

Energy Dependent Separable Optical Potentials for (d, p) Reactions

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Abstract

An important ingredient for applications of nuclear physics to, e. g., astrophysics or nuclear energy are the cross sections for reactions of neutrons with rare isotopes. Since direct measurements are often not possible, indirect methods like (d, p) reactions must be used instead. Those (d, p) reactions may be viewed as effective three-body reactions and described with Faddeev techniques. An additional challenge posed by (d, p) reactions involving heavier nuclei is the treatment of the Coulomb force. To avoid numerical complications in dealing with the screening of the Coulomb force, recently a new approach using the Coulomb distorted basis in momentum space was suggested. In order to implement this suggestion separable representations of neutron- and proton-nucleus optical potentials, which are not only complex but also energy dependent, need to be introduced. Including excitations of the nucleus in the calculation requires a multichannel optical potential, and thus separable representations thereof.

Keywords: *Energy dependent separable representation of optical potentials, multi-channel optical potentials, nonlocal optical potentials, (d, p) reactions*

1 Introduction

Nuclear reactions are an important probe to learn about the structure of unstable nuclei. Due to the short lifetimes involved, direct measurements are usually not possible. Therefore indirect measurements using (d, p) reactions have been proposed (see, e. g., Refs. [1–3]). Deuteron induced reactions are particularly attractive from an experimental perspective since deuterated targets are readily available. From a theoretical perspective they are equally attractive because the scattering problem can be reduced to an effective three-body problem [4]. Traditionally deuteron-induced single-neutron transfer (d, p) reactions have been used to study the shell structure in stable nuclei, nowadays experimental techniques are available to apply the same approaches to exotic beams (see, e. g., Ref. [5]). Deuteron induced (d, p) or (d, n) reactions in inverse kinematics are also useful to extract neutron or proton capture

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rates on unstable nuclei of astrophysical relevance. Given the many ongoing experimental programs worldwide using these reactions, a reliable reaction theory for (d, p) reactions is critical.

One of the most challenging aspects of solving the three-body problem for nuclear reactions is the repulsive Coulomb interaction. While for very light nuclei, exact calculations of (d, p) reactions based on momentum-space Faddeev equations in the Alt–Grassberger–Sandhas (AGS) [6] formulation can be carried out [7] by using a screening and renormalization procedure [8,9], this technique leads to increasing technical difficulties when moving to computing (d, p) reactions with heavier nuclei [10]. Therefore, a new formulation of the Faddeev–AGS equations which does not rely on the screening procedure, was presented in Ref. [11]. Here the Faddeev–AGS equations are cast in the momentum-space Coulomb-distorted partial-wave representation instead of the plane-wave basis. Thus all operators, specifically the interactions in the two-body subsystems, must be evaluated in the Coulomb basis, which is a nontrivial task (performed recently for the neutron-nucleus interaction [12]). The formulation of Ref. [11] requires the interactions in the subsystems to be of separable form.

Separable representations of the forces between constituents forming the subsystems in the Faddeev approach have a long tradition, specifically when considering the nucleon-nucleon (NN) interaction (see, e. g., Ref. [13–15]) or meson-nucleon interactions [16,17]. Here the underlying potentials are Hermitian, and a scheme for deriving separable representations suggested by Ernst–Shakin–Thaler [18] (EST) is well suited, specifically when working in momentum space. It has a nice property that the on-shell and half-off-shell transition matrix elements of the separable representation are exact at predetermined energies, the so-called EST support points. However, when dealing with neutron-nucleus (nA) or proton-nucleus (pA) phenomenological optical potentials, which are in general complex to account for absorptive channels that are not explicitly treated, as well as energy-dependent, extensions of the EST scheme have to be made.

2 Separable representation of single channel energy dependent optical potentials

The pioneering work by Ernst, Shakin and Thaler [18] constructed separable representations of Hermitian potentials. To apply this formalism to optical potentials, it needs to be extended to handle complex potentials [19]. We briefly recall the most important features, namely that a separable representation for a complex, energy-independent potential U_l in a fixed partial wave of orbital angular momentum l is given by [19]

$$u_l = \sum_{ij} U_l |\psi_{l,i}^+\rangle \lambda_{ij}^{(l)} \langle \psi_{l,j}^- | U_l, \quad (1)$$

where $|\psi_{l,i}^+\rangle$ is a solution of the Hamiltonian $H = H_0 + U_l$ with outgoing boundary conditions at energy E_i , and $|\psi_{l,i}^-\rangle$ is a solution of the Hamiltonian $H = H_0 + U_l^*$ with incoming boundary conditions. The energies E_i are referred to as EST support points. The free Hamiltonian H_0 has eigenstates $|k_i\rangle$ with $k_i^2 = 2\mu E_i$, μ being the reduced mass of the neutron-nucleus system. The EST scheme constrains the matrix

