Large-scale shell-model challenges within the RIB era

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- A. Gargano (INFN)
- N. Itaco (SUN and INFN)
- L. C. (INFN)

Framework

- Large-scale shell-model calculations are, at present, a consolidated tool to investigate nuclear properties.
- The new physics coming from RIBs facilities provides a challenging ground, since they are approaching the nuclear driplines.
- The computational complexity of dealing with large model spaces and many interacting valence nucleons is the main problematic to be tackled.

Large-scale shell model

PHYSICAL REVIEW C VOLUME 50, NUMBER 1 JULY 1994

Full pf shell model study of A=48 nuclei

E. Caurier and A. P. Zuker
Groupe de Physique Théorique, Centre de Recherches Nucléaires, Institut National de Physique Nucléaire et de Physique des Particles, Centre National de la Recherche Scientifique, Université Louis Pasteur Boîte Postale 20, F-67037 Strasbourg-Cedex, France

A. Poves and G. Martínez-Pinedo
Departamento de Física Teórica C-XI, Universidad Autónoma de Madrid, E-28049 Madrid, Spain
(Received 16 December 1993)

VOLUME 77, NUMBER 16 PHYSICAL REVIEW LETTERS 14 OCTOBER 1996

Nuclear Shell Model by the Quantum Monte Carlo Diagonalization Method

Michio Honma,[1] Takahiro Mizusaki,[2] and Takaharu Otsuka[2,3]
[1]*Center for Mathematical Sciences, University of Aizu, Tsuruga, Ikki-machi, Aizu-Wakamatsu, Fukushima 965, Japan*
[2]*Department of Physics, University of Tokyo, Hongo, Tokyo 113, Japan*
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(Received 29 April 1996)

Large-scale shell model: shell model calculations performed within a model space made up by a number of orbitals larger than usual.

An extended model space enables to study exotic (for shell model) properties: collective motion, deformation, clustering, etc.

Collective behavior

PRL **110,** 242701 (2013) PHYSICAL REVIEW LETTERS week ending 14 JUNE 2013

Quadrupole Collectivity in Neutron-Rich Fe and Cr Isotopes

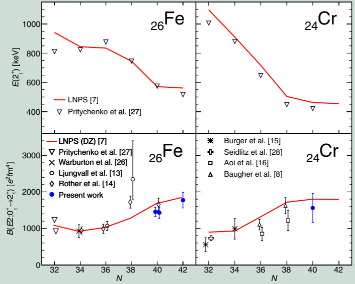
H. L. Crawford,[1] R. M. Clark,[1] P. Fallon,[1] A. O. Macchiavelli,[1] T. Baugher,[2,3] D. Bazin,[2] C. W. Beausang,[4] J. S. Berryman,[2] D. L. Bleuel,[5] C. M. Campbell,[1] M. Cromaz,[1] G. de Angelis,[6] A. Gade,[2,3] R. O. Hughes,[4] I. Y. Lee,[1] S. M. Lenzi,[7] F. Nowacki,[8] S. Paschalis,[1] M. Petri,[1] A. Poves,[9] A. Ratkiewicz,[2,3] T. J. Ross,[4] E. Sahin,[6] D. Weisshaar,[2] K. Wimmer,[2,10] and R. Winkler[2]

Onset of collectivity at $N = 40$

Model space

- 4 proton orbitals:
  $0f_{7/2}, 1p_{3/2}, 1p_{1/2}, 0f_{5/2}$
- 5 neutron orbitals:
  $1p_{3/2}, 1p_{1/2}, 0f_{5/2}, 0g_{9/2}, 1d_{5/2}$

NATHAN shell-model code



Novel collective feaures

RAPID COMMUNICATIONS

PHYSICAL REVIEW C **89**, 031301(R) (2014)

Novel shape evolution in exotic Ni isotopes and configuration-dependent shell structure

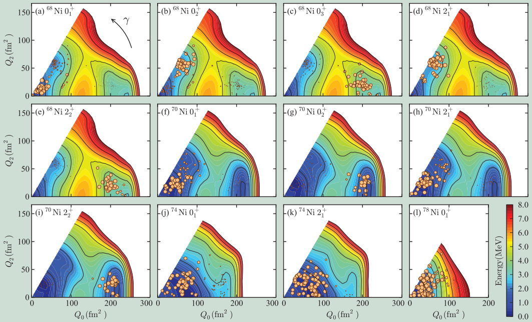
Yusuke Tsunoda,[1] Takaharu Otsuka,[1,2,3] Noritaka Shimizu,[2] Michio Honma,[4] and Yutaka Utsuno[5]

Shape evolution in Ni isotopes

Model space

- 6 proton orbitals:
  $0f_{7/2}, 1p_{3/2}, 1p_{1/2}, 0f_{5/2}, 0g_{9/2}, 1d_{5/2}$
- 6 neutron orbitals:
  $0f_{7/2}, 1p_{3/2}, 1p_{1/2}, 0f_{5/2}, 0g_{9/2}, 1d_{5/2}$

Monte Carlo shell model



Islands of inversion

PHYSICAL REVIEW C **90**, 014302 (2014)

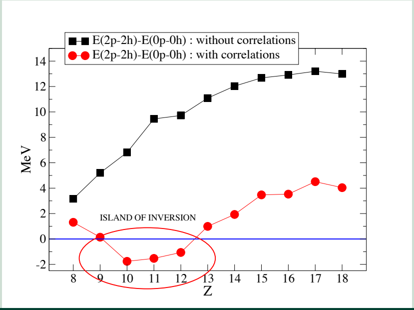
Merging of the islands of inversion at $N = 20$ and $N = 28$

E. Caurier,[1] F. Nowacki,[1] and A. Poves[2,3]
[1]*IPHC, IN2P3-CNRS and Université Louis Pasteur, F-67037 Strasbourg, France*
[2]*Departamento de Física Teórica and IFT-UAM/CSIC, Universidad Autónoma de Madrid, E-28049 Madrid, Spain*
[3]*Isolde (CERN) 1211 Genève 23, Switzerland*

Merging of the $N = 20$ and $N = 28$ islands of inversion in Mg isotopes

Model space:
full sdfp orbitals

NATHAN
shell-model code



Shell evolution

RAPID COMMUNICATIONS

PHYSICAL REVIEW C **91**, 021303(R) (2015)

Quenching of the neutron $N = 82$ shell gap near ^{120}Sr with monopole-driving core excitations

Han-Kui Wang,[1,2] Kazunari Kaneko,[3] and Yang Sun[2,4,*]

[1] *School of Physics and Mechanical and Electrical Engineering, Zhoukou Normal University, Henan 466000, People's Republic of China*

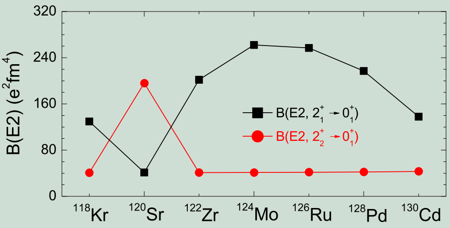
[2] *Department of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai 200240, People's Republic of China*

[3] *Department of Physics, Kyushu Sangyo University, Fukuoka 813-8503, Japan*

[4] *IFSA Collaborative Innovation Center, Shanghai Jiao Tong University, Shanghai 200240, People's Republic of China*

