



Direct Reactions on Nuclei

Ch. Elster

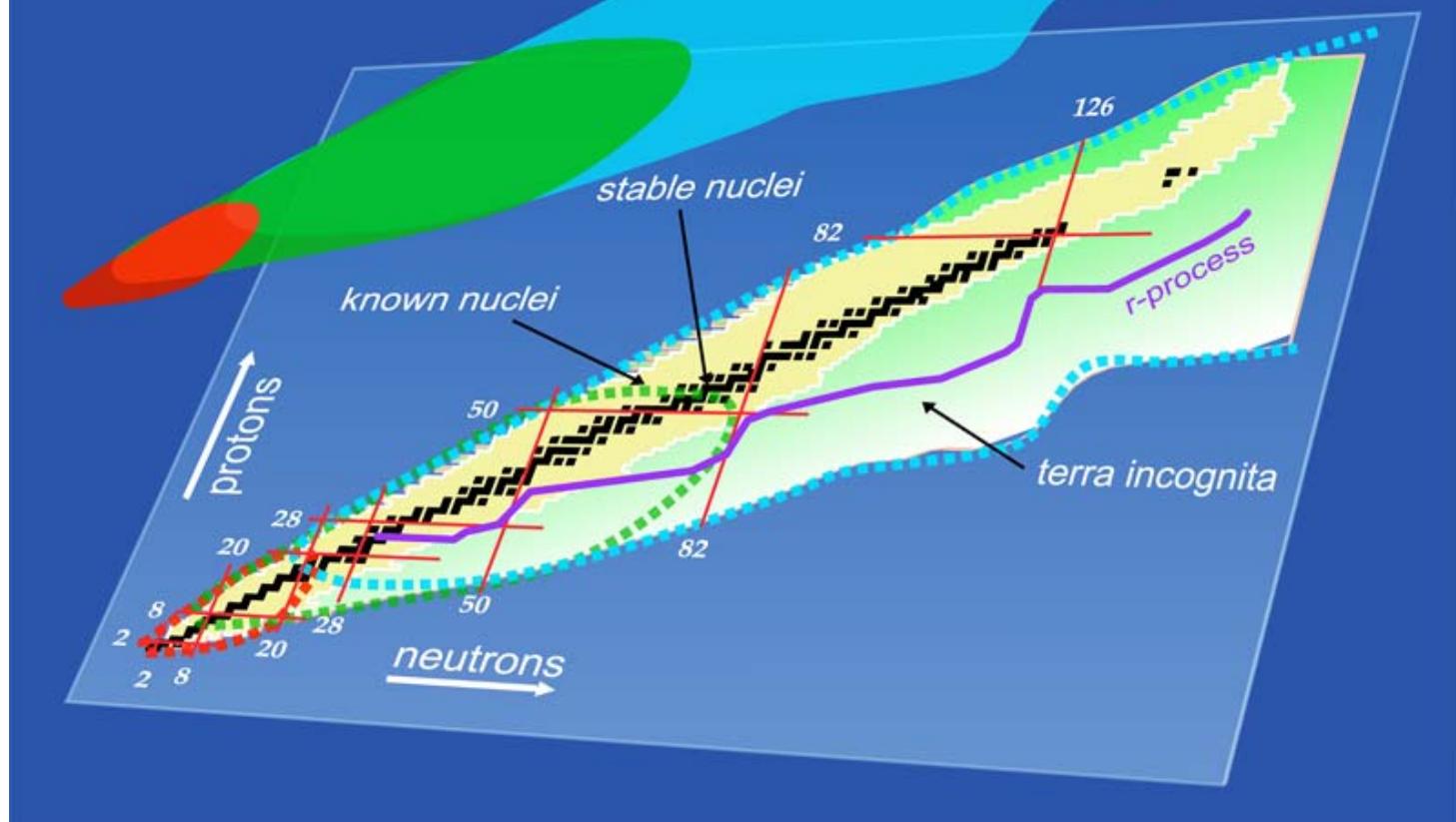
PhD Thesis of L. Hlophe

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Nuclear Landscape

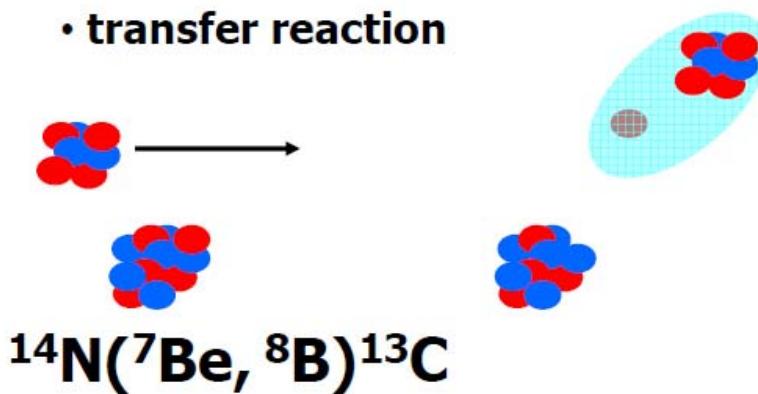
Ab initio
Configuration Interaction
Density Functional Theory



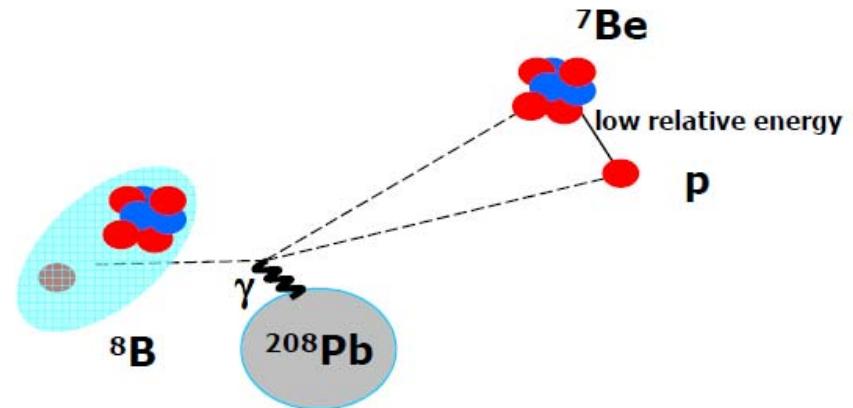
Indirect Methods: Nuclear Reactions

- direct measurement ${}^7\text{Be}(\text{p},\gamma){}^8\text{B}$

- transfer reaction

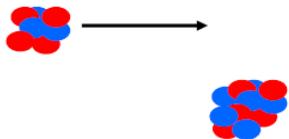


- Coulomb dissociation



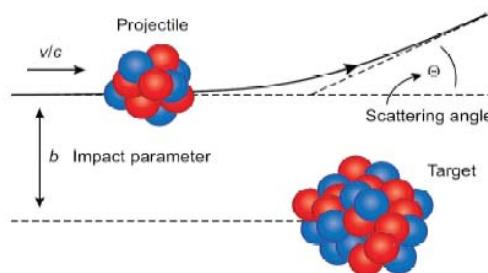
Why Reactions?

Elastic:



Traditionally used to extract optical potentials,
rms radii, density distributions

Inelastic:



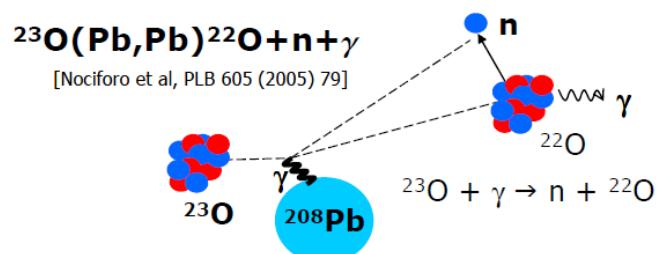
Traditionally used to extract
electromagnetic transitions
or nuclear deformations.

Transfer:

Traditionally used to extract spin, parity, spectroscopic factors
example: $^{132}\text{Sn}(\text{d},\text{p})^{133}\text{Sn}$

Traditionally used to study two-nucleon correlations and pairing
example: $^{11}\text{Li}(\text{p},\text{t})^{9}\text{Li}$

Breakup:

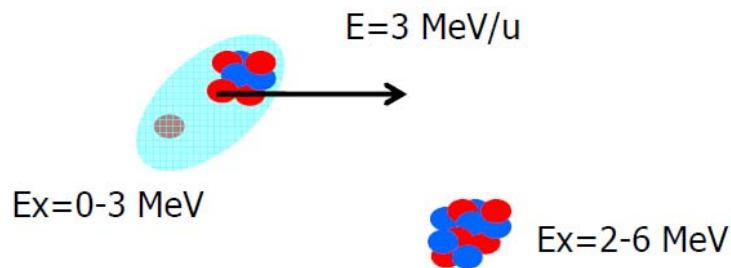


Theory Challenge:

In the continuum theory can solve the few-body problem exactly.

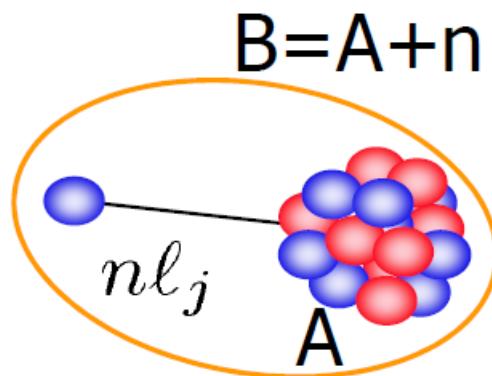
Reaction theories need to map onto the many-body problem!

It is not easy to develop effective field theories for reactions:



There is not always a clear separation of scales.