$\lambda_{ij}^{(l)}$ with the conditions

$$\begin{aligned}\delta_{kj} &= \sum_i \langle \psi_{l,k}^- | U_l | \psi_{l,i}^+ \rangle \lambda_{ij}^{(l)}, \\ \delta_{ik} &= \sum_j \lambda_{ij}^{(l)} \langle \psi_{l,j}^- | U_l | \psi_{l,k}^+ \rangle,\end{aligned}\tag{2}$$

where the subscript $i = 1, \dots, N$ and N indicates the rank of the separable potential. Those two constraints of Eq. (2) on $\lambda_{ij}^{(l)}$ are an essential feature of the EST scheme and ensure that at the EST support points E_i both, the original U and the separable potential u , have identical wave functions or half-shell t -matrices. The corresponding separable t -matrix takes the form

$$t_l(E) = \sum_{ij} U_l | \psi_{l,i}^+ \rangle \tau_{ij}^{(l)}(E) \langle \psi_{l,j}^- | U_l,\tag{3}$$

with

$$\left(\tau_{ij}^{(l)}(E) \right)^{-1} = \langle \psi_{l,i}^- | U_l - U_l g_0(E) U_l | \psi_{l,j}^+ \rangle.\tag{4}$$

Here $g_0(E) = (E - H_0 + i\varepsilon)^{-1}$ is the free propagator. The form factors are given as half-shell t -matrices

$$T_l(E_i) | k_i \rangle \equiv U_l | \psi_{l,i}^+ \rangle,\tag{5}$$

and are obtained through solving a momentum space Lippmann–Schwinger (LS) equation. However, when applying the same formulation to an energy-dependent complex potential $U(E)$, one obtains

$$u_l = \sum_{ij} U_l(E_i) | \psi_{l,i}^+ \rangle \lambda_{ij}^{(l)} \langle \psi_{l,j}^- | U_l(E_j),\tag{6}$$

with the constraints

$$\begin{aligned}\delta_{kj} &= \sum_i \langle \psi_{l,k}^- | U_l(E_i) | \psi_{l,i}^+ \rangle \lambda_{ij}^{(l)}, \\ \delta_{ik} &= \sum_j \lambda_{ij}^{(l)} \langle \psi_{l,j}^- | U_l(E_j) | \psi_{l,k}^+ \rangle.\end{aligned}\tag{7}$$

Omitting the partial wave index l , the two constraints on λ can be written in matrix form as

$$\mathcal{U}^t \lambda = \mathbf{1} = \lambda \mathcal{U},\tag{8}$$

with

$$\mathcal{U}_{ij} = \langle \psi_i^- | U(E_i) | \psi_j^+ \rangle.\tag{9}$$

For a separable potential of rank $N > 1$ it is obvious that the matrix \mathcal{U}_{ij} is not symmetric in the indices i and j . This leads to an asymmetric matrix λ and thus a t -matrix which violates reciprocity. Therefore, a different approach must be taken in order to construct separable representations for energy-dependent potentials. Here we note that although the potential u contains some of the energy dependence of $U(E)$ through the form factors calculated at the different fixed energy support points E_i , it has no explicit energy dependence. Thus, this separable construction needs to be considered as energy-independent EST representation.

A separable expansion for energy-dependent Hermitian potentials was suggested by Pearce [20]. It is straightforward to apply this suggestion to complex potentials by using the insights previously gained in Ref. [19]. In analogy, we define the EST separable representation for complex, energy-dependent potentials (eEST) by allowing an explicit energy dependence of the coupling matrix elements λ_{ij} .

$$u(E) = \sum_{ij} U(E_i) |\psi_i^+\rangle \lambda_{ij}(E) \langle \psi_j^- | U(E_j), \quad (10)$$

where the partial wave index l has been omitted for simplicity. In order to obtain a constraint on the matrix $\lambda(E)$, we require that the matrix elements of the potential $U(E)$ and its separable form $u(E)$ between the states $|\psi_i^+\rangle$ be the same at all energies E . This condition ensures that the potentials $U(E)$ and $u(E)$ lead to identical wave functions at the EST support points, just like in the energy-independent EST scheme.

The constraints on $\lambda_{ij}(E)$ become

$$\langle \psi_m^- | U(E) | \psi_n^+ \rangle = \langle \psi_m^- | u(E) | \psi_n^+ \rangle = \sum_i \langle \psi_m^- | U(E_i) | \psi_i^+ \rangle \lambda_{ij}(E) \langle \psi_j^- | U(E_j) | \psi_n^+ \rangle. \quad (11)$$

The corresponding separable t -matrix then takes the form

$$t(E) = \sum_{ij} U(E_i) |\psi_i^+\rangle \tau_{ij}(E) \langle \psi_j^- | U(E_j). \quad (12)$$

