $\mathbf{k}$ is the momentum of photon in the r. f. defined by Eq. (14), $M_i = M_f = M_D$ are the masses of the initial and final NN system (deuteron).

The internal deuteron wave function is

$$|\chi_i\rangle = \frac{1}{r} \sum_{l=0,2} u_l(r) | l, 1; J = 1 M_J \rangle_{\mathbf{r}}; \qquad (17)$$

it is normalized: $\langle \chi_i | \chi_i \rangle = 1$. We use the momentum space wave function

$$|\chi_i\rangle = \frac{1}{q} \sum_{l=0,2} u_l(q)\, | l, 1; 1 M_J \rangle_{\mathbf{q}}, \qquad (18)$$

where

$$u(q) \equiv u_0(q) = \sqrt{\frac{2}{\pi}} \int dr \sin(qr) u(r), \qquad (19)$$

$$w(q) \equiv u_2(q) = \sqrt{\frac{2}{\pi}} \int dr \left[\left(\frac{3}{(qr)^2} - 1 \right) \sin(qr) - \frac{3}{qr} \cos(qr) \right] w(r). \qquad (20)$$

Transformations from the Breit r. f. (14) to the initial and final c. m. frame of the NN system are the boosts along vector $\mathbf{h}$ (axis z). Projection of the total deuteron angular momentum onto z axis are unaffected by these boosts. The initial deuteron moves in the Breit r. f. in the direction opposite to $\mathbf{h}$. Its internal wave function with the spirality Λ_i is

$$|\Lambda_i\rangle = \frac{1}{q} \sum_{l=0,2} u_l(q)\, | l, 1; 1, M_J = -\Lambda_i \rangle. \qquad (21)$$

The wave function of the final deuteron with the spirality Λ_f is

$$|\Lambda_f\rangle = \frac{1}{q} \sum_{l=0,2} u_l(q)\, | l, 1; 1, M_J = \Lambda_f \rangle. \qquad (22)$$

A conventional parametrization of the deuteron (spin-1 particle) EM current operator (CO) matrix element is [2, 3, 31]:

$$\begin{aligned} &(4P_i^0 P_f^0)^{1/2} \langle P_f, \chi_f | J^\mu | P_i, \chi_i \rangle \\ &\quad = -\left\{ G_1(Q^2)\, (\boldsymbol{\xi}_f^* \cdot \boldsymbol{\xi}_i) - G_3(Q^2)\, \frac{(\boldsymbol{\xi}_f^* \cdot \Delta P)(\boldsymbol{\xi}_i \cdot \Delta P)}{2M_D^2} \right\} (P_i^\mu + P_f^\mu) \\ &\qquad - G_2(Q^2)\, [\xi_i^\mu\, (\boldsymbol{\xi}_f^* \cdot \Delta\ \mathbf{P}) - \xi_f^{*\mu} (\boldsymbol{\xi}_i \cdot \Delta \mathbf{P})], \end{aligned} \qquad (23)$$

where $(a \cdot b) = a^0 b^0 - (\mathbf{a} \cdot \mathbf{b})$, form factors $G_i(Q^2)$, $i = 1, 2, 3$, are the functions of $Q^2 = -\Delta P^2$, $\Delta P = P_f - P_i$.

In the Breit r.f. $\mathbf{P}_f = -\mathbf{P}_i$, $P_i^0 = P_f^0 \equiv P^0 = M_D/\sqrt{1-h^2}$, $\Delta P = (0, 2\mathbf{P}_f)$, $P_i^\mu + P_f^\mu = (2P^0, \mathbf{0})$, $\mathbf{P}_f/P^0 = \mathbf{h}$, $\mathbf{P}_f = \mathbf{h}M_D/\sqrt{1-h^2}$, $\Delta P^2 = -4h^2M_D^2/(1-h^2)$, $Q^2 \equiv -\Delta P^2$, $h^2 = (\mathbf{h}\cdot\mathbf{h})$,