Study of the $N = 82$ shell evolution as a function of the neutron shell gap

- 6 proton orbitals: $0f_{5/2}, 1p_{3/2}, 1p_{1/2}, 0g_{9/2}, 0g_{7/2}, 1d_{5/2}$
- 7 neutron orbitals: $0g_{7/2}, 1d_{5/2}, 1d_{3/2}, 2s_{1/2}, 0h_{11/2}, 1f_{7/2}, 2p_{3/2}$
- NuShellX code



- In calculations [1] both proton model space is spanned by the four orbitals $0f_{7/2}, 1p_{3/2}, 1p_{1/2}, 0f_{5/2}$ and the five neutron ones $1p_{3/2}, 1p_{1/2}, 0f_{5/2}, 0g_{9/2}, 1d_{5/2}$ outside ^{48}Ca core, and the shell model basis is truncated so to retain up to $14p - 14h$ excitations across the $Z = 28$ and $N = 40$ gaps.
- In calculations [2] both proton and neutron model spaces are spanned by the six orbitals $0f_{7/2}, 1p_{3/2}, 1p_{1/2}, 0f_{5/2}, 0g_{9/2}, 1d_{5/2}$ outside ^{40}Ca core. In the m-scheme the dimension of the basis is $\simeq 10^{24}$, reduced to 50 by the importance sampling of the shell-model basis performed within the Monte Carlo Shell Model (MCSM) approach.
- In calculations [3] only neutron $N = 20$ cross-shell excitations are taken into account. Shell model basis has a dimension up to 10^{10}
- In calculations [4] only one valence-neutron is allowed to occupy the $1f_{7/2}, 2p_{3/2}$.

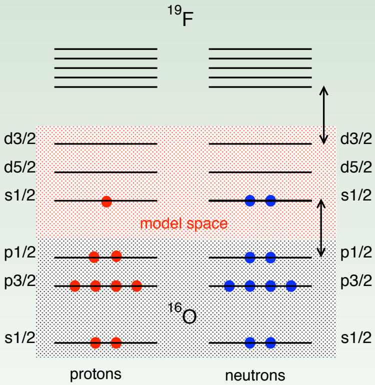
Calculations with a large number of valence nucleons need to employ reduction/truncation schemes.

Those schemes need to be under control, convergence properties and theoretical error estimates are an important tool to understand the reliability of the shell-model calculations.

The realistic shell model

- The derivation of the shell-model hamiltonian using the many-body theory may provide a reliable approach
- The model space may be “shaped” according to the computational needs of the diagonalization of the shell-model hamiltonian
- In such a case, the effects of the neglected degrees of freedom are taken into account by the effective hamiltonian H_{eff} theoretically

An example: ^{19}F



- 9 protons & 10 neutrons interacting
- spherically symmetric mean field (e.g. harmonic oscillator)
- 1 valence proton & 2 valence neutrons interacting in a truncated model space

The degrees of freedom of the core nucleons and the excitations of the valence ones above the model space are not considered explicitly.

Effective shell-model hamiltonian

The shell-model hamiltonian has to take into account in an effective way all the degrees of freedom not explicitly considered

Two alternative approaches

- phenomenological
- microscopic

$$V_{NN} \quad (+V_{NNN}) \Rightarrow \text{many-body theory} \Rightarrow H_{\text{eff}}$$

Definition

The eigenvalues of H_{eff} belong to the set of eigenvalues of the full nuclear hamiltonian

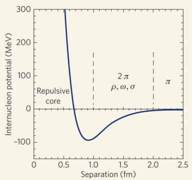
Workflow for a realistic shell-model calculation

1. Choose a realistic *NN* potential (*NNN*)
2. Determine the model space better tailored to study the system under investigation
3. Derive the effective shell-model hamiltonian by way of the many-body theory
4. Calculate the physical observables (energies, e.m. transition probabilities, ...)

Realistic nucleon-nucleon potential: V_{NN}

Several realistic potentials $\chi^2/datum \simeq 1$:
CD-Bonn, Argonne V18, Nijmegen, ...

Strong short-range repulsion



How to handle the short-range repulsion ?

- Brueckner G matrix
- EFT inspired approaches

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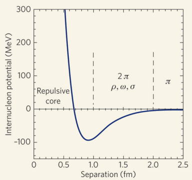

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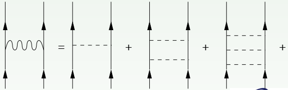

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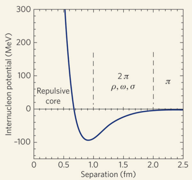

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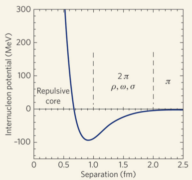

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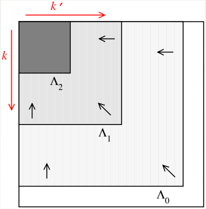

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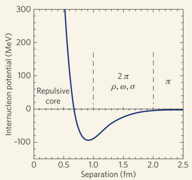

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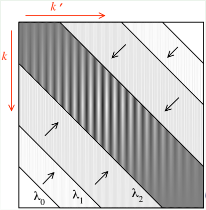

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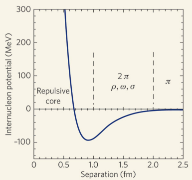
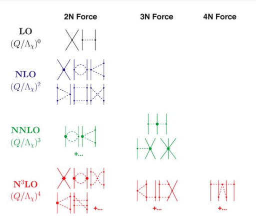




The shell-model effective hamiltonian

A-nucleon system Schrödinger equation

$$H|\Psi_\nu\rangle = E_\nu|\Psi_\nu\rangle$$

with

$$H = H_0 + H_1 = \sum_{i=1}^{A}(T_i + U_i) + \sum_{i<j}(V_{ij}^{NN} - U_i)$$

Model space

$$|\Phi_i\rangle = [a_1^\dagger a_2^\dagger \ldots a_n^\dagger]_i|c\rangle \Rightarrow P = \sum_{i=1}^{d}|\Phi_i\rangle\langle\Phi_i|$$

Model-space eigenvalue problem

$$H_{\mathrm{eff}}P|\Psi_\alpha\rangle = E_\alpha P|\Psi_\alpha\rangle$$

The shell-model effective hamiltonian

$$\left(\begin{array}{c|c} PHP & PHQ \\ \hline QHP & QHQ \end{array}\right) \underset{Q\mathcal{H}P=0}{\overset{\mathcal{H}=X^{-1}HX}{\Longrightarrow}} \left(\begin{array}{c|c} P\mathcal{H}P & P\mathcal{H}Q \\ \hline 0 & Q\mathcal{H}Q \end{array}\right)$$

$$H_{\rm eff} = P\mathcal{H}P$$

Suzuki & Lee $\Rightarrow$ $X = e^{\omega}$ with $\omega = \left(\begin{array}{c|c} 0 & 0 \\ \hline Q\omega P & 0 \end{array}\right)$

$$H_1^{\rm eff}(\omega) = PH_1P + PH_1Q\frac{1}{\epsilon - QHQ}QH_1P - \\ -PH_1Q\frac{1}{\epsilon - QHQ}\omega H_1^{\rm eff}(\omega)$$