Often attempted meeting point between Structure and Reactions:



overlap function

$$I_{I_A:I_B}(\mathbf{r}) = \langle \Phi_{I_A}^A(\xi_A) | \Phi_{I_B}^B(\xi_A, \mathbf{r}) \rangle$$

spectroscopic factor ($S_{n/j}$):
norm of overlap function

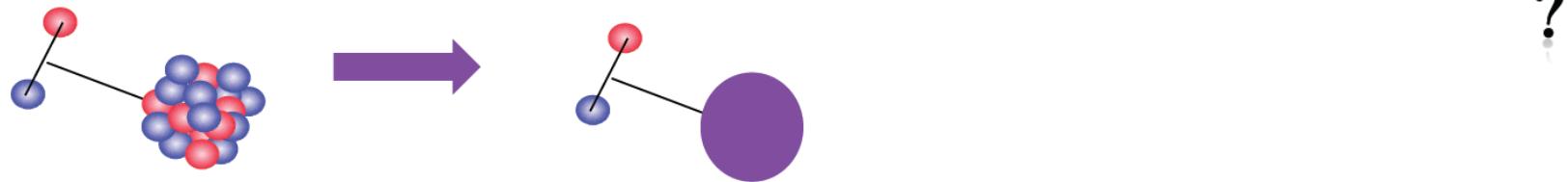
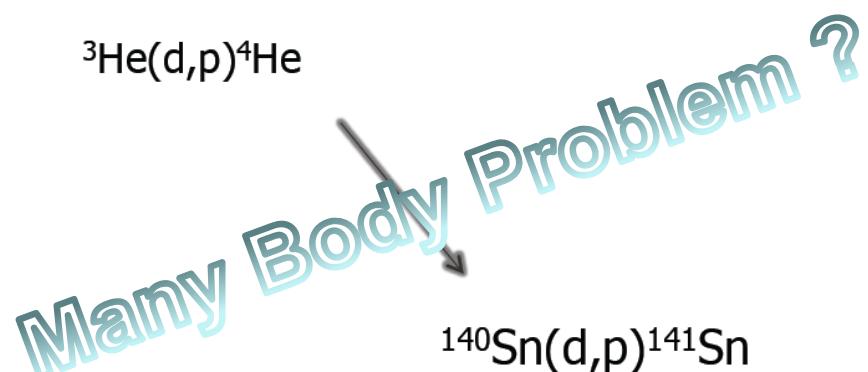
Extracting spectroscopic factors can provide some handle on structure theory.

However: spectroscopic factors are not observables

Instead: asymptotic normalization coefficients

Direct Reactions with Nuclei: Dominated by few degrees of freedom

- Elastic & inelastic scattering
- Few-particle transfer
(stripping, pick-up)
- Charge exchange
- Knockout



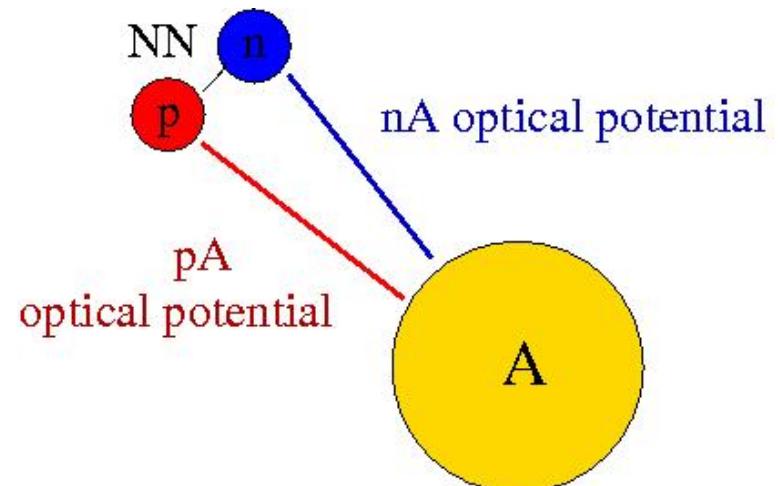
Task:

- Isolate important degrees of freedom in a reaction
- Keep track of important channels
- Connect back to the many-body problem

(d,p) Reactions: Effective Three-Body Problem

Hamiltonian for the effective few-body problem:

$$H = H_0 + V_{np} + V_{nA} + V_{pA}$$

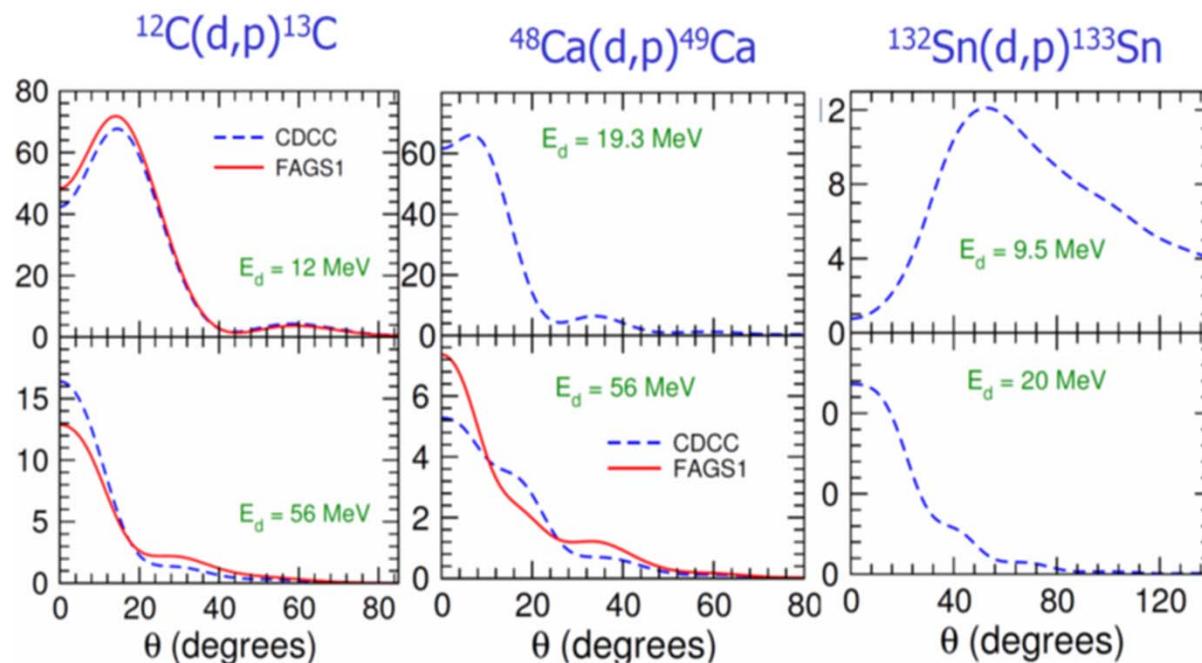


(d,p) Reactions as three-body problem



Deltuva and Fonseca, Phys. Rev. C79, 014606 (2009).

Elastic, breakup, rearrangement channels are included and fully coupled (compared to e.g. CDCC calculations)



Issues:

- current momentum space implementation of Coulomb interaction (shielding) does **not** converge for $Z \geq 20$
- CDCC and FAGS do not agree in breakup up

Generalization of Faddeev-AGS approach needed :

Theory: A.M. Mukhamedzhanov, V.Eremenko and A.I. Sattarov,
Phys.Rev. C86 (2012) 034001

Generalized Faddeev formulation of (d,p) reactions with:

- (a) explicit inclusion of the Coulomb interaction (no screening)
- (b) explicit inclusion of target excitations



Explicit inclusion of Coulomb interaction:

Formulation of Faddeev equations in Coulomb basis instead of plane waves

Needs: Formulation with separable interactions to avoid pinch singularities



Target excitations:

Including specific excited states

→ Formulation with separable interactions also useful.



Faddeev Equations in Coulomb Basis :

Three-body scattering state: $|\Psi\rangle = |\phi\rangle + \sum_{a=1}^3 g_0^C(E) V_a^S |\Psi\rangle$

$$g_0^C(E) = (E - H_0 - V_{pA}^C + i0)^{-1}$$

Faddeev components ψ_a

$$|\Psi\rangle \equiv \sum_a |\psi_a\rangle = \sum_a \underbrace{\delta_{a,1} |\phi_1\rangle + g_0^C(E) V_a^S |\Psi\rangle}_{|\psi_a\rangle}$$

e.g. $|\phi_1\rangle$ is initial state $A + (pn)$

Faddeev-type equations

$$\begin{cases} |\psi_1\rangle = |\phi_1\rangle + g_0^C(E) V_1^S \sum_a |\psi_a\rangle \\ |\psi_2\rangle = g_0^C(E) V_2^S \sum_a |\psi_a\rangle \\ |\psi_3\rangle = g_0^C(E) V_3^S \sum_a |\psi_a\rangle \end{cases} \Rightarrow \begin{cases} |\psi_1\rangle = |\phi_1\rangle + g_0^C(E) t_1 \sum_{a \neq 1} |\psi_a\rangle \\ |\psi_2\rangle = g_0^C(E) t_2 \sum_{a \neq 2} |\psi_a\rangle \\ |\psi_3\rangle = g_0^C(E) t_3 \sum_{a \neq 3} |\psi_a\rangle \end{cases}$$


proof of principle work is here

A(d,p)B Reaction using Coulomb Green's functions

Coulomb-modified Green's function

$$V^C = \frac{Z_1 Z_2 \alpha^2}{r} \quad g_0^C \equiv [E - H_0 + i0 - V_{pA}^C]^{-1}$$
$$g_0^C = \frac{|\psi_{\mathbf{k},\eta}^C\rangle\langle\psi_{\mathbf{k},\eta}^C|}{E - \overline{H}_0 + i0}$$

- * É.I.Dolinskii and A.M.Mukhamedzhanov. Sov. J. of Nucl. Phys. **3** (1966), 180.
- * C.R.Chinn *et al.* Phys. Rev. C **44** (1991), 1569.

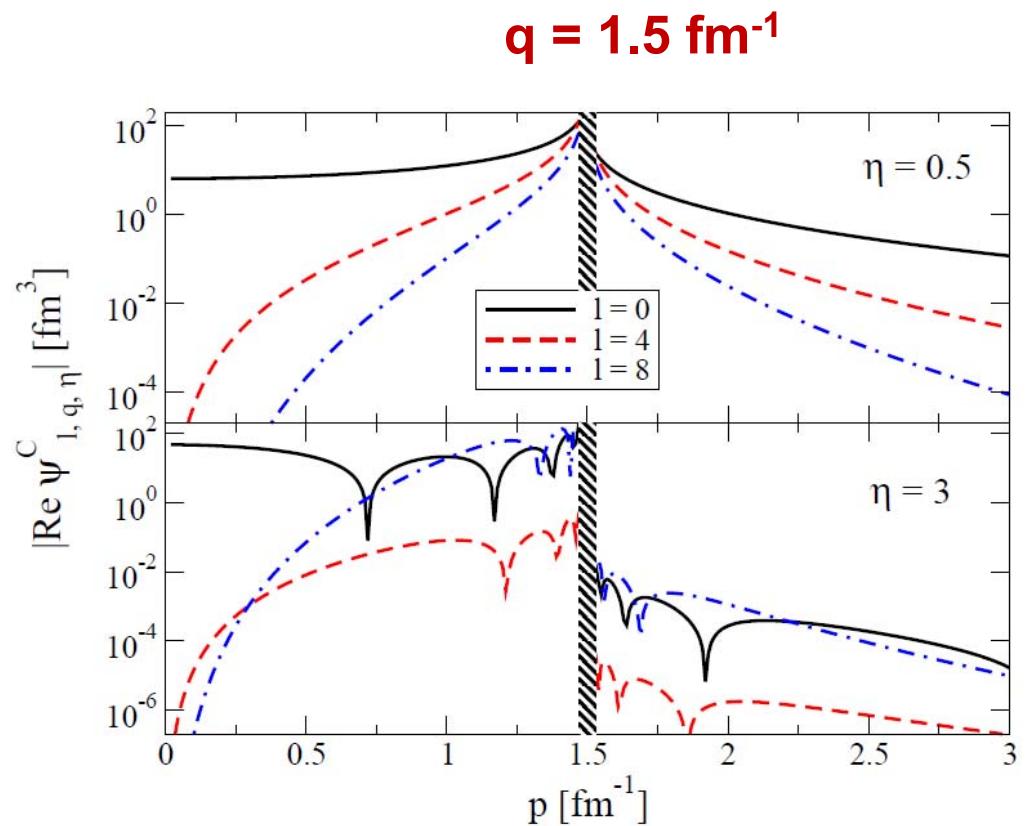
All matrix elements must be calculated in the Coulomb basis

Not trivial !

