KAKENHI grand 25870168

Priority Issue 9
to be Tackled by Using Post K Computer
"Elucidation of the Fundamental Laws and
Evolution of the Universe" (hp160211, hp150224)

JICFuS

NTSE 2016 @ Pacific National University,
Khabarovsk, Russia, 2016/09/21

Large-scale shell-model studies for exotic nuclei and nuclear level densities

Noritaka Shimizu
Center for Nuclear Study, the University of Tokyo

CENTER *for* NUCLEAR STUDY

東京大学
THE UNIVERSITY OF TOKYO

Contents

- Introduction of the activities in Tokyo nuclear theory group : large-scale nuclear structure calculations
  - Zr isotopes by Monte Carlo shell model
  - no-core Monte Carlo shell model + JISP16
    - Be isotopes + intrinsic density
  - UMOA + Daejeon 16

- Stochastic estimation of level density in large-scale shell model calculations (LSSM)
  - numerical framework of the stochastic estimation of level density
  - benchmark test
  - application : parity equilibration of ^{58}Ni level density

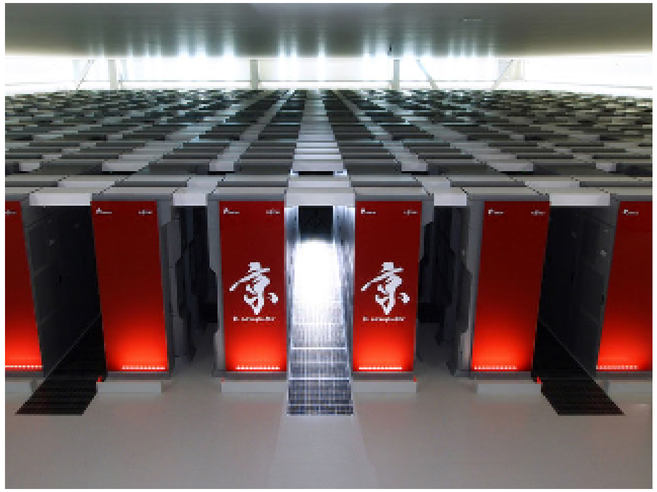
Computational science projects in Japan

- 9 projects
  - 1st : medicine design
  - …
  - 8th: manufacturing
  - 9th: fundamental science
    - particle physics, <u>nuclear physics</u>, astrophysics, …

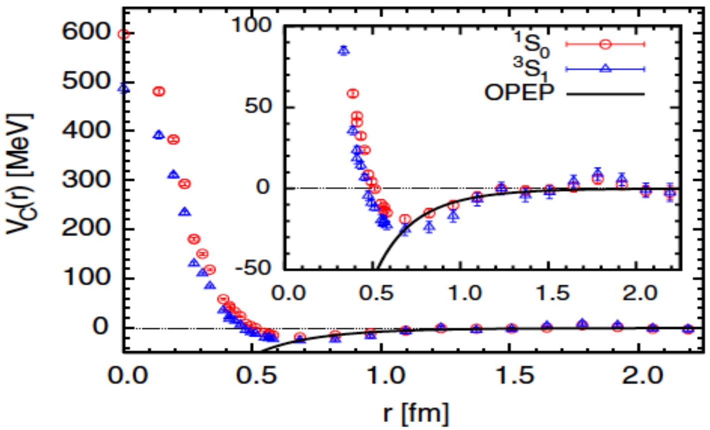
JICFuS

"K computer" launched in 2012

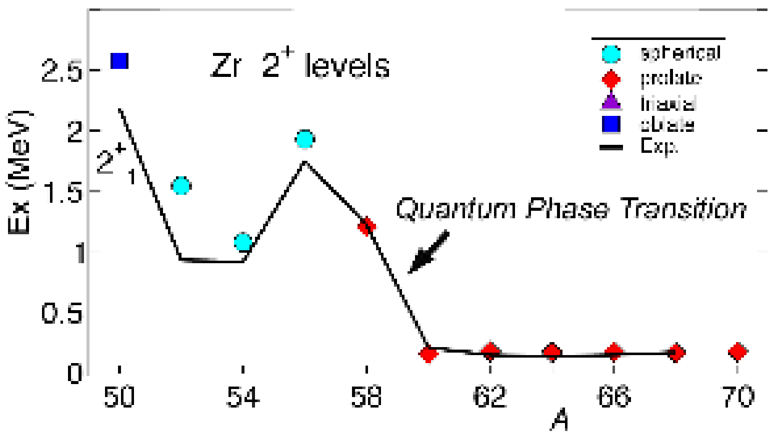
Post-K project in 2021



Hadrons : Lattice QCD calculation to derive the potential



<u>Nuclear structure</u> : LSSM calc. for exotic nuclei



Nuclear theory group in the University of Tokyo

Department of Physics

- Takaharu Otsuka
- Takashi Abe
- J. Menendez, T. Miyagi, S. Yoshida

Center for Nuclear Study

- Noritaka Shimizu
- T. Ichikawa, T. Togashi, N. Tsunoda, Y. Tsunoda, T. Yoshida
- Yutaka Utsuno (JAEA)

For no-core shell model calculations,
collab. with James Vary and Pieter Maris (Iowa State U)

Monte Carlo Shell Model

Otsuka, Honma, Mizusaki, Shimizu, Utsuno, PPNP47, 319 (2001)

a tool to go beyond the conventional Lanczos diagonalization method for Large-Scale Shell Model calculations

Huge *M*-scheme dimension

$$\mathbf{H} = \begin{pmatrix} * & * & * & * & * & \cdot & \cdot \\ * & * & * & * & \cdot & \cdot & \cdot \\ * & * & * & \cdot & \cdot & \cdot & \\ * & * & \cdot & \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & \cdot & & & \\ \cdot & & & & & & \end{pmatrix} \xrightarrow{\text{diagonalization}} \begin{pmatrix} \varepsilon_1 & & & & & 0 \\ & \varepsilon_2 & & & & \\ & & \varepsilon_3 & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ 0 & & & & & \cdot \end{pmatrix}$$

Conventional Shell Model

all Slater determinants

Stochastically selected basis

$$\mathbf{H} \approx \begin{pmatrix} * & * & * & \cdot \\ * & * & * & \cdot \\ * & * & \cdot & \\ \cdot & \cdot & & \end{pmatrix} \xrightarrow{\text{diagonalization}} \begin{pmatrix} \varepsilon'_1 & & 0 \\ & \varepsilon'_2 & \\ & & \cdot \\ 0 & & & \cdot \end{pmatrix}$$

$$\boxed{|\phi(\sigma)\rangle = \prod e^{\Delta\beta \cdot h(\sigma)} \cdot |\phi^{(0)}\rangle}$$

Monte Carlo Shell Model

bases important for a specific eigenstate

$\hat{h}(\sigma)$ one-body Hamiltonian

σ... auxiliary fields

random numbers

$$|\Psi\rangle = \sum_{k=1}^{N_{MCSM}} f_k P^{J,\pi} |\phi_k\rangle \qquad |\phi_k\rangle = \prod_{\alpha=1}^{N} \left(\sum_{i=1}^{N_{sp}} c_i^\dagger D_{i\alpha}^{(k)} \right) |-\rangle$$

MCSM basis, deformed Slater det.

+ Further optimization + extrapolation with energy variance

= "advanced Monte Carlo shell model" N. Shimizu *et al.*, PTEP 2012

MCSM wave function

- Efficient description of nuclear many-body states based on the basic picture of nuclear structure
  = intrinsic state + rotation + superposition

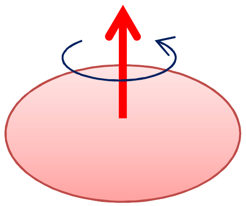
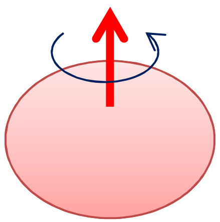
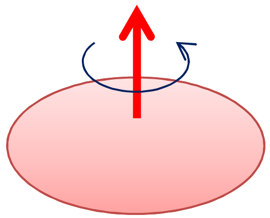
$$|\Psi^{IM\pi}(N_b)\rangle = \underbrace{\sum_{d=1}^{N_b} f^{(d)}}_{\text{superposition}} \underbrace{\sum_{K=-I}^{I} g_K^{(d)} \hat{P}^{\pi} \hat{P}^{I}_{MK}}_{\text{projection onto good quantum numbers}} \underbrace{|\Phi(D^{(d)})\rangle}_{\text{optimized Slater determinant}}$$

MCSM basis dimension ≈ 100

superposition → mixing with different shapes

projection onto good quantum numbers → rotation

optimized Slater determinant → deformed intrinsic state

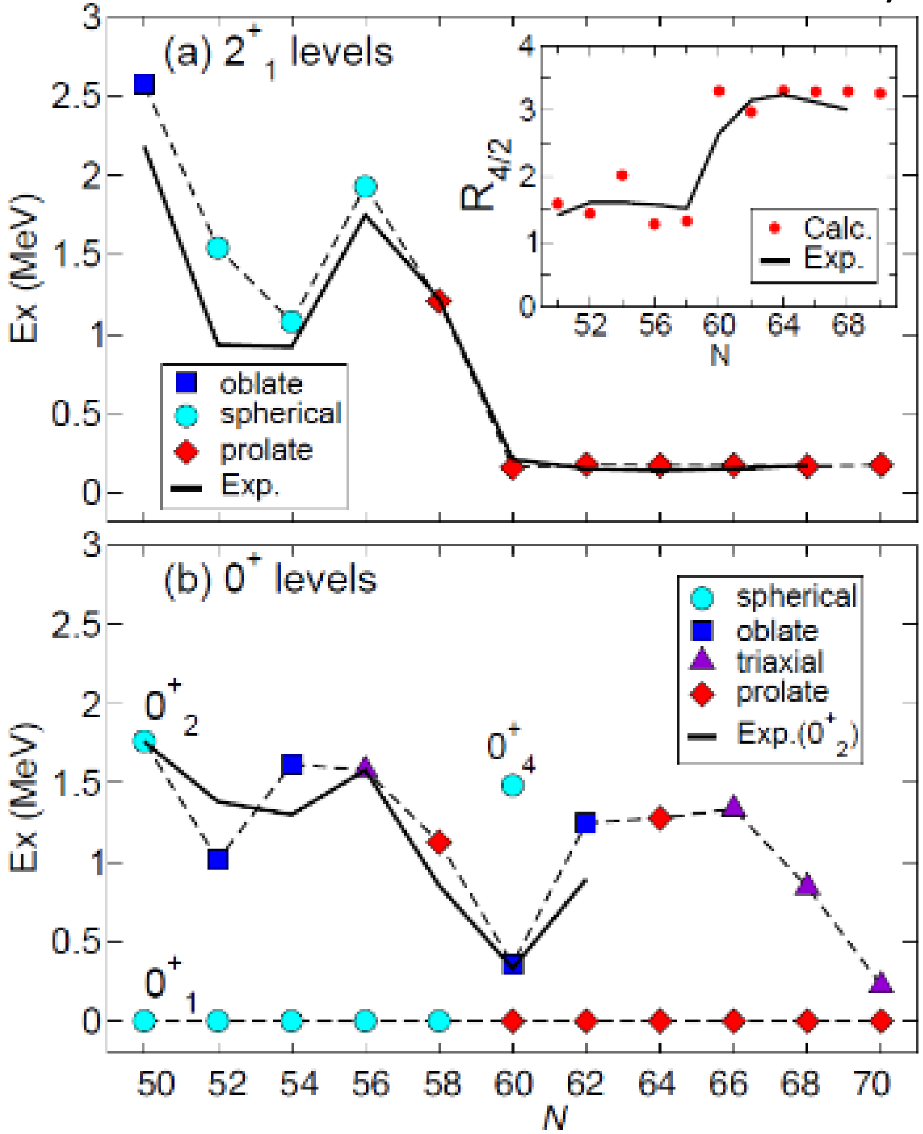
Wave function = $f^{(1)}$  + $f^{(2)}$  + $f^{(3)}$  + ...

