Effects of QRPA Correlations on Nuclear Matrix Elements of Neutrinoless Double-Beta Decay through Overlap Matrix

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Abstract

We show an improvement of the quasiparticle random-phase approximation (QRPA) approach for calculating the nuclear matrix elements (NMEs) of the neutrinoless double-beta decay. One of the techniques of obtaining the NME is to calculate the overlaps of the QRPA excited states obtained from the initial and final states, and these overlaps should be calculated using the QRPA ground states defined as the vacuum of the QRPA quasibosons. The significant difference from the usual method is that a normalization factor arises from this definition, and this factor is much larger than one.

Keywords: *QRPA; nuclear matrix elements; neutrinoless double-beta decay*

1 Introduction

The determination of the effective neutrino mass is one of the most important subjects in physics now due to multiple reasons. The neutrino has been assumed to be massless in the standard theory, however, the neutrino oscillations showed that the neutrino is actually massive [1–4]. Mass of elementary particle is a basic physical constant that we cannot leave unknown. The effective neutrino mass affects the fluctuation of the mass distribution in the universe [5]. The neutrino also plays an important role in the energy and momentum transport in the supernova explosion [6].

The neutrino oscillations also provided us with most of the matrix elements of the transformation matrix between the mass eigen and flavor states. This information is, however, not sufficient for determining the absolute neutrino masses. One of few methods which can provide the expectation value of neutrino mass (effective neutrino mass) is the neutrinoless double-beta ($0\nu\beta\beta$) decay which occurs if the neutrino is a Majorana particle. This decay can occur in nuclei if the mass of the nucleus with (proton number, neutron number) = $(Z+2,N-2)$ (daughter nucleus) is smaller than that with (Z,N) (parent nucleus). For experiments, other conditions are necessary to be satisfied: e. g., a suppression of the single-beta decay, a separation of the spectrum of the two-neutrino double-beta decay from that of the neutrinoless one, and a productibility of the parent nucleus. Many experiments are now in preparation for observing the $0\nu\beta\beta$ decay, see, e. g., Ref. [7].

A challenging problem for nuclear theory is to calculate the respective nuclear matrix elements (NMEs) accurately; these are the transition matrix elements between the initial and final nuclear states in the $0\nu\beta\beta$ decay, and the decay probability is

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proportional to the absolute value of the NME squared and the effective neutrino mass squared. Currently, we are facing a problem that the calculated values significantly depend on the methods and differ by a factor of 2 approximately [8]. The decay probability is also proportional to the so-called phase-space factor which is the electron component of the transition matrix element, and the calculation of this factor is well-established. Several methods have been used for calculating the NME. An approach using the quasiparticle random-phase approximation (QRPA) is one of the oldest methods (see, e. g., Ref. [9]), and many improvements have been made, e. g., an introduction of the deformation (see, e. g., Ref. [10]), the extension to the renormalized QRPA (see, e. g., Ref. [11]), an extension of the wave-function space (see, e. g., Ref. [12]), and an introduction of the effective-operator method in calculating the matrix elements of the transition operator (see, e. g., Ref. [13]). The above discrepancy problem is not yet solved in spite of these improvements. In particular, the QPRA values are larger than the shell-model values by a factor of 2 in more than several decay instances systematically.

In this paper, we introduce another improvement of the QRPA approach, which has not been exploited. The NME is obtained as the trace of the product of four matrices: the matrix of the transition operator (neutrino potential), two transition matrices from the initial and final states to the intermediate states, and the overlap matrix of two intermediate states which are obtained by two QRPA calculations based on the initial and final states. In our new method, the overlap matrix is calculated using the QRPA ground state defined as the vacuum of the QRPA quasibosons. The equation of the QRPA ground state has been known since some decades ago (see, e. g., Ref. [14]), however, our study is the first application of that formulation to the NME.

2 Two QRPA approaches

The decay probability of the $0\nu\beta\beta$ decay is given by

$$1/T_{0\nu}(0^+ \to 0^+) = \left|M^{(0\nu)}\right|^2 G_{01}\left(\langle m_\nu\rangle/m_e\right)^2, \quad (1)$$

where $T_{0\nu}(0^+ \to 0^+)$ denotes the half-life of the decay, and 0^+ indicates the initial and final states having $J^\pi = 0^+$. $M^{(0\nu)}$ and G_{01} are the NME and the phase-space factor, respectively. $\langle m_\nu\rangle$ is the effective neutrino mass, and m_e is the electron mass. Under the closure approximation replacing the intermediate-state energy with an average value $\bar{E}$, the equation of the NME can be written

$$M^{(0\nu)} \simeq \sum_{pp'nn'}\sum_{b_f b_i}\langle pp'|V(\bar{E})|nn'\rangle\langle 0^+_{pn,f}|c^\dagger_{p'}c_{n'}|b_f\rangle\langle b_f|b_i\rangle\langle b_i|c^\dagger_p c_n|0^+_{pn,i}\rangle, \quad (2)$$

