Ab Initio Description of the Tetraneutron with Realistic NN Interactions within the NCSM-SS-HORSE Approach

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Abstract

We continue the study of the tetraneutron resonance within the democratic SS-HORSE extension of the *ab initio* No-Core Shell Model [16] using modern NN interactions. With Daejeon16 and SRG-evolved chiral Idaho N3LO NN interactions we obtain the S-matrix pole corresponding to the tetraneutron resonance with energy between 0.7 and 1.0 MeV and width between 1.1 and 1.7 MeV. However we do not obtain a low-lying narrow resonance with the original Idaho N3LO but, instead, we obtain a very low-lying virtual state with the energy of 15 keV.

Keywords: *Tetraneutron; resonant states; realistic NN-interactions; No-Core Shell Model; SS-HORSE method; democratic decay*

1 Introduction

Interest in the tetraneutron was revived by a recent experiment [1] where a few events were detected which were interpreted as a resonant state in the four-neutron system with an energy of 0.83 ± 0.65 (stat.) ± 1.25 (syst.) MeV and a width not exceeding 2 MeV. As indicated in a historical review of the studies of few-neutron systems of Ref. [2], this is the first observation of the tetraneutron resonance which has been sought for more than fifty years [3]. The possibility of a bound tetraneutron state was proposed 15 year ago in Ref. [4] in the ^{14}Be breakup reaction $^{14}\mathrm{Be} \to {}^{10}\mathrm{Be} + 4n$. This experimental result, however, has not been confirmed.

The state-of-the-art theoretical studies conclude [5–14] that the tetraneutron cannot be bound without a significant altering of modern nuclear forces that will spoil a description of other nuclei. There are some indications on the existence of a low-lying tetraneutron resonance based on an artificial binding of the tetraneutron by

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strengthening the NN interaction [5] or by putting the four-neutron system in a trap [15] and by extrapolating these bound states to the case when the strengthening parameter is continuously reduced or the trap is continuously removed. Such extrapolations cannot predict the width of the resonance and should not be regarded as a firm proof of the resonant state. Existing calculations that explicitly account for the continuum [6,9–14] using various approaches [hyperspherical harmonics (HH), Faddeev–Yakubovsky equations, no-core Gamow shell model, complex scaling, etc.] with various realistic inter-nucleon forces resulted in the absence of a low-lying tetraneutron resonance narrow enough to be detected experimentally.

However, in our recent theoretical study [16], we obtained the tetraneutron resonance with the energy $E_r = 0.8$ MeV and the width $\Gamma = 1.4$ MeV. To the best of our knowledge, this is an only theoretical prediction consistent with the experimental finding of Ref. [1]. These calculations utilized the NN interaction JISP16 [17] and were performed within the so-called SS-HORSE (single state harmonic oscillator representation of scattering equations) extension [18–22] of the no-core shell model (NCSM) [23] adapted in Ref. [16] to the description of democratic decays (also known as a true four-body scattering or $4 \to 4$ scattering) [24, 25]. So, it is important to understand whether this low-lying resonance should be associated with the JISP16 NN interaction which was used in the tetraneutron studies only in Ref. [16] or with the new democratic NCSM-SS-HORSE approach able to describe correctly some specific features of the four-particle decay which are likely beyond the scope of other methods.

To this end, we perform here the NCSM-SS-HORSE calculations of the tetraneutron resonance with additional contemporary NN interactions. In particular, we adopt a new NN interaction Daejeon16 [26] which is better fitted to observables in light nuclei than JISP16. We also adopt the chiral NN interaction Idaho N3LO [27], both unperturbed ('bare') and softened by the methods of the Similarity Renormalization Group (SRG) [28, 29] with flow parameters $\Lambda = 1.5$ fm^{-1} and 2.0 fm^{-1}. We note that the Daejeon16 interaction was obtained by applying phase-equivalent transformations to the SRG-evolved with $\Lambda = 1.5$ fm^{-1} Idaho N3LO which adjust the interaction to describe light nuclei without referring to three-nucleon forces.