The shell-model effective hamiltonian

Folded-diagram expansion

$\hat{Q}$-box vertex function

$$\hat{Q}(\epsilon) = PH_1P + PH_1Q\frac{1}{\epsilon - QHQ}QH_1P$$

$\Rightarrow$ Recursive equation for $H_{\rm eff}$ $\Rightarrow$ iterative techniques (Krenciglowa-Kuo, Lee-Suzuki, ...)

$$H_{\rm eff} = \hat{Q} - \hat{Q}'\int\hat{Q} + \hat{Q}'\int\hat{Q}\int\hat{Q} - \hat{Q}'\int\hat{Q}\int\hat{Q}\int\hat{Q}\cdots,$$

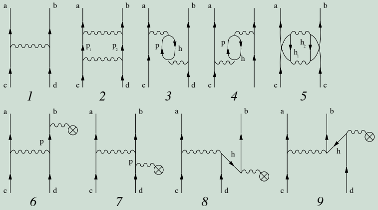
The perturbative approach to the shell-model H^{eff}

$$\hat{Q}(\epsilon) = PH_1P + PH_1Q\frac{1}{\epsilon - QHQ}QH_1P$$

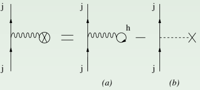
The $\hat{Q}$-box can be calculated perturbatively

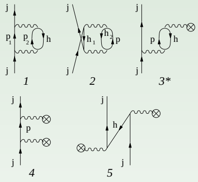
$$\frac{1}{\epsilon - QHQ} = \sum_{n=0}^{\infty}\frac{(QH_1Q)^n}{(\epsilon - QH_0Q)^{n+1}}$$

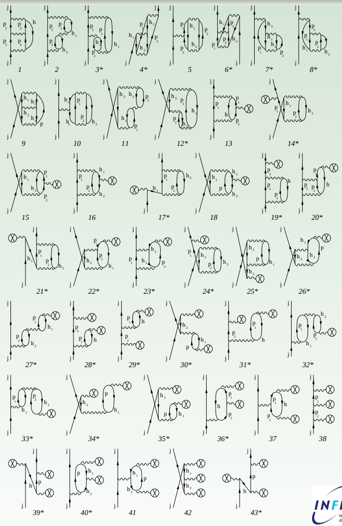
The diagrammatic expansion of the $\hat{Q}$-box



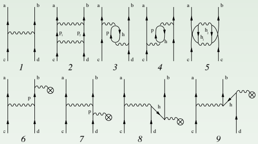
$\hat{Q}$-box perturbative expansion: 1-body diagrams

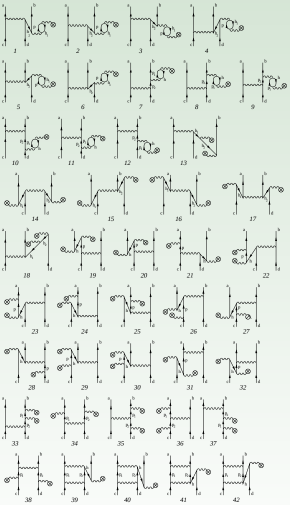






$\hat{Q}$-box perturbative expansion: 2-body diagrams





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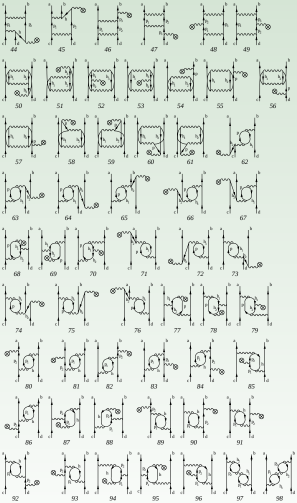
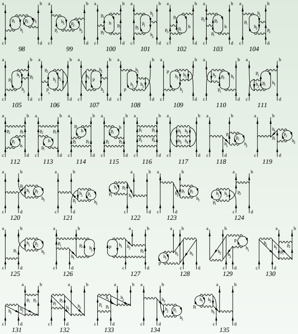
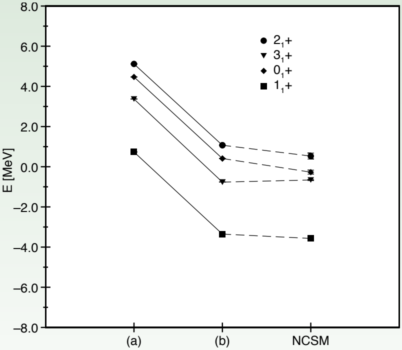




Benchmark calculation

A benchmark calculation has been performed for light *p*-shell nuclei, comparing realistic shell-model calculations with those of NCSM, starting from chiral two-body potential N^{3}LO.



(a) not translationally invariant Hamiltonian

(b) purely intrinsic hamiltonian

L.C., A. Covello, A. Gargano, N. Itaco, and T. T. S. Kuo, Ann. Phys. ***327*** *, 2125-2151 (2012)*

Our recipe for realistic shell model

- Input V_{NN}: $V_{\rm low-k}$ derived from the high-precision *NN* CD-Bonn potential with a cutoff: $\Lambda = 2.6$ fm^{-1}.
- $H_{\rm eff}$ obtained calculating the *Q*-box up to the 3rd order in $V_{\rm low-k}$.
- Effective electromagnetic operators are consistently derived by way of the the MBPT

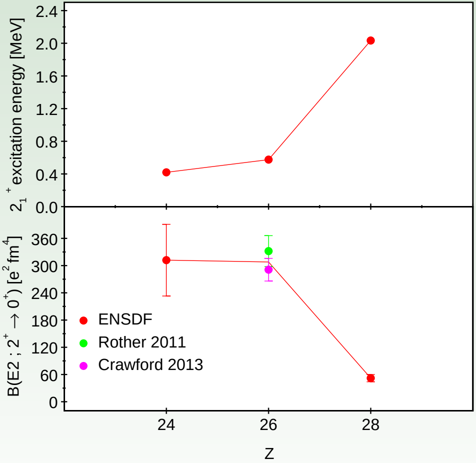
Double-step approach

However, it may occur that H_{eff} can be diagonalized for a certain class of nuclei, but not for other with a larger number of valence nucleons

Recently, we have started to explore the possibility to perform a double-step approach to the renormalization of the shell-model hamiltonian

More precisely, after we have derived H_{eff} in a certain model space P, starting from this one we generate a new $H_{\mathrm{eff}}^{\mathrm{new}}$ acting in a truncated subspace $P^{\mathrm{new}} \subset P$

First example: the collectivity at $N = 40$



The collectivity at $N = 40$: the model space

Within the shell-model framework the key role for the onset/disappearance of the $N = 40$ collectivity is played by the interaction between the quadrupole partners $\nu 0g_{9/2}, \nu 1d_{5/2}$

In order to study this phenomenon we have chosen to perform a sort of "differential diagnosis", employing as the proton model space the $\pi 0f_{7/2}, \pi 1p_{3/2}$ orbitals, and two different neutron model spaces:

- Model space I: $1p_{3/2}, 1p_{1/2}, 0f_{5/2}, 0g_{9/2}$
- Model space II: $1p_{3/2}, 1p_{1/2}, 0f_{5/2}, 0g_{9/2}, 1d_{5/2}$

The double-step procedure

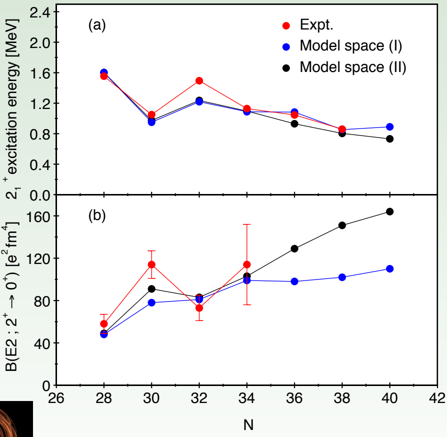
In order to make this comparison as much consistent as possible, we have followed this procedure

1. We have derived first $H_{\rm eff}$ - within the MBPT - in a very large model space outside the ^{40}Ca closed core, and spanned by six proton and neutron *pfgd* orbitals.
2. Then, we derive from this "mother hamiltonian" two new effective hamiltonians - again using MBPT - defined in the smaller model spaces (I) and (II).
3. Single-particle energies are taken for experimental data.

