Large-Scale Shell-Model Studies for Exotic Nuclei: Probing Shell Evolution

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Abstract

We report on recent advances in large-scale shell-model calculations for exotic nuclei, focusing on how shell evolution is probed from strongly correlated nuclei. We choose the $N \sim 28$ region as a typical case. The effective interaction is constructed on the basis of the monopole-based universal interaction which consists of a phenomenological central force and the bare tensor force. It is demonstrated that the proton spin-orbit splitting is significantly reduced in going from $N = 20$ to $N = 28$ due to the tensor force by comparing spectroscopic factors for the one-proton removal from ^{48}Ca. This narrowing spin-orbit splitting causes large deformation in ^{42}Si, as a consequence of the tensor-force-driven Jahn–Teller effect. It is predicted that the new $N = 34$ magic number found recently in ^{54}Ca enhances toward lower-Z isotopes and produces a new doubly-magic nucleus ^{48}Si.

Keywords: *Shell model; magic number; shell evolution; tensor force*

1 Introduction

Since a nucleus is a strongly correlated system, large-scale structure calculations are required to describe it accurately. The nuclear shell model is regarded as one of the most popular approaches for this purpose, including every possible correlation within the single-particle model space assumed. The usual shell model typically takes full one major shell for the single-particle space. Minor effects from the outside of the model space are taken into account by renormalizing the Hamiltonian and operators used in the calculation. The renormalized nuclear force and operators are called effective interaction and effective operators, respectively.

Constructing a good effective interaction is crucial for the descriptive power of the shell model. Microscopic effective interactions derived from the bare nucleon-nucleon forces are usually subject to empirical modification for better description. In particular, the monopole interaction is known to be critical [1, 2]. The monopole

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interaction v_{ij}^{m} works to change the single-particle energy of the orbit i with the orbit j occupied. Hence, the monopole interaction is responsible for the evolution of shell structure, often referred to as *shell evolution* [3]. Recently, shell evolution has received much attention in the study of exotic nuclei, because a number of phenomena that indicate the modification of shell structure have been observed. The breakdown of the conventional magic numbers is a good example. While the normal magic numbers $N = 8, 20, 28$ are known to disappear in very neutron-rich nuclei [4–6], new magic numbers $N = 16, 34$ have been discovered quite recently [7, 8].

An understanding of the source of the monopole interaction that causes the shell evolution is thus very important. Almost ignored over the years, the tensor force has been revisited for a decade as an essential ingredient of the effective interaction since Otsuka *et al.* pointed out that the tensor force produces significant changes of the spin-orbit splitting (Otsuka effect) [9]. Later, taking into account a phenomenological central force, the monopole-based universal interaction (V_{MU}) has been proposed [10] to describe the shell evolution in a unified manner. Although the V_{MU} seems to give a rather reasonable evolution through a simple mean-field estimate, the experimental energy levels are not the pure single-particle states obtained from this approach.

The shell evolution thus should be probed with reliable many-body calculations such as the shell model. In this conference, we survey recent advances in the understanding of the shell evolution for exotic nuclei via large-scale shell-model calculations. We covered two major topics: the shell evolution in the $N \sim 28$ region based on the conventional shell-model calculation and the shape coexistence in Ni isotopes based on the advanced Monte Carlo shell-model calculation. Since the methodology of the advanced Monte Carlo shell model was also introduced in Abe's talk [11] in this conference and part of the results for Ni isotopes were reported in the proceedings of NTSE-2013 by Otsuka *et al.* [12], here we concentrate on the first subject.

2 Structure of exotic nuclei in the $N \sim 28$ region

Recently many intriguing phenomena concerning shell evolution has been observed in the neutron-rich $N \sim 28$ region. While $N = 28$ is known to be a good magic number for pf-shell nuclei, this magicity breaks down in ^{42}Si [6]. On the other hand, a new magic number $N = 34$ has been found very recently [8], more than a decade after its prediction [13]. Here we present our shell-model results for this region using an effective interaction based on V_{MU}.