The next Section presents a brief description of the SS-HORSE method and its application to calculating democratic four-body decays within the NCSM. The tetraneutron calculation results with the Daejeon16, SRG-evolved and 'bare' Idaho N3LO interactions are given in Sections 3, 4, and 5 respectively. The last Section summarizes these studies.

2 SS-HORSE method for the $4 \to 4$ scattering

We use here the same theoretical approach as in our previous paper [16]. That is, we utilize the democratic decay approximation [24,25] to describe the four-neutron decay channel within the NCSM-SS-HORSE approach. A decay of a system into A particles is called '*democratic*' if none of subsystems built of these A particles has a bound state. In particular, the tetraneutron presents a nice example of nuclear system decaying through a four-body democratic channel only, and the study of the tetraneutron of Ref. [30] is one of the first applications of the democratic decay approximation.

It is natural to study democratic decays within the HH method, which introduces a 'democratic' collective coordinate, the hyperradius $\rho = \sqrt{\sum_{i=1}^{A}(\mathbf{r}_i - \mathbf{R})^2}$ ($\mathbf{r}_i$ are the coordinates of individual nucleons and $\mathbf{R}$ is the center-of-mass coordinate), and describes the dynamics of a system in terms of this coordinate. Formally, the democratic decay channel involves a superposition of an infinite number of HH with hypermomenta $K = K_{\min}$, $K_{\min} + 2, ...$, where $K_{\min}$ is the minimal hypermomentum consistent with the Pauli principle for a given nucleus; however, in practical applications, one usually uses a restricted set of HH adequate for the description of the decay channel. We use here the minimal approximation for the tetraneutron decay mode, i. e., we retain only the HH with hypermomentum $K = K_{\min} = 2$. This approximation relies on the fact that the decay in the hyperspherical states with $K > K_{\min}$ is strongly suppressed by a large hyperspherical centrifugal barrier $\mathcal{L}(\mathcal{L}+1)/\rho^2$, where the effective angular momentum

$$\mathcal{L} = K + \frac{3A-6}{2} = K + 3. \tag{1}$$

Note, the minimal approximation is used for the description of the decay channel only, i. e., for the description of the wave function asymptotics, while all possible HH are retained in the NCSM basis. The accuracy of this approximation was confirmed in studies of democratic decays in cluster models [31–34].

The NCSM utilizes the harmonic oscillator basis, and a natural extension of the NCSM to the continuum can be achieved within the J-matrix [35] (also known as the HORSE [36]) formalism in scattering theory, in particular, in an efficient SS-HORSE version [18–22] of this formalism. The general theory of the democratic decay within the HORSE formalism was proposed in Refs. [37, 38]; a derivation of the democratic SS-HORSE version along the lines suggested in Refs. [18, 19] is strightforward [16].

Within the minimal approximation, the S-matrix of the four-body decay is expressed through the hyperspherical phase shift δ as

$$S = e^{2i\delta}. \tag{2}$$

The SS-HORSE formalism provides the following expression for the phase shifts at the eigenenergies E_ν of the NCSM Hamiltonian [16, 18, 19]:

$$\tan\delta(E_\nu) = -\frac{S_{\mathbb{N}+2,\mathcal{L}}(E_\nu/\hbar\Omega)}{C_{\mathbb{N}+2,\mathcal{L}}(E_\nu/\hbar\Omega)}. \tag{3}$$

Here, $S_{N\mathcal{L}}$ and $C_{N\mathcal{L}}$ are linearly-independent solutions of the infinite tridiagonal free Hamiltonian matrix in the hyperspherical harmonic oscillator basis for which analytical expressions can be found in Refs. [37, 38], and $\mathbb{N}$ is the maximal total quanta of many-body oscillator states included in the NCSM basis,

$$\mathbb{N} = N_{\max} + N_{\min}, \tag{4}$$

$N_{\min} = 2$ is the quanta of the lowest possible oscillator state of the $4n$ system, and $N_{\max}$ is the maximal excitation quanta in the NCSM basis.

