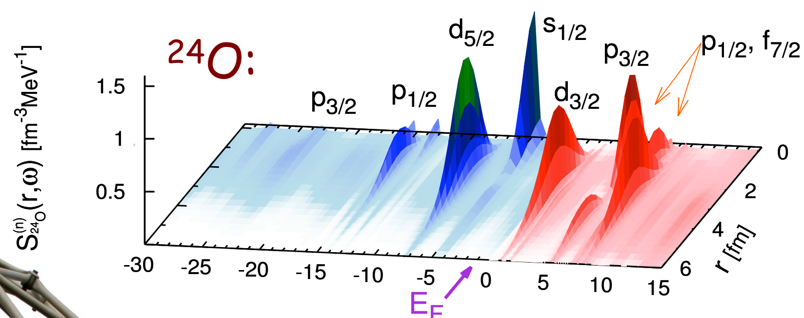
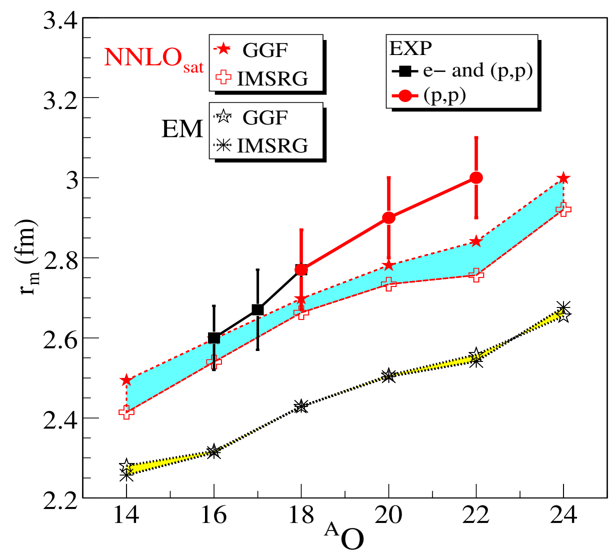
UNIVERSITY OF SURREY

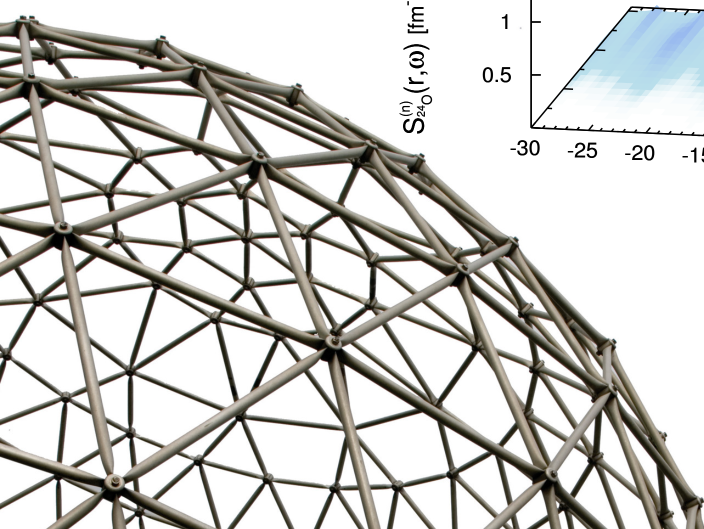
Self-Consistent Green’s function Computations of Medium-Mass Isotopes

Carlo Barbieri — University of Surrey

Sept. 19th, 2016



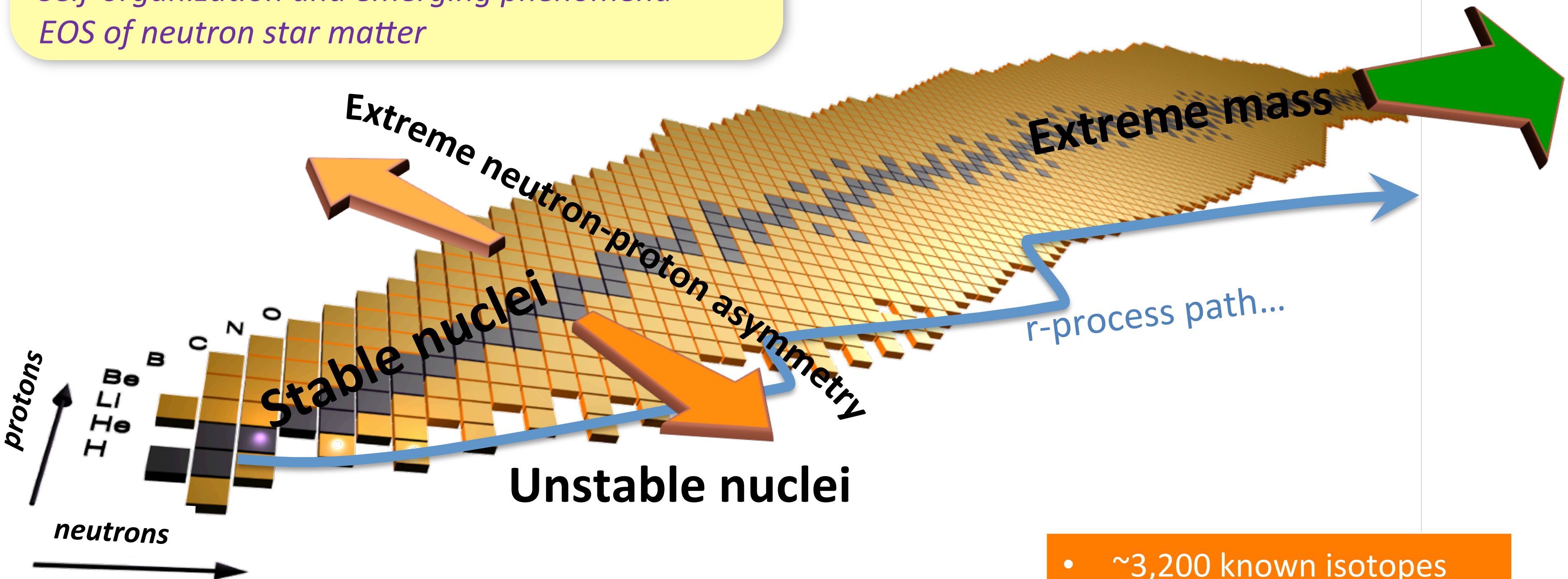




Current Status of low-energy nuclear physics

Composite system of interacting fermions
Binding and limits of stability
Coexistence of individual and collective behaviors
Self-organization and emerging phenomena
EOS of neutron star matter

Experimental programs
RIKEN, FAIR, FRIB



- ~3,200 known isotopes
- ~7,000 predicted to exist
- Correlation characterised in full for ~283 stable

Nature **473**, 25 (2011); **486**, 509 (2012)

Current Status of low-energy nuclear physics

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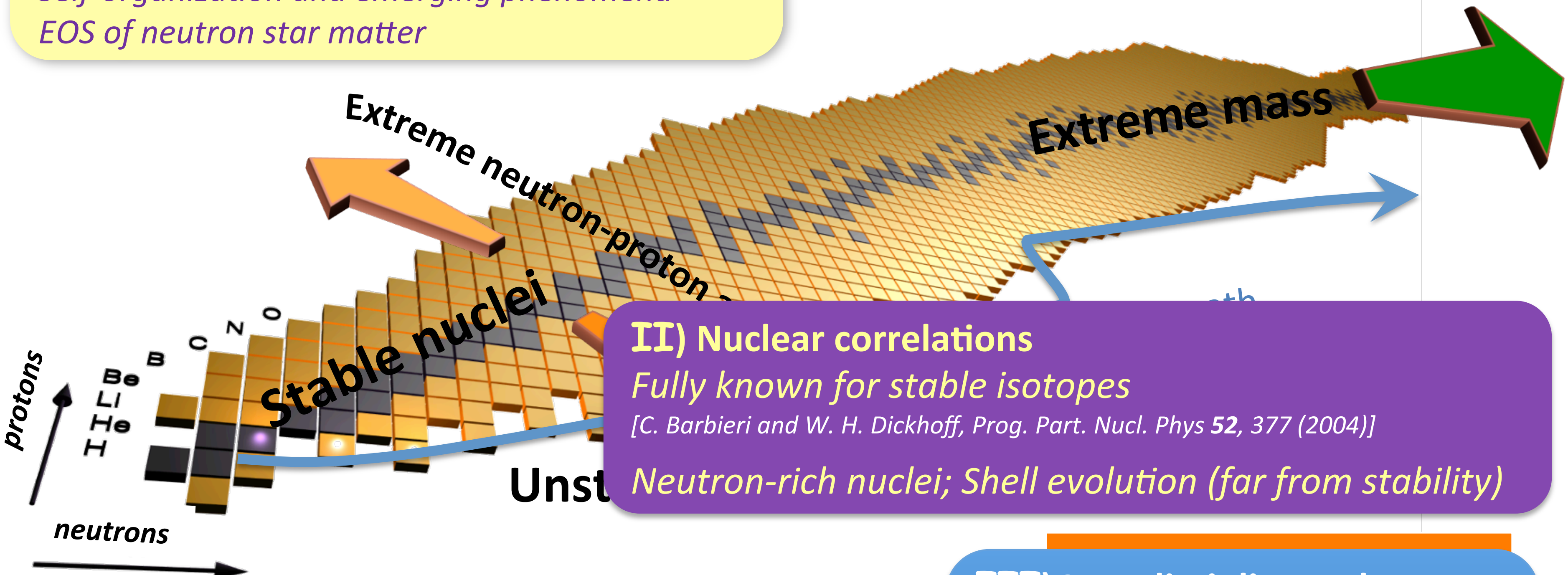


II) Nuclear correlations
Fully known for stable isotopes
[C. Barbieri and W. H. Dickhoff, Prog. Part. Nucl. Phys ***52****, 377 (2004)]*
Neutron-rich nuclei; Shell evolution (far from stability)

I) Understanding the nuclear force
QCD-derived; 3-nucleon forces (3NFs)
First principle (ab-initio) predictions

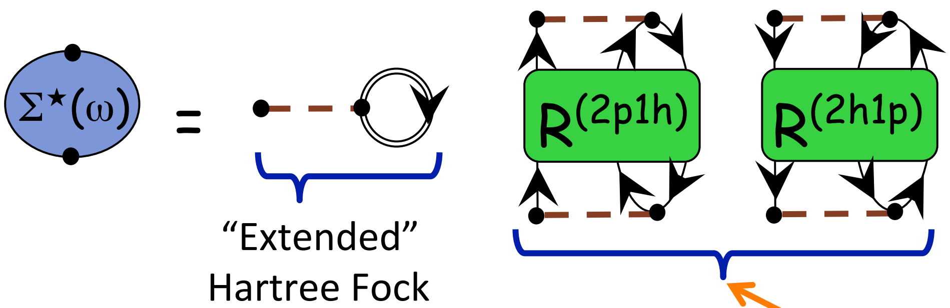
III) Interdisciplinary character
Astrophysics
Tests of the standard model
Other fermionic systems:
ultracold gasses; molecules;

Ab-Initio SCGF approaches

The FRPA Method in Two Words

Particle vibration coupling is the main cause driving the distribution of particle strength—on both sides of the Fermi surface…

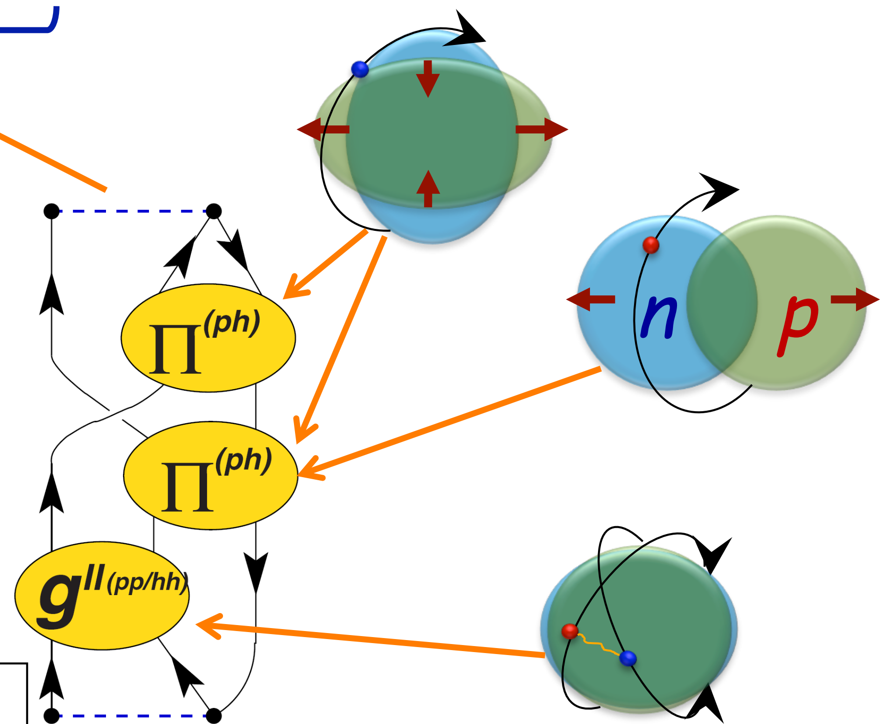
CB et al.,
Phys. Rev. C **63**, *034313 (2001)*
Phys. Rev. A **76**, *052503 (2007)*
Phys. Rev. C **79**, *064313 (2009)*

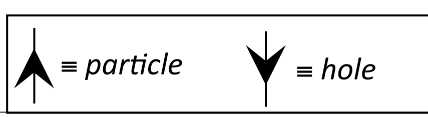


- A complete expansion requires *all* *types* of particle-vibration coupling

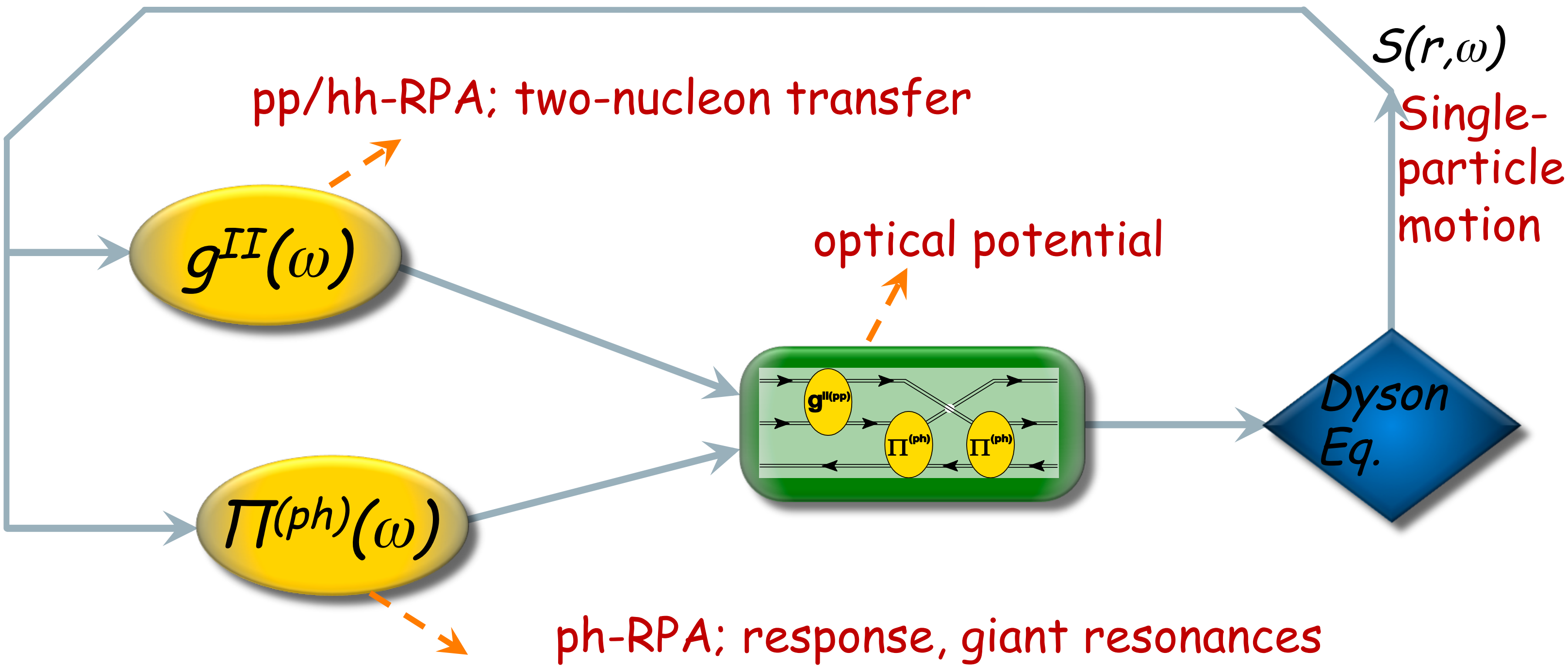
  …these modes are all resummed exactly and to all orders in a *ab-initio* many-body expansion.

- The Self-energy $\Sigma^{\star}(\omega)$ yields *both* single-particle states and scattering





Self-Consistent Green's Function Approach



- Global picture of nuclear dynamics
- Reciprocal correlations among effective modes
- Guaranties *macroscopic conservation laws*

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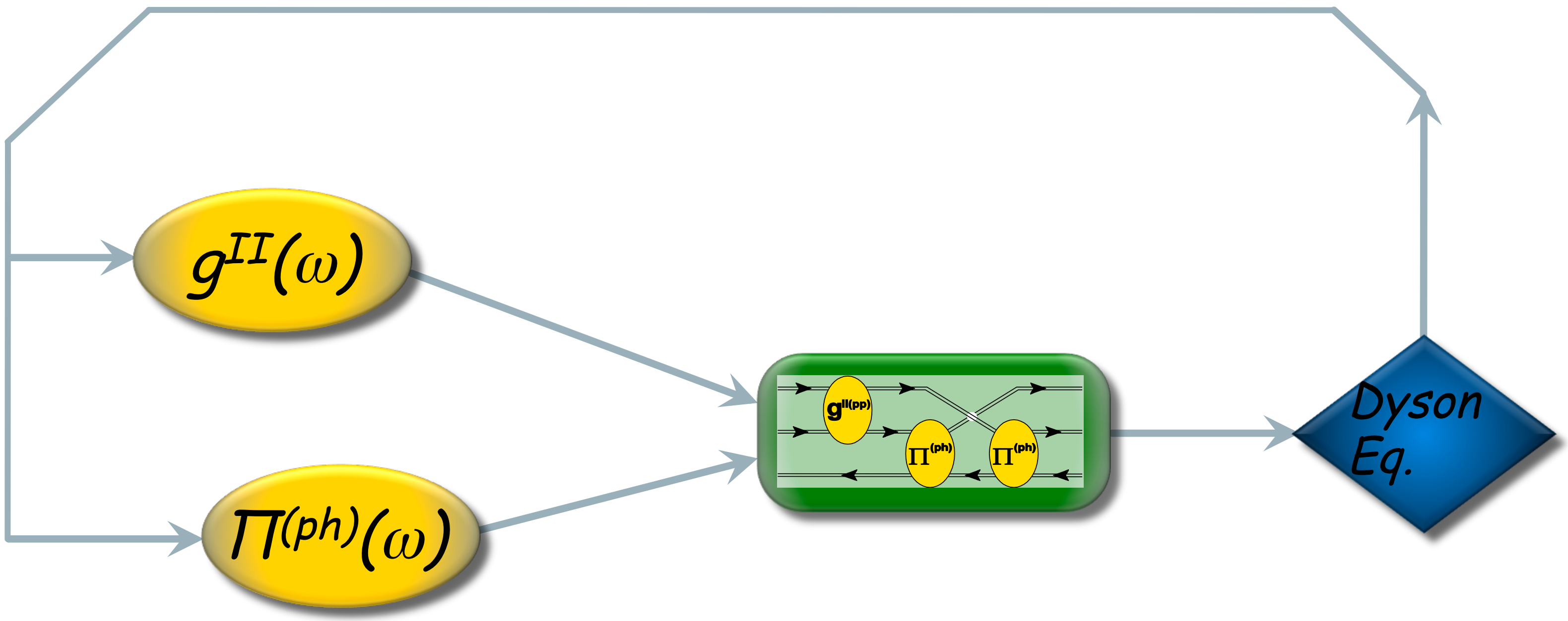

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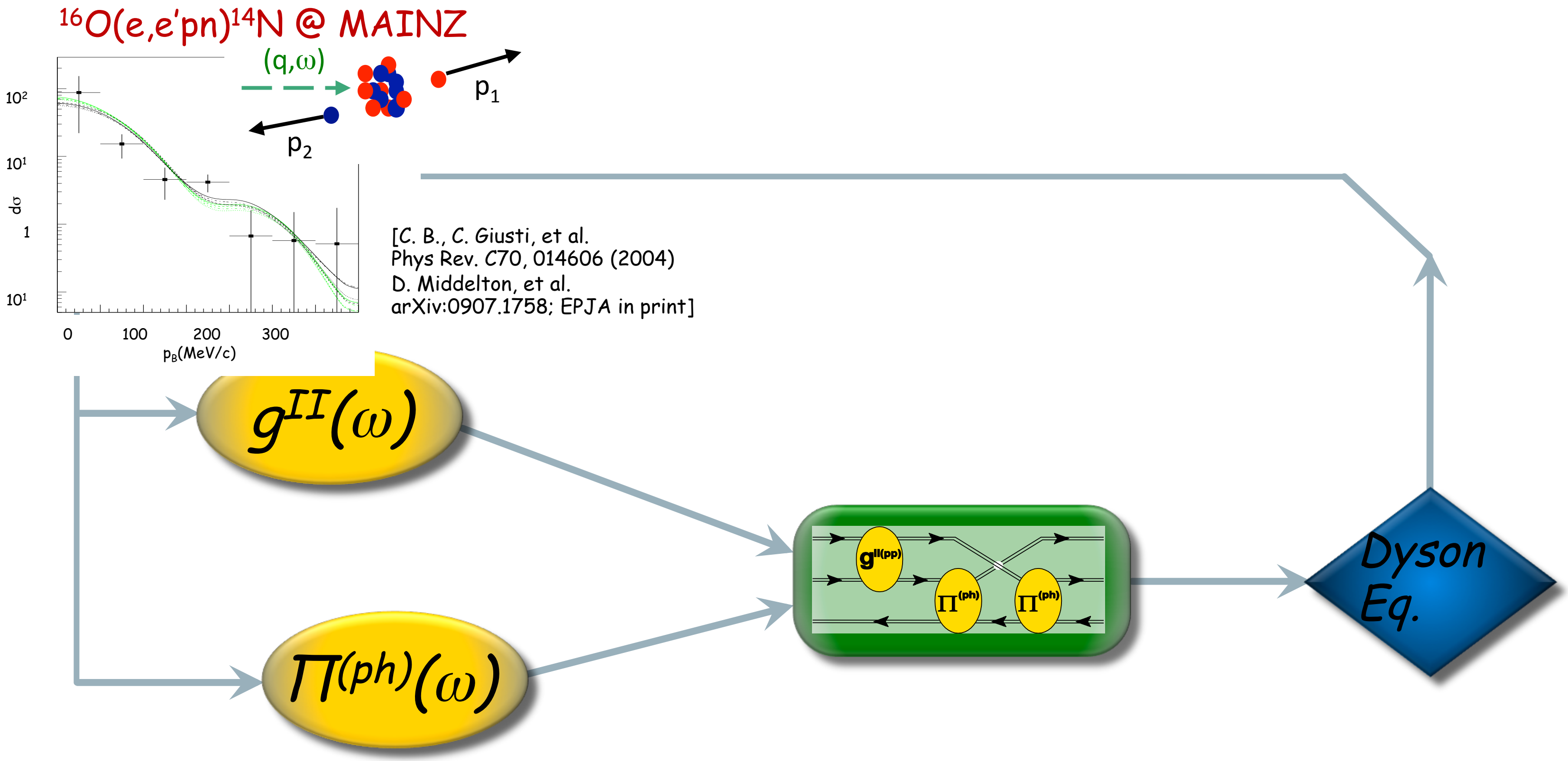

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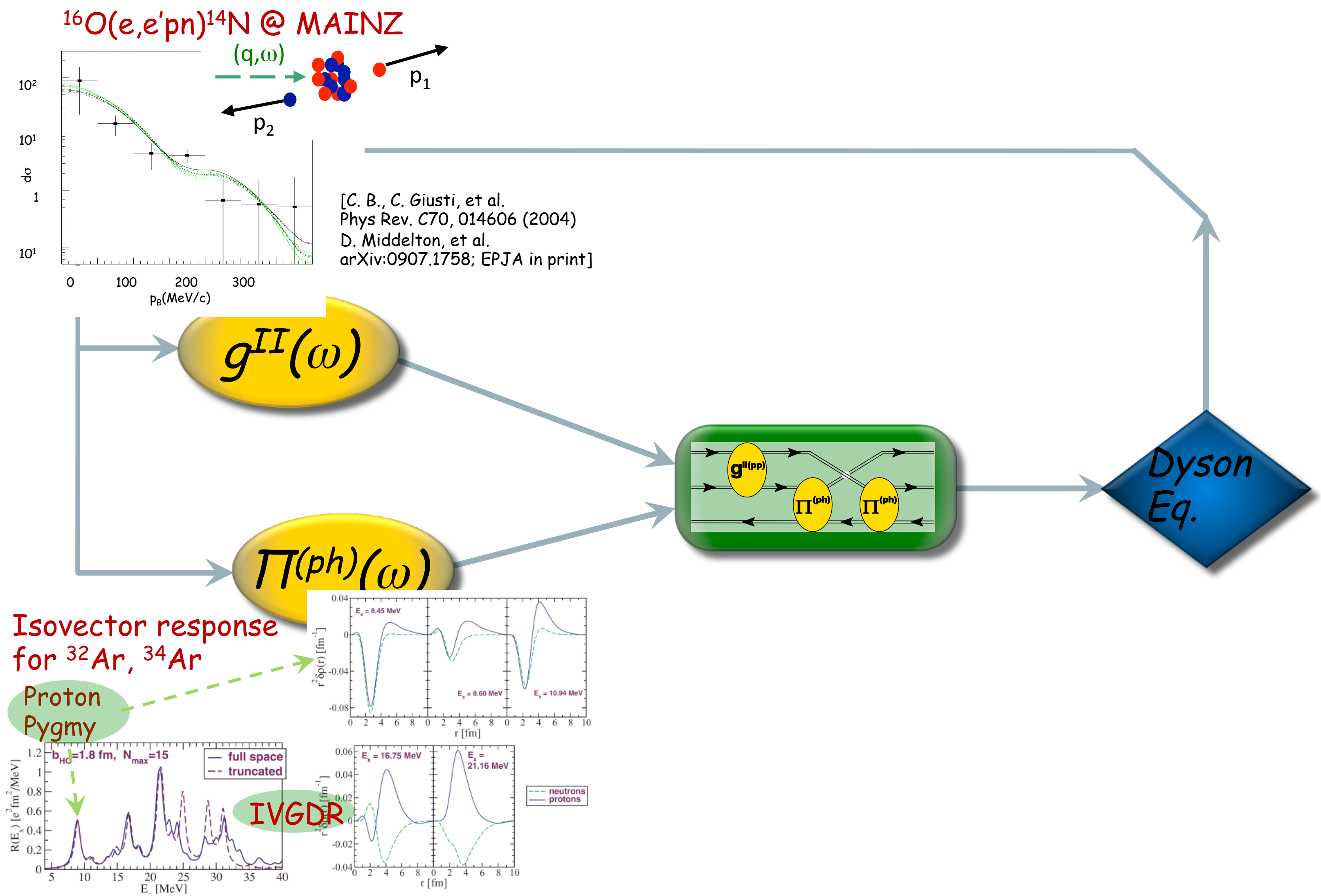

[C. B., K. Langanke, et al., Phys Rev. C77, 024304 (2008)]

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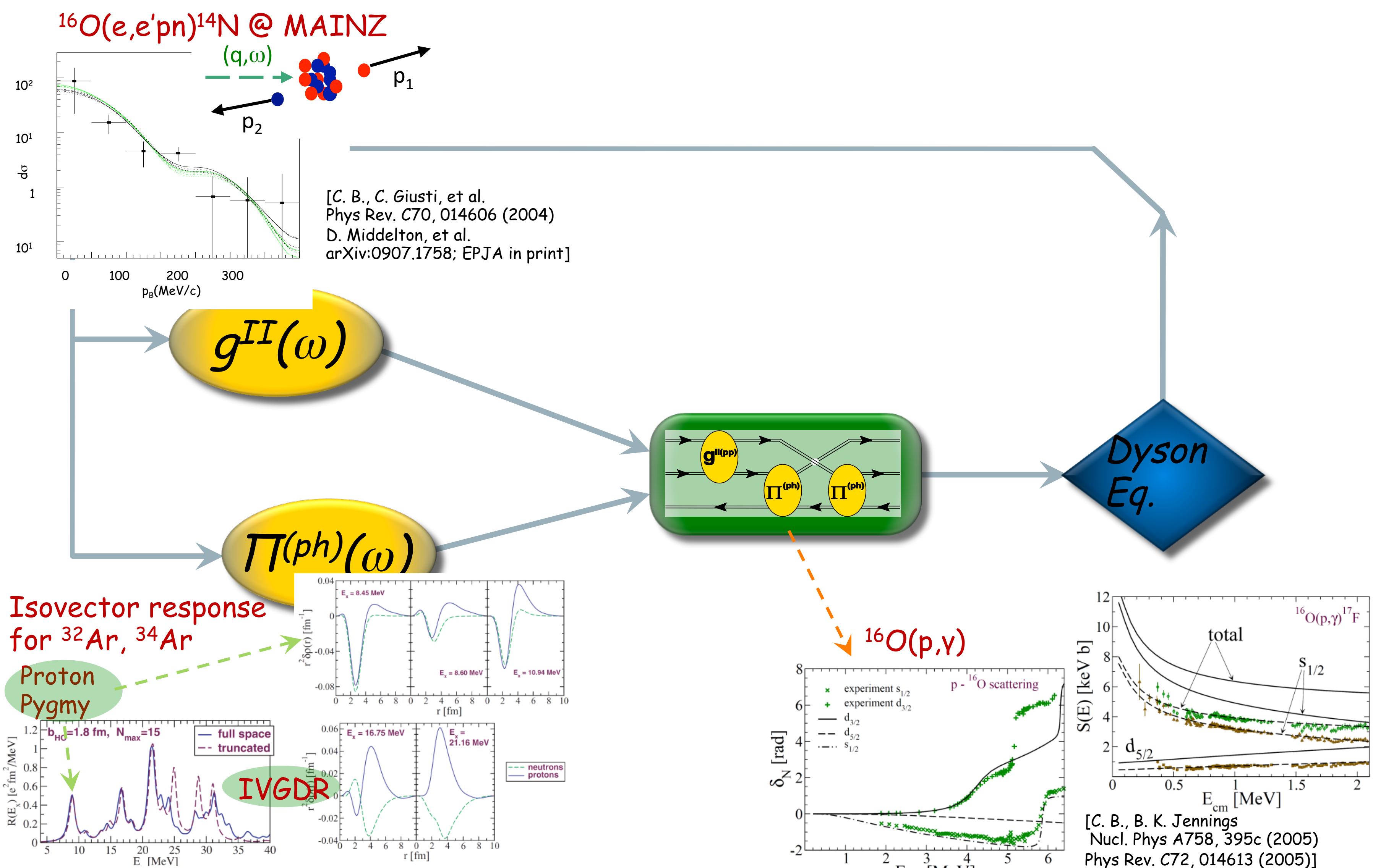
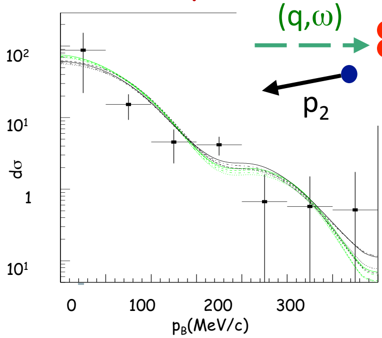


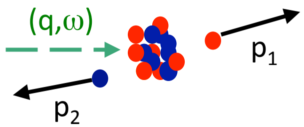
[C. B., K. Langanke, et al., Phys Rev. C77, 024304 (2008)]

SURREY

Self-Consistent Green's Function Approach

$^{16}O(e,e'pn)^{14}N$ @ MAINZ



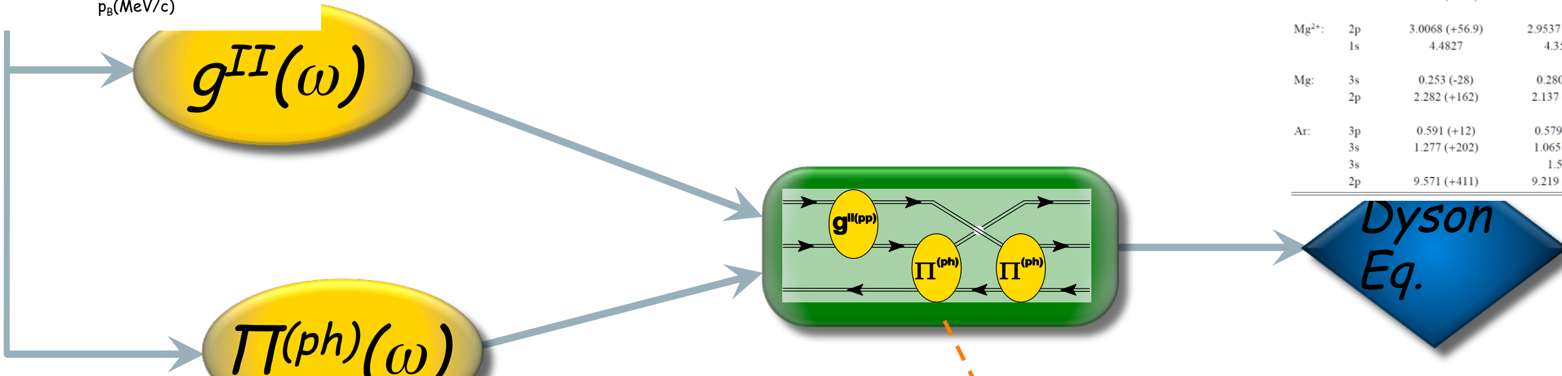


[C. B., C. Giusti, et al.
Phys Rev. C70, 014606 (2004)
D. Middelton, et al.
arXiv:0907.1758; EPJA in print]