2.1 Shell evolution from $N = 20$ to $N = 28$

In this section, we perform shell-model calculations in the valence orbits consisting of the sd and pf shells. Since it is impossible to carry out shell-model calculations in the full $sd + pf$ model space, we introduce the truncation of the model space by not allowing nucleon excitation across the $N(Z) = 20$ shell gap. This truncation is valid except the "island of inversion" region [5] around ^{32}Mg, where the ground state is dominated by $2p$–$2h$ excitation across the $N = 20$ shell gap. Since the present study concentrates on neutron-rich nuclei having $Z \leq 20$ and $N \simeq 28$, the truncation should work well.

The cross-shell interaction, i. e. two-body matrix elements connecting the sd shell and the pf shell, plays a key role in the structure of neutron-rich nuclei evolving from the ^{40}Ca core. We use a refined V_{MU} interaction for this part. The refinement aims (1) to include the two-body spin-orbit force and (2) to better fit the semi-empirical GXPF1 interaction [14] than the original V_{MU} interaction. More details about the refinement can be found in Ref. [15].

We discuss the shell evolution in going from $N = 20$ to $N = 28$. Since the neutron $0f_{7/2}$ orbit is occupied, the monopole interaction concerning the $0f_{7/2}$ orbit

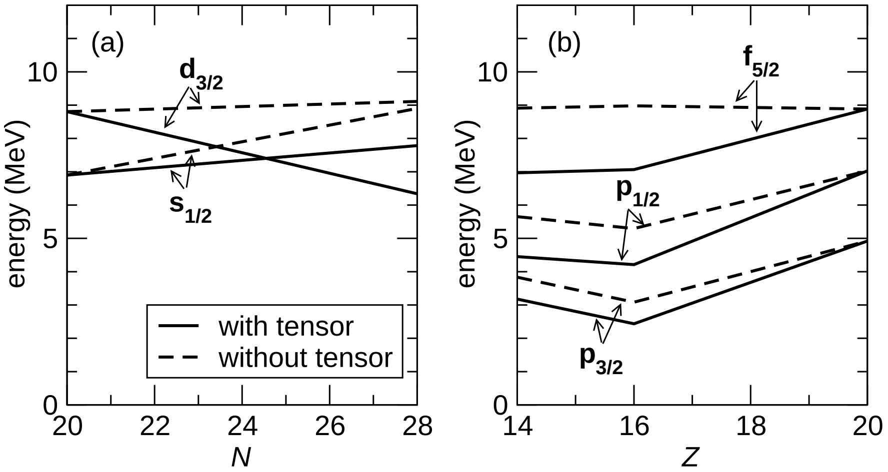

Figure 1: (a) Proton effective single-particle energies of the $1s_{1/2}$ and $0d_{3/2}$ orbits relative to the $0d_{5/2}$ in Si isotopes calculated with the present shell-model interaction. (b) Neutron effective single-particle energies of the $1p_{3/2}$, $1p_{1/2}$ and $0f_{5/2}$ relative to the $0f_{7/2}$ orbit in $N = 28$ isotones calculated with the present shell-model interaction. For both figures, the solid and dashed lines correspond to the Hamiltonian with and without the tensor force, respectively.

is relevant. Since the $0f_{7/2}$ orbit is a $j_>$ $(j = l + 1/2)$ orbit, the tensor force works to pull down the $0d_{3/2}$ orbit and to push up the $0d_{5/2}$ orbit according to the Otsuka effect [9]. As a result, the spin-orbit splitting for protons decreases as shown in Fig. 1(a). The change of the spin-orbit splitting can be probed from the distribution of proton spectroscopic factors. As presented in Ref. [16], the proton-hole strengths measured from the $(e, e'p)$ reaction clearly support the reduction of the spin-orbit splitting obtained in the shell-model calculation including the tensor force. Thus, the present shell-model interaction describes the shell evolution quite well.