Challenge: momentum space Coulomb functions

- ‘Regular’ representation for $p \neq k$.
- ‘Pole-proximity’ representation for $p \approx k$:

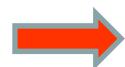
$$\psi_{l,k,\eta}^C(p) \sim \frac{1}{(p - k + i0)^{1+i\eta}}.$$



Eremenko *et al.* Comp. Phys. Comm. **187**, 195 (2015)

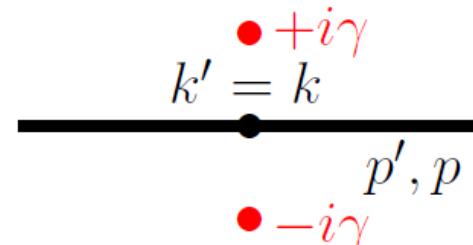
Challenge: Matrix elements in the Coulomb basis

$$t_{a,l}^C(k', k, E) = \int dp' dp \psi_{l,k',\eta'}^C(p')^\dagger t_{a,l}(p', p, E) \psi_{l,k,\eta}^C(p)$$



Pinch singularity in the elastic channel:

- $E = 2\mu k^2 = 2\mu k'^2$
- Since $\gamma \rightarrow +0$, singularities of $\psi_{l,k',\eta'}^{C\dagger}$ and $\psi_{l,k,\eta}^C$ are pinching the integration contour



If t-matrix separable

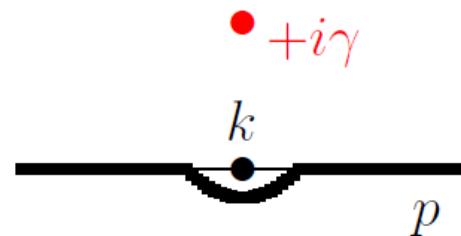
$$t_l^C(k, k', E) = \sum_{zy} u_{l,z}^C(k) \lambda_{l,zy}(E) u_{l,y}^C(k')^\dagger$$

$$u_{l,z}^C(k) = \int \frac{dp p^2}{2\pi^2} u_{l,z}(p) \psi_{l,k,\eta}^C(p)^*$$



No pinch singularity!

- Two independent integrals over p and p'
- Cauchy's theorem
- Numerical calculations use Gel'fand-Shilov regularization



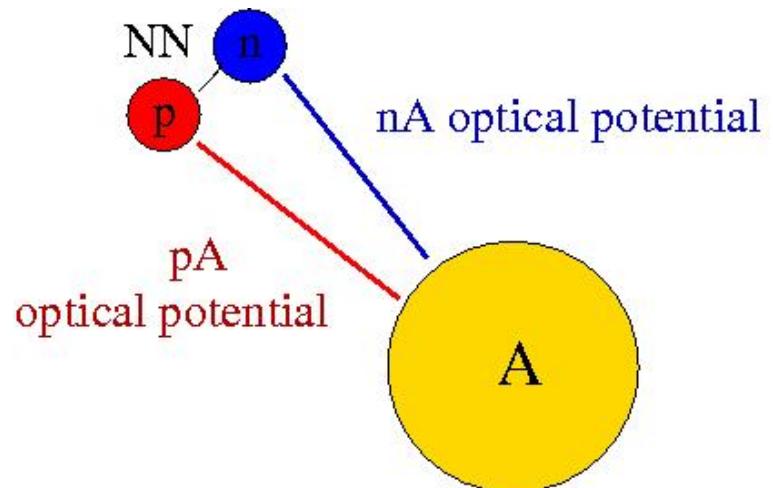
Effective Three-Body Problem

Hamiltonian for effective three-body problem:

$$H = H_0 + V_{np} + V_{nA} + V_{pA}$$

Needed:

Separable two-body transition matrices
in all channels (NN, nA, pA)



Challenge:

pA and nA effective interactions

- are **complex** and **energy dependent**
- Vary considerably over mass regions of the nuclear chart
- Many nuclei are deformed, i.e. additional degrees of freedom (rotational,vibrational)

Starting point: Ernst-Shakin-Thaler (EST) representation

D. J. Ernst, C. M. Shakin, and R. M. Thaler, Phys.Rev. C9, 1780 (1974)

Separable expansion of Hermitian operator, V , in basis $\{|\psi_i\rangle\}$:

Projection operator: $P = \sum_i |\psi_i\rangle\langle\psi_i|$

Projecting V onto P -subspace yields $v = VP(PVP)^{-1}PV$

In the limit $P \rightarrow 1$ we obtain $v = V$

EST scheme: choose basis vectors to be $|\psi_i^{(+)}\rangle$, the outgoing solutions of $(H_0 + V)|\psi_i^{(+)}\rangle = E_i|\psi_i^{(+)}\rangle$, so that $v = \sum_{ij} V|\psi_i^{(+)}\rangle\lambda_{ij}\langle\psi_j^{(+)}|V$

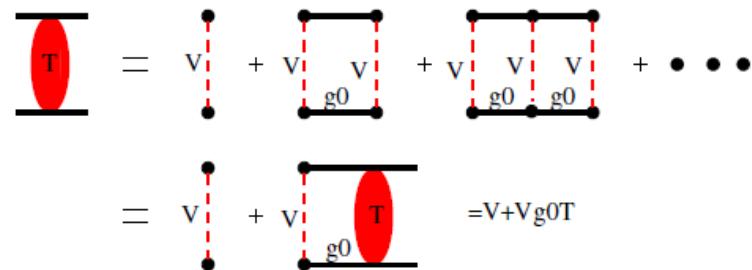
Constraints $\Rightarrow \delta_{kj} = \sum_i \langle\psi_k^{(+)}|V|\psi_i^{(+)}\rangle\lambda_{ij}$

$\delta_{ik} = \sum_j \lambda_{ij}\langle\psi_j^{(+)}|V|\psi_k^{(+)}\rangle$

Constraints ensure that at EST support points, E_i , wavefunctions corresponding to v and V are identical

Explicit Construction in Momentum Space:

1. Form factor (half-shell t -matrix): $T(E)|k_i\rangle \equiv V|\psi^{(+)}\rangle$ given by LS equation



2. Separable t -matrix elements are given by

$$\langle k'|t(E)|k\rangle = \sum_{ij} \langle k'|V|\psi_i^{(+)}\rangle \tau_{ij}(E) \langle \psi_j^{(-)}|V(E_j)|k\rangle$$

$$\tau_{ij}^{-1}(E) = \langle \psi_i^{(-)}|V(E_i)|\psi_j^{(+)}\rangle - \langle \psi_i^{(-)}|V(E_i)g_0(E)V(E_j)|\psi_j^{(+)}\rangle$$

Choosing scattering wave functions as basis is easy in momentum space

Evaluating $\tau_{ij}^{-1}(E)$ involves only integrals over half-shell t -matrices, numerically straightforward

Quite different from the original EST work in coordinate space!

Complex potentials

(L. Hlophe, et al. Phys. Rev. C88, 064608 (2013))

Given a time-reversal operator, \mathcal{K} , reciprocity is fulfilled if

$$\mathcal{K}v\mathcal{K}^\dagger = v^\dagger$$

Separable potential v does not satisfy this relation for complex potentials

Remedy: use ‘in’ states $|\psi_i^{(-)}\rangle$, eigenstates of $H' = H_0 + V^*$

For non-Hermitian potential V : $v = \sum_{ij} V |\psi_i^{(+)}\rangle \lambda_{ij} \langle \psi_j^{(-)}| V$

Constraints $\Rightarrow \delta_{kj} = \sum_i \langle \psi_k^{(-)} | V | \psi_i^{(+)} \rangle \lambda_{ij}$

$$\delta_{ik} = \sum_j \lambda_{ij} \langle \psi_j^{(-)} | V | \psi_k^{(+)} \rangle$$

v fulfills reciprocity
since λ is symmetric,
i.e. $\lambda_{ij} = \lambda_{ji}$

t -matrix: $t(E) = \sum_{ij} V |\psi_i^{(+)}\rangle \tau_{ij}(E) \langle \psi_j^{(-)}| V$

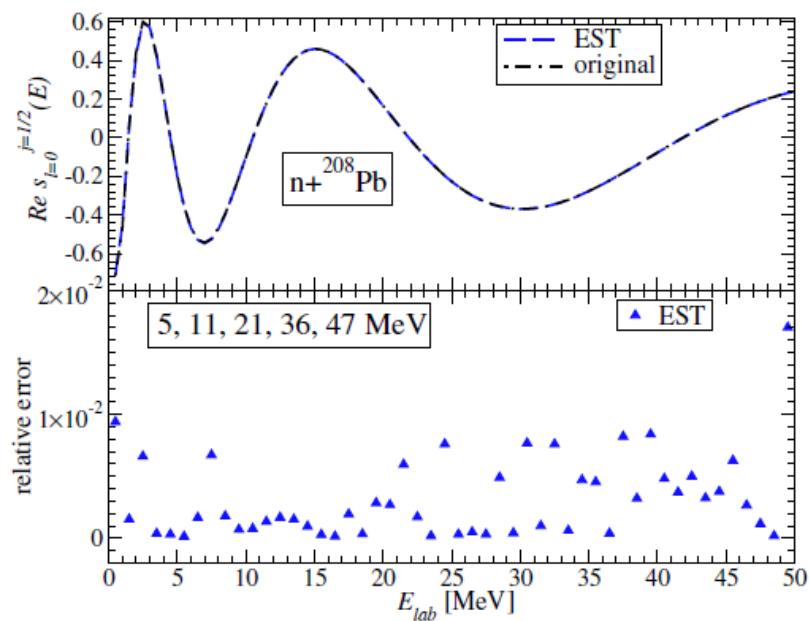
Highlights for n -A potential: general set of EST support points

system	partial wave(s)	rank	EST support point(s) [MeV]
n+ ⁴⁸ Ca	$l \geq 10$	1	40
	$l \geq 8$	2	29, 47
	$l \geq 6$	3	16, 36, 47
	$l \geq 0$	4	6, 15, 36, 47
n+ ¹³² Sn and n+ ²⁰⁸ Pb	$l \geq 16$	1	40
	$l \geq 13$	2	35, 48
	$l \geq 11$	3	24, 39, 48
	$l \geq 6$	4	11, 21, 36, 45
	$l \geq 0$	5	5, 11, 21, 36, 47