$f^{(d)}, g_K^{(d)}, D^{(d)}$: variational parameters

MCSM enables us to discuss “intrinsic deformation”

Recent achievement:
Quantum Phase Transition in the Shape of Zr isotopes

T. Togashi, Y. Tsunoda, T. Otsuka and N. Shimizu,
Phys. Rev. Lett. accepted, arXiv:nucl-th 1606.09056



drastic change of Ex(2^+) at N=60

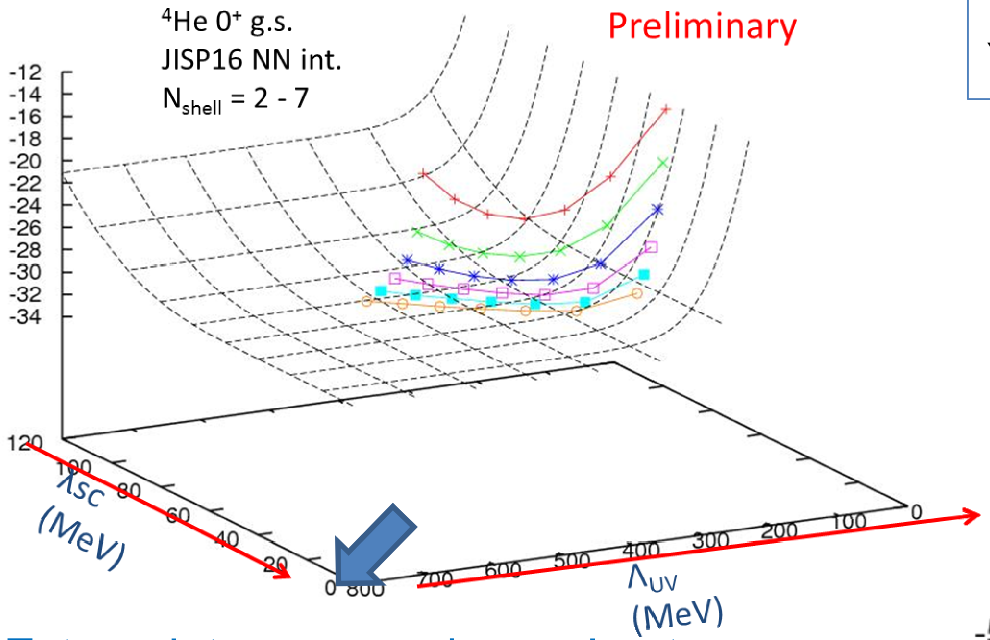
"quantum phase transition" from spherical to prolate deformation is shown by the analysis of Monte Carlo shell model (model space: 28<Z<70, 40<N<94, realistic effective int.)

"Type-II shell evolution" plays an important role.

T. Otsuka and Y. Tsunoda,
J. Phys. G 43, 024009 (2016)

No-core shell model calculations with MCSM + JISP16

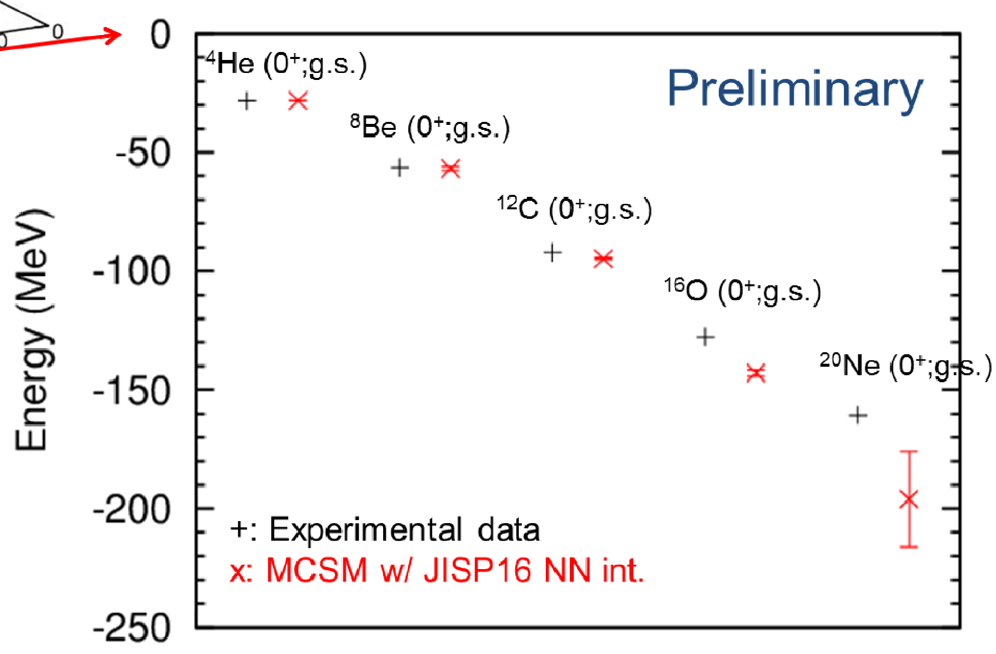
T. Abe, P. Maris, T. Otsuka, N. Shimizu, Y. Utsuno & J. P. Vary



Binding energy of light nuclei :

up to 7 major shells as model space

Extrapolate energy eigenvalue to infinitely large model space using ultra-violet and infrared “cutoff”



Infrared cutoff

$$\lambda_{sc} = \sqrt{m\hbar\omega/(N+3/2)} = \lambda^2/\Lambda$$

Ultraviolet cutoff

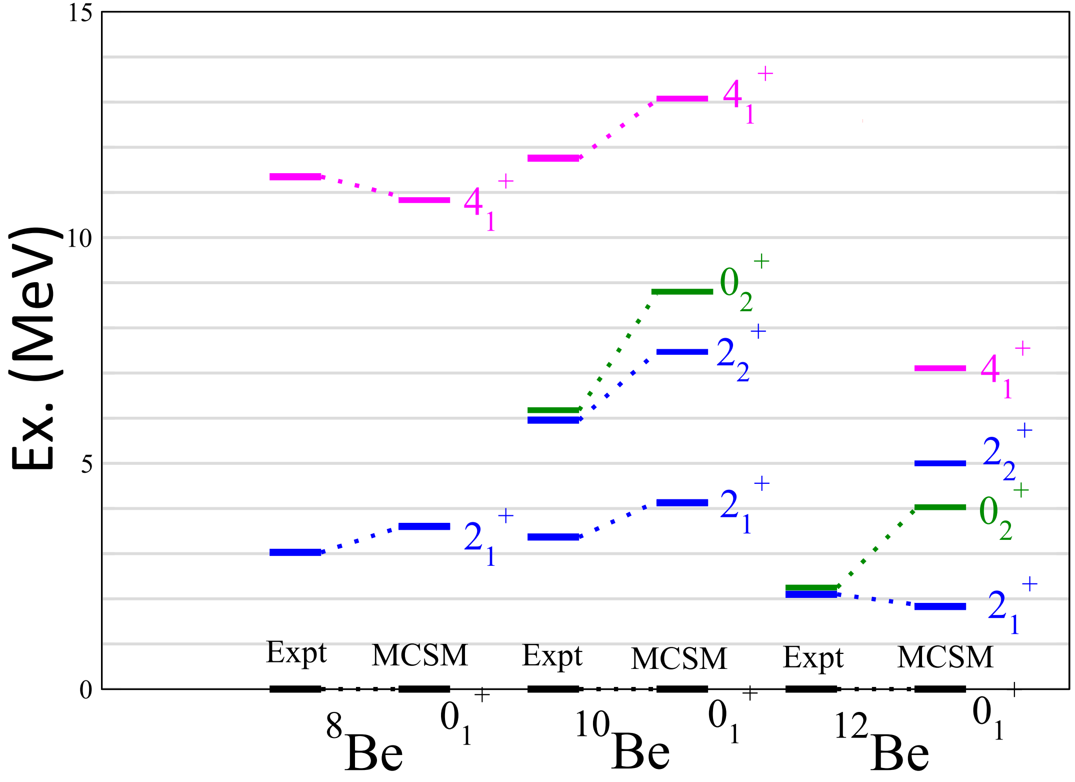
$$\Lambda = \sqrt{m(N+3/2)\hbar\omega}$$

JISP16: Shirokov *et al.*, PLB 644, 37 (2011)

no-core Monte Carlo shell model + JISP16

T. Yoshida, N. Shimizu, T Abe and T. Otsuka

Energy levels of ^{8,10,12}Be



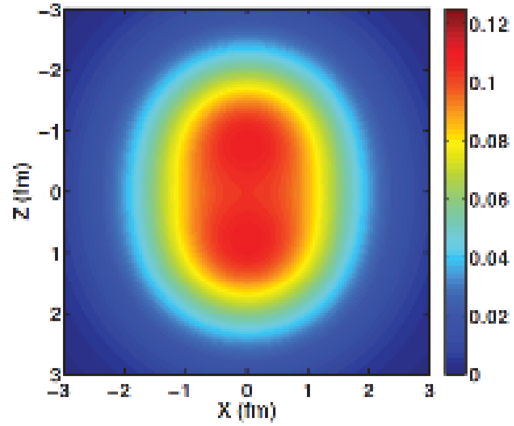
shell-model calculations with $N_{shell} = 6$ and JISP16 interaction

overall tendency is good, rather overestimated

Expt. ^{8}Be,^{10}Be: Tilley *et al.*, 2004,
^{12}Be: Shimoura *et al.*, 2003

Intrinsic density in Monte Carlo shell model

cf. Translationally invariant density
^{8}Li 2+ Cockrell *et al.,* PRC86 (2012)