where b_i and b_f denote the proton-neutron QRPA excited states based on the initial and final states, respectively. $V(\bar{E})$ is the transition operator of the $0\nu\beta\beta$ decay. The symbols pp' and nn' denote the proton and neutron single-particle states, respectively, and c_p and $c^\dagger_p$ denote the annihilation and creation operators, respectively. $|0^+_{pn,i}\rangle$ ($|0^+_{pn,f}\rangle$) is the initial (final) state obtained by the proton-neutron QRPA. Using the closure relation with respect to the intermediate states, we also have

$$M^{(0\nu)} \simeq \sum_{pp'nn'}\sum_{b_f b_i}\langle pp'|V(\bar{E})|nn'\rangle\langle 0^+_{\mathrm{like},f}|c^\dagger_{p'}c^\dagger_p|b_f\rangle\langle b_f|b_i\rangle\langle b_i|c_n c_{n'}|0^+_{\mathrm{like},i}\rangle, \quad (3)$$

where b_i and b_f denote those obtained by the like-particle QRPA. In our first attempt, we use this like-particle QRPA version because it is known that this QRPA is a good approximation in the well-deformed heavy mass ($A \sim 150$) region.

3 Formulation of QRPA ground state and overlap of QRPA states

Hereafter, we omit the subscript "like" in the symbols that we use. The QRPA ground state as the vacuum of the QRPA quasibosons is written as

$$|0_i^+\rangle = \prod_{K\pi} \frac{1}{\mathcal{N}_i^{K\pi}} \exp\left[v_i^{(K\pi)}\right] |0_{\mathrm{HFB},i}^+\rangle, \tag{4}$$

where $K\pi$ denotes the combination of the K quantum number and parity of the nuclear state, and $|0_{\mathrm{HFB},i}^+\rangle$ is the Hartree–Fock–Bogoliubov (HFB) (initial) ground state. $\mathcal{N}_i^{K\pi}$ is the normalization factor. In the quasiboson approximation ignoring exchange terms, we get

$$v_i^{(K\pi)} = \sum_{\mu\nu\mu'\nu'} \frac{1}{1+\delta_{K0}} \left(Y^{i,K\pi} \frac{1}{X^{i,K\pi}} \right)^{\dagger}_{\mu\nu,\mu'\nu'} a_\mu^{i\dagger} a_\nu^{i\dagger} a_{\mu'}^{i\dagger} a_{\nu'}^{i\dagger}, \tag{5}$$

$X^{i,K\pi}$ and $Y^{i,K\pi}$ are matrices consisting of forward and backward amplitudes of the QRPA state,

$$\begin{aligned} |b_i\rangle &= O_b^{i\dagger}|0_i^+\rangle \\ &= \sum_{\mu\nu\mu'\nu'} \left(X_{\mu\nu,b}^{i,K\pi} a_\mu^{i\dagger} a_\nu^{i\dagger} - Y_{-\mu-\nu,b}^{i,K\pi} a_{-\nu}^{i} a_{-\mu}^{i} \right) |0_i^+\rangle, \end{aligned} \tag{6}$$

and quasiparticle creation and annihilation operators are $a_\mu^\dagger$ and a_μ, respectively. This quasiparticle basis is obtained by the HFB calculation determining $|0_{\mathrm{HFB},i}^+\rangle$. The index "$-\mu$" indicates that the K quantum number of this quasiparticle state is opposite to that of the state μ. $O_b^{i\dagger}$ is the creation operator of the QRPA state.

The overlap of the two QRPA states based on the initial and final states can be calculated by the expansion and truncation with respect to $v_i^{(K\pi)}$ and $v_f^{(K\pi)\dagger}$,

$$\begin{aligned} \langle b_f|b_i\rangle &\simeq \frac{1}{\mathcal{N}_f\mathcal{N}_i} \prod_{K\pi} \langle 0_{\mathrm{HFB},f}^+| \exp\left[v_f^{(K\pi)\dagger}\right] O_b^f O_b^{i\dagger} \exp\left[v_i^{(K\pi)}\right] |0_{\mathrm{HFB},i}^+\rangle \\ &\simeq \frac{1}{\mathcal{N}_f\mathcal{N}_i} \Bigg\{ \langle 0_{\mathrm{HFB},f}^+| O_b^f O_b^{i\dagger} |0_{\mathrm{HFB},i}^+\rangle \\ &\quad + \sum_{K\pi} \Big(\langle 0_{\mathrm{HFB},f}^+| v_f^{(K\pi)\dagger} O_b^f O_b^{i\dagger} |0_{\mathrm{HFB},i}^+\rangle + \langle 0_{\mathrm{HFB},f}^+| O_b^f O_b^{i\dagger} v_i^{(K\pi)} |0_{\mathrm{HFB},i}^+\rangle \Big) \Bigg\}. \end{aligned} \tag{7}$$

It has been checked that the next-order terms are negligible in test calculations using ^{26}Mg and ^{25}Si [15]. This truncation can be applied because many high-energy excitations over the Fermi surface region do not contribute to the overlap in which two ground states are states of different nuclei. Thus we can assume that this truncation is also applicable to those heavier nuclei which are interesting for the $0\nu\beta\beta$ decay studies. Note that the normalization factors need higher-order truncations; we calculate up to the fourth order in the expansion of $\mathcal{N}_i^2$ and $\mathcal{N}_f^2$ with the quasiboson approximation.