Ionization energies/ affinities, in atoms

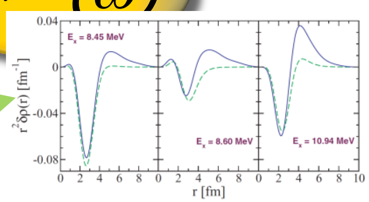
[CB, D. Van Neck,
AIP Conf.Proc.1120,104 ('09) & in prep]

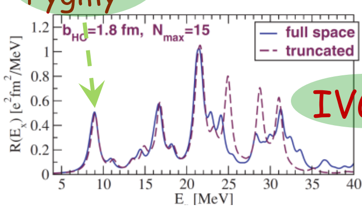
		Hartree-Fock	FRPAc	Experiment [16, 17]
He:	1s	0.918 (+14)	0.9008 (-2.9)	0.9037
Be^{2+}:	1s	5.6672 (+116)	5.6551 (-0.5)	5.6556
Be:	2s	0.3093 (-34)	0.3224 (-20.2)	0.3426
	1s	4.733 (+200)	4.5405 (+8)	4.533
Ne:	2p	0.852 (+57)	0.8037 (+11)	0.793
	1s	1.931 (+149)	1.7967 (+15)	1.782
Mg^{2+}:	2p	3.0068 (+56.9)	2.9537 (+3.8)	2.9499
	1s	4.4827	4.3589	
Mg:	3s	0.253 (-28)	0.280 (-1)	0.281
	2p	2.282 (+162)	2.137 (+17)	2.12
Ar:	3p	0.591 (+12)	0.579 (≈0)	0.579
	3s	1.277 (+202)	1.065 (-10)	1.075
	3s		1.544	
	2p	9.571 (+411)	9.219 (+59)	9.160



Isovector response for ^{32}Ar, ^{34}Ar

Proton Pygmy





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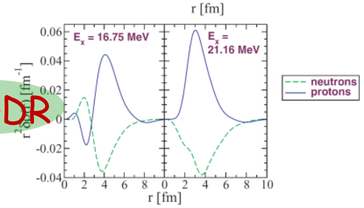

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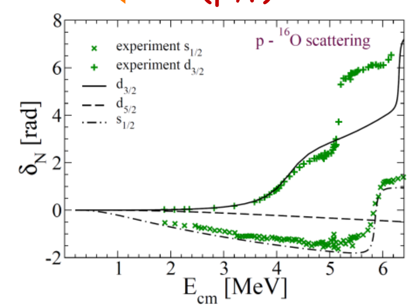
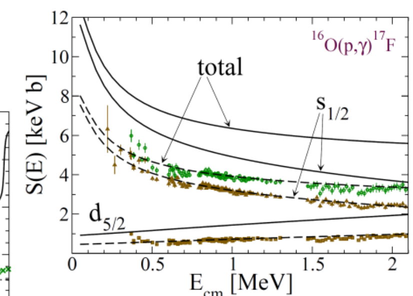
$^{16}O(p,\gamma)$





[C. B., B. K. Jennings
Nucl. Phys A758, 395c (2005)
Phys Rev. C72, 014613 (2005)]

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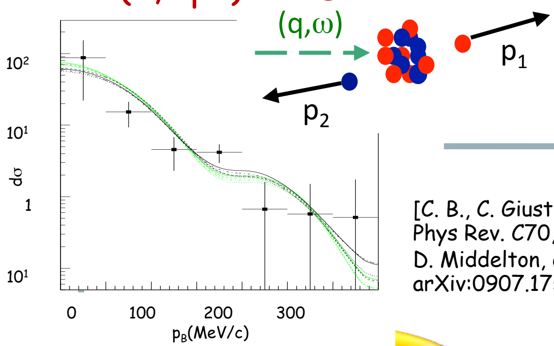

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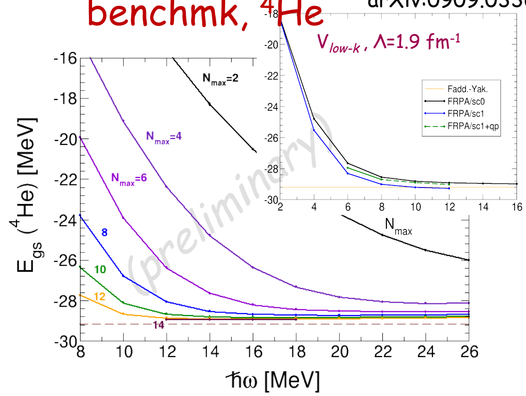
Binding energy benchmk, ^4He

[C. B., arXiv:0909.0336]



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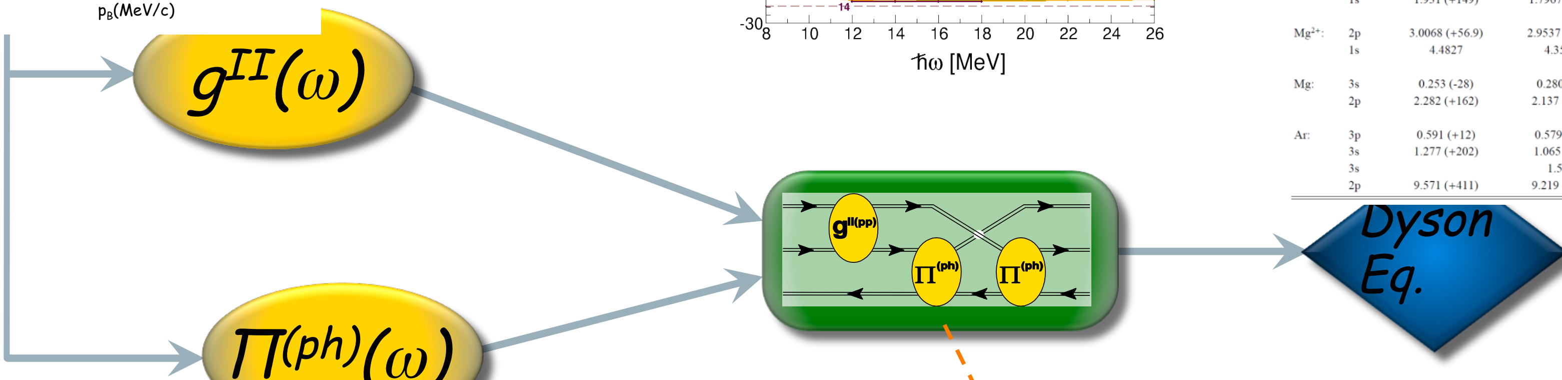

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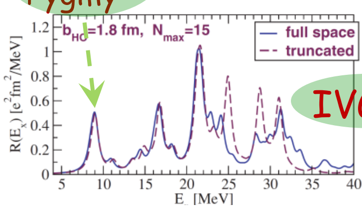

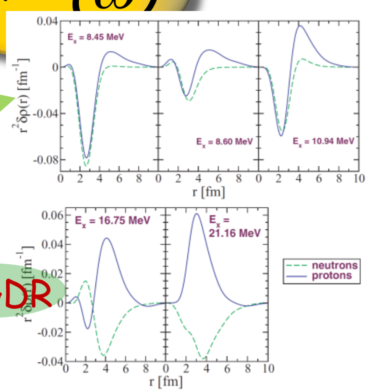

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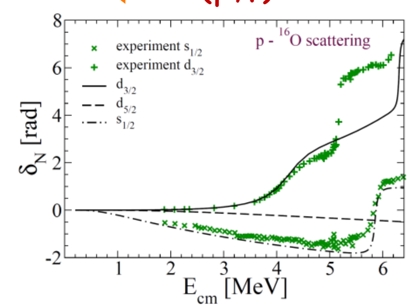

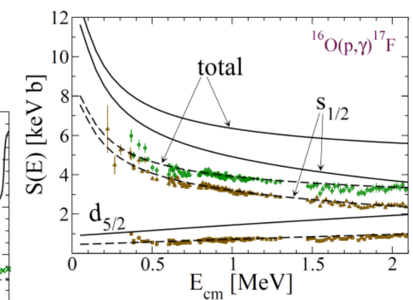

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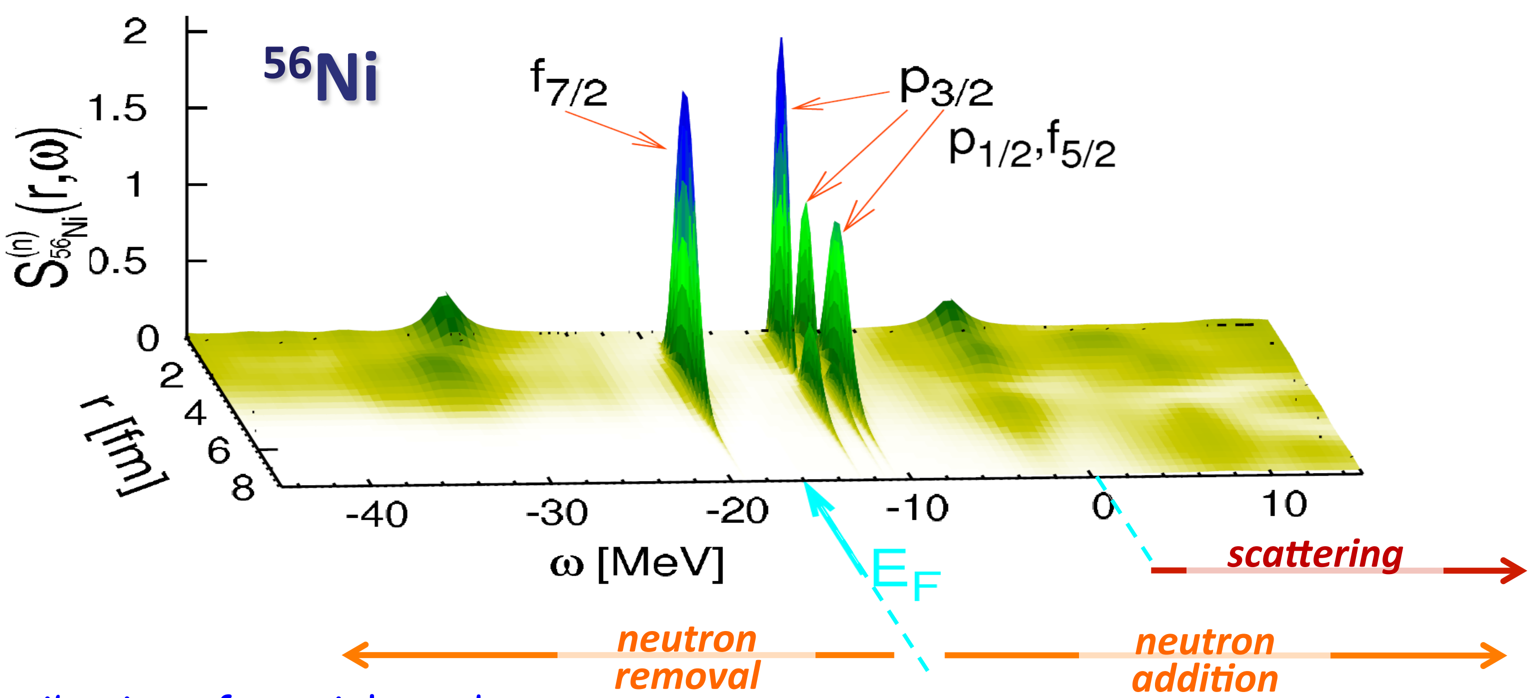
One-nucleon spectral function

$$S^{p,h}(r,\omega) = \mp\frac{1}{\pi}\,\mathrm{Im}\,g(r=r';\omega)$$



Distribution of particle and hole neutron states in ^{56}Ni

W. Dickhoff, CB, Prog. Part. Nucl. Phys. 53, 377 (2004)
CB, M.Hjorth-Jensen, Pys. Rev. C**79**, 064313 (2009)

Gorkov and its implementation

Gorkov and symmetry breaking approaches

V. Somà, CB, T. Duguet, , Phys. Rev. C **89**, 024323 (2014)
V. Somà, CB, T. Duguet, Phys. Rev. C **87**, 011303R (2013)
V. Somà, T. Duguet, CB, Phys. Rev. C **84**, 064317 (2011)

- Ansatz $\boxed{\ldots \approx E_0^{N+2} - E_0^N \approx E_0^N - E_0^{N-2} \approx \ldots \approx 2\mu}$

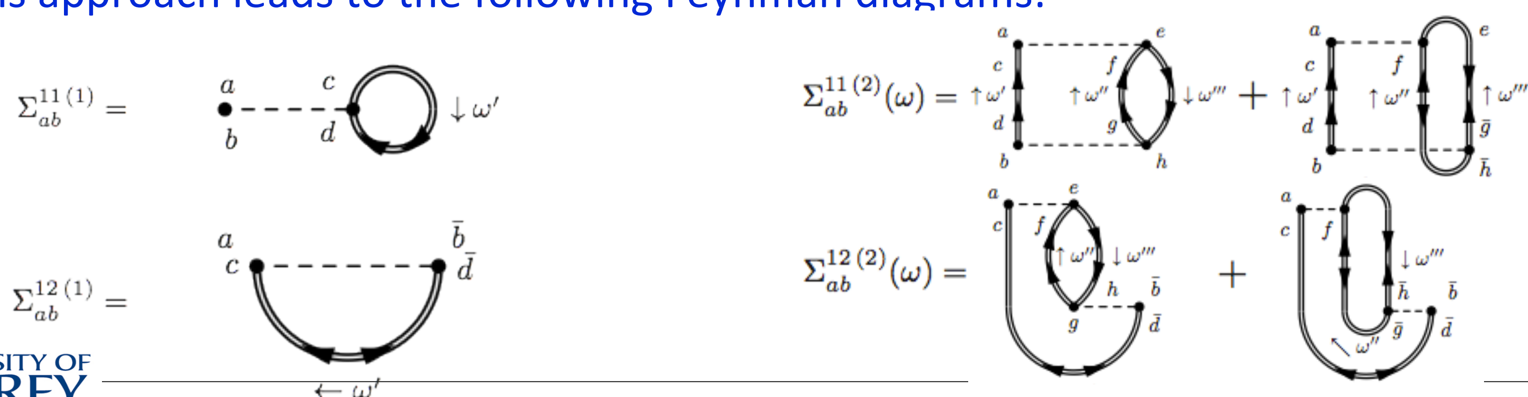
- Auxiliary many-body state $|\Psi_0\rangle \equiv \sum_N^{\text{even}} c_N \, |\psi_0^N\rangle$

  → Mixes various particle numbers

  → Introduce a "grand-canonical" potential $\Omega = H - \mu N$

$\Rightarrow$ $|\Psi_0\rangle$ minimizes $\Omega_0 = \langle \Psi_0 | \Omega | \Psi_0 \rangle$ under the constraint $N = \langle \Psi_0 | N | \Psi_0 \rangle$

- This approach leads to the following Feynman diagrams:



Espressions for 1st & 2nd order diagrams

[*V. Somà*, T. Duguet, CB, Pys. Rev. C84, 046317 (2011)]

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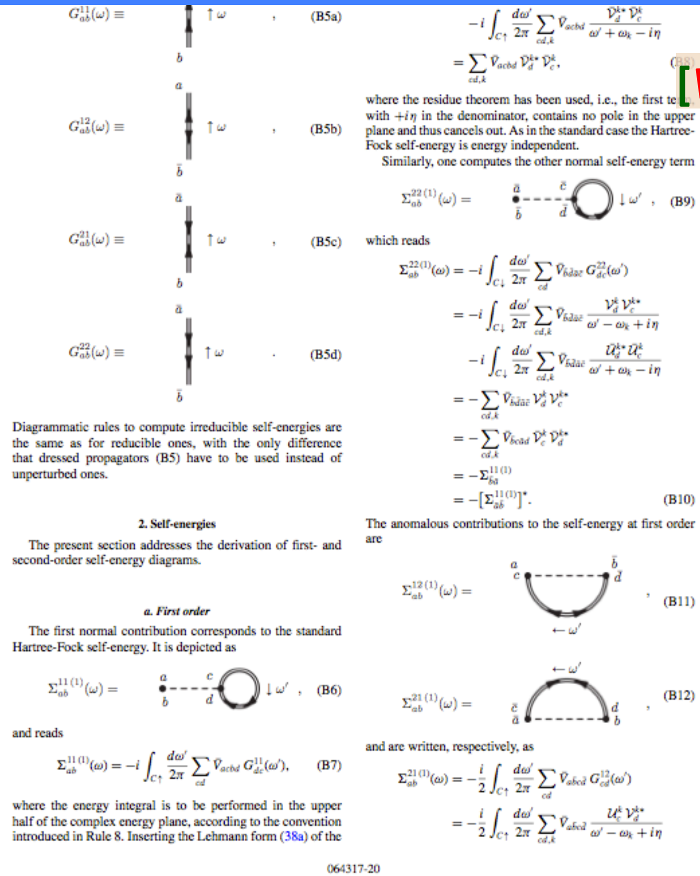

$$G^{11}_{ab}(\omega) \equiv \quad \uparrow \omega \quad , \qquad \text{(B5a)}$$

b

a

$$G^{12}_{ab}(\omega) \equiv \quad \uparrow \omega \quad , \qquad \text{(B5b)}$$

$\bar{b}$

$\bar{a}$

$$G^{21}_{ab}(\omega) \equiv \quad \uparrow \omega \quad , \qquad \text{(B5c)}$$

b

$\bar{a}$

$$G^{22}_{ab}(\omega) \equiv \quad \uparrow \omega \quad . \qquad \text{(B5d)}$$

$\bar{b}$

Diagrammatic rules to compute irreducible self-energies are the same as for reducible ones, with the only difference that dressed propagators (B5) have to be used instead of unperturbed ones.

2. Self-energies

The present section addresses the derivation of first- and second-order self-energy diagrams.

a. First order

The first normal contribution corresponds to the standard Hartree-Fock self-energy. It is depicted as

$$\Sigma^{11\,(1)}_{ab}(\omega) = \quad \downarrow \omega' \quad , \qquad \text{(B6)}$$

and reads

$$\Sigma^{11\,(1)}_{ab}(\omega) = -i \int_{C\uparrow} \frac{d\omega'}{2\pi} \sum_{cd} \bar{V}_{acbd}\, G^{11}_{dc}(\omega'), \qquad \text{(B7)}$$

where the energy integral is to be performed in the upper half of the complex energy plane, according to the convention introduced in Rule 8. Inserting the Lehmann form (38a) of the

$$-i \int_{C\uparrow} \frac{d\omega'}{2\pi} \sum_{cd,k} \bar{V}_{acbd} \frac{\mathcal{V}^{k*}_{d} \mathcal{V}^{k}_{c}}{\omega' + \omega_k - i\eta}$$

$$= \sum_{cd,k} \bar{V}_{acbd}\, \mathcal{V}^{k*}_{d} \mathcal{V}^{k}_{c}, \qquad \text{(B8)}$$

where the residue theorem has been used, i.e., the first te[rm], with $+i\eta$ in the denominator, contains no pole in the upper plane and thus cancels out. As in the standard case the Hartree-Fock self-energy is energy independent.

Similarly, one computes the other normal self-energy term

$$\Sigma^{22\,(1)}_{ab}(\omega) = \quad \downarrow \omega' \quad , \qquad \text{(B9)}$$

which reads

$$\Sigma^{22\,(1)}_{ab}(\omega) = -i \int_{C\downarrow} \frac{d\omega'}{2\pi} \sum_{cd} \bar{V}_{\bar{b}\bar{d}\bar{a}\bar{c}}\, G^{22}_{dc}(\omega')$$

$$= -i \int_{C\downarrow} \frac{d\omega'}{2\pi} \sum_{cd,k} \bar{V}_{\bar{b}\bar{d}\bar{a}\bar{c}} \frac{\mathcal{V}^{k}_{d} \mathcal{V}^{k*}_{c}}{\omega' - \omega_k + i\eta}$$

$$-i \int_{C\downarrow} \frac{d\omega'}{2\pi} \sum_{cd,k} \bar{V}_{\bar{b}\bar{d}\bar{a}\bar{c}} \frac{\bar{\mathcal{U}}^{k*}_{d} \bar{\mathcal{U}}^{k}_{c}}{\omega' + \omega_k - i\eta}$$

$$= -\sum_{cd,k} \bar{V}_{\bar{b}\bar{d}\bar{a}\bar{c}}\, \mathcal{V}^{k}_{d} \mathcal{V}^{k*}_{c}$$

$$= -\sum_{cd,k} \bar{V}_{bcad}\, \mathcal{V}^{k}_{c} \mathcal{V}^{k*}_{d}$$

$$= -\Sigma^{11\,(1)}_{ba}$$

$$= -\left[\Sigma^{11\,(1)}_{ab}\right]^{*}. \qquad \text{(B10)}$$

The anomalous contributions to the self-energy at first order are

$$\Sigma^{12\,(1)}_{ab}(\omega) = \quad \leftarrow \omega' \quad , \qquad \text{(B11)}$$

$$\Sigma^{21\,(1)}_{ab}(\omega) = \quad \leftarrow \omega' \quad , \qquad \text{(B12)}$$

and are written, respectively, as

$$\Sigma^{21\,(1)}_{ab}(\omega) = -\frac{i}{2} \int_{C\uparrow} \frac{d\omega'}{2\pi} \sum_{cd} \bar{V}_{ab\bar{c}\bar{d}}\, G^{12}_{cd}(\omega')$$

$$= -\frac{i}{2} \int_{C\uparrow} \frac{d\omega'}{2\pi} \sum_{cd,k} \bar{V}_{ab\bar{c}\bar{d}} \frac{\mathcal{U}^{k}_{c} \mathcal{V}^{k*}_{d}}{\omega' - \omega_k + i\eta}$$

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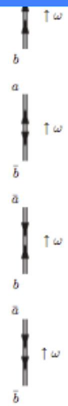
$$G^{11}_{ab}(\omega) \equiv \quad \uparrow \omega \quad , \qquad \text{(B5a)}$$

$$G^{12}_{ab}(\omega) \equiv \quad \uparrow \omega \quad , \qquad \text{(B5b)}$$

$$G^{21}_{ab}(\omega) \equiv \quad \uparrow \omega \quad , \qquad \text{(B5c)}$$

$$G^{22}_{ab}(\omega) \equiv \quad \uparrow \omega \quad . \qquad \text{(B5d)}$$



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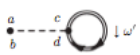

and reads

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where the energy integral is to be performed in the upper half of the complex energy plane, according to the convention introduced in Rule 8. Inserting the Lehmann form (38a) of the

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$$= \sum_{cd,k} \bar{V}_{acbd}\, \mathcal{V}^{k*}_d \mathcal{V}^k_c,$$

where the residue theorem has been used, i.e., the first te[…] with $+i\eta$ in the denominator […] plane and thus cancels out. As […] Fock self-energy is energy in[…]

Similarly, one computes th[…]

$$\Sigma^{22\,(1)}_{ab}(\omega) = \ldots$$

which reads

$$\Sigma^{22\,(1)}_{ab}(\omega) = -i \int_{C\downarrow} \ldots$$

$$= -i \int_{C\downarrow} \ldots$$

$$-i \int_{C\downarrow} \ldots$$

$$= -\sum_{cd,k} \bar{V}_{\bar{b}\bar{d}\bar{a}} \ldots$$

$$= -\sum_{cd,k} \bar{V}_{\bar{b}c\bar{a}} \ldots$$

$$= -\Sigma^{11\,(1)}_{\bar{b}\bar{a}}$$

$$= -[\Sigma^{11\,(1)}_{\bar{a}\bar{b}}]^{\ldots}$$

The anomalous contributions are

$$\Sigma^{12\,(1)}_{ab}(\omega) = \ldots$$

$$\Sigma^{21\,(1)}_{ab}(\omega) = \ldots$$

and are written, respectively, […]

$$\Sigma^{21\,(1)}_{ab}(\omega) = -\frac{i}{2} \int_{C\uparrow} \ldots$$

$$= -\frac{i}{2} \int_{C\uparrow} \ldots$$

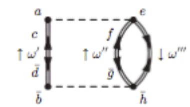
for what concerns the first contribution, which reads

$$\Sigma^{12\,(2')}_{ab}(\omega) = -\int \frac{d\omega'}{2\pi}\frac{d\omega''}{2\pi} \sum_{cdefgh} \bar{V}_{aecf}\, \bar{V}_{\bar{h}\bar{b}\bar{g}\bar{d}}\, G^{12}_{cd}(\omega')\, G^{11}_{fh}(\omega'')\, G^{11}_{ge}(\omega'+\omega''-\omega)$$

$$= -\sum_{cdefgh,k_1k_2k_3} \bar{V}_{aecf}\, \bar{V}_{\bar{h}\bar{b}\bar{g}\bar{d}} \left\{ \frac{\mathcal{U}^{k_1}_c \mathcal{V}^{k_1*}_d \mathcal{U}^{k_2}_f \mathcal{U}^{k_2*}_h \bar{\mathcal{V}}^{k_3*}_g \bar{\mathcal{V}}^{k_3}_e}{\omega - (\omega_{k_1}+\omega_{k_2}+\omega_{k_3}) + i\eta} + \frac{\bar{\mathcal{V}}^{k_1*}_c \bar{\mathcal{U}}^{k_1}_d \bar{\mathcal{V}}^{k_2*}_f \bar{\mathcal{V}}^{k_2}_h \mathcal{U}^{k_3}_g \mathcal{U}^{k_3*}_e}{\omega + (\omega_{k_3}+\omega_{k_1}+\omega_{k_2}) - i\eta} \right\} \qquad \text{(B26)}$$

and

$$\Sigma^{12\,(2'')}_{ab}(\omega) = \quad \uparrow \omega' \quad \uparrow \omega'' \quad \downarrow \omega''' \quad , \qquad \text{(B27)}$$



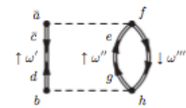
for the second contribution, yielding

$$\Sigma^{12\,(2'')}_{ab}(\omega) = \frac{1}{2}\int \frac{d\omega'}{2\pi}\frac{d\omega''}{2\pi} \sum_{cdefgh} \bar{V}_{aecf}\, V_{\bar{h}\bar{b}\bar{g}\bar{d}}\, G^{12}_{cd}(\omega')\, G^{12}_{fg}(\omega'')\, G^{21}_{he}(\omega'+\omega''-\omega)$$

$$= \frac{1}{2}\sum_{cdefgh,k_1k_2k_3} \bar{V}_{aecf}\, V_{\bar{h}\bar{b}\bar{g}\bar{d}} \left\{ \frac{\mathcal{U}^{k_1}_c \mathcal{V}^{k_1*}_d \mathcal{U}^{k_2}_f \mathcal{V}^{k_2*}_g \bar{\mathcal{U}}^{k_3*}_h \bar{\mathcal{V}}^{k_3}_e}{\omega - (\omega_{k_1}+\omega_{k_2}+\omega_{k_3}) + i\eta} + \frac{\bar{\mathcal{V}}^{k_1*}_c \bar{\mathcal{U}}^{k_1}_d \bar{\mathcal{V}}^{k_2*}_f \bar{\mathcal{U}}^{k_2}_g \mathcal{V}^{k_3}_h \mathcal{U}^{k_3*}_e}{\omega + (\omega_{k_3}+\omega_{k_1}+\omega_{k_2}) - i\eta} \right\}. \qquad \text{(B28)}$$

Finally,

$$\Sigma^{21\,(2')}_{ab}(\omega) = \quad \uparrow \omega' \quad \uparrow \omega'' \quad \downarrow \omega''' \quad , \qquad \text{(B29)}$$



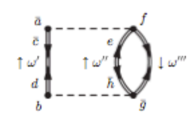
reads as

$$\Sigma^{21\,(2')}_{ab}(\omega) = -\int \frac{d\omega'}{2\pi}\frac{d\omega''}{2\pi} \sum_{cdefgh} \bar{V}_{\bar{c}f\bar{a}e}\, \bar{V}_{gdhb}\, G^{21}_{cd}(\omega')\, G^{11}_{eg}(\omega'')\, G^{11}_{hf}(\omega'+\omega''-\omega)$$

$$= -\sum_{cdefgh,k_1k_2k_3} \bar{V}_{\bar{c}f\bar{a}e}\, \bar{V}_{gdhb} \left\{ \frac{\mathcal{V}^{k_1}_c \mathcal{U}^{k_1*}_d \mathcal{U}^{k_2}_e \mathcal{U}^{k_2*}_g \bar{\mathcal{V}}^{k_3*}_h \bar{\mathcal{V}}^{k_3}_f}{\omega - (\omega_{k_1}+\omega_{k_2}+\omega_{k_3}) + i\eta} + \frac{\bar{\mathcal{U}}^{k_1*}_c \bar{\mathcal{V}}^{k_1}_d \bar{\mathcal{V}}^{k_2*}_e \bar{\mathcal{V}}^{k_2}_g \mathcal{U}^{k_3}_h \mathcal{U}^{k_3*}_f}{\omega + (\omega_{k_3}+\omega_{k_1}+\omega_{k_2}) - i\eta} \right\}, \qquad \text{(B30)}$$

while

$$\Sigma^{21\,(2'')}_{ab}(\omega) = \quad \uparrow \omega' \quad \uparrow \omega'' \quad \downarrow \omega''' \quad , \qquad \text{(B31)}$$



is expressed as

$$\Sigma^{21\,(2'')}_{ab}(\omega) = \frac{1}{2}\int \frac{d\omega'}{2\pi}\frac{d\omega''}{2\pi} \sum_{cdefgh} \bar{V}_{\bar{c}f\bar{a}e}\, \bar{V}_{\bar{g}d\bar{h}b}\, G^{21}_{cd}(\omega')\, G^{12}_{eh}(\omega'')\, G^{21}_{gf}(\omega'+\omega''-\omega)$$

$$= \frac{1}{2}\sum_{cdefgh,k_1k_2k_3} \bar{V}_{\bar{c}f\bar{a}e}\, \bar{V}_{\bar{g}d\bar{h}b} \left\{ \frac{\mathcal{V}^{k_1}_c \mathcal{U}^{k_1*}_d \mathcal{U}^{k_2}_e \mathcal{V}^{k_2*}_h \bar{\mathcal{U}}^{k_3*}_g \bar{\mathcal{V}}^{k_3}_f}{\omega - (\omega_{k_1}+\omega_{k_2}+\omega_{k_3}) + i\eta} + \frac{\bar{\mathcal{U}}^{k_1*}_c \bar{\mathcal{V}}^{k_1}_d \bar{\mathcal{V}}^{k_2*}_e \bar{\mathcal{U}}^{k_2}_h \mathcal{V}^{k_3}_g \mathcal{U}^{k_3*}_f}{\omega + (\omega_{k_3}+\omega_{k_1}+\omega_{k_2}) - i\eta} \right\}. \qquad \text{(B32)}$$

Espressions for 1st & 2nd order diagrams

[*V. Somà*, T. Duguet, CB, Pys. Rev. C84, 046317 (2011)]

$G^{11}_{ab}(\omega) \equiv$ (B5a)

$G^{12}_{ab}(\omega) \equiv$ (B5b)

$G^{21}_{ab}(\omega) \equiv$ (B5c)

$G^{22}_{ab}(\omega) \equiv$ (B5d)

Diagrammatic rules to compute irreducible self-energies are the same as for reducible ones, with the only difference that dressed propagators (B5) have to be used instead of unperturbed ones.