Mean-field approx. + angular-momentum projection

$$\left|\Psi\right\rangle = P^{J}\boxed{\left|\phi(D)\right\rangle}$$

Slater determinant
Intrinsic state

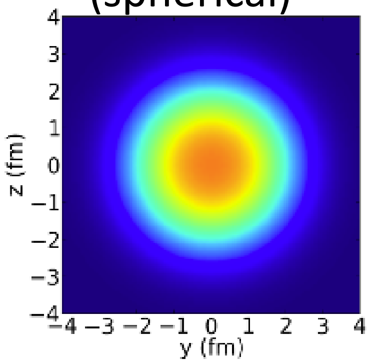
Monte Carlo shell model

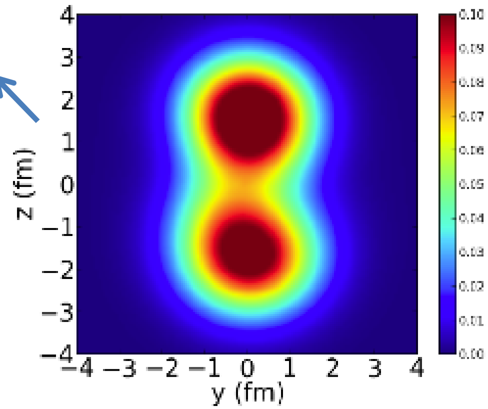
$$\left|\Psi\right\rangle = \sum_{k=1}^{N_{MCSM}} v_k P^{J,\pi}\left|\phi_k\right\rangle = P^{J,\pi}\boxed{\left(\sum_{k=1}^{N_{MCSM}} v_k\left|\phi_k\right\rangle\right)}$$

labo-system
(spherical)

Angular-momentum
projector

Intrinsic state





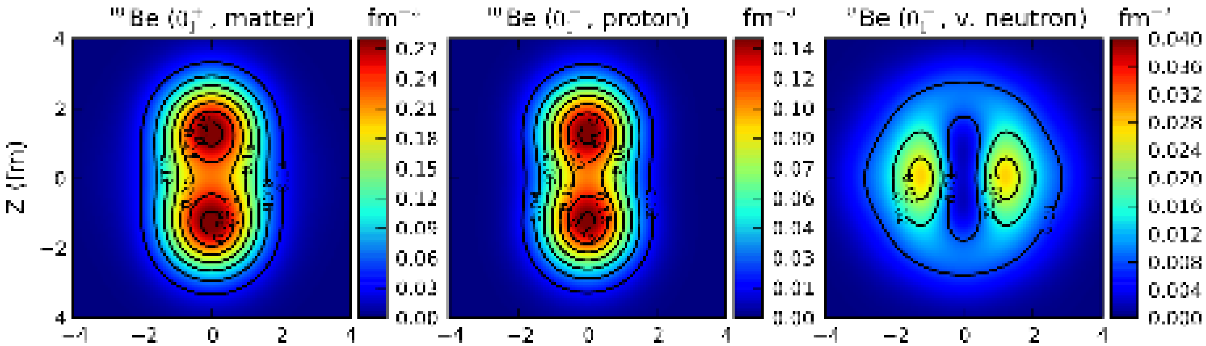
Density profile of ^{8}Be ground state

Intrinsic density by Monte Carlo shell model : Molecular orbits in ^{10}Be

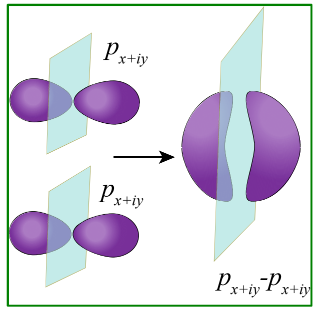
T. Yoshida, N. Shimizu, T Abe and T. Otsuka

Matter | Proton | Valence neutron (neutron – proton)

^{10}Be 0_1^+

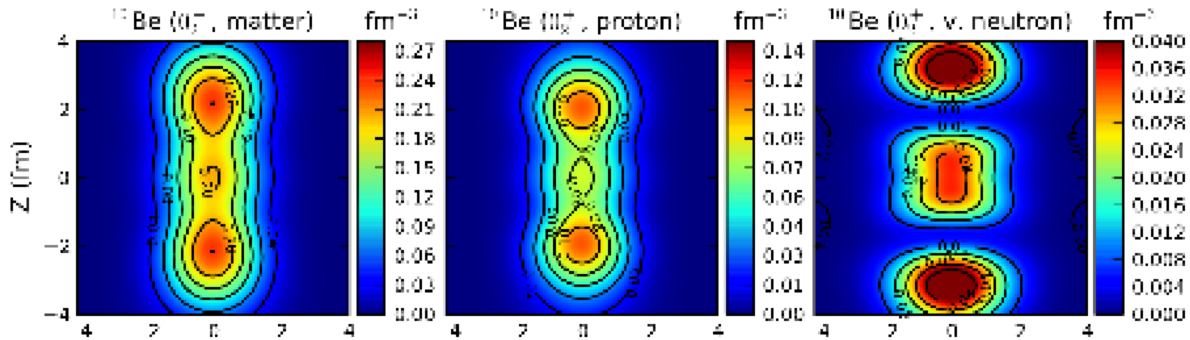


Pi molecular orbit

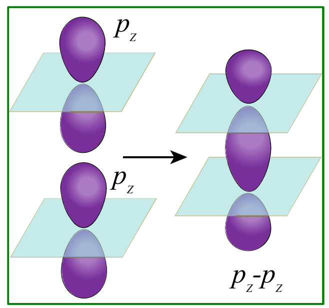


Excited state

^{10}Be 0_2^+



Sigma molecular orbit



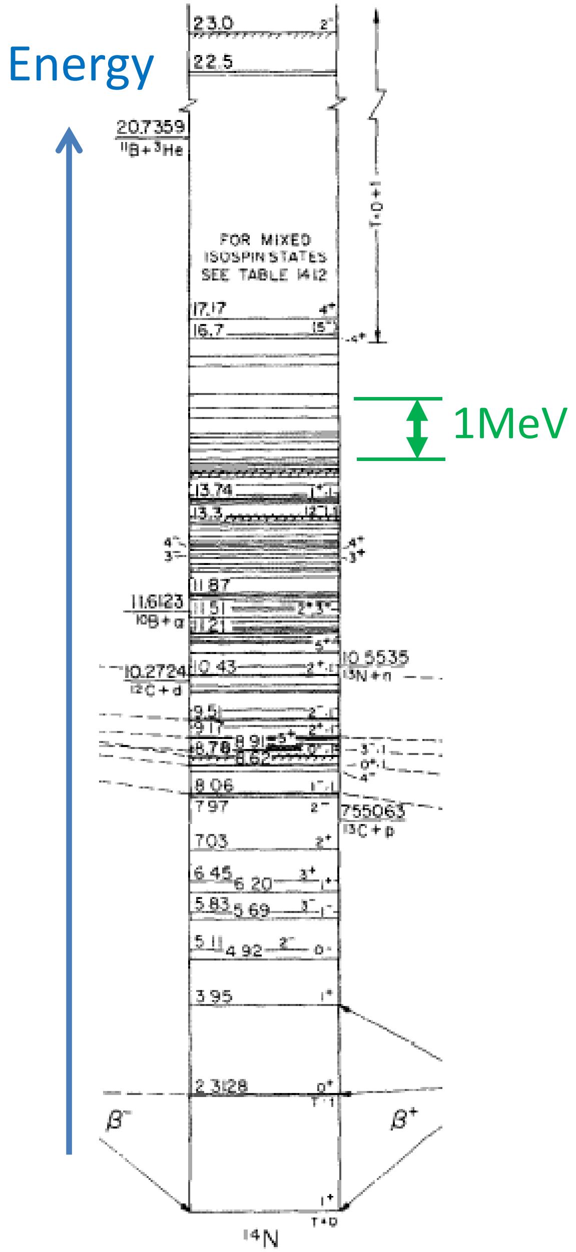
Elongation effect for 2-alpha distance

Appearance of sigma molecular orbit

Stochastic estimation of nuclear level density in the nuclear shell model: An application to parity-dependent level density in ^{58}Ni

Noritaka Shimizu (U. Tokyo), Yutaka Utsuno (JAEA),
Yasunori Futamura (U. Tsukuba), Tetsuya Sakurai (U. Tsukuba),
Takahiro Mizusaki (U. Senshu), and Takaharu Otsuka (U. Tokyo)
Phys. Lett. B **753**, 13 (2016)

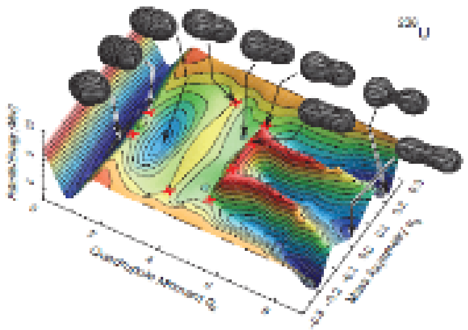
Nuclear level density

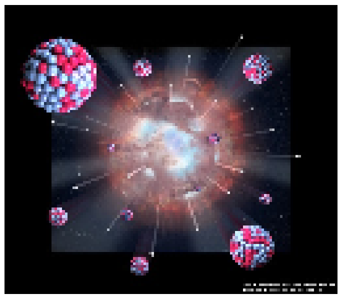


- Statistical model is important where the level density is large

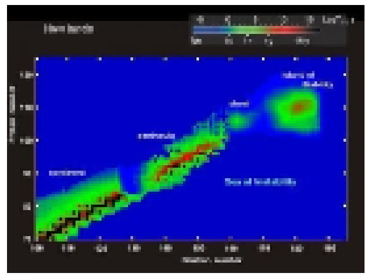
- Hauser-Feshbach theory
  - input: Level density, γ-ray strength function, optical potential
  - output: neutron-capture cross section

