Three-Nucleon Reactions with Recently Derived Nuclear Potentials

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Abstract

Elastic nucleon-deuteron scattering is investigated at low and medium energies within the formalism of the Faddeev equations. We present various observables for this process obtained using the recently developed JISP16 nucleon-nucleon interaction and the chiral two-nucleon N^{4}LO force with the semi-local regularization. Comparison with data demonstrates, in general, good behavior of these two interactions at low energies but also reveals inadequacies of the JISP16 force for some observables. The origin of the observed problems lies in drawbacks of the P-wave interactions implemented in the JISP16 model.

Keywords: *Elastic nucleon-deuteron scattering, nucleon-nucleon force, few-body systems*

1 Introduction

The complex structure of nuclear interactions is one of the reasons why nuclear physics is still a significant intellectual challenge. Unfortunately, a derivation of nuclear system properties directly from Quantum Chromodynamics is still beyond the realms of possibility, despite the first ongoing attempts [1, 2]. This situation implies that the effective models of nuclear interactions are considered and used in practical *ab initio* calculations. Most of such models are semi-phenomenological and among the most advanced ones let us mention the Nijmegen [3, 4], the Argonne V18 (AV18) [5] and the Charge-Dependent Bonn (CD Bonn) [6, 7] forces. These potentials depend on several dozens of free parameters to be fixed from experimental data in the two-nucleon sector. These semi-phenomenological models describe experimental data for proton-proton (pp) and neutron-proton (np) scattering up to the two-nucleon energy of about 350 MeV very well, yielding $\chi^2/data'99 = 1.01$ in the case of the CD Bonn [8] and $\chi^2/data'99 = 1.35$ for the AV18 [8]

The nucleon-nucleon potential JISP16 [9] is one of the newest semi-phenomenological forces. This force is a successor of the J-matrix Inverse Scattering Potential [10] which in turn follows the Inverse Scattering Tridiagonal Potential (ISTP) developed within the inverse scattering methods in [11]. The free parameters of the JISP6 force

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have been fixed from the bound and resonance states of nuclei up to $A = 6$ [10]. Correspondingly, bound and resonance states of nuclei up to ^{16}O have been used to determine the JISP16 parameters [9]. The JISP forces also describe two-nucleon scattering data with a precision comparable to other modern potentials, reaching χ^2=1.03(1.05) for the JISP6 with the neutron-proton data'1992(1999). The main motivation behind developing the JISP16 model was a derivation of the two-body interaction which, at least partially, accommodates effects of many-body forces. This should result in a substantial improvement of the convergence of the nuclear structure calculations, especially ones performed within the No-Core Shell Model [12]. This aim has been achieved and indeed the JISP16 force works very well in investigations of bound and resonant states, as was documented for example in Refs. [13–15]. In this contribution we use the JISP16 interaction to study the elastic nucleon-deuteron scattering performing the first test of this force in few-body reactions.

The chiral approach to the nuclear forces has been developed simultaneously with the semi-phenomenological methods described above. The nuclear interaction is constructed in a framework of the effective field theory for nucleon and pion fields with incorporated chiral symmetry, see, e. g., Ref. [16] for a detailed review. Within this approach, it is possible to derive the nuclear interaction perturbatively by expanding the Lagrangian in powers ν of the parameter $\left(\frac{Q}{\Lambda_\chi}\right)$, where Q is the scale of typical values of nucleon momenta in the initial and final states, and $\Lambda_\chi \approx 1$ GeV is the scale of chiral symmetry breaking. The ν parameter can be related to geometrical properties (like the number of vertices, number of loops, etc.) of the graphs representing a given contribution to the potential. The resulting dominant contribution to the nucleon-nucleon interaction comes from the one-pion exchange force. On top of that also two-nucleon contact terms are present. They describe a short-ranged nucleon-nucleon interaction which in the semi-phenomenological models is represented by heavier meson (like σ, ρ or ω) exchanges.