$$\langle\chi_f|j^0(\mathbf{h})|\chi_i\rangle = -G_1(Q^2)(\boldsymbol{\xi}'^*\cdot\boldsymbol{\xi}) + 2G_3(Q^2)\frac{(\boldsymbol{\xi}_f^*\cdot\ \mathbf{h})(\boldsymbol{\xi}_i\cdot\mathbf{h})}{1-h^2} + G_2(Q^2)[\xi_i^0(\boldsymbol{\xi}_f^*\cdot\mathbf{h}) - \xi_f^{0*}(\boldsymbol{\xi}_i\cdot\mathbf{h})], \quad (24)$$

$$\langle\chi_f|\mathbf{j}(\mathbf{h})|\chi_i\rangle = G_2(Q^2)[\xi_i(\boldsymbol{\xi}_f^*\cdot\mathbf{h}) - \xi_f^*(\boldsymbol{\xi}_i\cdot\mathbf{h})] = G_2(Q^2)[\mathbf{h}\times[\boldsymbol{\xi}_i\times\boldsymbol{\xi}_f^*]]. \quad (25)$$

It has been shown [15] that these expressions are equivalent to choosing j^ν as

$$j^0(\mathbf{h}) = G_C(Q^2) + \frac{2}{(1-h^2)}\,G_Q(Q^2)\left[\frac{2}{3}h^2 - (\mathbf{h}\cdot\mathbf{J})^2\right], \quad (26)$$

$$\mathbf{j}(\mathbf{h}) = -\frac{\imath}{\sqrt{1-h^2}}\,G_M(Q^2)\,(\mathbf{h}\times\mathbf{J}), \quad (27)$$

where $\mathbf{J}$ is the total angular momentum (spin) of the deuteron; G_C, G_Q and G_M are its charge monopole, charge quadruple and magnetic dipole FFs.

Spiral deuteron polarizations in the initial and final states are

$$\xi_i^\Lambda = \begin{cases} (0, \pm1, -\imath, 0)/\sqrt{2} & (\Lambda = \pm), \\ (-Q/2, 0, 0, P_0)/M_D = (-h, 0, 0, 1)/\sqrt{1-h^2} & (\Lambda = 0), \end{cases} \quad (28)$$

$$\xi_f^\Lambda = \begin{cases} (0, \mp1, -\imath, 0)/\sqrt{2} & (\Lambda = \pm), \\ (Q/2, 0, 0, P_0)/M_D = (h, 0, 0, 1)/\sqrt{1-h^2} & (\Lambda = 0). \end{cases} \quad (29)$$

A virtual photon polarization is

$$\epsilon^\lambda = \begin{cases} (0, \mp1, -\imath, 0)/\sqrt{2} & (\lambda = \pm), \\ (1, 0, 0, 0) & (\lambda = 0). \end{cases} \quad (30)$$

FFs G_i are expressed as

$$\begin{aligned} G_C &= G_1 + \frac{2}{3}\eta G_Q, \\ G_Q &= G_1 - G_M + (1+\eta)G_3, \\ G_1 &= G_C - \frac{2h^2}{3(1-h^2)}G_Q, \\ G_3 &= G_Q\left(1 - \frac{h^2}{3}\right) - G_C(1-h^2) + G_M(1-h^2), \end{aligned} \quad (31)$$

where $\eta = Q^2/4M_D^2 = h^2/(1-h^2)$. Supposing $Q^2 = 0$, we have $G_Q = G_1 - G_M + G_3$ and $G_C = G_1$. Form factors $G_C(0) = e$, $G_M(0) = \mu_D e/2M_D$ and $G_Q = Q_D e/M_D^2$ provide deuteron charge, magnetic and quadruple momenta respectively.

Denoting helicity amplitudes as $j^\lambda_{\Lambda_f\Lambda_i} \equiv \langle\Lambda_f|\left(\epsilon^\lambda_\mu\cdot j^\mu(\mathbf{h})\right)|\Lambda_i\rangle$, we arrive at

$$j^0_{00}(Q^2) = G_C + \frac{4}{3}\frac{h^2}{1-h^2}G_Q, \quad (32)$$

$$j^0_{+-}(Q^2) = j^0_{-+}(Q^2) = G_C - \frac{2}{3}\frac{h^2}{1-h^2}G_Q, \quad (33)$$

$$\frac{j^+_{+0}(Q^2) + j^+_{0-}(Q^2)}{2} = -\frac{h}{\sqrt{1-h^2}}G_M \quad (34)$$

and

$$j^+_{+0}(Q^2) = j^-_{-0}(Q^2) \approx j^+_{0-}(Q^2) = j^-_{0+}(Q^2). \quad (35)$$