Varying $N_{\max}$ and $\hbar\Omega$ in the NCSM calculations, we obtain the phase shifts and S-matrix in some energy interval. Parametrizing the S-matrix in this energy interval, we obtain information about its nearby poles and hence resonances in the system.

Note, the phase shifts used for the parametrization should form some curve as a function of energy. However some phase shifts calculated by Eq. (3), especially those corresponding to the NCSM results obtained in small enough model spaces, deviate from the common curve signaling that convergence is not achieved. Therefore, before parametrizing the phase shifts, one needs to preselect the NCSM results retaining only those that are sufficiently converged so as to lie on the common curve.

Due to S-matrix symmetry properties [39,40], the hyperspherical phase shift $\delta(E)$ should be an odd function of momentum $k \sim \sqrt{E}$,

$$\delta(E) = v_1\sqrt{E} + v_3\big(\sqrt{E}\big)^3 + ... + v_9\big(\sqrt{E}\big)^9 + v_{11}\big(\sqrt{E}\big)^{11} + ... \tag{5}$$

On the other hand, at low energies, i. e., in the limit $k \to 0$, the phase shifts should behave as $\delta \sim k^{2\mathcal{L}+1}$ [39,40]. In the case of $4 \to 4$ scattering, $\mathcal{L} = K_{\min} + 3 = 5$ and therefore the expansion (5) starts at the eleventh power, i. e.,

$$v_1 = v_3 = ... = v_9 = 0. \tag{6}$$

To parametrize the phase shifts, we use the equation

$$-\arctan\frac{S_{N_{\max}+4,5}(E/\hbar\Omega)}{C_{N_{\max}+4,5}(E/\hbar\Omega)} = \sum_p \delta_p(E) + \phi(E), \tag{7}$$

which is obtained by rewriting Eq. (3) with the help of Eq. (4). Here $\phi(E)$ is a background phase, which is expected to be a smooth function parametrized as a Padé approximant,

$$\phi(E) = -\frac{w_1\sqrt{E} + w_3\Big(\sqrt{E}\Big)^3 + c\Big(\sqrt{E}\Big)^5}{1 + w_2E + w_4E^2 + w_6E^3 + dE^4}. \tag{8}$$

The sum in the rhs of Eq. (7) presents rapidly changing with E contributions from pole terms associated with the S-matrix poles located close to the origin of the complex momentum plane, in particular, resonant poles ($p = r$), false (redundant) poles at a positive imaginary momentum ($p = f$) which does not correspond to a bound state [39,40], or virtual state poles at a negative imaginary momentum ($p = v$) [39,40]. The respective phase shifts are

$$\delta_r(E) = -\arctan\frac{a\sqrt{E}}{E - b^2}, \tag{9a}$$

$$\delta_f(E) = -\arctan\sqrt{\frac{E}{|E_f|}}, \tag{9b}$$

$$\delta_v(E) = \arctan\sqrt{\frac{E}{|E_v|}}. \tag{9c}$$

The resonance energy E_r and width Γ are expressed through parameters a and b as

$$E_r = b^2 - a^2/2, \qquad \Gamma = a\sqrt{4b^2 - a^2}. \tag{10}$$

We attempted various fits including one, two, or three pole terms in Eq. (7) aimed to obtain a smooth background phase $\phi(E)$. The parameters w_1, w_2, w_3, w_4, and w_6

should guarantee that Eq. (6) is satisfied [note, the pole terms contribute to the low-order expansion terms in Eq. (5)]. The parameters c and d entering Eq. (8) together with the parameters a, b, E_f, and E_v of the included pole terms are used as fit parameters.