The chiral potentials, starting from the smallest possible value of $\nu = 0$ [the leading order (LO)], $\nu = 2$ [the next-to-leading order (NLO)], have been completely constructed up to $\nu = 5$ (N^{4}LO). Moreover, some dominant contributions arising at N^{5}LO have been also derived [17]. The most advanced two- and consistent three-nucleon chiral forces have been derived by the Bochum/Jülich group [18–22]. The newest version of the two-body force presented in Refs. [21, 22] includes all terms of the chiral expansion up to N^{4}LO and benefits from an improved way in which values of the low-energy constants in the long-range part of the interaction are established. Namely, in Refs. [21, 22] the values of these constants are taken directly from the pion-nucleon scattering without additional fine tuning applied in the previous model. Secondly, an improved regularization method has been used. In the older model, matrix elements of the potential V, $\langle \vec{p}|V|\vec{p'}\rangle$, were multiplied by the exponential factor $\exp[-(p^6 + p'^6)/\Lambda^6]$, where $\vec{p'}$ and $\vec{p}$ are the relative momenta of nucleons in the initial and final states, respectively, $p' =\mid \vec{p'} \mid$, $p =\mid \vec{p} \mid$ and $\Lambda \approx 550$ MeV is the regularization parameter. Such a non-local regularization implemented in the same way for all partial waves, leads to unwelcome artifacts in the long-range part of the nucleon-nucleon force and does not completely eliminate unwanted short-range components of the two-pion exchange. The same non-local regularization has been also utilized for the three-nucleon ($3N$) force [23] affecting the description of observables in the $3N$ sector, see Refs. [24] and [25] for applications in the nucleon-deuteron elastic scattering and in the neutron induced deuteron breakup, respectively. The extensive tests of electroweak processes using the older chiral models, can be found in Refs. [26–28]. These works have revealed that the cut-off dependence of the nuclear forces employed there is too strong (especially at N^3LO) and precludes precise conclusions about the investigated processes.

Within the improved model of Refs. [21,22], the semi-local regularization has been applied. It means that the long-range component of the interaction in the coordinate space is multiplied by the function $f(r) = [1 - \exp(-r^2/R^2)]^6$ while the contact interactions are regularized using a non-local Gaussian regulator in the momentum space. The values of the cut-off parameter R are chosen in the 0.8–1.2 fm range, however they do not describe the two-nucleon phase shifts equally well — the best description (up to $E_{\rm lab} = 300$ MeV) is obtained for R of 0.9 and 1.0 fm. The first applications of this newest two-body chiral force to the studies of the elastic nucleon-deuteron scattering have been announced in Ref. [29] and investigations of various electroweak processes have been described in Ref. [30]. These first tests demonstrate a good quality of the chiral interaction, a weak regulator dependence, a fast chiral convergence and a good behavior at high energies. In this paper we present results which are based on the N^4LO chiral nucleon-nucleon force [21, 22] with semi-local regularization and choose the regulator $R = 0.9$ fm.

A transition from the two- to the three-nucleon system entails substantial complications of theoretical and numerical methods required for a precise analysis of scattering processes. Even the simplest three-nucleon reaction, the elastic nucleon-deuteron scattering, reveals the differences between various models of the nucleon-nucleon force. A review of numerous observables and their sensitivity to the interaction details can be found in Refs. [31] and [32]. In the case of the chiral forces an additional uncertainty of theoretical predictions stems from the regularization methods which employ unknown *a priori* regularization parameters. However, as demonstrated in Ref. [29], the model of Refs. [21, 22] shows at N^4LO only a weak dependence on the regulator values from the range suggested by the two-body phase shift analysis.

The elastic nucleon-deuteron scattering process has been also intensively investigated experimentally, see, e. g., Refs. [33–35] for recent reviews. The comparison of theoretical predictions for the elastic nucleon-deuteron scattering obtained within various theoretical approaches [31, 32, 36–38], shows that the three-nucleon force is important for this process at energies above approx 30 MeV. However, in this work, we restrict ourselves only to the nucleon-nucleon interactions and present just a single set of predictions obtained with the Urbana IX three-nucleon force combined with the AV18 nucleon-nucleon interaction to give the reader an idea about a magnitude of expected three-nucleon force effects. The lack of some contributions in the present-day models of the three-nucleon force is considered as a probable source of remaining discrepancies observed in the nucleon-deuteron scattering at low and medium energies.

The elastic nucleon-deuteron scattering process can also be used to study relativistic effects observed in the cross section at energies around 200 MeV. Inclusion of such relativistic features as the relativistic correction to the nucleon-nucleon force, the boost of the potential and Wigner spin rotations, leads to noticeable effects, especially at backward scattering angles [39]. However up to now, the existing models of the three-nucleon force even combined with the relativistic ingredients are not fully able to explain the data. A comparison with the proton-deuteron data also indicates that neglecting the Coulomb force in the theoretical analysis increases the observed discrepancies, especially at low energies. The differential cross section for the proton-deuteron elastic scattering at very forward angles at energies below approximately 20 MeV is a good example. The inclusion of the Coulomb force improves the description of the proton-deuteron data in that region of angles [40].