2. Self-energies

The present section addresses the derivation of first- and second-order self-energy diagrams.

a. First order

The first normal contribution corresponds to the standard Hartree-Fock self-energy. It is depicted as

$\Sigma^{11\,(1)}_{ab}(\omega) =$ (B6)

and reads

$$\Sigma^{11\,(1)}_{ab}(\omega) = -i \int_{C\uparrow} \frac{d\omega'}{2\pi} \sum_{cd} \bar{V}_{acbd}\, G^{11}_{dc}(\omega'), \qquad (B7)$$

where the energy integral is to be performed in the upper half of the complex energy plane, according to the convention introduced in Rule 8. Inserting the Lehmann form (38a) of the

$$-i \int_{C\uparrow} \frac{d\omega'}{2\pi} \sum_{cd,k} \bar{V}_{acbd} \frac{\mathcal{V}^{k*}_d \mathcal{V}^k_c}{\omega' + \omega_k - i\eta}$$
$$= \sum_{cd,k} \bar{V}_{acbd}\, \mathcal{V}^{k*}_d \mathcal{V}^k_c,$$

where the residue theorem has been used, i.e., the first te with $+i\eta$ in the denominator plane and thus cancels out. As Fock self-energy is energy in Similarly, one computes th

$\Sigma^{22\,(1)}_{ab}(\omega) =$

which reads

$\Sigma^{22\,(1)}_{ab}(\omega) = -i \int_{C\downarrow}$ …

The anomalous contributions are

$\Sigma^{12\,(1)}_{ab}(\omega) =$

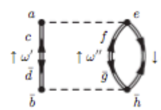
$\Sigma^{21\,(1)}_{ab}(\omega) =$

and are written, respectively,

for what concerns the first contribution, which reads

$$\Sigma^{12\,(2')}_{ab}(\omega) = -\int \frac{d\omega'}{2\pi}\frac{d\omega''}{2\pi} \sum_{cdefgh} \bar{V}_{aecf}\, \bar{V}_{\bar{h}\bar{b}\bar{g}\bar{d}}\, G^{12}_{cd}(\omega')\, G^{11}_{fh}(\omega'')\, G^{11}_{ge}(\omega'+\omega''-\omega)$$
$$= -\sum_{cdefgh,k_1k_2k_3} \bar{V}_{aecf}\, \bar{V}_{\bar{h}\bar{b}\bar{g}\bar{d}} \left\{ \frac{\mathcal{U}^{k_1}_c \mathcal{V}^{k_1*}_d \mathcal{U}^{k_2}_f \mathcal{U}^{k_2*}_h \mathcal{V}^{k_3*}_g \mathcal{V}^{k_3}_e}{\omega - (\omega_{k_1}+\omega_{k_2}+\omega_{k_3}) + i\eta} + \frac{\mathcal{V}^{k_1*}_c \mathcal{U}^{k_1}_d \mathcal{V}^{k_2*}_f \mathcal{V}^{k_2}_h \mathcal{U}^{k_3}_g \mathcal{U}^{k_3*}_e}{\omega + (\omega_{k_1}+\omega_{k_2}+\omega_{k_3}) - i\eta} \right\} \qquad (B26)$$

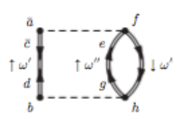
and

$\Sigma^{12\,(2'')}_{ab}(\omega) =$  , (B27)

for the second contribution, yielding

$$\Sigma^{12\,(2'')}_{ab}(\omega) = \frac{1}{2}\int \frac{d\omega'}{2\pi}\frac{d\omega''}{2\pi} \sum_{cdefgh} \bar{V}_{aecf}\, \bar{V}_{\bar{h}\bar{b}\bar{g}\bar{d}}\, G^{12}_{cd}(\omega')\, G^{12}_{fg}(\omega'')\, G^{21}_{he}(\omega'+\omega''-\omega)$$
$$= \frac{1}{2}\sum_{cdefgh,k_1k_2k_3} \bar{V}_{aecf}\, \bar{V}_{\bar{h}\bar{b}\bar{g}\bar{d}} \left\{ \frac{\mathcal{U}^{k_1}_c \mathcal{V}^{k_1*}_d \mathcal{U}^{k_2}_f \mathcal{V}^{k_2*}_g \mathcal{U}^{k_3*}_h \mathcal{V}^{k_3}_e}{\omega - (\omega_{k_1}+\omega_{k_2}+\omega_{k_3}) + i\eta} + \frac{\mathcal{V}^{k_1*}_c \mathcal{U}^{k_1}_d \mathcal{V}^{k_2*}_f \mathcal{U}^{k_2}_g \mathcal{V}^{k_3}_h \mathcal{U}^{k_3*}_e}{\omega + (\omega_{k_1}+\omega_{k_2}+\omega_{k_3}) - i\eta} \right\}. \qquad (B28)$$

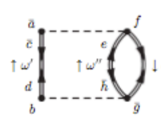
Finally,

$\Sigma^{21\,(2')}_{ab}(\omega) =$  , (B29)

reads as

$$\Sigma^{21\,(2')}_{ab}(\omega) = -\int \frac{d\omega'}{2\pi}\frac{d\omega''}{2\pi} \sum_{cdefgh} \bar{V}_{\bar{c}\bar{f}\bar{a}\bar{e}}\, \bar{V}_{gdhb}\, G^{21}_{cd}(\omega')\, G^{11}_{eg}(\omega'')\, G^{11}_{hf}(\omega'+\omega''-\omega)$$
$$= -\sum_{cdefgh,k_1k_2k_3} \bar{V}_{\bar{c}\bar{f}\bar{a}\bar{e}}\, \bar{V}_{gdhb} \left\{ \frac{\mathcal{V}^{k_1}_c \mathcal{U}^{k_1*}_d \mathcal{U}^{k_2}_e \mathcal{U}^{k_2*}_g \mathcal{V}^{k_3*}_h \mathcal{V}^{k_3}_f}{\omega - (\omega_{k_1}+\omega_{k_2}+\omega_{k_3}) + i\eta} + \frac{\mathcal{U}^{k_1*}_c \mathcal{V}^{k_1}_d \mathcal{V}^{k_2*}_e \mathcal{V}^{k_2}_g \mathcal{U}^{k_3}_h \mathcal{U}^{k_3*}_f}{\omega + (\omega_{k_1}+\omega_{k_2}+\omega_{k_3}) - i\eta} \right\}, \qquad (B30)$$

while

$\Sigma^{21\,(2'')}_{ab}(\omega) =$  , (B31)

is expressed as

$$\Sigma^{21\,(2'')}_{ab}(\omega) = \frac{1}{2}\int \frac{d\omega'}{2\pi}\frac{d\omega''}{2\pi} \sum_{cdefgh} \bar{V}_{\bar{c}\bar{f}\bar{a}\bar{e}}\, \bar{V}_{\bar{g}\bar{d}\bar{h}b}\, G^{21}_{cd}(\omega')\, G^{12}_{eh}(\omega'')\, G^{21}_{gf}(\omega'+\omega''-\omega)$$
$$= \frac{1}{2}\sum_{cdefgh,k_1k_2k_3} \bar{V}_{\bar{c}\bar{f}\bar{a}\bar{e}}\, \bar{V}_{\bar{g}\bar{d}\bar{h}b} \left\{ \frac{\mathcal{V}^{k_1}_c \mathcal{U}^{k_1*}_d \mathcal{U}^{k_2}_e \mathcal{V}^{k_2*}_h \mathcal{U}^{k_3*}_g \mathcal{V}^{k_3}_f}{\omega - (\omega_{k_1}+\omega_{k_2}+\omega_{k_3}) + i\eta} + \frac{\mathcal{U}^{k_1*}_c \mathcal{V}^{k_1}_d \mathcal{V}^{k_2*}_e \mathcal{U}^{k_2}_h \mathcal{V}^{k_3}_g \mathcal{U}^{k_3*}_f}{\omega + (\omega_{k_1}+\omega_{k_2}+\omega_{k_3}) - i\eta} \right\}. \qquad (B32)$$

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contribution, which reads (B26) (B27) , yielding (B28) (B29) (B30) (B31) (B32)
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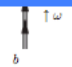
UNIVERSITY OF SURREY

Espressions for 1st & 2nd order diagrams

[*V. Somà*, T. Duguet, CB, Pys. Rev. C84, 046317 (2011)]

$G^{11}_{ab}(\omega) \equiv$  , (B5a)

$G^{12}_{ab}(\omega) \equiv$  , (B5b)

$$-i\int_{C\uparrow}\frac{d\omega'}{2\pi}\sum_{cd,k}\bar{V}_{acbd}\frac{\mathcal{V}^{k*}_d\,\mathcal{V}^k_c}{\omega'+\omega_k-i\eta}$$

$$=\sum_{cd,k}\bar{V}_{acbd}\,\mathcal{V}^{k*}_d\,\mathcal{V}^k_c,$$

where the residue theorem has been used, i.e., the first te with $+i\eta$ in the denominator ... plane and thus cancels out. As Fock self-energy is energy in

Similarly, one computes th

$\Sigma^{22\,(1)}_{ab}(\omega) =$

5. Block-diagonal structure of self-energies

a. First order

The goal of this subsection is to discuss how the block-diagonal form of the propagators and interaction matrix elements reflects in the various self-energy contributions, starting with the first-order normal self-energy $\Sigma^{11\,(1)}$. Substituting Eqs. (C27a) and (C19) into Eq. (B7), and introducing the factor

$$f^{n_a n_b n_c n_d}_{\alpha\beta\gamma\delta} \equiv \sqrt{1+\delta_{\alpha\beta}\,\delta_{n_a n_b}}\,\sqrt{1+\delta_{\gamma\delta}\,\delta_{n_c n_d}}, \tag{C28}$$

one obtains

$$\begin{aligned}
\Sigma^{11\,(1)}_{ab} &= \sum_{cd,k}\bar{V}_{acbd}\,\mathcal{V}^{k*}_d\,\mathcal{V}^k_c\\
&= \sum_{n_c n_d n_k}\sum_{\gamma}\sum_{m_c}\sum_{JM} f^{n_a n_c n_b n_d}_{\alpha\gamma\beta\gamma}\, C^{JM}_{j_a m_a j_c m_c}\, C^{JM}_{j_b m_b j_c m_c}\,\bar{V}^{J\,[\alpha\gamma\beta\gamma]}_{n_a n_c n_b n_d}\,\mathcal{V}^{n_k *}_{n_d\,[\gamma]}\,\mathcal{V}^{n_k}_{n_c\,[\gamma]}\\
&= \delta_{\alpha\beta}\,\delta_{m_a m_b}\sum_{n_c n_d}\sum_{\gamma}\sum_{J} f^{n_a n_c n_b n_d}_{\alpha\gamma\alpha\gamma}\,\frac{2J+1}{2j_a+1}\,\bar{V}^{J\,[\alpha\gamma\alpha\gamma]}_{n_a n_c n_b n_d}\,\rho^{[\gamma]}_{n_d n_c}\\
&\equiv \delta_{\alpha\beta}\,\delta_{m_a m_b}\,\Sigma^{11\,[\alpha]\,(1)}_{n_a n_b}\\
&\equiv \delta_{\alpha\beta}\,\delta_{m_a m_b}\,\Lambda^{[\alpha]}_{n_a n_b},
\end{aligned}\tag{C29}$$

where the block-diagonal normal density matrix is introduced through $\rho_{ab} \equiv \delta_{\alpha\beta}\,\delta_{m_a m_b}\,\rho^{[\alpha]}_{n_a n_b}$, such that

$$\rho^{[\alpha]}_{n_a n_b} = \sum_{n_k}\mathcal{V}^{n_k}_{n_b\,[\alpha]}\,\mathcal{V}^{n_k *}_{n_a\,[\alpha]}, \tag{C30}$$

and properties of Clebsch-Gordan coefficients has been used. The fact that the interaction conserves parity and charge yields $\delta_{\pi_a\pi_b}$ and $\delta_{q_a q_b}$, leading to $\delta_{\alpha\beta} = \delta_{j_a j_b}\,\delta_{\pi_a\pi_b}\,\delta_{q_a q_b}$. Similarly, for $\Sigma^{22\,(1)}$,

$$\begin{aligned}
\Sigma^{22\,(1)}_{ab} &= -\sum_{cd,k}\bar{V}_{bcad}\,\bar{\mathcal{V}}^k_c\,\bar{\mathcal{V}}^{k*}_d\\
&= -\delta_{\alpha\beta}\,\delta_{m_a m_b}\sum_{n_c n_d}\sum_{\gamma}\sum_{J} f^{n_b n_c n_a n_d}_{\alpha\gamma\alpha\gamma}\,\frac{2J+1}{2j_a+1}\,\bar{V}^{J\,[\alpha\gamma\alpha\gamma]}_{n_b n_c n_a n_d}\,\rho^{[\gamma]}_{n_d n_c}\\
&\equiv \delta_{\alpha\beta}\,\delta_{m_a m_b}\,\Sigma^{22\,[\alpha]\,(1)}_{n_a n_b}\\
&= -\delta_{\alpha\beta}\,\delta_{m_a m_b}\,\Lambda^{[\alpha]}_{n_b n_a}\\
&= -\delta_{\alpha\beta}\,\delta_{m_a m_b}\,[\Lambda^{[\alpha]}_{n_a n_b}]^*.
\end{aligned}\tag{C31}$$

Let us consider the anomalous contributions to the first-order self-energy. Substituting Eqs. (C27b) and (C19) into Eq. (B12) one derives

$$\begin{aligned}
\Sigma^{12\,(1)}_{ab} &= \frac{1}{2}\sum_{cd,k}\bar{V}_{a\bar{b}c\bar{d}}\,\mathcal{V}^{k*}_c\,\mathcal{U}^k_d\\
&= -\frac{1}{2}\sum_{n_c n_d n_k}\sum_{\gamma}\sum_{m_c}\sum_{JM} f^{n_a n_b n_c n_d}_{\alpha\beta\gamma\gamma}\,\eta_b\eta_c\,C^{JM}_{j_a m_a j_b -m_b}\,C^{JM}_{j_c m_c j_c -m_c}\,\bar{V}^{J\,[\alpha\beta\gamma\gamma]}_{n_a n_b n_c n_d}\,\mathcal{V}^{n_k *}_{n_c\,[\gamma]}\,\mathcal{U}^{n_k}_{n_d\,[\gamma]}\\
&= -\frac{1}{2}\sum_{n_c n_d}\sum_{\gamma}\sum_{m_c}\sum_{J} f^{n_a n_b n_c n_d}_{\alpha\beta\gamma\gamma}\,\eta_b\eta_c\,C^{J0}_{j_a m_a j_b -m_b}\,C^{J0}_{j_c m_c j_c -m_c}\,\bar{V}^{J\,[\alpha\beta\gamma\gamma]}_{n_a n_b n_c n_d}\,\tilde{\rho}^{[\gamma]}_{n_c n_d}\\
&= -\frac{1}{2}\sum_{n_c n_d}\sum_{\gamma} f^{n_a n_b n_c n_d}_{\alpha\beta\gamma\gamma}\,\eta_b\pi_c(-1)^{2j_c}\,C^{00}_{j_a m_a j_b -m_b}\,\sqrt{2j_c+1}\,\bar{V}^{0\,[\alpha\beta\gamma\gamma]}_{n_a n_b n_c n_d}\,\tilde{\rho}^{[\gamma]}_{n_c n_d}\\
&= \delta_{\alpha\beta}\,\delta_{m_a m_b}\,\frac{1}{2}\sum_{n_c n_d}\sum_{\gamma} f^{n_a n_b n_c n_d}_{\alpha\alpha\gamma\gamma}\,\pi_a\pi_c(-1)^{2j_c}\,\frac{\sqrt{2j_c+1}}{\sqrt{2j_a+1}}\,\bar{V}^{0\,[\alpha\alpha\gamma\gamma]}_{n_a n_b n_c n_d}\,\tilde{\rho}^{[\gamma]}_{n_c n_d}\\
&\equiv \delta_{\alpha\beta}\,\delta_{m_a m_b}\,\Sigma^{12\,[\alpha]\,(1)}_{n_a n_b}\\
&\equiv \delta_{\alpha\beta}\,\delta_{m_a m_b}\,\tilde{h}^{[\alpha]}_{n_a n_b},
\end{aligned}\tag{C32}$$

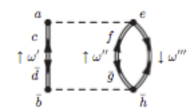
where the block-diagonal anomalous density matrix is introduced through $\tilde{\rho}_{ab} \equiv \delta_{\alpha\beta}\,\delta_{m_a m_b}\,\tilde{\rho}^{[\alpha]}_{n_a n_b}$, such that

$$\tilde{\rho}^{[\alpha]}_{n_a n_b} = \sum_{n_k}\mathcal{U}^{n_k}_{n_b\,[\alpha]}\,\mathcal{V}^{n_k *}_{n_a\,[\alpha]}. \tag{C33}$$

for what concerns the first contribution, which reads

$$-\int\frac{d\omega'}{2\pi}\frac{d\omega''}{2\pi}\sum_{cdefgh}\bar{V}_{aecf}\,\bar{V}_{\bar{h}\bar{b}g\bar{d}}\,G^{12}_{cd}(\omega')\,G^{11}_{fh}(\omega'')\,G^{11}_{ge}(\omega'+\omega''-\omega)$$

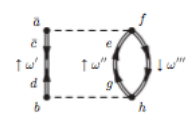
$$-\sum_{cdefgh,k_1k_2k_3}\bar{V}_{aecf}\,\bar{V}_{\bar{h}\bar{b}g\bar{d}}\left\{\frac{\mathcal{U}^{k_1}_c\,\mathcal{V}^{k_1}_d\,\mathcal{U}^{k_2}_f\,\mathcal{U}^{k_2*}_h\,\mathcal{V}^{k_3*}_g\,\mathcal{V}^{k_3}_e}{\omega-(\omega_{k_1}+\omega_{k_2}+\omega_{k_3})+i\eta}+\frac{\mathcal{V}^{k_1*}_c\,\mathcal{U}^{k_1}_d\,\mathcal{V}^{k_2*}_f\,\mathcal{V}^{k_2}_h\,\mathcal{U}^{k_3}_g\,\mathcal{U}^{k_3*}_e}{\omega+(\omega_{k_1}+\omega_{k_2}+\omega_{k_3})-i\eta}\right\} \tag{B26}$$

$\Sigma^{12\,(2'')}_{ab}(\omega) =$  , (B27)

yielding

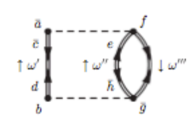
$$\int\frac{d\omega'}{2\pi}\frac{d\omega''}{2\pi}\sum_{cdefgh}\bar{V}_{aecf}\,\bar{V}_{\bar{h}\bar{b}g\bar{d}}\,G^{12}_{cd}(\omega')\,G^{12}_{fg}(\omega'')\,G^{21}_{he}(\omega'+\omega''-\omega)$$

$$\sum_{cdefgh,k_1k_2k_3}\bar{V}_{aecf}\,\bar{V}_{\bar{h}\bar{b}g\bar{d}}\left\{\frac{\mathcal{U}^{k_1}_c\,\mathcal{V}^{k_1*}_d\,\mathcal{U}^{k_2}_f\,\mathcal{V}^{k_2}_g\,\bar{\mathcal{U}}^{k_3*}_h\,\bar{\mathcal{V}}^{k_3}_e}{\omega-(\omega_{k_1}+\omega_{k_2}+\omega_{k_3})+i\eta}+\frac{\bar{\mathcal{V}}^{k_1*}_c\,\bar{\mathcal{U}}^{k_1}_d\,\bar{\mathcal{V}}^{k_2*}_f\,\bar{\mathcal{U}}^{k_2}_g\,\mathcal{V}^{k_3}_h\,\mathcal{U}^{k_3*}_e}{\omega+(\omega_{k_1}+\omega_{k_2}+\omega_{k_3})-i\eta}\right\}. \tag{B28}$$

$\Sigma^{21\,(2')}_{ab}(\omega) =$  , (B29)

$$\int\frac{d\omega'}{2\pi}\frac{d\omega''}{2\pi}\sum_{cdefgh}\bar{V}_{\bar{c}f\bar{a}e}\,\bar{V}_{gdhb}\,G^{21}_{cd}(\omega')\,G^{11}_{eg}(\omega'')\,G^{11}_{hf}(\omega'+\omega''-\omega)$$

$$\sum_{cdefgh,k_1k_2k_3}\bar{V}_{\bar{c}f\bar{a}e}\,\bar{V}_{gdhb}\left\{\frac{\mathcal{V}^{k_1}_c\,\mathcal{U}^{k_1*}_d\,\mathcal{U}^{k_2}_e\,\mathcal{U}^{k_2*}_g\,\mathcal{V}^{k_3*}_h\,\mathcal{V}^{k_3}_f}{\omega-(\omega_{k_1}+\omega_{k_2}+\omega_{k_3})+i\eta}+\frac{\mathcal{U}^{k_1*}_c\,\mathcal{V}^{k_1}_d\,\mathcal{V}^{k_2*}_e\,\mathcal{V}^{k_2}_g\,\mathcal{U}^{k_3}_h\,\mathcal{U}^{k_3*}_f}{\omega+(\omega_{k_1}+\omega_{k_2}+\omega_{k_3})-i\eta}\right\}. \tag{B30}$$

$\Sigma^{21\,(2'')}_{ab}(\omega) =$  , (B31)

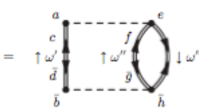
$$\int\frac{d\omega'}{2\pi}\frac{d\omega''}{2\pi}\sum_{cdefgh}\bar{V}_{\bar{c}f\bar{a}e}\,\bar{V}_{g\bar{d}hb}\,G^{21}_{cd}(\omega')\,G^{12}_{eh}(\omega'')\,G^{21}_{gf}(\omega'+\omega''-\omega)$$

$$\sum_{cdefgh,k_1k_2k_3}\bar{V}_{\bar{c}f\bar{a}e}\,\bar{V}_{g\bar{d}hb}\left\{\frac{\mathcal{V}^{k_1}_c\,\mathcal{U}^{k_1*}_d\,\mathcal{U}^{k_2}_e\,\mathcal{V}^{k_2*}_h\,\bar{\mathcal{U}}^{k_3*}_g\,\bar{\mathcal{V}}^{k_3}_f}{\omega-(\omega_{k_1}+\omega_{k_2}+\omega_{k_3})+i\eta}+\frac{\bar{\mathcal{U}}^{k_1*}_c\,\bar{\mathcal{V}}^{k_1}_d\,\bar{\mathcal{V}}^{k_2*}_e\,\bar{\mathcal{U}}^{k_2}_h\,\mathcal{V}^{k_3}_g\,\mathcal{U}^{k_3*}_f}{\omega+(\omega_{k_1}+\omega_{k_2}+\omega_{k_3})-i\eta}\right\}. \tag{B32}$$

contribution, which reads

$$-\int\frac{d\omega'}{2\pi}\frac{d\omega''}{2\pi}\sum_{cdefgh}\bar{V}_{aecf}\,\bar{V}_{\bar{h}\bar{b}g\bar{d}}\,G^{12}_{cd}(\omega')\,G^{11}_{fh}(\omega'')\,G^{11}_{ge}(\omega'+\omega''-\omega)$$

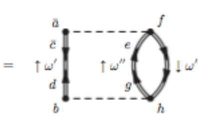
$$-\sum_{cdefgh,k_1k_2k_3}\bar{V}_{aecf}\,\bar{V}_{\bar{h}\bar{b}g\bar{d}}\left\{\frac{\mathcal{U}^{k_1}_c\,\mathcal{V}^{k_1*}_d\,\mathcal{U}^{k_2}_f\,\mathcal{U}^{k_2*}_h\,\mathcal{V}^{k_3*}_g\,\mathcal{V}^{k_3}_e}{\omega-(\omega_{k_1}+\omega_{k_2}+\omega_{k_3})+i\eta}+\frac{\bar{\mathcal{V}}^{k_1*}_c\,\bar{\mathcal{U}}^{k_1}_d\,\bar{\mathcal{V}}^{k_2*}_f\,\bar{\mathcal{V}}^{k_2}_h\,\mathcal{U}^{k_3}_g\,\mathcal{U}^{k_3*}_e}{\omega+(\omega_{k_1}+\omega_{k_2}+\omega_{k_3})-i\eta}\right\} \tag{B26}$$

$\Sigma^{12\,(2'')}_{ab}(\omega) =$  , (B27)

, yielding

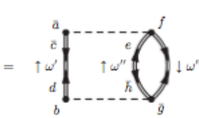
$$\frac{1}{2}\int\frac{d\omega'}{2\pi}\frac{d\omega''}{2\pi}\sum_{cdefgh}\bar{V}_{aecf}\,\bar{V}_{\bar{h}\bar{b}g\bar{d}}\,G^{12}_{cd}(\omega')\,G^{12}_{fg}(\omega'')\,G^{21}_{he}(\omega'+\omega''-\omega)$$

$$\frac{1}{2}\sum_{cdefgh,k_1k_2k_3}\bar{V}_{aecf}\,\bar{V}_{\bar{h}\bar{b}g\bar{d}}\left\{\frac{\mathcal{U}^{k_1}_c\,\mathcal{V}^{k_1*}_d\,\mathcal{U}^{k_2}_f\,\mathcal{V}^{k_2*}_g\,\bar{\mathcal{U}}^{k_3*}_h\,\bar{\mathcal{V}}^{k_3}_e}{\omega-(\omega_{k_1}+\omega_{k_2}+\omega_{k_3})+i\eta}+\frac{\bar{\mathcal{V}}^{k_1*}_c\,\bar{\mathcal{U}}^{k_1}_d\,\bar{\mathcal{V}}^{k_2*}_f\,\bar{\mathcal{U}}^{k_2}_g\,\mathcal{V}^{k_3}_h\,\mathcal{U}^{k_3*}_e}{\omega+(\omega_{k_1}+\omega_{k_2}+\omega_{k_3})-i\eta}\right\}. \tag{B28}$$

$\Sigma^{21\,(2')}_{ab}(\omega) =$  , (B29)

$$-\int\frac{d\omega'}{2\pi}\frac{d\omega''}{2\pi}\sum_{cdefgh}\bar{V}_{\bar{c}f\bar{a}e}\,\bar{V}_{gdhb}\,G^{21}_{cd}(\omega')\,G^{11}_{eg}(\omega'')\,G^{11}_{hf}(\omega'+\omega''-\omega)$$

$$-\sum_{cdefgh,k_1k_2k_3}\bar{V}_{\bar{c}f\bar{a}e}\,\bar{V}_{gdhb}\left\{\frac{\mathcal{V}^{k_1}_c\,\mathcal{U}^{k_1*}_d\,\mathcal{U}^{k_2}_e\,\mathcal{U}^{k_2*}_g\,\mathcal{V}^{k_3*}_h\,\mathcal{V}^{k_3}_f}{\omega-(\omega_{k_1}+\omega_{k_2}+\omega_{k_3})+i\eta}+\frac{\bar{\mathcal{U}}^{k_1*}_c\,\bar{\mathcal{V}}^{k_1}_d\,\bar{\mathcal{V}}^{k_2*}_e\,\bar{\mathcal{V}}^{k_2}_g\,\mathcal{U}^{k_3}_h\,\mathcal{U}^{k_3*}_f}{\omega+(\omega_{k_1}+\omega_{k_2}+\omega_{k_3})-i\eta}\right\}, \tag{B30}$$

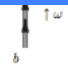
$\Sigma^{21\,(2'')}_{ab}(\omega) =$  , (B31)

$$\frac{1}{2}\int\frac{d\omega'}{2\pi}\frac{d\omega''}{2\pi}\sum_{cdefgh}\bar{V}_{\bar{c}f\bar{a}e}\,\bar{V}_{g\bar{d}hb}\,G^{21}_{cd}(\omega')\,G^{12}_{eh}(\omega'')\,G^{21}_{gf}(\omega'+\omega''-\omega)$$

$$-\frac{1}{2}\sum_{cdefgh,k_1k_2k_3}\bar{V}_{\bar{c}f\bar{a}e}\,\bar{V}_{g\bar{d}hb}\left\{\frac{\mathcal{V}^{k_1}_c\,\mathcal{U}^{k_1*}_d\,\mathcal{U}^{k_2}_e\,\mathcal{V}^{k_2*}_h\,\bar{\mathcal{U}}^{k_3*}_g\,\bar{\mathcal{V}}^{k_3}_f}{\omega-(\omega_{k_1}+\omega_{k_2}+\omega_{k_3})+i\eta}+\frac{\bar{\mathcal{U}}^{k_1*}_c\,\bar{\mathcal{V}}^{k_1}_d\,\bar{\mathcal{V}}^{k_2*}_e\,\bar{\mathcal{U}}^{k_2}_h\,\mathcal{V}^{k_3}_g\,\mathcal{U}^{k_3*}_f}{\omega+(\omega_{k_1}+\omega_{k_2}+\omega_{k_3})-i\eta}\right\}. \tag{B32}$$

Espressions for 1st & 2nd order diagrams

[*V. Somà*, T. Duguet, CB, Pys. Rev. C84, 046317 (2011)]

$G^{11}_{ab}(\omega) \equiv$  $\uparrow \omega$, (B5a)

$G^{12}_{ab}(\omega) \equiv$  $\uparrow \omega$, (B5b)

5. Block-diagonal structu

a. First or

The goal of this subsection is to discuss how the block-diagona reflects in the various self-energy contributions, starting with the fir and (C19) into Eq. (B7), and introducing the factor

$$f^{n_a n_b n_c n_d}_{\alpha\beta\gamma\delta} \equiv \sqrt{1+\delta_{\alpha\beta}\,\delta_{n_a n}}$$

one obtains

$$\Sigma^{11\,(1)}_{ab} = \sum_{cd,k} \bar{V}_{acbd}\, \mathcal{V}^{k*}_d\, \mathcal{V}^k_c$$

$$= \ldots \equiv \delta_{\alpha\beta}\,\delta_{m_a m_b}\,\Sigma^{11\,[\alpha]\,(1)}_{n_a n_b} \equiv \delta_{\alpha\beta}\,\delta_{m_a m_b}\,\Lambda^{[\alpha]}_{n_a n_b},$$

where the block-diagonal normal density matrix is introduced throu ... and properties of Clebsch-Gordan coefficients has been used. The ... leading to $\delta_{\alpha\beta} = \delta_{j_a j_b}\,\delta_{\pi_a\pi_b}\,\delta_{q_a q_b}$. Similarly, for $\Sigma^{22\,(1)}$

$$\Sigma^{22\,(1)}_{ab} = -\sum_{cd,k} \bar{V}_{bcad}\, \mathcal{V}^k_c\, \mathcal{V}^{k*}_d = \ldots \equiv \delta_{\alpha\beta}\,\delta_{m_a m_b}\,\Sigma^{22\,[\alpha]\,(1)}_{n_a n_b} = -\delta_{\alpha\beta}\,\delta_{m_a m_b}\,\Lambda^{[\alpha]}_{n_a n_b} = -\delta_{\alpha\beta}\,\delta_{m_a m_b}\,[\Lambda^{[\alpha]}_{n_a n_b}]^*.$$

Let us consider the anomalous contributions to the first-order self-en derives

$$\Sigma^{12\,(1)}_{ab} = \frac{1}{2}\sum_{cd,k} \bar{V}_{abcd}\, \mathcal{V}^{k*}_c\, \mathcal{U}^k_d = \ldots \equiv \delta_{\alpha\beta}\,\delta_{m_a m_b}\,\Sigma^{12\,[\alpha]\,(1)}_{n_a n_b} \equiv \delta_{\alpha\beta}\,\delta_{m_a m_b}\,\tilde{h}^{[\alpha]}_{n_a n_b},$$

where the block-diagonal anomalous density matrix is introduced th

$$\tilde{\rho}^{[\alpha]}_{n_a n_b} = \sum_{n_k} \mathcal{U}^{n_k}_{n_b\,[\alpha]}\, \mathcal{V}^{n_k}_{n_a\,[\alpha]}. \quad \text{(C33)}$$

$$-i\int_{C\uparrow} \frac{d\omega'}{2\pi} \sum_{cd,k} \bar{V}_{acbd}\, \frac{\mathcal{V}^{k*}_d\, \mathcal{V}^k_c}{\omega' + \omega_k - i\eta}$$

