Deuteron Wave Function and Neutron Form Factors from Elastic Electron-Deuteron Scattering

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Abstract

A study of the deuteron structure in the framework of relativistic quantum mechanics is presented. The point-form (PF) relativistic quantum mechanics (RQM) is applied to elastic eD scattering. The deuteron wave function and neutron form factors are fitted to electromagnetic deuteron form factors. We also compare results obtained with various realistic deuteron wave functions stemming from Nijmegen-I, Nijmegen-II, JISP16, CD-Bonn, Paris, Argonne, Idaho and Moscow (with forbidden states) potentials. It is shown that the electromagnetic deuteron form factors may be described without exchange currents.

Keywords: *Nucleon; potential; deuteron; eD elastic scattering; electromagnetic form factors; relativistic quantum mechanics; point-form dynamics*

1 Introduction

Being the simplest nucleus, the deuteron provides the most direct test of various nucleon-nucleon interaction models and relevant degrees of freedom. In this context, the deuteron electromagnetic studies with electron or photon probes are the simplest in theoretical and experimental aspects. These studies provide a picture of deuteron electromagnetic structure in terms of deuteron electromagnetic (EM) form factors (FFs). These FFs are functions of the square of the four-momentum transferred by a probe ($q^2 = -Q^2$).

During the last two decades a considerable advance has been made in the experimental knowledge of deuteron electromagnetic structure. On the other hand, there is a substantial diversity of opinions regarding an appropriate general theoretical approach. Anyhow, it seems natural as well as confirmed by a general data analysis [1] that in the space-like region of Q^2 (corresponding to the elastic scattering) a successful theory may be obtained from a relativistic description of the NN channel only which is supplemented by minor modifications of the short-range structure of the deuteron EM current operator. Here, again, there are various approaches to the relativistic description as well as to the EM current operator structure [1, 2].

A conventional assumption is that a majority of the existing data of eD elastic scattering are described to high precision within a single-photon exchange approximation and by three electromagnetic deuteron FFs [1, 3–5]. The deuteron FFs are

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calculated in the conventional model using the deuteron wave function (S and D components) and nucleon FFs. FFs may be chosen so as to be equal in the $Q^2 = 0$ (static) limit to the deuteron charge, magnetic and quadruple moments. The first two FFs are described within the conventional nuclear model in terms of nucleon degrees of freedom only. However the deuteron static electric quadrupole moment is not reproduced well enough in modern NN potential calculations. It is generally accepted that meson exchange contributions should be accounted for to get an agreement with the data.

The single-photon exchange approximation assumes that the electron and deuteron exchange a single virtual photon. It is believed that this approximation should be valid to a high precision due to a small value of the fine-structure constant. Therefore the elastic eD scattering allows to extract the deuteron EM FF dependencies on the transferred 4-momentum Q in the space-like region. To extract these dependencies, it is required to measure three independent observables of the eD elastic scattering in this region. Two of them (structure functions A and B) are extracted from the unpolarized differential cross section and the third one is extracted from polarization measurements.

Simple non-relativistic calculations with various realistic NN potentials (NN channel only) at low $Q \lesssim 0.7$ GeV/$c \approx 3.5$ Fm^{-1} are in close harmony and agree well enough with the eD elastic scattering data [1, 2]. A disagreement between calculations increases with the rise of $Q > 3.5$ Fm^{-1}. None of the calculations describes data for $Q \gtrsim 3.5$ Fm^{-1}. This disagreement indicates that relativistic effects and contributions of other channels may be essential at $Q > 3.5$ Fm^{-1} [1, 2]. Indeed, an inclusion of relativistic and meson exchange corrections results in a good description of the data [6, 7]. However there is another problem that is not emphasized usually. Important ingredients of the calculations are the nucleon FF dependencies on the Q_i^2 transferred to an individual nucleon. These dependencies are extracted experimentally. The proton FFs are extracted from direct measurements with proton targets. Nevertheless, even in this case, there is a notable discrepancy between the values of the proton FF ratio, G_{Ep}/G_{Mp}, extracted from polarization and cross section experiments. The cross sections are necessary to extract the absolute values of G_{Ep} and G_{Mp} while the polarization transfer measurements provide information only about the ration G_{Ep}/G_{Mp}. The discrepancy begins at $Q \approx 1$ GeV/$c \approx 5$ Fm^{-1}. It may be explained by the hard two-photon exchange process (TPE), and the data reveal some evidence for this explanation [8]. It also should be noted that model calculations and analyses demonstrate that the TPE significantly changes G_{Ep} while the G_{Mp} alteration is at a few percent ($\sim$3%) level [9, 10]. The latest analytical fit to the proton FFs is a simultaneous fit to the polarization and cross section data. The cross section data are corrected by an additive term assuming some phenomenological TPE corrections [11].

A free neutron target is not available. The neutron FFs are extracted from the measurements of eD or $e\,^3$He scattering. Therefore the data analysis is affected by uncertainties stemming from an assumed nuclear theoretical model describing the target nucleus and reaction.

