From Gogny Force to Shell-Model Calculations

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Abstract

We use the Gogny force to calculate the two-body matrix elements of the nucleon-nucleon (NN) interaction for shell-model calculations. As the first step, we ignore the effective three-body force which is usually written in a density-dependent two-body form in the Gogny force, therefore the parameters of the Gogny force should be readjusted. In the present work, we investigate *sd*-shell nuclei. The comparison with experimental data shows that the shell-model calculations starting from the Gogny force can be successful. The present calculations are preliminary, and further work is going on. The aim is to obtain consistent descriptions in both shell-model and mean-field calculations using a unified Gogny force.

Keywords: *Gogny force; two-body matrix elements; shell model; sd-shell nuclei; spectra*

1 Introduction

In shell-model calculations, one of the most crucial tasks is how to calculate the two-body matrix elements (TBME's) of the nucleon-nucleon (NN) interaction. One can start from a realistic bare NN interaction and make necessary renormalization to soften it for shell-model calculations. For core shell-model calculations, a further renormalization is needed to include effects from excluded space (including core polarization). The renormalization approximations would bring restrictions on calculations. For example, the renormalization using the folded-diagram expansion restricts the shell-model calculations within a single shell. This means that the model space cannot include configurations from different shells due to perturbation approximation used [1]. On the other hand, such calculations are complicated mathematically and time-consuming computationally, and cannot be accurately quantitatively compared with experimental data.

As another way, one can use phenomenological interaction matrix elements obtained by fitting to experimental data, e.g., the USD interaction for the *sd* shell [2,3]. However such phenomenological method involves a big number of parameters to be determined by fitting data. This is a big task in fitting, particularly for heavy mass regions, and also requires enough experimental data available.

In the present work, we use the Gogny force [4,5] to calculate the effective TBME's. The Gogny force has been successfully used in mean-field calculations, involving

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only 14 parameters. Similar works using the Skyrme force were done for the sd-shell nuclei ^{18}O and ^{36}Ar [6], and for p-shell nuclei [7]. These calculations indicate that the phenomenological Skyrme force which has been widely employed in mean-field calculations can be used for the calculations of effective TBME's in shell model. Such phenomenological calculations of TBME's involve much less parameters than in the case of the USD interaction.

2 The model and calculations

The Gogny force is written as [4,5]

$$V(1,2) = \sum_{j=1,2} e^{-(\mathbf{r}_1-\mathbf{r}_2)^2/\mu_j^2}(W_j + B_j P_\sigma - H_j P_\tau - M_j P_\sigma P_\tau) \\ + t_0\,(1 + x_0 P_\sigma)\,\delta(\mathbf{r}_1 - \mathbf{r}_2)\left[\rho\left(\frac{\mathbf{r}_1+\mathbf{r}_2}{2}\right)\right]^\alpha \\ + iW_0 \overleftarrow{\nabla}_{12}\,\delta(\mathbf{r}_1-\mathbf{r}_2)\times\overrightarrow{\nabla}_{12}\cdot(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2),$$

where P_σ (P_τ) is a two-body spin (isospin) exchange operator. Totally there are 14 parameters with μ_1 and μ_2 describing the short- and intermediate-range properties of the nuclear interaction, respectively.

As the first step of our work, we ignore the three-body-force density-dependent term, $t_0\,(1+x_0 P_\sigma)\,\delta(\mathbf{r}_1-\mathbf{r}_2)\,\left[\rho\left(\frac{\mathbf{r}_1+\mathbf{r}_2}{2}\right)\right]^\alpha$. Such approximation should be reasonable in shell model calculations in which the two-body interaction provides the most important contributions. Now only 11 parameters are involved in calculations of TBME's. Because the existing Gogny parameters have been evaluated by mean-field model fitting to nuclear structure data mainly which included the three-body force, the Gogny parameters should be refitted for the shell-model calculations ignoring the three-body force.

In the present work, to test the validity of the Gogny shell model, we focus on the sd-shell nuclei which have been well investigated by shell-model calculations. Calculations are performed in the spherical harmonic oscillator basis with $\hbar\omega \approx 45A^{-1/3} - 25A^{-2/3}$ [3]. The sd-shell single-particle energies are the same as in the USDB shell model [3]: $e(d_{5/2}) = -3.9257$ MeV; $e(s_{1/2}) = -3.2079$ MeV; $e(d_{3/2}) = 2.1117$ MeV.