It is interesting to note that the first-order anomalous self-energies involve only $J = 0$ matrix elements as a direct result of dealing with a $J = 0$ many-body state. The other anomalous term is similarly obtained from Eq. (B13) and reads

$$\Sigma^{21\,(1)}_{ab} = \frac{1}{2}\sum_{cd,k} \bar{V}_{\bar{c}\bar{d}\bar{a}b}\, \bar{\mathcal{U}}^{k*}_c\, \bar{\mathcal{V}}^k_d$$

$$= -\frac{1}{2}\sum_{n_c n_d n_k}\sum_{\gamma}\sum_{m_c}\sum_{JM} f^{n_a n_b n_c n_d}_{\alpha\beta\gamma\gamma}\,\eta_a\eta_c\, C^{JM}_{j_c -m_c\, j_c m_c}\, C^{JM}_{j_a -m_a\, j_b m_b}\, \bar{V}^{J\,[\gamma\gamma\alpha\beta]}_{n_c n_d n_a n_b}\, \mathcal{U}^{n_k}_{n_c\,[\gamma]}\, \mathcal{V}^{n_k *}_{n_d\,[\gamma]}$$

$$= \delta_{\alpha\beta}\,\delta_{m_a m_b}\,\frac{1}{2}\sum_{n_c n_d n_k}\sum_{\gamma} f^{n_a n_b n_c n_d}_{\alpha\alpha\gamma\gamma}\,\pi_a\,\pi_c(-1)^{2j_c}\frac{\sqrt{2j_c+1}}{\sqrt{2j_a+1}}\, \bar{V}^{0\,[\gamma\gamma\alpha\alpha]}_{n_c n_d n_a n_b}\, \tilde{\rho}^{[\gamma]}_{n_d n_c}$$

$$\equiv \delta_{\alpha\beta}\,\delta_{m_a m_b}\,\Sigma^{21\,[\alpha]\,(1)}_{n_a n_b} \equiv \delta_{\alpha\beta}\,\delta_{m_a m_b}\,\tilde{h}^{[\alpha]\,\dagger}_{n_a n_b}. \quad \text{(C34)}$$

b. Second order

Block-diagonal forms of second-order self-energy contributions (77) and (79) can be obtained by considering explicitly the angular momentum couplings of the three quasiparticles to their total momentum $J_{\rm tot}$, separately for the six objects $\mathcal{M}$, $\mathcal{N}$, $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{R}$, and $\mathcal{S}$. One proceeds first coupling particles 1 and 2 to some momentum J_c, which is afterward coupled to particle 3 to give $J_{\rm tot}$. The recoupled $\mathcal{M}$ term is computed as follows:

$$\mathcal{M}^{k_1k_2k_3}_{\alpha\,(J_c J_{\rm tot})} = \sum_{m_1 m_2 m_3 M_c} C^{J_c M_c}_{j_{k_1} m_{k_1}\, j_{k_2} m_{k_2}}\, C^{J_{\rm tot} M_{\rm tot}}_{J_c M_c\, j_{k_3} m_{k_3}}\, \mathcal{M}^{k_1k_2k_3}_{a}$$

$$= \sum_{m_1 m_2 m_3 M_c}\sum_{rst} C^{J_c M_c}_{j_{k_1} m_{k_1}\, j_{k_2} m_{k_2}}\, C^{J_{\rm tot} M_{\rm tot}}_{J_c M_c\, j_{k_3} m_{k_3}}\, \bar{V}_{atrs}\, \mathcal{U}^{k_1}_r\, \mathcal{U}^{k_2}_s\, \bar{\mathcal{V}}^{k_3}_t$$

$$= \sum_{m_1 m_2 m_3 M_c}\sum_{rst}\sum_{J_r M_r} \delta_{\kappa_1\rho}\,\delta_{m_{k_1} m_r}\,\delta_{\kappa_2\sigma}\,\delta_{m_{k_2} m_s}\,\delta_{\kappa_3\tau}\,\delta_{m_{k_3},-m_t}\,(-\eta_t)\, f^{n_a n_t n_r n_s}_{\alpha\tau\rho\sigma}$$

$$\times\, C^{J_c M_c}_{j_{k_1} m_{k_1}\, j_{k_2} m_{k_2}}\, C^{J_{\rm tot} M_{\rm tot}}_{J_c M_c\, j_{k_3} m_{k_3}}\, C^{J_r M_r}_{j_a m_a\, j_t m_t}\, C^{J_r M_r}_{j_r m_r\, j_s m_s}\, \bar{V}^{J_r\,[\alpha\tau\rho\sigma]}_{n_a n_t n_r n_s}\, \mathcal{U}^{n_{k_1}}_{n_r\,[\rho]}\, \mathcal{U}^{n_{k_2}}_{n_s\,[\sigma]}\, \mathcal{V}^{n_{k_3}}_{n_t\,[\tau]}$$

$$= \sum_{m_1 m_2 m_3 M_c}\sum_{n_r n_s n_t}\sum_{J_r M_r} \eta_{k_3}\, f^{n_a n_t n_r n_s}_{\alpha\kappa_3\kappa_1\kappa_2}\, C^{J_c M_c}_{j_{k_1} m_{k_1}\, j_{k_2} m_{k_2}}\, C^{J_{\rm tot} M_{\rm tot}}_{J_c M_c\, j_{k_3} m_{k_3}}\, C^{J_r M_r}_{j_a m_a\, j_{k_3} -m_{k_3}}\, C^{J_r M_r}_{j_{k_1} m_{k_1}\, j_{k_2} m_{k_2}}\, \bar{V}^{J_r\,[\alpha\kappa_3\kappa_1\kappa_2]}_{n_a n_t n_r n_s}\, \mathcal{U}^{n_{k_1}}_{n_r\,[\kappa_1]}\, \mathcal{U}^{n_{k_2}}_{n_s\,[\kappa_2]}\, \mathcal{V}^{n_{k_3}}_{n_t\,[\kappa_3]}$$

$$= \sum_{m_3 M_c}\sum_{n_r n_s n_t} \eta_{k_3}\, f^{n_a n_t n_r n_s}_{\alpha\kappa_3\kappa_1\kappa_2}\, \frac{\sqrt{2J_c+1}}{\sqrt{2j_a+1}}\,(-1)^{J_c+2j_{k_3}-j_a}\, C^{J_{\rm tot} M_{\rm tot}}_{J_c M_c\, j_{k_3} m_{k_3}}\, C^{j_a m_a}_{J_c M_c\, j_{k_3} m_{k_3}}\, \bar{V}^{J_c\,[\alpha\kappa_3\kappa_1\kappa_2]}_{n_a n_t n_r n_s}\, \mathcal{U}^{n_{k_1}}_{n_r\,[\kappa_1]}\, \mathcal{U}^{n_{k_2}}_{n_s\,[\kappa_2]}\, \mathcal{V}^{n_{k_3}}_{n_t\,[\kappa_3]}$$

$$= -\delta_{J_{\rm tot} j_a}\,\delta_{M_{\rm tot} m_a} \sum_{n_r n_s n_t} \pi_{k_3}\, f^{n_a n_t n_r n_s}_{\alpha\kappa_3\kappa_1\kappa_2}\, \frac{\sqrt{2J_c+1}}{\sqrt{2j_a+1}}\,(-1)^{J_c+j_{k_3}-j_a}\, \bar{V}^{J_c\,[\alpha\kappa_3\kappa_1\kappa_2]}_{n_a n_t n_r n_s}\, \mathcal{U}^{n_{k_1}}_{n_r\,[\kappa_1]}\, \mathcal{U}^{n_{k_2}}_{n_s\,[\kappa_2]}\, \mathcal{V}^{n_{k_3}}_{n_t\,[\kappa_3]}$$

$$\equiv \delta_{J_{\rm tot} j_a}\,\delta_{M_{\rm tot} m_a}\, \mathcal{M}^{n_{k_1} n_{k_2} n_{k_3}}_{n_a\,[\alpha\kappa_3\kappa_1\kappa_2]\,J_c}, \quad \text{(C35)}$$

where general properties of Clebsch-Gordan coefficients have been used. Similarly, one derives the $\mathcal{N}$ term

$$\mathcal{N}^{k_1k_2k_3}_{\alpha\,(J_c J_{\rm tot})} = \delta_{J_{\rm tot} j_a}\,\delta_{M_{\rm tot} m_a} \sum_{n_r n_s n_t} \pi_{k_3}\, f^{n_a n_t n_r n_s}_{\alpha\kappa_3\kappa_1\kappa_2}\, \frac{\sqrt{2J_c+1}}{\sqrt{2j_a+1}}\,(-1)^{J_c+j_{k_3}-j_a}\, \bar{V}^{J_c\,[\alpha\kappa_3\kappa_1\kappa_2]}_{n_a n_t n_r n_s}\, \mathcal{V}^{n_{k_1}}_{n_r\,[\kappa_1]}\, \mathcal{V}^{n_{k_2}}_{n_s\,[\kappa_2]}\, \mathcal{U}^{n_{k_3}}_{n_t\,[\kappa_3]}$$

$$\equiv \delta_{J_{\rm tot} j_a}\,\delta_{M_{\rm tot} m_a}\, \mathcal{N}^{n_{k_1} n_{k_2} n_{k_3}}_{n_a\,[\alpha\kappa_3\kappa_1\kappa_2]\,J_c}. \quad \text{(C36)}$$

One can show that the same result is obtained by recoupling directly $\bar{\mathcal{N}}$ as follows:

$$\bar{\mathcal{N}}^{k_1k_2k_3}_{\alpha\,(J_c J_{\rm tot})} = \sum_{m_1 m_2 m_3 M_c} C^{J_c M_c}_{j_{k_1} m_{k_1}\, j_{k_2} m_{k_2}}\, C^{J_{\rm tot} M_{\rm tot}}_{J_c M_c\, j_{k_3} m_{k_3}}\, \bar{\mathcal{N}}^{k_1k_2k_3}_{a}$$

$$= \sum_{m_1 m_2 m_3 M_c}\sum_{rst} C^{J_c M_c}_{j_{k_1} m_{k_1}\, j_{k_2} m_{k_2}}\, C^{J_{\rm tot} M_{\rm tot}}_{J_c M_c\, j_{k_3} m_{k_3}}\, \bar{V}_{\bar{a}\bar{t}\bar{r}\bar{s}}\, \bar{\mathcal{V}}^{k_1}_r\, \bar{\mathcal{V}}^{k_2}_s\, \mathcal{U}^{k_3}_t$$

$$= \sum_{m_1 m_2 m_3 M_c}\sum_{rst}\sum_{J_r M_r} \delta_{\kappa_1\rho}\,\delta_{m_{k_1},-m_r}\,\delta_{\kappa_2\sigma}\,\delta_{m_{k_2},-m_s}\,\delta_{\kappa_3\tau}\,\delta_{m_{k_3} m_t}\,\eta_a\eta_t\, f^{n_a n_t n_r n_s}_{\alpha\tau\rho\sigma}$$

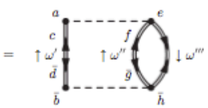
$$\times\, C^{J_c M_c}_{j_{k_1} m_{k_1}\, j_{k_2} m_{k_2}}\, C^{J_{\rm tot} M_{\rm tot}}_{J_c M_c\, j_{k_3} m_{k_3}}\, C^{J_r M_r}_{j_a -m_a\, j_t -m_t}\, C^{J_r M_r}_{j_r -m_r\, j_s -m_s}\, \bar{V}^{J_r\,[\alpha\tau\rho\sigma]}_{n_a n_t n_r n_s}\, \mathcal{V}^{n_{k_1}}_{n_r\,[\rho]}\, \mathcal{V}^{n_{k_2}}_{n_s\,[\sigma]}\, \mathcal{U}^{n_{k_3}}_{n_t\,[\tau]}$$

contribution, which reads

$$-\int \frac{d\omega'}{2\pi}\frac{d\omega''}{2\pi}\sum_{cdefgh} \bar{V}_{aecf}\, \bar{V}_{\bar{h}\bar{b}\bar{g}\bar{d}}\, G^{12}_{cd}(\omega')\, G^{11}_{fh}(\omega'')\, G^{11}_{ge}(\omega'+\omega''-\omega)$$

$$-\sum_{cdefgh,k_1k_2k_3} \bar{V}_{aecf}\, \bar{V}_{\bar{h}\bar{b}\bar{g}\bar{d}} \left\{ \frac{\mathcal{U}^{k_1}_c\, \mathcal{V}^{k_1*}_d\, \mathcal{U}^{k_2}_f\, \mathcal{U}^{k_2*}_h\, \mathcal{V}^{k_3*}_g\, \mathcal{V}^{k_3}_e}{\omega-(\omega_{k_1}+\omega_{k_2}+\omega_{k_3})+i\eta} + \frac{\bar{\mathcal{V}}^{k_1*}_c\, \bar{\mathcal{U}}^{k_1}_d\, \bar{\mathcal{V}}^{k_2*}_f\, \bar{\mathcal{V}}^{k_2}_h\, \mathcal{U}^{k_3}_g\, \mathcal{U}^{k_3*}_e}{\omega+(\omega_{k_3}+\omega_{k_1}+\omega_{k_2})-i\eta} \right\} \quad \text{(B26)}$$

$$\Sigma^{12\,(2'')}_{ab}(\omega) = \quad \text{(B27)}$$

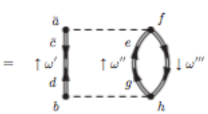


, yielding

$$\frac{1}{2}\int \frac{d\omega'}{2\pi}\frac{d\omega''}{2\pi}\sum_{cdefgh} \bar{V}_{aecf}\, \bar{V}_{\bar{h}\bar{b}\bar{g}\bar{d}}\, G^{12}_{cd}(\omega')\, G^{12}_{fg}(\omega'')\, G^{21}_{he}(\omega'+\omega''-\omega)$$

$$\frac{1}{2}\sum_{cdefgh,k_1k_2k_3} \bar{V}_{aecf}\, \bar{V}_{\bar{h}\bar{b}\bar{g}\bar{d}} \left\{ \frac{\mathcal{U}^{k_1}_c\, \mathcal{V}^{k_1*}_d\, \mathcal{U}^{k_2}_f\, \mathcal{V}^{k_2*}_g\, \bar{\mathcal{U}}^{k_3*}_h\, \bar{\mathcal{V}}^{k_3}_e}{\omega-(\omega_{k_1}+\omega_{k_2}+\omega_{k_3})+i\eta} + \frac{\bar{\mathcal{V}}^{k_1*}_c\, \bar{\mathcal{U}}^{k_1}_d\, \bar{\mathcal{V}}^{k_2*}_f\, \bar{\mathcal{U}}^{k_2}_g\, \mathcal{V}^{k_3}_h\, \mathcal{U}^{k_3*}_e}{\omega+(\omega_{k_3}+\omega_{k_1}+\omega_{k_2})-i\eta} \right\}. \quad \text{(B28)}$$

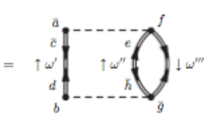
$$\Sigma^{21\,(2')}_{ab}(\omega) = \quad \text{(B29)}$$



$$-\int \frac{d\omega'}{2\pi}\frac{d\omega''}{2\pi}\sum_{cdefgh} \bar{V}_{\bar{c}f\bar{a}e}\, \bar{V}_{gdhb}\, G^{21}_{cd}(\omega')\, G^{11}_{eg}(\omega'')\, G^{11}_{hf}(\omega'+\omega''-\omega)$$

$$-\sum_{cdefgh,k_1k_2k_3} \bar{V}_{\bar{c}f\bar{a}e}\, \bar{V}_{gdhb} \left\{ \frac{\mathcal{V}^{k_1}_c\, \mathcal{U}^{k_1*}_d\, \mathcal{U}^{k_2}_e\, \mathcal{U}^{k_2*}_g\, \mathcal{V}^{k_3*}_h\, \mathcal{V}^{k_3}_f}{\omega-(\omega_{k_1}+\omega_{k_2}+\omega_{k_3})+i\eta} + \frac{\bar{\mathcal{U}}^{k_1*}_c\, \bar{\mathcal{V}}^{k_1}_d\, \bar{\mathcal{V}}^{k_2*}_e\, \bar{\mathcal{V}}^{k_2}_g\, \mathcal{U}^{k_3}_h\, \mathcal{U}^{k_3*}_f}{\omega+(\omega_{k_3}+\omega_{k_1}+\omega_{k_2})-i\eta} \right\}, \quad \text{(B30)}$$

$$\Sigma^{21\,(2'')}_{ab}(\omega) = \quad \text{(B31)}$$



$$\frac{1}{2}\int \frac{d\omega'}{2\pi}\frac{d\omega''}{2\pi}\sum_{cdefgh} \bar{V}_{\bar{c}f\bar{a}e}\, \bar{V}_{\bar{g}d\bar{h}b}\, G^{21}_{cd}(\omega')\, G^{12}_{eh}(\omega'')\, G^{21}_{gf}(\omega'+\omega''-\omega)$$

$$-\frac{1}{2}\sum_{cdefgh,k_1k_2k_3} \bar{V}_{\bar{c}f\bar{a}e}\, \bar{V}_{\bar{g}d\bar{h}b} \left\{ \frac{\mathcal{V}^{k_1}_c\, \mathcal{U}^{k_1*}_d\, \mathcal{U}^{k_2}_e\, \mathcal{V}^{k_2*}_h\, \bar{\mathcal{U}}^{k_3*}_g\, \bar{\mathcal{V}}^{k_3}_f}{\omega-(\omega_{k_1}+\omega_{k_2}+\omega_{k_3})+i\eta} + \frac{\bar{\mathcal{U}}^{k_1*}_c\, \bar{\mathcal{V}}^{k_1}_d\, \bar{\mathcal{V}}^{k_2*}_e\, \bar{\mathcal{U}}^{k_2}_h\, \mathcal{V}^{k_3}_g\, \mathcal{U}^{k_3*}_f}{\omega+(\omega_{k_3}+\omega_{k_1}+\omega_{k_2})-i\eta} \right\}. \quad \text{(B32)}$$

Espressions for 1st & 2nd order diagrams

[V. Somà, T. Duguet, CB, Pys. Rev. C84, 046317 (2011)]

which recovers relation (72a). The remaining quantities [see Eqs. (69) and (70)] are related to $\mathcal{M}$ and $\mathcal{N}$ by permutations of $\{k_1, k_2, k_3\}$ indices and can be obtained from Eqs. (C35) and (C36) by taking into account the different recoupling of j_{k_1}, j_{k_2} and j_{k_3} to $J_{\rm tot}$ and J_c as follows:

$$\mathcal{P}^{k_1k_2k_3}_{a(J_c J_{\rm tot})} = \sum_{J_d} (-1)^{J_c+J_d+j_{k_2}+j_{k_3}} \sqrt{2J_c+1}\sqrt{2J_d+1} \begin{Bmatrix} j_{k_2} & j_{k_1} & J_c \\ j_{k_3} & J_{\rm tot} & J_d \end{Bmatrix} \mathcal{M}^{k_1k_3k_2}_{a(J_d J_{\rm tot})} \quad \text{(C38)}$$

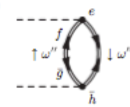
$$\mathcal{Q}^{k_1k_2k_3}_{a(J_c J_{\rm tot})} = \sum_{J_d} (-1)^{J_c+J_d+j_{k_2}+j_{k_3}} \sqrt{2J_c+1}\sqrt{2J_d+1} \begin{Bmatrix} j_{k_2} & j_{k_1} & J_c \\ j_{k_3} & J_{\rm tot} & J_d \end{Bmatrix} \mathcal{N}^{k_1k_3k_2}_{a(J_d J_{\rm tot})} \quad \text{(C39)}$$

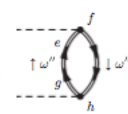
$$\mathcal{R}^{k_1k_2k_3}_{a(J_c J_{\rm tot})} = \sum_{J_d} (-1)^{2j_{k_1}+2J_d} \sqrt{2J_c+1}\sqrt{2J_d+1} \begin{Bmatrix} j_{k_1} & j_{k_2} & J_c \\ j_{k_3} & J_{\rm tot} & J_d \end{Bmatrix} \mathcal{M}^{k_3k_2k_1}_{a(J_d J_{\rm tot})} \quad \text{(C40)}$$

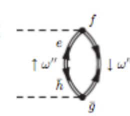
$$\mathcal{S}^{k_1k_2k_3}_{a(J_c J_{\rm tot})} = \sum_{J_d} (-1)^{2j_{k_1}+2J_d} \sqrt{2J_c+1}\sqrt{2J_d+1} \begin{Bmatrix} j_{k_1} & j_{k_2} & J_c \\ j_{k_3} & J_{\rm tot} & J_d \end{Bmatrix} \mathcal{N}^{k_3k_2k_1}_{a(J_d J_{\rm tot})} \quad \text{(C41)}$$

These terms are finally put together to form the different contributions to second-order self-energies. Let us consider $\Sigma^{11(2')}_{ab}$ as an example [see Eq. (75)]. By inserting Eqs. (C35) and (C36) and summing over all possible total and intermediate angular momenta, one has

$$\Sigma^{11(2')}_{ab} = \frac{1}{2} \sum_{J_{\rm tot}M_{\rm tot}J_c} \sum_{k_1k_2k_3} \left\{ \frac{\mathcal{M}^{k_1k_2k_3}_{a(J_cJ_{\rm tot})} \left(\mathcal{M}^{k_1k_2k_3}_{b(J_cJ_{\rm tot})}\right)^*}{\omega - (\omega_{k_1}+\omega_{k_2}+\omega_{k_3}) + i\eta} + \frac{\mathcal{N}^{k_1k_2k_3}_{a(J_cJ_{\rm tot})} \left(\mathcal{N}^{k_1k_2k_3}_{b(J_cJ_{\rm tot})}\right)^*}{\omega + (\omega_{k_1}+\omega_{k_2}+\omega_{k_3}) - i\eta} \right\}$$

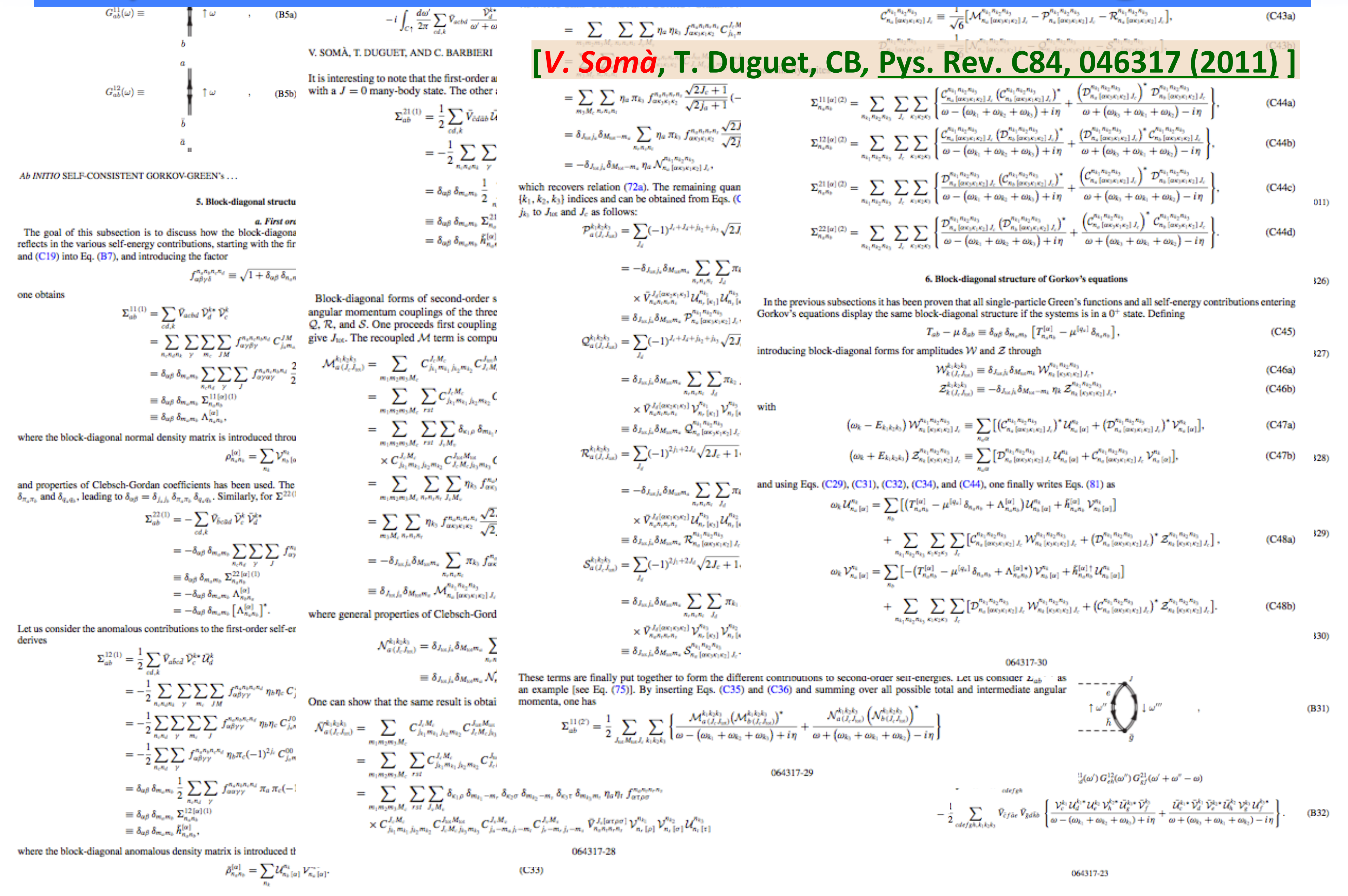

(B27)


(B29)


(B31)

Espressions for 1st & 2nd order diagrams

[V. Somà, T. Duguet, CB, Pys. Rev. C84, 046317 (2011)]



Gorkov equations

[V. Somà, T. Duguet, CB, Pys. Rev. C**84**, 046317 (2011)]

$$\sum_b \begin{pmatrix} t_{ab} - \mu_{ab} + \Sigma^{11}_{ab}(\omega) & \Sigma^{12}_{ab}(\omega) \\ \Sigma^{21}_{ab}(\omega) & -t_{ab} + \mu_{ab} + \Sigma^{22}_{ab}(\omega) \end{pmatrix}\Bigg|_{\omega_k} \begin{pmatrix} \mathcal{U}^k_b \\ \mathcal{V}^k_b \end{pmatrix} = \omega_k \begin{pmatrix} \mathcal{U}^k_a \\ \mathcal{V}^k_a \end{pmatrix}$$

$$\Downarrow$$

$$\boxed{\begin{pmatrix} T-\mu+\Lambda & \tilde{h} & \mathcal{C} & -\mathcal{D}^\dagger \\ \tilde{h}^\dagger & -T+\mu-\Lambda & -\mathcal{D}^\dagger & \mathcal{C} \\ \mathcal{C}^\dagger & -\mathcal{D} & E & 0 \\ -\mathcal{D} & \mathcal{C}^\dagger & 0 & -E \end{pmatrix} \begin{pmatrix} \mathcal{U}^k \\ \mathcal{V}^k \\ \mathcal{W}_k \\ \mathcal{Z}_k \end{pmatrix} = \omega_k \begin{pmatrix} \mathcal{U}^k \\ \mathcal{V}^k \\ \mathcal{W}_k \\ \mathcal{Z}_k \end{pmatrix}}$$

Energy *independent* eigenvalue problem

with the normalization condition $\sum_a \left[\left|\mathcal{U}^k_a\right|^2 + \left|\mathcal{V}^k_a\right|^2 \right] + \sum_{k_1 k_2 k_3} \left[\left|\mathcal{W}^{k_1 k_2 k_3}_k\right|^2 + \left|\mathcal{Z}^{k_1 k_2 k_3}_k\right|^2 \right] = 1$

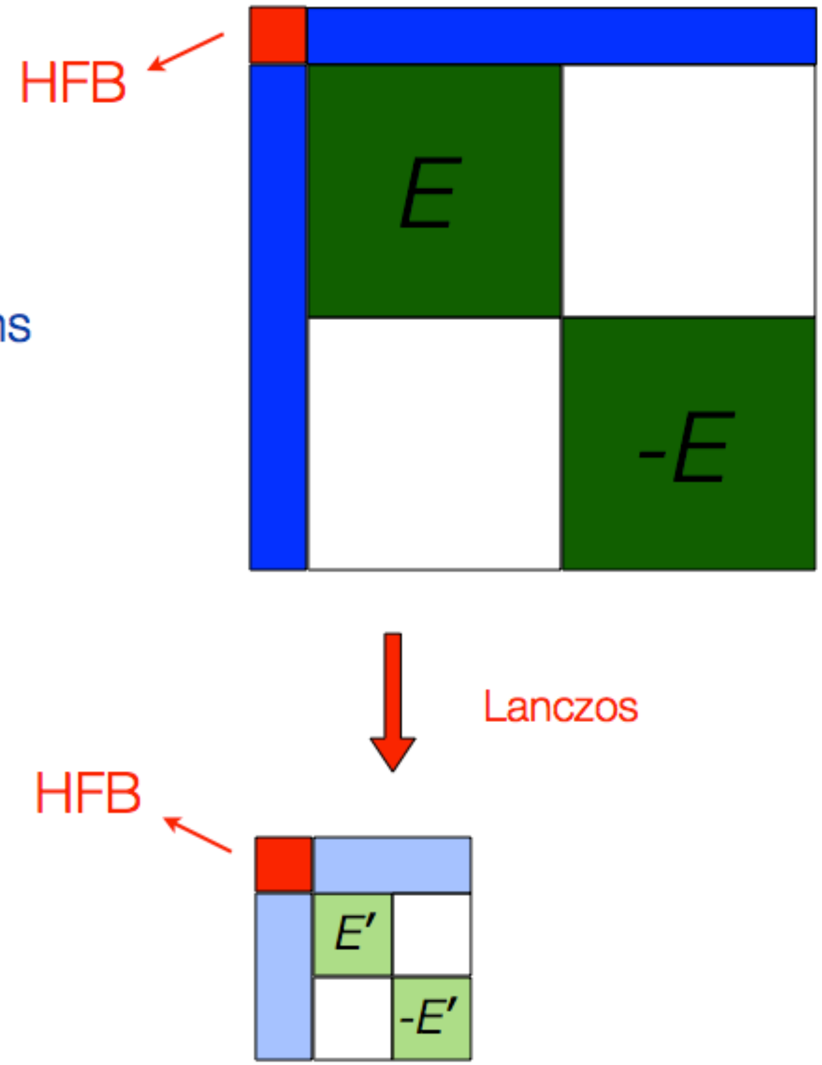
Lanczos reduction of self-energy

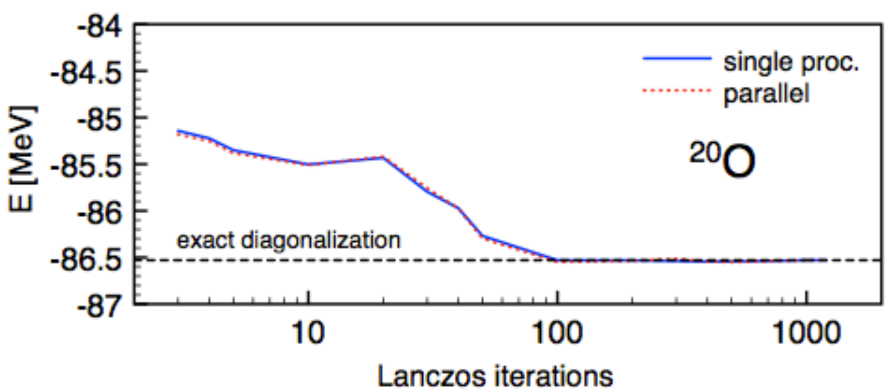
V. Somà, CB, T. Duguet, , Phys. Rev. C **89**, 024323 (2014)

$$\begin{pmatrix} T-\mu+\Lambda & \tilde{h} & \mathcal{C} & -\mathcal{D}^\dagger \\ \tilde{h}^\dagger & -T+\mu-\Lambda & -\mathcal{D}^\dagger & \mathcal{C} \\ \mathcal{C}^\dagger & -\mathcal{D} & E & 0 \\ -\mathcal{D} & \mathcal{C}^\dagger & 0 & -E \end{pmatrix} \begin{pmatrix} \mathcal{U}^k \\ \mathcal{V}^k \\ \mathcal{W}_k \\ \mathcal{Z}_k \end{pmatrix} = \omega_k \begin{pmatrix} \mathcal{U}^k \\ \mathcal{V}^k \\ \mathcal{W}_k \\ \mathcal{Z}_k \end{pmatrix}$$

➡ Conserves moments of spectral functions

➡ Equivalent to exact diagonalization for $N_L \rightarrow \dim(E)$





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Lanczos reduction of self-energy

V. Somà, CB, T. Duguet, , Phys. Rev. C **89**, 024323 (2014)

$$\begin{pmatrix} T-\mu+\Lambda & \tilde{h} & \mathcal{C} & -\mathcal{D}^\dagger \\ \tilde{h}^\dagger & -T+\mu-\Lambda & -\mathcal{D}^\dagger & \mathcal{C} \\ \mathcal{C}^\dagger & -\mathcal{D} & E & 0 \\ -\mathcal{D} & \mathcal{C}^\dagger & 0 & -E \end{pmatrix} \begin{pmatrix} \mathcal{U}^k \\ \mathcal{V}^k \\ \mathcal{W}_k \\ \mathcal{Z}_k \end{pmatrix} = \omega_k \begin{pmatrix} \mathcal{U}^k \\ \mathcal{V}^k \\ \mathcal{W}_k \\ \mathcal{Z}_k \end{pmatrix}$$

➟ Conserves moments of spectral functions

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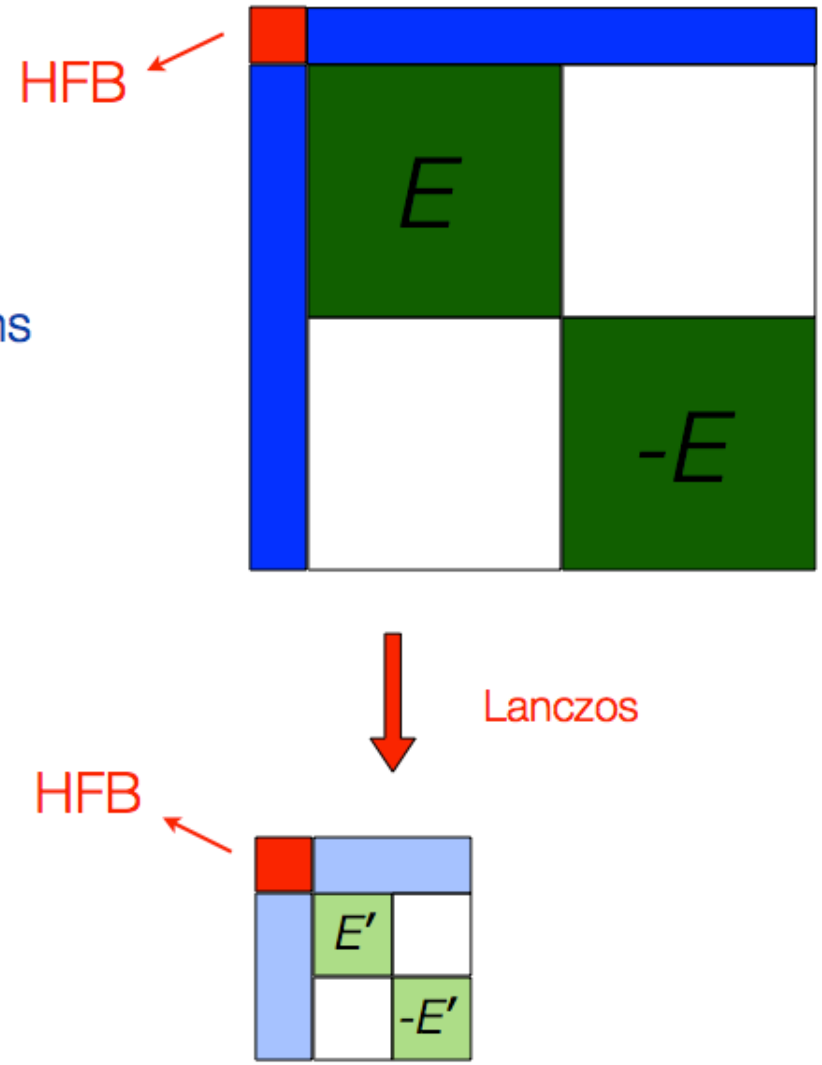
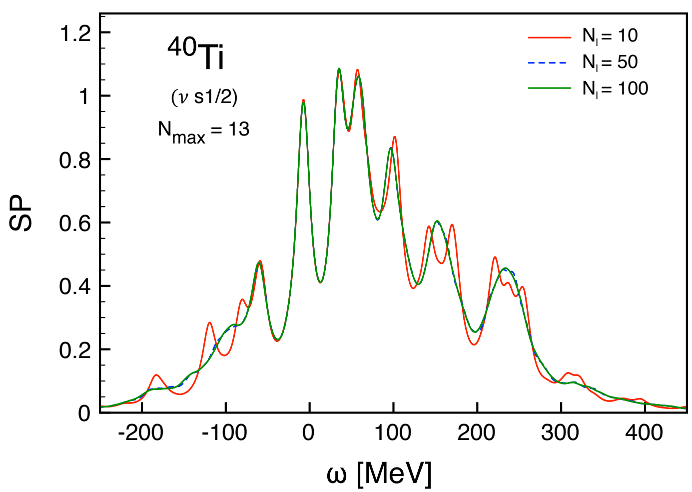