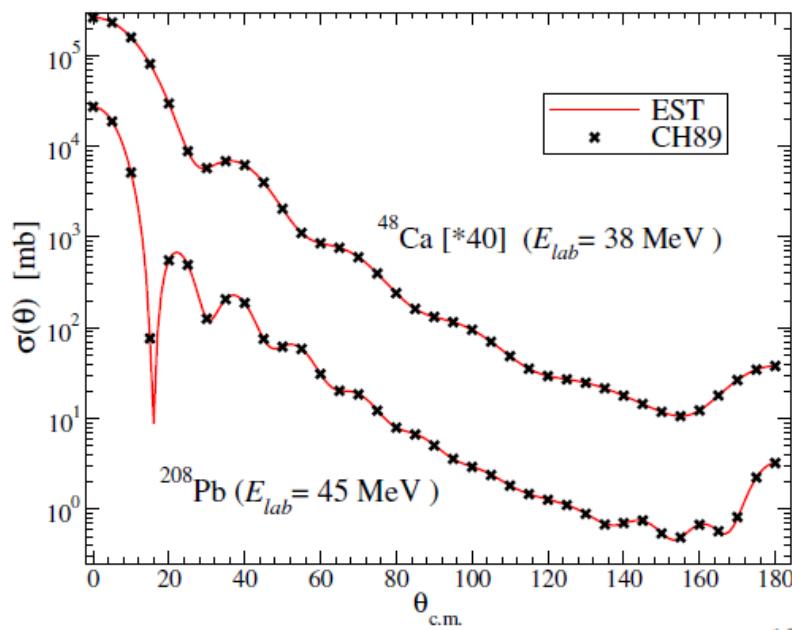
These EST support points provide good description of the s -matrices and cross sections from 0 to 50 MeV

Universal set

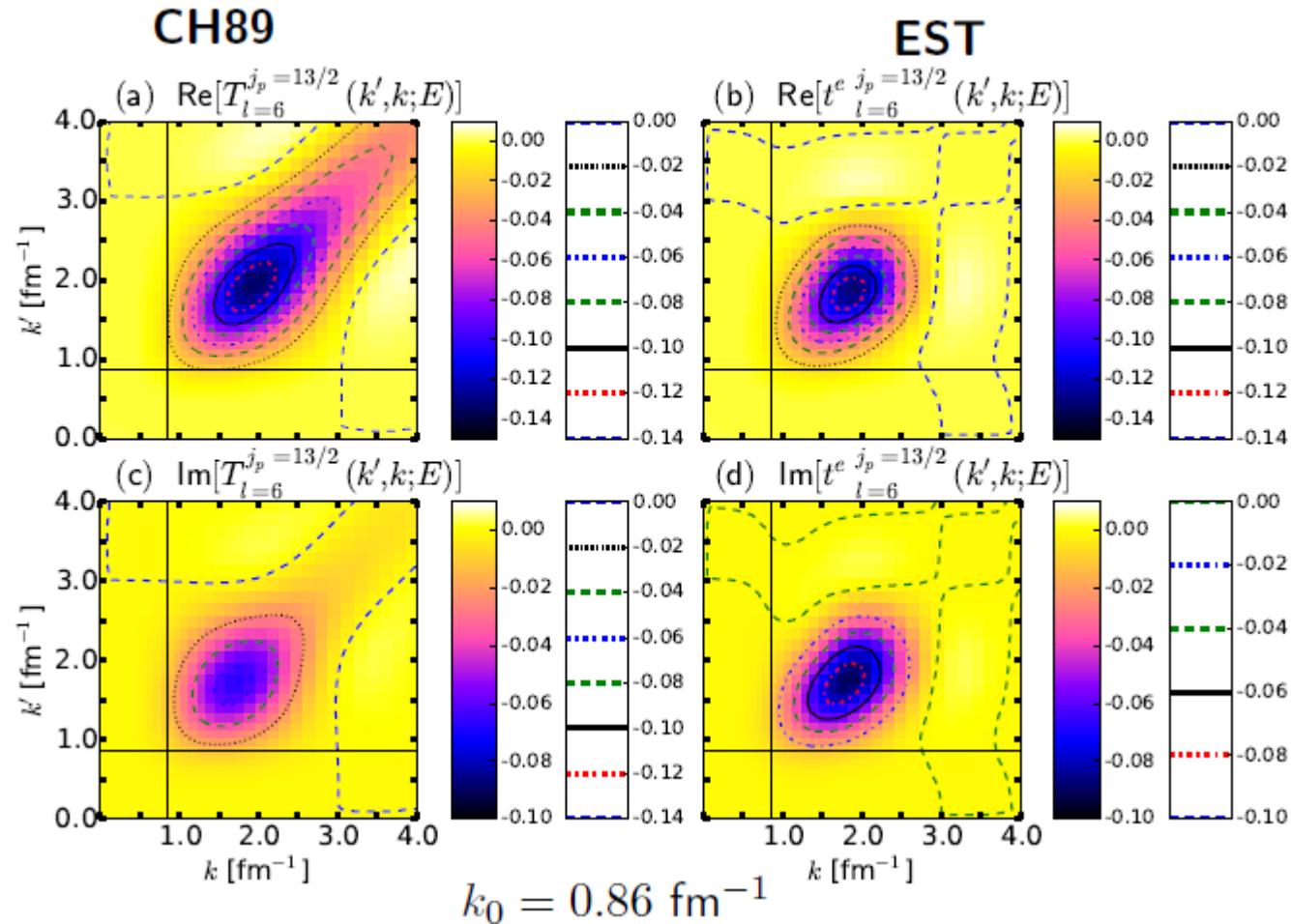
Elastic scattering s -matrix



Differential cross section



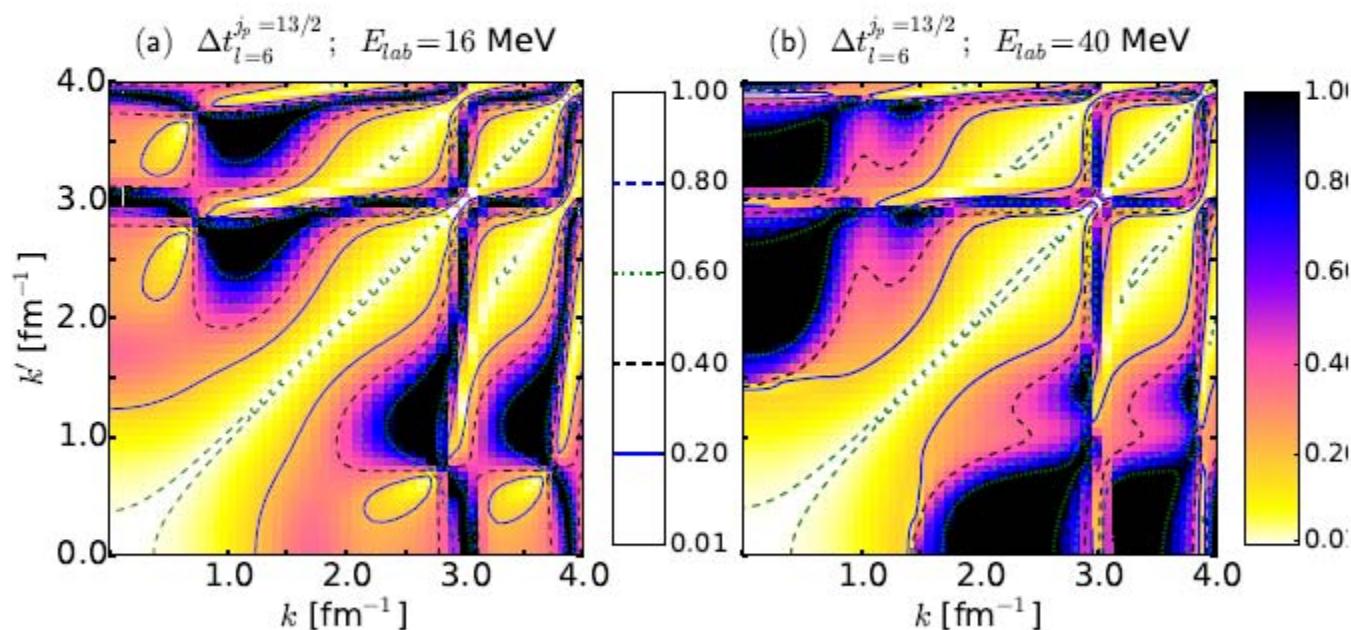
Off shell t -matrix: $n + {}^{48}\text{Ca}$, $l = 6$, $E_{\text{lab}} = 16$ MeV





Off-shell t -matrix

Asymmetry: $\Delta t_l^{j_p}(k', k; E) = \left| \frac{t_l^{j_p}(k', k; E) - t_l^{j_p}(k, k'; E)}{[t_l^{j_p}(k', k; E) + t_l^{j_p}(k, k'; E)]/2} \right|$



Not completely symmetric!