Nuclear engineering





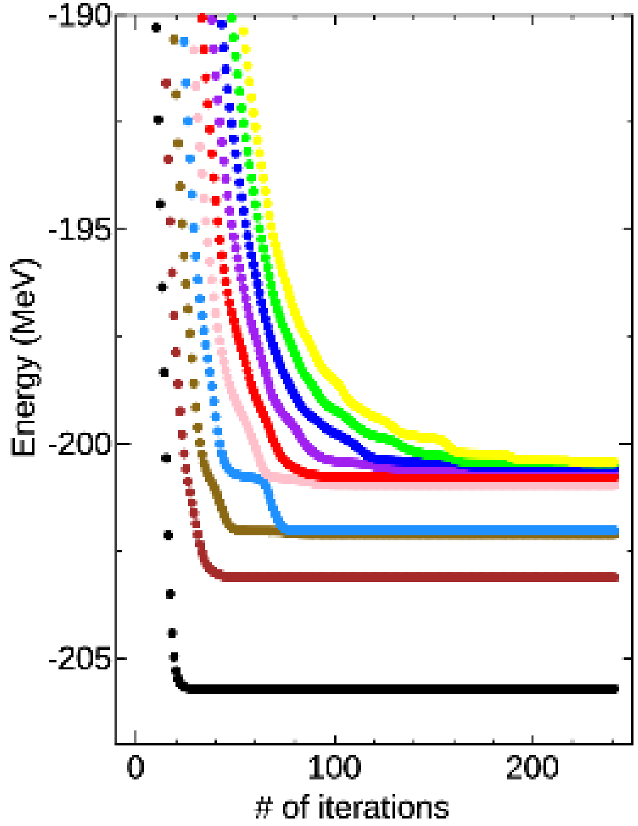
Nucleosynthesis



superheavy nuclei

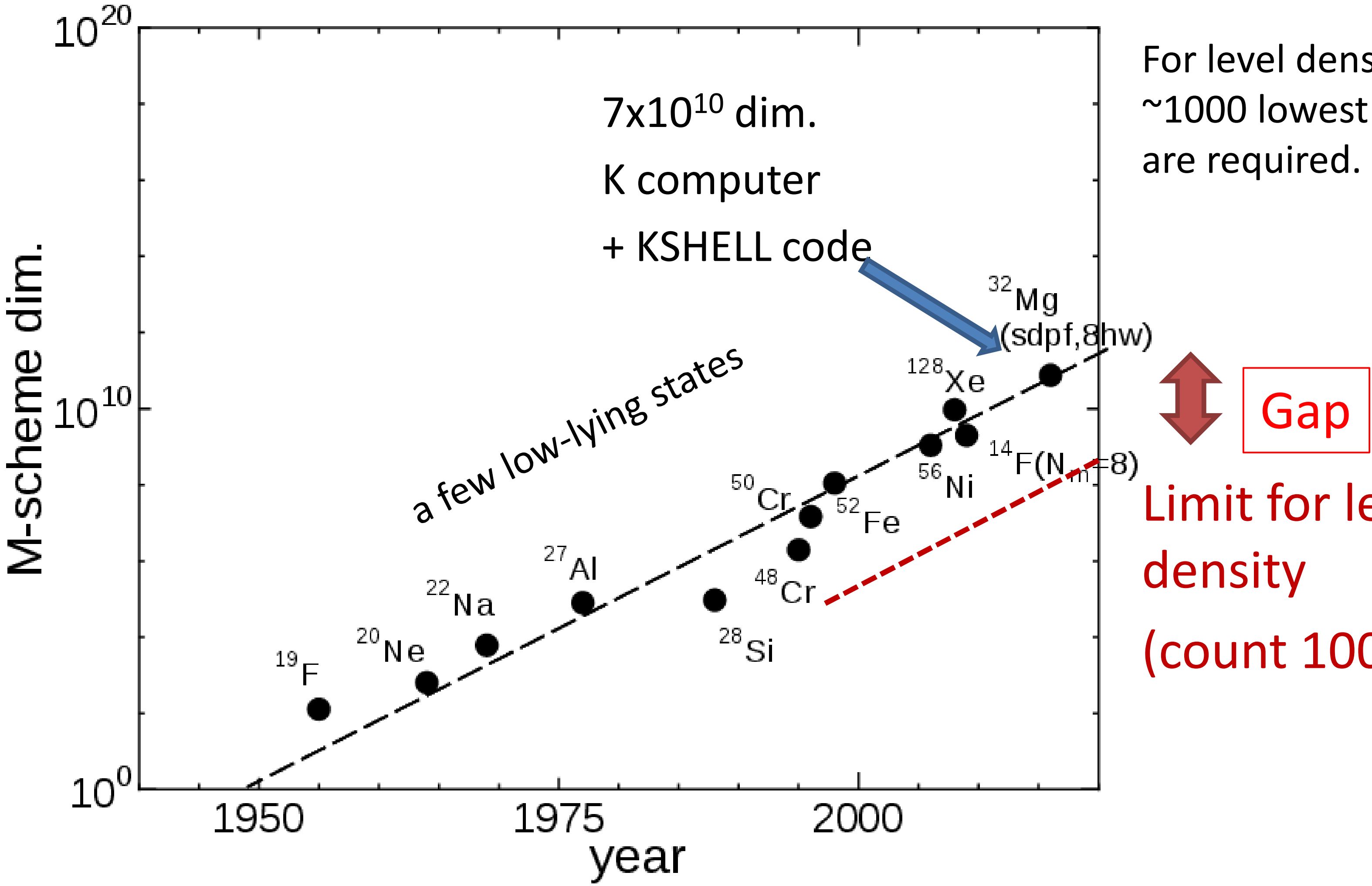
nuclear level density in LSSM

- Level density is a key input for Hauser-Feshbach theory. An empirical formula is usually used to obtain the level density.
  ⇒ microscopic model wanted
- Nuclear shell-model calculation is one of the most powerful tools to describe level density including various many-body correlations.
- Shell Model Monte Carlo (Y. Alhassid, H. Nakada *et al*.) is a most powerful tool to estimate level density. However, "sign problem" prevents us from using general realistic interaction.
- We propose a novel efficient method to compute level density in shell-model calculations avoiding direct count.



Convergence of eigenvalues in conventional Lanczos method
(^{56}Ni, pf-shell)
Difficult to obtain highly-excited high-density region

Current limitation of large-scale shell-model calc.



For level density,
~1000 lowest eigenenergies
are required.

Limit for level density

(count 1000 states)

Level density in nuclear shell-model calc.

- Level density ⇒ count the number of eigenvalues of huge sparse matrix in a certain energy region

JSIAM Letters Vol.2 (2010) pp.127–130 ©2010 Japan Society for Industrial and Applied Mathematics

JSIAM Letters

Parallel stochastic estimation method of eigenvalue distribution

Yasunori Futamura[1], Hiroto Tadano[1] and Tetsuya Sakurai[1]

[1] Graduate School of Systems and Information Engineering, University of Tsukuba, odai, Tsukuba-shi, Ibaraki 305-8573, Japan

E-mail *futamura@mma.cs.tsukuba.ac.jp*

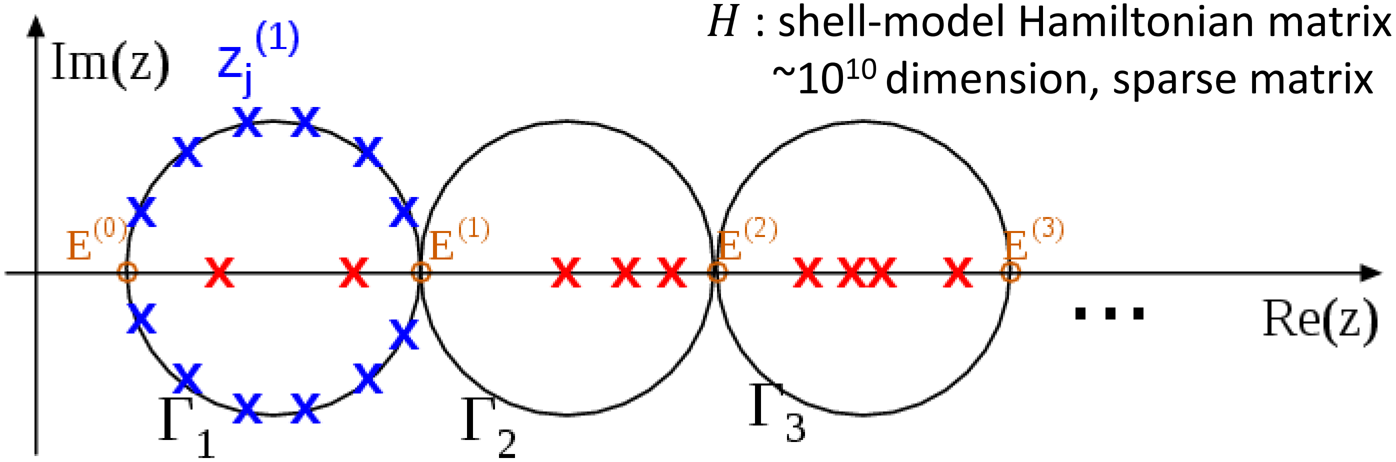
Received July 29, 2010, Accepted October 22, 2010

z-Pares

Counting eigenvalues : Cauchy integral

Count poles of $f(z) = \mathrm{Tr}\left((z-H)^{-1}\right) = \sum_i \frac{1}{z - E_i}$

H : shell-model Hamiltonian matrix in M-scheme
~10^{10} dimension, sparse matrix



$$\mu_k = \frac{1}{2\pi i} \oint_{\Gamma_k} dz \; \mathrm{tr}((z-H)^{-1})$$

$$\simeq \sum_j w_j \boxed{\mathrm{tr}((z_j^{(k)} - H)^{-1}),}$$

discretize

Next step : Stochastic estimation of trace

Hutchinson estimator

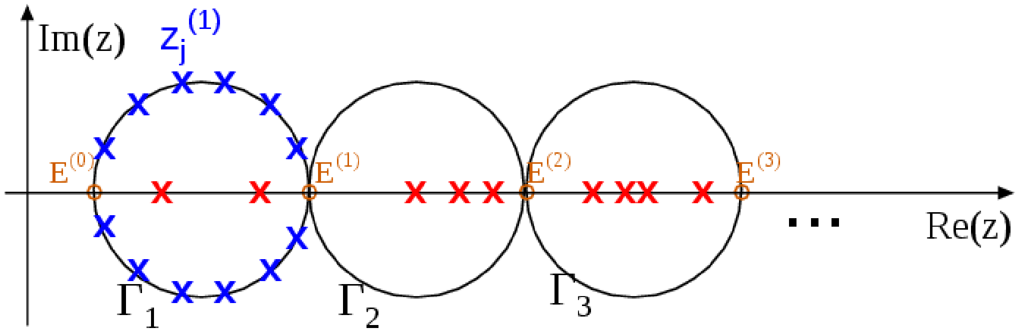
Estimate the matrix trace in Monte Carlo way

$$\mathrm{Tr}(A) \cong \frac{1}{N_s}\sum_{i=1}^{N_s} v_i^T A v_i$$

- matrix elements of the vector, v_i , are taken as -1 or 1 randomly with equal probability.
- well known in applied mathematics
- statistical error is small if non-diagonal matrix elements of A are small. (exact if A is diagonal)
- $v_i^T A v_i \ = \mathrm{Tr}(A) + \underbrace{\sum_{k\neq k'} (v_i)_k A_{kk'} (v_i)_{k'}}_{\text{Non-diagonal part}}$

Ref. M. F. Hutchinson, Comm. Stat. Sim. **19**, 433 (1990)

Stochastic estimation of “trace”



$$\mathrm{Tr}\left((z-H)^{-1}\right)=\sum_i \frac{1}{z-E_i}$$

D: Whole dimension

- estimating $\mathrm{tr}(z-H)^{-1}$

$$=\sum_i^{\boxed{D}} \boldsymbol{e}_i^T (z-H)^{-1}\boldsymbol{e}_i$$

$$\mu_k=\oint_{\Gamma_k}\mathrm{tr}\left((z\text{-}H)^{-1}\right)dz$$

N_s: Number of samples

- Stochastic sampling

$$\simeq \frac{1}{N_s}\sum_s^{\boxed{N_s}} \boxed{\boldsymbol{v}_i^T (z-H)^{-1}\boldsymbol{v}_i}$$

N_s=32 show Next step : Matrix inverse

$\boldsymbol{v}_i^T (z - H)^{-1} \boldsymbol{v}_i$

solve linear equations to avoid the inverse of matrix

solve $\boldsymbol{v}_i \;= (z - H)\,\boldsymbol{x}_i$ and obtain $\boldsymbol{v}_i^T \boldsymbol{x}_i$