Substituting Eqs. (10)–(12) into the LS equation leads to constraint for the matrix $\tau(E)$ such that

$$R(E) \cdot \tau(E) \equiv \mathcal{M}(E), \quad (13)$$

where

$$R_{ij}(E) = \langle \psi_i^- | U(E_i) | \psi_j^+ \rangle - \sum_n \mathcal{M}_{in}(E) \langle \psi_n^- | U(E_n) g_0(E) U(E_j) | \psi_j^+ \rangle, \quad (14)$$

with

$$\mathcal{M}_{in}(E) \equiv [\mathcal{U}^e(E) \cdot \mathcal{U}^{-1}]_{in}. \quad (15)$$

The matrix elements of \mathcal{U} are defined in Eq. (9), and

$$\mathcal{U}_{ij}^e(E) \equiv \langle \psi_i^- | U(E) | \psi_j^+ \rangle. \quad (16)$$

For energy-independent potentials, $\mathcal{U}^e(E)$ becomes \mathcal{U} and the matrix $\mathcal{M}(E)$ is the unit matrix. The matrix element $\mathcal{U}_{ij}^e(E)$ is explicitly given as

$$\begin{aligned} \mathcal{U}_{ij}^e(E) &= U(k_i, k_j, E) + \int_0^\infty dp p^2 T(p, k_i; E_i) g_0(E_i, p) U(p, k_j, E) \\ &\quad + \int_0^\infty dp p^2 U(k_i, p, E) g_0(E_j, p) T(p, k_j; E_j) \\ &\quad + \int_0^\infty dp p^2 \int_0^\infty dp' p'^2 T(p, k_i; E_i) g_0(E_i, p) U(p, p', E) g_0(E_j, p') T(p', k_j; E_j), \quad (17) \end{aligned}$$

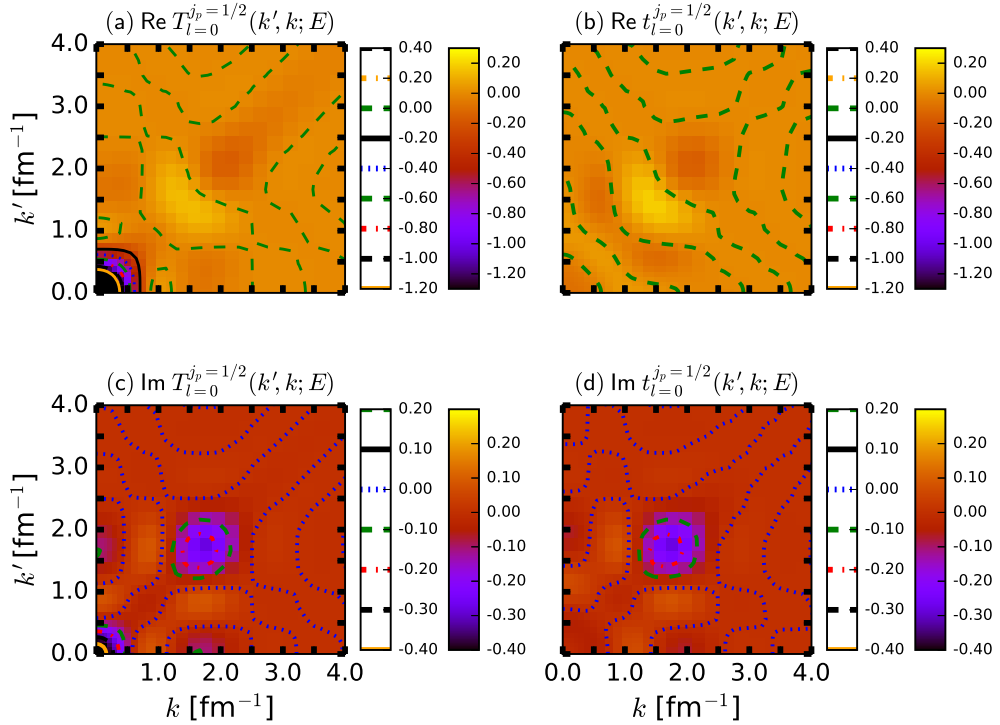


Figure 1: s -wave off-shell t -matrix for the $n + {}^{48}\text{Ca}$ system calculated from the CH89 optical potential [23] as a function of the off-shell momenta k and k' at 20 MeV incident neutron laboratory kinetic energy. (a) and (c) — real and imaginary t -matrix elements corresponding to the CH89 global optical potential; (b) and (d) — real and imaginary parts of the eEST separable representation of the off-shell t -matrix. The on-shell momentum is $k = 0.978 \text{ fm}^{-1}$.

where $g_0(E, p) = [E - p^2/2\mu + i\varepsilon]^{-1}$. For the evaluation of $U_{ij}^e(E)$ for all energies E within the relevant energy regime, the form factors $T(p', k_j; E_j)$ are needed at the specified EST support points and the matrix elements of the potential $U(p', p, E)$ at all energies. The explicit derivation of the above expressions is given in Refs. [21, 22] together with suggestions to simplify the calculation of $U(p', p, E)$. The difference between the energy-dependent separable representation and its energy-independent counterpart is illustrated in Fig. 1 for the s -wave off-shell t -matrix for the $n + {}^{48}\text{Ca}$ system.