The paper is organized as follows: we briefly summarize our theoretical approach in Section 2 and present the results for the elastic nucleon-deuteron scattering in Section 3. We summarize in Section 4.

2 Formalism

Working in momentum space, in the nonrelativistic regime, and assuming only two-body interactions, we obtain observables for the elastic nucleon-deuteron scattering from an auxiliary state $T|\psi\rangle$ which fits a Faddeev-like equation [31]

$$T|\psi\rangle = tP|\psi\rangle + tPG_0T|\psi\rangle. \tag{1}$$

Allowing also for the three-body potential, leads to a more complicated Faddeev equation [32] with two additional terms involving the three-nucleon force:

$$T|\psi\rangle = tP|\psi\rangle + tPG_0T|\psi\rangle + (1+tG_0)V_4^{(1)}(1+P)|\psi\rangle + (1+tG_0)V_4^{(1)}(1+P)T|\psi\rangle. \tag{2}$$

In Eqs. (1) and (2), the initial state $|\psi\rangle$ is composed of a deuteron and a momentum eigenstate of the projectile nucleon, P is a permutation operator which takes into account the identity of the nucleons and G_0 is the free three-nucleon propagator. The $2N$ interaction V together with the two-nucleon free propagator $\tilde{G}_0$ enters Eqs. (1) and (2) through a solution of the Lippmann–Schwinger equation for the t-matrix:

$$t = V + V\tilde{G}_0 t. \tag{3}$$

In Eq. (2), the $V_4^{(1)}$ is a part of the three-nucleon force which is symmetric under the exchange of nucleons 2 and 3.

We solve Eqs. (1) and (2) in the partial wave scheme. We use the $|p, q, \alpha\rangle$ basis states with $p = |\vec{p}|$ and $q = |\vec{q}|$ being the magnitudes of the relative Jacobi momenta $\vec{p}$ and $\vec{q}$. Further, α represents the set of discrete quantum numbers for three-nucleon system in the jI-coupling:

$$\alpha = \big((l, s)j; (\lambda, \frac{1}{2})I; (j, I)JM_J; (t, \frac{1}{2})TM_T\big). \tag{4}$$

Here l, s, j and t denote the orbital angular momentum, total spin, total angular momentum and total isospin of the 2-3 subsystem. Further, λ and I are the orbital and total angular momenta of particle 1 with respect to the centre of mass of the 2-3 subsystem. Finally, J, M_J, T and M_T are the the total angular momentum of the $3N$ system, its projection on the quantization axis, the total $3N$ isospin and its projection, respectively.

Using the completeness relation for the $|p, q, \alpha\rangle$ states,

$$\sum_\alpha \int dp\, p^2 \int dq\, q^2\, |p, q, \alpha\rangle\langle p, q, \alpha| = 1, \tag{5}$$

Eq. (1) can be rewritten as

$$\langle p,q,\alpha|T|\psi\rangle = \sum_{\alpha'}\int dp'\,p'^2\int dq'\,q'^2\,\langle p,q,\alpha|t|p',q',\alpha'\rangle\langle p',q',\alpha'|P|\psi\rangle$$
$$+\sum_{\alpha'}\int dp'\,p'^2\int dq'\,q'^2\,\langle p,q,\alpha|t|p',q',\alpha'\rangle\langle p',q',\alpha'|PG_0T|\psi\rangle. \quad (6)$$

This form reveals that while solving Eq. (6), the two-nucleon force matrix elements present in the t-operator, clearly interfere which can significantly affect the observables. We solve Eq. (6) by generating its Neumann series and summing it up using the Padé method [31]. In the results presented here we use all partial waves with $j \leq 4$ and $J \leq \frac{25}{2}$. These values are sufficient to obtain fully converged solutions at the energies considered here. More details about our numerical performance can be found in Ref. [31]. Results presented in the next Section have been obtained using, in the case of all interaction models, only the neutron-proton force (including the neutron-neutron subsystem). The reason for this is that the JISP16 model assumes charge independence. This assumption has only a small influence on the magnitudes of the observables presented here.