$\boldsymbol{v}_i \;= A\,\boldsymbol{x}_i$

Complex Orthogonal Conjugate Gradient (COCG)

$$
\begin{aligned}
x_{k+1} &= x_k + \alpha_k p_k, \\
r_{k+1} &= r_k - \alpha_k \boxed{A p_k}, \\
p_{k+1} &= r_{k+1} + \beta_k p_k,
\end{aligned}
$$

$$\alpha_k = r_k^T r_k / p_k^T A p_k$$

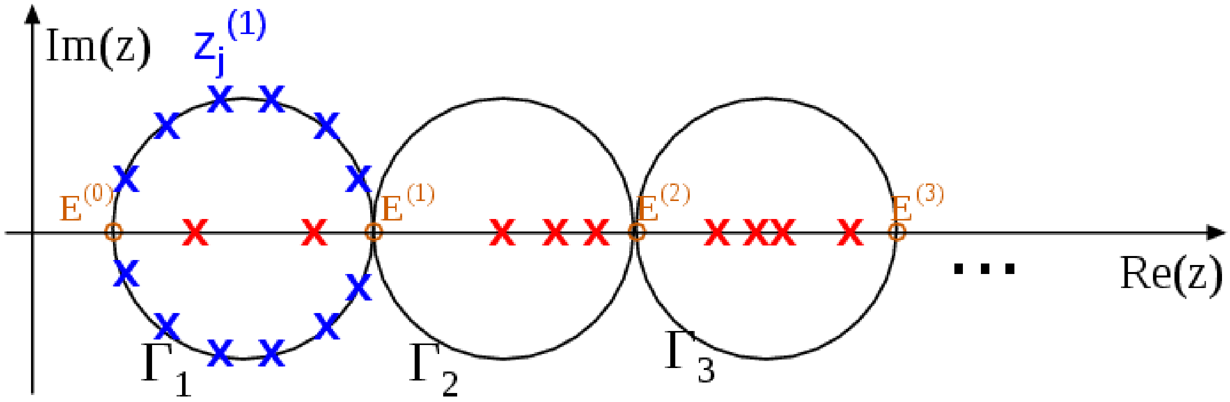
$$\beta_k = r_{k+1}^T r_{k+1} / r_k^T r_k$$

The matrix appears only in matrix-vector product

$\Rightarrow$ efficient for sparse matrix

In practice, we adopt Block COCG (BCOCG) with block bilinear form.

Introduction to shift algorithm



too many $z_j^{(i)}$ to be computed ...

⇒ Shift algorithm

solve $\boldsymbol{v}_i = (z_{ref} - \sigma - H)\,\boldsymbol{x}_i$ σ: shift

using solution of solve $\boldsymbol{v}_i = (z_{ref} - H)\,\boldsymbol{x}_i$

Shift invariance of Krylov subspace

- Purpose : solve $A\boldsymbol{x} = \boldsymbol{b}$ and $(A-\sigma)\boldsymbol{x} = \boldsymbol{b}$ simultaneously

  (σ: constant number, "shift")

- Krylov subspace

$$\mathcal{K}r(A,b) = \{b, Ab, A^2b, A^3b, \dots, A^{n-1}b\}$$

- COCG solution of $A\boldsymbol{x} = b$ is always on the Krylov subspace
- Is the solution of $(A-\sigma)\boldsymbol{x} = b$?

  $\Rightarrow$ It is on the same Krylov subspace

$$\mathcal{K}r(A-\sigma,b) = \{b, (A-\sigma)b, (A-\sigma)^2b, \dots, (A-\sigma)^{n-1}b\}$$
$$= \mathcal{K}r(A,b)$$

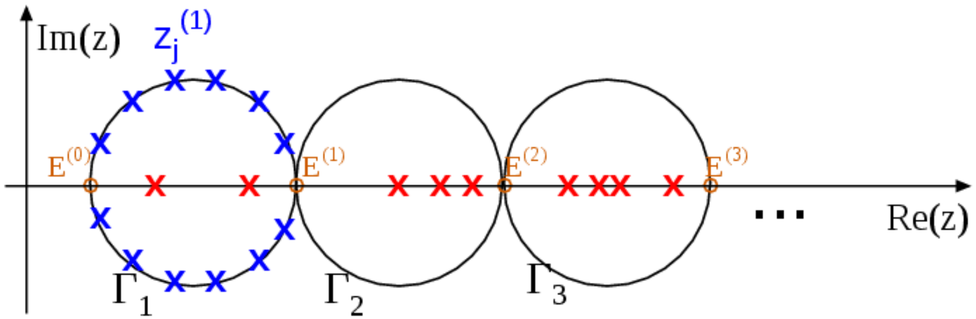
- Especially in COCG method, the residual of the shifted COCG is conserved except for its norm

$$r_k^\sigma = \frac{1}{\pi_k^\sigma} r_k$$

$$\begin{aligned} \pi_{k+1}^\sigma &= (1+\alpha_k\sigma)\pi_k^\sigma + \frac{\alpha_k\beta_{k-1}}{\alpha_{k-1}}(\pi_k^\sigma - \pi_{k-1}^\sigma), \\ \alpha_k^\sigma &= \frac{\pi_k^\sigma}{\pi_{k+1}^\sigma}\alpha_k, \\ \beta_k^\sigma &= \left(\frac{\pi_k^\sigma}{\pi_{k+1}^\sigma}\right)\beta_k \end{aligned}$$

Recipe for estimating level density

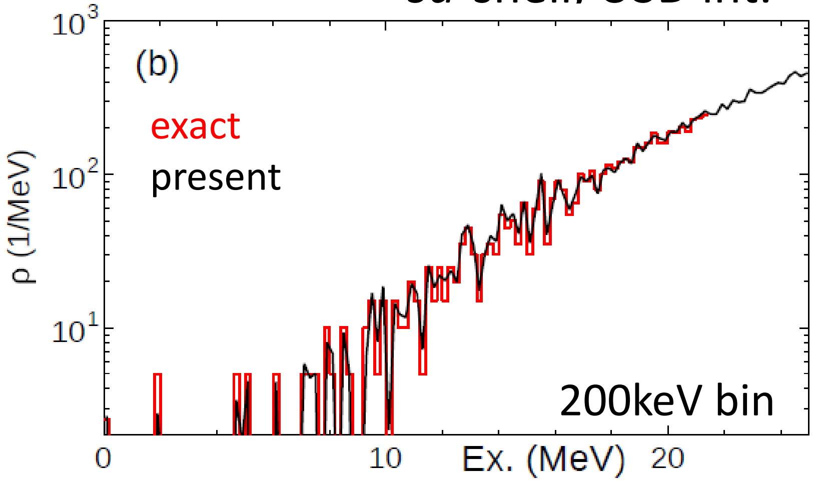
$$\mu_k = \frac{1}{2\pi i}\oint_{\Gamma_k} dz \, \mathrm{tr}((z-H)^{-1})$$



- Prepare mesh points $z_j^{(k)}$ to calculate the contour integral of z.
- Prepare random vectors $\boldsymbol{v}_i$ to estimate trace with Hutchinson estimator.
- Solve $(z-H)\boldsymbol{x}_i = \boldsymbol{v}_i$ by Block COCG method to obtain
  $\boldsymbol{v}_i^T(z-H)^{-1}\boldsymbol{v}_i$
- $\boldsymbol{v}_i^T\left(z_j^{(k)}-H\right)^{-1}\boldsymbol{v}_i$ for any j, k are calculated by shift algorithm

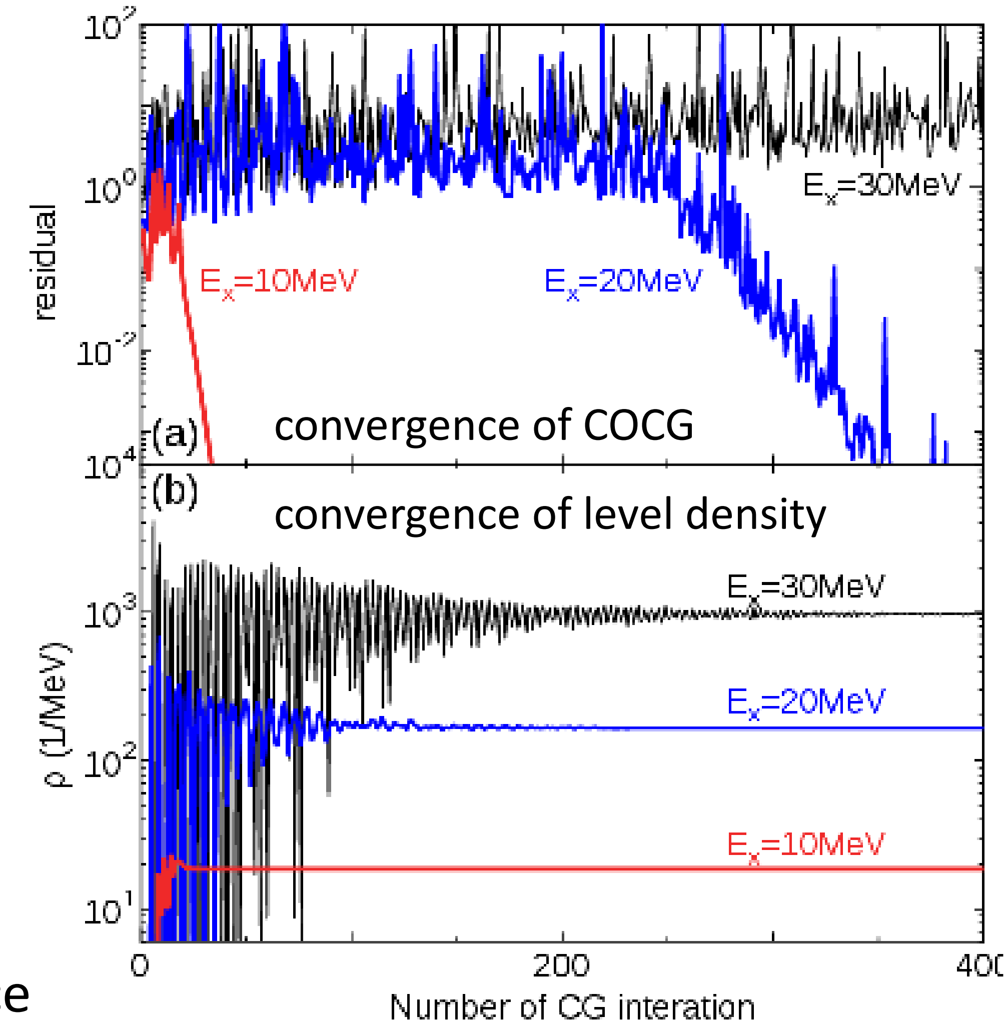
Benchmark test

Exact vs. stochastic estimate (N_s=32) for the level density in ^{28}Si *sd*-shell. USD int.



Stochastic estimation well reproduces the exact value!