To apply the formulation to proton-nucleus scattering, one first realizes that the proton-nucleus potential consists of the point Coulomb force V^c together with a short-ranged nuclear as well as a short-ranged Coulomb interaction representing the charge distribution of the nucleus which we refer to as $U^s(E)$. While the point Coulomb potential has a simple analytical form, an optical potential is employed to model the short-range nuclear potential. The extension of the energy-independent EST separable representation to proton-nucleus optical potentials was carried out in Ref. [24]. In that work it was shown that the form factors of the separable representation are solutions of the LS equation in the Coulomb basis, and that they are obtained using

methods introduced in Refs. [25, 26]. It was also demonstrated that the extension of the energy-independent EST separable representation scheme to proton-nucleus scattering involves two steps. First, the nuclear wave functions $|\psi_{l,i}^{(+)}\rangle$ are replaced by Coulomb-distorted nuclear wavefunctions $|\psi_{l,i}^{sc(+)}\rangle$. Second, the free resolvent $g_0(E)$ is replaced by the Coulomb Green's function, $g_c(E) = (E - H_0 - V^c + i\varepsilon)^{-1}$, and third, the energy-dependent scheme must be generalized.

Upon suppressing the index l we obtain a constraint similar to Eq. (13),

$$R^c(E) \cdot \tau^c(E) = \mathcal{M}_{ij}^c(E), \quad (18)$$

with the matrix elements of $R^c(E)$ satisfying

$$R_{ij}^c(E) = \langle \psi_i^{sc(-)} | U^s(E_i) | \psi_j^{sc(+)} \rangle - \sum_i \mathcal{M}_{in}^c(E) \langle \psi_n^{sc(-)} | U^s(E_n) g_c(E) U^s(E_j) | \psi_j^{sc(+)} \rangle. \quad (19)$$

The matrix $\mathcal{M}^c(E)$ is the Coulomb distorted counterpart of $\mathcal{M}(E)$ of Eq. (15), and is defined as

$$\mathcal{M}_{in}^c(E) = [\mathcal{U}^{e,sc}(E) \cdot (\mathcal{U}^{sc})^{-1}]_{in}, \quad (20)$$

with

$$\begin{aligned} \mathcal{U}_{ij}^{sc} &\equiv \langle \psi_i^{sc(-)} | U^s(E_i) | \psi_j^{sc(+)} \rangle, \\ \mathcal{U}_{ij}^{e,sc}(E) &\equiv \langle \psi_{k_i}^{sc(-)} | U^s(E) | \psi_{k_j}^{sc(+)} \rangle. \end{aligned} \quad (21)$$

If the potential is energy-independent the matrix $\mathcal{M}^c(E)$ becomes a unit matrix just like $\mathcal{M}(E)$. Further details for the explicit evaluation are given in Refs. [21, 22].

In order to illustrate the quality of the separable representation of energy-dependent optical potentials for neutron as well as proton elastic scattering, the differential cross sections for proton scattering off ^{48}Ca at laboratory kinetic energy 38 MeV and ^{208}Pb at 45 MeV are shown in Fig. 2 and compared to the equivalent coordinate space calculations. We observe that the separable representation provides an excellent description on both cases. The power of a separable representation based on the EST scheme lies in the choice of the basis, namely here the half-shell t -matrices calculated at specific energies. This basis contains a lot of information about the system considered, and thus only a small number of basis states, represented by the rank of the separable potential, are needed to have this excellent representation.

3 Coordinate space separable representation of single channel optical potentials

A formal scheme for deriving separable representations for Hermitian potentials was given by Ernst, Shakin, and Thaler in Ref. [18], and an application of the scheme to a two-body coordinate space potential representing an s-wave bound and scattering state in can be found in Ref. [28]. The authors chose to carry out their construction of the separable representation in the coordinate space, which makes the procedure more cumbersome compared to the momentum space construction we employ, leading

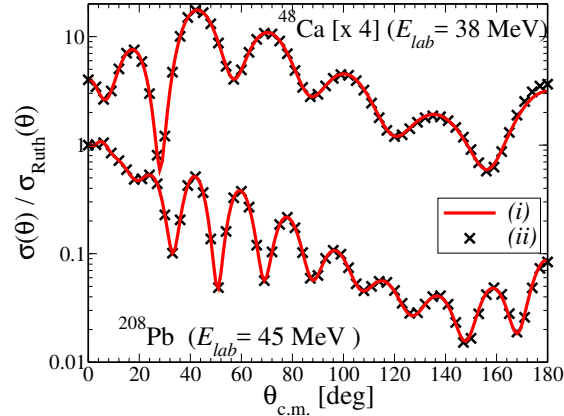


Figure 2: Unpolarized differential cross section for elastic scattering of protons by ^{48}Ca (upper) and ^{208}Pb (lower) as a function of the c. m. angle. For ^{48}Ca the cross section is calculated at a laboratory kinetic energy of 38 MeV and is scaled by a factor of 4. The calculation for ^{208}Pb is carried out at $E_{lab} = 45$ MeV. Solid curves (i) — the cross section calculated in momentum space based on the rank-5 separable representation of the phenomenological optical potential CH89 [23], crosses (ii) — the corresponding coordinate space calculations [27].

to a momentum space separable representation of either the transition matrix or the potential.