A conventional procedure for the neutron FF extraction may be exemplified as follows. The electric neutron FF was measured in Ref. [12] up to $Q^2 = 3.4$ GeV$^2/c^2$ using the $\overrightarrow{^3\mathrm{He}}(\vec{e}, e'n)pp$ reaction. The extraction includes calculations of the asymmetries in the quasi-elastic processes $\overrightarrow{^3\mathrm{He}}(\vec{e}, e'n)pp$ and $\overrightarrow{^3\mathrm{He}}(\vec{e}, e'p)np$. These calculations were performed using the generalized eikonal approximation and included spin-dependent final-state interactions (FSI), meson-exchange currents and the ^{3}He wave function resulting from the AV18 potential. Finally, to extract the G_{En}, a linearly interpolated G_{Mn} from Ref. [13] was used. The procedure of Ref. [13] is a measurement of the ratio R of the cross sections of the quasielastic $^2\mathrm{H}(e,e'n)p$ and $^2\mathrm{H}(e,e'p)n$ reactions. To extract the G_{Mn} from R, the authors use: 1) Cross section calculations within the Plane Wave Impulse Approximation (PWIA) for $Q^2 > 1.0$ GeV$^2/c^2$, the AV18 deuteron wave function, and the Glauber theory for FSI; 2) Calculations of a nuclear correction factor which is the ratio of the results of the full calculation and those within the PWIA but without FSI; 3) The proton cross section (a parameterization of Ref. [10]); 4) Parameterizations of G_{En} of Ref. [14,15]. Obviously, it looks like a vicious circle. There is a model-insensitive technique for the G_{En} extraction of Ref. [16] but it is applicable only to the $d(\vec{e},e'\vec{n})p$ reaction in the quasi-free kinematics. This technique was utilized to extract the G_{En} at $Q^2 = 0.255$ GeV^2 ($Q \approx 2.6$ Fm^{-1}) [17] resulting in $G_{En} = 0.066 \pm 0.036 \pm 0.009$. The "scale" uncertainty (± 0.009) was estimated using independent measurements of G_{Mn} and the so-called "Arenhövel's model". That sounds convincing and may mean that the Argonne V14 or other contemporary phenomenological potentials were used in the estimation. However there is no an estimation of the model systematic error and the whole procedure is again an obvious vicious circle.

There are various relativistic models [6,18–21] for calculations of the deuteron EM FFs. All of them look reasonable but they may produce different results. That is not a contradiction. This issue is discussed in the last part of the paper. In this paper, we extend our previous investigations where we described the elastic NN scattering up to laboratory energy of 3 GeV [22] and electromagnetic reactions with two nucleons: a bremsstrahlung in the pp scattering $pp \to pp\gamma$ [23], the deuteron photodisintegration $\gamma D \to np$ [24–26], the exclusive deuteron electrodisintegration [27] and the eD elastic scattering [28,29]. We apply manifestly covariant relativistic quantum mechanics (RQM) [30] in the point-form (PF). The PF is one of three forms of the RQM proposed by Dirac [31]. The other two forms in common use are the instant and the front forms. Each form is associated with a subgroup of the Poincaré group. This subgroup is considered to be free of interactions. All these forms are unitary equivalent [32], however each form has particular advantages. For example, the PF has some simplifying features [33]. Only in the PF all generators of the homogeneous Lorentz group are free of interactions. That means a manifest covariance that clearly simplifies the boost transformations. Therefore the spectator approximation (SA) of an electromagnetic process preserves its spectator character in any reference frame (r. f.) [34–36]. There are two equivalent SAs of the EM current operator in composite systems within the PF RQM [33,34]. The PF RQM SA has been applied to calculations of EM FFs of composite systems [20,37–41] with satisfactory results.

2 Potential model in PF RQM

A general method of allowing for interactions in generators of the Poincaré group was proposed by Bakamjian and Thomas [42]. We present here only the results of PF RQM necessary for our eD calculations. We use formalism and notation of Ref. [34] for calculation of matrix elements of the EM current operator.

Let p_i be the 4-momentum of nucleon i, $P \equiv (P^0, \mathbf{P}) = p_1 + p_2$ is the system

4-momentum, M is the system mass and $G = P/M$ is the system 4-velocity. The wave function of two particles with 4-momentum P is expressed through a tensor product of external and internal parts

$$|P,\chi\rangle = U_{12}\,|P\rangle \otimes |\chi\rangle, \tag{1}$$

where the internal wave function $|\chi\rangle$ satisfies Eqs. (7)–(8). The unitary operator

$$U_{12} = U_{12}(G,\mathbf{q}) = \prod_{i=1}^{2} D[\mathbf{s}_i; \alpha(p_i/m)^{-1}\alpha(G)\,\alpha(q_i/m)] \tag{2}$$

relates the "internal" Hilbert space with the Hilbert space of two-particle states [34]. $D[\mathbf{s};u]$ is a SU(2) operator corresponding to the element $u \in$SU(2), $\mathbf{s}$ are the SU(2) generators. In our case of spin $s = 1/2$ particles, we deal with the fundamental representation, i. e., $\mathbf{s}_i \equiv \frac{1}{2}\boldsymbol{\sigma}_i$ [$\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ are the Pauli matrices] and $D[\mathbf{s};u] \equiv u$. The momenta of the particles in their c. m. frame are

$$q_i = L[\alpha(G)]^{-1} p_i, \tag{3}$$

where $L[\alpha(G)]$ is the Lorentz boost to the frame moving with the 4-velocity G. The matrix

$$\alpha(g) = (g^0 + 1 + \boldsymbol{\sigma}\cdot\mathbf{g})/\sqrt{2(g^0+1)} \tag{4}$$

corresponds to a 4-velocity $g \equiv \left(g^0, \mathbf{g}\right)$.

