Time-dependent Basis Function Approach to Nuclear Scattering

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NTSE-2016, Khabarovsk, Russia, Sept. 20, 2016

Outline

- Background and motivation
- Methodology
  - Test case: deuteron dissociation due to Coulomb field
- Conclusion and outlook

Basis Functions *in Ab initio* Shell Model

- Traditionally basis function approach has been widely used in *ab initio* shell model

$$H|\psi\rangle = E|\psi\rangle$$

- By choosing a basis, one casts the quantum many-body problem into the eigenvalue problem of the Hamiltonian matrix
- Eigenvalues → mass spectrum
- Eigenvectors → wavefunctions
- See Many Fermion Dynamics – nuclear physics (MFDn) for a well-established implementation

P. Maris, M. Sosonkina, J.P. Vary, E.G. Ng and C. Yang, ICCS 2010, Procedia Computer Science 1, 97(2010).

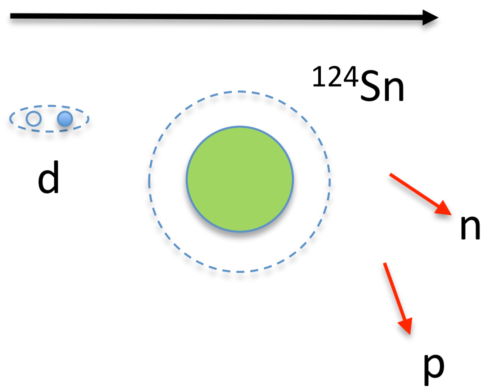
Motivation for Time-dependent Extension

- Technological advances in supercomputing make solving time-dependent Schrödinger equation within reach
  - Moore's law
  - It's time to development approach for the future
- Higher precision results from more differential measurements available due to progress in experimental nuclear physics
- More in-detail and more precise nonperturbative study of the dynamics in nuclear scattering is needed, esp. for strong/time-dependent fields

Example: d+^{124}Sn

- In the lab-frame:

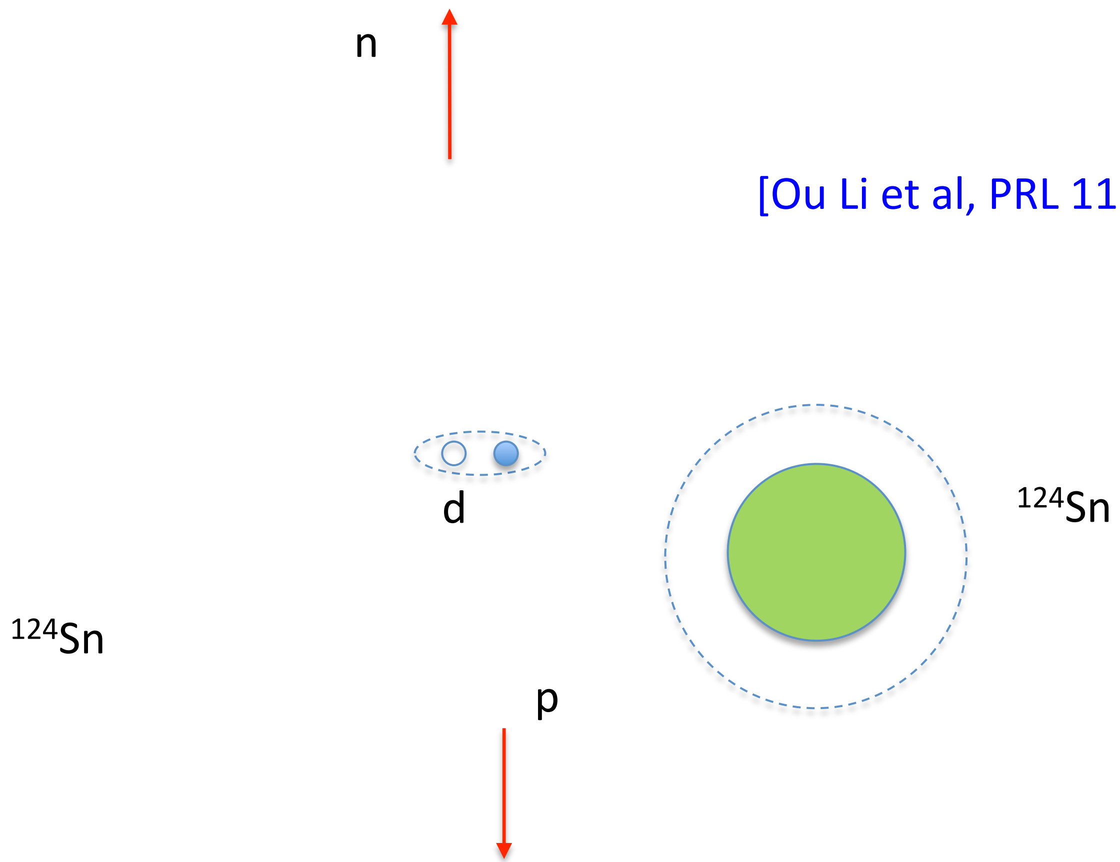
[Ou Li et al, PRL 115, 212501 (2015)]



- Physical motivation: study EoS of nuclear matter, esp. symmetry energy term
- Relative momentum between n and p in the final state is affected by the symmetry energy term in the target nuclear potential
- Quantum Molecular Dynamics (QMD) simulation results available
- Experimentally measurable (at RIKEN)

In Deuteron Center-of-Mass Frame

[Ou Li et al, PRL 115, 212501 (2015)]



- The target approaches the projectile and the classical background field is time-dependent
- Trajectory is estimated by QMD calculation
- Background potential is obtained by Skyrme energy density functional approach

Time-dependent Basis Function

Ab initio Shell Model

$H|\psi\rangle = E|\psi\rangle$

Time-dependent Basis Function

$i\,d/dt\,|\psi(t)\rangle = H(t)|\psi(t)\rangle$

- Natural extension of *ab initio* method to time-dependent regime
- Handle background fields explicitly depending on time
- Amplitude level -> quantum interference effects
- Non-perturbative -> strong field physics
- "Snapshots" of the system under investigation
- Take the advantage of high-performance supercomputing

tBF vs tBLFQ

[Zhao, Ilderton, Maris, Vary, PRD 88, 035205 (2013)]

- tBF: time-dependent Basis Function
  - For low-energy quantum mechanics
- tBLFQ: time-dependent Basis Light-front Quantization
  - For relativistic quantum field theory

tBF

$$id/dt\,|\psi(t)\rangle = H(t)|\psi(t)\rangle$$

tBLFQ

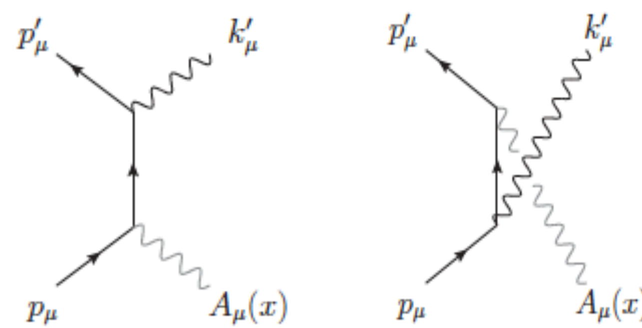
$$i\frac{\partial}{\partial x^{+}}\left|\psi(x^{+})\right\rangle = \frac{1}{2}P^{-}\left|\psi(x^{+})\right\rangle$$

- tBLFQ uses the Hamiltonian of quantum field theory
  - Relativity is naturally built-in
- tBLFQ basis consists of multiple Fock sectors such as

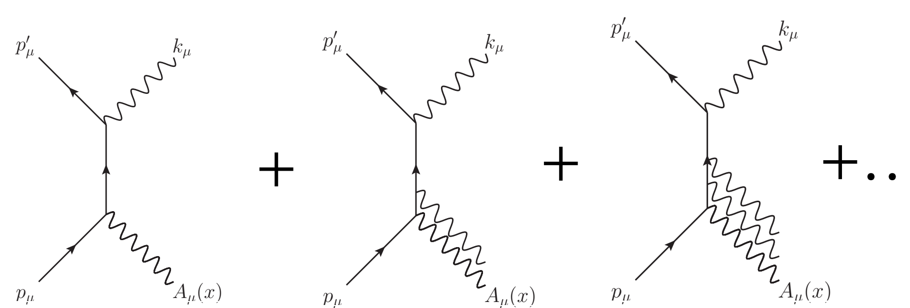
$$|e\downarrow p\rangle = a|e\rangle + b|e\gamma\rangle + c|e\gamma\gamma\rangle + d|eee\rangle + \ldots.$$

Application to Strong QED: Nonlinear Compton Scattering

- $e + n\gamma(laser) \rightarrow e' + \gamma'$
- 10^{20} photons in a laser: model as background field
- Perturbation theory:
- $\sigma \propto \text{Klein-Nishina} \times \tilde{A}^2$

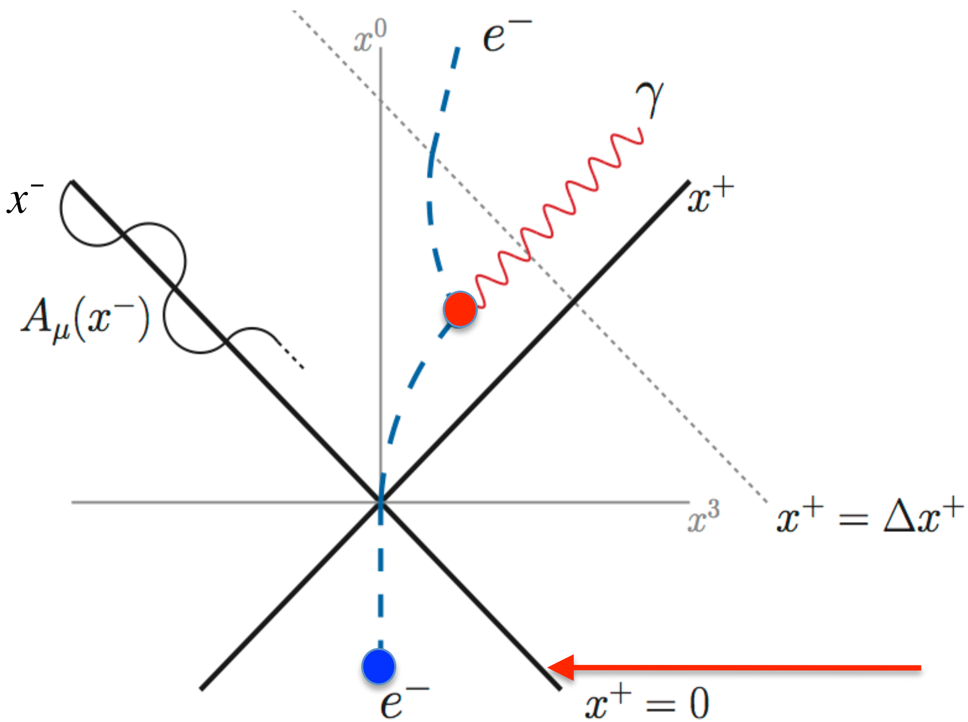


- At high intensity: non-perturbative treatment needed



Setup for Nonlinear Compton Scattering

- Space-time structure



[Zhao, Ilderton, Maris, Vary, PRD 88, 035205 (2013)]