Since coordinate space techniques have long tradition in nuclear physics, it can be useful to consider an EST based separable representation of potentials in coordinate space. Separable potentials are inherently nonlocal. Using the EST formulation leads to a well defined behavior of this non-locality. However, instead of implementing the EST construction in the coordinate space, one can carry out the entire scheme in the momentum space and then Fourier transform the momentum space result to the coordinate space. This is quite simple since it involves only a one-dimensional Fourier transform of the form factors.

To illustrate the coordinate space realization of the EST separable representation, we show in Fig. 3 the form factors $h_{l,i}$ as a function of the momentum p for the $n + ^{48}\text{Ca}$ system in panel (a) together with their Fourier transformed counterparts in the coordinate space in panel (b). The index i refers to the EST support points used. The form factors are well behaved functions in the momentum space as well as in the coordinate space. In Fig. 4 the s -wave form factors for the $n + ^{208}\text{Pb}$ system are shown, and we note that for the heavier nucleus ^{208}Pb they extend to larger values of r as should be expected considering the larger size of the heavier nucleus.

The separable representation of the coordinate space potential in a given partial wave is obtained by summing over the rank of the potential according to Eq. (1). The resulting nonlocal separable coordinate space representation of the CH89 optical potential is shown in Fig. 5 for the $n + ^{48}\text{Ca}$ system for the $s_{1/2}$ and $p_{3/2}$ channels. The non-locality is symmetric in r and r' as required by reciprocity and its extension in r and r' is given by the fall-off behavior of the form factors. It also shows a more intricate behavior than the often employed Perey–Buck Gaussian-type [29] non-

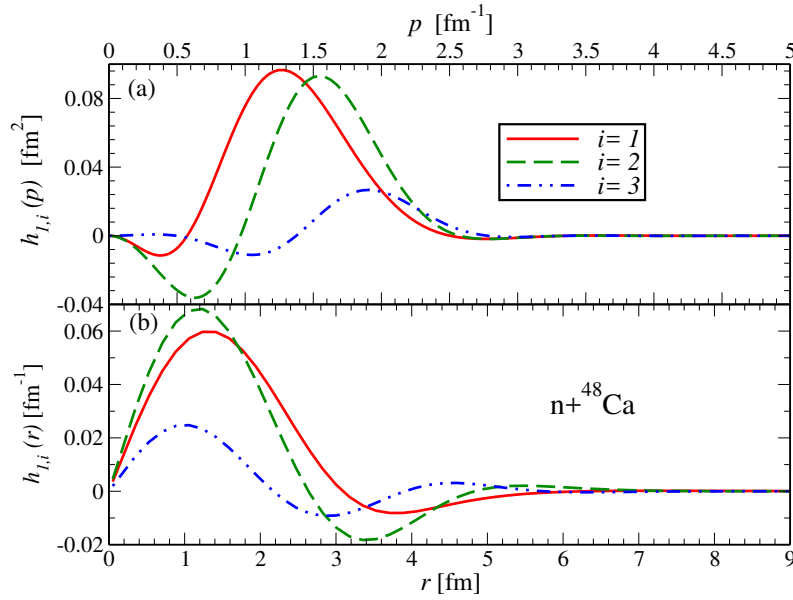


Figure 3: $p_{3/2}$ form factors $h_{1,i}$ for the $n+^{48}\text{Ca}$ system obtained from the CH89 optical potential [23]. Panel (a) illustrates the form factors as a function of the momentum p while panel (b) depicts its Fourier transform as a function of the position coordinate r . The indices $i = 1, 2$, and 3 correspond to the support points 5, 21, and 47 MeV.