The external part of the wave function is defined as

$$\langle G|P'\rangle \equiv \frac{2}{M'} G'^{\,0}\, \delta^3(\mathbf{G} - \mathbf{G}'). \tag{5}$$

Its scalar product is

$$\langle P''|P'\rangle = \int \frac{d^3\mathbf{G}}{2G^0}\, \langle P''|G\rangle\langle G|P'\rangle = 2\sqrt{M'^2 + \mathbf{P}'^2}\, \delta^3(\mathbf{P}'' - \mathbf{P}'), \tag{6}$$

where $G^0(\mathbf{G}) \equiv \sqrt{1+\mathbf{G}^2}$. The internal part $|\chi\rangle$ is characterized by the total angular momentum J of the system and by momentum $\mathbf{q} = \mathbf{q}_1 = -\mathbf{q}_2$ of one of the particles in the c. m. frame.

The 4-momentum $\hat{P} = \hat{G}\hat{M}$ incorporates the interaction V_{int}, where $\hat{M}$ is sum of the free mass operator M_{free} and the interaction, $\hat{M} = M_{free} + V_{int}$. The wave function is an eigenfunction of the system mass operator $\hat{M}$. We represent this wave function as a product of the external and internal parts. The internal wave function $|\chi\rangle$ is also an eigenfunction of the mass operator and for the system of two nucleons with masses $m_1 \approx m_2 \approx m = 2m_1m_2/(m_1+m_2)$ satisfies the following equation:

$$\hat{M}|\chi\rangle \equiv \left[2\sqrt{\mathbf{q}^2 + m^2} + V_{int}\right]|\chi\rangle = M|\chi\rangle. \tag{7}$$

This equation can be rewritten as

$$\left[\mathbf{q}^2 + mV\right]|\chi\rangle = q^2|\chi\rangle, \tag{8}$$

where V acts on internal variables only, and

$$q^2 = \frac{M^2}{4} - m^2. \tag{9}$$

The operators V_{int} and V (and therefore $\hat{M}$ and M_{free}) commute with the spin operator J and with the 4-velocity operator $\hat{G}$. The generators of space-time rotations are interaction-free. Most of formal results of non-relativistic scattering theory are valid in the case of two relativistic particles [30]. For example, the relative orbital angular momentum and spins are coupled in the c. m. frame in the same manner as in the non-relativistic case. Equation (8) is identical in form to the Schrödinger equation. Relativistic corrections affect the deuteron binding energy only and may be easily accounted for by replacing the experimental deuteron binding of 2.2246 MeV by an effective value of 2.2233 MeV. The origin of this relativistic correction is the following. Let ε be the deuteron binding energy. Then $M = 2m - \varepsilon$ and $q^2 = \frac{M^2}{4} - m^2 = -m\varepsilon\left(1 - \frac{\varepsilon}{4m}\right)$. Comparing with the non-relativistic relationship $q^2 = -m\varepsilon$, we identify the factor $\left(1 - \frac{\varepsilon}{4m}\right)$ as a relativistic correction. There is no similar correction in the scattering domain since $q^2 = mE_{lab}/2$ is an exact relativistic relationship (E_{lab} is the laboratory energy) used in the partial wave analysis. The above correction is negligible for our problem.

The deuteron wave function $|P_i, \chi_i\rangle$ is normalized,

$$\langle P_f, \chi_f | P_i, \chi_i\rangle = 2P_i^0\, \delta^3(\mathbf{P}_i - \mathbf{P}_f)\langle \chi_f|\chi_i\rangle. \tag{10}$$

3 *eD* elastic scattering

There is a convenient r. f. for calculation of current operator matrix elements in PF RQM introduced by F. Lev [34] (it coincides with the Breit r. f. in the case of elastic *ed* scattering). The Lev's r. f. is defined by the following condition for all EM reactions with two nucleons:

$$\mathbf{G}_f + \mathbf{G}_i = 0, \tag{11}$$

where $\mathbf{G}_f = \mathbf{P}_f/M_D$, $\mathbf{G}_i = \mathbf{P}_i/M_D$ are the final and initial 4-velocities of the deuteron and M_D is its mass. The matrix element of the current operator is [34]:

$$\langle P_f, \chi_f|\hat{J}^\mu(x)|P_i, \chi_i\rangle = 2(M_f M_i)^{1/2}\exp(\imath(P_f - P_i)x)\langle\chi_f|\hat{j}^\mu(\mathbf{h})|\chi_i\rangle, \tag{12}$$

where the internal current operator $\hat{j}^\mu(\mathbf{h})$ defines an action of current operator in the internal space of the NN system,

$$\mathbf{h} = \frac{2(M_i M_f)^{1/2}}{(M_i + M_f)^2}\,\mathbf{k} = \frac{\mathbf{k}}{2M_D} \tag{13}$$

is a vector-parameter [34] ($0 \le h \le 1$), $\mathbf{k}$ is the momentum of photon in the Lev's r. f., $M_i = M_f = M_D$ are the masses of the initial and final NN systems (deuteron).