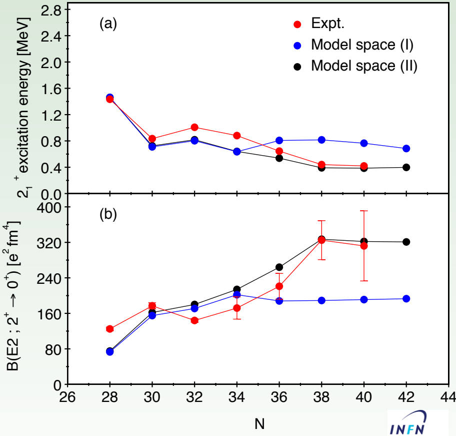
L. C., A. Covello, A. Gargano, and N. Itaco, Phys. Rev. C **89**, *024319 (2014)*

Collectivity at $N = 40$

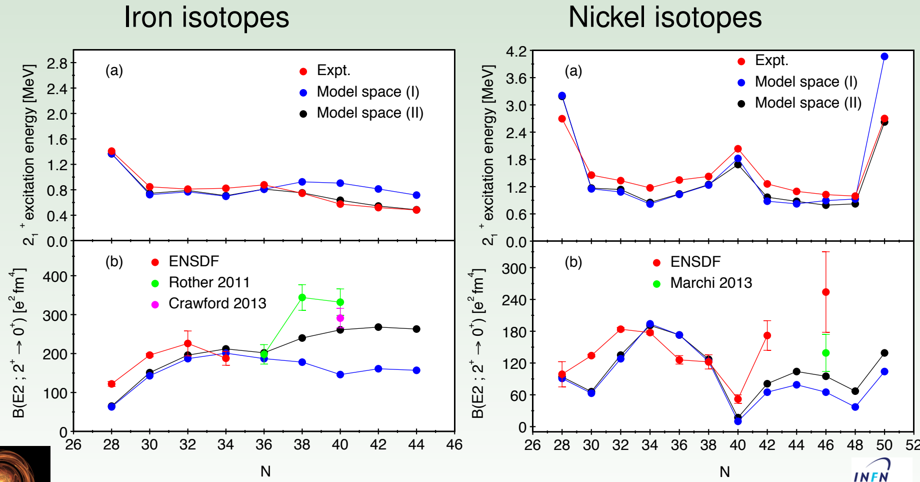
Titanium isotopes



Chromium isotopes



Collectivity at $N = 40$



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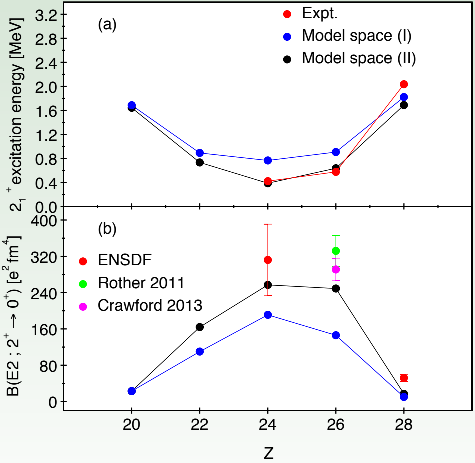


A second example: quadrupole collectivity around $Z = 50$

Our interest: to study the quadrupole collectivity due to $Z = 50$ cross-shell excitations in even-mass isotopic chains above ^{88}Sr.

Model space of the "mother hamiltonian":

Proton orbitals	Neutron orbitals
$1p_{1/2}$	
$0g_{9/2}$	
$1d_{5/2}$	$1d_{5/2}$
$0g_{7/2}$	$0g_{7/2}$
$1d_{3/2}$	$1d_{3/2}$
$2s_{1/2}$	$2s_{1/2}$
$0h_{11/2}$	$0h_{11/2}$

A modest proposal

A "Poor Man's Approach" to lighten the computational complexity of diagonalizing the "mother hamiltonian" H^{75} defined in a large shell-model space:

- First step: analyze the evolution of the effective single-particle energies (ESPE) of the "mother hamiltonian", and locate the relevant degrees of freedom (single-particle orbitals).
- Second step: perform a unitary transformation of the "mother hamiltonian" into a reduced model space, so to obtain an effective hamiltonian that is computationally.

Single-particle energies, effective two-body matrix elements, and effective electromagnetic operators are all derived from theory

L. C., A. Covello, A. Gargano, N. Itaco, and T. T. S. Kuo, Phys. Rev. C **91**, *041301 (2015)*

L. C., A. Gargano, and N. Itaco, Phys. Rev. C **93**, *064328 (2016)*

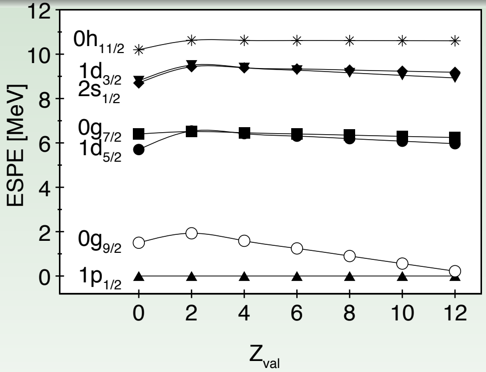
Single-particle properties with H^{75}

orbital	proton s.p.e.
$1p_{1/2}$	0.0
$0g_{9/2}$	1.5
$0g_{7/2}$	5.7
$1d_{5/2}$	6.4
$1d_{3/2}$	8.8
$2s_{1/2}$	8.7
$0h_{11/2}$	10.2
orbital	neutron s.p.e.
$1d_{5/2}$	0.0
$0g_{7/2}$	1.5
$2s_{1/2}$	2.2
$1d_{3/2}$	3.4
$0h_{11/2}$	5.1

$n_a l_a j_a \; n_b l_b j_b$	$\langle a \| e_p \| b \rangle$
$0g_{9/2}\; 0g_{9/2}$	1.62
$0g_{9/2}\; 0g_{7/2}$	1.67
$0g_{9/2}\; 1d_{5/2}$	1.60
$0g_{7/2}\; 0g_{7/2}$	1.73
$0g_{7/2}\; 1d_{5/2}$	1.74
$0g_{7/2}\; 1d_{3/2}$	1.76
$1d_{5/2}\; 1d_{5/2}$	1.73
$1d_{5/2}\; 1d_{3/2}$	1.72
$1d_{5/2}\; 2s_{1/2}$	1.76
$1d_{3/2}\; 1d_{3/2}$	1.74
$1d_{3/2}\; 2s_{1/2}$	1.76
$0h_{11/2}\; 0h_{11/2}$	1.72