The deuteron FFs are associated with unpolarized structure functions [32]:

$$A(Q^2) = G_C^2(Q^2) + \frac{2}{3}\,\eta\, G_M^2(Q^2), \tag{36}$$

$$B(Q^2) = \frac{4}{3}\,\eta\,(1+\eta)\, G_M^2(Q^2). \tag{37}$$

These quantities are extracted from the elastic eD scattering with unpolarized particles. A tensor polarization observable $t_{20}(Q^2,\theta)$ is conventionally used as an additional quantity needed for definition of all three FFs.

In the present paper, we use the EM CO obtained within SA in Ref. [15] without expanding it in powers of h and calculate its matrix elements in the momentum space. Therefore we use the following expansion of $\hat{j}^{\mu}(\mathbf{h}) \approx \hat{j}^{\mu}_{SA}(\mathbf{h})$ [27] for the matrix element calculations:

$$\hat{j}^{\mu}_{SA}(\mathbf{h}) = \big(1 + (\mathbf{A}_2\cdot\mathbf{s}_2)\big)\big(B_1^{\mu} + (\mathbf{C}_1^{\mu}\cdot\mathbf{s}_1)\big)\mathbf{I}_1(\mathbf{h}) + \big(1 + (\mathbf{A}_1\cdot\mathbf{s}_1)\big)\big(B_2^{\mu} + (\mathbf{C}_2^{\mu}\cdot\mathbf{s}_2)\big)\mathbf{I}_2(\mathbf{h}), \tag{38}$$

where $\mathbf{A}_i$, B_i^{μ} and $\mathbf{C}_i^{\mu}$ are some vector functions of $\mathbf{h}$ and $\mathbf{q}(q,\theta,\phi)$. In the spherical coordinate system (q,θ,ϕ), the dependence of these functions on ϕ appears as $e^{\pm im\phi}$ $(m = 0, 1, 2)$. The angle ϕ is analytically integrated out giving trivial equalities (35).

4 Results

In our calculations, we use as an input momentum space deuteron wave functions and nucleon EM FFs. The momentum space deuteron wave functions stemming from Nijmegen-I (NijmI), Nijmegen-I (NijmII) [33], JISP16 [34], CD-Bonn [29], Paris [35], Argonne18 [30] (the momentum space deuteron wave function is grabbed from Ref. [36]) and Moscow (with forbidden states) [22] potentials are shown in Figs. 1. We use two versions of Moscow type potential: Moscow06 [22] and Moscow14. The latter one was obtained by the author in the same manner outlined in Ref. [22] but with deuteron asymptotic constants fitted to describe static deuteron form factors. Parameters of both Moscow potentials may be obtained upon request from the author (e-mail: `nikolakhokhlov@yandex.ru`). The S wave functions of all potentials but JISP16 change sign at $q \approx 2$ fm^{-1}, and D wave functions change sign at $q \approx 6{-}8$ fm^{-1}. The S and D wave functions of Argonne18, Paris and NijmII are close at $q \lesssim 5$ fm^{-1}. The S wave functions of CD-Bonn and NijmI are close at $q \lesssim 5$ fm^{-1}. The JISP16 wave functions decrease rapidly at q larger than approximately 2 fm^{-1} without changing sign.

Our results for deuteron EM FFs are presented in Table 1 and in Figs. 2, 3, 4. The results for Argonne18, Paris and NijmII are close manifesting the closeness of their wave functions at $q \lesssim 5$ fm^{-1}. NijmI and CD-Bonn provide more distinct results. Our calculations demonstrate that G_M obtained with all potentials changes sign at rather low Q that is not seen experimentally. Nevertheless CD-Bonn and NijmI result in a reasonable description of G_M at $Q < 7$ fm. Moscow potentials provide the best description of charge form factor G_C.