Spectral strength

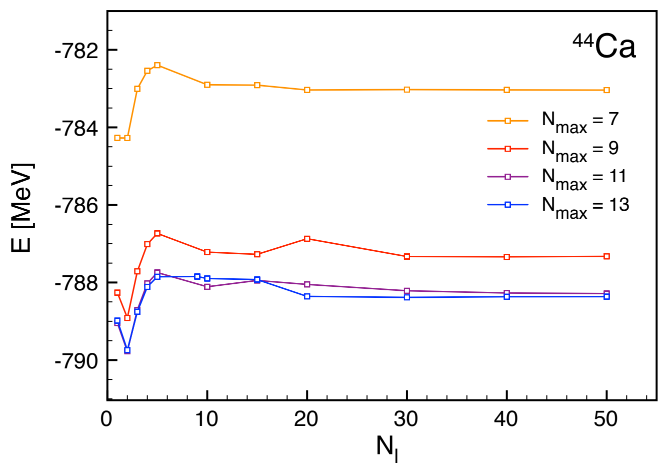


Testing Krylov projection

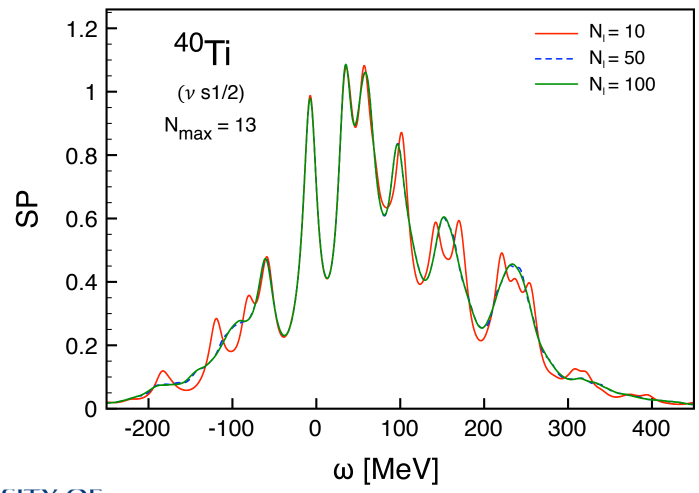
V. Somà, CB, T. Duguet, , Phys. Rev. C **89**, 024323 (2014)

- Energy and spectra independent of the projection
- Same behavior for all model spaces

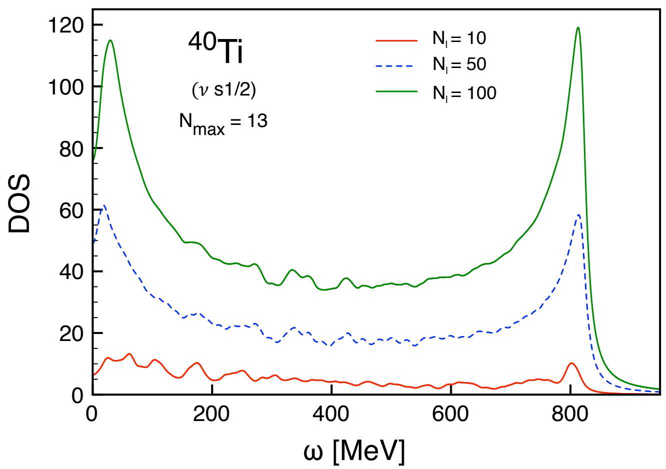
Favourable scaling



Spectral strength



Deinsity of states



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Approaches in GF theory

Truncation scheme:	Dyson formulation (closed shells)	Gorkov formulation (semi-/doubly-magic)
1st order:	Hartree-Fock	HF-Bogolioubov
2nd order:	2nd order	2nd order (w/ pairing)
. . .	. . .	
3rd and all-orders sums, P-V coupling:	ADC(3) FRPA etc…	G-ADC(3) …work in progress

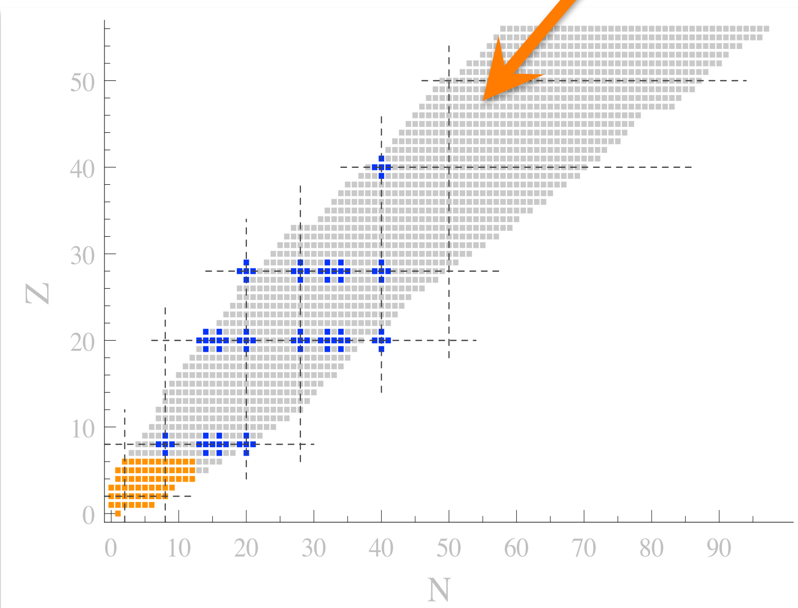
Approaches in GF theory

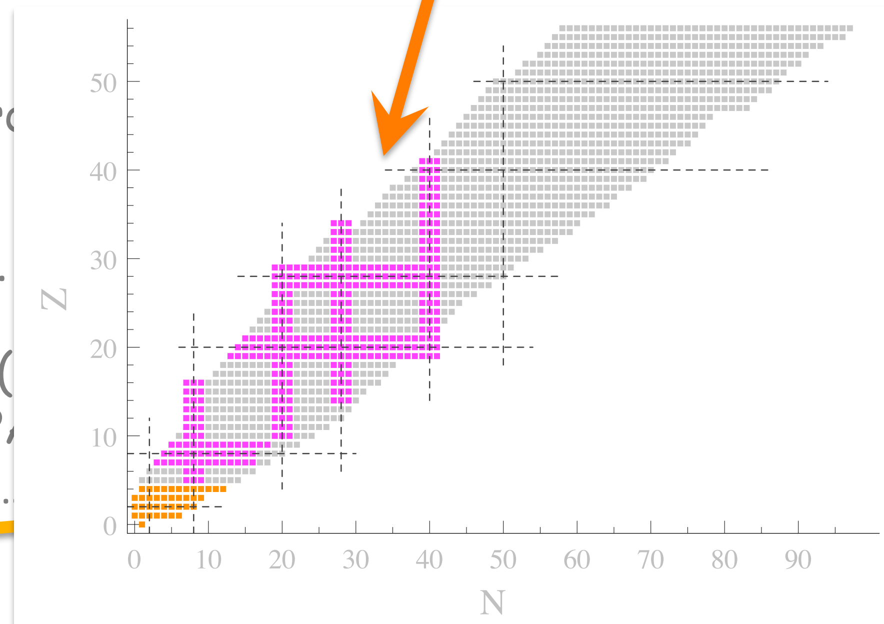
Truncation scheme:

Dyson formulation (closed shells)

Gorkov formulation (semi-/doubly-magic)

1st order: Hartree-Fock HF-Bogolioubov



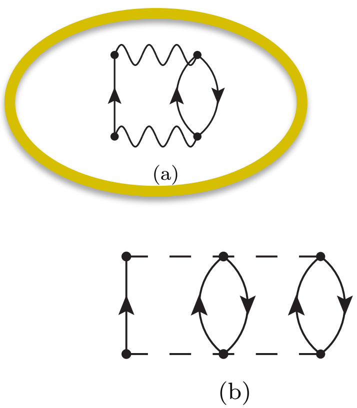


pics. credits: V. Somà

Inclusion of NNN forces

A. Carbone, CB, et al., Phys. Rev. **C88**, 054326 (2013)
and F. Raimondi, CB, in preparation (2016).

- Second order PT diagrams with 3BFs:



- Third order PT diagrams with 3BFs:

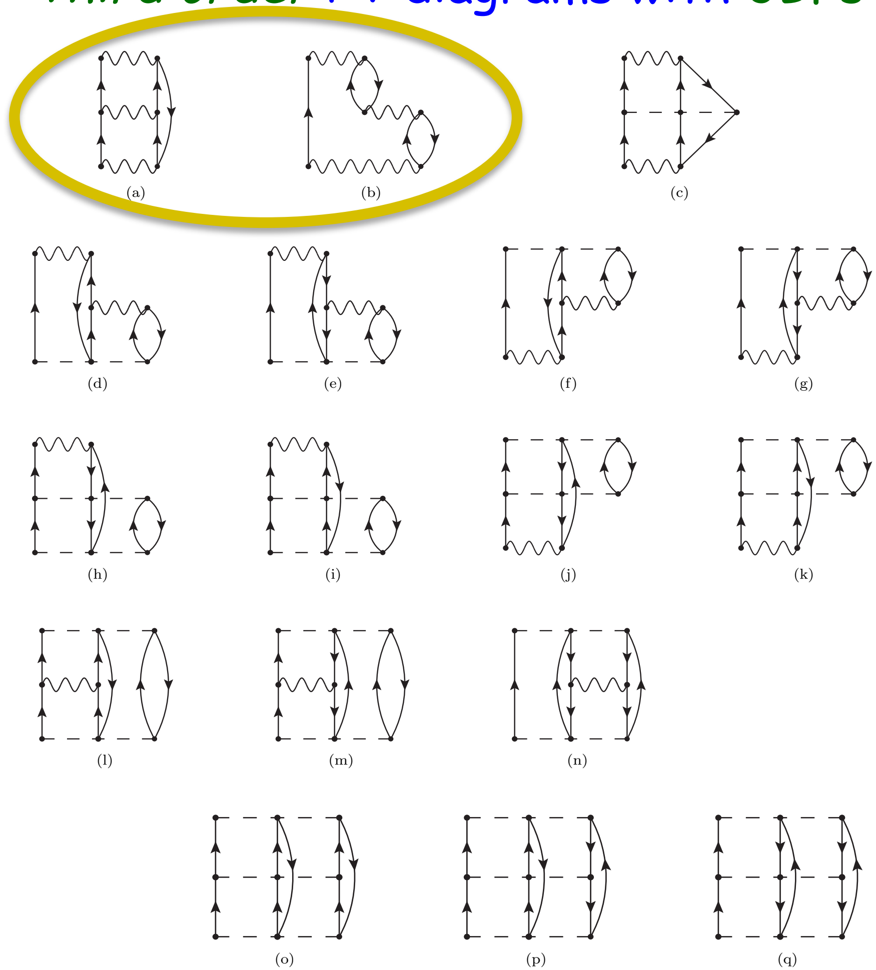


FIG. 5. 1PI, *skeleton* and *interaction irreducible* self-energy diagrams appearing at 3^{rd}-order in perturbative expansion (7), making use of the effective hamiltonian of Eq. (9).

➔ Use of irreducible 2-body interactions

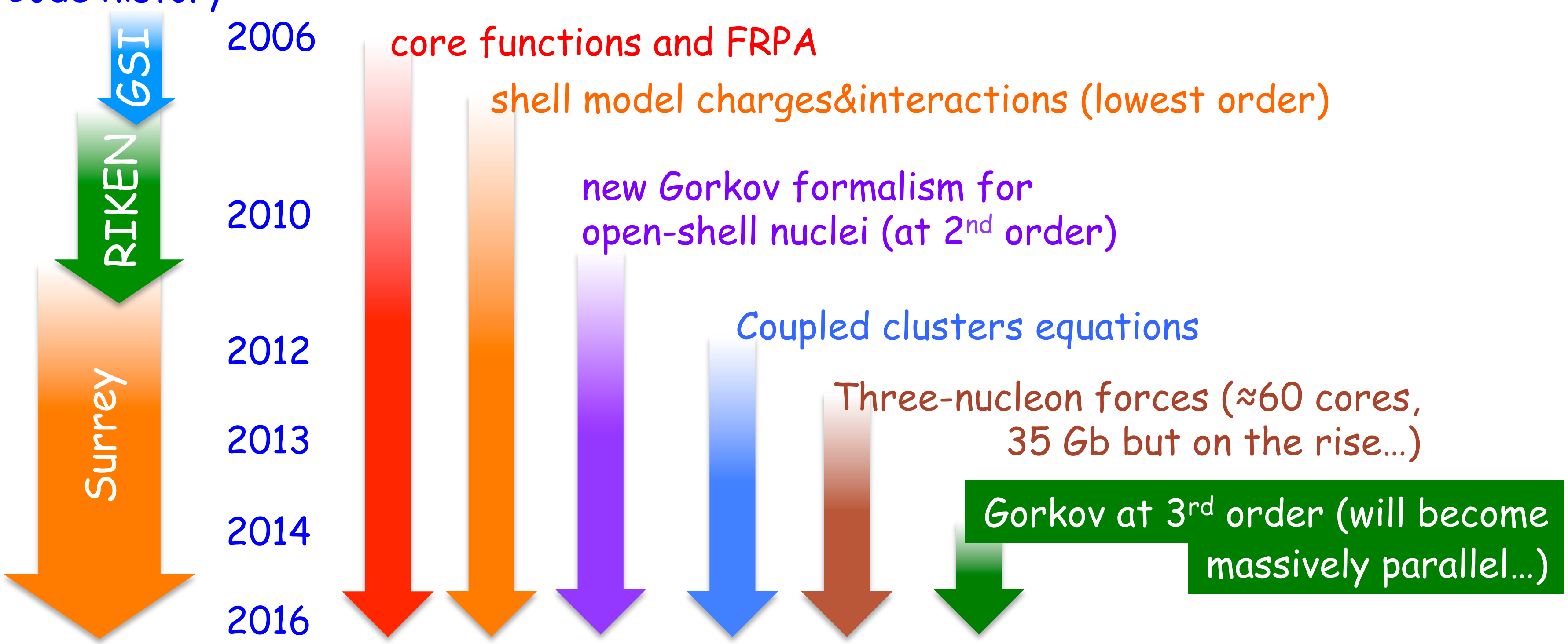
➔ Need to correct the Koltun sum rule (for energy)

Ab-initio Nuclear Computation & BcDor code

BoccaDorata code:
(<u>C. Barbieri</u> *2006-16*
V. Somà 2010-15
A. Cipollone 2011-14)

- Provides a *C++ class library* for handling many-body propagators (≈40,000 lines, MPI&OpenMP based).
- Allows to solve for nuclear spectral functions, many-body propagators, RPA responses, coupled cluster equations and effective interaction/charges for the shell model.

Code history:

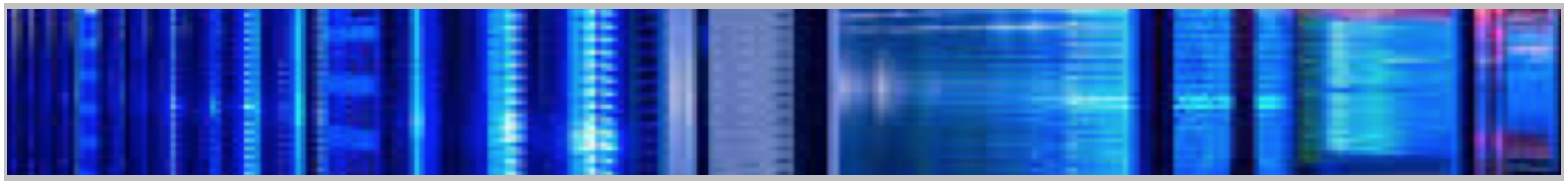


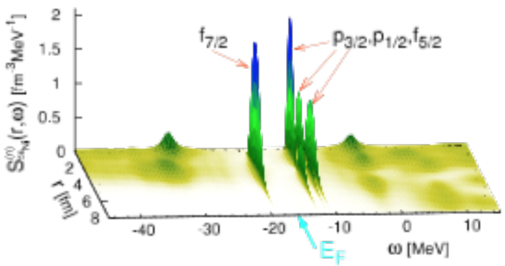
… applications …

Ab-initio Nuclear Computation & BcDor code

http://personal.ph.surrey.ac.uk/~cb0023/bcdor/

Computational Many-Body Physics





Download

Documentation

Welcome

From here you can download a public version of my self-consistent Green's function (SCGF) code for nuclear physics. This is a code in J-coupled scheme that allows the calculation of the single particle propagators (a.k.a. one-body Green's functions) and other many-body properties of spherical nuclei.

This version allows to:

- Perform Hartree-Fock calculations.
- Calculate the the correlation energy at second order in perturbation theory (MBPT2).
- Solve the Dyson equation for propagators (self consistently) up to second order in the self-energy.
- Solve coupled cluster CCD (doubles only!) equations.

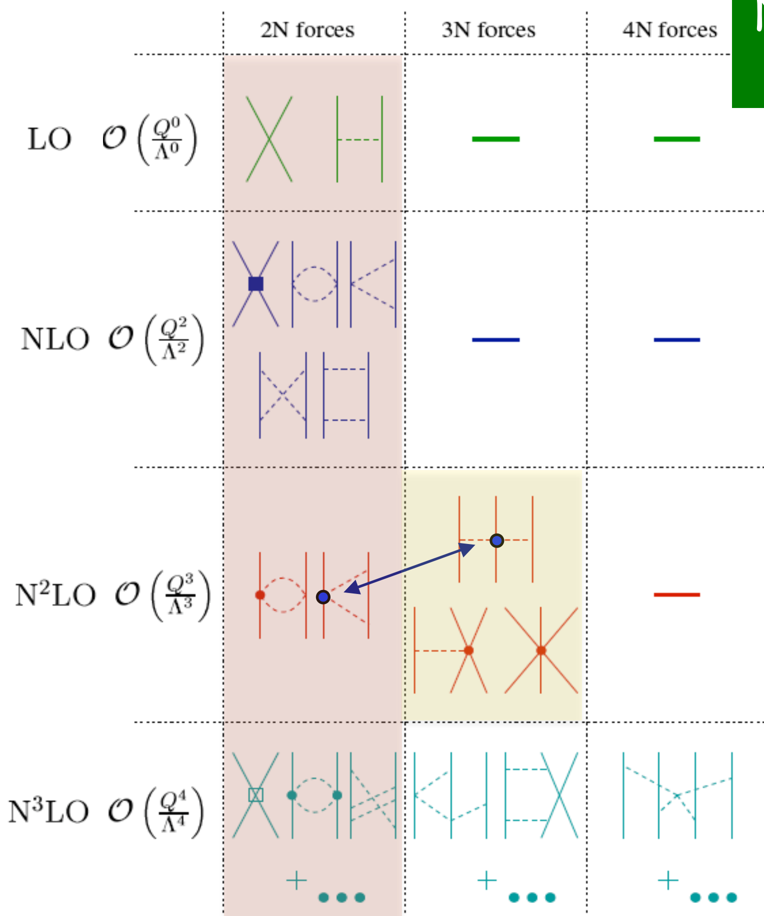
When using this code you are kindly invited to follow the creative commons license agreement, as detailed at the weblinks below. In particular, we kindly ask you to refer to the publications that led the development of this software.

Relevant references (which can also help in using this code) are:

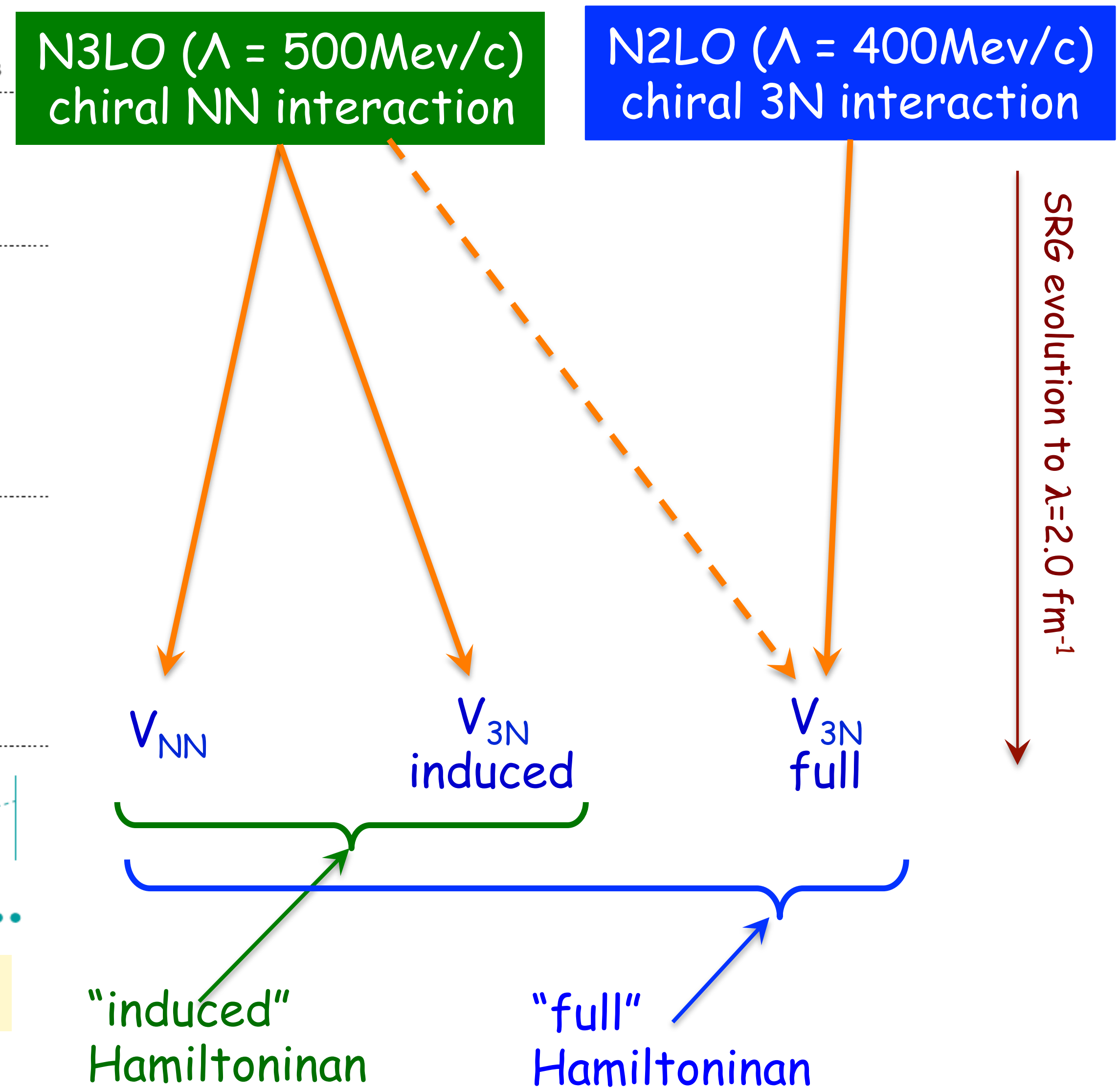
Prog. Part. Nucl. Phys. 52, p. 377 (2004),
Phys. Rev. A76, 052503 (2007),
Phys. Rev. C79, 064313 (2009),
Phys. Rev. C89, 024323 (2014)

Chiral interactions for mid-mass isotopes

Chiral Nuclear forces - SRG evolved

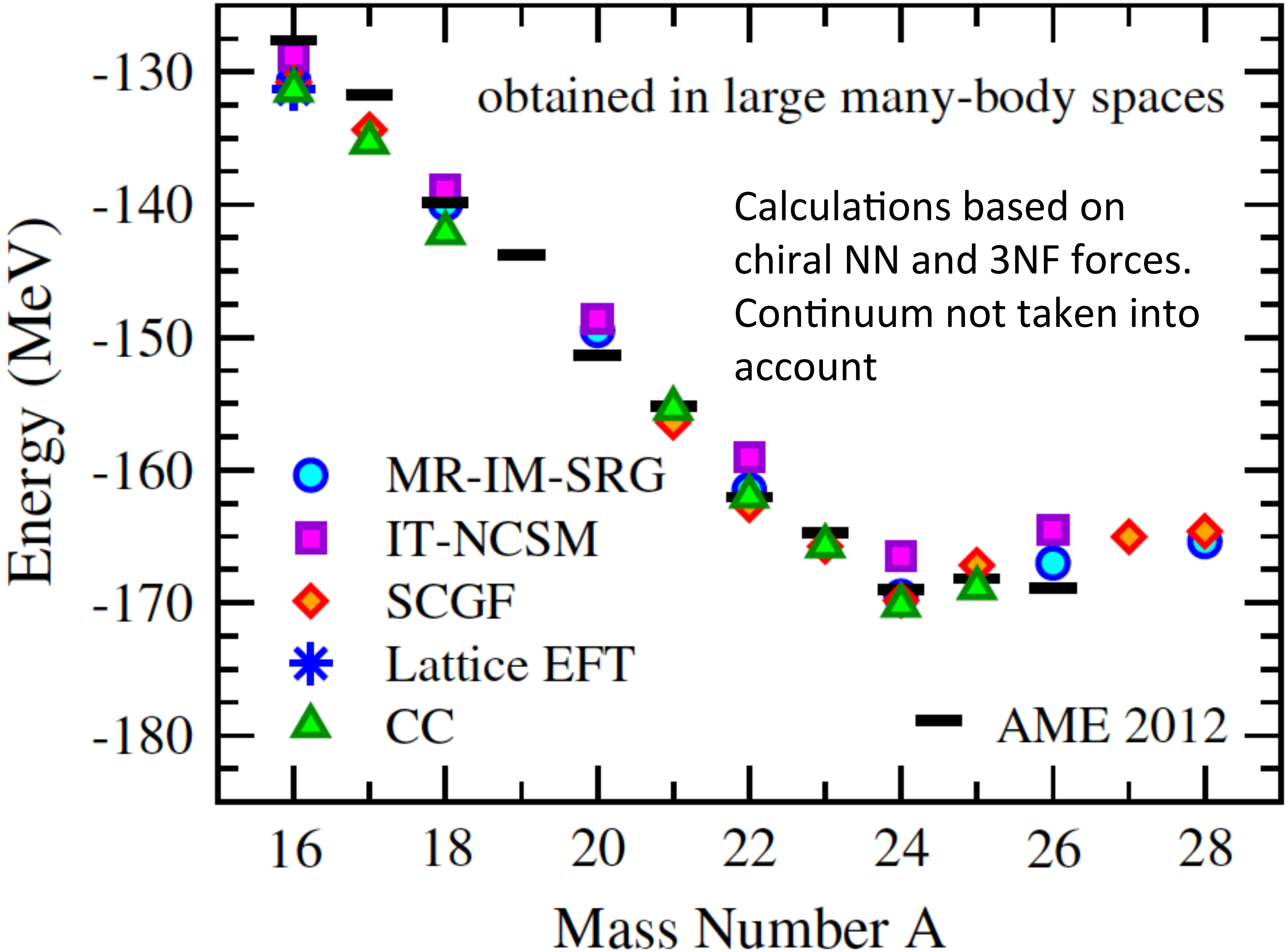


(3NFs arise naturally at N2LO)



[Jurgenson, Navrátil, Furnstahl,
Phys. Rev. Lett. **103**, 082501 (2009);
Hebeler, Phys. Rev. C 85, 021002 (2012)]

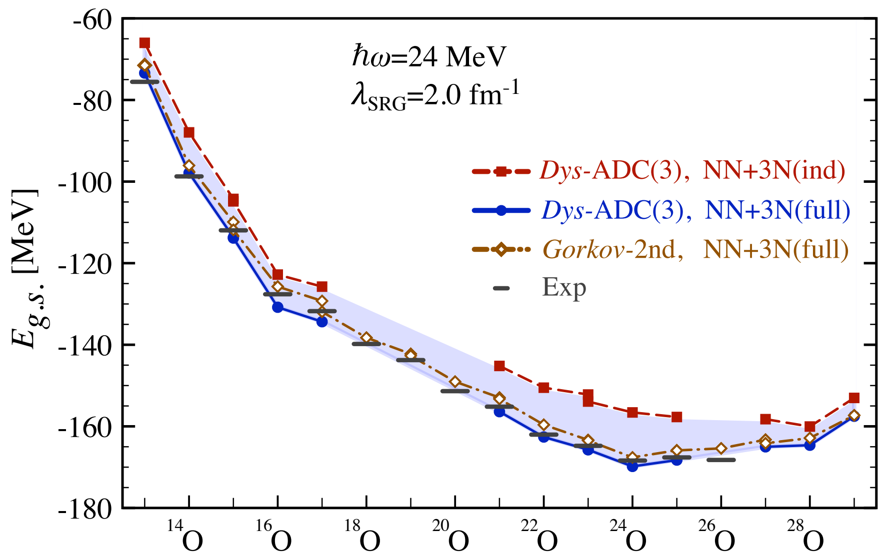
Benchmark of ab-initio methods in the oxygen isotopic chain

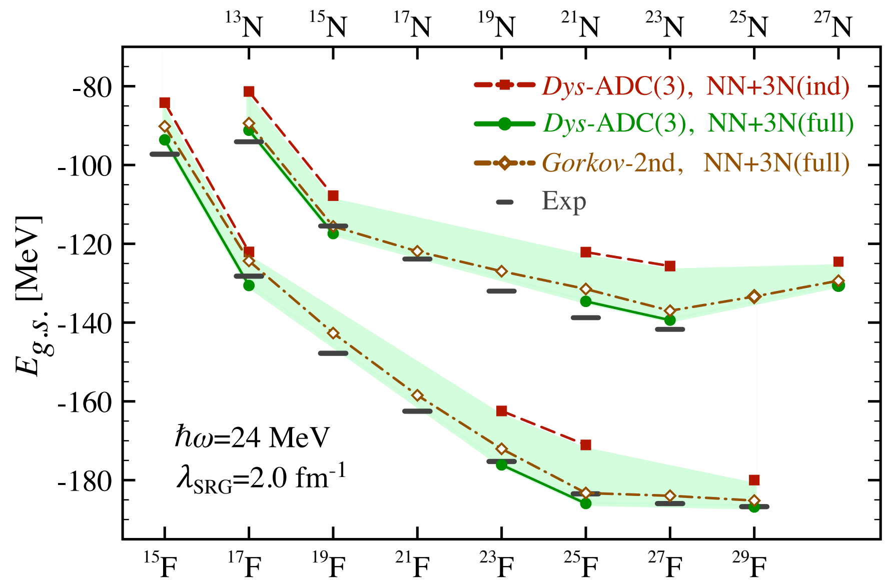


Hebeler, Holt, Menendez, Schwenk, Ann. Rev. Nucl. Part. Sci. in press (2015)

Results for the N-O-F chains

A. Cipollone, CB, P. Navrátil, Phys. Rev. Lett. **111**, 062501 (2013)
and Phys. Rev. C **92**, 014306 (2015)





→ 3NF crucial for reproducing binding energies and driplines around oxygen

→ cf. microscopic shell model [Otsuka et al, PRL**105**, 032501 (2010).]