We choose five nuclei, ^{18}O, ^{18}F, ^{20}Ne, ^{22}Na, ^{24}Mg, for the fit of the 11 parameters. We fit the lowest level for each given spin in their spectra. The NuShellX code [8] is used for the shell-model calculations. We adopt the Monte Carlo simulated annealing algorithm in the process of parameter fittings. The fitted parameters are listed and compared with existing mean-field Gogny parameters in Table 1.

With the parameters determined, we calculate the TBME's for the sd shell and perform shell-model calculations using the NuShellX code [8] for this mass region. Figures 1–3 display some of spectroscopic calculations of sd-shell nuclei. We see that a good agreement has been obtained. The Gogny force involving only 11 parameters can describe well the effective shell-model interaction, providing a simple way to calculate the TBME's. This will be very useful.

3 Summary

We have used the Gogny force which is a non-zero-range effective interaction, to calculate the TBME's of the shell-model NN interaction. As the first step, we ignore the three-body force but readjust the Gogny parameters. In the present work, we focus on sd-shell nuclei. Five nuclei, ^{18}O, ^{18}F, ^{20}Ne, ^{22}Na and ^{24}Mg were chosen in

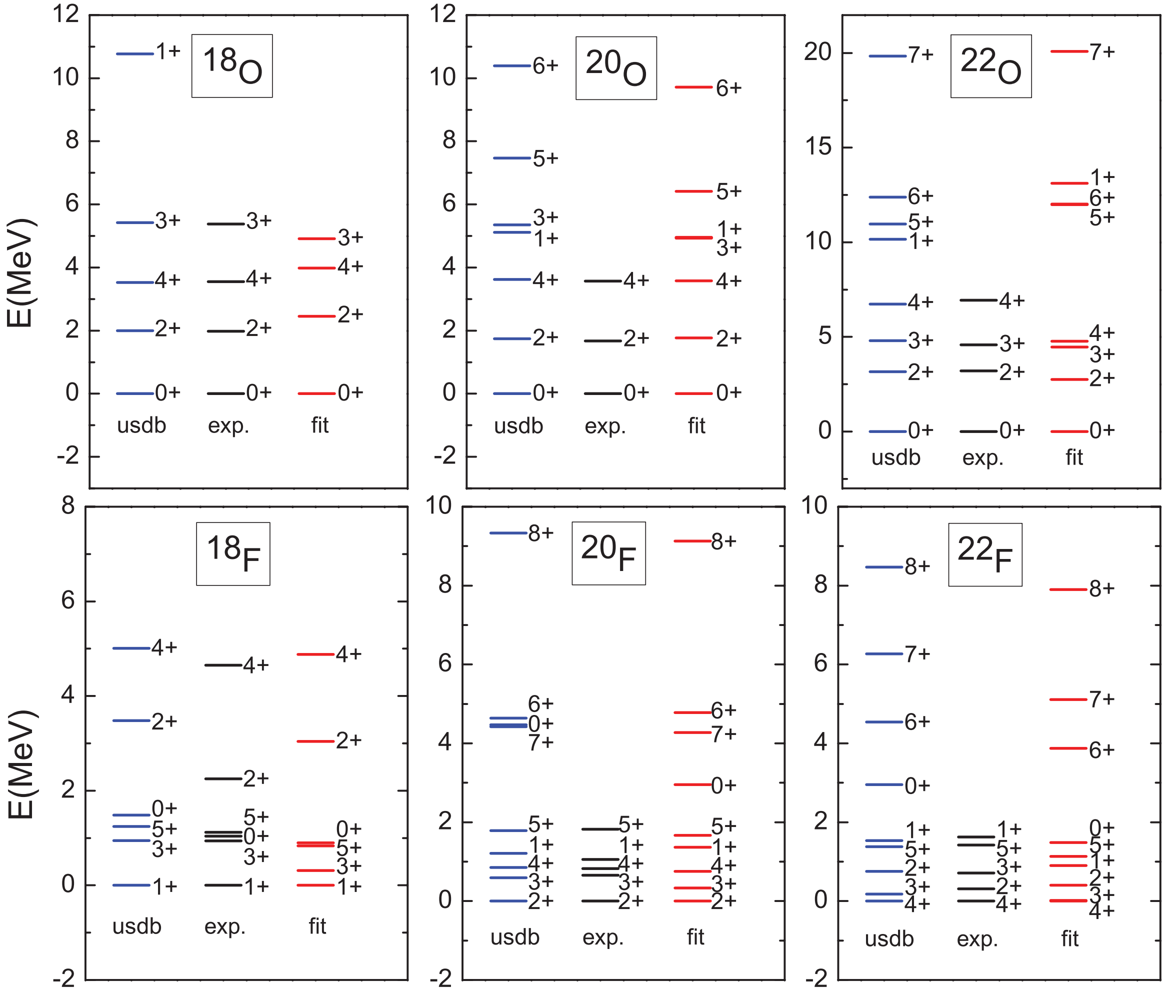

Figure 1: Shell-model calculations using TBME's calculated with the Gogny force with the refitted Gogny parameters given in Table 1, compared with the USDB calculations [3] and experimental data.

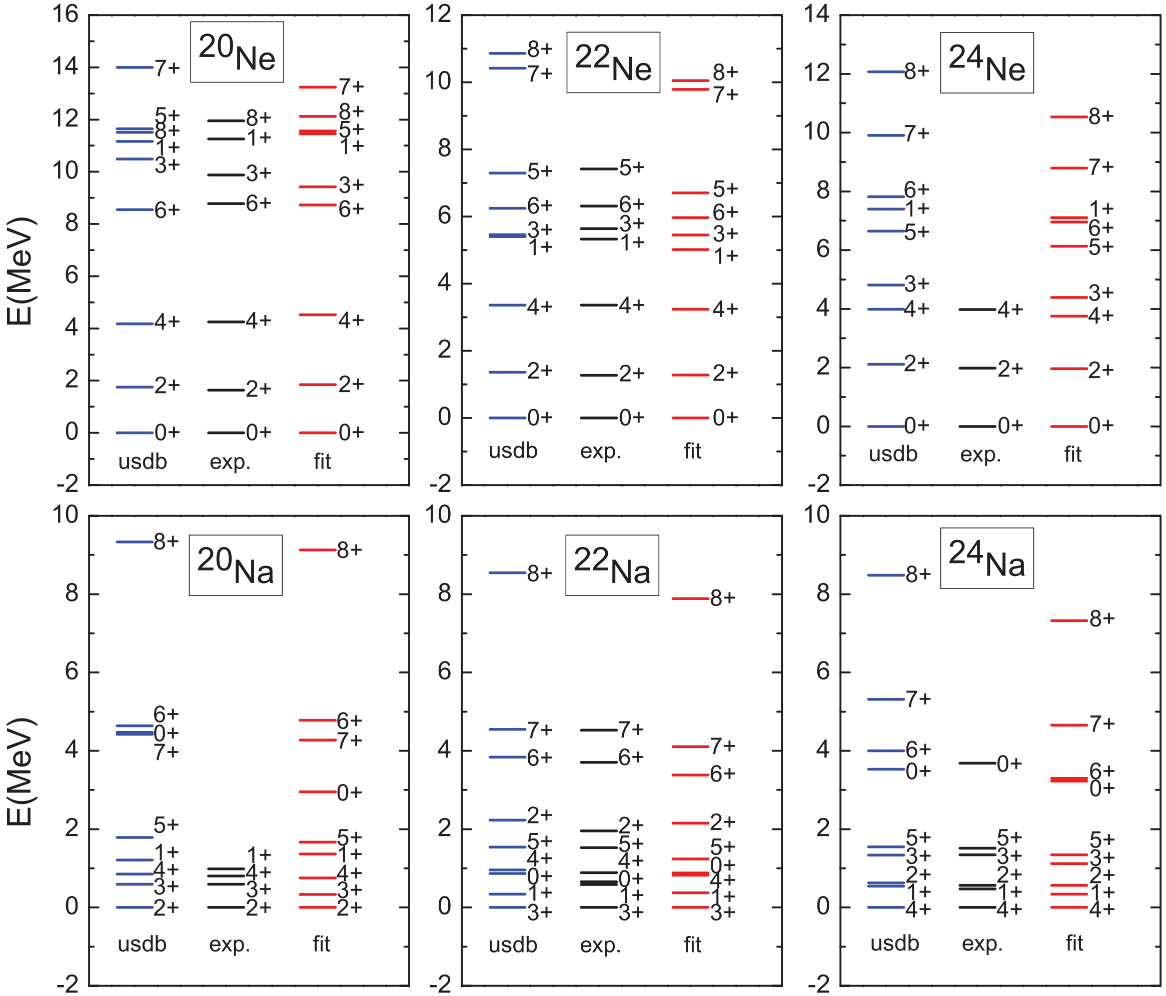

Figure 2: Same as Fig. 1 but for another set of nuclei.