Complex, energy dependent potentials

L. Hlophe et al. PRC 93, 034601 (2016)

Revisit: EST Constraints $\Rightarrow \delta_{kj} = \sum_i \langle \psi_k^{(-)} | V(E_i) | \psi_i^{(+)} \rangle \lambda_{ij}$

$$\delta_{ik} = \sum_j \lambda_{ij} \langle \psi_j^{(-)} | V(E_k) | \psi_k^{(+)} \rangle$$

Energy dependence of V breaks symmetry: $\lambda_{ij} \neq \lambda_{ji} \Rightarrow$ violation of reciprocity

New: Energy dependent EST (eEST)

Define $v(E) = \sum_{ij} V(E_i) | \psi_i^+ \rangle \lambda_{ij}(E) \langle \psi_j^- | V(E_j)$

Constraint $\Rightarrow \langle \psi_m^- | V(E) | \psi_n^+ \rangle = \langle \psi_m^- | v(E) | \psi_n^+ \rangle$

\Downarrow
Matrix elements of $v(E)$ and $V(E)$ are identical at all energies

Both EST constraints are satisfied and $\lambda_{ij} = \lambda_{ji}$

EST vs eEST separable representation

- EST and eEST schemes have same form factors
- Evaluation of $\tau_{ij}(E)$ is different

EST:

$$R(E) \cdot \tau(E) \equiv 1$$

$$R_{ij}(E) = \langle \psi_i^- | V(E_i) | \psi_j^+ \rangle - \langle \psi_i^- | V(E_i) g_0(E) V(E_j) | \psi_j^+ \rangle$$

eEST:

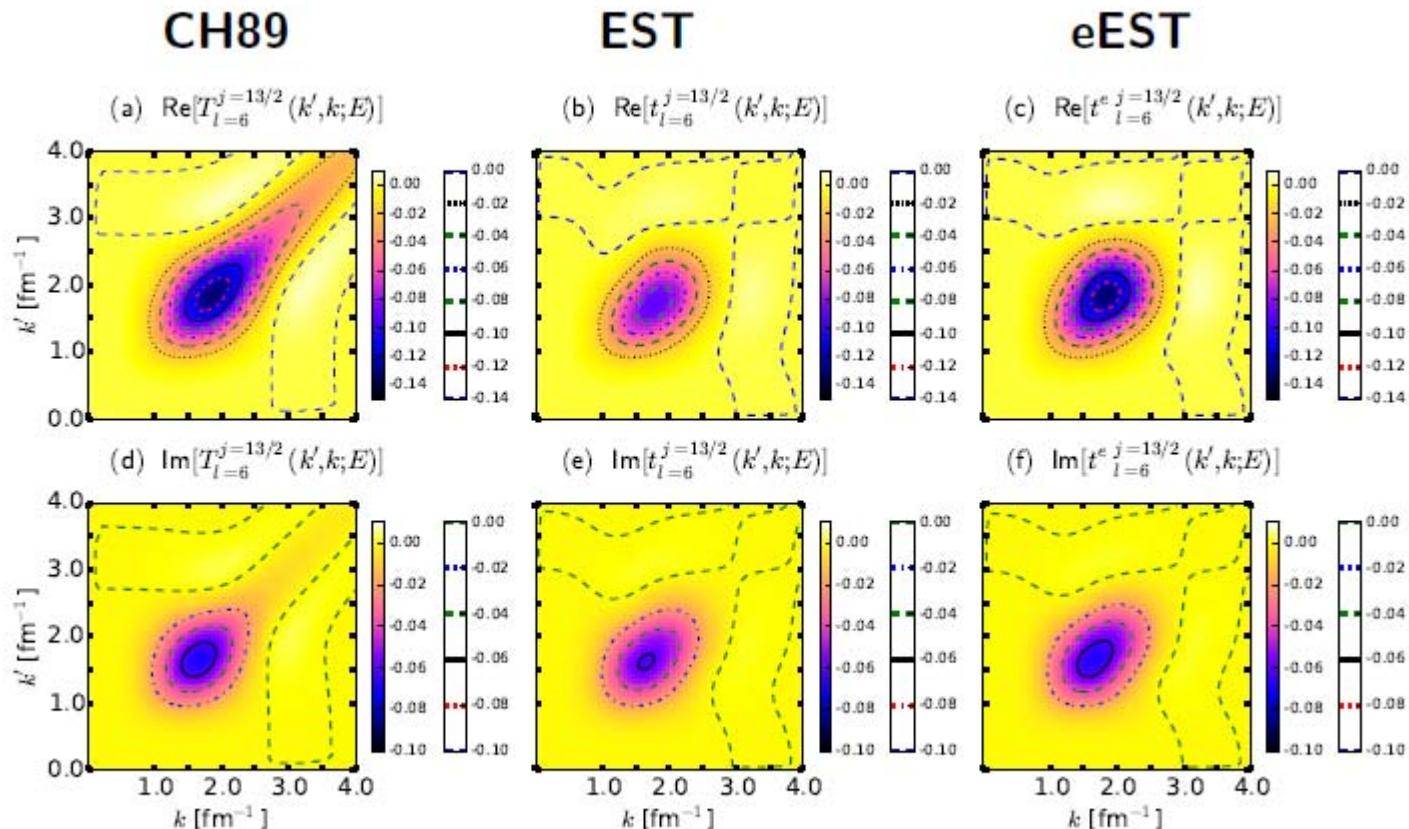
$$R(E) \cdot \tau(E) \equiv \mathcal{M}(E)$$

$$R_{ij}(E) = \langle \psi_i^- | V(E_i) | \psi_j^+ \rangle - \sum_n \mathcal{M}_{in}(E) \langle \psi_n^- | V(E_n) g_0(E) V(E_j) | \psi_j^+ \rangle$$

- For energy independent potentials $\mathcal{M}_{in}(E) = \delta_{in}$ then EST and eEST schemes are identical

On-shell: no visible difference

Off shell t -matrix: $n + {}^{48}\text{Ca}$, $l = 6$, $E_{\text{lab}} = 16$ MeV

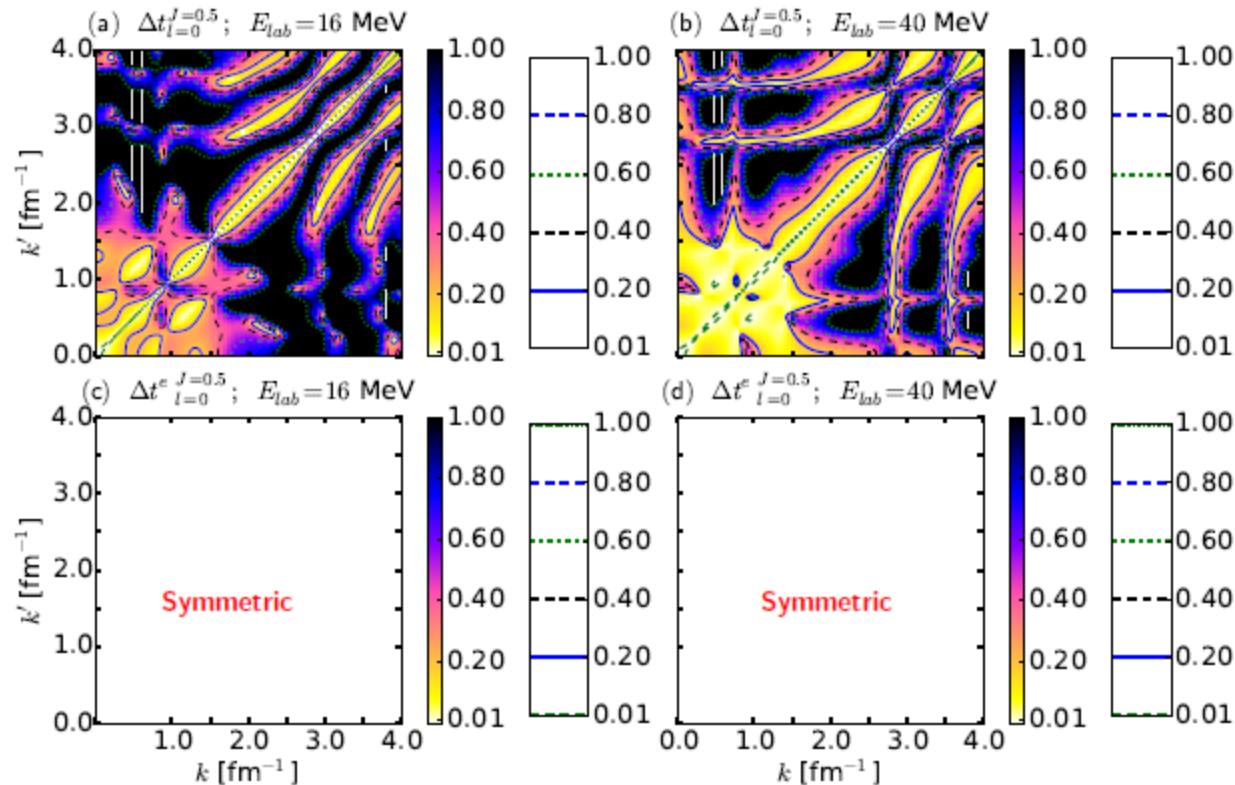


- t -matrix in $[\text{fm}^2]$, on shell momentum at $k_0 = 0.86 \text{ fm}^{-1}$

EST

eEST

$$\text{Asymmetry: } \Delta t_l^{jp}(k', k; E) = \left| \frac{t_l^{jp}(k', k; E) - t_l^{jp}(k, k'; E)}{[t_l^{jp}(k', k; E) + t_l^{jp}(k, k'; E)]/2} \right|$$



- On shell momentum: $k_0 = 0.86 \text{ fm}^{-1}$ at 16 MeV,
 $k_0 = 1.36 \text{ fm}^{-1}$ at 40 MeV

EST and eEST schemes for proton-nucleus scattering

(L. Hlophe, et al., Phys. Rev. C90, 061602 (2014))

- (1) Coulomb-distorted nuclear scattering states, $|\psi_i^{sc(+)}\rangle$ are used in the separable expansion
- (2) Free propagator $g_0(E) = (E - H_0 + i\varepsilon)^{-1}$ replaced by Coulomb Green's function $g_c(E) = (E - H_0 - V^c + i\varepsilon)^{-1}$

◆ Separable t -matrix becomes

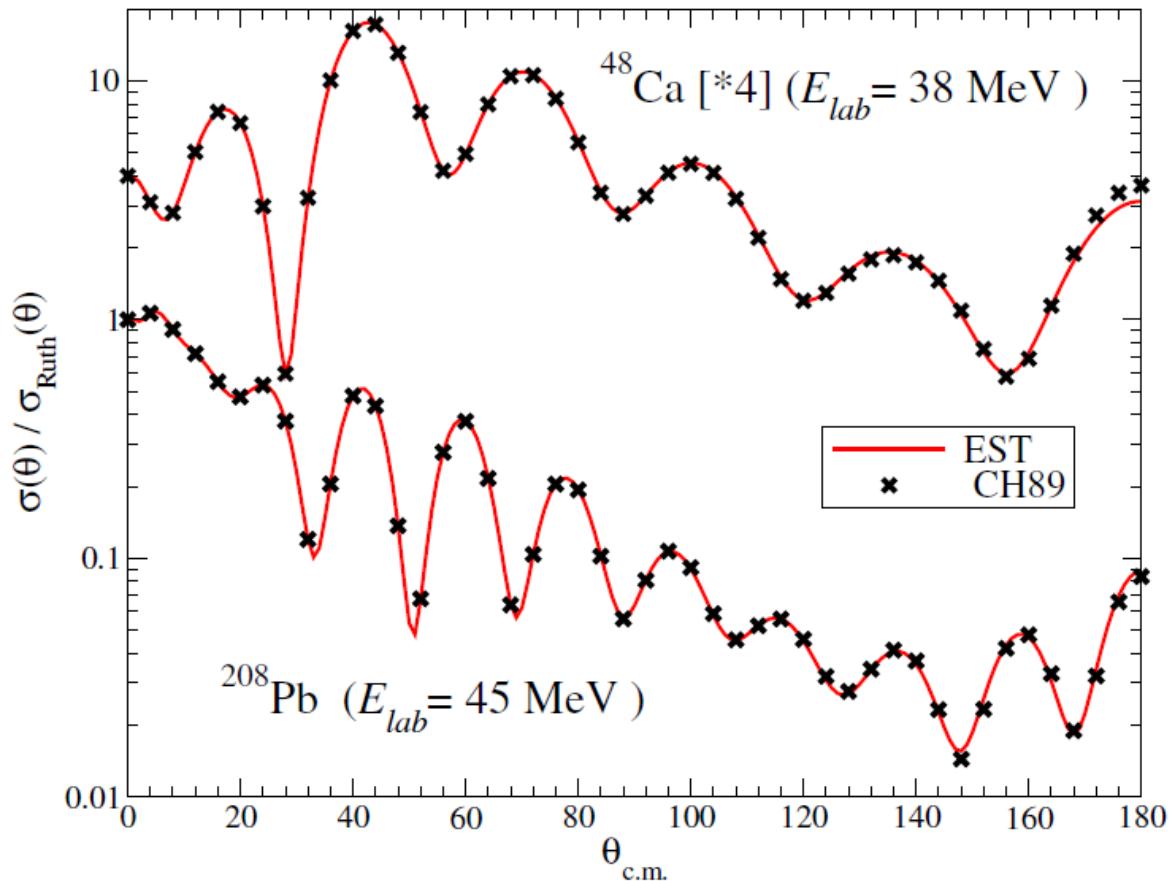
$$t^{sc}(E) = \sum_{ij} \textcolor{red}{V^s(E_i)} |\psi_i^{sc(+)}\rangle \tau_{ij}^c(E) \langle \psi_j^{sc(-)} | \textcolor{red}{V^s(E_j)}$$