The internal deuteron wave function

$$|\chi_i\rangle = \frac{1}{r}\sum_{l=0,2} u_l(r)|l, 1; J = 1M_J\rangle_{\mathbf{r}} \tag{14}$$

is normalized: $\langle\chi_i|\chi_i\rangle = 1$. This configuration space wave function has a physical sense only in the non-relativistic limit. In our calculations we use the momentum space wave function:

$$|\chi_i\rangle = \frac{1}{q}\sum_{l=0,2} u_l(q)|l, 1; 1M_J\rangle_{\mathbf{q}}, \tag{15}$$

where

$$u(q) \equiv u_0(q) = \sqrt{\frac{2}{\pi}} \int dr\, \sin(qr)\, u(r), \tag{16}$$

$$w(q) \equiv u_2(q) = \sqrt{\frac{2}{\pi}} \int dr \left[\left(\frac{3}{(qr)^2} - 1 \right) \sin(qr) - \frac{3}{qr} \cos(qr) \right] w(r). \tag{17}$$

A transformation of the Breit r. f. (11) into the final (initial) c. m. frame of the NN system is the boost along (in the backward direction than) vector $\mathbf{h}$ (axis z). The projection of the total deuteron angular momentum onto the z is unaffected by these boosts. The initial deuteron in the Breit r. f. moves in the direction opposite to $\mathbf{h}$. Its internal wave function with the spirality Λ_i is

$$|\Lambda_i\rangle = \frac{1}{q} \sum_{l=0,2} u_l(q) |l, 1; 1, M_J = -\Lambda_i\rangle. \tag{18}$$

The wave function of the final deuteron with the spirality Λ_f is

$$|\Lambda_f\rangle = \frac{1}{q} \sum_{l=0,2} u_l(q) |l, 1; 1, M_J = \Lambda_f\rangle. \tag{19}$$

A conventional parametrization of the EM current operator matrix element for a spin-1 particle (deuteron) is [1, 2, 43]

$$\begin{aligned} &(4P_i^0 P_f^0)^{1/2} \langle P_f, \chi_f | J^\mu | P_i, \chi_i \rangle \\ &\quad = -\left\{ G_1(Q^2)(\boldsymbol{\xi}_f^* \cdot \boldsymbol{\xi}_i) - G_3(Q^2) \frac{(\boldsymbol{\xi}_f^* \cdot \Delta P)(\boldsymbol{\xi}_i \cdot \Delta P)}{2M_D^2} \right\} (P_i^\mu + P_f^\mu) \\ &\qquad - G_2(Q^2)[\xi_i^\mu (\boldsymbol{\xi}_f^* \cdot \Delta\ \mathbf{P}) - \xi_f^{*\mu} (\boldsymbol{\xi}_i \cdot \Delta \mathbf{P})], \end{aligned} \tag{20}$$

where $(a \cdot b) = a^0 b^0 - (\mathbf{a} \cdot \mathbf{b})$, EM FFs $G_i(Q^2)$, $i = 1, 2, 3$ are functions of $Q^2 = -\Delta P^2$, $\Delta P = P_f - P_i$. In the Breit r. f. $\mathbf{P}_f = -\mathbf{P}_i$, $P_i^0 = P_f^0 \equiv P^0 = M_D/\sqrt{1 - h^2}$, $\Delta P = (0, 2\mathbf{P}_f)$, $P_i^\mu + P_f^\mu = (2P^0, \mathbf{0})$, $\mathbf{P}_f / P^0 = \mathbf{h}$, $\mathbf{P}_f = \mathbf{h} M_D / \sqrt{1 - h^2}$, $Q^2 \equiv -\Delta P^2$, $\Delta P^2 = -4h^2 M_D^2/(1 - h^2)$, $h^2 = (\mathbf{h} \cdot \mathbf{h})$. Matrix elements of the internal current operator are

$$\begin{aligned} \langle \chi_f | j^0(\mathbf{h}) | \chi_i \rangle = -G_1(Q^2)(\boldsymbol{\xi}'^* \cdot \boldsymbol{\xi}) + 2G_3(Q^2) \frac{(\boldsymbol{\xi}_f^* \cdot\ \mathbf{h})(\boldsymbol{\xi}_i \cdot \mathbf{h})}{1 - h^2} \\ + G_2(Q^2)[\xi_i^0 (\boldsymbol{\xi}_f^* \cdot \mathbf{h}) - \xi_f^{0*} (\boldsymbol{\xi}_i \cdot \mathbf{h})], \end{aligned} \tag{21}$$

$$\langle \chi_f | \mathbf{j}(\mathbf{h}) | \chi_i \rangle = G_2(Q^2)[\xi_i (\boldsymbol{\xi}_f^* \cdot \mathbf{h}) - \xi_f^* (\boldsymbol{\xi}_i \cdot \mathbf{h})] = G_2(Q^2)[\mathbf{h} \times [\boldsymbol{\xi}_i \times \boldsymbol{\xi}_f^*]]. \tag{22}$$

It has been shown [34] that these expressions are equivalent to chosing j^ν as

$$j^0(\mathbf{h}) = G_C(Q^2) + \frac{2G_Q(Q^2)}{(1 - h^2)} \left[\frac{2}{3} h^2 - (\mathbf{h} \cdot \mathbf{J})^2 \right], \tag{23}$$

$$\mathbf{j}(\mathbf{h}) = -\frac{\imath}{\sqrt{1 - h^2}} G_M(Q^2)(\mathbf{h} \times \mathbf{J}), \tag{24}$$

where $\mathbf{J}$ is the total angular momentum (spin) of the deuteron; G_C, G_Q and G_M are its charge monopole, charge quadruple and magnetic dipole FFs.