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N3LO (Λ = 500Mev/c) chiral NN interaction evolved to 2N + 3N forces (2.0fm^{-1})
N2LO (Λ = 400Mev/c) chiral 3N interaction evolved (2.0fm^{-1})

Inversion of $d_{3/2}$—$s_{1/2}$ at N=28

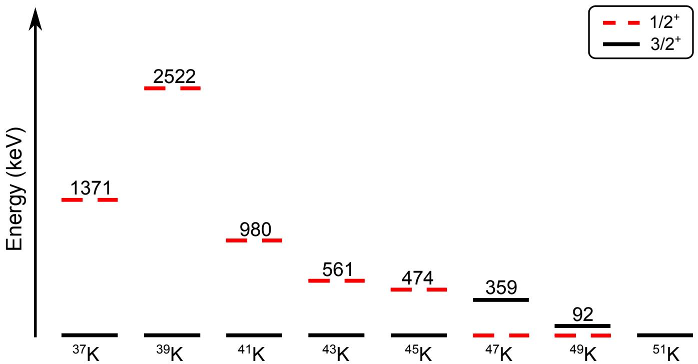


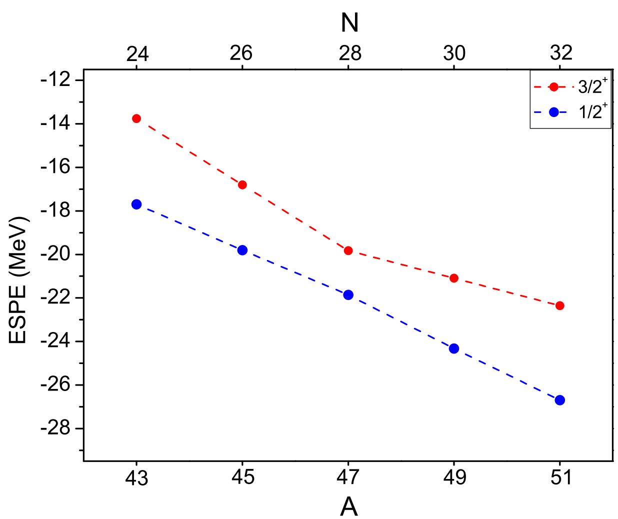
FIG. 1. (color online) Experimental energies for $1/2^+$ and $3/2^+$ states in odd-A K isotopes. Inversion of the nuclear spin is obtained in ^{47,49}K and reinversion back in ^{51}K. Results are

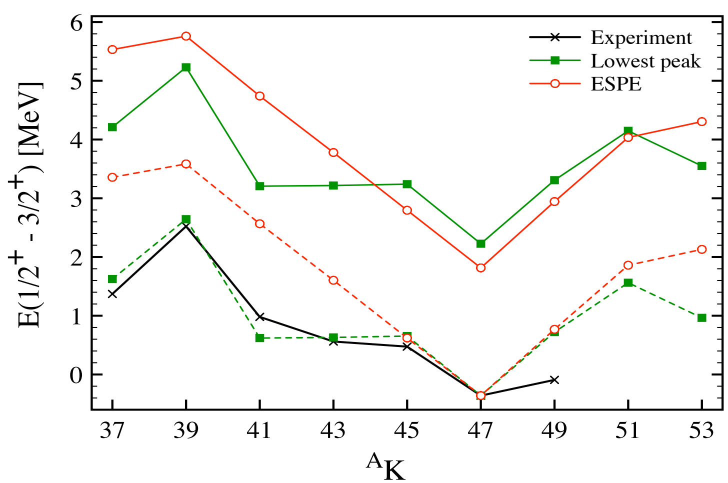
J. Papuga, et al., Phys. Rev. Lett. **110**, 172503 (2013);
J. Papuga, CB, et al., Phys. Rev. C **90**, 034321 (2014)

^{A}K isotopes

Laser spectroscopy @ ISOLDE

Change in separation described by chiral NN[EM(500)]+3NF[N2LO(400)]:





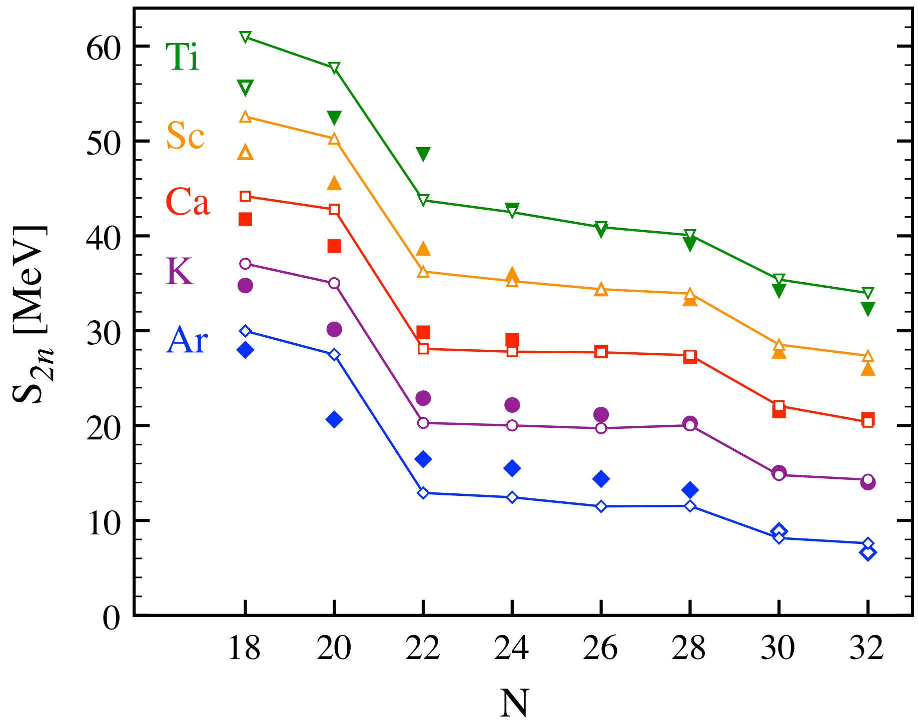
ESPE: "centroid" energies

(Gorkov calculations at 2nd order)

Neighbouring Ar, K, Ca, Sc, and Ti chains

V. Somà, CB *et al.* Phys. Rev. C**89**, 061301R (2014)

Two-neutron separation energies predicted by chiral NN[EM(500)]+3NF[N2LO(400)]:



→ First *ab-initio* calculation over a contiguous portion of the nuclear chart—open shells are now possible through the Gorkov-GF formalism

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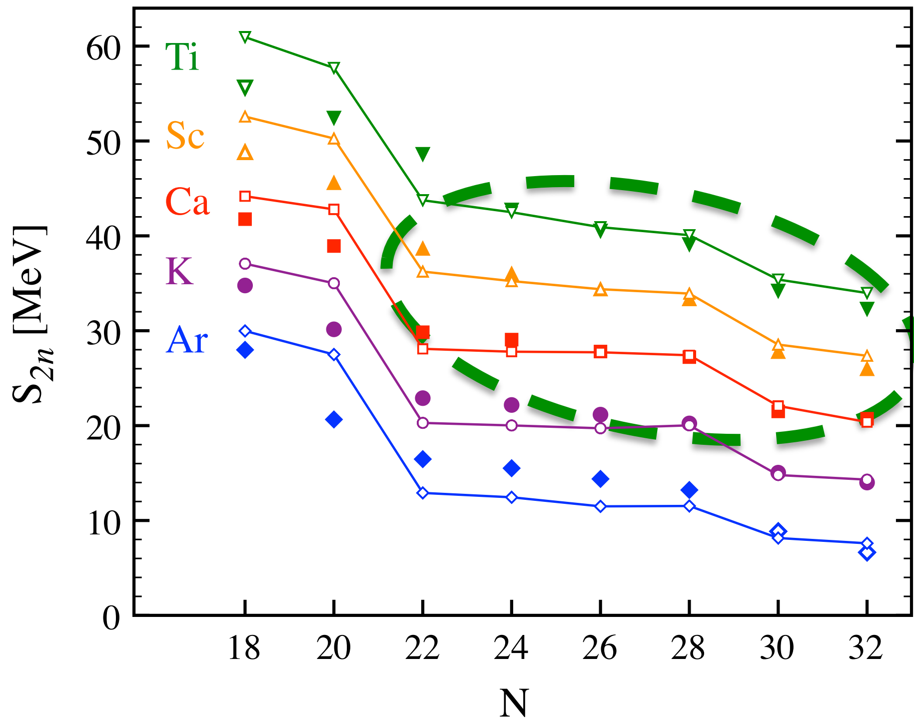
Neighbouring Ar, K, Ca, Sc, and Ti chains

V. Somà, CB *et al.* Phys. Rev. C**89**, 061301R (2014)

Two-neutron separation energies predicted by chiral NN[EM(500)]+3NF[N2LO(400)]:



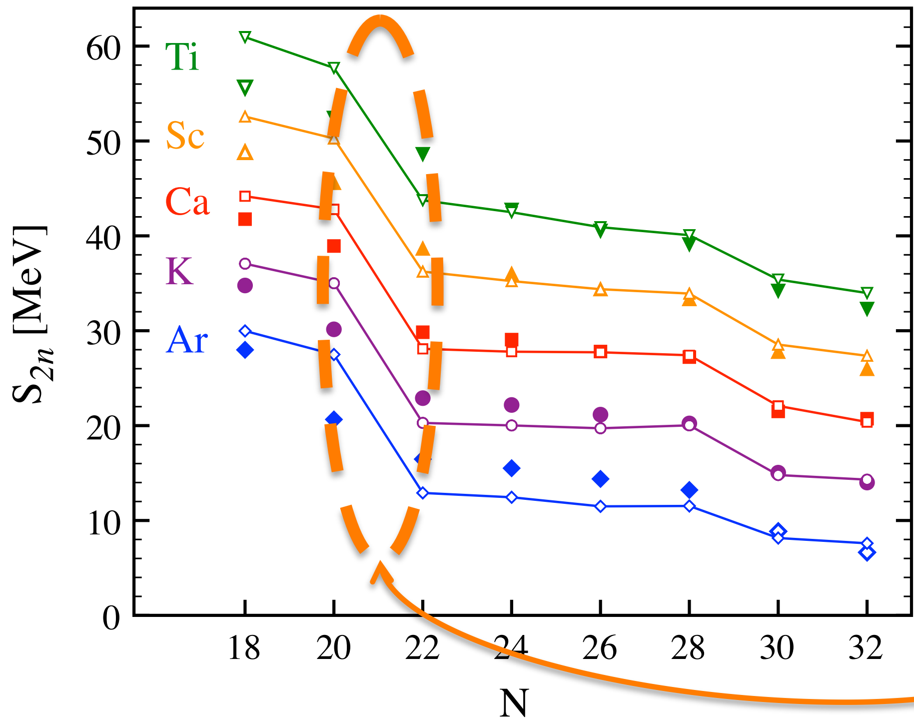
Works well in the *pf* shell

→ First *ab-initio* calculation over a contiguous portion of the nuclear chart—open shells are now possible through the Gorkov-GF formalism

Neighbouring Ar, K, Ca, Sc, and Ti chains

V. Somà, CB *et al.* Phys. Rev. C**89**, 061301R (2014)

Two-neutron separation energies predicted by chiral NN[EM(500)]+3NF[N2LO(400)]:



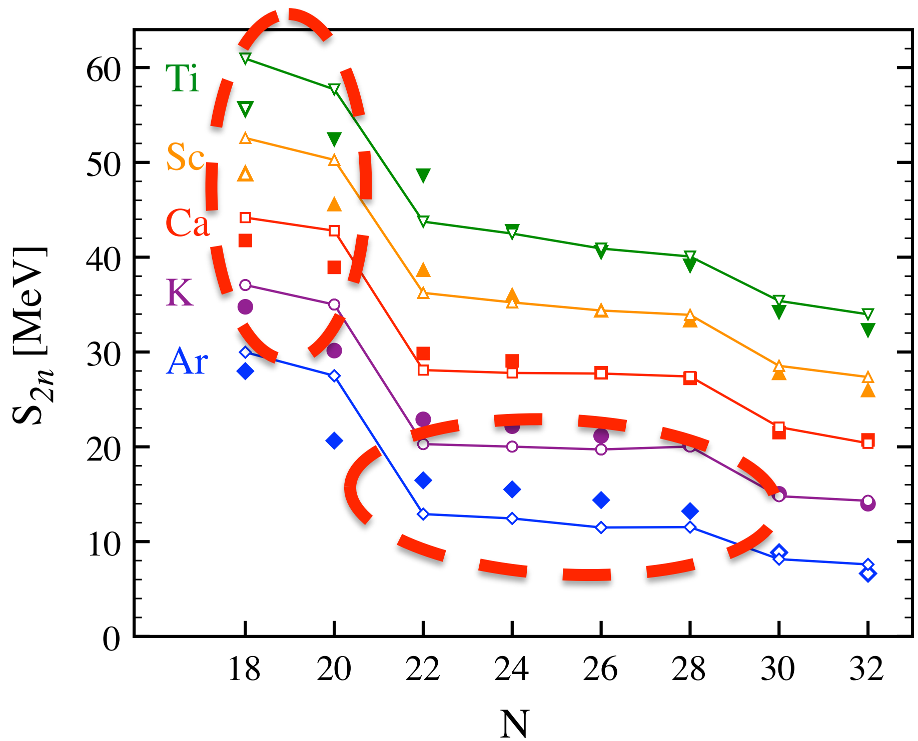
Over estimated
N=20 and Z=20 gaps

→ First *ab-initio* calculation over a contiguous portion of the nuclear chart—open shells are now possible through the Gorkov-GF formalism

Neighbouring Ar, K, Ca, Sc, and Ti chains

V. Somà, CB *et al.* Phys. Rev. C**89**, 061301R (2014)

Two-neutron separation energies predicted by chiral NN[EM(500)]+3NF[N2LO(400)]:

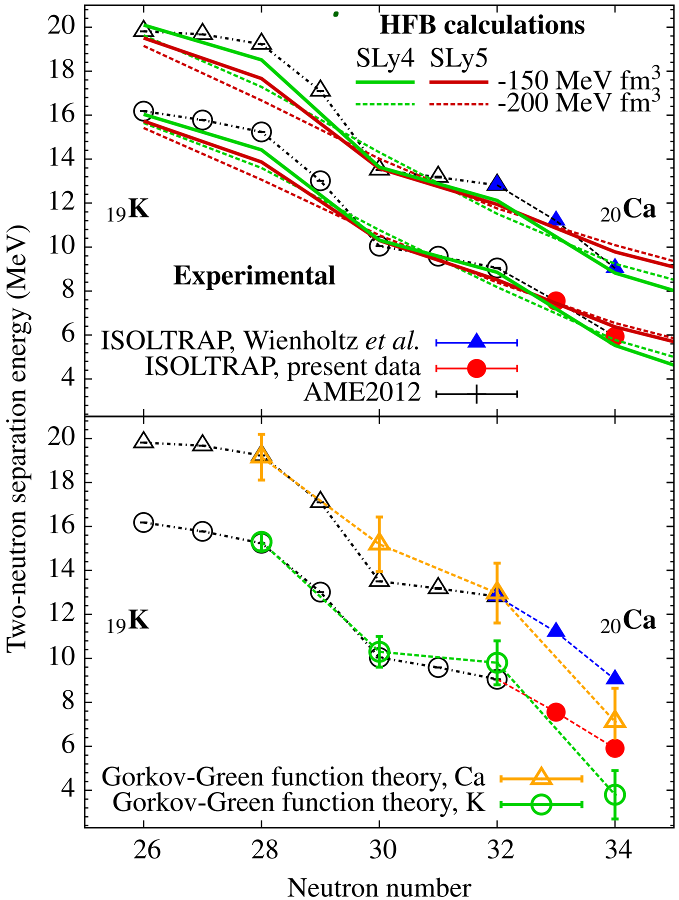


Lack of deformation due to quenched cross-shell quadrupole excitations

→ First *ab-initio* calculation over a contiguous portion of the nuclear chart—open shells are now possible through the Gorkov-GF formalism

Two-neutron separation energies for neutron rich K isotopes

*M. Rosenbusch, CB, et al., PRL**114**, 202501 (2015)*



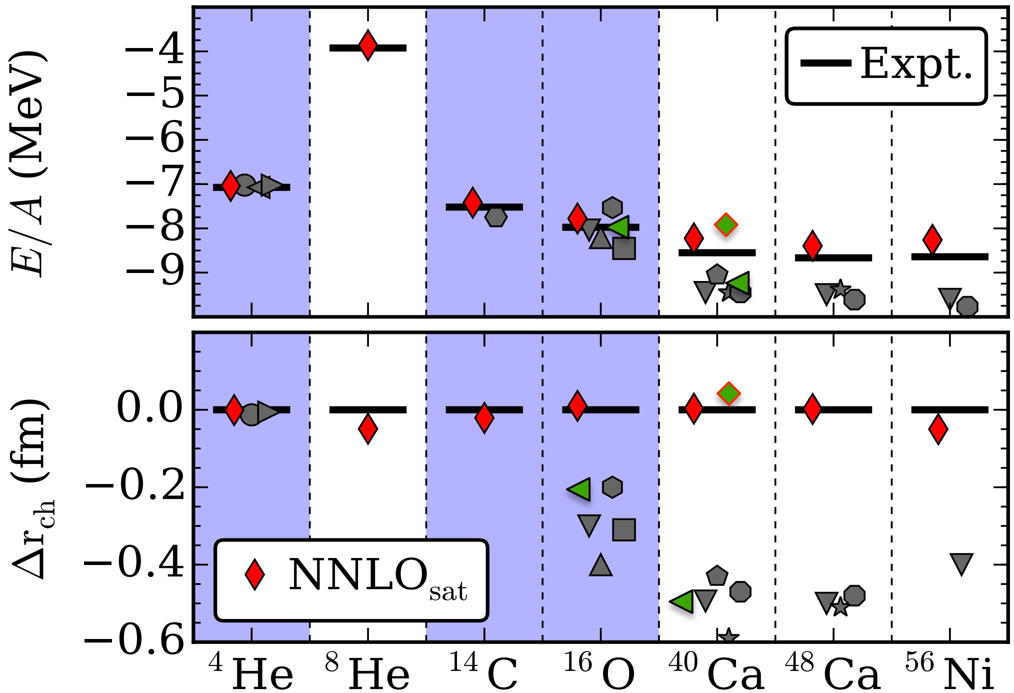
Measurements @ ISOLTRAP

Theory tend to overestimate the gap at N=34, but overall good

→ Error bar in predictions are from extrapolating the many-body expansion to convergence of the model space.

NNLO-sat : a global fit up to A≈24

A. Ekström *et al.* Phys. Rev. C**91**, 051301(R) (2015)



- Constrain NN phase shifts

- Constrain radii and energies up to A≲24

➔ Provides saturation up to large masses!

NNLOsat (V2 + W3) -- *Grkv 2nd ord.*

From **SCGF:**

V2-N3LO(500) + W3-NNLO(400MeV/c) w/ SRG at 2.0 fm^{-1}
A. Cipollone, CB, P. Navrátil, Phys. Rev. Lett. **111**, 062501 (2013)
V. Somà, CB *et al.* Phys. Rev. C**89**, 061301R (2014)

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PRL **117**, 052501 (2016) PHYSICAL REVIEW LETTERS week ending 29 JULY 2016

Radii and Binding Energies in Oxygen Isotopes: A Challenge for Nuclear Forces

V. Lapoux,[1,*] V. Somà,[1] C. Barbieri,[2] H. Hergert,[3] J. D. Holt,[4] and S. R. Stroberg[4]

- New fits of chiral interactions (NNLOsat) highly improve comparison to data

- Deficiencies remain for neutron rich isotopes

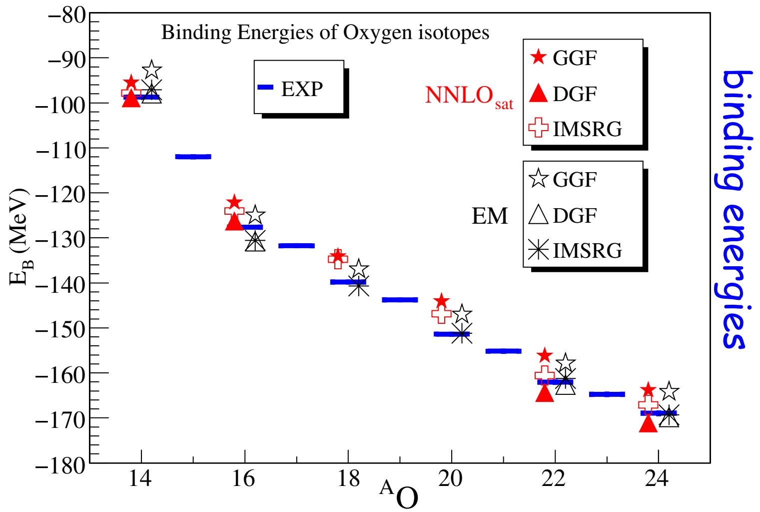
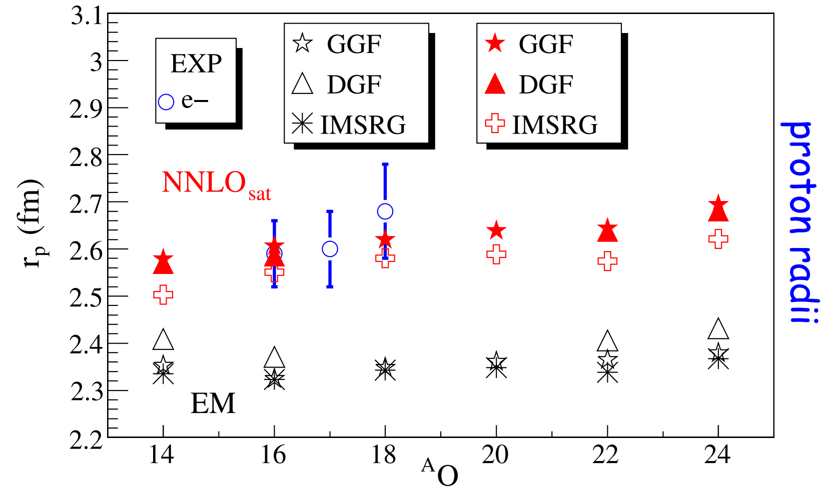
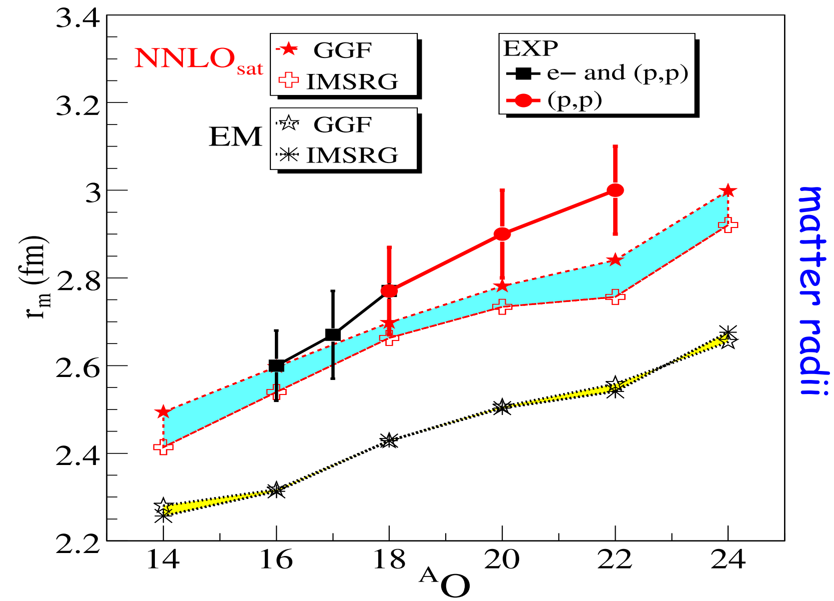


FIG. 1. Oxygen binding energies. Results from SCGF and IMSRG calculations performed with EM [20–22] and NNLO$_{sat}$ [26] interactions are displayed along with available experimental data.





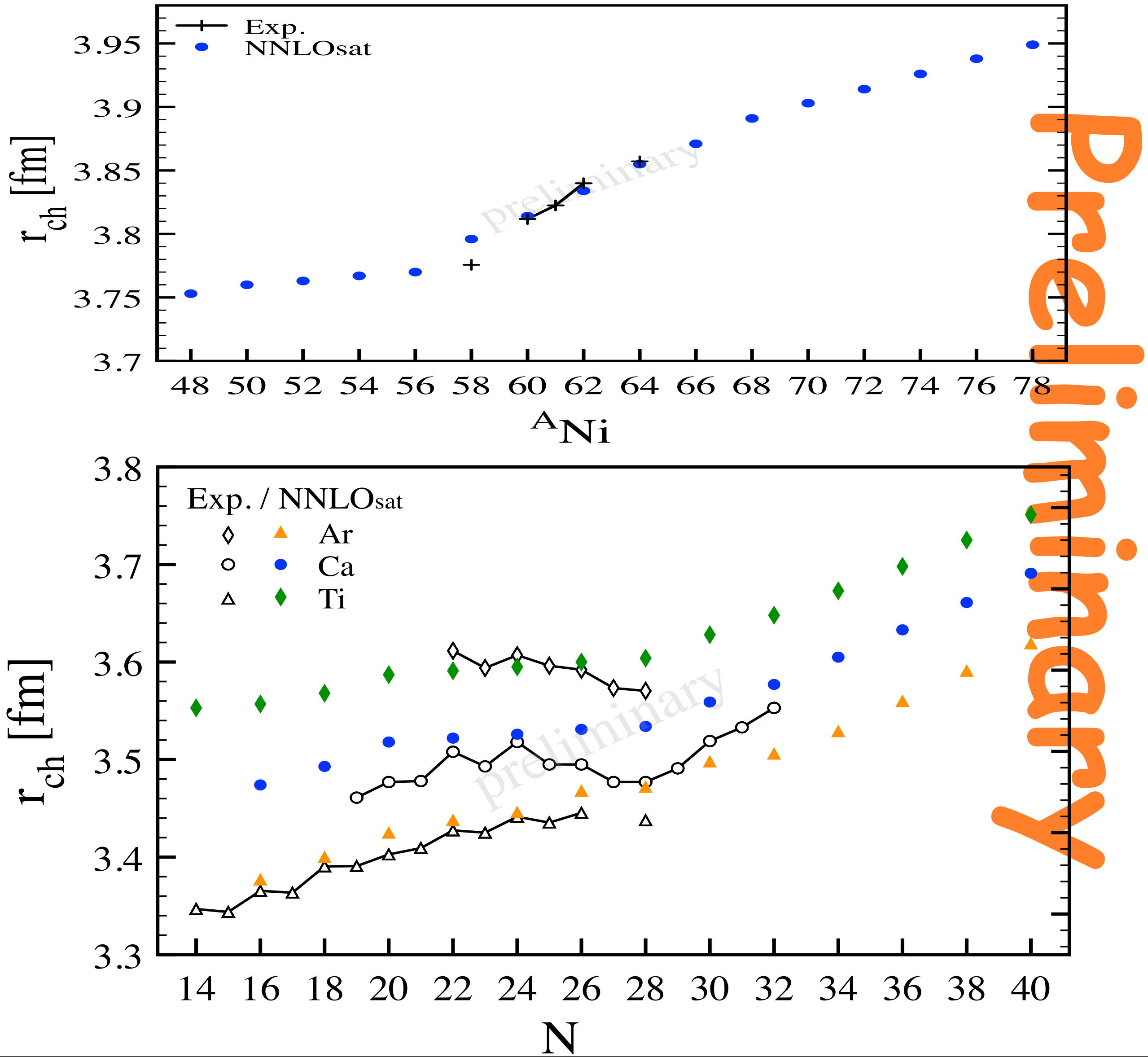
charge radii in the pf shell

Size of radii not prefect but remains overall correct throughout the ***pf*** shell with NNLO-sat.

This suggests that saturation is indeed under control.

➔ Improvements of many-body truncations beyond 2nd order Gorkov will also be relevant.

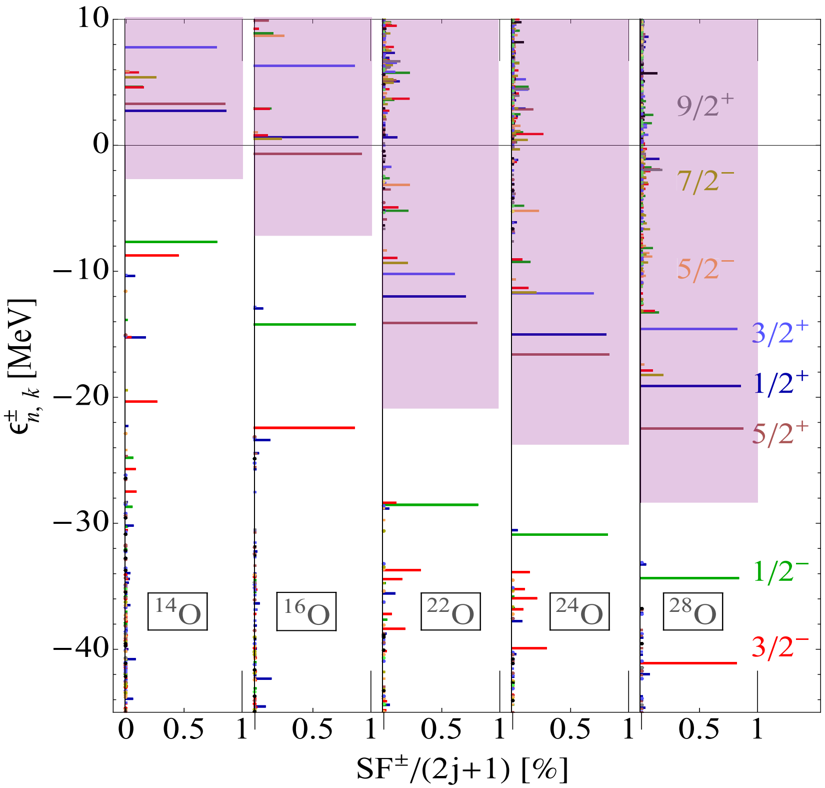
(work in progress!)



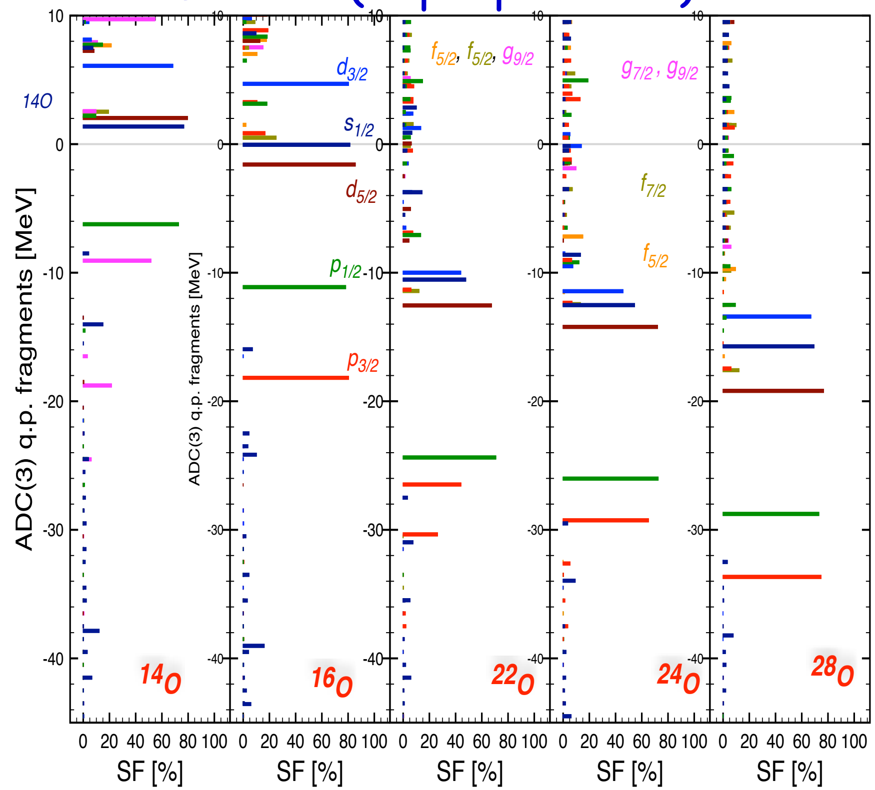
Proton spectral strength in Oxygen

A. Cipollone, CB, P. Navrátil, Phys. Rev. Lett. **111**, 062501 (2013)
and Phys. Rev. C **92**, 014306 (2015)
and ***in preparation***

EM-N3LO(500)/3NF-NNLO(400)



NNLO-sat (in preparation)



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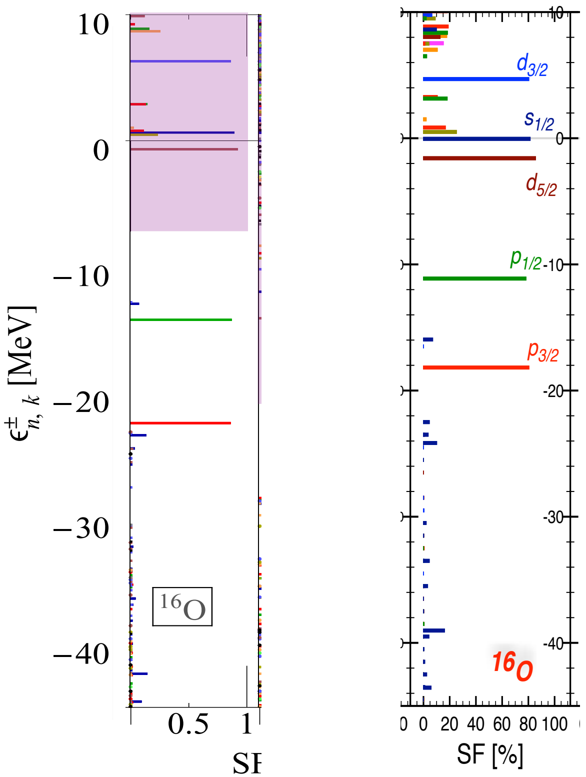
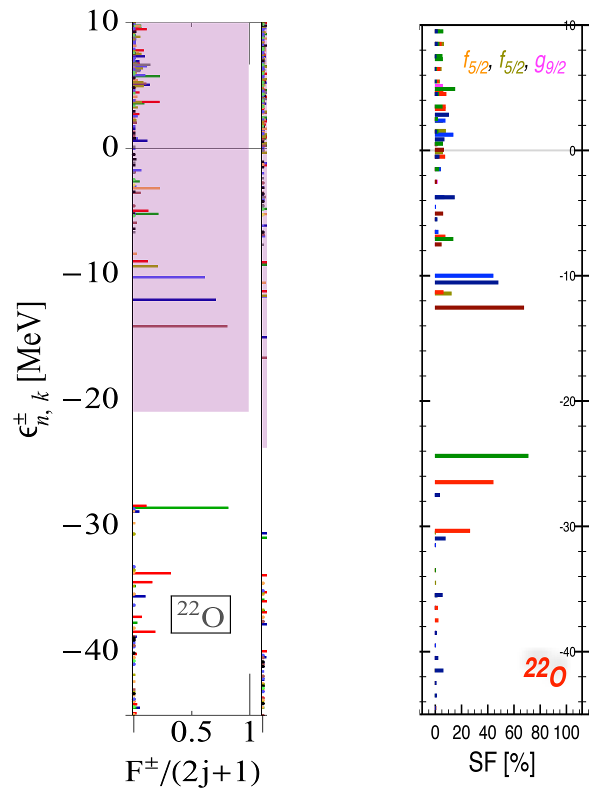
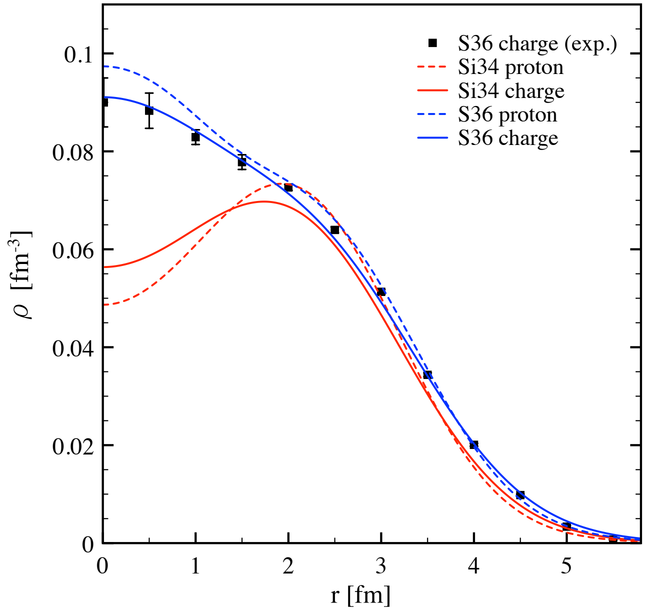
More in detail:





Bubble nuclei... ^{34}Si prediction

[Simon Lecluse, V. Somà, T. Duguet, CB, P. Navrátil]