An essential factor affecting our calculations is the nucleon FF dependency on the momentum transferred to the individual nucleon, $Q_p^2 \approx Q_n^2 \neq Q^2$. These FFs have been measured at discrete values of $Q^2_{i=p,n}$ while we need a continuous dependency on Q_i. In our calculations, we utilize phenomenological nucleon FF dependencies on Q_i^2 of Ref. [54]. It should be noted that the neutron EM FFs are extracted from experimental data on $^2\vec{\mathrm{H}}(\vec{e},e'n)p$ and other processes with deuteron and triton using various models of mechanism of these possesses and nuclei. Therefore these FFs are model dependent.

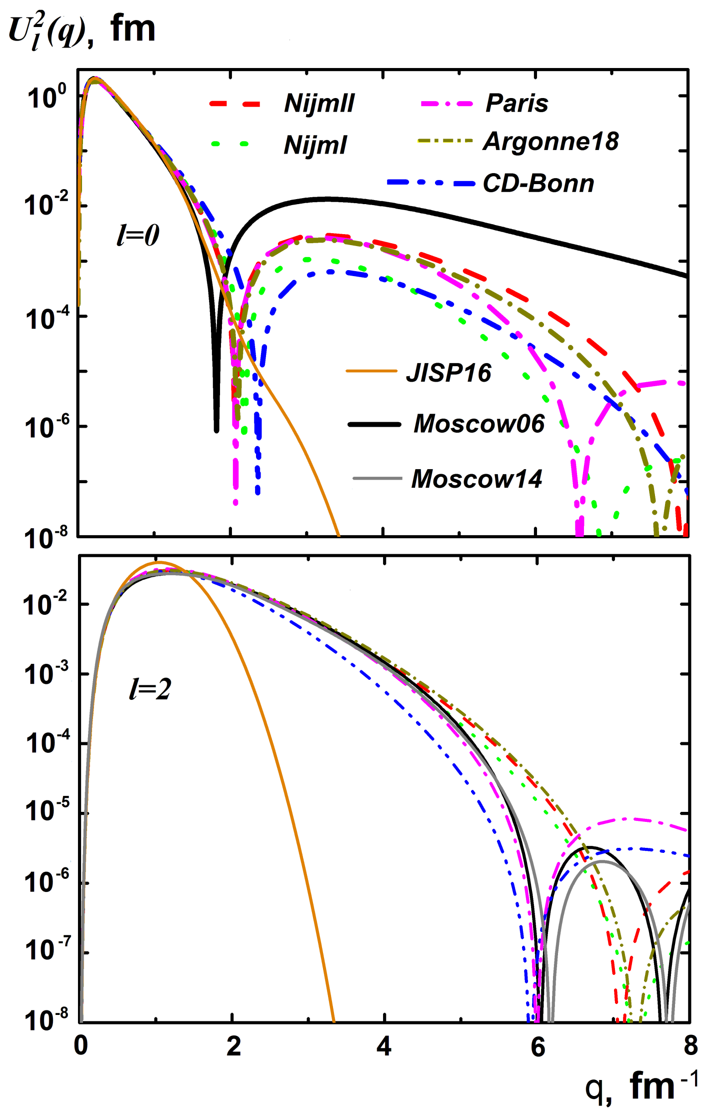

Figure 1: Momentum space deuteron wave functions used in calculations.

Table 1: Static deuteron form factors. The results of relativistic (nonrelativistic) calculations are given before (after) slash.

	$G_M(0) = \frac{M_d}{m_p}\mu_d$	$G_Q(0) = M_d^2 Q_d$
Exp	1.7148	25.83
NijmI	1.697/1.695	24.8/24.6
NijmII	1.700/1.695	24.7/24.5
Paris	1.696/1.694	25.6/25.2
CD-Bonn	1.708/1.704	24.8/24.4
Argonne18	1.696/1.694	24.7/24.4
JISP16	1.720/1.714	26.3/26.1
Moscow06	1.711/1.699	24.5/24.2
Moscow14	1.716/1.700	26.0/25.8