For each set of parameters, we solve Eq. (7) to find the energies $\mathcal{E}^{(i)} = \mathcal{E}(N^i_{\max}, \hbar\Omega^i)$ for each combination of $N^i_{\max}$ and $\hbar\Omega^i$ values and search for the parameter set minimizing the rms deviation

$$\Xi = \sqrt{\frac{1}{D}\sum_{i=1}^{D}\left(E_0^{(i)} - \mathcal{E}^{(i)}\right)^2} \tag{11}$$

of $\mathcal{E}^{(i)}$ from the selected set of the lowest NCSM eigenenergies E_0^i obtained with the same $N^i_{\max}$ and $\hbar\Omega^i$.

3 Results with Daejeon16

We performed the NCSM calculations using the code MFDn [41, 42] for the tetraneutron with $N_{\max}$ values ranging from 2 to 20 and $\hbar\Omega$ values ranging from 1 to 50 MeV. As in Refs. [16, 18–22], we select for the phase shift parametrization the NCSM results generating the phase shifts according to Eq. (3) that form approximately a common curve as a function of energy E. Additionally, we do not include in the analysis the NCSM eigenenergies above 7 MeV thus improving the description of the resonance region. The eigenenergy selection is shown by the shaded area in left panel of Fig. 1. The right panel of Fig. 1 shows the phase shifts obtained directly from the selected NCSM results using Eq. (3).

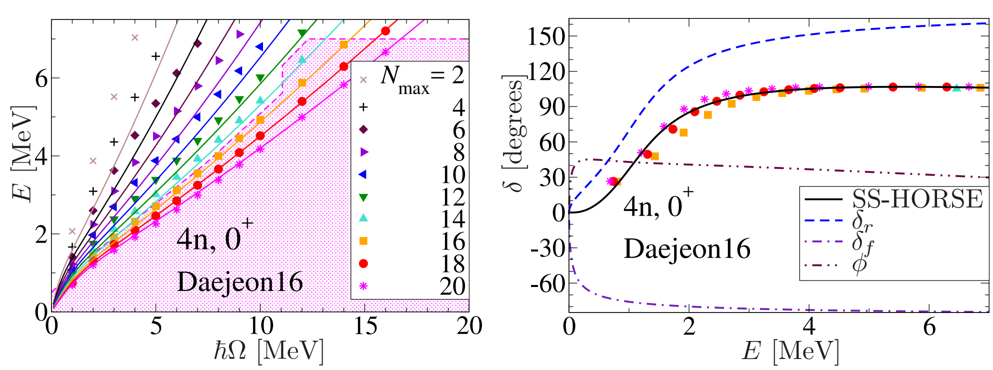

Figure 1: Left panel: the lowest 0^+ tetraneutron states obtained in the NCSM with the Daejeon16 NN interaction (symbols) with various $N_{\max}$ as functions of $\hbar\Omega$ and the energies $\mathcal{E}^{(i)}$ (solid curves) obtained from the phase shifts parametrization; the shaded area shows the NCSM result selection for the phase shift parametrization. Right panel: the $4 \to 4$ phase shift parametrization (solid curve) and phase shifts obtained directly from the selected NCSM results using Eq. (3) (symbols); contributions to the phase shifts of the resonant pole, the false pole and the background phase are shown by dashed, dashed-dotted and dashed-double-dotted curves respectively.

Table 1: Tetraneutron resonance energy E_r and width Γ and other fit parameters including the energy of the false pole E_f and of the virtual state $|E_v|$ as well as the rms deviation of energies Ξ characterizing the quality of the fit, obtained with JISP16 [16], Daejeon16, SRG-evolved with flow parameters $\Lambda = 1.5$ and 2.0 fm^{-1} Idaho N3LO, and 'bare' Idaho N3LO NN interactions.