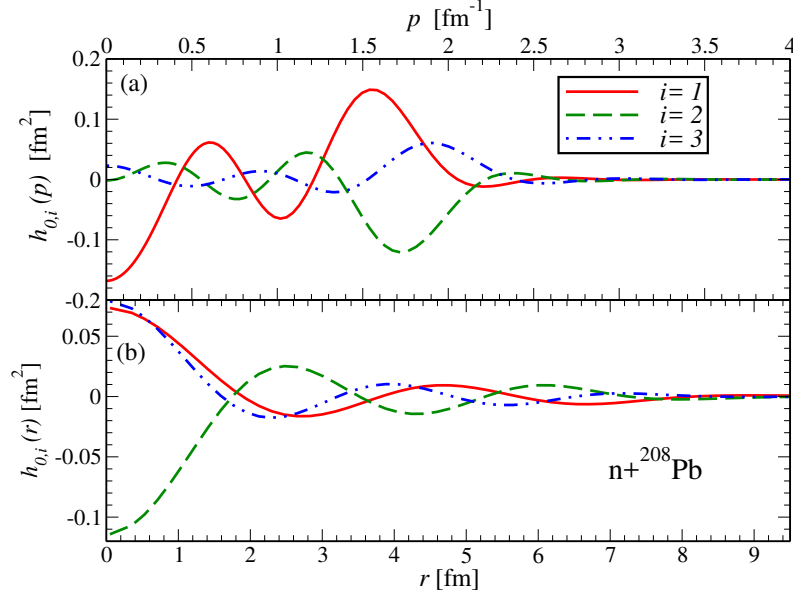


Figure 4: s -wave form factors $h_{0,i}$ for the $n+^{208}\text{Pb}$ system obtained from the CH89 optical potential [23]. See Fig. 3 for details.

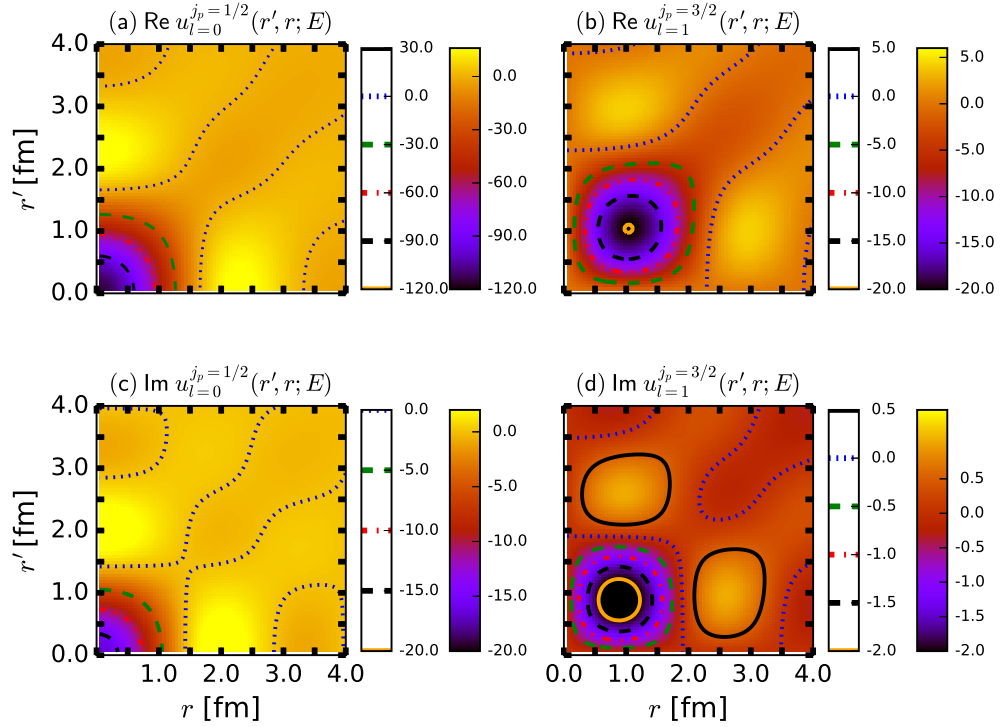


Figure 5: Off-shell potential elements $u_l^{j_p}(r', r, E)$ of the separable representation of the CH89 optical potential [23] for the $n + {}^{48}\text{Ca}$ system as a function of the coordinates r and r' at $E = 20$ MeV incident neutron laboratory kinetic energy. (a) and (c) — real and imaginary potential matrix elements for the $s_{1/2}$ partial wave; (b) and (d) — real and imaginary parts of the $p_{3/2}$ separable potential.

locality construct. Employing the nonlocal separable representation in solving the integro-differential Schrödinger equation [30] reveals that the resulting coordinate space wave function exactly agree with the wavefunctions obtained from solving the Schrödinger equation with the local CH89 optical potential [31].