- Two effects: acceleration and radiation

Advantages

- "Snapshots" of the nucleon systems, revealing nuclear dynamics in real time
- Quantum interference is kept during time-evolution
- Study nucleon systems in strong/time-dependent background field
- Nonperturbative effects
- Close connection to light-front quantum field theory -> systematically expandable to quantum field theory treatment

Time-dependent Basis Function Approach

- BLFQ: for quantum field eigenspectrum
- tBLFQ: for quantum field evolution

BLFQ

$$P^- |\psi\rangle = P^-_\psi |\psi\rangle$$

tBLFQ

$$i\frac{\partial}{\partial x^+}|\psi(x^+)\rangle = \frac{1}{2}P^-|\psi(x^+)\rangle$$

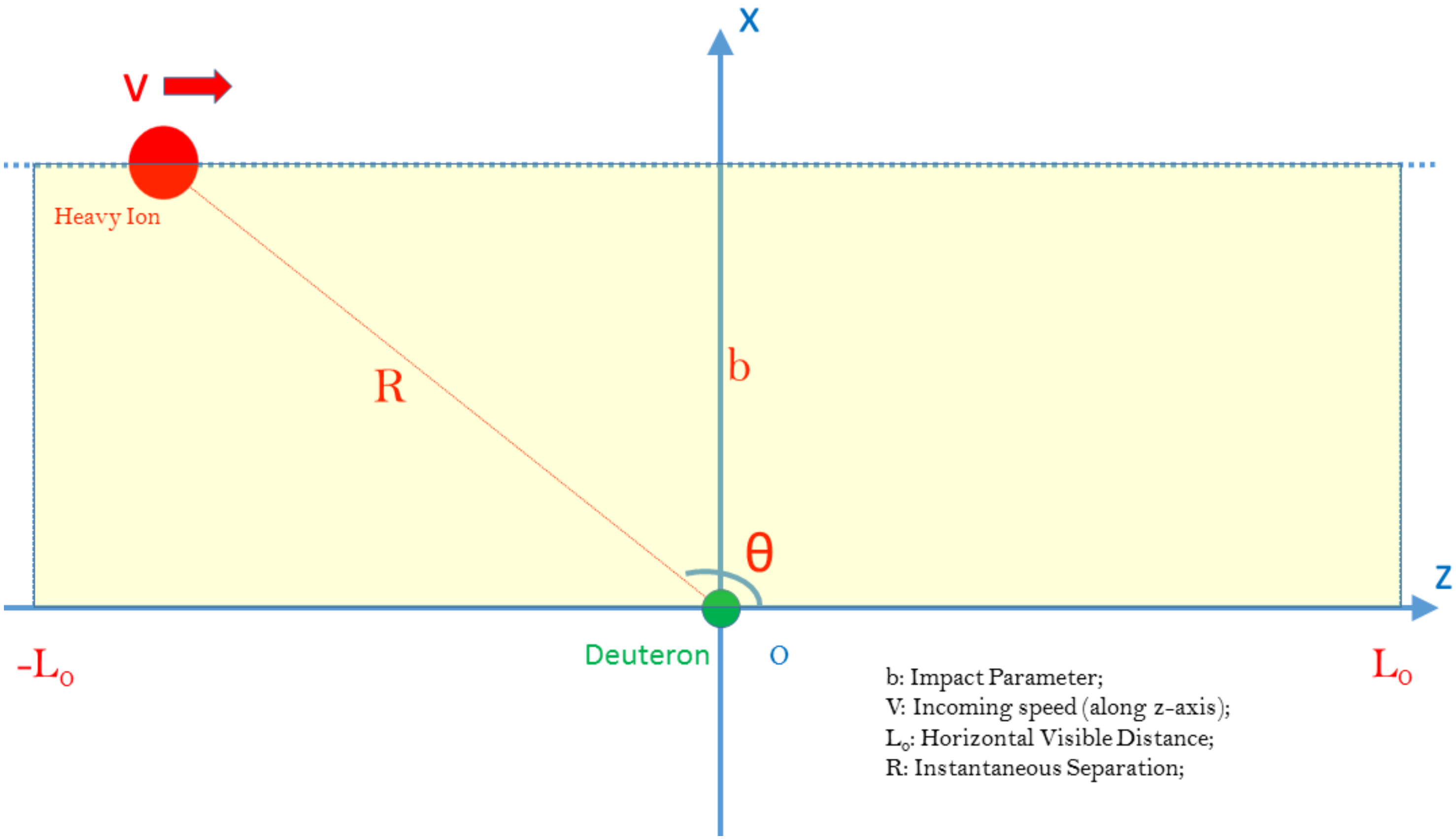
- Real-time framework: BLFQ → tBLFQ
- tBLFQ is designed for:
  - time-dependence in dynamical processes
  - in strong/time-dependent background field

General Procedure for tBF

1. Write down the Hamiltonian
2. Adopt the interaction picture
3. Prepare the initial ('in') state
4. Evolve the initial state until the background field subsides
5. Project the scattering final state onto 'out' states (constructed out of QED eigenstates) and obtain S-matrix element

$$S = {}_I\langle out \,|\, T\exp\left(-i\int_{-\infty}^{\infty} V_I\right) |\, in\rangle_I$$

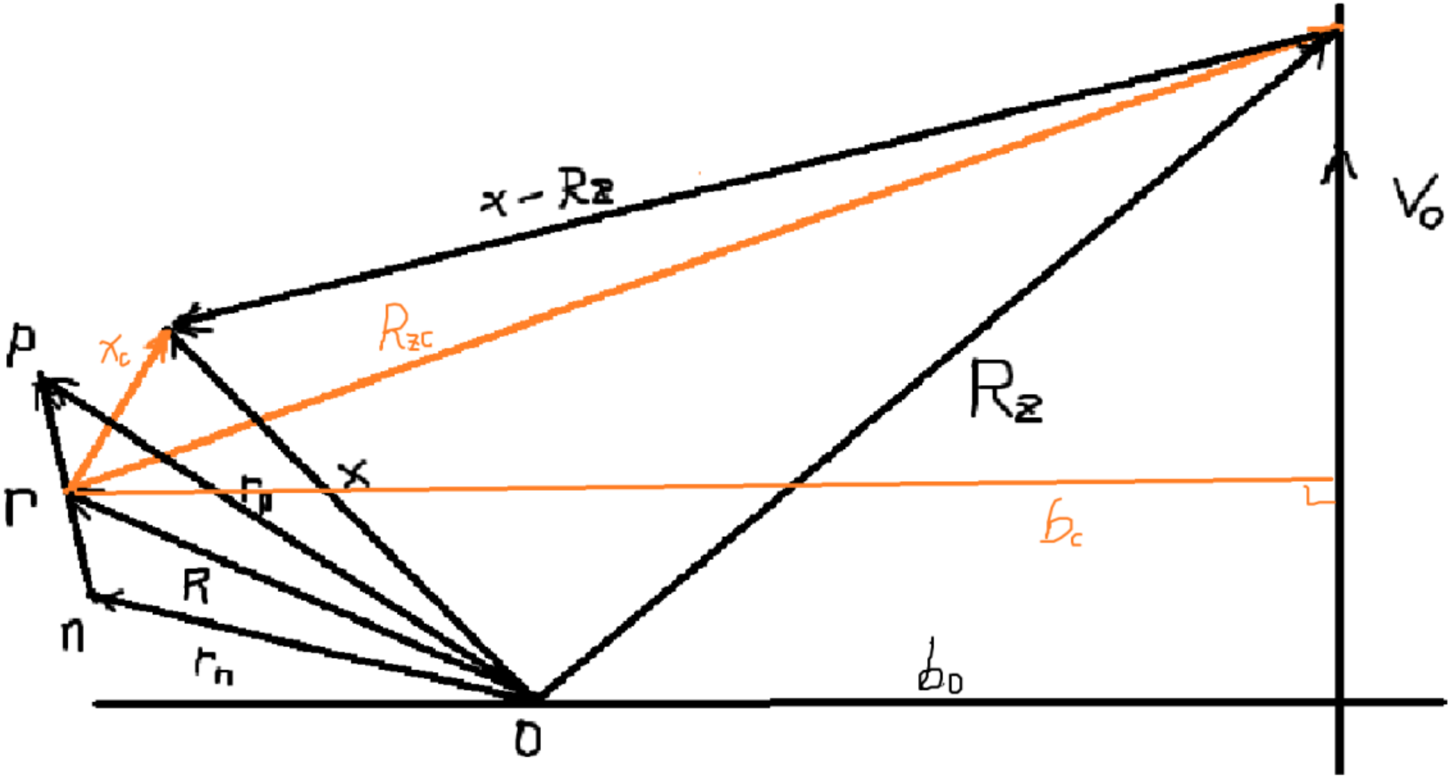
Test Case: Deuteron Dissociation in Coulomb Field



The Coulomb field is due to a passing-by heavy-ion with constant velocity v

Approximation: we neglect the center-of-mass motion of the deuteron R; we are interested in the relative motion between p and n only r.

Coordinate System



$$\vec{R} = \frac{\vec{r}_p + \vec{r}_n}{2} \qquad \vec{r} = \vec{r}_p - \vec{r}_n$$

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Hamiltonian

$$
\begin{aligned}
H_{Full} &= H_0 + V_{int} \\
H_0 &= KE + V_{QCD} \\
V_{int}(t) &= \int A_\mu J^\mu \, d\vec{r}
\end{aligned}
$$

H_{Full}: total Hamiltonian

H_0: (time-independent) Hamiltonian for deuteron

V_{int}: (time-dependent) interaction between the heavy-ion and deuteron

KE: we keep the kinetic energy of relative motion only

V_{QCD}: nucleon-nucleon interaction

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Neutron-Proton Stationary States