3 Results

In the following we discuss results for various observables in the neutron-deuteron elastic scattering process at two laboratory energies of the incoming neutron: $E = 5$ MeV and $E = 65$ MeV. We present our predictions for the differential cross section $\frac{d\sigma}{d\Omega}$, neutron analyzing power $A_Y(N)$, deuteron vector analyzing power iT_{11} and deuteron tensor analyzing power T_{21} in Figs. 1–4. In all figures, the black solid, red dashed, blue

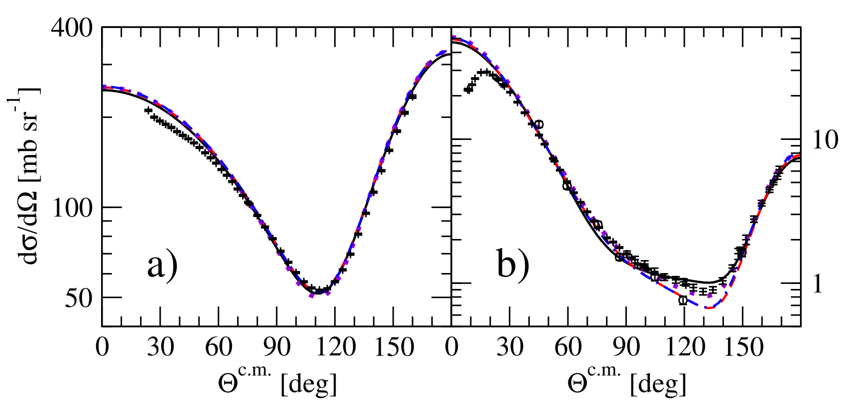

Figure 1: Differential cross section $d\sigma/d\Omega$ of elastic neutron-deuteron scattering at initial neutron laboratory energy $E = 5$ MeV (left) and $E = 65$ MeV (right). The black solid, red dashed, violet dotted and blue dash-dotted curves represent predictions based on the JISP16, AV18, AV18 + Urbana IX and chiral N^{4}LO (with regularization parameter R=0.9 fm) forces, respectively. The data at $E = 5$ MeV are from Ref. [41] and the data at $E = 65$ MeV are from Ref. [42] (pd crosses) and [43] (nd circles).

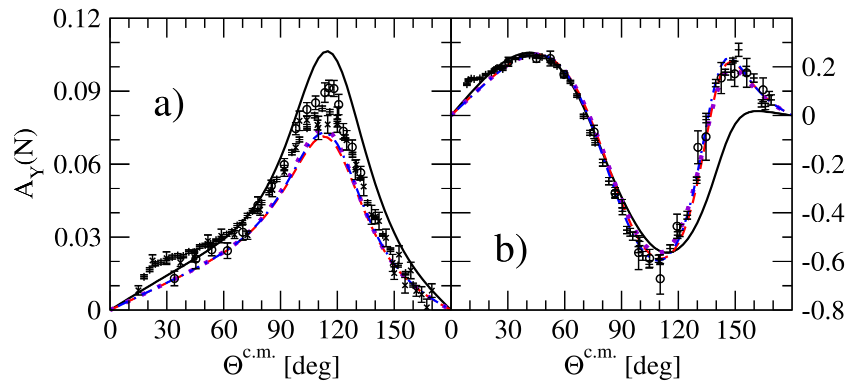

Figure 2: Nucleon analyzing power $A_Y(N)$ at the same energies as in Fig. 1. Curves are the same as in Fig. 1. Data at $E = 5$ MeV are from Ref. [41] (*pd* crosses), [44] (*nd* circles) and [45] (x-es). Data at $E = 65$ MeV are from Ref. [42] (*pd* crosses) and Ref. [43] (*nd* circles).

dash-dotted and violet dotted curves represent the predictions obtained with JISP16, AV18, chiral N^{4}LO with R=0.9 fm and AV18 + Urbana IX forces, respectively.

All these interaction models lead to very similar results for the differential cross section at $E = 5$ MeV delivering excellent data description, as is documented in Fig. 1. Note, the discrepancy between the predictions and the proton-deuteron data clearly visible at very forward scattering angles, originates from neglecting the Coulomb force in our theoretical calculations. A deviation among various predictions is seen at $E = 65$ MeV. While the chiral and the AV18 results are practically indistinguishable and underestimate the data, the JISP16 results are on the opposite side of the data. Only the AV18 + Urbana IX predictions correctly describe the data.