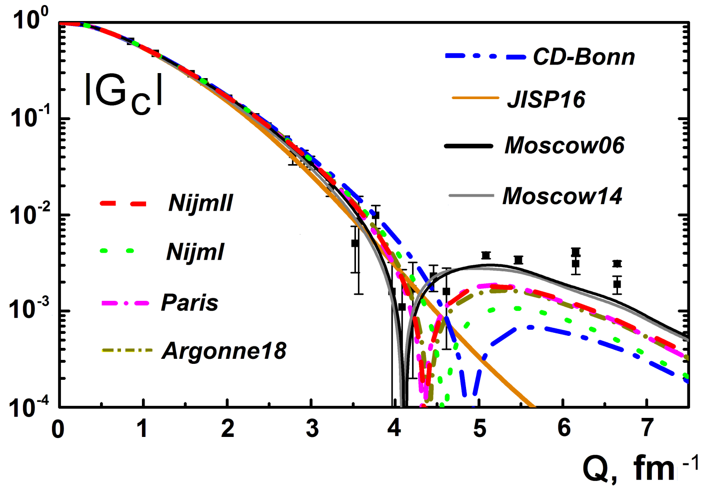

Figure 2: Deuteron form factor G_C as a function of Q. Experimental points are from compilation [3] where they were calculated using data for A, B and t_{20} obtained in Refs. [37–53].

We see a good overall agreement between the theory and experiment at $Q < 5$ fm^{-1}. Discrepancies at larger Q are comparable with differences of results for different potentials. Model calculations [55] show that meson exchange currents may provide a significant contribution to EM processes in the *np*-system. We do not take into account these currents. However it is not clear how these currents can be derived consistently with the short-range NN interaction of the QCD origin. In addition, the EM FFs of nucleons are not described by meson degrees of freedom at intermediate and high energies [56].

To complete this line of our investigation, we plan to calculate neutron EM FFs compatible with Moscow potential model which has not been used for the extraction of these FFs.

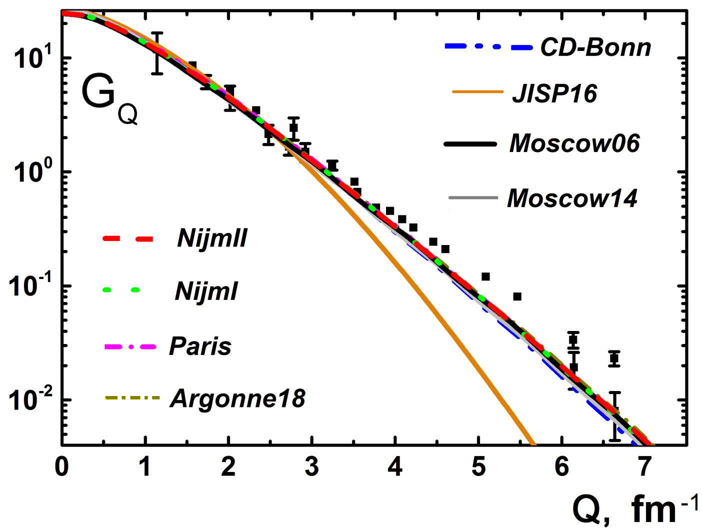

Figure 3: Deuteron form factor G_Q as a function of Q. See Fig. 2 for details.

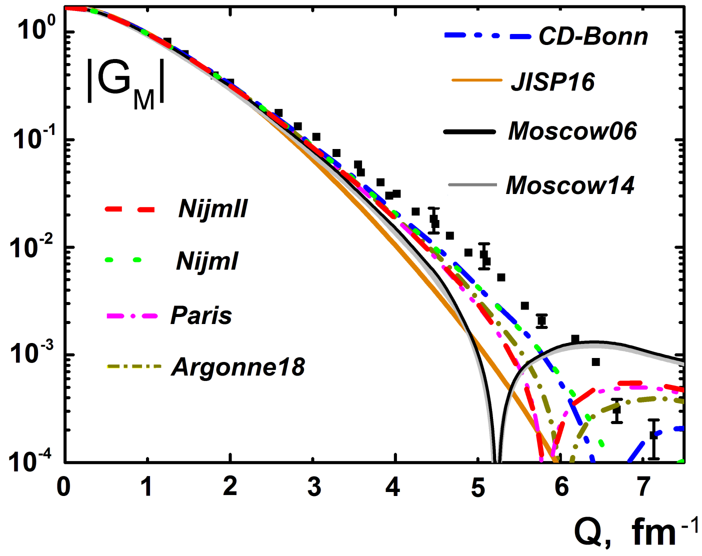

Figure 4: Deuteron form factor G_M as a function of Q. See Fig. 2 for details.

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