- In interaction picture, time evolution is computed in the basis formed by the eigenstates of H_0
- The background field V_{int} induces transitions among the "tower" of eigenstates of H_0
- We need to solve the eigenvalue problem of H_0 and get a "tower" of eigenstates

$$H_0 |\psi\rangle = E|\psi\rangle$$

- We take V_{QCD} to be JISP16 NN interaction

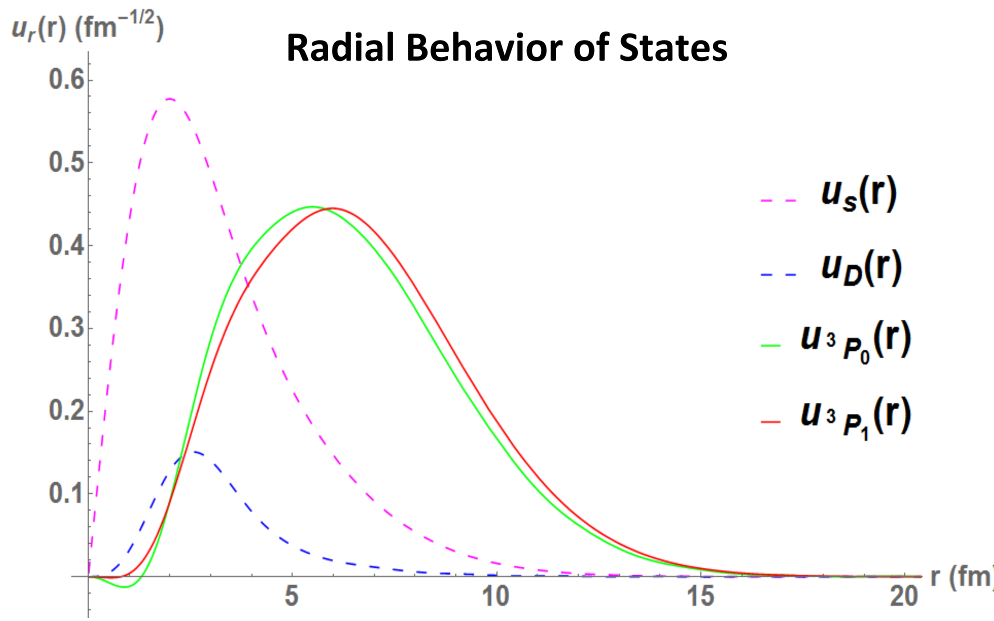
[A. M. Shirokov et al, PLB 644, 33 (2007)]

Deuteron and Its Excitation Spectrum

- As test problem, we retain 3 channels: $({}^3S_1, {}^3D_1) \quad {}^3P_0 \quad {}^3P_1$
- Deuteron has one bound state – ground state, all the excited states are scattering states in continuum; we regulate the scattering states by putting the deuteron system in a HO trap with ω=5MeV
- Obtained level system:

Block	States	Energy
Block 1	$1st; ({}^3S_1, {}^3D_1); M_j = -1$	E=-0.65289 MeV
	$1st; ({}^3S_1, {}^3D_1); M_j = 0$	
	$1st; ({}^3S_1, {}^3D_1); M_j = +1$	
Block 2	$1st;\ {}^3P_0; M_j = 0$	E= 12.0733 MeV
Block 3	$1st;\ {}^3P_1; M_j = -1$	E= 12.7585 MeV
	$1st;\ {}^3P_1; M_j = 0$	
	$1st;\ {}^3P_1; M_j = +1$	

N_{max}=60, b=5MeV



Background Field

- We neglect magnetic interaction and keep only electric interaction

$$V_{int}(t) = \int A_\mu J^\mu \, d\vec{r}$$
$$= \int \rho(\vec{r},t)\varphi(\vec{r},t)\, d\vec{r} - \int \vec{j}(\vec{r},t)\cdot\vec{A}(\vec{r},t)\, d\vec{r}$$

- We perform multipole expansion on Coulomb field

$$\varphi(\vec{x},t) = \frac{Ze}{|\vec{x} - \vec{R}_Z(t)|}$$
$$= Ze\sum_{\lambda\mu}\frac{4\pi}{2\lambda+1}Y^*_{\lambda\mu}(\hat{R}_Z(t))\, Y_{\lambda\mu}(\hat{x})\frac{x^\lambda}{R_Z^{\lambda+1}(t)}$$

$R_Z(t) = b + v\,t$ is the location of the source

Same time-dependence for same λ

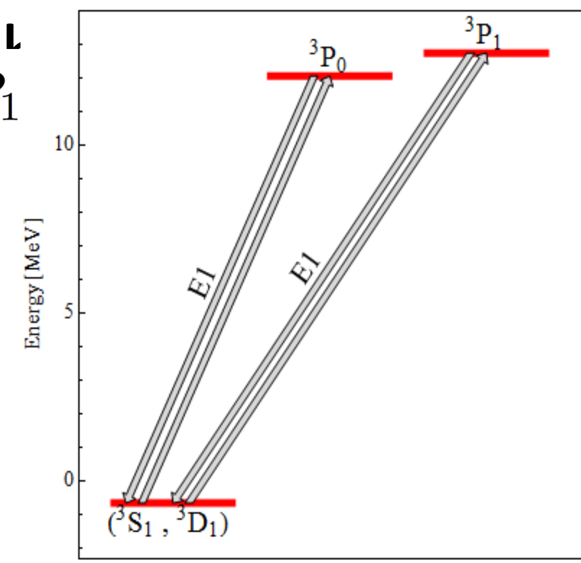
E1 Transitions

- Since $R\downarrow Z \gg x$, we consider E1 transitions onl

$$V_{int}(1\mu;\ t) = \frac{4\pi}{3} Ze^2 \sum_{\mu=-1}^{+1} \frac{Y^*_{1\mu}(\hat{R}_Z(t))}{R^2_Z(t)} r_p Y_{1\mu}(\hat{r}_p)$$

- E1 transitions shi $(^3S_1, ^3D_1)$ $^3P_0$ $^3P_1$
- Since we include

ch $^3P_0$ ↔ $(^3S_1, ^3D_1)$ ↔ $^3P_1$ attern:



- In interaction picture: $V\downarrow I(t) = exp(iH\downarrow 0\ t)\, V\downarrow int(t)\, exp(-iH\downarrow 0\ t)$

E1 Transition Matrix Elements

$$V_{I;jk}(E1;\ t) \;=\; \frac{4\pi}{3} Ze^2\ e^{i(E_j - E_k)t} \sum_{\mu} \frac{Y^*_{1\mu}(\hat{R}(t))}{|R(t)|^2} \int d\vec{r}\ \psi^*_j(\vec{r})\ \frac{r}{2}\ Y_{1\mu}(\vec{r})\psi_k(\vec{r})$$

$$|i\rangle = \begin{pmatrix} 1st;\ (^3S_1, ^3D_1);\ M_j = -1 \\ 1st;\ (^3S_1, ^3D_1);\ M_j = 0 \\ 1st;\ (^3S_1, ^3D_1);\ M_j = +1 \\ 1st;\ ^3P_0;\ M_j = 0 \\ 1st;\ ^3P_1;\ M_j = -1 \\ 1st;\ ^3P_1;\ M_j = 0 \\ 1st;\ ^3P_1;\ M_j = +1 \end{pmatrix}$$

Block 1 — E=-0.65289 MeV

Block 2 — E= 12.0733 MeV

Block 3 — E= 12.7585 MeV

$$Z = 50$$

$$b = 7.5fm$$

$$\alpha = 1/137.04$$

$jV\downarrow I\,(t=0)\,i$ =

0	0	0	-0.000738817	0.00135126	-0.00135126	0.
0	0	0	-0.000738817	0.00135126	0.	-0.00135126
0	0	0	-0.000738817	0.	0.00135126	-0.00135126
0.000738817	-0.000738817	0.000738817	0	0	0	0
0.00135126	-0.00135126	0.	0	0	0	0
0.00135126	0.	-0.00135126	0	0	0	0
0.	0.00135126	-0.00135126	0	0	0	0

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Choosing Initial State

- Initial state can be chosen according to experimental setup
- Our choice:

$$\begin{pmatrix} 1st; (^3S_1, ^3D_1); M_j = -1 \\ 1st; (^3S_1, ^3D_1); M_j = 0 \\ 1st; (^3S_1, ^3D_1); M_j = +1 \\ 1st;\ ^3P_0; M_j = 0 \\ 1st;\ ^3P_1; M_j = -1 \\ 1st;\ ^3P_1; M_j = 0 \\ 1st;\ ^3P_1; M_j = +1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

- At RIKEN, polarized deuteron beams are available

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Solve Time-dependent Schrödinger Equation

- Time-dependent Schrödinger equation in interaction picture

$$i\frac{d}{dt}|\psi(t)\rangle_I = V_I(t)|\psi(t)\rangle_I$$

- Formal solution:

$$|\psi(t)\rangle_I = T\exp\left(-i\int_{-\infty}^{\infty} V_I\right)|\psi(-\infty)\rangle_I$$

Euler vs MSD Method

Euler:

$$T \;\; exp\left[-i\frac{1}{\hbar}\int_0^t \; V_I(t) \; dt \right] \quad \xrightarrow{\sum \delta t} \quad \left[1 - \frac{i}{\hbar}V_I(t_n)\delta t\right]\left[1 - \frac{i}{\hbar}V_I(t_{n-1})\delta t\right] \;\cdots\; \left[1 - \frac{i}{\hbar}V_I(t_1)\delta t\right]$$

Multi-step differencing (MSD):

$$|\psi, t + \delta t >_I \quad \approx \quad |\psi, t - \delta t >_I - \frac{2i}{\hbar} \; V_I(t) \;\, \delta t \;\, |\psi, t >_I$$

We employ MSD2 for better numerical stability compared to Euler method, since MSD is accurate up to $(V_I \, \delta t)^2$ while Euler is up to $(V_I \, \delta t)^1$

Higher order MSDs such as MSD4 or MSD6 are available

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First-Order Perturbation Theory

First-order perturbation theory:

$$|\psi;\ t >_I = T_+\ exp\left[-i\frac{1}{\hbar}\int_0^t\ V_I(t)\ dt\right]|\psi;\ 0 >_I$$

$$\rightarrow \left[1-\frac{i}{\hbar}V_I(t_n)\delta t\right]\left[1-\frac{i}{\hbar}V_I(t_{n-1})\delta t\right]\ \cdots\ \left[1-\frac{i}{\hbar}V_I(t_1)\delta t\right]|\psi;\ 0 >_I$$

$$\rightarrow \left[1-\frac{i}{\hbar}\delta t\Big(V_I(t_n)+V_I(t_{n-1})+\cdots+V_I(t_1)\Big)\right]|\psi;\ 0 >_I$$

Parameters Used in Numerical Calculation

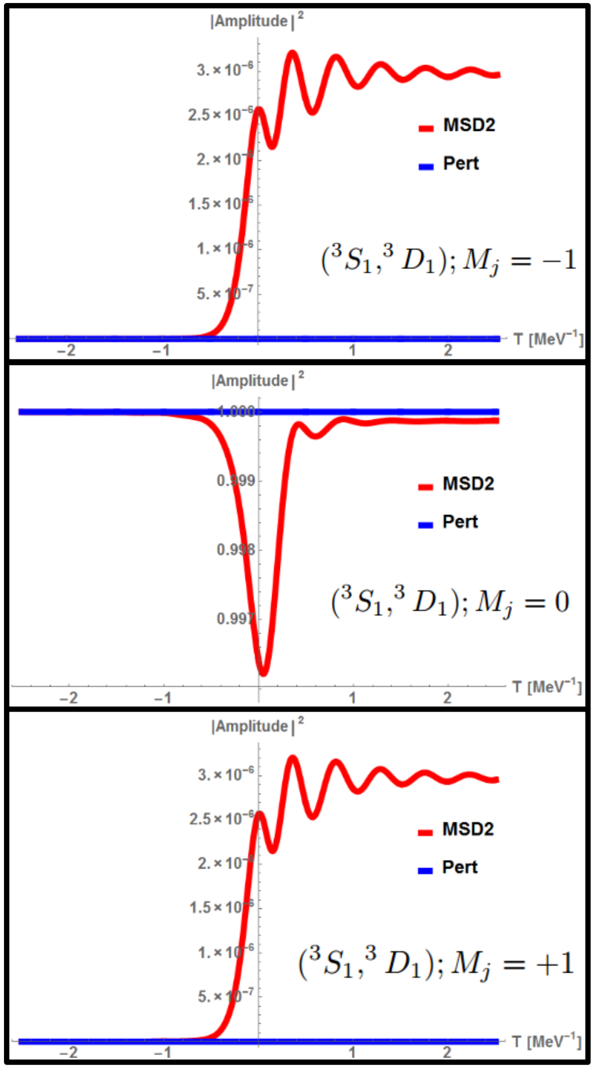
$$
\begin{aligned}
Z &= 50 \quad \text{(Sn)} \\
b &= 7.5fm \\
\alpha &= 1/137.04 \\
\delta T &= 0.001\ [MeV]^{-1}
\end{aligned}
$$