Interaction	JISP16, Ref. [16]	Daejeon16	Idaho N3LO, SRG $\Lambda = 1.5$ fm^{-1}	Idaho N3LO, SRG $\Lambda = 2.0$ fm^{-1}	Idaho N3LO
a (MeV$^{\frac{1}{2}}$)	0.701	0.749	0.613	0.662	—
b^2 (MeV)	1.09	1.28	0.970	1.07	—
c (MeV$^{-\frac{5}{2}}$)	−27.0	−16.2	−31.6	−28.1	4960
d (MeV^{-4})	0.281	0.717	0.720	0.776	2330
E_r (MeV)	0.844	0.997	0.783	0.846	—
Γ (MeV)	1.38	1.60	1.15	1.29	—
E_f (keV)	−54.9	−63.4	−52.1	−54.5	—
$\lvert E_v\rvert$ (keV)	—	—	—	—	15.2
Ξ (keV)	43.8	47.9	29.0	31.7	19.4

We can accurately describe the NCSM results using only one resonant pole term. However this parametrization, as in the case of JISP16 [16], results in a very rapid changes of the background phase signaling the presence of another S-matrix pole in the vicinity of zero energy. A description of the selected NCSM eigenenergies approximately with the same rms deviation is achieved also by a parametrization with two pole terms associated with a resonant state and a false state. This parametrization essentially decreases the variation of the background phase and appears to be acceptable from the physical viewpoint. The resulting phase shifts are presented in the right panel of Fig. 1 while the fit parameters including the resonance energy and width and the energy of the false pole are given in Table 1. It is seen that the Daejeon16 NN interaction suggests a low-lying resonance in the system of four neutrons with energy about 1 MeV and width about 1.6 MeV consistent with the experimental observations of Ref. [1].

For comparison, we present in Table 1 also the results of Ref. [16] obtained with the JISP16 interaction with the same two-pole parametrization. It is seen that JISP16 and Daejeon16 interactions provide very similar predictions not only for the tetraneutron resonance energy and width but also for other fit parameters.

4 Results with SRG-evolved Idaho N3LO

As it was already noted, the Daejeon16 interaction was fitted to the observables in light nuclei by applying phase-equivalent transformations to the SRG-evolved Idaho N3LO NN interaction with flow parameter $\Lambda = 1.5$ fm^{-1}. Therefore it is interesting to investigate the effect of this adjustment of the NN interaction, which makes it possible to calculate nuclei without an explicit use of three-nucleon forces, on the tetraneutron resonance. We perform the tetraneutron calculations with the SRG-evolved Idaho N3LO NN interaction with flow parameters $\Lambda = 1.5$ fm^{-1} and $\Lambda = 2.0$ fm^{-1} to examine also the dependence of the tetraneutron resonance energy and width on the