The polarization observables are more sensitive to the details of nuclear interactions. In the case of the neutron analyzing power $A_Y(N)$ shown in Fig. 2, the difference between the JISP16 predictions and those based on the AV18 or the chiral interactions appears already at $E = 5$ MeV. At the maximum of the $A_Y(N)$, the JISP16 overpredicts the experimental data while the AV18 and the chiral results are below the data. At small scattering angles all theoretical models underestimate the experimental results. At $E = 65$ MeV all models of nuclear forces give very similar results, in agreement with the data, for the scattering angles below approximately $\theta^{c.m.} = 110°$. At larger angles, the JISP16 predictions differ from the remaining ones suggesting a poor data description. For this observable, at both energies, the three-nucleon force effects are negligible, thus the AV18 + Urbana IX predictions practically overlap with those employing the AV18 nucleon-nucleon force alone.

A big difference between the JISP16 results and those based on the other models used here can be observed in the case of the deuteron vector analyzing power iT_{11} at $E = 5$ MeV (see Fig. 3). At the maximum of the iT_{11}, the JISP16 predictions are twice as big as the others. This picture changes when moving to $E = 65$ MeV where all predictions are much closer to each other although some difference between predictions based on the JISP16 model and other results remains, especially in the $90° < \theta^{c.m.} < 150°$ range. The three-nucleon force effects are small at both energies and the AV18 and chiral predictions follow the data at $E = 65$ MeV. The

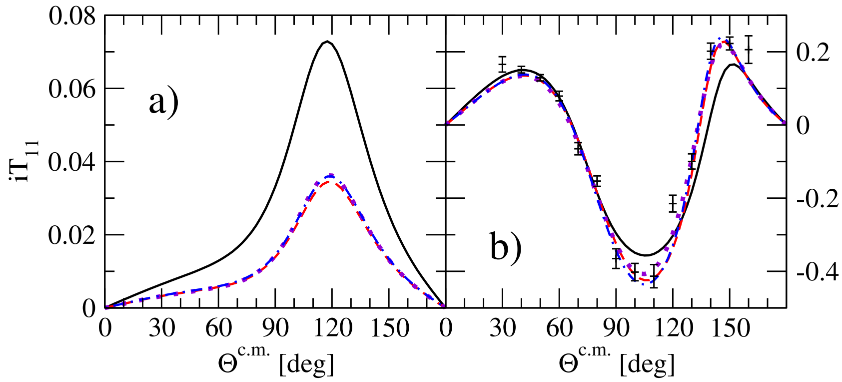

Figure 3: Deuteron analyzing power iT_{11} at the same energies as in Fig. 1. Curves are the same as in Fig. 1. Data at $E = 65$ MeV are from Ref. [46].

explanation of the puzzling behavior of the JISP16 potential at the lower energy has required a more detailed study and is discussed below.

Finally, in Fig. 4, T_{21} is shown as an example of the deuteron tensor analyzing powers. At both energies all interaction models predict qualitatively similar values of T_{21} and are in agreement with the data at $E = 65$ MeV. A closer look at Fig. 4b reveals that the JISP16 results are closer to the data at forward and at the very backward scattering angles while at medium angles the chiral N^{4}LO model delivers the best data description. The three-nucleon force effects are again small and the AV18 + Urbana IX predictions usually overlap with those for the N^{4}LO force.

The puzzling behavior of the JISP16 model in the case of the deuteron vector analyzing power iT_{11} has encouraged us to study this case in more detail. We present in Fig. 5 (in a restricted range of scattering angles) the results of calculations performed in such a way that for solving Eq. (6) the individual t-matrix elements for the JISP16 force in the two-nucleon subspace are replaced in given channels (defined by the l, s and j quantum numbers) by the same matrix elements taken from the chiral N^{4}LO interaction. Thus the mixed interaction is used: in all partial waves the JISP16 force