$v = 0.1c$

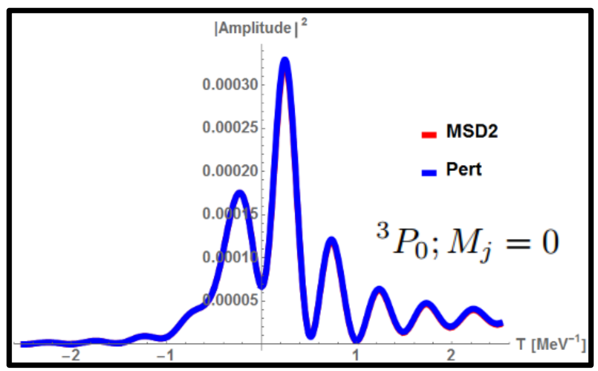
2T= 5 MeV^{-1};

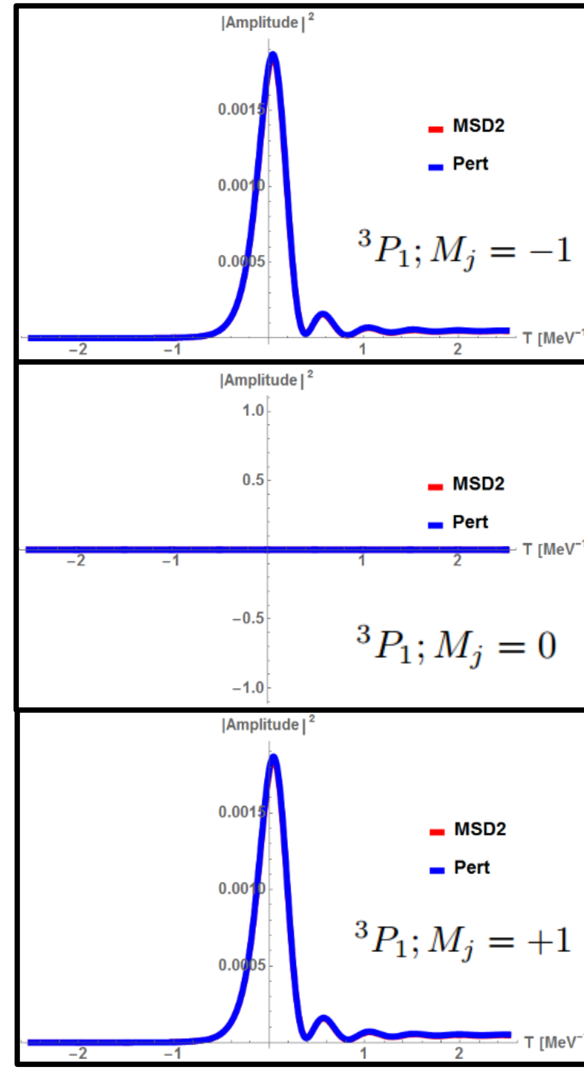
M_j = 0 Initial State;

7 states Evolution

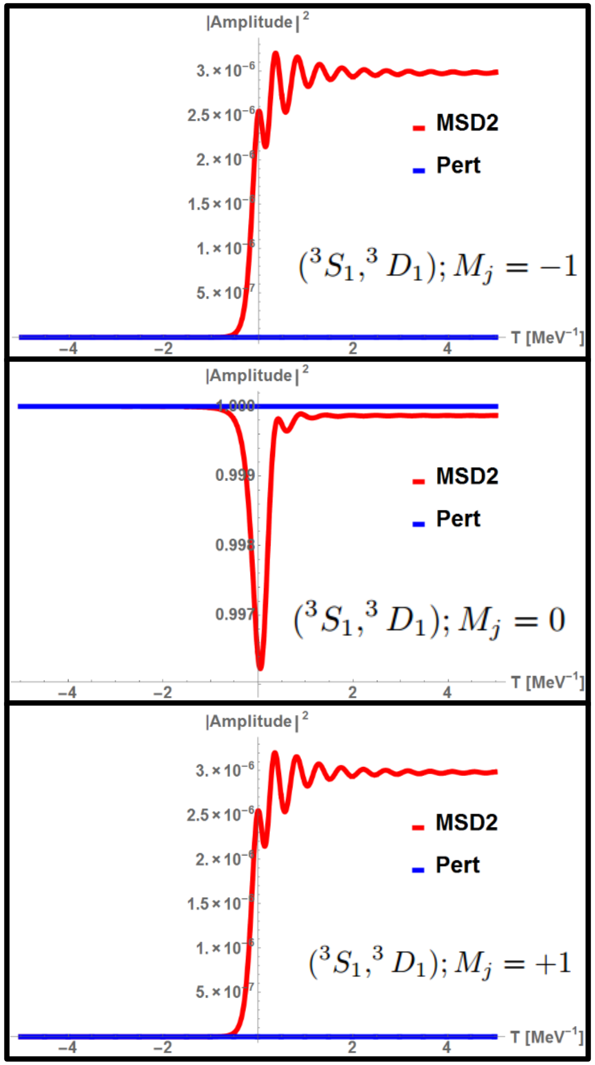


$$\begin{pmatrix} (^3S_1, ^3D_1); M_j = -1 \\ (^3S_1, ^3D_1); M_j = 0 \\ (^3S_1, ^3D_1); M_j = +1 \\ ^3P_0; M_j = 0 \\ ^3P_1; M_j = -1 \\ ^3P_1; M_j = 0 \\ ^3P_1; M_j = +1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$





- Fluctuations are signatures of quantum virtual processes
- Long-term fully quantal treatment vs. classical treatment reveals net quantum effects (work in progress)
- Feeding to states forbidden by first-order perturbation theory

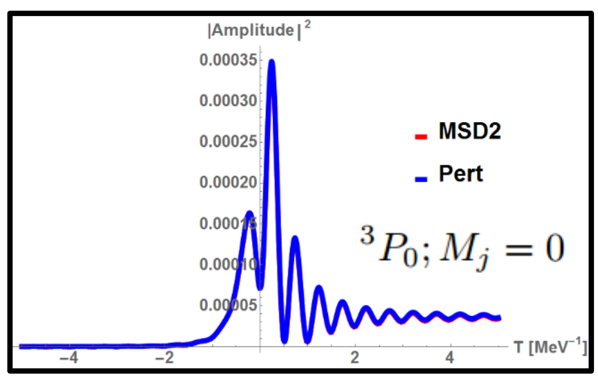


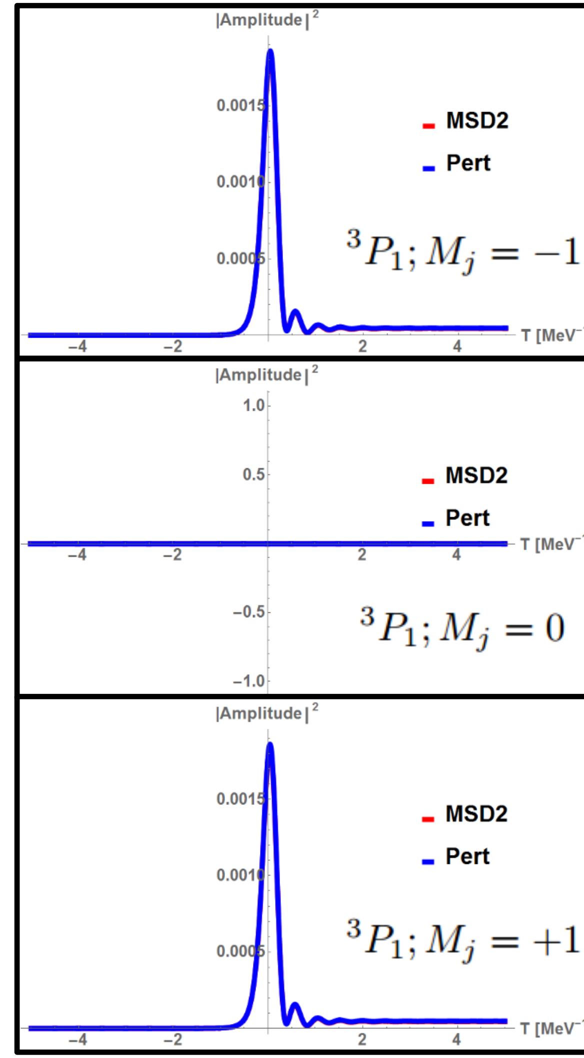
2T= 10 MeV^{-1}

M_j = 0 Initial State;

7 states evolution

$$\begin{pmatrix} (^3S_1, ^3D_1); M_j = -1 \\ (^3S_1, ^3D_1); M_j = 0 \\ (^3S_1, ^3D_1); M_j = +1 \\ ^3P_0; M_j = 0 \\ ^3P_1; M_j = -1 \\ ^3P_1; M_j = 0 \\ ^3P_1; M_j = +1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$





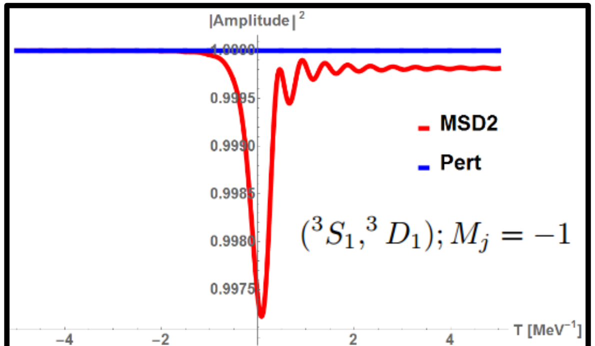
- Transition rates diminish when the heavy ion is far way