Table 1: Readjusted Gogny parameters, compared with the existing mean-field values of the Gogny parameters.

	D1' [4]	D1 [4]	D1M [5]	D1N [9]	D1S [10]	FIT
μ_1 (fm)	0.7000	0.7000	0.5000	0.8000	0.7000	0.6537
μ_2 (fm)	1.2000	1.2000	1.0000	1.2000	1.2000	0.9837
W_1 (MeV)	−402.40	−402.40	−12797.57	−2047.61	−1720.30	−1951.8104
W_2 (MeV)	−21.30	−21.30	490.95	293.02	103.64	272.4556
B_1 (MeV)	−100.00	−100.00	14048.85	1700.00	1300.00	3280.8417
B_2 (MeV)	−11.77	−11.77	−752.27	−300.78	−163.48	−607.9833
H_1 (MeV)	−496.20	−496.20	−15144.43	−2414.93	−1813.53	−2433.3474
H_2 (MeV)	37.27	37.27	675.12	414.59	162.81	273.1813
M_1 (MeV)	−23.56	−23.56	11963.89	1519.35	1397.60	2562.6194
M_2 (MeV)	−68.81	−68.81	−693.57	−316.84	−223.93	−647.3149
W_0 (MeV $\cdot$ fm^5)	130.00	115.00	115.36	115.00	−130.00	460.8928

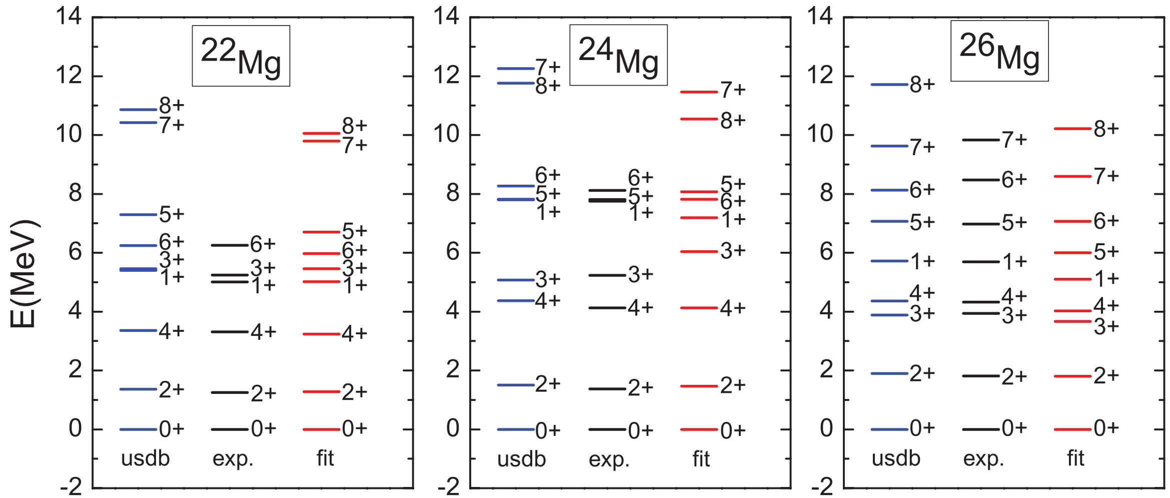

Figure 3: Same as Figs. 1 and 2 but for another set of nuclei.

the fitting of the parameters. With such determined parameters, we calculated the TBME's for the sd shell and performed shell-model calculations of nuclear spectra in this mass region. Good results have been obtained.

The further work should be to extend the method on other mass regions, for example, pf shell and $sd + pf$ shells. The Gogny force provides a powerful way to calculate the TBME's. We should also take the three-body force into account in the next work. By taking the existing Gogny parameters, we can test whether the Gogny force can give an unified description within both mean field and shell models.

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