- High excitation energy
  ⇒ high level density
  ⇒ slow COCG convergence
- Level density converges much faster than COCG



Benchmark of spin-dependent level density at large-scale problem

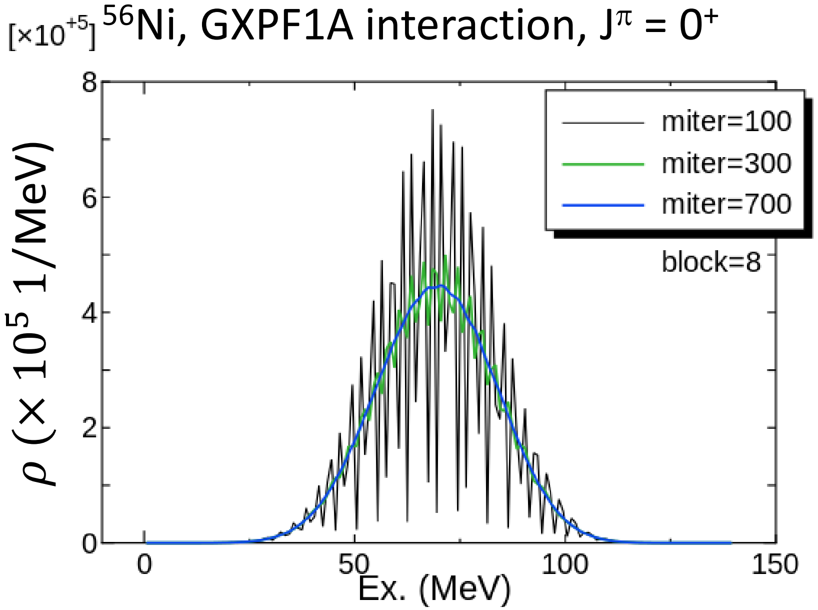
- Spin-dependent level density: replace initial sample vectors of *M*-scheme, $\boldsymbol{v}_i$, by angular-momentum projected vectors $P^J \boldsymbol{v}_i$

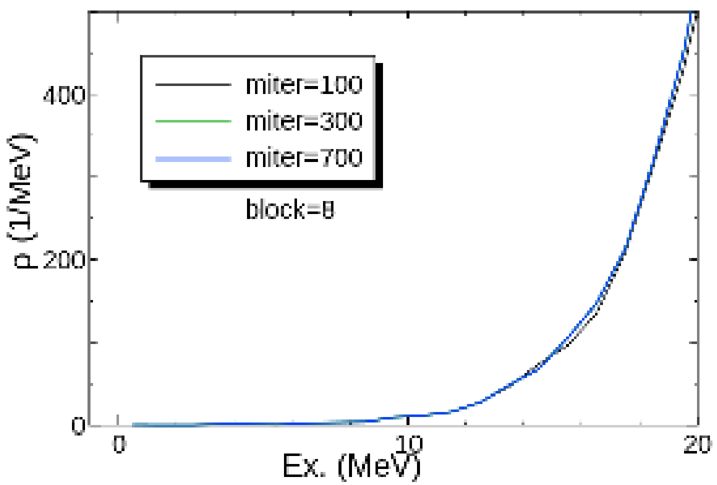
$$\rho(E) \approx \frac{1}{N} \sum_i \oint_\Gamma \vec{v}_i^{\,T} (z-H)^{-1} \vec{v}_i \, dz$$

- ^{56}Ni in pf-shell
  1.0×10^9 *M*-scheme dimension,
  1.5×10^7 *J*-scheme dimension
  impossible to obtain level density
  by conventional Lanczos method

- To converge the level density in whole energy region, 700 COCG iterations are required.

- 100 COCG iterations are enough for low-energy region

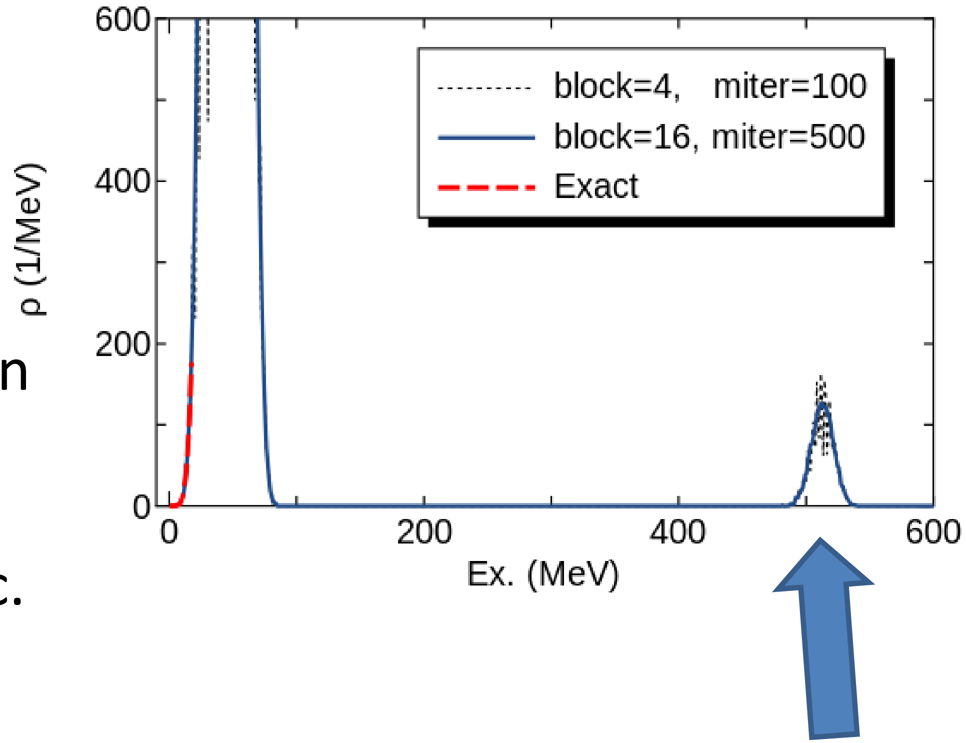




Subtract spurious center-of-mass states（Lawson method)

$$H' = H + \beta_{\rm CM} H_{\rm CM}$$

- ^{48}Ca J=1$^-$, *sd-pf-sdg* shell, 1ħω truncation



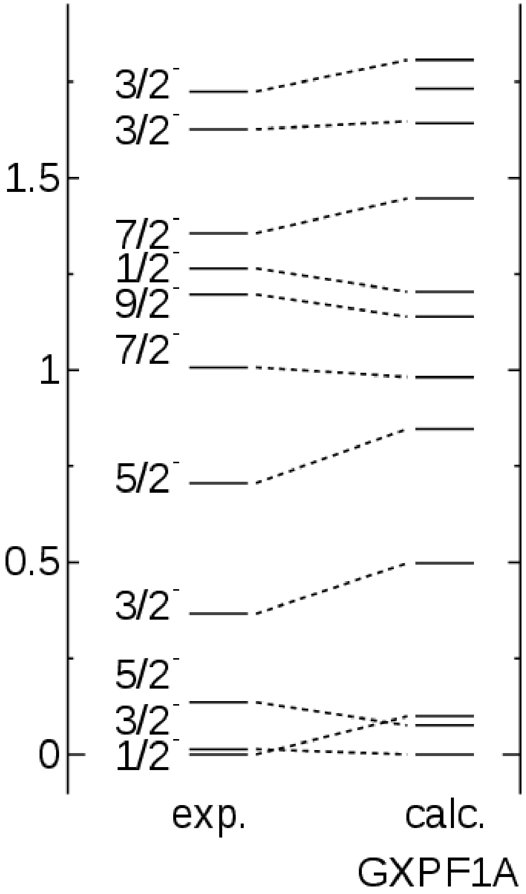
Spurious state by center-of-mass excitation

- Lift up spurious center-of-mass excited states by Lawson method clearly.

- agree well with the exact calc.

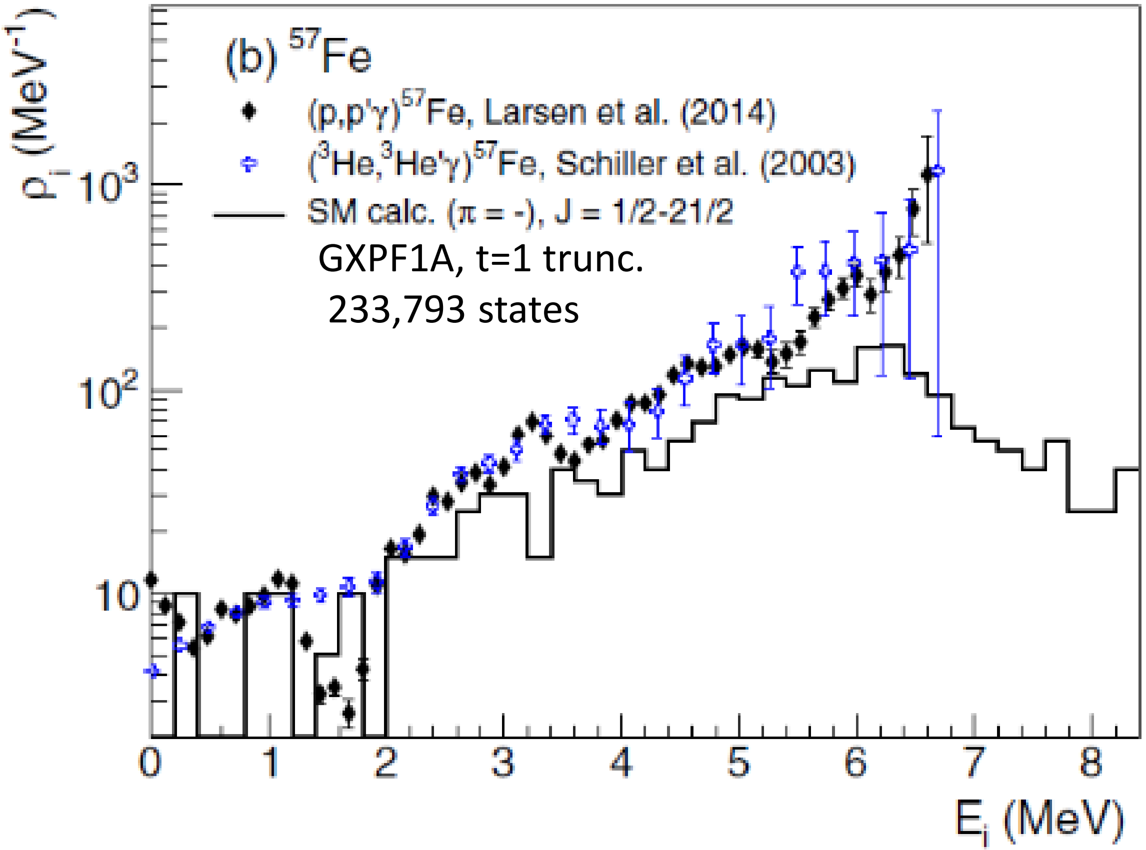
comparison with experiment: level density of ^{57}Fe

low-lying levels



Level density

exp. vs SM Lanczos method



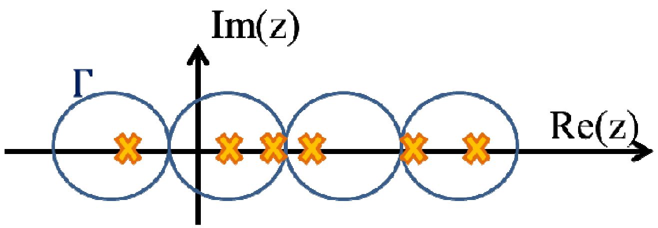
B. A. Brown and A. C. Larsen, PRL 113, 252502 (2014)

Realistic interaction gives a unified description of low-lying spectroscopy and level density in low-energy region

Level density by large-scale shell model calc.