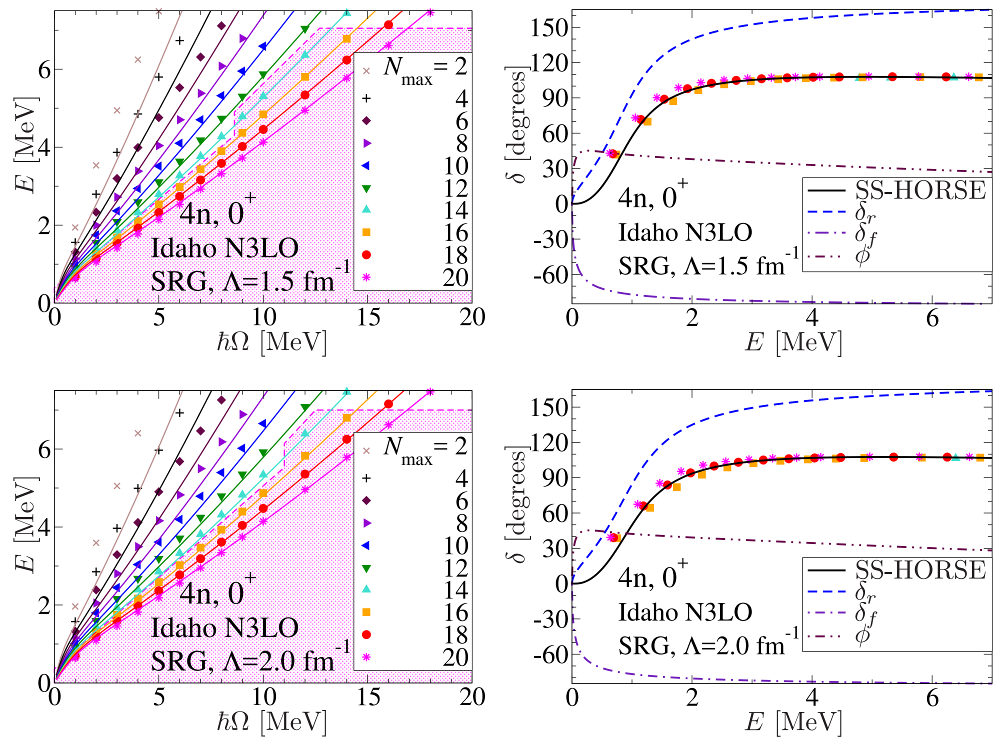

Figure 2: NCSM results for the lowest 0^+ tetraneutron states and the $4 \to 4$ phase shifts obtained with SRG-evolved Idaho N3LO NN interactions with flow parameters $\Lambda = 1.5$ fm^{-1} (upper panel) and $\Lambda = 2.0$ fm^{-1} (lower panel). See Fig. 1 for details.

flow parameters Λ.

It is interesting to note here that the SRG-evolved chiral Idaho N3LO with these values of flow parameters Λ, as well as the JISP16 and Daejeon16 interactions, provide without three-nucleon forces ground states of the bound $A = 3$ and $A = 4$ systems close to experiment while the original Idaho N3LO significantly underbinds these systems (see, e. g., Ref. [23, 43]). For example, the SRG-evolved chiral Idaho N3LO with our adopted flow parameters Λ, as well as the JISP16 and Daejeon16 interactions, all provide ground state energies of $A = 3$ nuclei and ^{4}He within 100 keV of experiment. On the other hand, the original Idaho N3LO underbinds ^{3}H by 620 keV and underbinds ^{4}He by 2.9 MeV.

We perform the calculations similar to those presented in the previous Section. In particular, we use the same set of N_{max} and $\hbar\Omega$ values in the NCSM calculations and make similar though not identical selections of the NCSM results for the phase shift parametrizations. We again come to a conclusion that physically reasonable parametrizations should include pole terms corresponding to resonant and false states, which suggest low-energy tetraneutron resonances. The results are presented in Fig. 2 and Table 1.

It is seen that we obtain the results similar to those obtained with JISP16 and Daejeon16 interactions. It is interesting that the interaction with $\Lambda = 2.0$ fm^{-1}

results in the values of the resonance energy and width as well as in the energy of the false state nearly identical to those obtained with JISP16. Decreasing Λ to 1.5 fm^{-1} causes small decreases of the resonant energy and width which become nearly 30% smaller than the width obtained with Daejeon16 while the difference in resonance energies is about 20%. These differences between the Daejeon16 and SRG-evolved with $\Lambda = 1.5$ fm^{-1} interactions may serve as a rough estimate of the three-body force effects in the tetraneutron since the Daejeon16, being fitted to light nuclei, mimics three-body force effects by off-shell properties of this two-nucleon only interaction.

A close look at the phase shift parametrizations in Figs. 1 and 2 reveals that the discrete energies, E_0^i, in the region of the extracted resonance are not as well converged as those outside this region. This slower convergence is reasonable in light of the low energy of the resonance which results from the delicate cancelation of small kinetic and small potential contributions to the values of E_0^i.