2.2 Disappearance of the $N = 28$ magic number and tensor-force-driven Jahn–Teller effect

It is very interesting to investigate how the shell evolution affects the nuclear collectivity such as deformation. The $N = 28$ nucleus ^{42}Si provides a good example in this context. In Fig. 2, the 2^+_1 and 4^+_1 energies in neutron-rich Si isotopes are compared with experiment. The shell-model calculation incorporating the tensor force successfully reproduces these energies including those measured after our calculation [17]. Two shell-model calculations, with and without the tensor force, give quite different results for ^{42}Si. The calculation with the tensor force reproduces the measured 2^+_1 level which is located very low, whereas the calculation without it leads to much higher energy. This means that the disappearance of the $N = 28$ magic number in ^{42}Si is caused by the tensor force. The calculated $B(E2)$ value and potential energy surface indicate that the very low 2^+_1 energy in ^{42}Si is due to a large deformation.

The reason why the tensor force induces the large deformation in ^{42}Si, is discussed in Ref. [16] in detail. As presented in Fig. 1, the tensor force reduces the spin-orbit splitting in j–j closed nuclei such as ^{42}Si. The neighboring orbits belonging to the same major shell, such as $0d_{5/2}$ and $1s_{1/2}$, are then located closer. These neighboring orbits are easily mixed, and the resulting mixed orbit has a freedom of deformation. Since the residual interaction is dominated by the $Q \cdot Q$ term, a deformed state is favored in order to minimize the Hamiltonian. This mechanism is the Jahn–Teller effect [18, 19] which is triggered by the shell evolution due to the tensor force, which

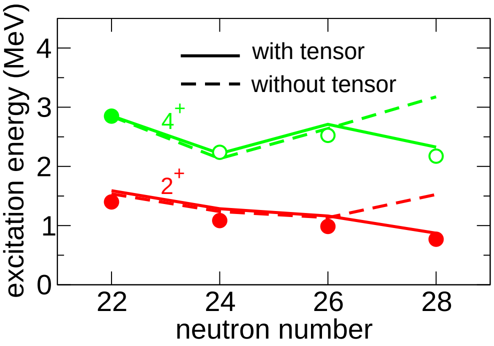

Figure 2: Comparison of theoretical predictions for the 2_1^+ and 4_1^+ energy levels in neutron-rich Si isotopes with experiment. The solid and dashed lines depict the shell-model calculations with and without the tensor force, respectively. The closed and open circles are the experimental data measured before and after our calculations.

we refer to as tensor-force-drive Jahn–Teller effect [16]. Since this is a general effect, it is very interesting to search for other cases in forthcoming experiments.

2.3 Evolution of the new $N = 34$ magic number

In 2001, the $N = 34$ magic number has been predicted to emerge around ^{54}Ca [13] in analogy with the appearance of the $N = 16$ magic number. According to the up-to-date shell-evolution mechanism, the $N = 34$ magic number appears due to a large attraction between the proton $0f_{7/2}$ and neutron $0f_{5/2}$ orbits that is favored by both central and tensor forces. Hence, the $0f_{5/2}$ orbit rises sharply with decreasing protons from the $0f_{7/2}$ orbit, positioning much higher than the $1p_{1/2}$ orbit in Ca isotopes. The $N = 34$ magic number is thus predicted to appear as a large sub-shell gap between $1p_{1/2}$ and $0f_{5/2}$. Much experimental effort has been devoted to detecting the predicted $N = 34$ magic number. Since it was extremely difficult to sufficiently produce the ^{54}Ca nucleus, the prediction has not been confirmed until the measurement of the 2_1^+ energy in ^{54}Ca was carried out in RIKEN in 2013 [8]. The measured 2_1^+ energy is much higher than those in singly-closed-shell nuclei such as $^{42-46,50}$Ca, demonstrating the occurrence of a new neutron magic number 34 in ^{54}Ca. What is amazing in this finding is that there is no a fingerprint of the $N = 34$ magic number in Ti ($Z = 22$) and Cr ($Z = 24$) isotopes, in a sharp contrast to the evolution of the $N = 32$ sub-shell closure [20–22]. This abrupt appearance of the $N = 34$ magic number is caused by a sharp evolution of the $0f_{5/2}$ orbit as a function of the proton number. Thus, the occurrence of the $N = 34$ magic number strongly validates the concept of shell evolution. The strength of the $N = 34$ shell gap is estimated to be $\sim$2.5 MeV from the shell-model calculations based on the GXPF1B semi-empirical interaction [23].