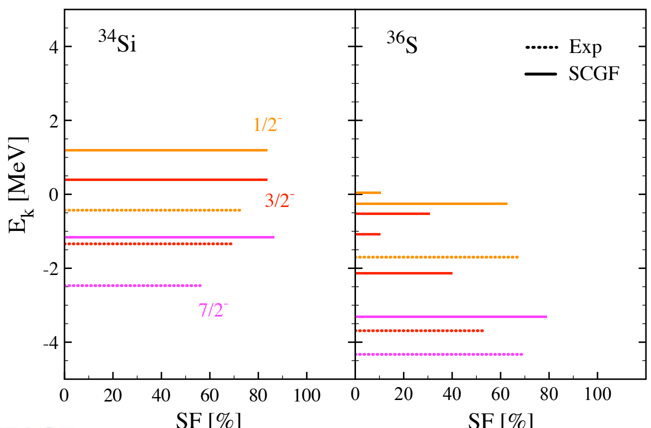


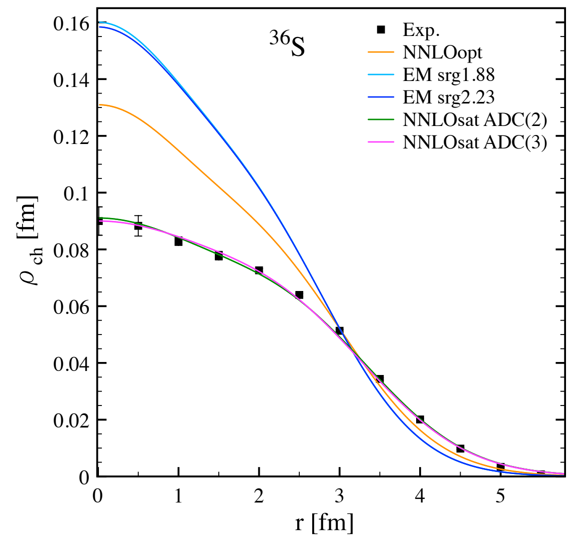
- ^{34}Si is unstable, charge distribution still unknown

- Suggested central depletion from mean-field simulations

- *Ab-initio* theory confirms predictions

Validated by charge distributions and neutron quasiparticle spectra:





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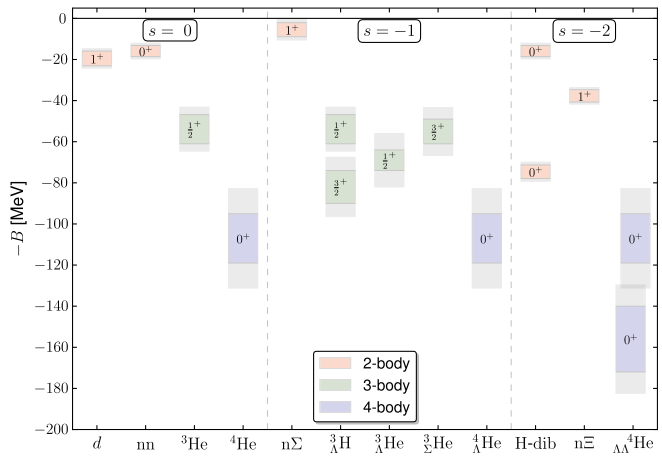
Study of nuclear interactions from Lattice QCD

Other paths in LQCD, see:



NPLQCD, PRD**87**, 034506 (2013)

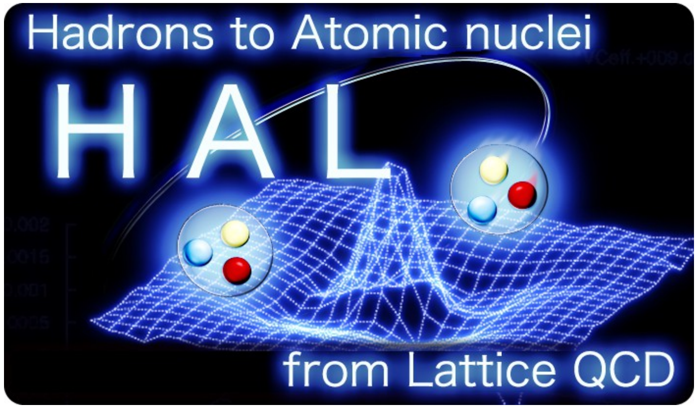
nuclei, the deuteron and the dineutron in 2 + 1 flavor QCD with $m_\pi = 0.51$ GeV and $m_N = 1.32$ GeV. The bound states are distinguished from the attractive scattering states by investigating the spatial volume dependence of the energy shift ΔE_L. In the infinite spatial volume limit we obtain

$$-\Delta E_\infty = \begin{cases} 43(12)(8) & \text{MeV for } {}^4\text{He}, \\ 20.3(4.0)(2.0) & \text{MeV for } {}^3\text{He}, \\ 11.5(1.1)(0.6) & \text{MeV for } {}^3\text{S}_1, \\ 7.4(1.3)(0.6) & \text{MeV for } {}^1\text{S}_0. \end{cases} \quad (17)$$

PACS-CS PRD **86**, 074514 (2012)

Study of nuclear interactions from Lattice QCD

In collaboration with:



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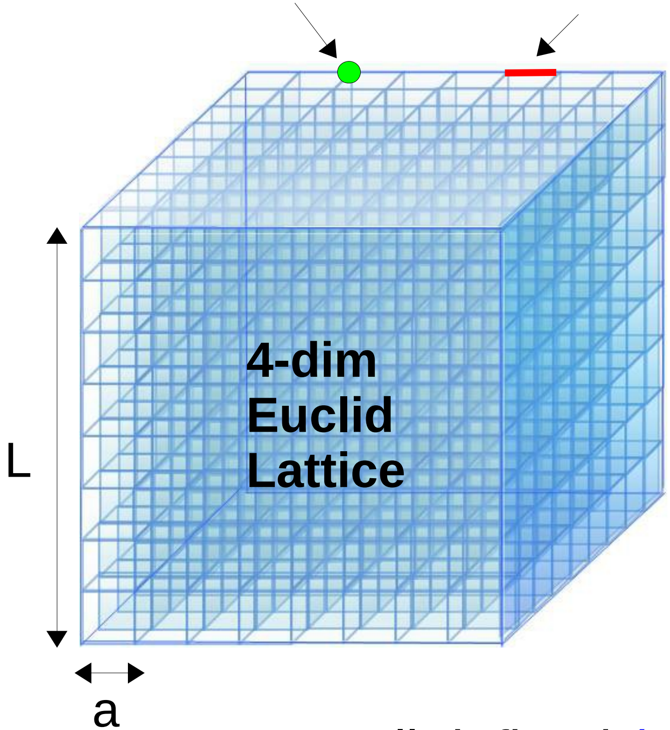
Lattice QCD

$$L = -\frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} + \bar{q}\gamma^\mu \left(i\partial_\mu - g t^a A^a_\mu \right) q - m \bar{q} q$$

quarks q
on the sites

gluons $U = e^{iaA_\mu}$
on the links



Vacuum expectation value

$$\langle O(\bar{q}, q, U) \rangle$$

path integral

$$= \int dU\, d\bar{q}\, dq\, e^{-S(\bar{q},q,U)}\, O(\bar{q}, q, U)$$

$$= \int dU \det D(U) e^{-S_U(U)}\, O(D^{-1}(U))$$

quark propagator

$$= \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} O(D^{-1}(U_i))$$

{ U_i } : ensemble of gauge conf. U
generated w/ probability $\det D(U)\, e^{-S_U(U)}$

- Well defined (regularized)
- Manifest gauge invariance
- Fully non-perturvative
- Highly predictive

The HAL-QCD Method

Define a general potential U(***r***,***r'***) which is and non-local but energy independent up to inelastic threshold, such that:

$$\frac{-\nabla^2}{2\mu}\varphi_{\vec{k}}(\vec{r}) \;+\int d\vec{r}\,'U(\vec{r},\vec{r}\,')\varphi_{\vec{k}}(\vec{r}\,') \;=\; E_{\vec{k}}\varphi_{\vec{k}}(\vec{r})$$

for the Nambu-Bethe-Salpeter (NBS) wave function,

$$\varphi_{\vec{k}}(\vec{r}) = \sum \langle 0|B_i(\vec{x}+\vec{r},t)B_j(\vec{x},t)|B=2,\vec{k}\rangle$$

<u>Operationally</u>, measure the 4-pt function on the QCD Lattice

$$\psi(\vec{r},t) = \sum_{\vec{x}}\langle 0|B_i(\vec{x}+\vec{r},t)B_j(\vec{x},t)\,J(t_0)|0\rangle = \sum_{\vec{k}} A_{\vec{k}}\varphi_{\vec{k}}(\vec{r})e^{-W_{\vec{k}}(t-t_0)} + \ldots$$

and extract U(***r***,***r'***) from: $\left\{2M_B - \frac{\nabla^2}{2\mu}\right\}\psi(\vec{r},t) \;+\int d\vec{r}\,'U(\vec{r},\vec{r}\,')\psi(\vec{r}\,',t) \;= -\frac{\partial}{\partial t}\psi(\vec{r},t)$

A *local potential* V(***r***) is then obtained through a derivative expansion of U(***r***,***r'***), which *must give the same observables* of the LQCD simulation:

$$U(\vec{r},\vec{r}\,') = \delta(\vec{r}-\vec{r}\,')V(\vec{r},\nabla) = \delta(\vec{r}-\vec{r}\,')\left\{V(\vec{r}) + \mathcal{O}(\nabla) + \mathcal{O}(\nabla^2) + \ldots\right\}$$

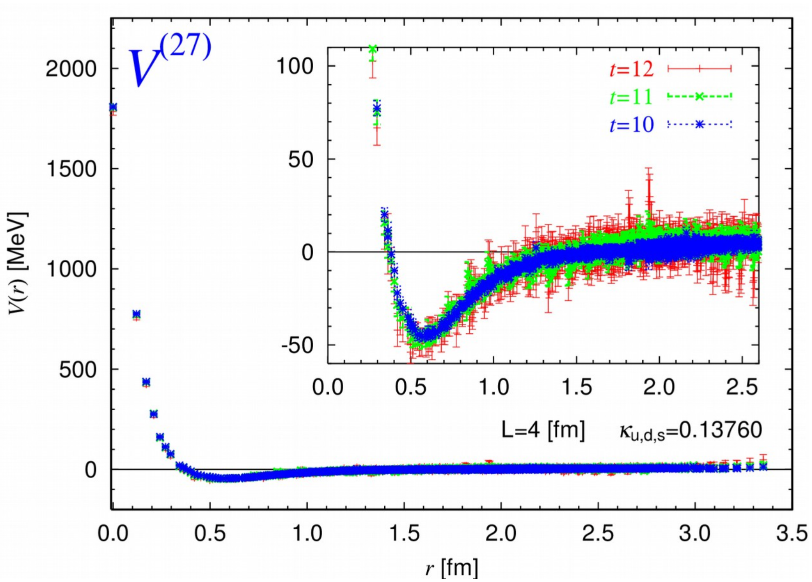
$$\Rightarrow\; V(\vec{r}) = \frac{1}{2\mu}\frac{\nabla^2\psi(\vec{r},t)}{\psi(\vec{r},t)} - \frac{\frac{\partial}{\partial t}\psi(\vec{r},t)}{\psi(\vec{r},t)} - 2M_B$$

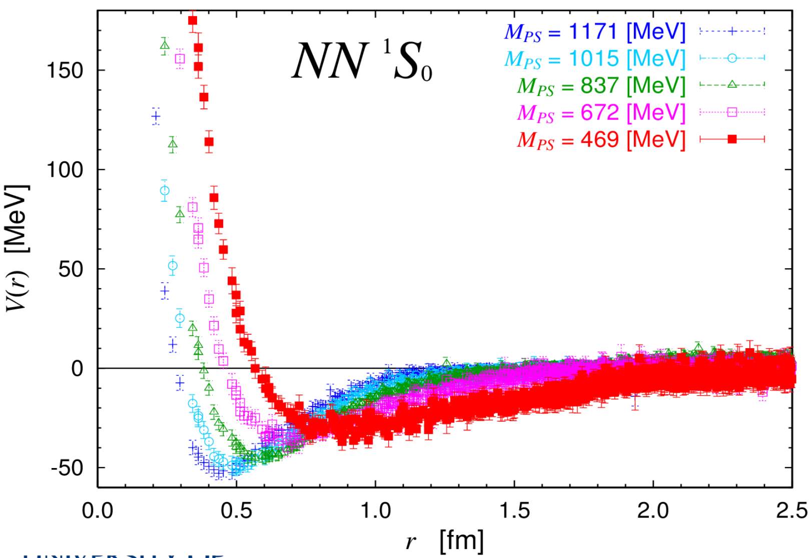
Tensor/Yukawa force in S-D

Spin-orbit force, P waves

Prog. Theor. Phys. 123 89 (2010); Phys. Lett. B712 , 437 (2012); Prog. Theor. Exp. Phys. 01A105 (2012)

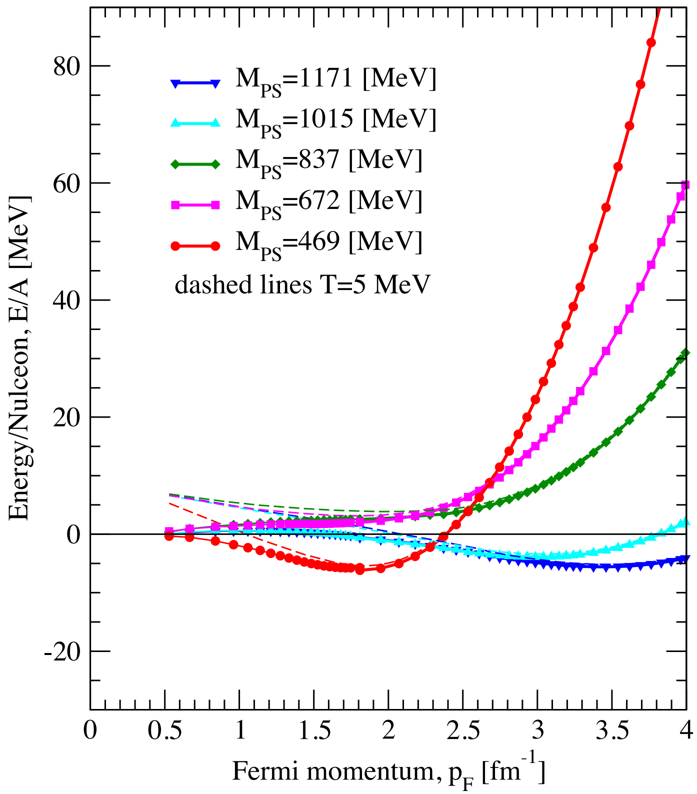
Two-Nucleon HAL potentials





Quark mass dependence of *V(r)* for NN partial wave (*$^1S_0$*, *$^3S_1$*, *$^3S_1$-$^3D_1$*)

➔ Potentials become stronger m_π as decreases.



T. Inoue *et al.*,
Phys. Rev. Lett. **111** 112503 (2013).

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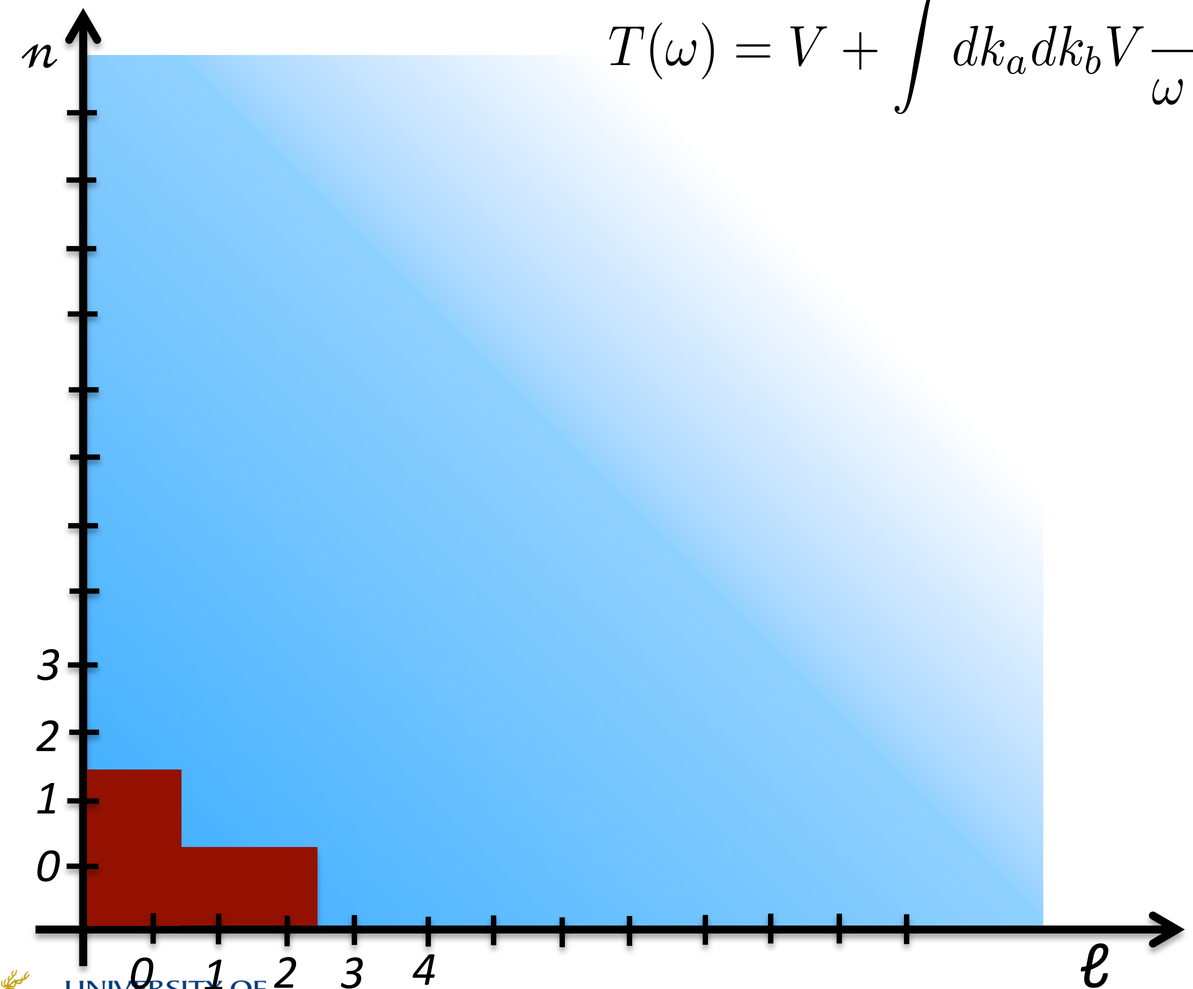
Prog. Theor. Exp. Phys. 01A105 (2012)

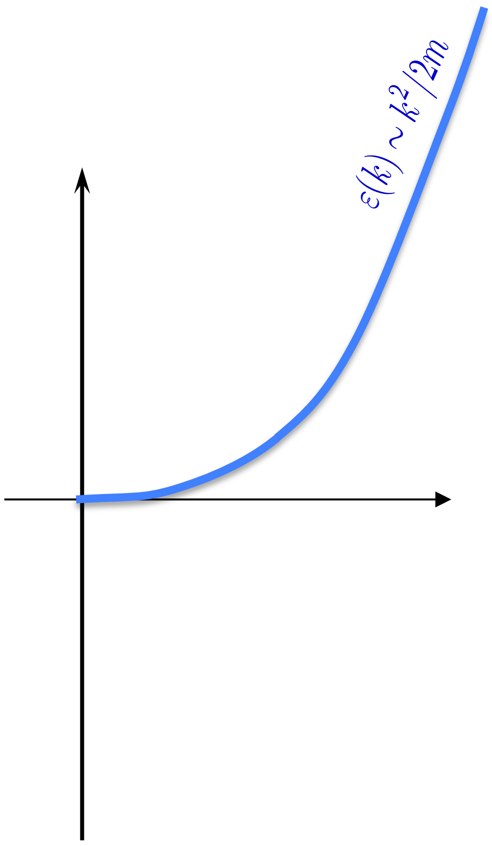
(Finite-T results by A. Carbone, priv. comm.)

Analysis of Brueckner HF

Scattering of two nucleon in free space:

$$T(\omega) = V + \int dk_a dk_b V \frac{1}{\omega - k_a^2/2m - k_b^2/2m + i\eta} T(\omega)$$

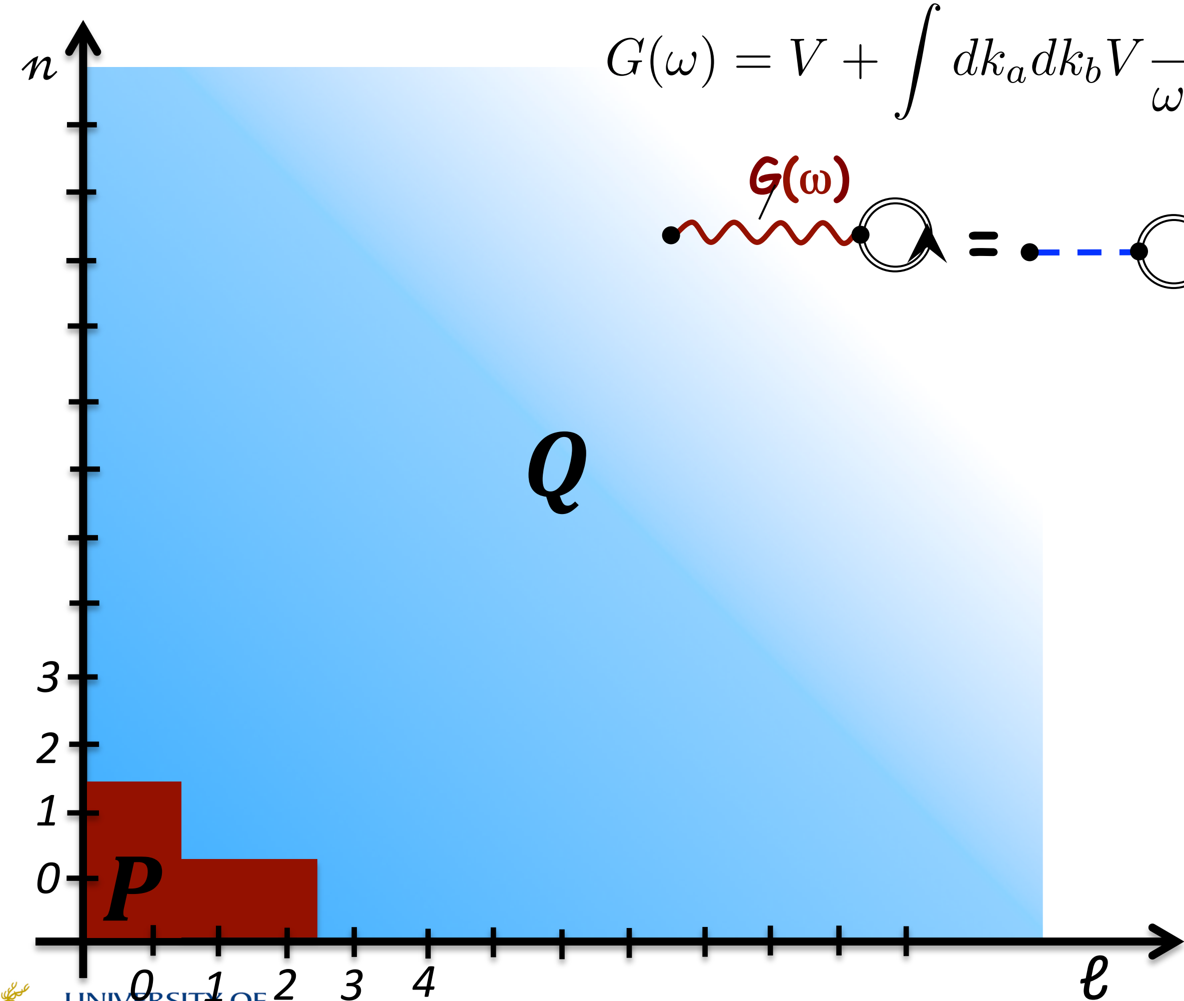


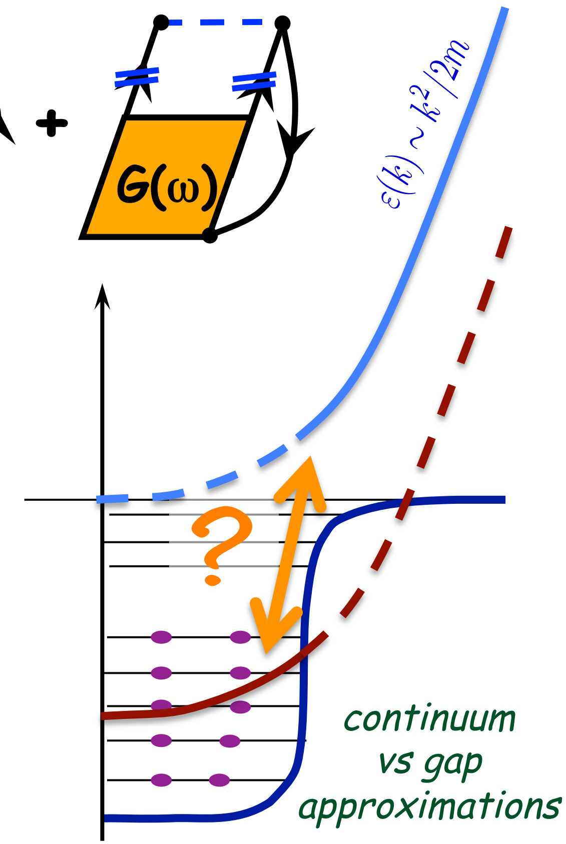


Analysis of Brueckner HF

Scattering of two nucleons outside the Fermi sea (➔BHF):

$$G(\omega) = V + \int dk_a dk_b V \frac{\hat{Q}}{\omega - \varepsilon(k_a) - \varepsilon(k_b) + i\eta} G(\omega)$$

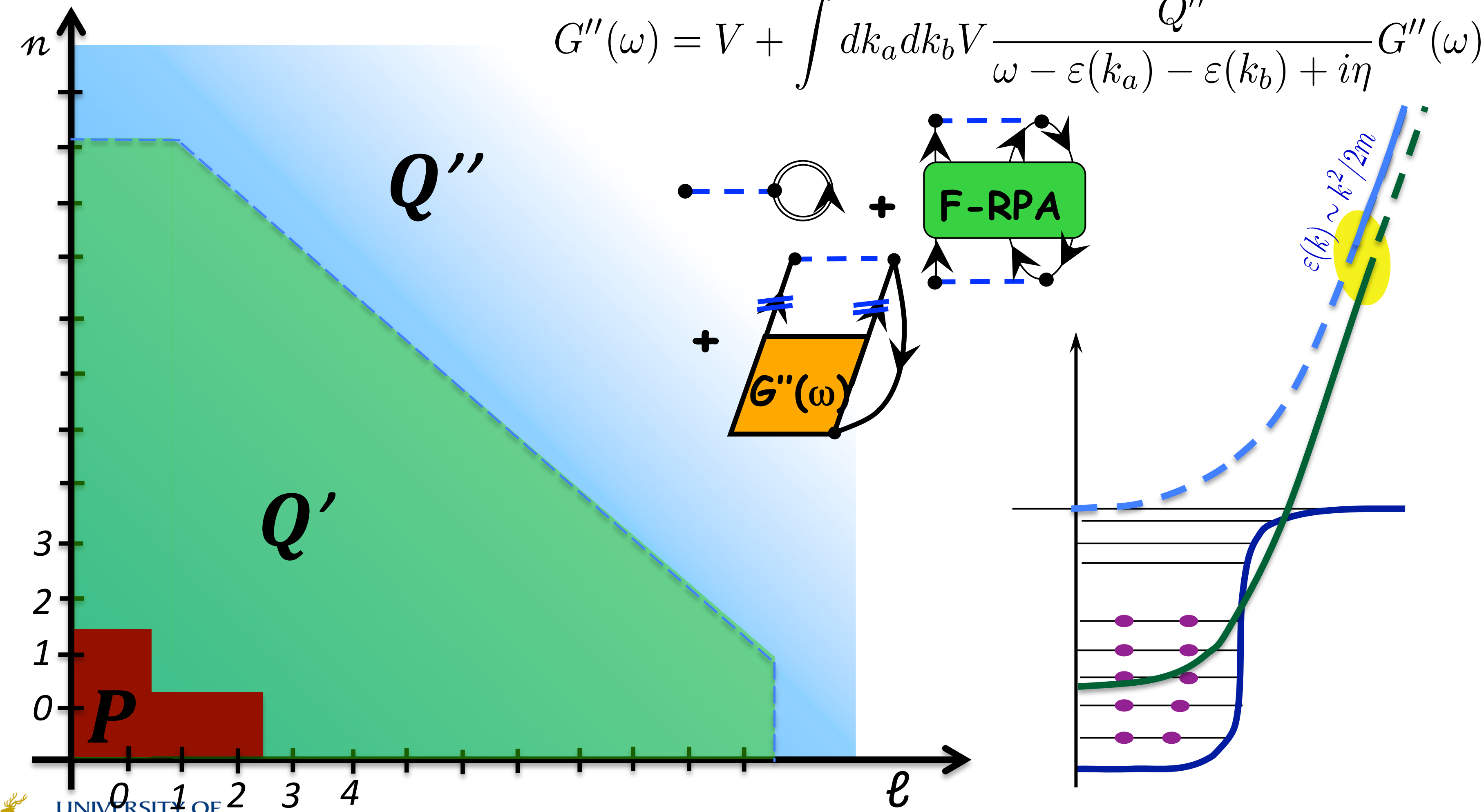




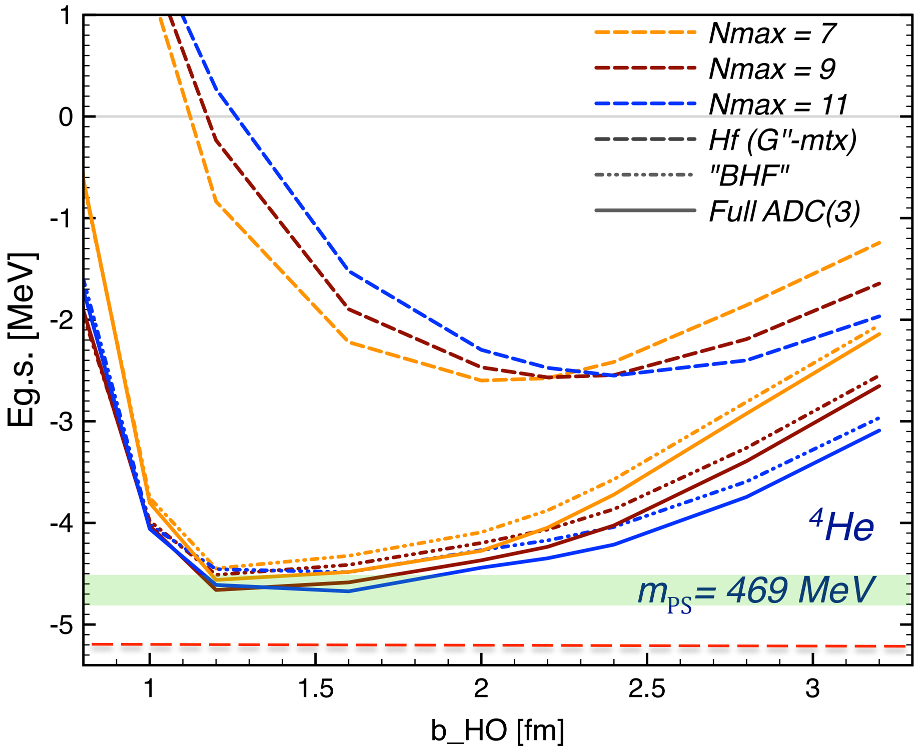
Mixed SCGF-Brueckner approach

Solve full many-body dynamics in model space (P+Q') and the Goldstone's ladders outside it (i.e. in Q'' only):

$$G''(\omega) = V + \int dk_a dk_b V \frac{\hat{Q}''}{\omega - \varepsilon(k_a) - \varepsilon(k_b) + i\eta} G''(\omega)$$



Benchmark on ^{4}He

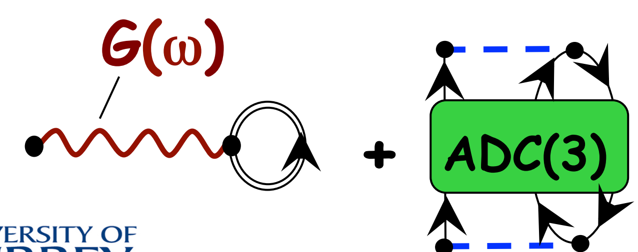


Can benchmark the Gmtx+ADC(3) method on light ^{4}He, where exact solutions are possible:

	G(ω) + ADC(3)	Exact
HALQCD @ m_π=469MeV	4.7(2) MeV	5.09 MeV[1]

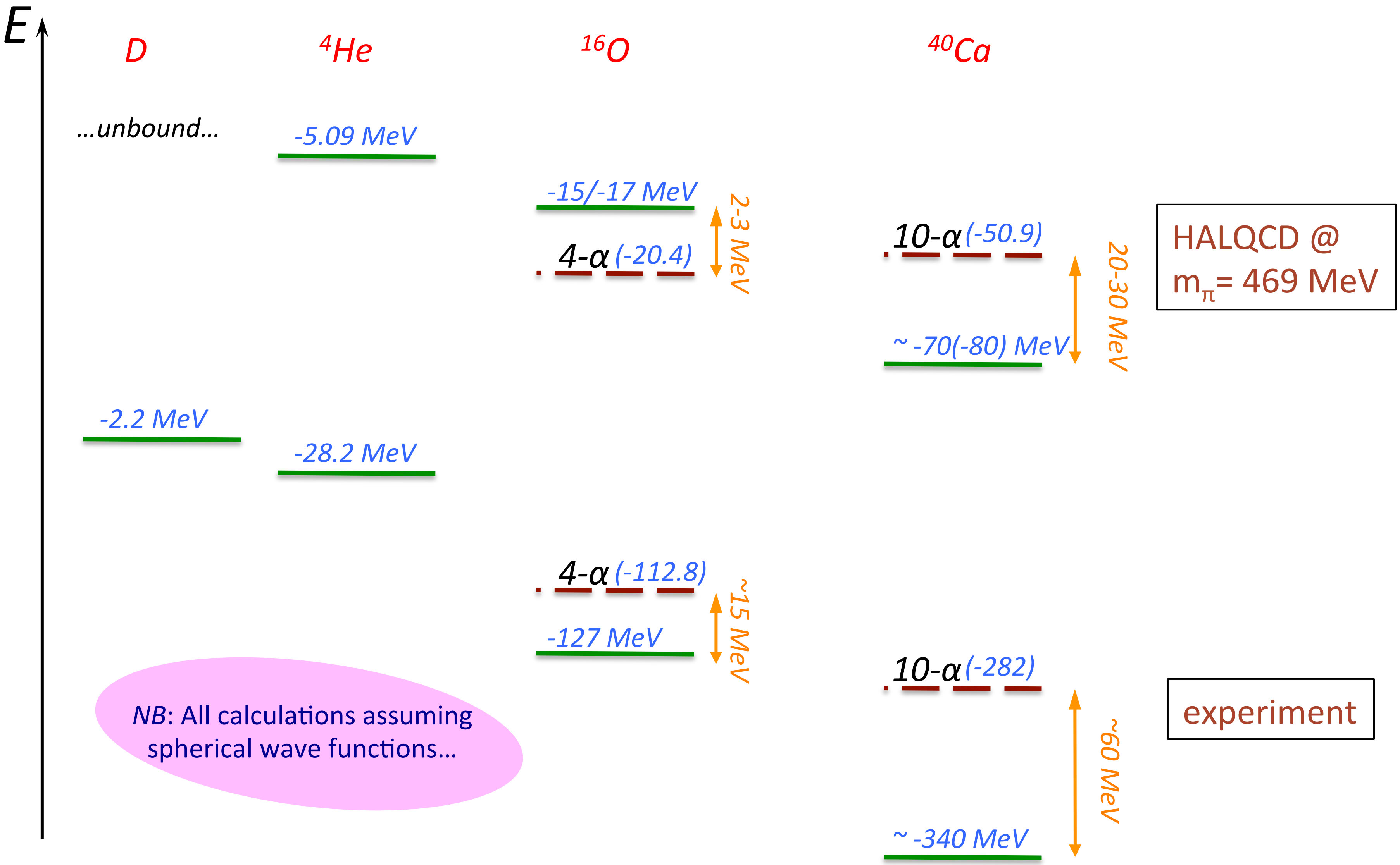
[1]H. Nemura *et al.*, Int. J. Mod. Phys. E **23**, 1461006 (2014)