Spiral polarizations of the deuteron in the initial and final states are

$$\xi_i^\Lambda = \begin{cases} (0, \pm 1, -\imath, 0)/\sqrt{2} & (\Lambda = \pm) \\ (-h, 0, 0, 1)/\sqrt{1-h^2} & (\Lambda = 0), \end{cases} \tag{25}$$

$$\xi_f^\Lambda = \begin{cases} (0, \mp 1, -\imath, 0)/\sqrt{2} & (\Lambda = \pm) \\ (h, 0, 0, 1)/\sqrt{1-h^2} & (\Lambda = 0). \end{cases} \tag{26}$$

A virtual photon polarization is

$$\epsilon^\lambda = \begin{cases} (0, \mp 1, -\imath, 0)/\sqrt{2} & (\lambda = \pm) \\ (1, 0, 0, 0) & (\lambda = 0). \end{cases} \tag{27}$$

The deuteron FFs G_i are expressed as

$$\begin{aligned} G_C &= G_1 + \frac{2}{3}\eta G_Q, \\ G_Q &= G_1 - G_M + (1+\eta)G_3, \\ G_1 &= G_C - \frac{2h^2}{3(1-h^2)}G_Q, \\ G_3 &= G_Q\left(1 - \frac{h^2}{3}\right) - G_C(1-h^2) + G_M(1-h^2), \end{aligned} \tag{28}$$

where $\eta = Q^2/4M_D^2 = h^2/(1-h^2)$. The form factors $G_C(0) = e$, $G_M(0) = \mu_D e/2M_D$ and $G_Q(0) = Q_D e/M_D^2$ provide the deuteron charge, magnetic and quadruple moments respectively.

Denoting the helicity amplitudes as $j^\lambda_{\Lambda_f \Lambda_i} \equiv \langle \Lambda_f | \big(\epsilon^\lambda_\mu \cdot j^\mu(\mathbf{h})\big) | \Lambda_i \rangle$, we have:

$$j^0_{00}(Q^2) = G_C + \frac{4}{3}\frac{h^2}{1-h^2}G_Q, \tag{29}$$

$$j^0_{+-}(Q^2) = j^0_{-+}(Q^2) = G_C - \frac{2}{3}\frac{h^2}{1-h^2}G_Q, \tag{30}$$

$$\frac{j^+_{+0}(Q^2) + j^+_{0-}(Q^2)}{2} = -\frac{h}{\sqrt{1-h^2}}G_M \tag{31}$$

and

$$j^+_{+0}(Q^2) = j^-_{-0}(Q^2) \approx j^+_{0-}(Q^2) = j^-_{0+}(Q^2). \tag{32}$$

The squares of deuteron FFs are extracted from the elastic eD scattering with unpolarized particles and an additional polarization observable (usually $t_{20}(Q^2, \theta)$).

In the present paper we use the SA of EM current operator of Ref. [34] without expanding it in powers of h and calculate its matrix elements in the momentum space. Therefore we use the following expansion of $\hat{j}^\mu(\mathbf{h}) \approx \hat{j}^\mu_{SA}(\mathbf{h})$ [27] in calculations:

$$\begin{aligned} \hat{j}^\mu_{SA}(\mathbf{h}) = {} & (1 + (\mathbf{A}_2 \cdot \mathbf{s}_2))\,(B_1^\mu + (\mathbf{C}_1^\mu \cdot \mathbf{s}_1))\,\mathbf{I}_1(\mathbf{h}) \\ & + (1 + (\mathbf{A}_1 \cdot \mathbf{s}_1))\,(B_2^\mu + (\mathbf{C}_2^\mu \cdot \mathbf{s}_2))\,\mathbf{I}_2(\mathbf{h}), \end{aligned} \tag{33}$$

where $\mathbf{A}_i$, B_i^μ, $\mathbf{C}_i^\mu$ are some cumbersome vector functions of $\mathbf{h}$ and $\mathbf{q}(q, \theta, \phi)$. In the spherical coordinate system (q, θ, ϕ), the ϕ dependence of these functions is $e^{\pm im\phi}$ $(m = 0, 1, 2)$. The analytical integration with respect to ϕ results in trivial equalities (32).

4 Results and Conclusions

We study the eD elastic scattering within the simplest model supposing that the NN channel is described in the PF RQM and using the SA for the NN EM current operator. Therefore we assume that the exchange current effects are negligible within the NN deuteron models at least for this reaction. The momentum space deuteron wave functions are transformed into the configuration space using Eqs. (16)–(17). We assume that the configuration space wave functions of Eq. (14) have a physical sense only in the non-relativistic limit. The deuteron wave functions stemming from Nijmegen-I (NijmI), Nijmegen-I (NijmII) [44], JISP16 [45], CD-Bonn [46], Paris [47], Argonne18 [48] (for this momentum space deuteron wave function we use a parametrization of Ref. [49]), Idaho [50] (thanks to Prof. David R. Entem for the respective computer code) and Moscow (with forbidden states) [22] potentials are shown in Figs. 1 and 2. The Moscow potential parameters and computer code generating Moscow potentials are available upon a request from the author (e-mail: `nikolakhokhlov@yandex.ru`).