4 Separable representation of multi-channel energy dependent optical potentials

To generalize the energy-dependent EST (eEST) scheme to multichannel potentials, we proceed analogously to Ref. [32] and replace the single-channel scattering wavefunctions with their multichannel counterparts, leading to a multichannel separable potential

$$u(E) = \sum_{\rho\sigma} \sum_{ij} \left(\sum_{\gamma JM} U(E_i) |\gamma JM \Psi_{\gamma\rho,i}^{J(+)}\rangle \right) \lambda_{ij}^{\rho\sigma}(E) \left(\sum_{\gamma JM} \langle \Psi_{\gamma\sigma,j}^{J(-)} | \gamma JM | U(E_j) \right). \quad (22)$$

The indices i and j stand for the EST support points. Using the definition of the multichannel half-shell t -matrix [33],

$$T(E_i)|\rho JM k_i^\rho\rangle = \sum_{\gamma} U(E_i)|\gamma JM \Psi_{\gamma\rho}^{J(+)}\rangle, \quad (23)$$

Eq. (22) can be recasted as

$$u(E) = \sum_{JM} \sum_{J'M'} \sum_{\rho\sigma} \sum_{ij} T(E_i)|\rho JM k_i^\rho\rangle \lambda_{ij}^{\rho\sigma}(E) \langle k_j^\sigma \sigma J' M' | T(E_j). \quad (24)$$

To determine the constraint on $u(E)$, we first generalize the matrices $\mathcal{U}^e(E)$ and \mathcal{U} to multichannel potentials. This is accomplished by replacing the single-channel scattering states by the multichannel scattering states so that

$$\mathcal{U}_{mn}^{e,\alpha\beta}(E) \equiv \sum_{\gamma\nu} \langle \Psi_{\gamma\alpha,m}^{J(-)} \gamma JM | U(E) | \nu JM \Psi_{\nu\beta,n}^{J(+)} \rangle = \sum_{\gamma\nu} \langle \Psi_{\gamma\alpha,m}^{J(-)} | U_{\gamma\nu}^J(E) | \Psi_{\nu\beta,n}^{J(+)} \rangle, \quad (25)$$

and

$$\mathcal{U}_{mn}^{\alpha\beta} \equiv \mathcal{U}_{mn}^{e,\alpha\beta}(E_m) = \sum_{\gamma\nu} \langle \Psi_{\gamma\alpha,m}^{J(-)} | U_{\gamma\nu}^J(E_m) | \Psi_{\nu\beta,n}^{J(+)} \rangle. \quad (26)$$

The J dependence of matrix elements $\mathcal{U}_{mn}^{e,\alpha\beta}(E)$ and $\mathcal{U}_{mn}^{\alpha\beta}$ is omitted for simplicity. On one hand, Eq. (26) shows that the matrix \mathcal{U} depends only on the support energies E_m and E_n . On the other hand, we see from Eq. (25) that $\mathcal{U}^e(E)$ depends on the projectile energy E as well as the support energies. The constraint on the separable potential is obtained by substituting the multichannel matrices \mathcal{U}^e and \mathcal{U} into Eq. (11) leading to

$$\mathcal{U}_{mn}^{e,\alpha\beta}(E) = \sum_{\rho\sigma} \sum_{ij} (\mathcal{U}^t)_{mi}^{\alpha\rho} \lambda_{ij}^{\rho\sigma}(E) \mathcal{U}_{jn}^{\sigma\beta} = [\mathcal{U}^t \cdot \lambda(E) \cdot \mathcal{U}]_{mn}^{\alpha\beta}. \quad (27)$$

To evaluate the separable multichannel t -matrix, we insert Eqs. (24)–(27) into a multichannel LS equation and obtain

$$\begin{aligned} t(E) &= \sum_{\rho\sigma} \sum_{ij} \left(\sum_{\gamma JM} U(E_i) |\gamma JM \Psi_{\gamma\rho,i}^{J(+)}\rangle \right) \tau_{ij}^{\rho\sigma}(E) \left(\sum_{\gamma JM} \langle \Psi_{\gamma\sigma,j}^{J(-)} \gamma JM | U(E_j) \right) \\ &= \sum_{JM} \sum_{J'M'} \sum_{\rho\sigma} \sum_{ij} T(E_i) |\rho JM k_i^\rho\rangle \tau_{ij}^{\rho\sigma}(E) \langle k_j^\sigma \sigma J' M' | T(E_j). \end{aligned} \quad (28)$$

The coupling matrix elements $\tau_{ij}^{\rho\sigma}(E)$ fit the equation

$$R(E) \cdot \tau(E) = \mathcal{M}(E), \quad (29)$$

where

$$\begin{aligned} R_{ij}^{\rho\sigma}(E) &= \langle k_i^\rho | T_{\rho\sigma}^J(E_i) + \sum_{\beta} T_{\rho\beta}^J(E_i) G_{\beta}(E_j) T_{\beta\sigma}^J(E_j) | k_j^\sigma \rangle \\ &\quad - \sum_{\beta\beta'} \sum_n \mathcal{M}_{in}^{\rho\beta} \langle k_n^\beta | T_{\beta\beta'}^J(E_n) G_{\beta'}(E) T_{\beta'\sigma}^J(E_j) | k_j^\sigma \rangle, \end{aligned} \quad (30)$$