➔ *Can expect accuracy on binding energies at about 10%*



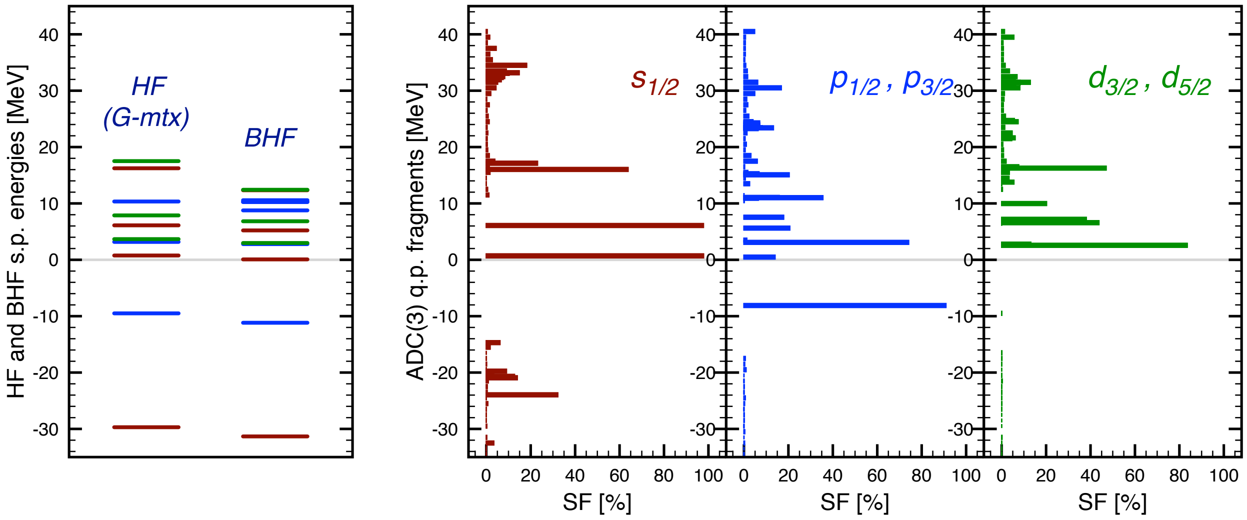
$$G''(\omega) = V + \int dk_a dk_b V \frac{\hat{Q}''}{\omega - \varepsilon(k_a) - \varepsilon(k_b) + i\eta} G''(\omega)$$

Results for binding



C. McIlroy, CB, et al., *in preparation*

Spectral strength in ^{16}O and ^{40}Ca:



Particle-hole gaps

^{16}O

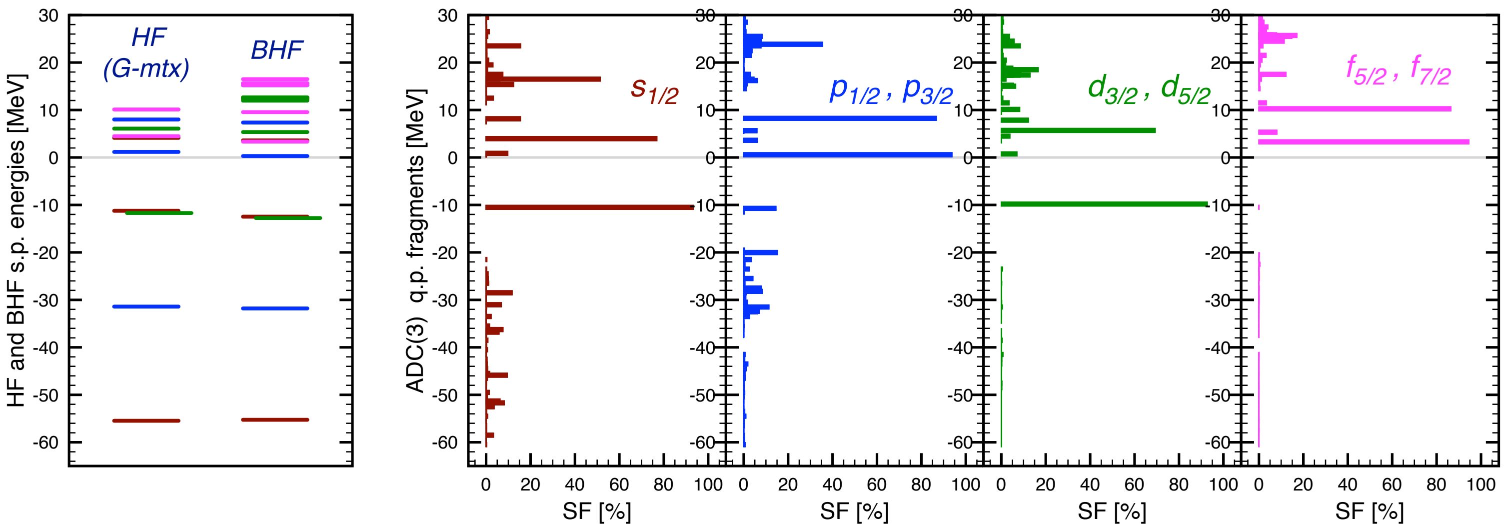
m_π= 469 MeV: ~8 MeV

Expt (phys m_π): 11.5 MeV

^{40}Ca

m_π= 469 MeV: ~10 MeV

Expt (phys m_π): 7.5 MeV



Summary

Thank you for your attention!!!

Mid-masses and chiral interactions:

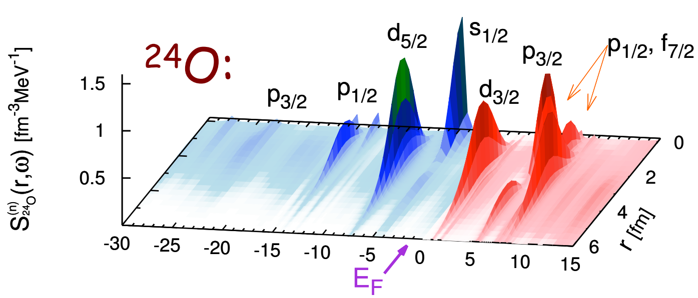
→ *Leading order 3NF are crucial to predict many important features that are observed experimentally (drip lines, saturation, orbit evolution, etc…)*

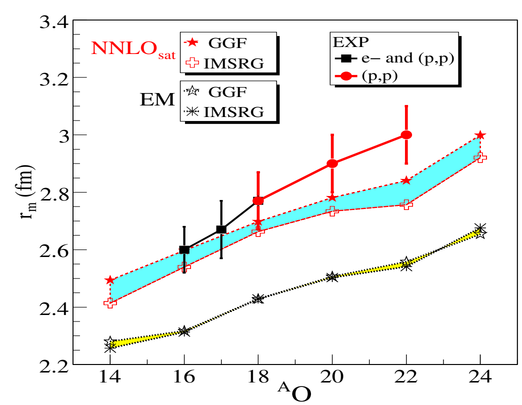
→ *Experimental binding is predicted accurately up to the lower sd shell (A≈30) but deteriorates for medium mass isotopes (Ca and above) with roughly 1 MeV/A over binding.*

→ *New fits of chiral interaction are promising for low-energy observables*

→ *Comparison of spectroscopic strength with experiment is much improved…*

→ *Nuclear forces from Lattice-QCD approaching physical pion mass*





Collaborators

A. Cipollone, C. McIlroy
A. Rios, A. Idini, F. Raimondi

A. Polls

V. Somà, T. Duguet

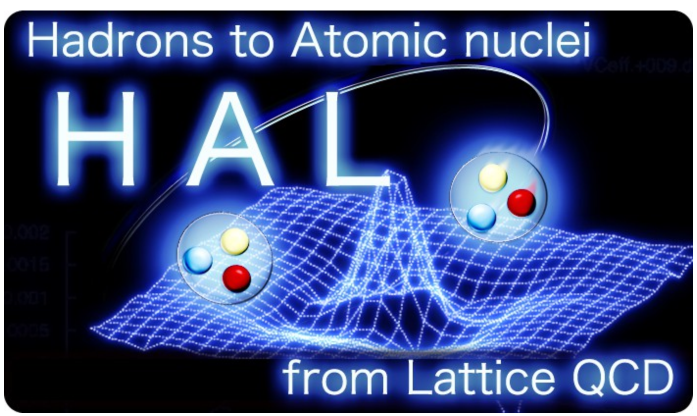
W.H. Dickhoff,
S. Waldecker

A. Carbone

D. Van Neck,

P. Navratil

M. Hjorth-Jensen



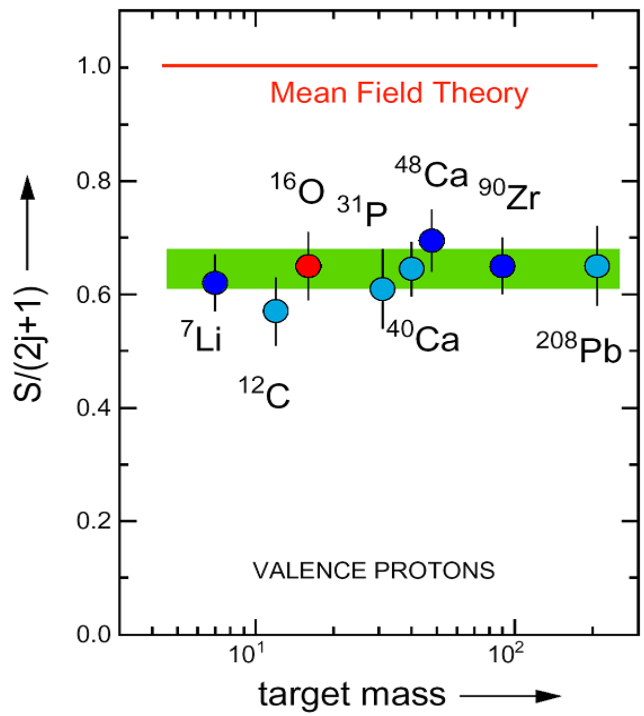
S. Aoki,	YITP Kyoto Univ.
T. Doi, **T. Hatsuda**, Y. Ikeda,	RIKEN Nishina
T. Inoue,	Nihon Univ.
N. Ishii, K. Murano,	RCNP Osaka Univ
H. Nemura, K. Sasaki	Univ. Tsukuba
F. Etminan	Univ. Birjand
T. Miyamoto,	Univ. Tsukuba
T. Iritani	Stony Brook Univ.
S. Gongyo	YITP Kyoto Univ.

Spectroscopic factors

Quenching of SF in stable nuclei

Nucl. Phys. A553 (1993) 297c

NIKHEF:



A common misconception about SRC:

"The quenching is constant over all stable nuclei, so it <u>must be</u> a short-range effect"

Actually, NO!

All calculations show that SRC have just a small effect at the Fermi surface. And the correlation to the <u>experimental p-h</u> gap is much more important.

[W. Dickhoff, CB, Prog. Part. Nucl. Phys. **52**, 377 *(2004)*]

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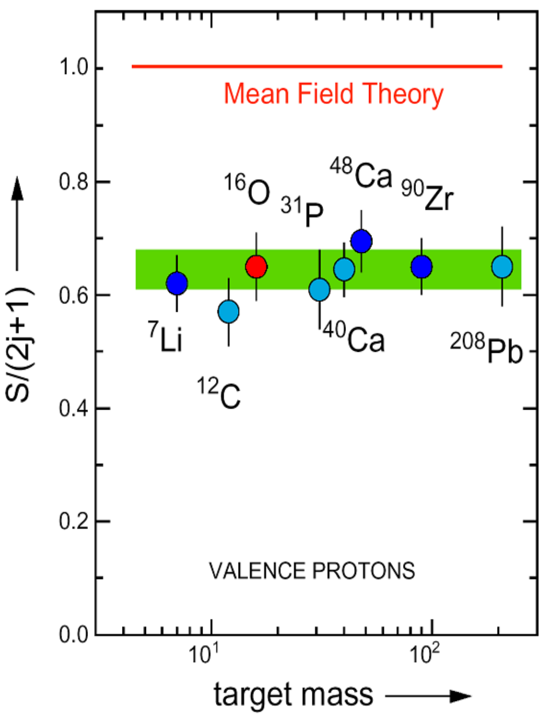
Quenching of SF in stable nuclei

NIKHEF:
Nucl. Phys. A553 (1993) 297c



	$S_{p1/2}$	$S_{p3/2}$
• Short-range correlations oriented methods:		
– VMC [Argonne, '94]	0.90	
– GF(SRC) [St.Louis-Tübingen '95]	0.91	0.89
– FHNC/SOC [Pisa '00]	0.90	
• Including particle-phonon couplings:		
– GF(FRPA) [St.Louis '01]	0.77	0.72
[CB et al., Phys. Rev. C**65**, (02)]		
• Experiment:	0.63	0.67 ±0.07 (estimated uncertainty)

SRC are present and verified experimentally

<u>BUT the are NOT the dominant mechanism for quenching SF!!!</u>

Dependence of Spect. Fact. from p-h gap

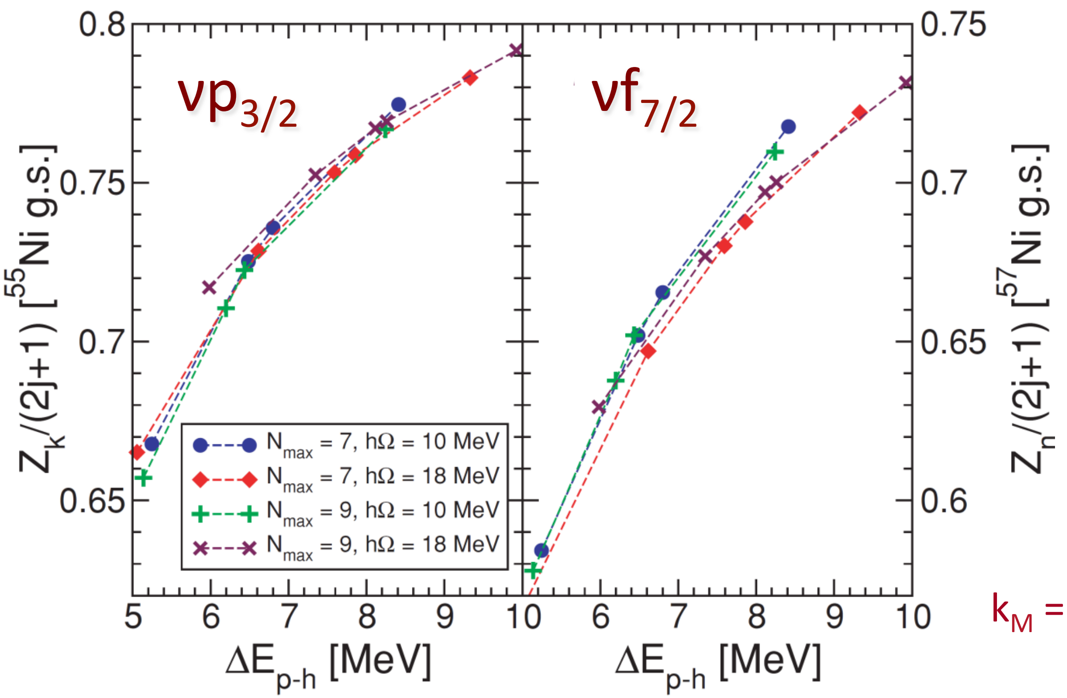
N3LO needs a monopole correction to fix the p-h gap:

$$\begin{cases} \Delta V^T_{fr} \to \Delta V^T_{fr} - (-1)^T \kappa_M , \\ \Delta V^T_{ff} \to \Delta V^T_{ff} - 1.5(1-T)\kappa_M , \end{cases}$$

$r \equiv p_{3/2}, p_{1/2}, f_{5/2}$

$f \equiv f_{7/2}$

Experimental Eph is found for k_M =0,57



k_M = 0.4-0.7 MeV

small k_M ← → large k_M

N3LO interaction + monopole corr.

[CB, M.Hjorth-Jensen, Pys.Rev.C79, 064313 (2009)]

Quenching of absolute spectroscopic factors

[CB, *Phys. Rev. Lett.* **103**, 202520 (2009)]

...with analogous conclusions for ^{48}Ca

Overall quenching of *spectroscopic factors* is driven by:

SRC → *~10%*

part-vibr. coupling → *dominant*

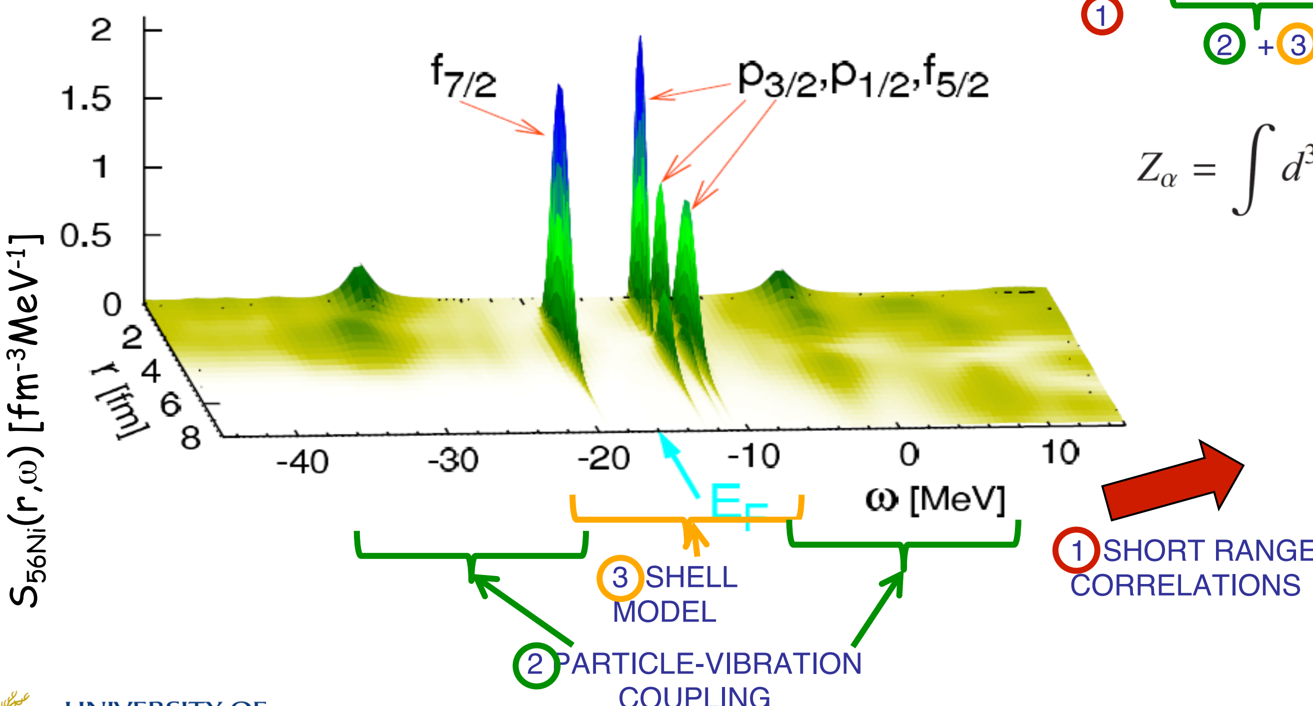
"shell-model" → *in open shell*

	10 osc. shells			Exp. [30]	$1p0f$ space		
	FRPA (SRC)	full FRPA	FRPA $+\Delta Z_\alpha$		FRPA	SM	ΔZ_α
^{57}Ni:							
$\nu 1p_{1/2}$	0.96	0.63	0.61		0.79	0.77	-0.02
$\nu 0f_{5/2}$	0.95	0.59	0.55		0.79	0.75	-0.04
$\nu 1p_{3/2}$	0.95	0.65	0.62	0.58(11)	0.82	0.79	-0.03
^{55}Ni:							
$\nu 0f_{7/2}$	0.95	0.72	0.69		0.89	0.86	-0.03
	1	2 + 3			3		

^{57}Ni

^{55}Ni

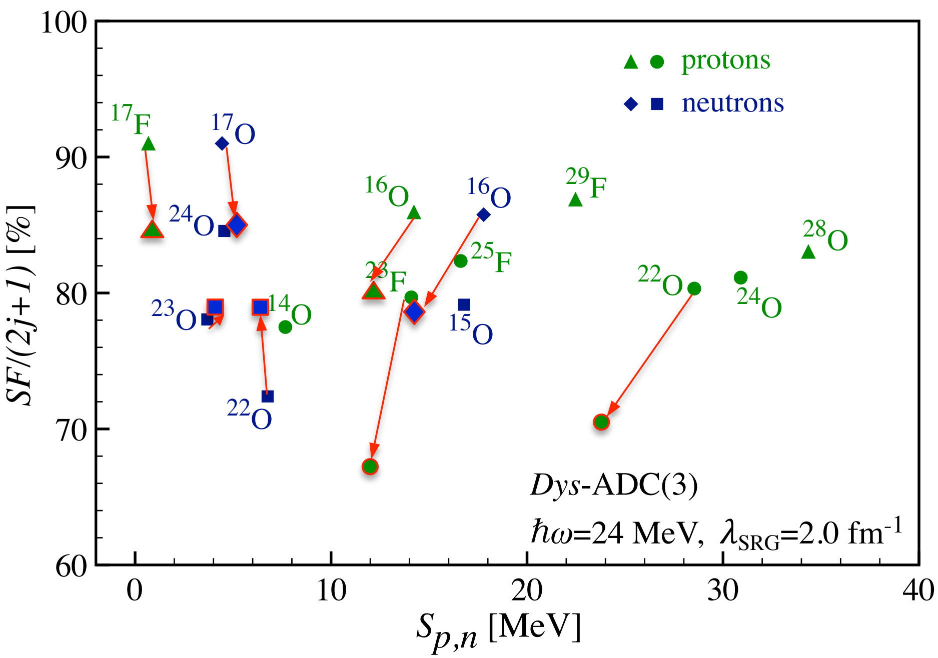
$$Z_\alpha = \int d^3r \, |\psi_\alpha^{overlap}(\mathbf{r})|^2 = \left. \frac{1}{1 - \frac{\partial \Sigma_{\hat\alpha\hat\alpha}(\omega)}{\partial \omega}} \right|_{\omega=\varepsilon_\alpha}$$



Z/N asymmetry dependence of SFs - Theory

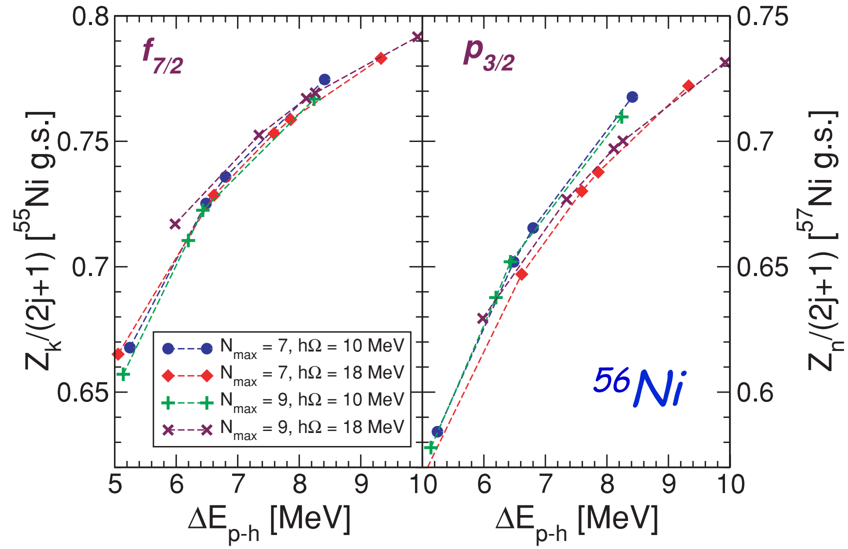
Ab-initio calculations explain (a very weak) the Z/N dependence but the effect is much lower than suggested by direct knockout

Rather the quenching is high correlated to the gap at the Femi surface.



A. Cipollone, CB, P Navrátil
Phys. Rev. C**92**, 014306 (2015)

Spectroscopic factor are strongly correlated to p-h gaps:



CB, M. Hjorth-Jensen,
Phys. Rev. C **79**, 064313 (2009)

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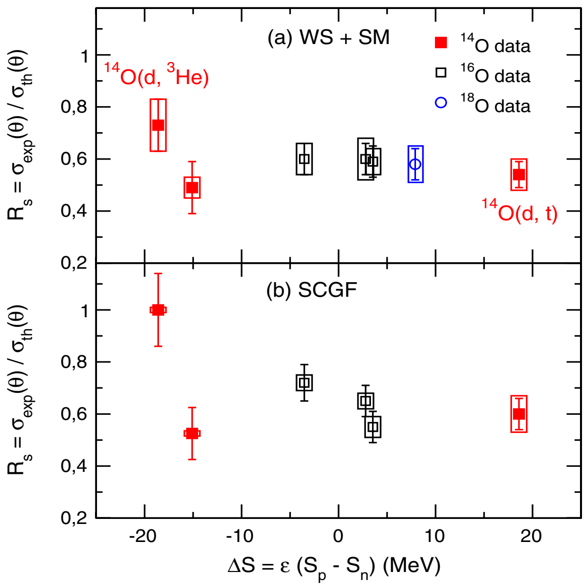
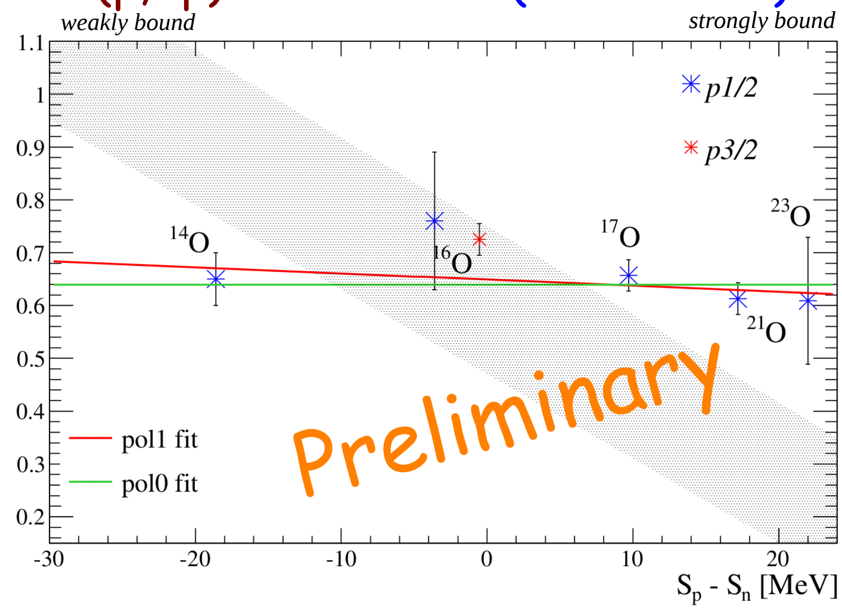
Z/N asymmetry dependence of SFs

$^{14}O(d,t)^{13}O$ and $^{14}O(d,^3He)^{13}N$ transfer reactions @ SPIRAL

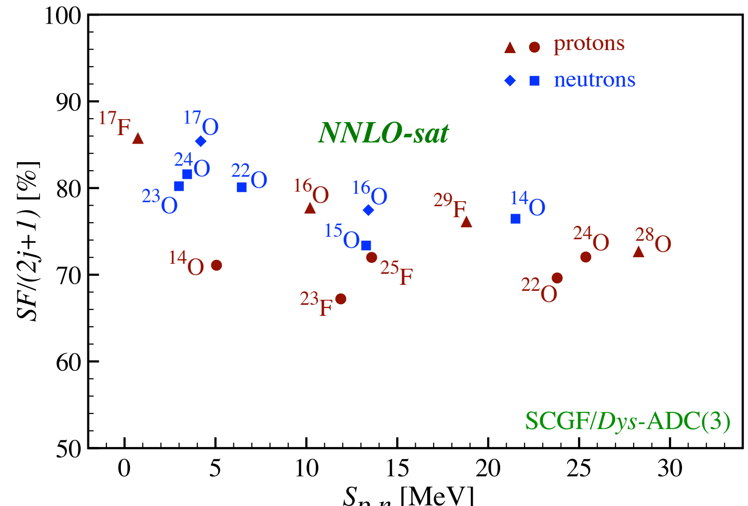


[F. Flavigny et al, PRL**110**, 122503 (2013)]

$^AO(p,2p)^{A-1}N$ at GSI (R3B-LAND)



Calculated spectroscopic factors are:
- independent of asymmetry
- correlated to p-h gaps



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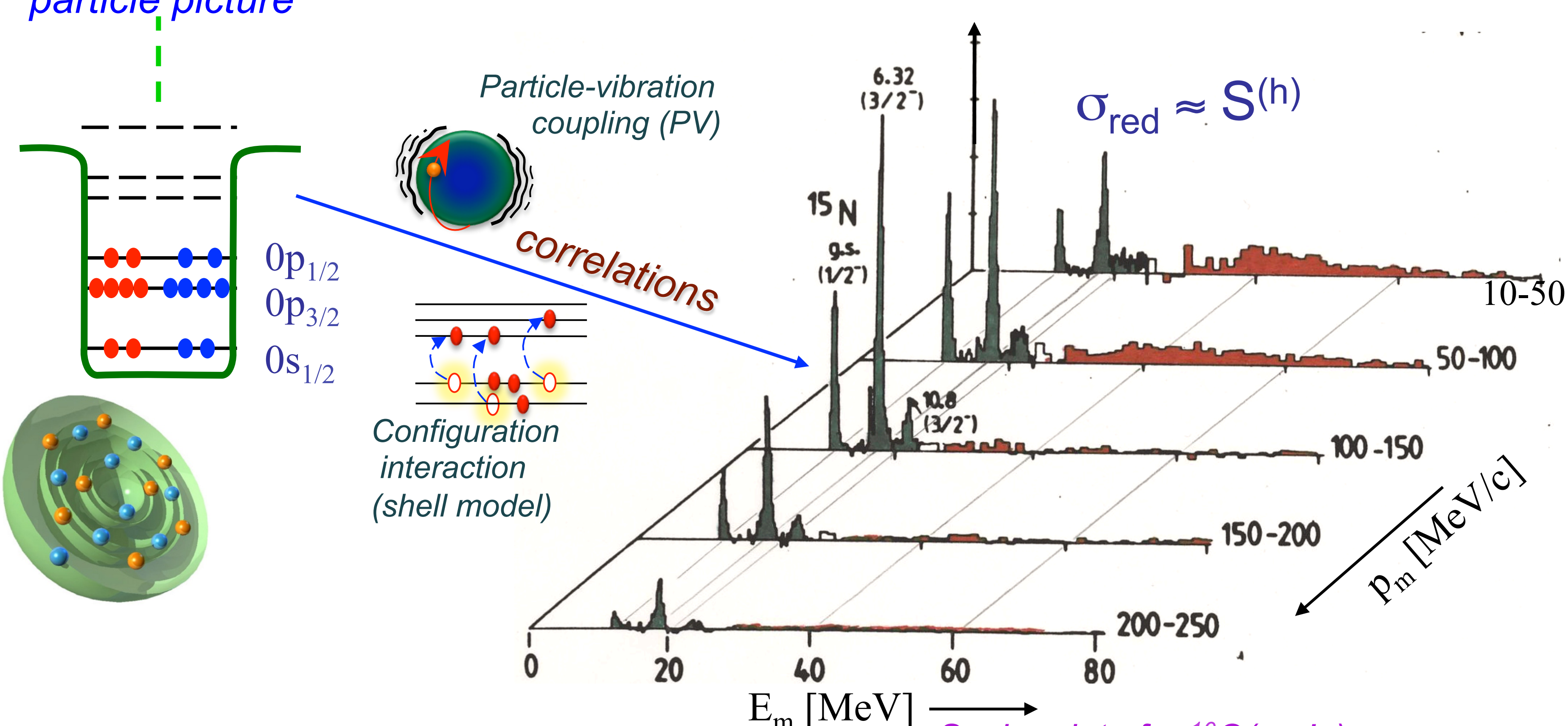
Concept of correlations

Spectral function: distribution of momentum (p_m) and energies (E_m)



Saclay data for $^{16}O(e,e'p)$

[Mougey et al., Nucl. Phys. A335, 35 (1980)]

Understood for a few stable closed shells:

*[CB and W. H. Dickhoff, Prog. Part. Nucl. Phys **52**, 377 (2004)]*