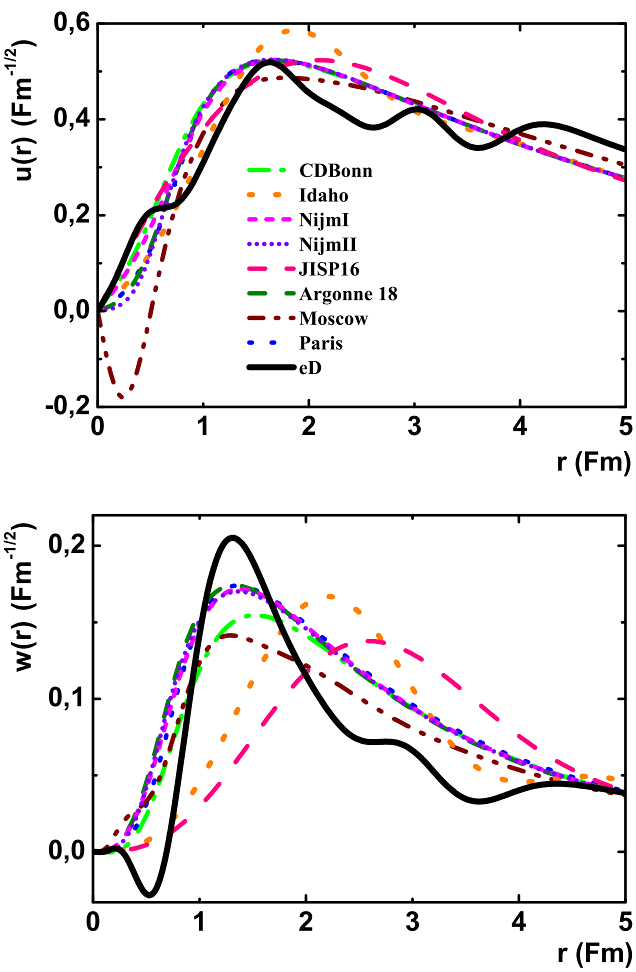

Figure 1: Configuration space deuteron wave functions used in calculations.

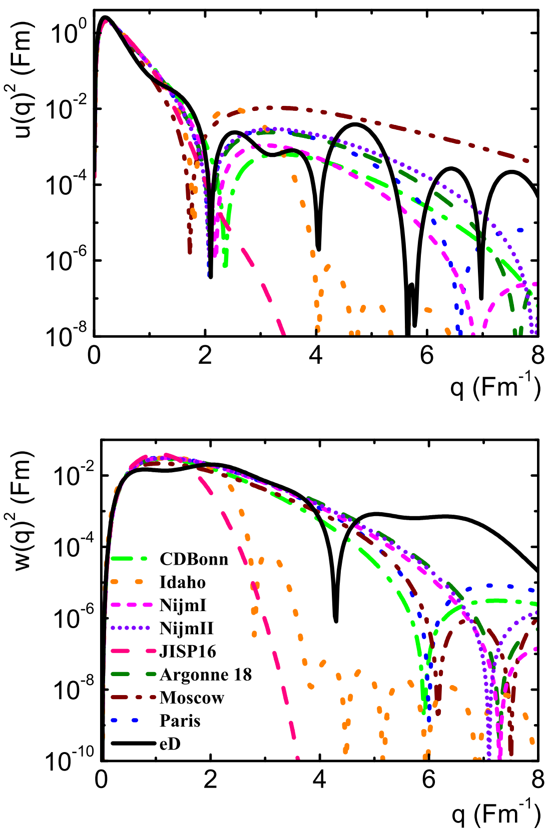

Figure 2: Momentum space deuteron wave functions used in the calculations.

In our previous studies [28, 29] we concentrated on the deuteron FF dependence on the nucleon FFs which was found to be considerable for $Q > 5$ Fm^{-1}. Now we examine a possibility of extracting the deuteron wave function and neutron EM FFs from the elastic eD scattering. Therefore we perform a fit of the deuteron wave function and nucleons EM FFs. We use the nucleon FF functional dependence on Q^2 of Ref. [10] and fit the parameters. The deuteron wave function parametrization is described bellow. Results of the fit denoted as eD are presented in Table 1 and figures.

We use the deuteron wave functions in the analytic form of Ref. [47] modified by a short range addend. The ansatz for the configuration space wave functions is

$$u(r) = \sum_{i=0}^{m} a_i^0\, R_{i,0}(r) + \sum_{j=1}^{n} C_j \exp(-m_j r), \tag{34}$$

$$w(r) = \sum_{i=0}^{k} a_i^2\, R_{i,2}(r) + \sum_{j=1}^{n} D_j \exp(-m_j r)\left[1 + \frac{3}{m_j r} + \frac{3}{(m_j r)^2}\right], \tag{35}$$

where the oscillator functions

$$R_{n,l}(r) = (-1)^n \sqrt{\frac{2n!}{r_0 \Gamma(n+l+3/2)}} \left(\frac{r}{r_0}\right)^{l+1} \exp\left(-\frac{r^2}{2r_0^2}\right) L_n^{l+\frac{1}{2}}\left(\frac{r^2}{r_0^2}\right), \tag{36}$$

L_n^α is the associated Laguerre polynomial, the oscillator radius $r_0 = 0.4$ fm.

Table 1: Static deuteron form factors. The slash-separated values are the results of relativistic/non-relativistic calculations.