$n_a l_a j_a \; n_b l_b j_b$	$\langle a \| e_n \| b \rangle$
$0g_{7/2}\; 0g_{7/2}$	0.94
$0g_{7/2}\; 1d_{5/2}$	0.96
$0g_{7/2}\; 1d_{3/2}$	0.95
$1d_{5/2}\; 1d_{5/2}$	0.94
$1d_{5/2}\; 1d_{3/2}$	0.97
$1d_{5/2}\; 2s_{1/2}$	0.79
$1d_{3/2}\; 1d_{3/2}$	0.96
$1d_{3/2}\; 2s_{1/2}$	0.79
$0h_{11/2}\; 0h_{11/2}$	0.87

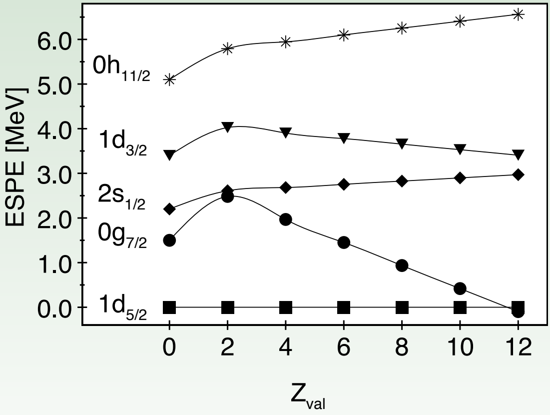
Proton ESPE



$$\mathrm{ESPE}(j) = \epsilon_j + \sum_{j'} V_{jj'} n_{j'}$$

with $V_{jj'} = \sum_J (2J+1) \langle jj' | V | jj' \rangle_J / \sum_J (2J+1)$

Neutron ESPE



Truncating the model space

- The evolution of proton and neutron ESPE suggests a possible reduction of both model spaces.
- By way of a unitary transformation we can derive a $H_{\rm eff}^{4n}$ defined in a reduced proton model space spanned only by 4 orbitals $1p_{1/2}, 0g_{9/2}, 0g_{7/2}, 1d_{5/2}$ and a neutron one spanned by both the 5 original orbitals or by only 2 orbitals $0g_{7/2}, 1d_{5/2}$.
- The physics of two valence-nucleon systems is exactly preserved.

The second step

Let us sketch out the derivation of H^{4n}.

The eigenvalue problem for H^{75} is:

$$H^{75}|\psi_k\rangle = E_k|\psi_k\rangle \qquad k = 1, ..., N$$

H^{75} is the sum of the unperturbed single-particle hamiltonian H_0 and the residual two-body potential V

$$H^{75} = H_0 + V \quad .$$

The model space is splitted up in two subspaces P^{4n} and $Q^{3,5-n}$. Since H_0 is diagonal:

$$H_0 = PH_0P + QH_0Q \quad .$$

The second step

The P-space eigenvalue problem is:

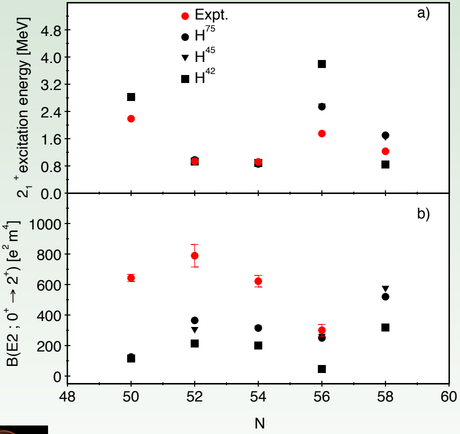
$$H^{4n}|\phi_k\rangle = \left(PH_oP + V^{4n}\right)|\phi_k\rangle = E_k|\phi_k\rangle \quad k = 1, ..., d$$

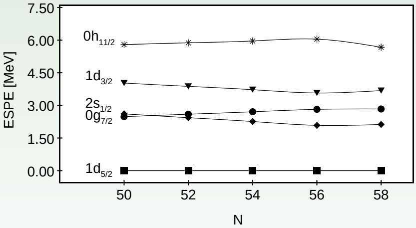
where $|\phi_k\rangle = P|\psi_k\rangle$.
The eigenvalue problem for H^{75} can be easily solved for the two valence-nucleon systems (^{90}Zr,^{90}Sr,^{90}Y), and consequently providing the E_k, ψ_k.
The solutions of the equation for the effective residual interaction $V^{4,n}$ are given by:

$$V^{4n} = \sum_{k=1}^{d} (E_k - E_0)|\phi_k\rangle\langle\tilde{\phi}_k| \quad ,$$

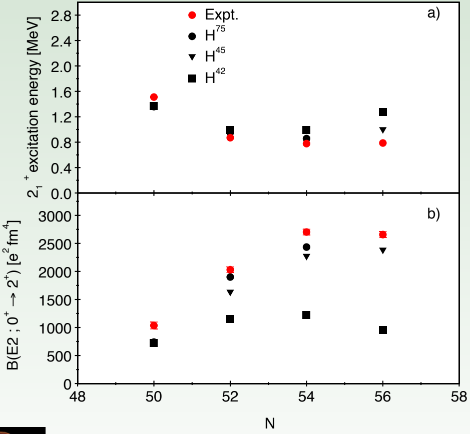
where $|\tilde{\phi}_k\rangle$ are biorthogonal states defined as $|\tilde{\phi}_k\rangle\langle\phi_{k'}| = \delta_{kk'}$

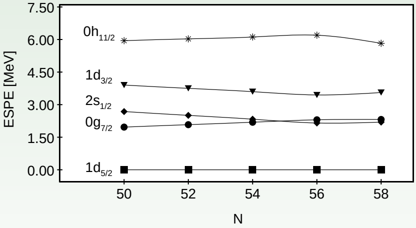
Results for Zr isotopes





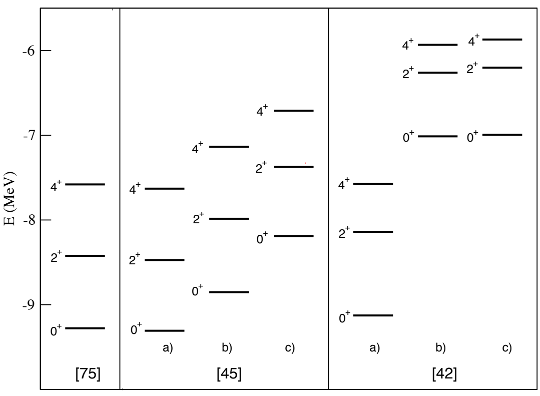
Results for Mo isotopes





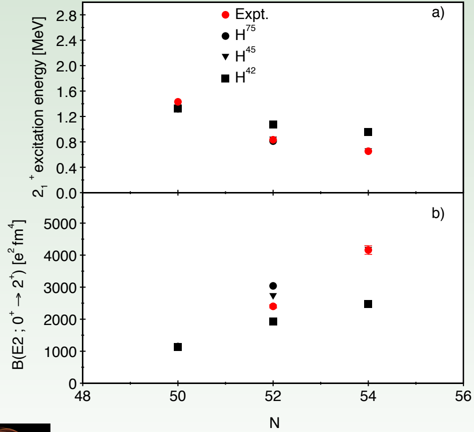
A closer look to ^{96}Mo

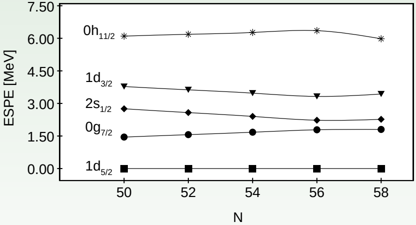
- $H^{75} \approx 10^8$
- $H^{45}_{\rm eff} \approx 10^7$
- $H^{42}_{\rm eff} \approx 10^5$