■ Form factor $\textcolor{red}{V^s(E_i)} |\psi_i^{sc(+)}\rangle \equiv T_l^{sc}(E) |\phi_{lk_0}^c\rangle$ evaluated from

$$\begin{aligned} \langle \phi_{lk}^c | T_l^{sc}(E) | \phi_{lk_0}^c \rangle &= \langle \phi_{lk}^c | U_l^{sc} | \phi_{lk_0}^c \rangle + \int dp p^2 \langle \phi_{lk}^c | U_l^{sc} | \phi_{lp}^c \rangle \\ &\times \langle \phi_{lp}^c | g_c(E) | \phi_{lp}^c \rangle \langle \phi_{lp}^c | T_l^{sc}(E) | \phi_{lk_0}^c \rangle \end{aligned}$$

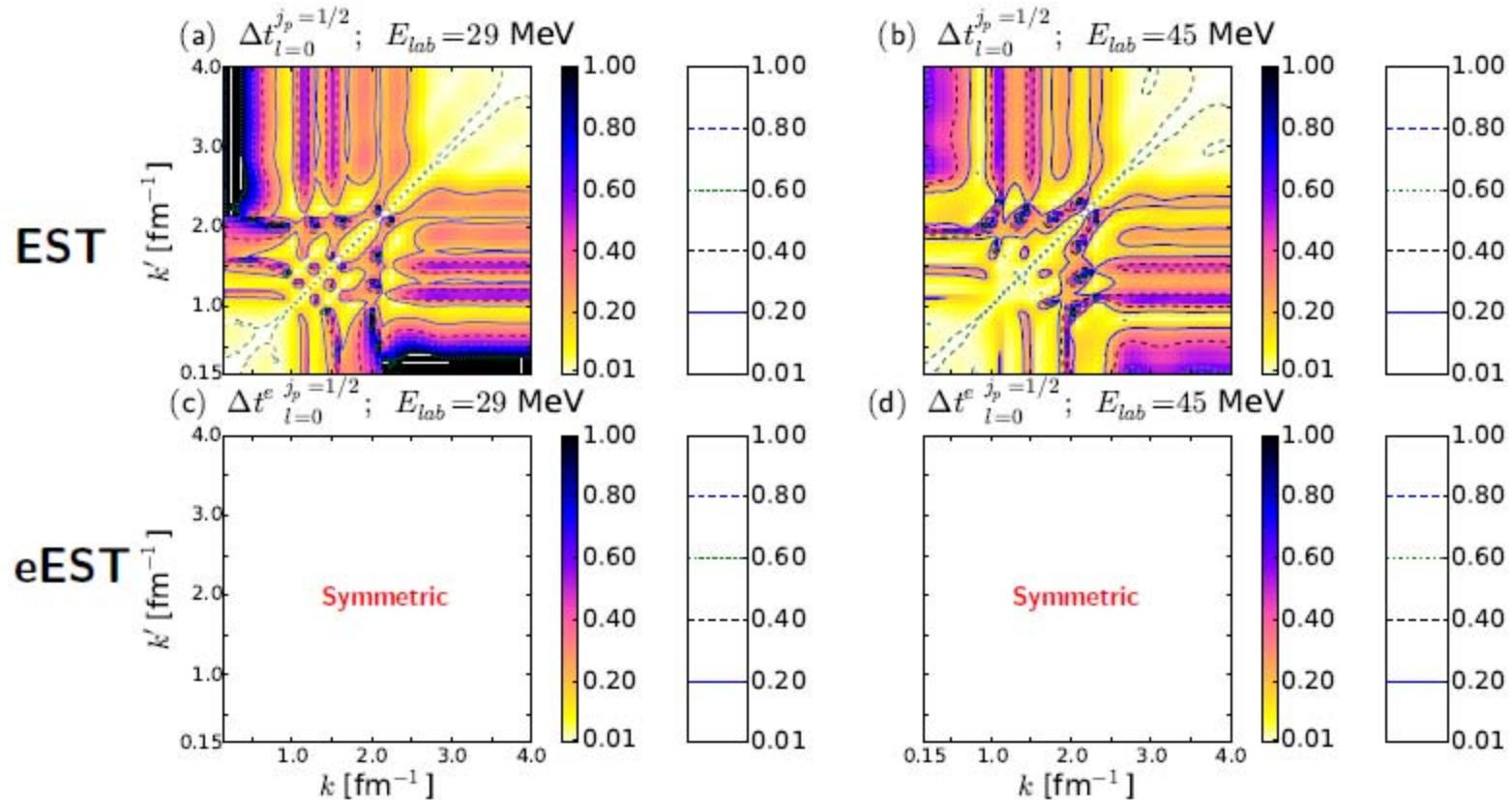
[Ch. Elster et al., J.Phys. G19, 2123 (1993)]

Proton-nucleus differential cross section



General set of EST support points also valid for $p+A$ optical potentials

Off shell t -matrix: $p + {}^{208}\text{Pb}208, l = 0$



- On shell momentum: $k_0 = 0.96 \text{ fm}^{-1}$ at 16 MeV,
 $k_0 = 1.36 \text{ fm}^{-1}$ at 40 MeV

Excitations of Nucleus: Multichannel Separable Potentials

- EST form factors become the multichannel half-shell t -matrices $T_{\alpha_0\rho}^J(E_i) \left| \phi_{l_\rho k_i^\rho} \right\rangle$
- Requires solutions of the multichannel t -matrix LS equation
- Multichannel separable t -matrix

$$t_{\alpha_0\alpha}^J(E) = \sum_{\rho\sigma} \sum_{ij} T_{\alpha_0\rho}^J(E_i) \left| \phi_{l_\rho k_i^\rho} \right\rangle \tau_{ij}^{\rho\sigma}(E) \left\langle \phi_{l_\sigma k_j^\sigma} \right| T_{\sigma\alpha}^J(E_j)$$

where $\tau_{ij}^{\rho\sigma}(E)$ is defined by

$$R(E) \cdot \tau(E) = \mathcal{M}(E),$$

$$\begin{aligned} R_{ij}^{\rho\sigma}(E) &= \left\langle \phi_{l_\rho k_i^\rho} \right| T_{\rho\sigma}^J(E_i) + \sum_{\beta} T_{\rho\beta}^J(E_i) G_{\beta}(E_j) T_{\beta\sigma}^J(E_j) \left| \phi_{l_\sigma k_j^\sigma} \right\rangle \\ &\quad - \sum_{\beta\beta'} \sum_n \mathcal{M}_{in}^{\rho\beta}(E) \left\langle \phi_{l_\beta k_n^\beta} \right| T_{\beta\beta'}^J(E_n) G_{\beta'}(E) T_{\beta'\sigma}^J(E_j) \left| \phi_{l_\sigma k_j^\sigma} \right\rangle \end{aligned}$$

Neutron Scattering from a deformed nucleus

- Deformed nuclei possess rotational excitation levels
- During a scattering process the low-lying rotational levels couple strongly to the ground state
- Employ a deformed optical model potential (DOMP)

$$R(\theta) = R_0 \left(1 + \sum_{\lambda \neq 0} \delta_\lambda Y_\lambda^0(\theta, 0) \right)$$

- DOMP is expanded in multipoles

$$\hat{U}(r, \theta) = \sum_{\lambda} \sqrt{4\pi} \textcolor{red}{U}_{\lambda}(r) Y_{\lambda}^0(\theta, 0)$$

$$\textcolor{red}{U}_{\lambda}(r) = \sqrt{\pi} \int_{-1}^{1} d \cos \theta \hat{U}(\tilde{r}) Y_{\lambda}^0(\theta, 0)$$