Generally, the results obtained with the SRG-evolved Idaho N3LO interactions are consistent with those from JISP16 and Daejeon16 interactions and with the experiment [1].

5 Results with original Idaho N3LO

Although the original Idaho N3LO interaction significantly underbinds the bound light nuclei with $A > 2$, we include results with this interaction since it does produce an excellent description the two-nucleon data. That is, normally, one includes a three-nucleon interaction with the original Idaho N3LO interaction to produce good binding results for the bound light nuclei with $A > 2$. It is also interesting to compare our results for this interaction with numerous studies of other authors that employed NN interactions with a strong short-range repulsion and did not obtain a narrow low-lying resonance in the tetraneutron.

We find that the same calculations with the 'bare' Idaho N3LO NN interaction alone, without a three-nucleon interaction, bring us to a very different conclusion about the tetraneutron resonance.

The NCSM calculations are performed in the same range of $N_{\max}$, $2 \leq N_{\max} \leq 20$, and $\hbar\Omega$, 1 MeV $\leq \hbar\Omega \leq$ 20 MeV; Fig. 3 shows the low-energy fraction (below 6 MeV) of the obtained NCSM results for the tetraneutron ground state together with the selection of eigenstates for the further SS-HORSE analysis. The phase shifts obtained directly from all NCSM results using Eq. (3) are shown in the left panel of Fig. 4. Contrary to other interactions discussed above, we have a convergence with the 'bare' N3LO only at low enough energies, below approximately 6 MeV, where the phase shifts with increasing $N_{\max}$ tend to a common curve formed by the phase shifts from the largest available model spaces. Therefore we select for the phase shift parametrization only the NCSM results with $N_{\max} = 16$, 18, and 20 lying below 6 MeV. Starting from the energies of 6 MeV, the convergence is clearly not achieved. One can speculate that the tendency of the phase shifts in this energy region suggests that the converged phase shift will form an additional smooth increase between 6 and 15 MeV that may indicate a presence of a wide resonant state with energy around 10 MeV, which, most probably, will be not possible to detect experimentally. There is also an indication that the convergence is achieved at energies around 20 MeV and higher which are of no interest for our analysis.

The behavior of the converged phase shifts in the right panel of Fig. 4 which

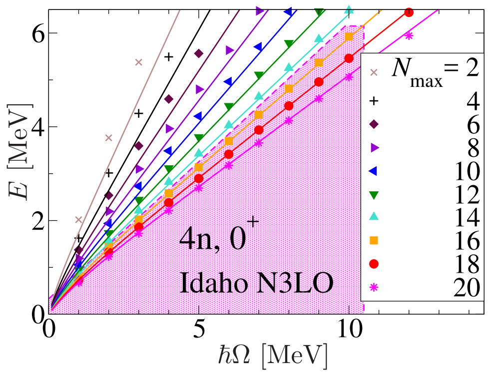

Figure 3: Low-energy fraction of the NCSM results obtained with 'bare' Idaho N3LO NN interaction. See Fig. 1 for details.

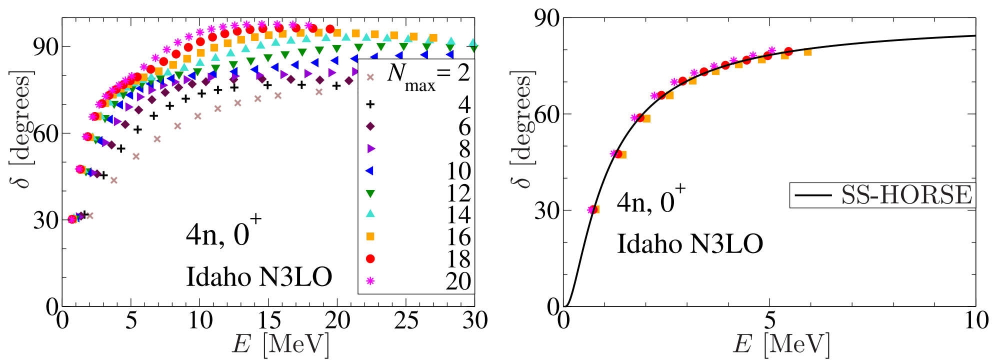

Figure 4: $4 \to 4$ phase shifts obtained directly from all available (left panel) and from the selected (right panel) NCSM results using Eq. (3) (symbols) together with the $4 \to 4$ phase shift parametrization (solid curve).