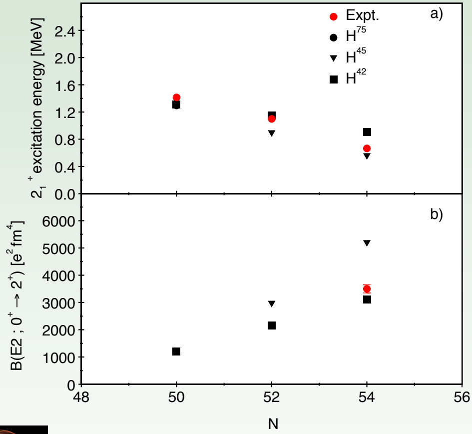
a) $H^{45}_{\rm eff}$ and $H^{42}_{\rm eff}$ with double-step procedure;
b) $\tilde{H}^{45}$, $\tilde{H}^{42}_{\rm eff}$ derived by way of the many-body perturbation theory;
c) the hamiltonian H^{75}, but constraining the calculations to model spaces [45], [42].

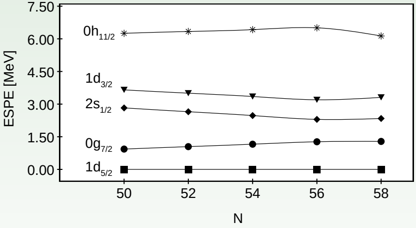
Results for Ru isotopes



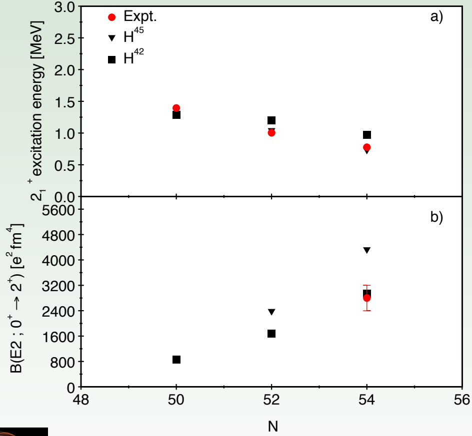


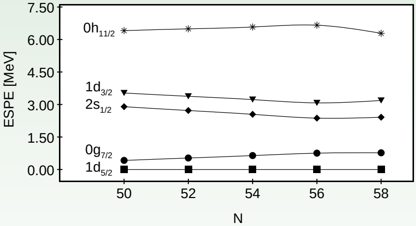
Results for Pd isotopes



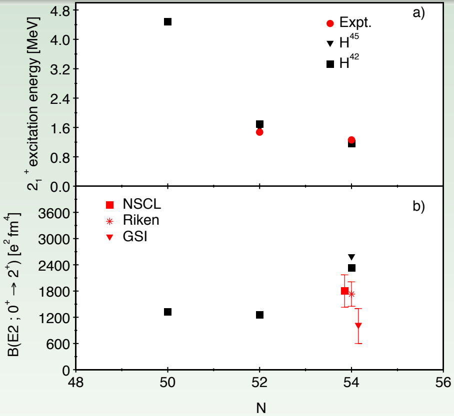


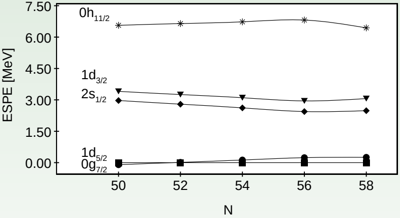
Results for Cd isotopes





Results for Sn isotopes





	^{100}Sn	^{102}Sn	^{104}Sn
H^{75}	10^9	10^{11}	10^{13}
$H^{45}_{\rm eff}$	10^5	10^8	10^9
$H^{42}_{\rm eff}$	10^5	10^7	10^8

Last example: open-shell nuclei around ^{132}Sn

Our goal: to study the structure of nuclei that are currently of exeperimental interest to detect the neutrinoless double-beta decay.

Model space of the "mother hamiltonian":

Proton orbitals	Neutron orbitals
$1d_{5/2}$	$1d_{5/2}$
$0g_{7/2}$	$0g_{7/2}$
$1d_{3/2}$	$1d_{3/2}$
$2s_{1/2}$	$2s_{1/2}$
$0h_{11/2}$	$0h_{11/2}$

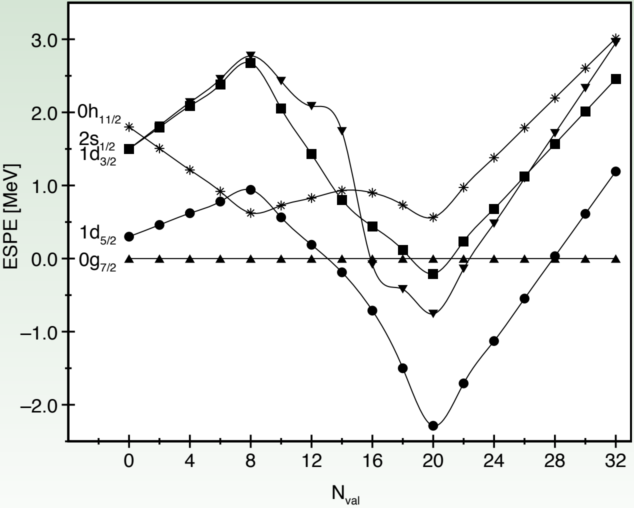
Single-particle properties with H^{55}

orbital	proton s.p.e.
$0g_{7/2}$	0.0
$1d_{5/2}$	0.3
$1d_{3/2}$	1.5
$2s_{1/2}$	1.5
$0h_{11/2}$	1.8
orbital	neutron s.p.e.
$0g_{7/2}$	0.0
$1d_{5/2}$	0.6
$1d_{3/2}$	1.5
$2s_{1/2}$	1.2
$0h_{11/2}$	2.7

$n_a l_a j_a \;\; n_b l_b j_b$	$\langle a \vert e_p \vert b \rangle$
$0g_{7/2}\;0g_{7/2}$	1.42
$0g_{7/2}\;1d_{5/2}$	1.40
$0g_{7/2}\;1d_{3/2}$	1.40
$1d_{5/2}\;1d_{5/2}$	1.21
$1d_{5/2}\;1d_{3/2}$	1.28
$1d_{5/2}\;2s_{1/2}$	1.23
$1d_{3/2}\;1d_{3/2}$	1.26
$1d_{3/2}\;2s_{1/2}$	1.29
$0h_{11/2}\;0h_{11/2}$	1.31