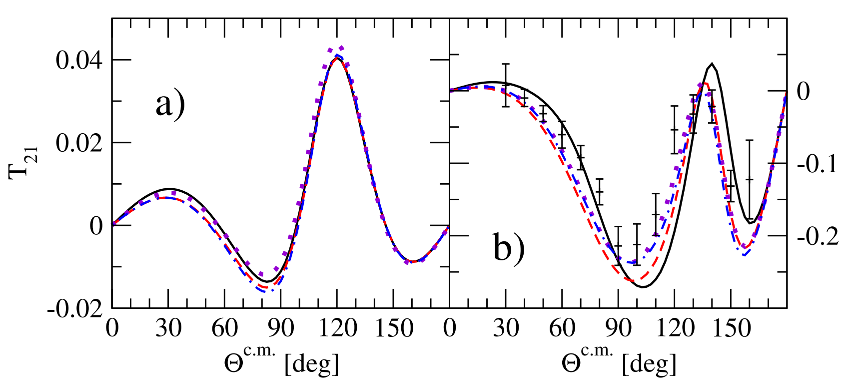

Figure 4: Deuteron tensor analyzing power T_{21} at the same energies as in Fig. 1. Curves are the same as in Fig. 1. Data at $E = 65$ MeV are from Ref. [46].

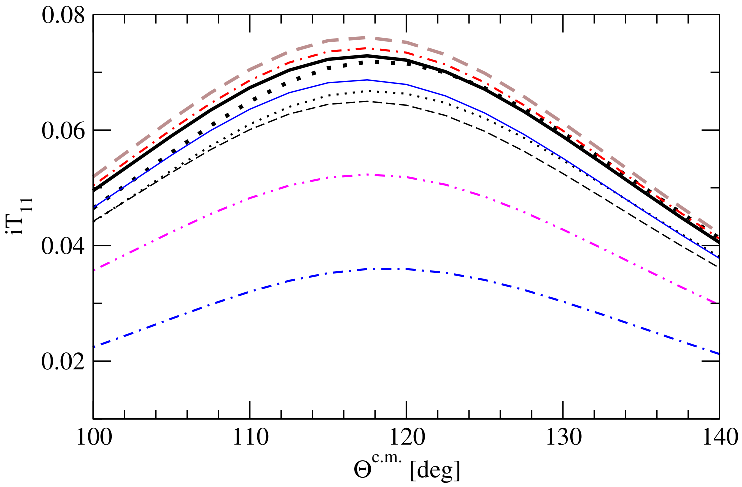

Figure 5: Deuteron vector analyzing power iT_{11} at $E = 5$ MeV. Predictions have been obtained with the JISP16 t-matrix with replacing its individual elements by the chiral N^{4}LO ones (see text for more details) in $^3P_0$ (thin black dashed curve), $^1S_0$ and $^3S_1$–$^3D_1$ (thick black dotted curve), $^1P_1$ (thin black dotted curve), $^3P_1$ (thick brown dashed curve), $^1D_2$ (red dash-dotted curve), $^3P_2$–$^3F_2$ (magenta dash-double-dotted curve) or $^3D_2$ (thin blue solid curve) partial wave. The thick black solid curve represents the JISP16 results and the thick blue dash-dotted curve shows the chiral N^{4}LO predictions.

is taken except for the one where it is replaced by the N^{4}LO force. The results given in Fig. 5 demonstrate that different partial waves contribute to the iT_{11} with different strengths. The biggest change is caused by replacing the $^3P_2$–$^3F_2$ t-matrix which reduces the difference between the JISP16 and the chiral N^{4}LO predictions by more than 50%. The $^3P_0$ and the $^1P_1$ channels are the next to produce the biggest changes in the iT_{11} values.

However, none of the exchanges of the individual partial wave in the t-matrix is able to explain completely the discrepancy between the JISP16 and the chiral N^{4}LO predictions. Thus, in Fig.6, we show what happens when not only a single individual partial wave is swapped but when a pair or more channels are replaced at the same time. We start from the simultaneous replacement of the $^3P_2$–$^3F_2$ and $^3P_0$ partial waves (the orange dashed curve). This reduces further the observed discrepancy by approximately 75%. A consecutive replacing of also the $^3P_1$ t-matrix does not introduce any visible shift but the interchanging in the $^1P_1$ partial wave on top of the $^3P_2 -^3 F_2$ and $^3P_0$ channels (the red dash-dotted curve) shifts the predictions in the proximity to the chiral results. Finally, exchanging all partial waves with $j \leq 2$ gives predictions overlapping with the pure N^{4}LO results. This shows trivially that at this energy the higher partial waves can be neglected, but it demonstrates also that the difference in the deuteron wave function supported by the JISP16 and N^{4}LO potentials, is unimportant in this case.