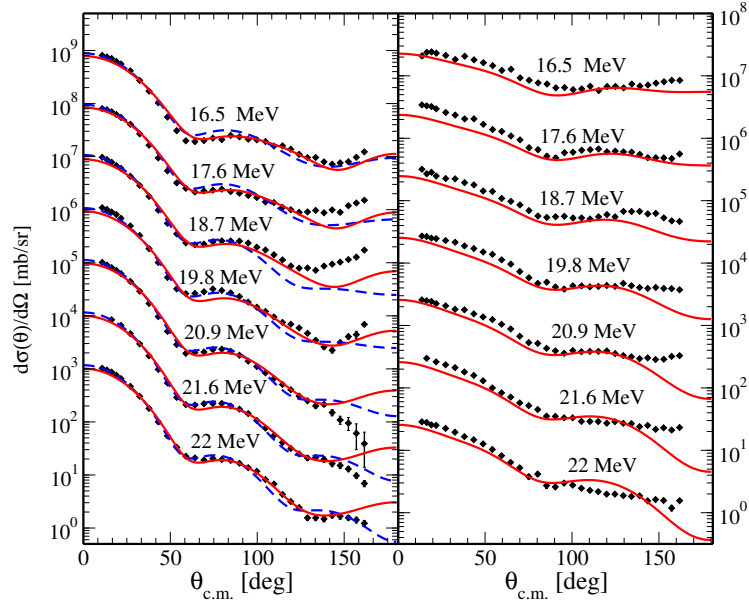


Figure 6: Differential cross sections for scattering in the $n + {}^{12}\text{C}$ system computed at various incident neutron energies with the eEST separable representation of the Olsson *et al.* DOMP [34] (solid curves). Left (right) panel shows the results for the elastic scattering (inelastic scattering to the 2^+ state of ${}^{12}\text{C}$). Dashed curves indicate the cross sections computed with the spherical OMP of Olsson *et al.* [34]. Filled diamonds are the data taken from Ref. [34]. The cross sections are scaled up by the multiples of 10, e. g., the results at 21.6 MeV are multiplied by 10, those at 20.9 MeV are multiplied by 100, etc.

and

$$\mathcal{M}_{ij}^{\rho\sigma}(E) = [\mathcal{U}^e(E) \cdot \mathcal{U}^{-1}]_{ij}^{\rho\sigma}. \quad (31)$$

The expression for the matrix $R_{ij}^{\rho\sigma}(E)$ is analogous to the one for the single-channel case except for the extra channel indices.

To illustrate the implementation of the multichannel eEST separable representation scheme, we consider the scattering of neutrons by the ${}^{12}\text{C}$ nucleus. The ${}^{12}\text{C}$ nucleus possesses selected excited states, with the first and second levels having $I^\pi = 2^+$ and $I^\pi = 4^+$ and located at 4.43 and 14.08 MeV above the 0^+ ground state. The collective rotational model [35] is assumed to describe the coupling between the ground state and these excited states. We consider here the elastic scattering and the inelastic scattering to the 2^+ rotational state. To test the multichannel eEST separable representation, we use the deformed optical potential model (DOMP) derived by Olsson *et al.* [34] and fitted to elastic and inelastic scattering data at laboratory kinetic energies between 16 and 22 MeV. In Fig. 6, the differential cross sections for the elastic and inelastic scattering in the $n + {}^{12}\text{C}$ system are shown at various incident neutron energies. The left hand panel shows the differential cross section of the elastic scattering, and the right hand panel presents the differential cross section of the inelastic scattering to the 2^+ state of ${}^{12}\text{C}$. The support points are at $E_{lab} = 6$ and 40 MeV. The

separable representation describes both differential cross sections very well. In addition, it is in good agreement with the coupled-channel calculations shown in Fig. 1 of Ref. [34]. The dashed curves indicate the cross sections computed with the spherical OMP of Olsson *et al.* [34].

5 Summary and outlook

In a series of steps we developed the input that will serve as a basis for Faddeev-AGS three-body calculations of (d,p) reactions which will not rely on the screening of the Coulomb force. To achieve this, Ref. [11] formulated the Faddeev-AGS equations in the Coulomb basis using separable interactions in the two-body subsystems. We developed separable representations of phenomenological optical potentials of the Woods-Saxon type for neutrons and protons. First we concentrated on the neutron-nucleus optical potentials and generalized the EST scheme [18] so that it can be applied to complex and energy-dependent optical potentials [19,21]. In order to consider the proton-nucleus optical potentials, we further extended the EST scheme so that it can be applied to the scattering of charged particles with a repulsive Coulomb force [24]. Finally we extended the EST formulation to incorporate multi-channel optical potentials [36].

The results demonstrate that separable representations based on a generalized EST scheme reproduce the standard coordinate space calculations of neutron and proton scattering cross sections very well. We also showed that the coordinate space separable representations can be obtained from the corresponding momentum space representations using Fourier transforms of the form factors. From those solutions, observables for (d,p) transfer reactions using the Faddeev-AGS formulation should be readily calculated. Work along these lines is in progress.

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