are increasing smoothly up to approximately 80° in a wide enough energy interval, suggests an absence of a narrow resonance; however, this phase shift increase may be caused by a wide resonance or by a low-lying virtual state as well as by some combination of S-matrix poles of different types. We have studied various possibilities and have come to the conclusion that the only way to describe the NCSM results with the unperturbed Idaho N3LO NN interaction is to introduce a single pole term associated with a virtual state with a very small energy of 15.2 keV. The fit parameters are listed in Table 1, the fitted hyperspherical phase sifts are depicted in the right panel of Fig. 4.

6 Summary and conclusions

We have studied in a democratic NCSM-SS-HORSE approach with various NN interactions a low-lying resonance in a system of four neutrons, which was recently observed in a RIKEN experiment [1]. We found that a narrow resonance consistent with experimental data is supported by soft NN interactions, in particular, by JISP16 and Daejeon16 interactions accurately describing the two-nucleon data and fitted to properties of light nuclei without making use of three-nucleon forses as well as by the SRG-evolved chiral Idaho N3LO NN interactions with flow parameters $\Lambda = 1.5$ fm^{-1}

and 2.0 fm^{-1}. All these interactions provide similar results indicating a resonance with energy between 0.7 and 1.0 MeV and width between 1.1 and 1.7 MeV. On the other hand, the original Idaho N3LO, which underbinds light nuclei in the absence of a three-body interaction, does not support a tetraneutron resonance but predicts a very low-lying tetraneutron virtual state with the energy of 15 keV. This is consistent with results of other theoretical studies of various authors who did not obtain a narrow low-lying tetraneutron resonance within various approaches with NN interactions with a strong short-distance repulsion. However, it appears that nobody before was searching for a virtual state in the four-neutron system.

Regarding the comparison with the experiment [1], we note that the experimentalists are not studying the S-matrix poles in the system of four neutrons but are studying cross sections of a complicated reaction $^4\mathrm{He}(^8\mathrm{He}, ^8\mathrm{Be})$, where the reaction mechanism plays a very important role. This reaction mechanism can reveal the tetraneutron resonance but, probably, at a somewhat shifted energy, or just mimic a resonance behavior in a system that has no low-lying resonance but a broad continuum structure as discussed in Ref. [9]. It is also possible that the virtual tetraneutron state can manifest itself as a resonant structure of the cross section due to some features of the reaction mechanism. Therefore it would be very interesting to study the reaction $^4\mathrm{He}(^8\mathrm{He}, ^8\mathrm{Be})$ in a realistic reaction-theory approach which will account for the pole structure of the tetraneutron.

It would also be interesting to study the tetraneutron with a combination of modern NN and three-nucleon forces. We experience technical difficulties in allowing for three-nucleon forces in our approach. In particular, we need matrix elements of a three-nucleon force in oscillator bases with large $N_{\max}$ and very small $\hbar\Omega$ values which presents a real challenge. This need arises since the NCSM results with large $N_{\max}$ and small $\hbar\Omega$ are of special importance for calculating low-energy behavior of the S-matrix and for locating its poles. We, however, hope to overcome this difficulty in future studies. The effects of the three-nucleon force on the tetraneutron resonance properties are roughly estimated to be around 20–30% by comparing the results obtained with Daejeon16 and SRG-evolved N3LO interactions.

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