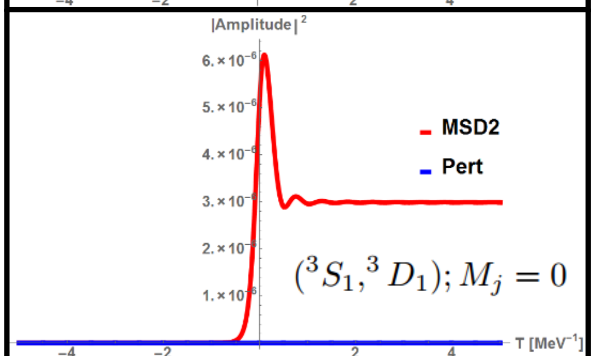
2T= 10 MeV^{-1}

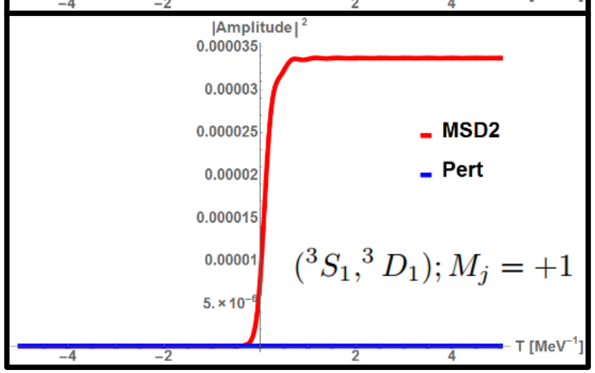
M_j = -1 Initial State;

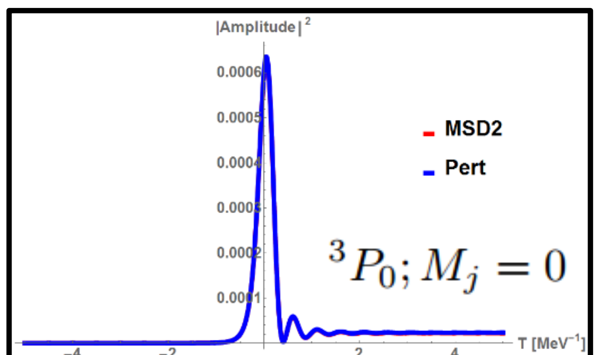
7 states evolution

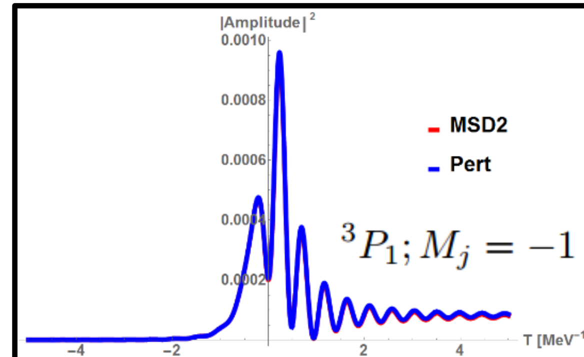
$$\begin{pmatrix} ({}^3S_1, {}^3D_1); M_j = -1 \\ ({}^3S_1, {}^3D_1); M_j = 0 \\ ({}^3S_1, {}^3D_1); M_j = +1 \\ {}^3P_0; M_j = 0 \\ {}^3P_1; M_j = -1 \\ {}^3P_1; M_j = 0 \\ {}^3P_1; M_j = +1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

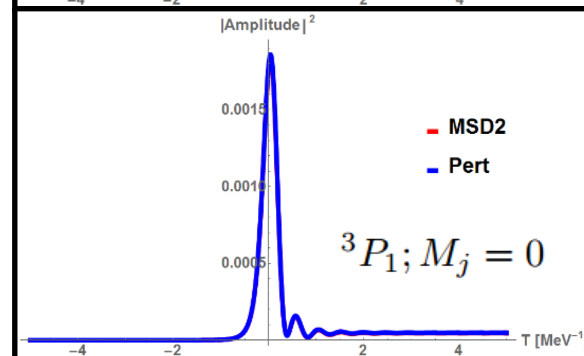


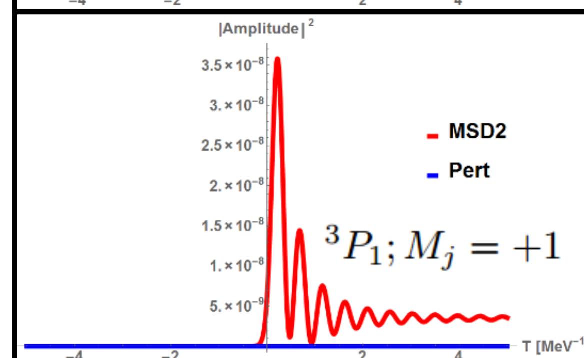


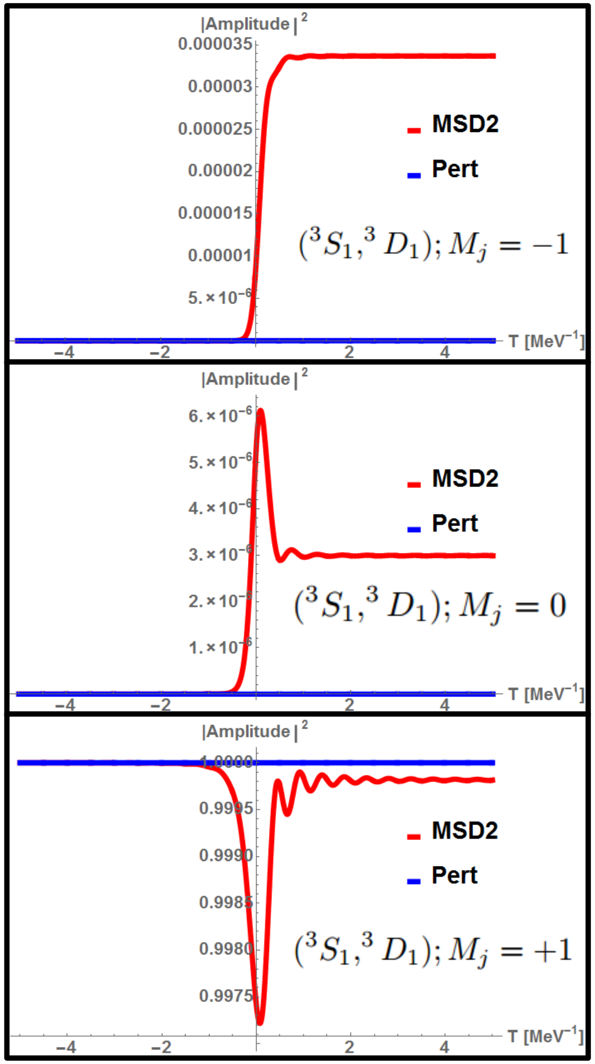










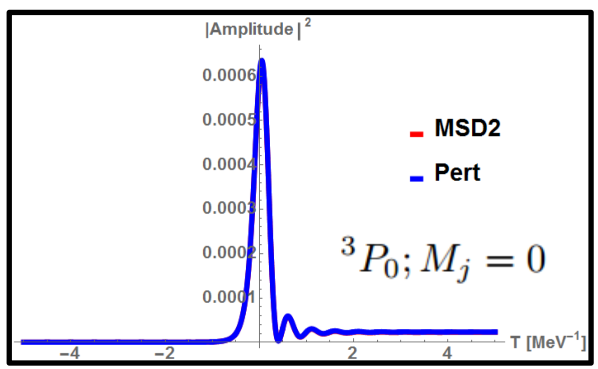


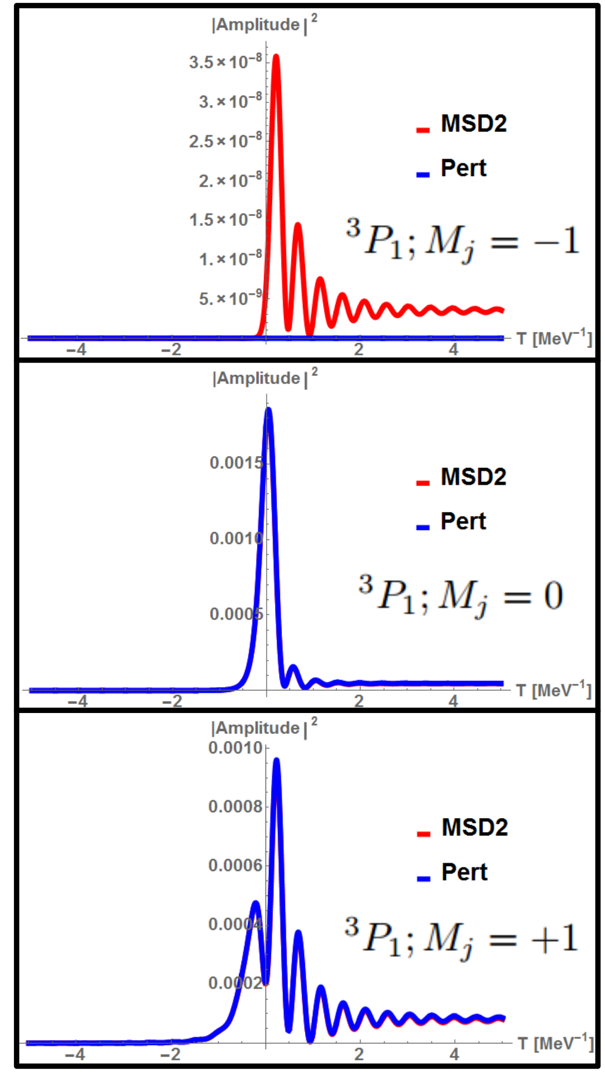
2T= 10 MeV^{-1}

M_j = +1 Initial State;

7 states Evolution

$$\begin{pmatrix} (^3S_1, ^3D_1); M_j = -1 \\ (^3S_1, ^3D_1); M_j = 0 \\ (^3S_1, ^3D_1); M_j = +1 \\ ^3P_0; M_j = 0 \\ ^3P_1; M_j = -1 \\ ^3P_1; M_j = 0 \\ ^3P_1; M_j = +1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