It is a very interesting issue how the $N = 34$ magic number behaves in more proton-deficient isotopes, Ar ($Z = 18$), S ($Z = 16$), and Si ($Z = 14$). Since the present interaction successfully describes the shell evolution without any direct adjustment to experimental data, it is considered to have a predictive power. Introducing a minor modification to the interaction aimed to almost exactly reproduce the proton single-hole-like spectra in K isotopes, we calculate the evolution of the $N = 34$ shell gap in Ar, S and Si isotopes. The resulting $N = 34$ gap is predicted to enhance at smaller proton numbers. For ^{48}Si, it becomes 3.9 MeV. This enlargement is mainly attributed to a large attraction of the central force between $\pi 0d_{3/2}$ and $\nu 0f_{5/2}$ compared to a repulsion of the tensor force. The effect of the enhanced $N = 34$ gap can be detected from experiment. Figure 3 shows the 2_1^+ energy levels in Ti, Ca, Ar, S, and Si isotopes

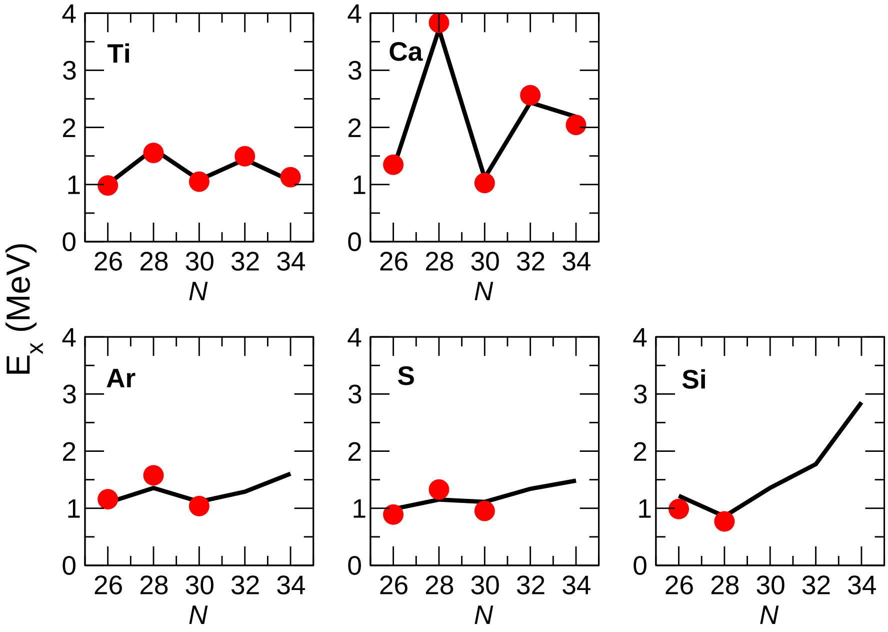

Figure 3: Comparison of theoretical predictions (lines) with experiment (circles) for the 2_1^+ levels in neutron-rich Ti, Ca, Ar, S and Si isotopes with $N = 26–34$.

with $N = 26–34$. The calculation predicts that the 2_1^+ level in the $N = 34$ nucleus ^{48}Si is located very high. As shown in Fig. 1 (a), the $Z = 14$ gap is also large in neutron-rich Si isotopes. As a result, ^{48}Si can be a new doubly-magic nucleus. Verifying this prediction in future RI-beam facilities will be a very attractive program.

3 Summary

We have investigated the shell evolution in the $N \sim 28$ exotic nuclei with large-scale shell-model calculations. The reduction of spin-orbit splitting due to the Otsuka effect [12] is probed by the distribution of spectroscopic strengths. This effect gives rise to a large deformation in the $j–j$ closed nucleus ^{42}Si. It is also predicted that the newly found $N = 34$ magic number enhances in ^{48}Si. The $N \sim 28$ region constitutes a typical case in which many-body properties are strongly affected by shell evolution. Thus, a high-performance computing in nuclear-structure calculations will be an indispensable tool for investigating unusual properties in exotic nuclei towards heavier-mass regions.

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