	$G_M(0)=\frac{M_d}{m_p}\mu_d$	$G_Q(0)=M_d^2 Q_d$
Exp	1.7148	25.83
NijmI	1.697/1.695	24.8/24.6
NijmII	1.700/1.695	24.7/24.5
Paris	1.696/1.694	25.6/25.2
CD-Bonn	1.708/1.704	24.8/24.4
Argonne18	1.696/1.694	24.7/24.4
JISP16	1.720/1.714	26.3/26.1
Moscow06	1.711/1.699	24.5/24.2
Moscow14	1.716/1.700	26.0/25.8
Idaho	1.714/1.700	26.22/25.98
eD	1.715/1.700	25.83/25.54

The momentum space wave functions can be easily obtained analytically by the Fourier transform. The boundary condition at $r=0$ leads to the constraints

$$C_n = -\sum_{j=1}^{n-1} C_j, \tag{37}$$

$$D_{n-2} = \frac{m_{n-2}^2}{(m_n^2 - m_{n-2}^2)(m_{n-1}^2 - m_{n-2}^2)} \times \left[-m_{n-1}^2 m_n^2 \sum_{j=1}^{n-3} \frac{D_j}{m_j^2} + (m_{n-1}^2 + m_n^2) \sum_{j=1}^{j=n-3} D_j - \sum_{j=1}^{n-3} D_j m_j^2 \right], \tag{38}$$

and two additional relations obtained by circular permutations of $n-2$, $n-1$, n. All parameters are available from the author upon a request.

Results of our calculations are presented in Figs. 1–5. We see a very good general agreement between the theory and experiment for $Q < 3$ Fm^{-1}. Discrepancies at larger Q values are comparable with discrepancies between results obtained with different interaction models (potentials). Some model calculations [69] demonstrate that the meson exchange currents may contribute significantly to EM processes in the np-system. The meson exchange currents are not accounted for in our calculations. It is not clear how to correlate these currents with the short-range behavior of NN interaction of the QCD origin. We have a number of deuteron models (Figs. 1 and 2) that obviously require different meson exchange currents. The nucleon EM FFs are not described by meson degrees of freedom at intermediate and high energies [70]; moreover, the neutron EM FFs cannot be measured directly. As discussed in the Introduction, all available data on the neutron EM FFs are model dependent. Any conclusion about the meson exchange current contribution looks unjustified without having solid data on the neutron EM FFs.

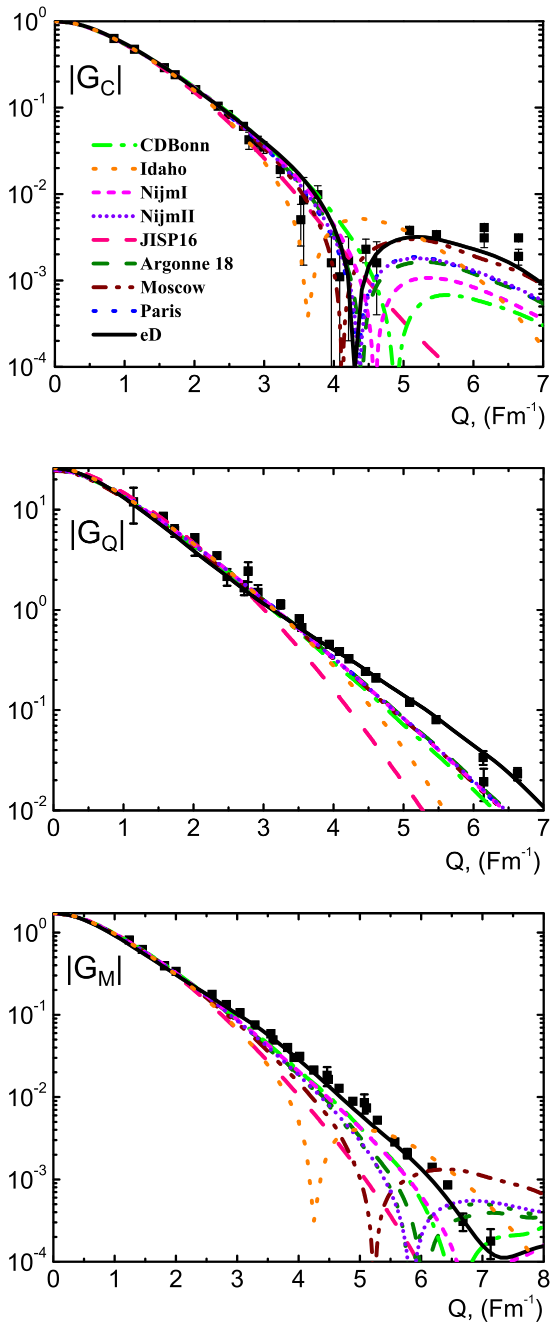

Figure 3: Deuteron form factors as a function of Q. Experimental data are the compilation of Ref. [2] where the results of experiments of Refs. [51–67] were analyzed.

Our calculations show that modest modifications of the deuteron wave function and nucleon FF parameterizations may simulate the effects of meson exchange currents in the eD elastic scattering. An analysis of Plachkov *et al.* [71] demonstrated that extracting the neutron electric FF is extremely model dependent (see Fig. 6). This analysis has been performed when the data on polarization experiments were