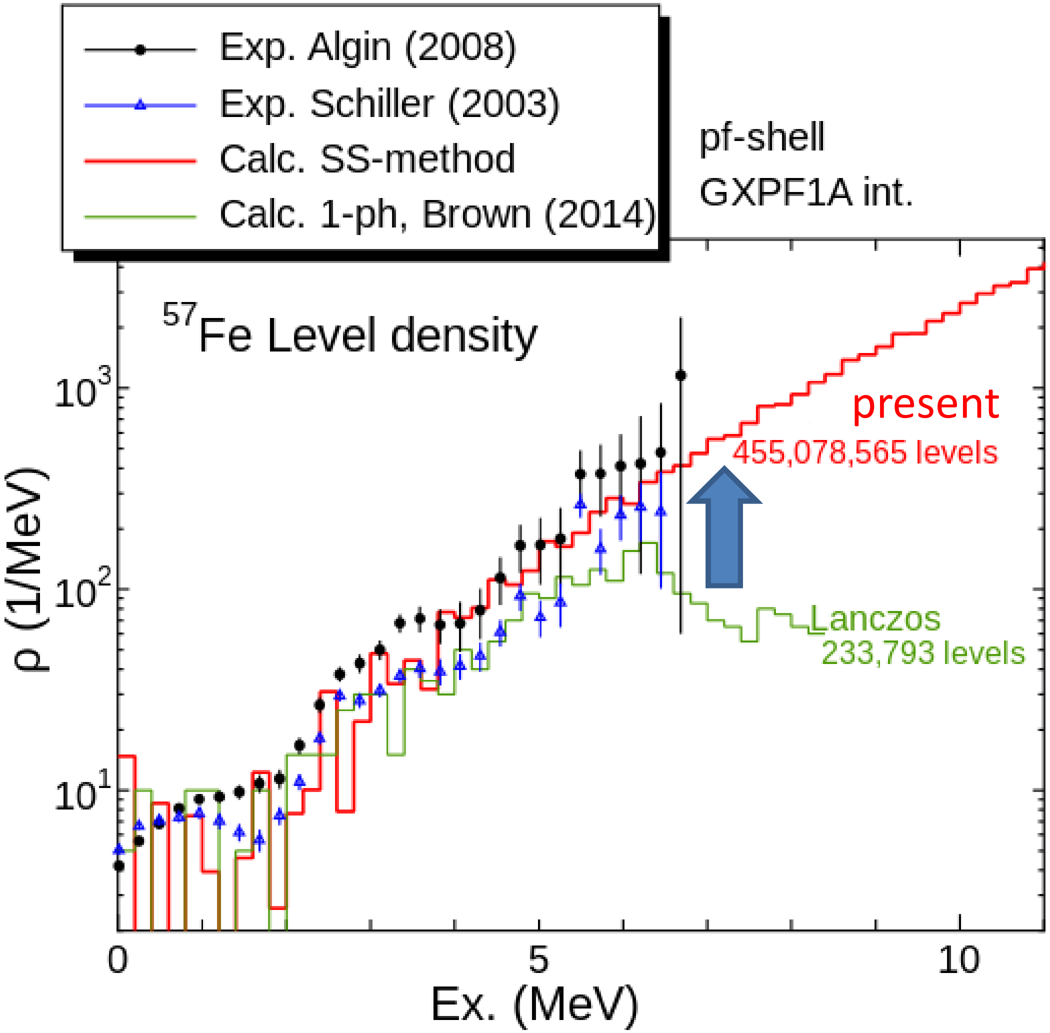
Stochastic estimation for density of states

$$\rho(E) = \oint_\Gamma \mathrm{tr}\left(\left(z\text{-}H\right)^{-1}\right) dz$$



$$\rho(E) \approx \frac{1}{N}\sum_i \oint_\Gamma \vec{v}_i \left(z\text{-}H\right)^{-1} \vec{v}_i \, dz$$

$$\approx \frac{1}{N}\sum_{i,j} w_j \vec{v}_i \left(z_j\text{-}H\right)^{-1} \vec{v}_i$$



SM calc. with realistic interaction provides us with a good description of level density

Puzzle: parity dependence of level density

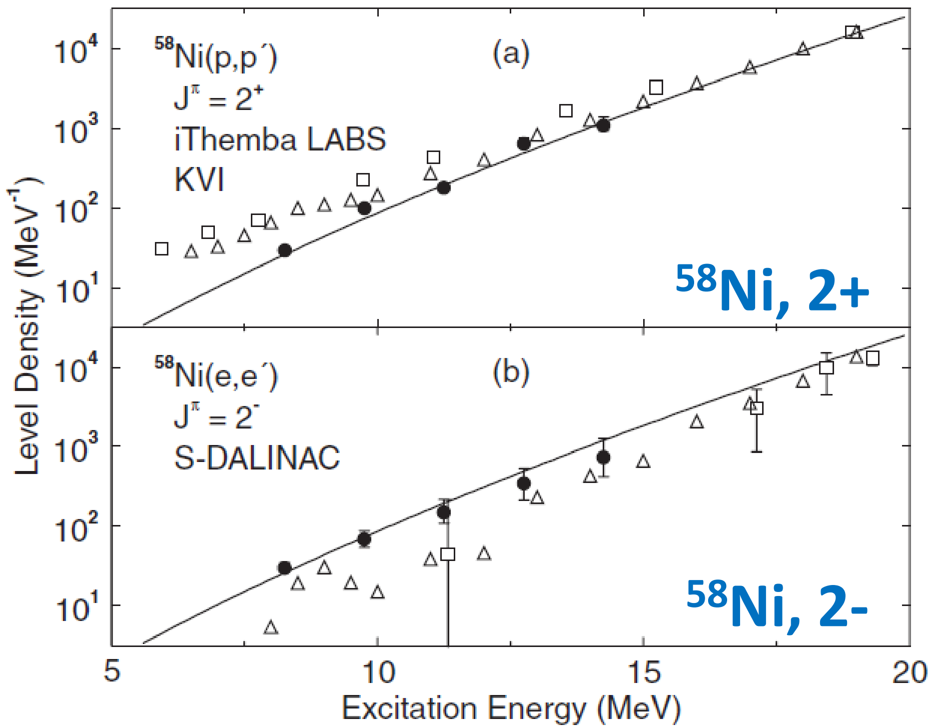
- Spin-parity dependence:
  crucial for application (due to selectivity)

● experiment
□ SMMC
△ HFB

- Parity-equilibration of level density is often assumed by empirical formulas

- Experiment shows that 2+ and 2- level densities of ^{58}Ni are similar value to each other (Kalmykov 2007)

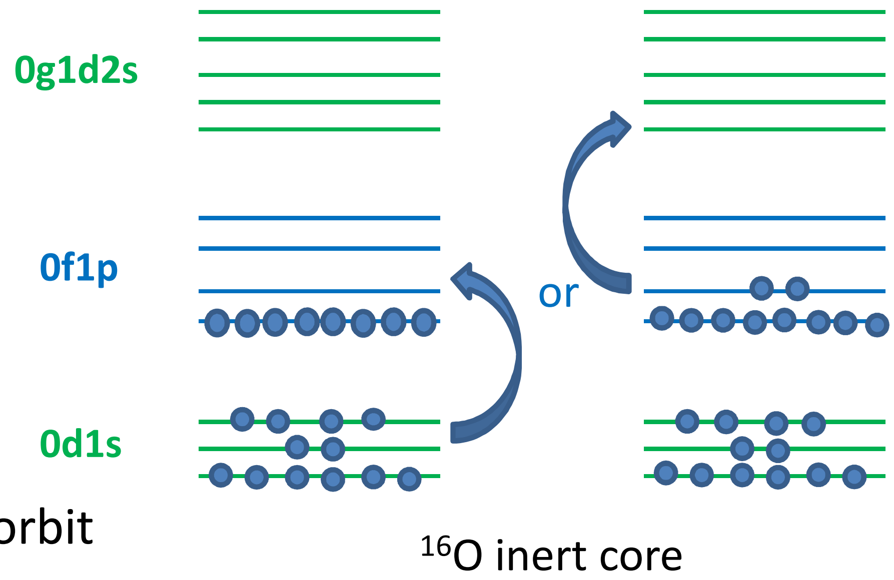
- Microscopic calculations (SMMC, HFB-based methods) overestimate 2+ level density and underestimate 2- level density



Ref. Y. Kalmykov, C. Ozen, K. Langanke, G. Martinez-Pinedo, P. von Neumann-Cosel and A. Richter, Phys. Rev. Lett. **99**, 202502 (2007).

Model space and interaction for LSSM calc. of *pf* -shell nuclei

- Target :
  - *pf*-shell nuclei (e.g. Ni isotopes)
- Valence shell
  - Full *sd-pf-sdg* shell
  - $0\hbar\omega$ for natural-parity states and
    $1\hbar\omega$ states for unnatural-parity states
  - up to 6-particle excitation from $0f_{7/2}$ orbit



- Effective interaction
  - SDPF-MU for *sd, pf* shells VMU for *sdg* shell (Y. Utsuno 2012)
    - SDPF-MU is successful in *sd-pf* shell calculations including exotic nuclei (e.g. ^{42}Si, ^{44}S)
  - $g_{9/2}$ SPE: fitted to $9/2^+_1$ in ^{51}Ti
- Removal of spurious center-of-mass motion
  - Lawson method: $H = H_{SM} + \beta H_{CM}$

Large-scale shell model calculations around ^{58}Ni

- Excitation energies and one-particle spectroscopic factors around ^{58}Ni
- Model space :
  - *sd, pf, sdg* shells
  - SDPF-MU for *sd, pf* shells VMU for *sdg* shell
  - 0ħω for natural-parity states, 1ħω for unnatural-parity states
  - up to 6-particle excitation from $0f_{7/2}$ orbit
  - 1.5×10^{10} *M*-scheme dim. for ^{58}Ni 2^- states