Olsson 89 DOMP: $n+^{12}C$ scattering

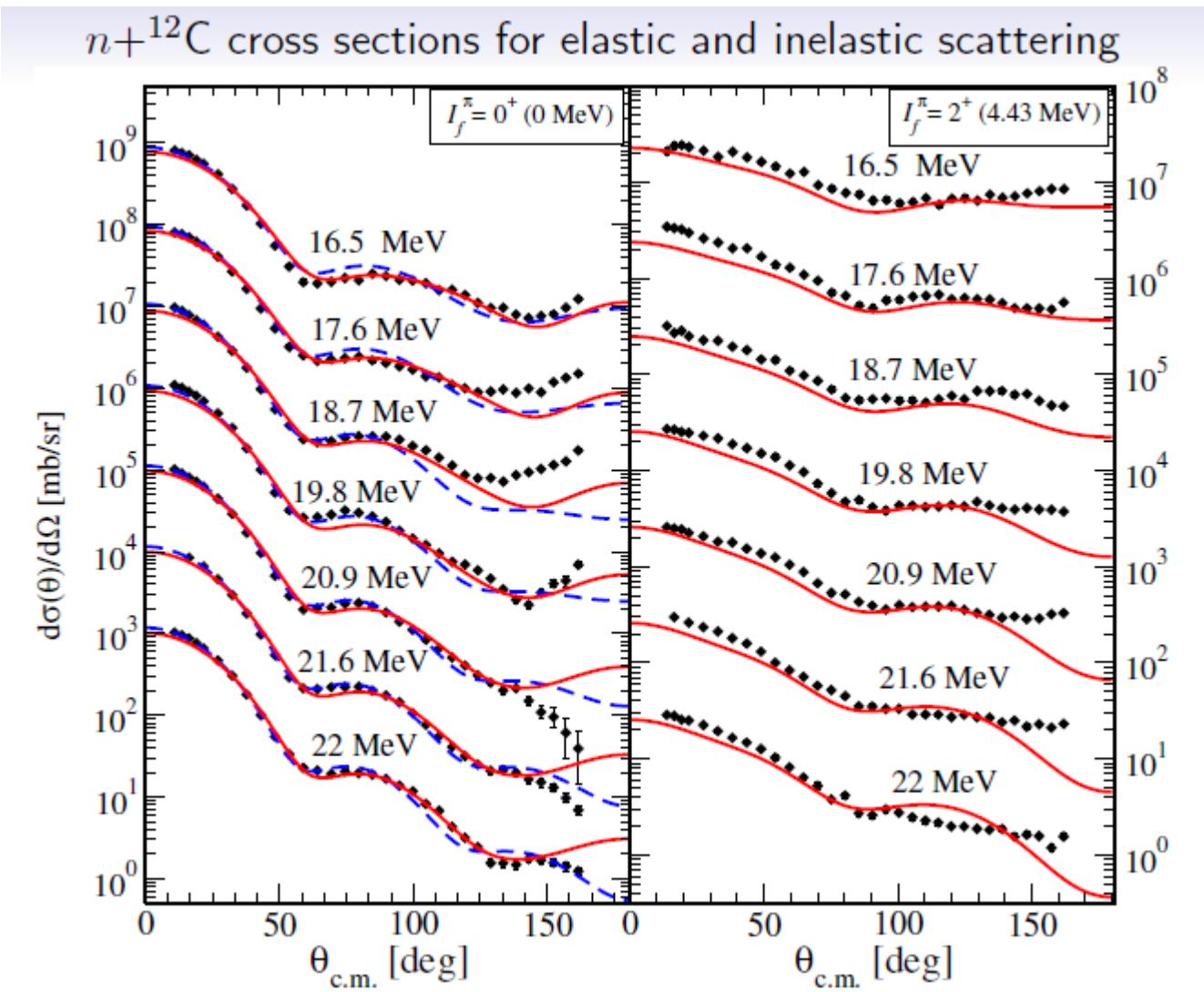
- ^{12}C rotational band: $I^\pi = 0^+, 2^+, 4^+, \dots$
- Scattering processes: elastic $0^+ \rightarrow 0^+$ and inelastic scattering $0^+ \rightarrow 2^+$
- Olsson 89 DOMP $n+^{12}C$ fitted to elastic and inelastic scattering data

[B. Olsson,*et al.*, Nucl. Phys. **A469**, 505, (1989).]

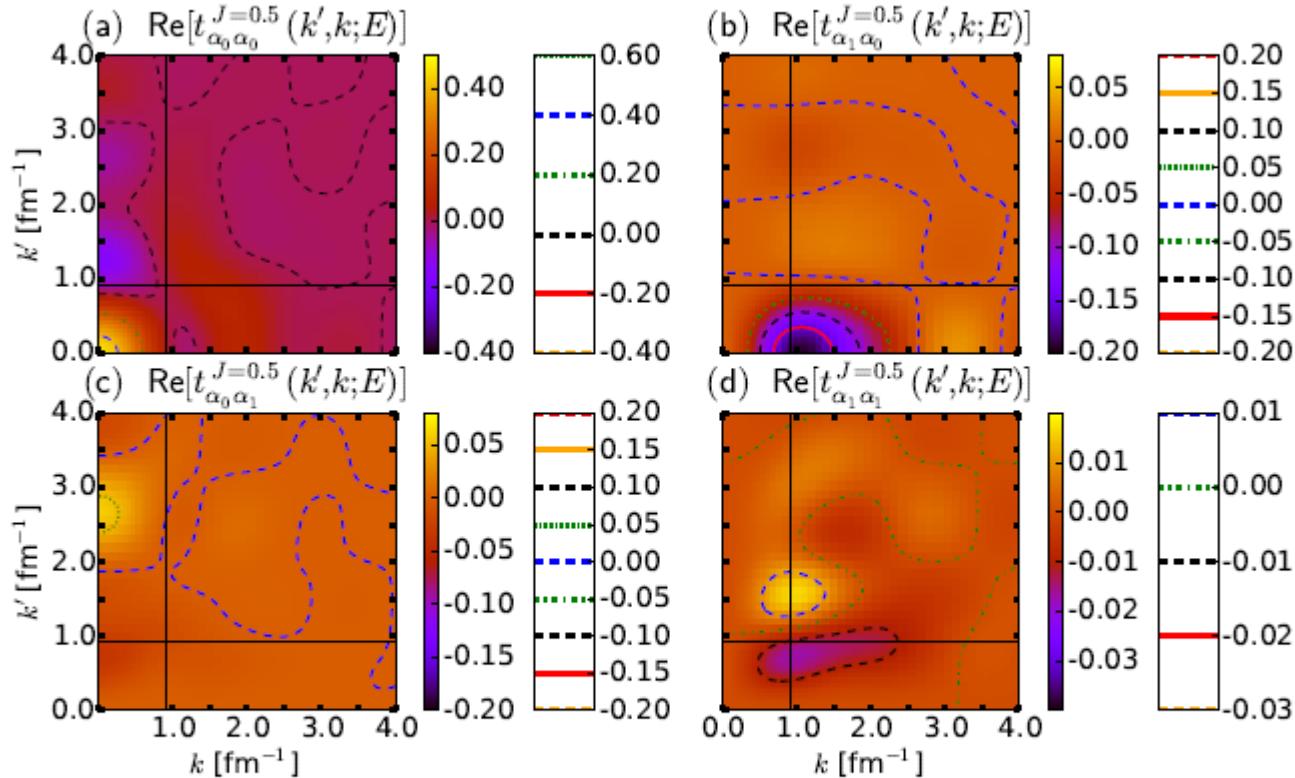
- Parameters:

Strength [MeV]	Radius [fm]	Diffusiveness [fm]
$V_r = 64.02 - 0.674E_n$	$R_r = 1.093A^{1/3}$	$a_r = 0.619$
$W_s = 1.16 + 0.251E_n$	$R_r = 1.319A^{1/3}$	$a_s = 0.327$
$V_{so} = 6.2$	$R_r = 1.050A^{1/3}$	$a_{so} = 0.550$

Reproduced the Olsson calculations with an eEST separable representation:

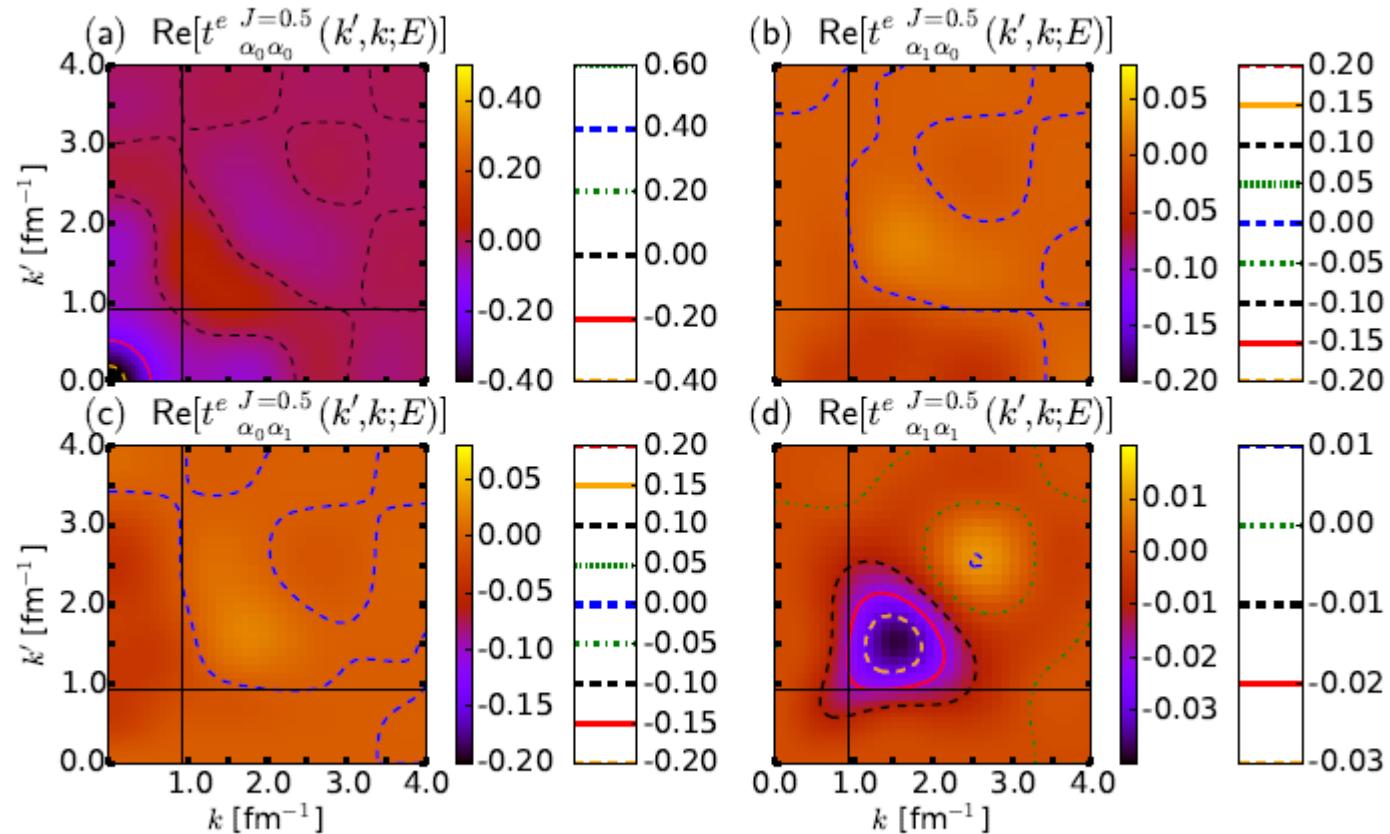


Off-shell EST separable t -matrix $n+^{12}\text{C}$, $E_{lab} = 20.9$

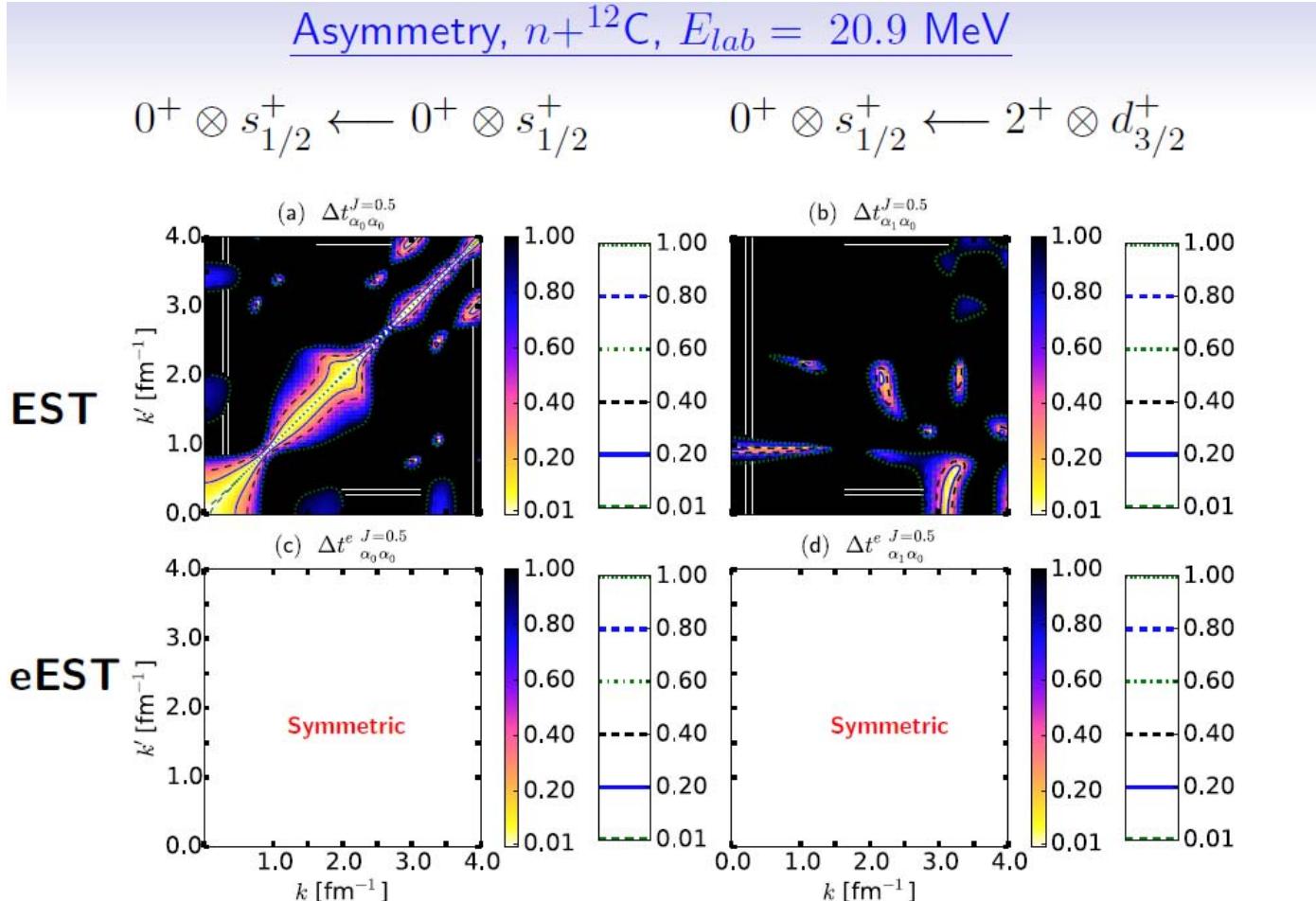


- EST support points: 6, 20, 40 MeV
- Asymmetry much more pronounced, off-diagonal on-shell t -matrix is not symmetric

Off-shell eEST separable t -matrix $n+^{12}\text{C}$, $E_{lab} = 20.9$



- eEST scheme leads to symmetric off-shell t -matrix



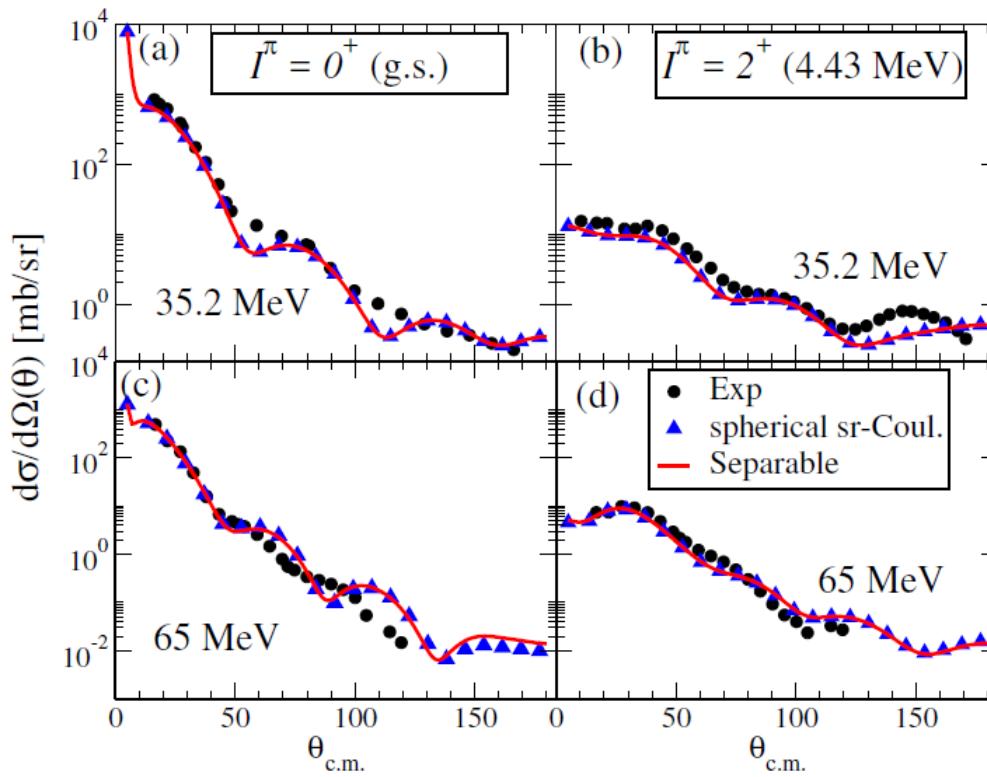
Asymmetry for EST more pronounced in multichannel scattering

eEST important when taking into account excitations

Straightforward extension to p+¹²C scattering

Similar to single-channel case:

- (1) Coulomb-distorted nuclear scattering states, $|\psi_i^{sc(+)}\rangle$ are used in the separable expansion
- (2) Free propagator $g_0(E) = (E - H_0 + i\varepsilon)^{-1}$ replaced by Coulomb Green's function $g_c(E) = (E - H_0 - V^c + i\varepsilon)^{-1}$



Potential and data from:

A. S. Meigooni, R. W. Finlay,
J. S. Petler, and J. P. Delaroche,
Nucl. Phys., vol. A445, pp. 304–332, 1985.

Future: Numerical implementation of Faddeev-AGS equations

With this we can solve the effective three-body problem for (d,p) reactions for nuclei across the nuclear chart



Can we test this picture?



Scattering $d+{}^4He$ can be calculated as many body problem
by NCSM+RGM

Benchmark elastic and breakup scattering for $d+{}^4He$



Only reactions with light nuclei will allow benchmarks

(i.e. with calculations by A. Deltuva)

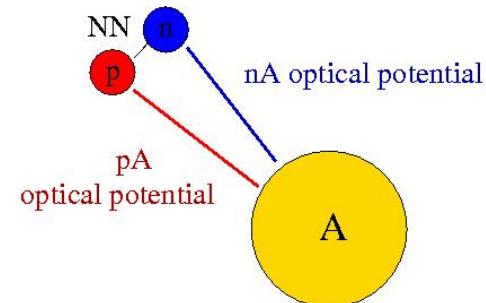
Further Challenge: Determine effective interactions V_{eff}

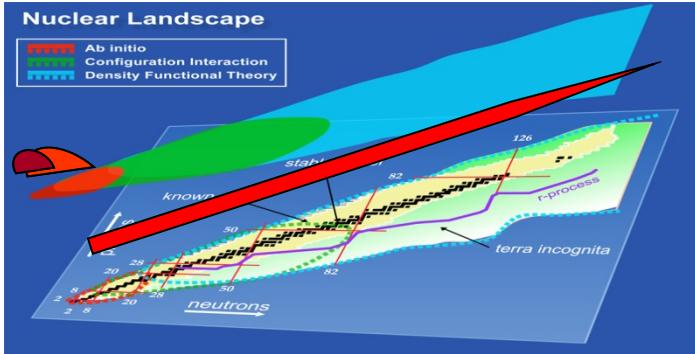
- V_{eff} is effective interaction between $N+A$ and should describe elastic scattering

Hamiltonian for effective few-body problem:

$$H = H_0 + V_{np} + V_{nA} + V_{pA}$$

- V_{np} is well understood
- V_{nA} and V_{pA} are effective interactions
- Most used: phenomenological approaches
 - Global optical potential fits to elastic scattering data
 - Most data available for stable nuclei
 - Extrapolation to exotic nuclei questionable
- Microscopic approaches need to be developed or existing ones refined and adapted for exotic nuclei
 - Microscopic approaches were developed for A being a closed shell nucleus.



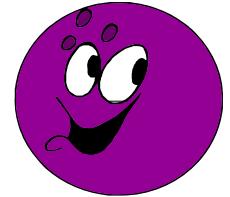


Goal for Reaction Theory:

Determine the topography of the nuclear landscape according to reactions described in definite schemes

- At present 'traditional' few-body methods are being successfully applied to a subset of nuclear reactions (with light nuclei)
 - Challenge: reactions with heavier nuclei
- Establish overlaps and benchmarks, where different approaches can be firmly tested.
- 'cross fertilization' of different fields (structure and reactions) carries a lot of promise for developing the theoretical tools necessary for R⁺⁻ physics.





p+A and n+A effective interactions (optical potentials)

- Renewed urgency in reaction theory community for microscopic input to e.g. (d,p) reaction models .
- Most likely complementary approaches needed for different energy regimes

● **In the multiple scattering approach not even the first order term is fully explored: all work concentrates on closed-shell nuclei**

Via $\langle \Phi_A | \Phi_A \rangle$ results from nuclear structure calculations enter

● **⇒ Structure and Reaction calculations can be treated with similar sophistication**

Older microscopic calculations concentrated on closed shell spin-0 nuclei (ground state wave functions were not available)

● Today one can start to explore **importance of open-shells in light nuclei** full complexity of the NN interactions enters

Experimental relevance: Polarization measurements for ${}^6\text{He} \rightarrow \text{p}$ at RIKEN