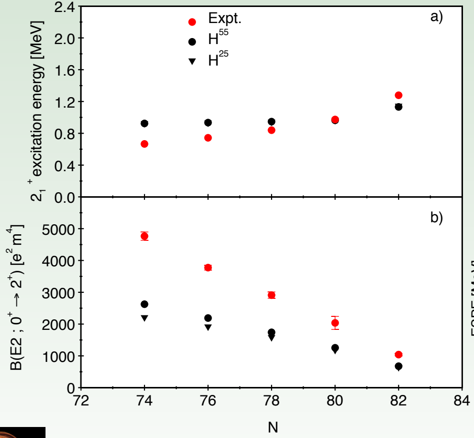
$n_a l_a j_a \;\; n_b l_b j_b$	$\langle a \vert e_n \vert b \rangle$
$0g_{7/2}\;0g_{7/2}$	1.00
$0g_{7/2}\;1d_{5/2}$	1.03
$0g_{7/2}\;1d_{3/2}$	0.98
$1d_{5/2}\;1d_{5/2}$	0.63
$1d_{5/2}\;1d_{3/2}$	0.65
$1d_{5/2}\;2s_{1/2}$	0.62
$1d_{3/2}\;1d_{3/2}$	0.69
$1d_{3/2}\;2s_{1/2}$	0.68
$0h_{11/2}\;0h_{11/2}$	0.68

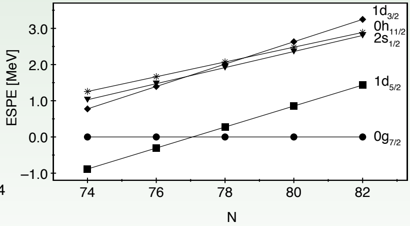
Proton ESPE



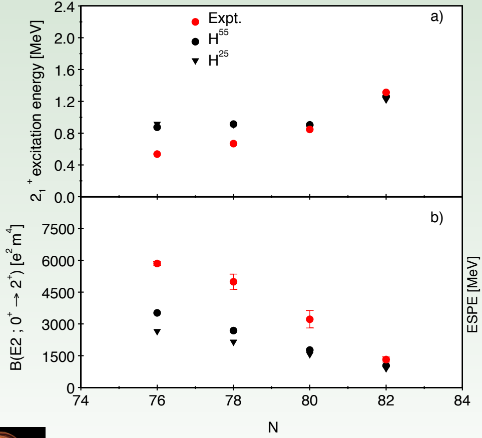
3.0
2.0
1.0
0.0
−1.0
−2.0
ESPE [MeV]
0h11/2
2s1/2
1d3/2
1d5/2
0g7/2
0
4
8
12
16
20
24
28
32
Nval

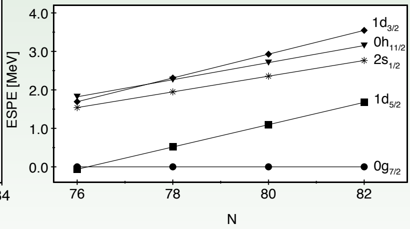
Results for heavy Te isotopes



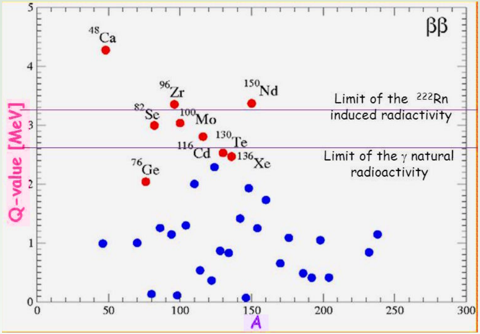


Results for heavy Xe isotopes



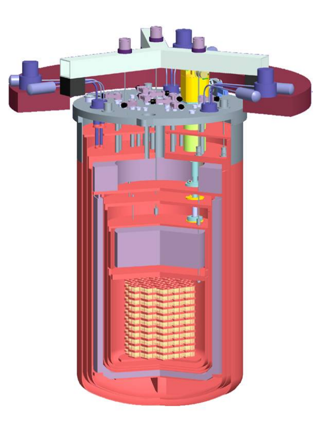


The detection of the $0\nu\beta\beta$-decay



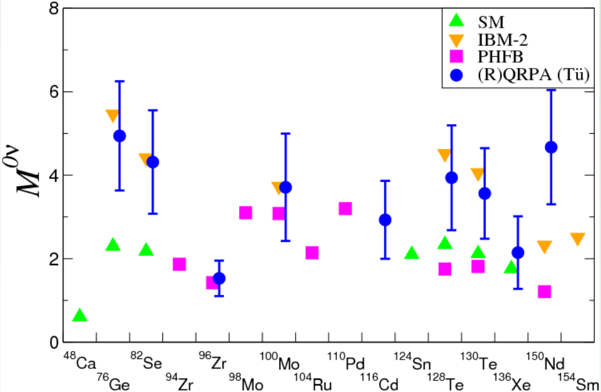
- First group: ^{76}Ge, ^{130}Te, and ^{136}Xe.
- Second group: ^{82}Se, ^{100}Mo, and ^{116}Cd.
- Third group: ^{48}Ca, ^{96}Zr, and ^{150}Nd.

CUORE@LNGS



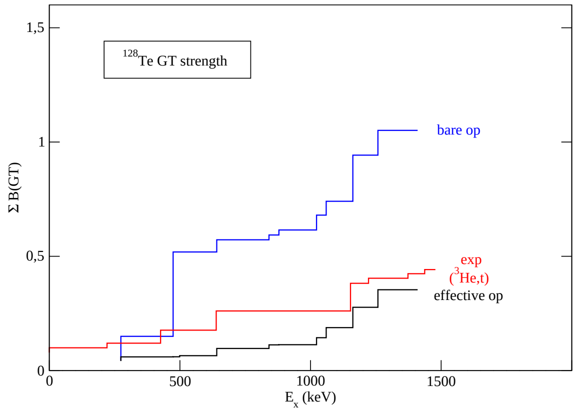
- TeO_2 crystals used as low heat capacity bolometers, arranged into towers and cooled in a large cryostat to approximately 10 m°K with a dilution refrigerator.
- The detectors are isolated from backgrounds by ultrapure low-radioactivity shielding.
- Temperature spikes from electrons emitted in Te $0\beta\beta$ are collected for spectrum analysis.

Nuclear structure calculations

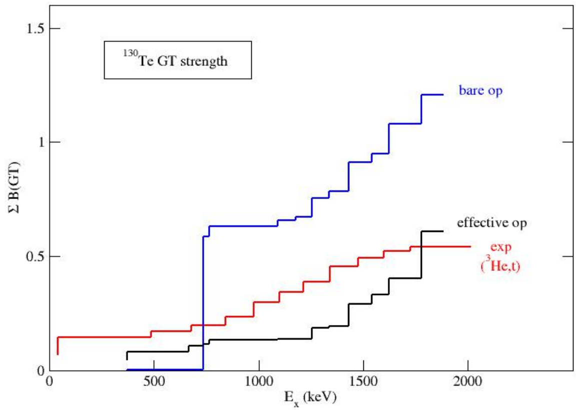


- The spread of nuclear structure calculations evidences inconsistencies among results obtained with different models

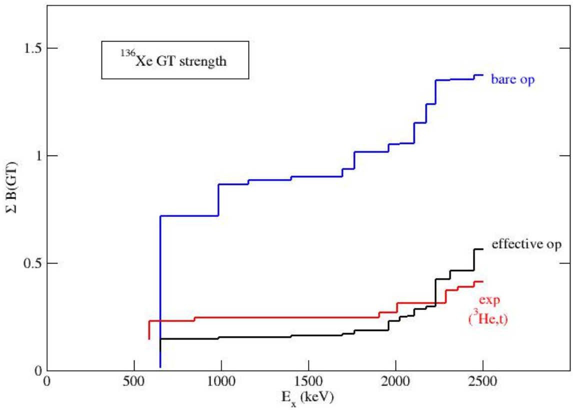
Gamow-Teller strengths for ^{128}Te



Gamow-Teller strengths for ^{130}Te



Gamow-Teller strengths for ^{136}Xe



Conclusions and outlook

- The introduction of a double-step procedure allows a reduction the complexity of the computational problem, and may be useful for other large scale shell-model calculations.
- Quadrupole collectivities in isotopic chains outside ^{48}Ca and ^{88}Sr cores are well reproduced.
- We are working to extend the procedure to consider also the truncation of the degrees of freedom of filled shell-model orbitals.
- The calculation of effective two-body operators are in order to improve the calculation of the electromagnetic-multipole transition rates.
- A parameter-free calculation of the NME for the neutrinoless double β-decay is in progress.