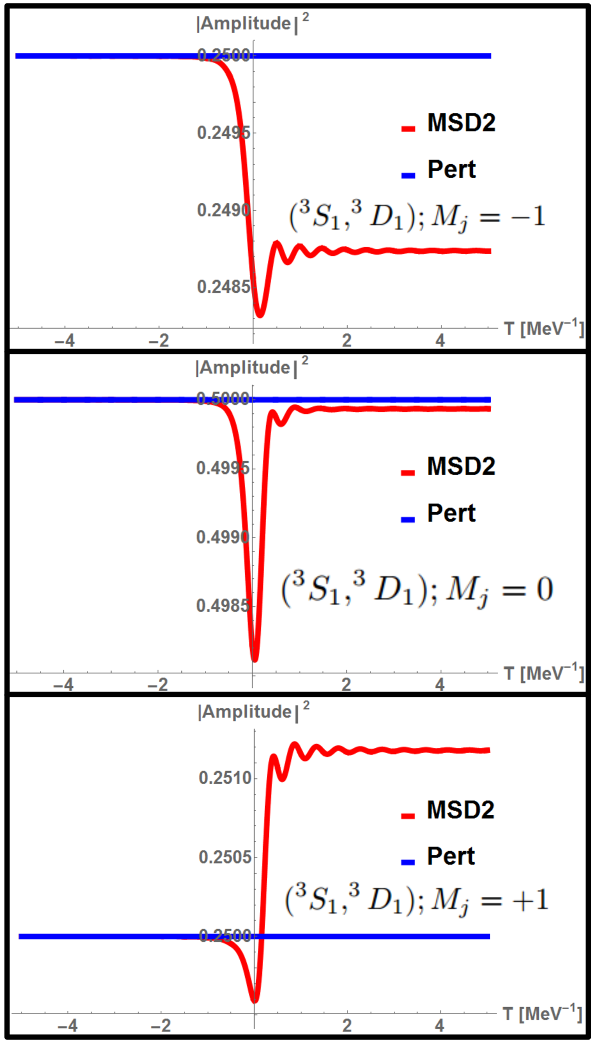




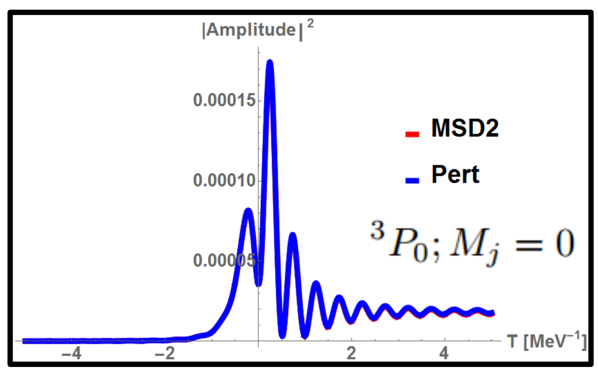
2T= 10 MeV^{-1}

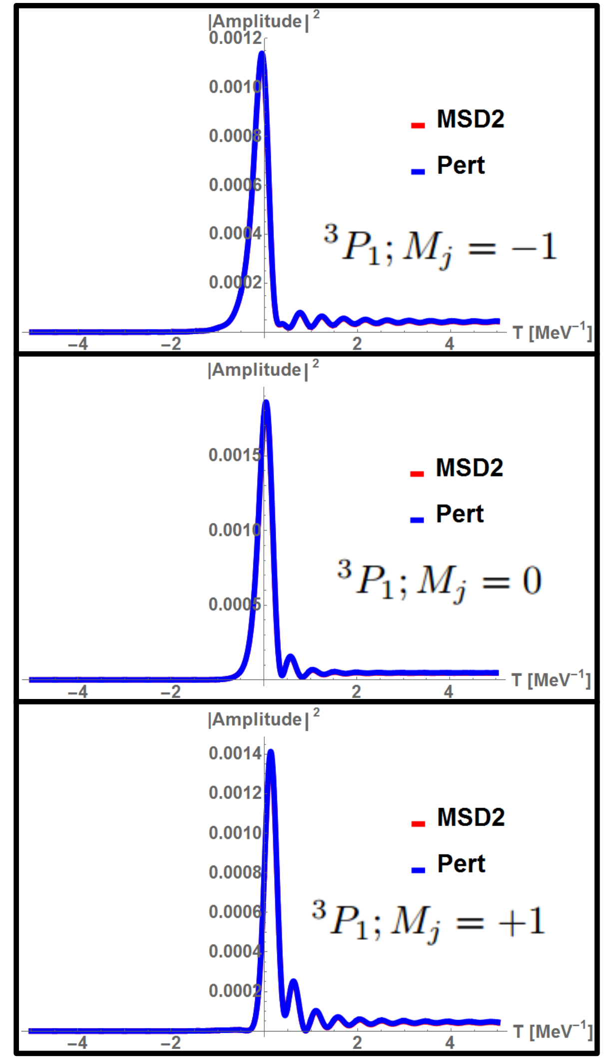
x^{+} *Polarization;*

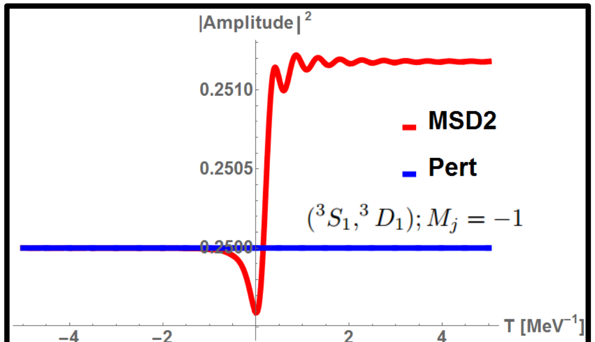
7 states Evolution

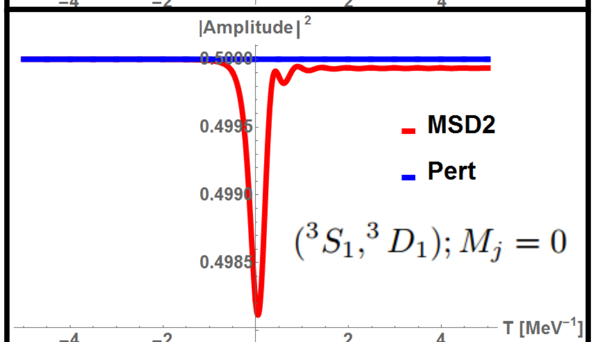


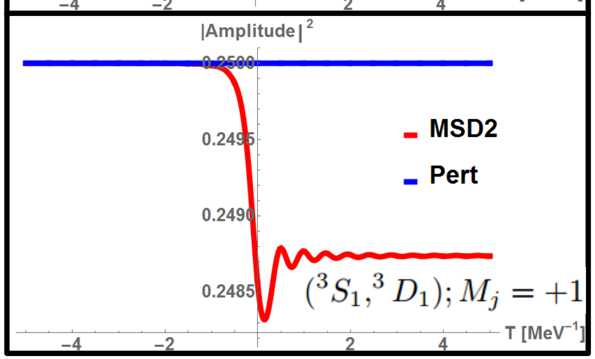
$$\begin{pmatrix} (^3S_1, ^3D_1); M_j = -1 \\ (^3S_1, ^3D_1); M_j = 0 \\ (^3S_1, ^3D_1); M_j = +1 \\ ^3P_0; M_j = 0 \\ ^3P_1; M_j = -1 \\ ^3P_1; M_j = 0 \\ ^3P_1; M_j = +1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{2}}{2} \\ \frac{1}{2} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$









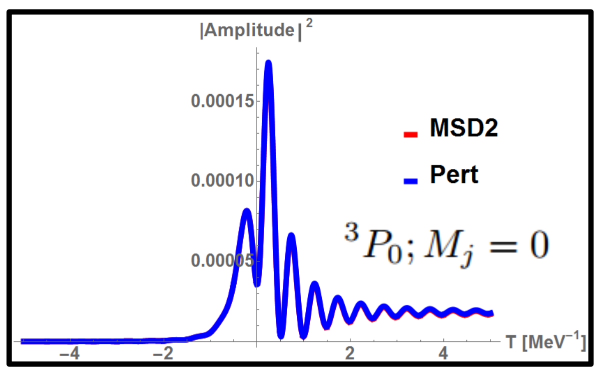


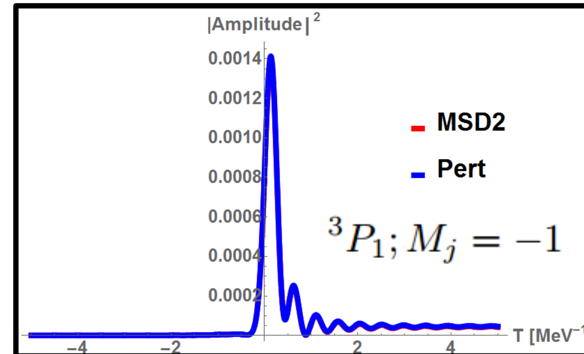
2T= 10 MeV⁻¹

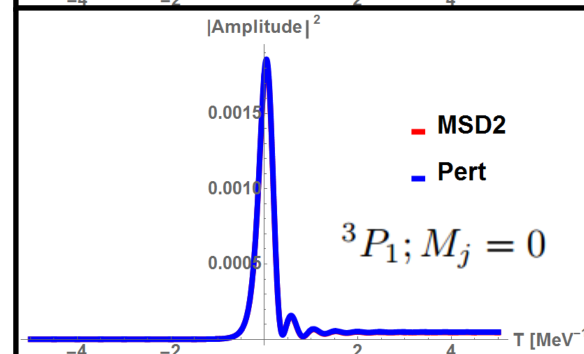
x *⁻⁻ Polarization;*

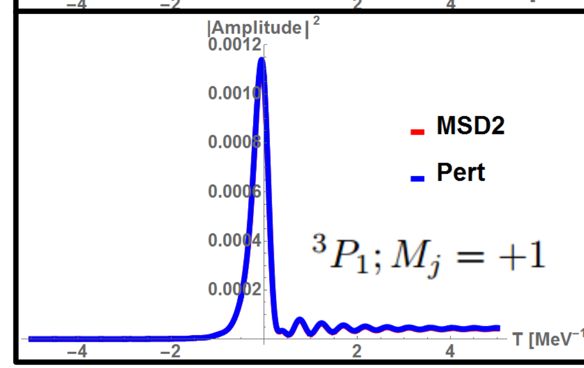
7 states Evolution

$$\begin{pmatrix} (^3S_1, ^3D_1); M_j = -1 \\ (^3S_1, ^3D_1); M_j = 0 \\ (^3S_1, ^3D_1); M_j = +1 \\ ^3P_0; M_j = 0 \\ ^3P_1; M_j = -1 \\ ^3P_1; M_j = 0 \\ ^3P_1; M_j = +1 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ -\frac{\sqrt{2}}{2} \\ \frac{1}{2} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