4 Calculation of QRPA ground state

We performed the QRPA and overlap calculations for ^{150}Nd $\to$ ^{150}Sm. The input is the Skyrme energy-density functional with the parameter set SkM* [16] and the volume pairing energy density functional [17]. The strength of the latter was determined so as to reproduce the pairing gaps of these nuclei obtained from the experimental mass differences. The HFB calculations were performed using a cylindrical box of the height ($z > 0$) and radius of 20 fm assuming the axial symmetry of nuclear states [18–20]. The cutoff energy of 60 MeV was introduced for the quasiparticle energy. The quasiparticle states of lower energies are used for calculating the density and pairing tensor. The calculated axial quadrupole deformation β is 0.279 for ^{150}Nd and 0.209 for ^{150}Sm.

We use $K = 0{-}8$ for getting the convergence of NME. The QRPA equation is solved by the so-called matrix formulation [21]. The size of the QRPA Hamiltonian matrix is near $58,000$ for $K = 0, 1$ and $\simeq 10,000{-}25,000$ for other K values. The size for $K = 0, 1$ is much larger than the others because it is necessary to separate the spurious states associated with symmetries of the Hamiltonian broken in the HFB states. We do not use the proton-neutron pairing energy density functional (see, e. g., Ref. [12]) because these nuclei are far from the $N = Z$ line.

There is a problem that we have to solve before proceeding to the calculation of NME. The QRPA correlation energy diverges because of the contact-interaction nature of the Skyrme and volume-pairing energy-density functionals. Since the normalization factors of the QRPA ground states are strongly correlated with the QRPA correlation energies through the backward amplitudes, the normalization factors also diverge or are too large. In order to avoid this problem, we first define the backward norm of the QRPA solution a,

$$\mathcal{N}^a_{\rm back} = \sum_{\mu\nu} \left| Y^a_{-\mu-\nu} \right|^2 . \tag{8}$$

Then we pick up the QRPA solutions with the largest backward norms so as to reproduce the semi-experimental correlation energy

$$E^{\rm exp}_{\rm cor} = E_{\rm exp} - E_{\rm HFB}, \tag{9}$$

where $E_{\rm exp}$ and $E_{\rm HFB}$ are the experimental energy (mass) and the HFB energy of the ground state, respectively. The QRPA correlation energy is calculated using the formula [22]

$$E^{\rm QRPA}_{\rm cor} \simeq \frac{1}{2} \sum_a (E_a - E^{\rm TDA}_a), \tag{10}$$

where E_a and $E^{\rm TDA}_a$ are the eigenenergies of the QPRA and Tamm–Dancoff approximation [21], respectively. We picked up 10 (18) QRPA solutions for ^{150}Nd (^{150}Sm) and obtained the QRPA correlation energies of -1.721 MeV for ^{150}Nd and -3.688 MeV for ^{150}Sm. The corresponding $E^{\rm exp}_{\rm cor}$ values are -1.696 MeV for ^{150}Nd and -3.661 MeV for ^{150}Sm (for the experimental masses, see Ref. [23]). Using this prescription, we obtained the product of the normalization factors of the initial and final QRPA ground states $\mathcal{N}_I \mathcal{N}_F = 1.860$; this implies that the NME is reduced significantly compared to the value obtained without the QRPA correlations in the ground states included in the overlaps. The calculation of the NME is now in progress.

5 Summary and future works

A new QRPA approach for calculating the NME has been presented; the QRPA ground state as the vacuum of the QRPA quasiboson is used in the overlaps of the intermediate QRPA states. The significant outcome of this approach is that the normalization factors are much larger than unity, and these normalization factors have an effect of reducing the NME. This calculation is a step toward obtaining the reliable NME.

We have many calculations to perform. First of all, the NME is necessary to calculate. One of the important tasks after the NME is to show that the two approaches using the proton-neutron and like-particle QRPAs provide us with the same NME, as indicated by Eqs. (2) and (3). The extension of the QRPA ground state to the product wave function of the proton-neutron and like-particle QRPA ground states will be necessary for showing this equivalence. It is also an important task to investigate whether the NMEs of the two-neutrino double-beta decay are reproduced by our new approach. The explicit QRPA ground-state wave function is also entering this NME, therefore the reduction effect by the normalization factors also applies. It is also an important question whether the new NME values are close to the shell-model ones. For this comparison, we need to calculate the decay instances other than ^{150}Nd $\rightarrow$ ^{150}Sm.

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