Present Prospects in Nuclear Structure

12th International Spring Seminar in Nuclear Physics, May 15-19 2017, Sant'Angelo d'Ischia

https://agenda.infn.it/event/spring2017

Test case: *p*-shell nuclei

- $V_{NN} \Rightarrow$ chiral N^{3}LO potential by Entem & Machleidt (smooth cutoff $\simeq 2.5$ fm^{-1})
- $H_{\rm eff}$ for two valence nucleons outside ^{4}He
- Single-particle energies and residual two-body interaction are derived from the theory. No empirical input

First, some convergence checks !

L.C., A. Covello, A. Gargano, N. Itaco, and T. T. S. Kuo, Ann. Phys. **327** *, 2125-2151 (2012)*

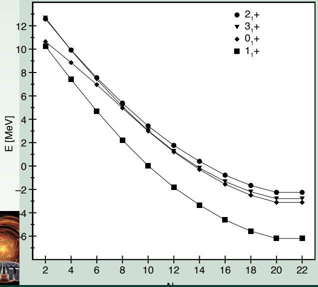
Convergence checks

The intermediate-state space Q

Q-space is truncated: intermediate states whose unperturbed excitation energy is greater than a fixed value E_{max} are disregarded

$$|\epsilon_0 - QH_0Q| \leq E_{\mathrm{max}} = N_{\mathrm{max}}\hbar\omega$$

^{6}Li yrast states

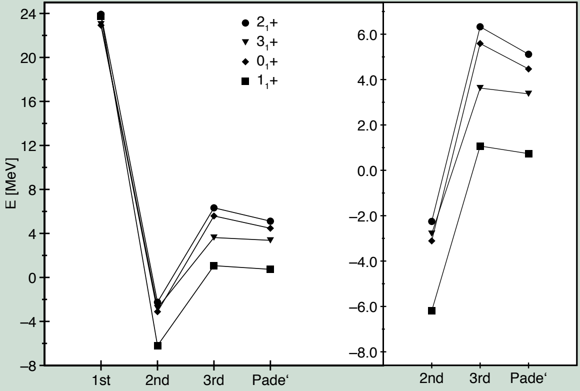


results stable for $N_{\mathrm{max}} \geq 20$

Convergence checks

Order-by-order convergence

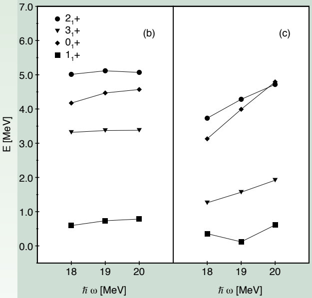
Compare results from $H^{\text{eff}}_{1\text{st}}$, $H^{\text{eff}}_{2\text{nd}}$, $H^{\text{eff}}_{3\text{rd}}$ and $H^{\text{eff}}_{\text{Padè}}$



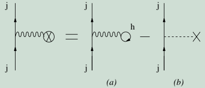
Convergence checks

Dependence on $\hbar\omega$

Auxiliary potential $U \Rightarrow$ harmonic oscillator potential



HF-insertions



- zero in a self-consistent basis
- neglected in most applications
- disregard of HF-insertions introduces relevant dependence on $\hbar\omega$

Benchmark calculation

Approximations are under control ... and what about the accuracy of the results ?

Compare the results with the “exact” ones

ab initio no-core shell model (NCSM)

P. Navrátil, E. Caurier, Phys. Rev. C **69**, 014311 (2004)

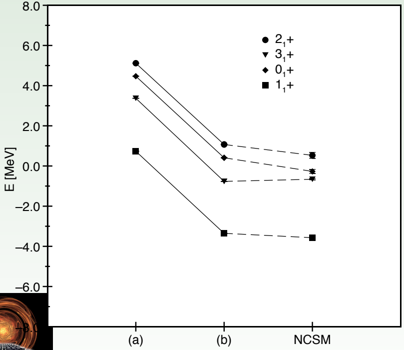
P. Navrátil *et al.*, Phys. Rev. Lett. **99**, 042501 (2007)

Benchmark calculation

To compare our results with NCSM we need to start from a translationally invariant Hamiltonian

$$H_{int} = \left(1 - \frac{1}{A}\right) \sum_{i=1}^{A} \frac{p_i^2}{2m} + \sum_{i<j=1}^{A} \left(V_{ij}^{NN} - \frac{\mathbf{p_i} \cdot \mathbf{p_j}}{mA} \right) =$$

$$= \left[\sum_{i=1}^{A} \left(\frac{p_i^2}{2m} + U_i \right) \right] + \left[\sum_{i<j=1}^{A} \left(V_{ij}^{NN} - U_i - \frac{p_i^2}{2mA} - \frac{\mathbf{p_i} \cdot \mathbf{p_j}}{mA} \right) \right]$$

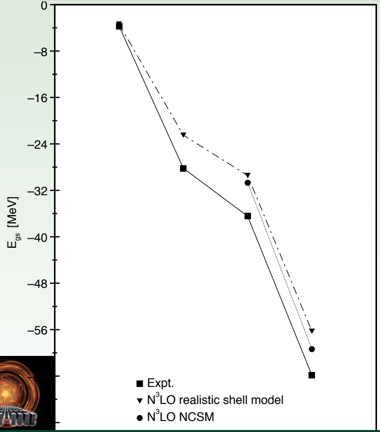


(a) not translationally invariant Hamiltonian
(b) purely intrinsic hamiltonian

Benchmark calculation

Remark

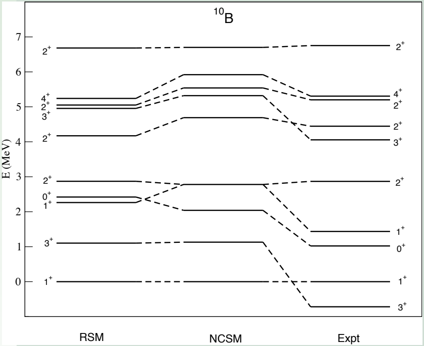
H^{eff} derived for 2 valence nucleon systems $\Rightarrow$ 3-, 4-, .. n-body components are neglected



- ground-state energies for $N = Z$ nuclei
- discrepancy grows with the number of valence nucleons

Benchmark calculation

^{10}B relative spectrum



- discrepancy $\leq$ 1 MeV
- minor role of many-body correlations