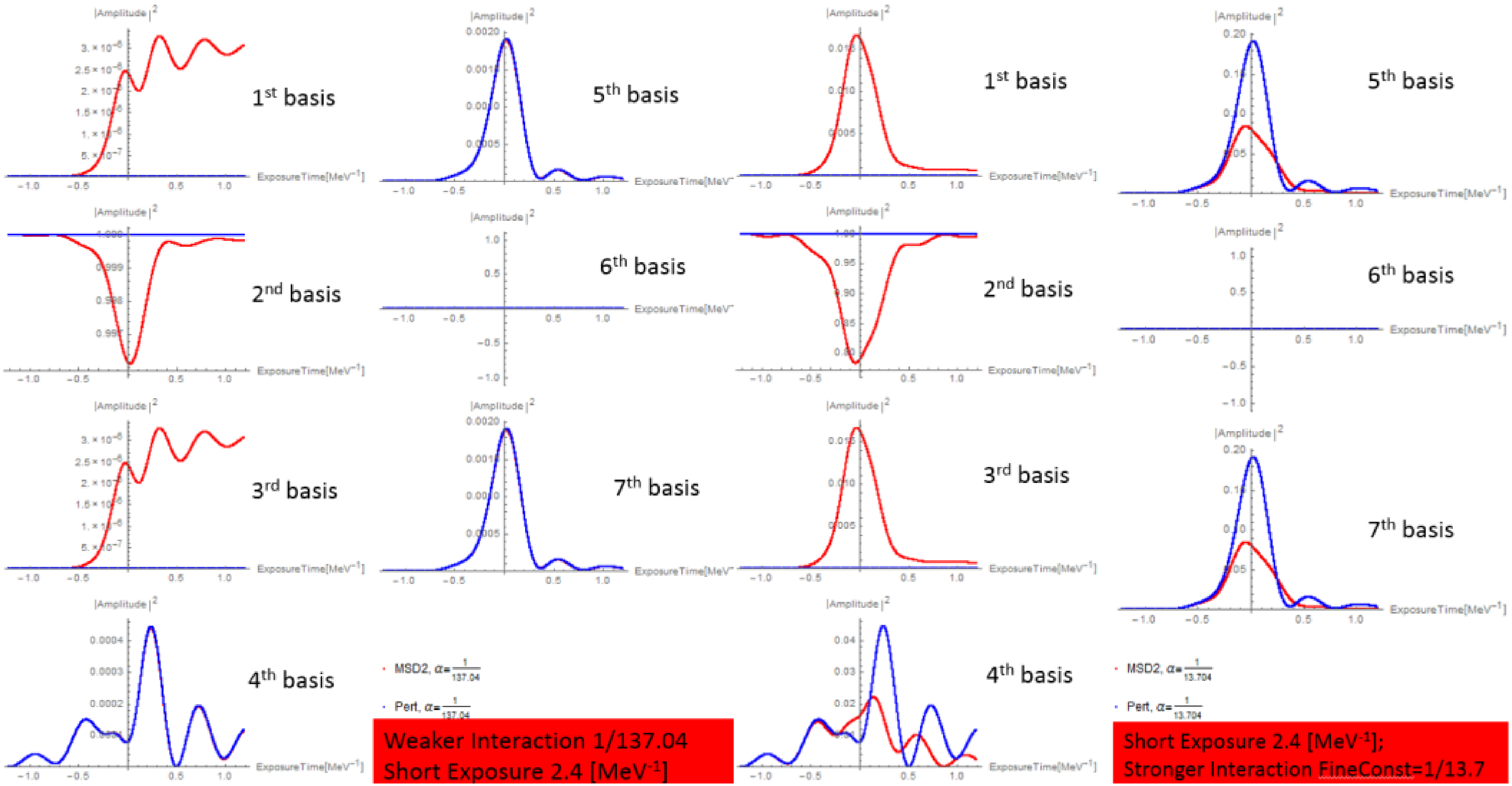






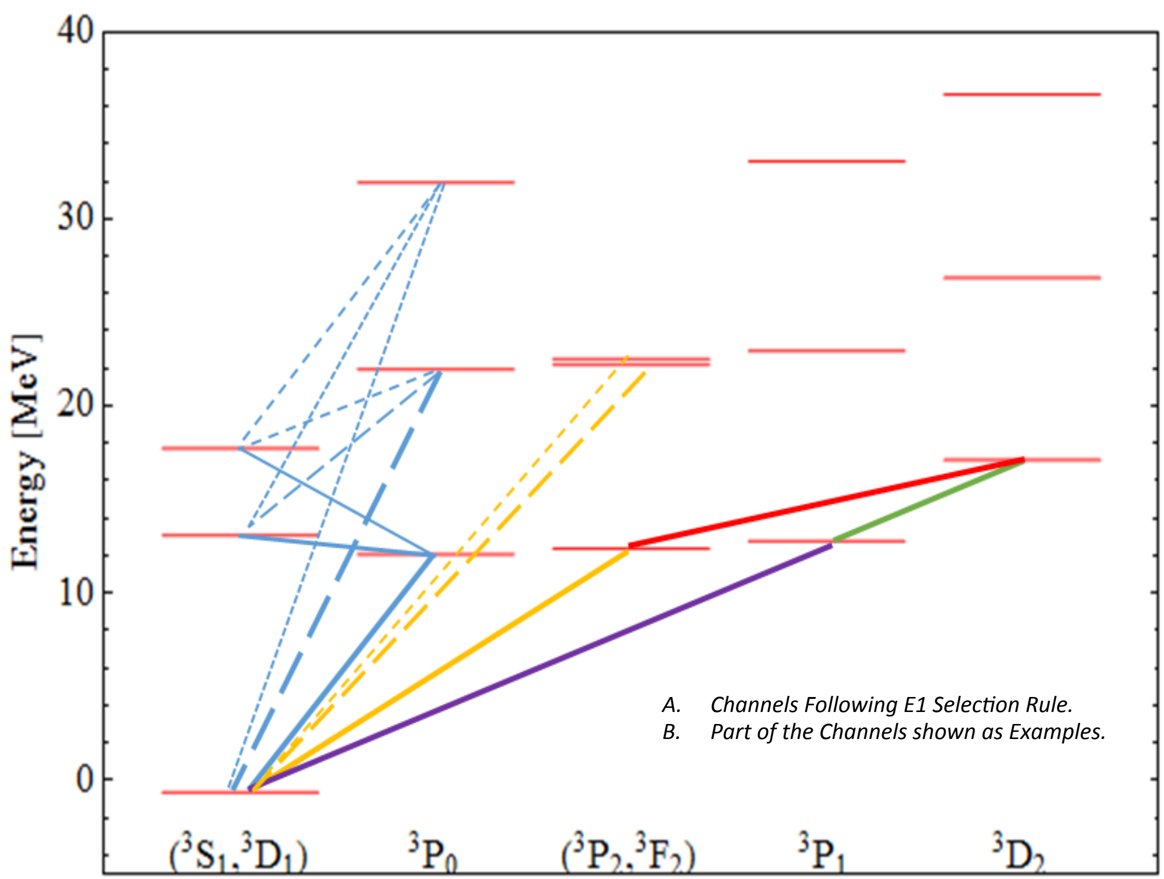


Dependence on Coupling Constant



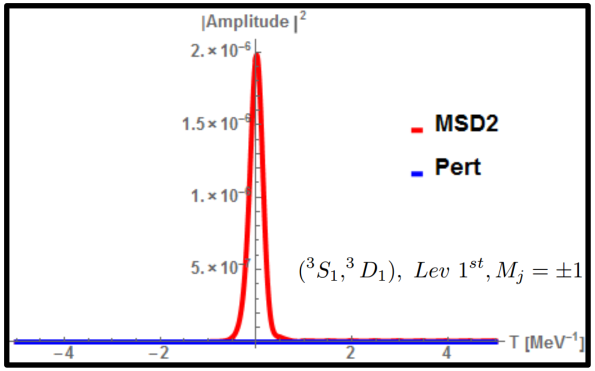
As Z increases, transitions deviate from first-order perturbation theory

51 Level-System Evolution (In the 5MeV HO trap)



Initial State Preparation

$$\begin{pmatrix} 1st;\,(^3S_1,^3D_1);\,M_j=-1 \\ 1st;\,(^3S_1,^3D_1);\,M_j=0 \\ 1st;\,(^3S_1,^3D_1);\,M_j=+1 \\ 2nd;\,(^3S_1,^3D_1);\,M_j=-1 \\ \vdots \\ 3rd;\,(^3S_1,^3D_1);\,M_j=+1 \\ 1st;\ ^3P_0;\,M_j=0 \\ \vdots \\ 3rd;\ ^3P_0;\,M_j=0 \\ 1st;\ ^3P_1;\,M_j=-1 \\ \vdots \\ 3rd;\ ^3P_1;\,M_j=+1 \\ 1st;\ (^3P_2,^3F_2);\,M_j=-2 \\ \vdots \\ 3rd;\ (^3P_2,^3F_2);\,M_j=+2 \\ 1st;\ 3D_2;\,M_j=-2 \\ \vdots \\ 3rd;\ 3D_2;\,M_j=+2 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ \vdots \\ \\ \\ \vdots \\ \\ \\ \vdots \\ \\ \\ \vdots \\ \\ \\ \vdots \\ \\ \vdots \end{pmatrix}$$