Nucl.	J^π	E_x (MeV)		C^2S		
		Cal.	Exp.	j	Cal.	Exp.
^{57}Co (^{58}Ni -p)	$7/2_1^-$	0	0	$\pi 0f_{7/2}^{-1}$	5.28	4.27, 5.53
	$1/2_1^+$	3.037	2.981	$\pi 1s_{1/2}^{-1}$	0.98	1.05, 1.31
	$3/2_1^+$	3.565	3.560	$\pi 0d_{3/2}^{-1}$	1.70	1.50, 2.33
^{57}Ni (^{58}Ni -n)	$3/2_1^-$	0	0	$\nu 1p_{3/2}^{-1}$	1.14	1.04, 1.25, 0.96
	$1/2_1^+$	5.581	5.580	$\nu 1s_{1/2}^{-1}$	0.51	0.62, 1.08
	$3/2_1^+$	5.579	4.372	$\nu 0d_{3/2}^{-1}$	0.29	0.01
	$3/2_2^+$	6.093	6.027	$\nu 0d_{3/2}^{-1}$	0.22	0.66, 0.54
^{59}Cu (^{58}Ni +p)	$3/2_1^-$	0	0	$\pi 1p_{3/2}^{+1}$	0.53	0.46, 0.49, 0.25
	$9/2_1^+$	3.139	3.023	$\pi 0g_{9/2}^{+1}$	0.26	0.24, 0.32, 0.27
^{59}Ni (^{58}Ni +n)	$3/2_1^-$	0	0	$\nu 1p_{3/2}^{+1}$	0.51	0.82, 0.33
	$9/2_1^+$	3.053	3.054	$\nu 0g_{9/2}^{+1}$	0.63	0.84, 0.39
	$5/2_1^+$	4.088	3.544	$\nu 1d_{5/2}^{+1}$	0.04	0.03
	$5/2_2^+$	4.595	4.506	$\nu 1d_{5/2}^{+1}$	0.30	0.23, 0.14
	$1/2_1^+$	4.399	5.149	$\nu 2s_{1/2}^{+1}$	0.00	0.09
	$1/2_2^+$	5.492	5.569	$\nu 2s_{1/2}^{+1}$	0.18	0.02
	$1/2_3^+$	5.589	5.692	$\nu 2s_{1/2}^{+1}$	0.02	0.13

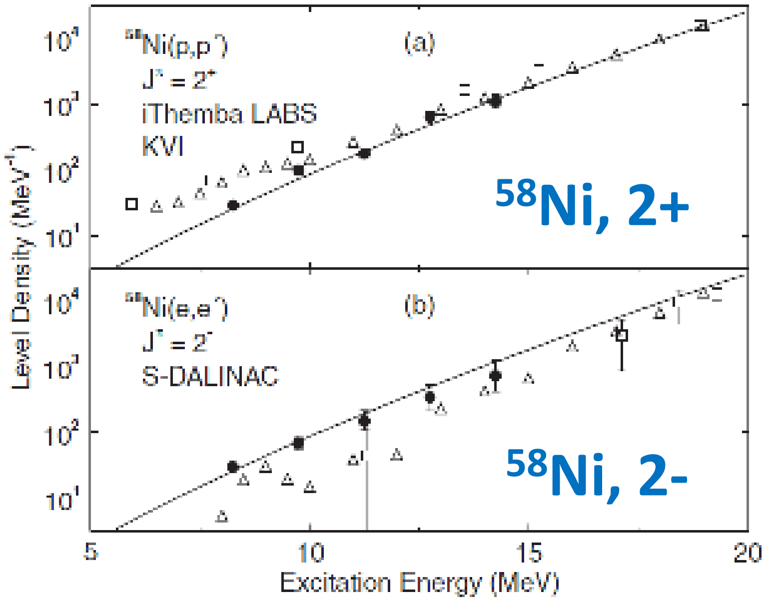
SDPF-MU int. Y. Utsuno *et al.*, PRC 86, 051301R (2012)
VMU int. T. Otsuka *et al.*, PRL 104, 012501 (2010)

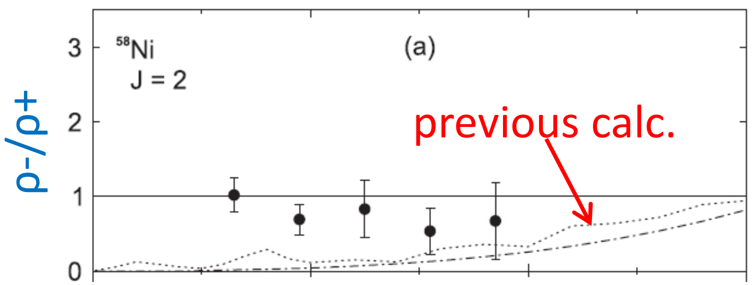
Single-particle characters of 58Ni ±p, n are successfully described by LSSM

Application: parity dependence of level density

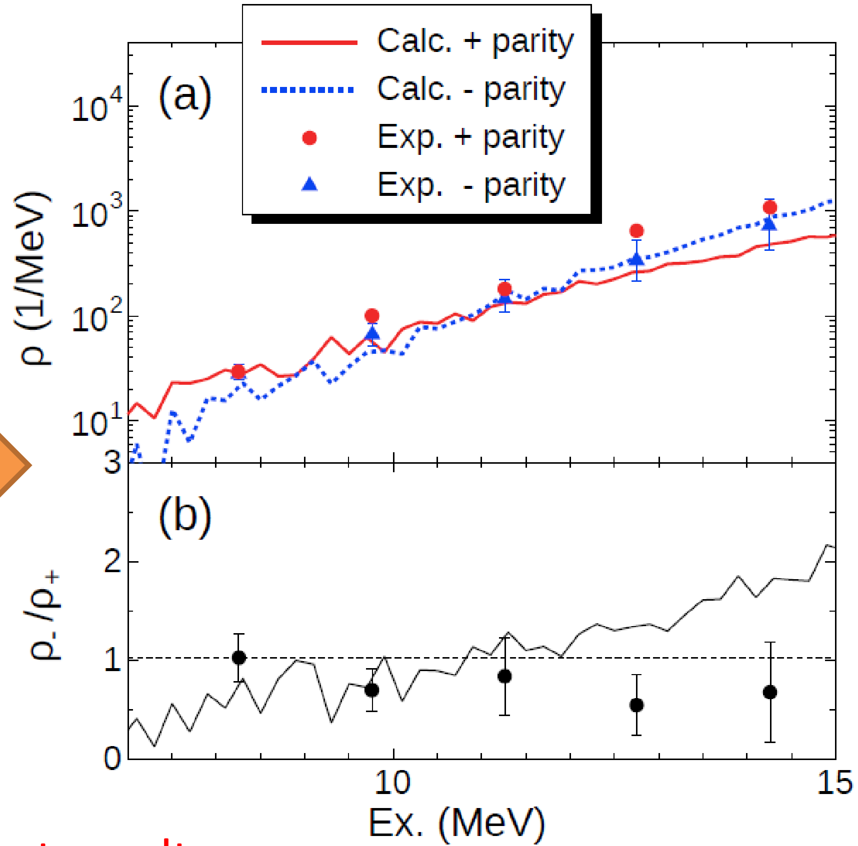
Contradiction between experiment and theory is resolved.

- No parity dependence (exp.) vs. strong parity dependence (prev. calc.)





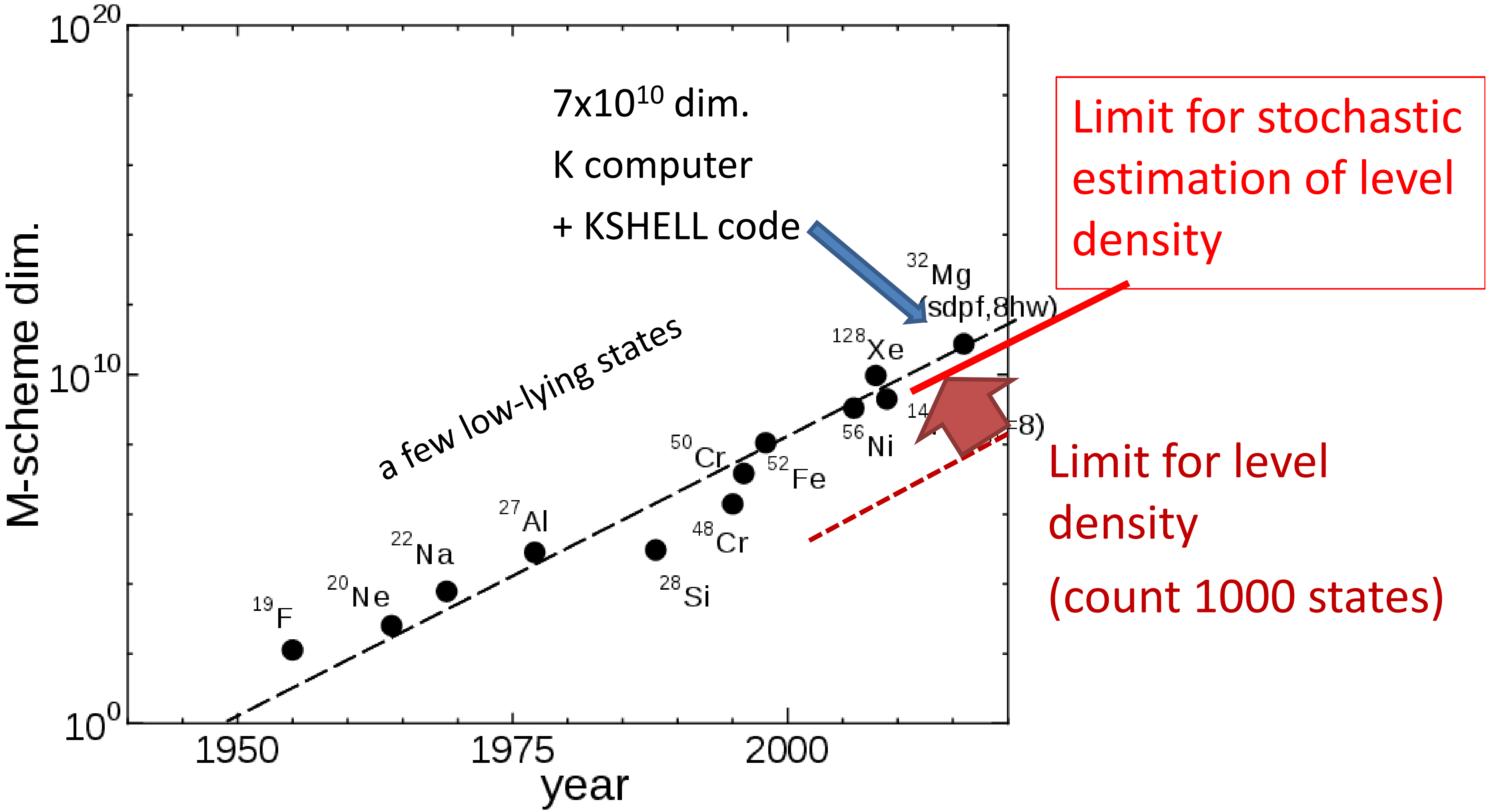
Ref. Y. Kalmykov *et al.*, PRL **99**, 202502 (2007).



Present result

Common setup to the study of exotic nuclei

Current limitation of large-scale shell-model calc.



Summary of LSSM level-density study

- We introduced a new stochastic estimation of level density in nuclear shell-model calculations utilizing conjugate gradient method.
  - Features: possible to treat any realistic interaction, spin-parity dependence, and removal of center-of-mass contamination.
  - Computational cost: similar order to obtaining a few low-lying states in Lanczos method
  - Benchmark : level densities by the new method agree well with the exact Lanczos results. Good convergence is seen.

- Parity equilibration of ^{58}Ni *J*=2 states of and low-lying spectroscopic information around ^{58}Ni are successfully described in a shell-model framework