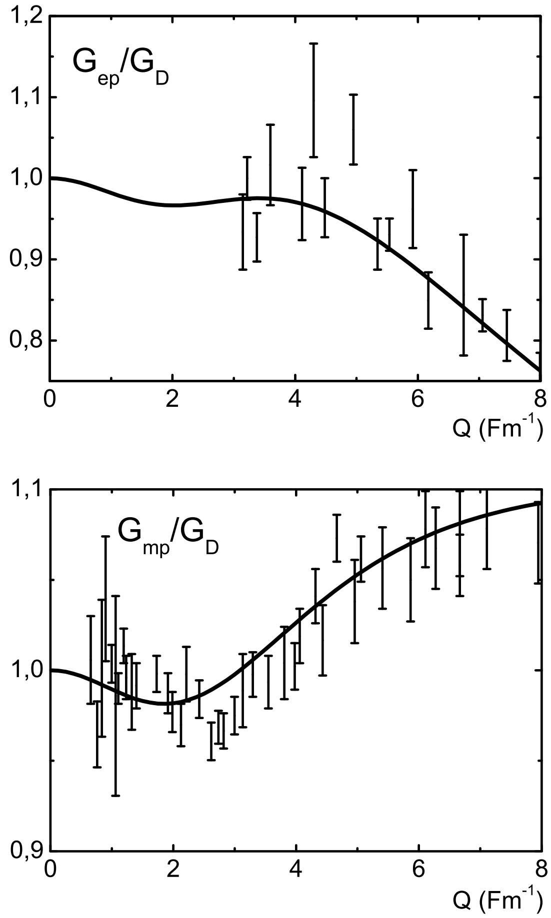

Figure 4: Proton EM FFs as a function of Q. Experimental data are the compilation of Ref. [68] of analyses of polarization experiments. Solid curves are proton FFs extracted from deuteron FFs by our fitting procedure.

scarce. Unfortunately, there is no a similar recent analysis of the neutron FFs based on the up-to-date experimental information. Such an analysis is complicated due to a large number of nuclear models and exchange currents used in modern literature. In addition, while extracting the neutron FFs, one faces the same difficulties as are inherent in the the proton FF extraction — their effect was never accurately estimated too. Therefore it is very possible that the error bars in the experimental data on the neutron FF presented in Fig. 5 are underestimated considerably, systematic errors of the NN interaction uncertainties are not properly accounted for. Our results for the nucleon FFs show that the extracted proton FFs are inside the experimental bars (Fig. 4), but the electric neutron FF (Fig. 5) may be 2–3 times larger than the results extracted using the so-called “Arenhövel’s model” [12]. In this case the magnetic neutron FF also changes (Fig. 5) but not drastically.

We plan to calculate the deuteron electrodisintegration to show that this reaction can be described in the “only NN channel” relativistic model, and our preliminary estimates are encouraging.

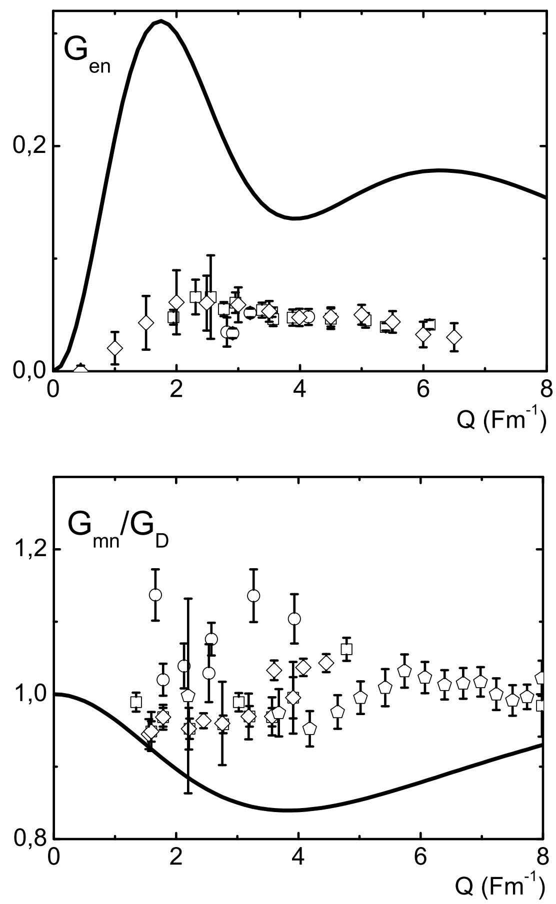

Figure 5: Neutron FFs as a function of Q. Experimental data are the compilation of Ref. [68] of model-dependent analyses of polarization experiments. Solid curves are neutron FFs extracted from deuteron FFs by our fitting procedure.

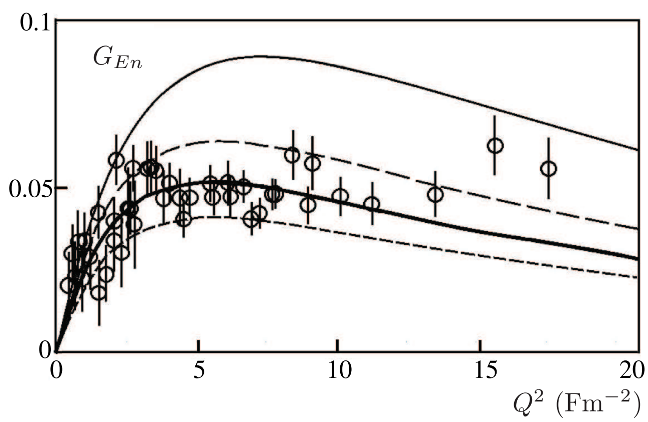

Figure 6: Data of Platchkov *et al.* [71] extracted from experiment using Paris potential. Curves are the fits to the same data extracted using Paris (thick solid), Reid soft core (short-dashed), Argonne V14 (long-dashed), and Nijmegen [72] (solid) potentials. The figure is grabbed from Ref. [71].

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