Tracking the SAME 7 states

1. $(^3S_1, ^3D_1)$ channel
2. $^3P_0$ channel
3. $^3P_1$ channel

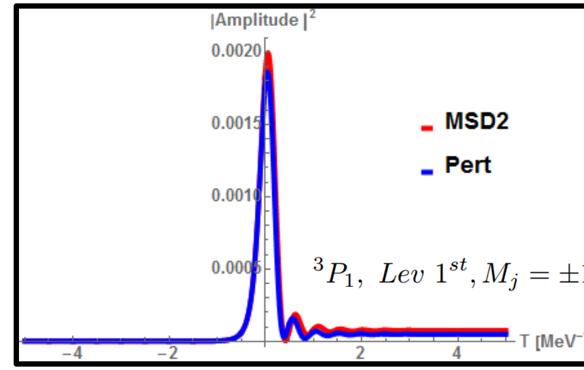
$$Z = 50$$

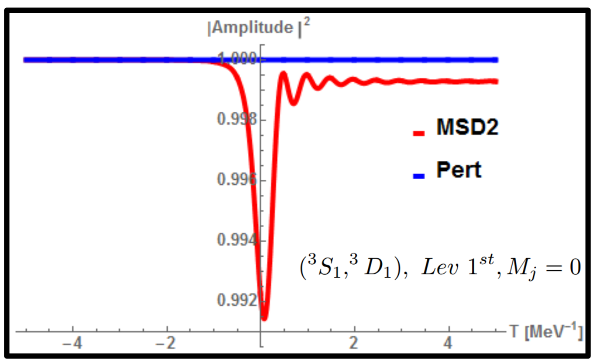
$$b = 7.5 fm$$

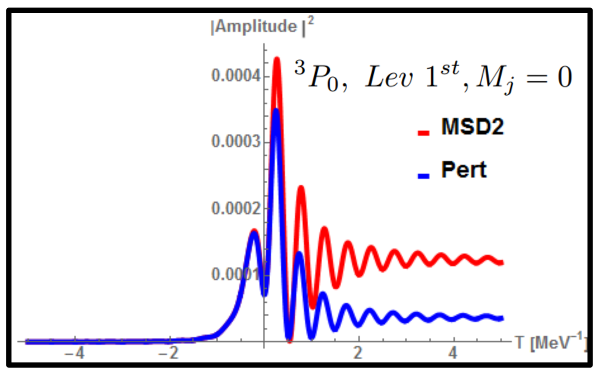
$$\alpha = 1/137.04$$

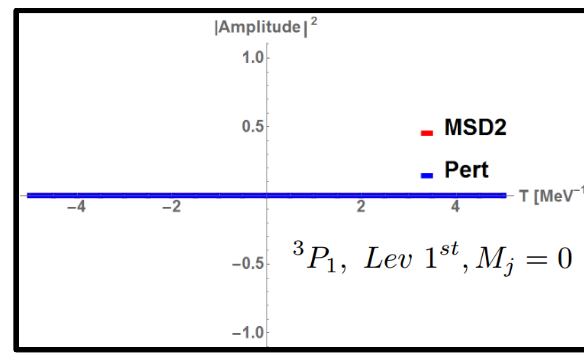
$$\delta T = 0.001\ [MeV]^{-1}$$

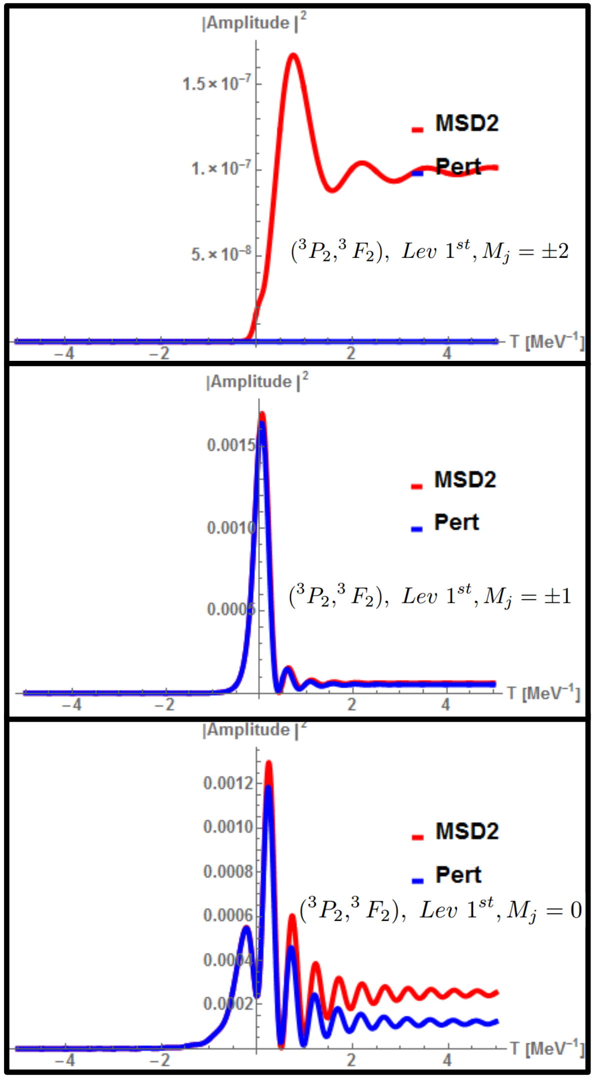
$$v = 0.1c$$









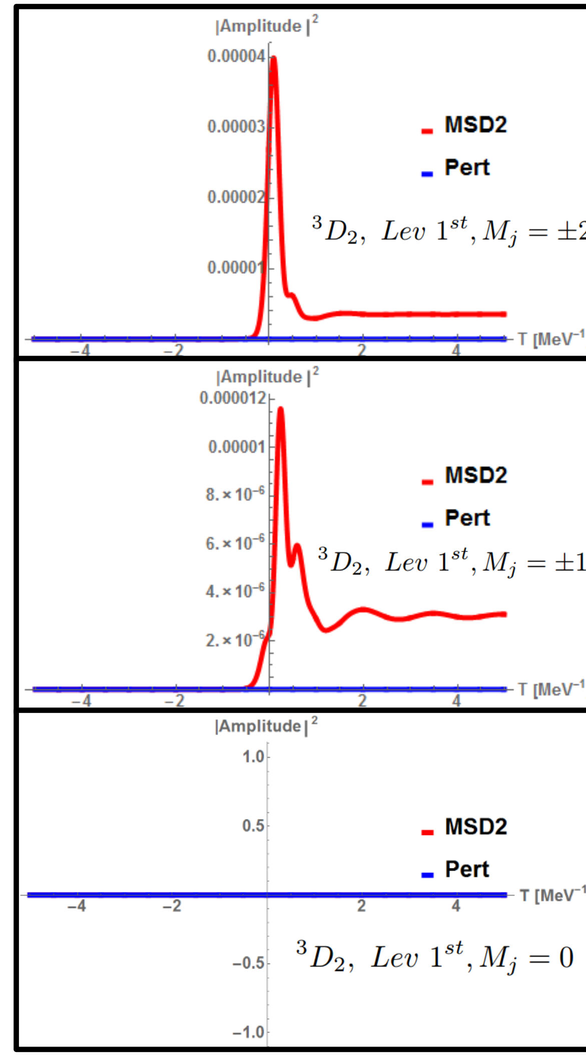


51 states evolution;

Evolution of lowest levels in Extra

1. $(^3P_2, ^3F_2)$ channel,
2. $^3D_2$ channel

Comment:
As the expansion of level system, deviation between MSD2 scheme and perturbation manifest.



Conclusion

- Time-dependent Basis Function (tBF) is motivated by progresses both in experimental nuclear physics and in supercomputing techniques
- tBF is an nonperturbative *ab initio* method for time-dependent problems
- tBF is particularly suitable for strong-field problems
- tBF operates on the level of amplitude
- tBF will hopefully provide further insights into fundamental questions in a more detailed and more differential manner

Outlook

- Observables: phase-space distributions, differential cross sections
- Perform calculation in larger basis space and study convergence with respect to states in continuum
- Compare with classical treatments
- Study the effects of E0, M1, E2, M2… transitions
- Study the sensitivity with respect to different NN interactions, such as Daejeon16 [A. M. Shirokov et al, PLB 761, 81 (2016)]
- Include strong force in the background field
- More realistic center-of-mass motion
  - Trajectory from QMD
  - Direct computation of cm motion (in future)

Thank you!

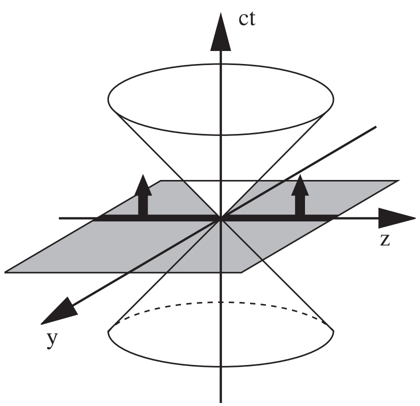
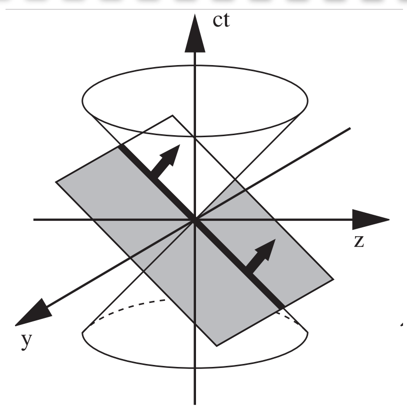
Ab initio Shell Model vs BLFQ

- Hamiltonian formalism
- Low-energy Nuclear Physics
- Quantum mechanics
- Nucleon degrees of freedom
- Nonrelativistic system
- Particle number is conserved
- Renormalization is tractable
- Galilean boost invariant
- Effective Hamiltonian: complicated

- Hamiltonian formalism
- Hadron Physics
- Quantum field theory
- Parton degrees of freedom
- Relativistic system
- Particle number is not conserved: multi-Fock sectors
- Renormalization is difficult: divergences beset
- Lorentz boost invariant
- Gauge theory: gauge symmetry
- Fundamental Hamiltonian

Light-front vs Equal-time Quantization

[Dirac 1949]

equal-time dynamics	VS	light-front dynamics
$t \equiv x^0$		$t \equiv x^+ = x^0 + x^3$
		
$i\frac{\partial}{\partial t}\lvert\varphi(t)\rangle = H\lvert\varphi(t)\rangle$		$i\frac{\partial}{\partial x^+}\lvert\varphi(x^+)\rangle = \frac{1}{2}P^-\lvert\varphi(x^+)\rangle$
$H = P^0$		$P^- = P^0 - P^3$
Kinematic Generators: P, J		Kinematic Generators: $P^\perp, P^+, E^\perp, K^3, J^3$

Common Variables in Light-front Dynamics

- Light-front time $x^+ = x^0 + x^3$
- Light-front Hamiltonian $P^- = P^0 - P^3$
- Longitudinal coordinate $x^- = x^0 - x^3$
- Longitudinal momentum $P^+ = P^0 + P^3$
- Transverse coordinate $x^\perp = x^{1,2}$
- Transverse momentum $P^\perp = P^{1,2}$
- Equal-time dispersion relation $P^0 = \sqrt{m^2 + \vec{P}^2}$
- Light-front dispersion relation $P^- = \dfrac{m^2 + P_\perp^2}{P^+}$

Basis Light-front Quantization

- Solve quantum field theory through

  eigenvalue problem of light-front Hamiltonian

$$P^{-}|\beta\rangle = P_{\beta}^{-}|\beta\rangle$$

  - P^{-} : light-front Hamiltonian
  - $|\beta\rangle$: light-front amplitude for mass eigenstates
  - P_{β}^{-} : eigenvalue (light-front energy) for eigenstate $|\beta\rangle$

- Evaluate observables for eigenstate $|\beta\rangle$

$$O \equiv \langle\beta|\hat{O}|\beta\rangle$$

Example: Obtain LF QED Hamiltonian

- QED Lagrangian $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\Psi}(i\gamma^\mu D_\mu - m_e)\Psi$
- Derived Light-front Hamiltonian

$$\left(A^+ = 0\right)$$

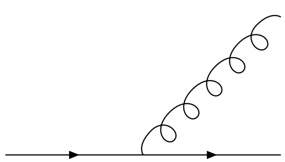
$$P^- = \int \mathrm{d}^2x^\perp \mathrm{d}x^- \; F^{\mu +}\partial_+ A_\mu + i\bar{\Psi}\gamma^+\partial_+\Psi - \mathcal{L}$$

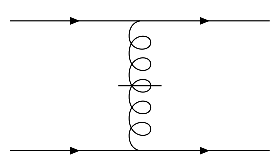
$$= \int \mathrm{d}^2x^\perp \mathrm{d}x^- \; \frac{1}{2}\bar{\Psi}\gamma^+\frac{m_e^2 + (i\partial^\perp)^2}{i\partial^+}\Psi + \frac{1}{2}A^j(i\partial^\perp)^2A^j$$

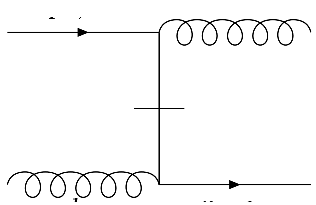
kinetic energy terms

$$+ ej^\mu A_\mu + \frac{e^2}{2}j^+\frac{1}{(i\partial^+)^2}j^+ + \frac{e^2}{2}\bar{\Psi}\gamma^\mu A_\mu\frac{\gamma^+}{i\partial^+}\gamma^\nu A_\nu\Psi$$

vertex interaction | instantaneous photon interaction | instantaneous fermion interaction

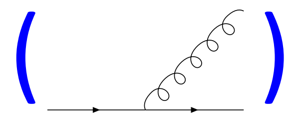
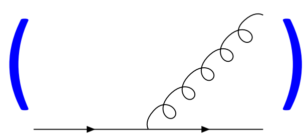






QED Hamiltonian in BLFQ Basis

- QED LF-Hamiltonian in a small basis: $|e\rangle + |e\gamma\rangle$, N_{max}=2, K=1.5

$\langle \alpha' \vert P^-_{QED} \vert \alpha \rangle$ (MeV)	$\vert e\rangle$	$\vert e\gamma\rangle$
$\vert e\rangle$	0.3482 (kinetic energy)	-0.0119 
$\vert e\gamma\rangle$	-0.0119 	0.9139 (kinetic energy)

- Eigenstates: $|e\rangle_{phys} = 0.9998|e\rangle + 0.0210|e\gamma\rangle$

$$|e\gamma\rangle_{scat} = -0.0210|e\rangle + 0.9998|e\gamma\rangle$$