It is needed to check how the replacement of the P-waves influences predictions for other observables. In Fig. 7 we give an example of the nucleon analyzing power $A_Y(N)$ and the deuteron tensor analyzing power T_{21} at $E = 5$ MeV. In both cases we observe

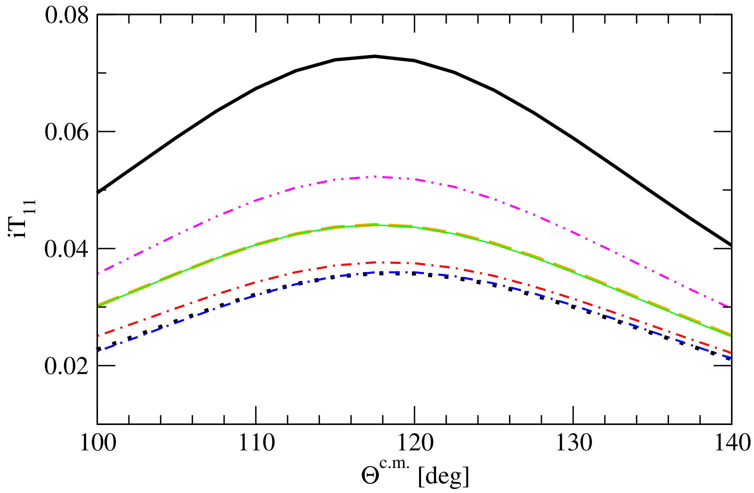

Figure 6: Deuteron vector analyzing power iT_{11} at $E = 5$ MeV. The thick black solid, thick blue dash-dotted and magenta dash-double-dotted curves are the same as in Fig. 5. Other curves present the predictions obtained with the JISP16 t-matrix with replacing some combinations of its matrix elements by the chiral N^{4}LO ones: in the $^3P_0$ and $^3P_2$–$^3F_2$ partial waves (orange dashed), in the $^3P_0$, $^3P_1$ and $^3P_2$–$^3F_2$ partial waves (thick green solid) and in the $^1P_1$, $^3P_0$ and $^3P_2$–$^3F_2$ partial waves (red dash-dotted). The thick dotted curve shows the results obtained with the JISP16 t-matrix replaced by the chiral N^{4}LO one in all partial waves with $j \leq 2$.

the anticipated behavior: when more partial waves are replaced, the predictions are shifted closer to the N^{4}LO ones. This is also true for the differential cross section (not shown here), however the changes are practically negligible for this observable.

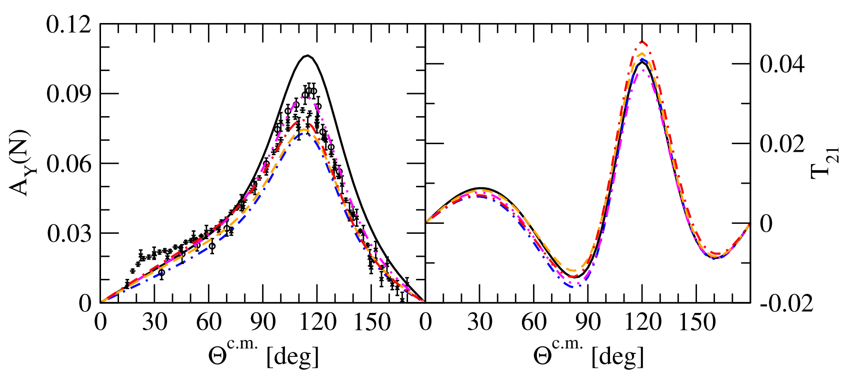

Figure 7: Nucleon analyzing power $A_Y(N)$ (left) and deuteron tensor analyzing power T_{21} (right) at $E = 5$ MeV. Curves are the same as in Fig. 6. Experimental data are the same as in Fig. 2.

4 Summary

In this contribution we present the first application of the JISP16 nucleon-nucleon interaction to the elastic nucleon-deuteron scattering process at incoming nucleon laboratory energies $E = 5$ MeV and $E = 65$ MeV. In addition to JISP16, the chiral N^4LO interaction with a semi-local regularization using the regulator value of $R = 0.9$ fm and the semi-phenomenological AV18 nucleon-nucleon force are used. We present also the results obtained with the AV18 two-body force supplemented by the three-nucleon Urbana IX interaction.

For most of the observables, the N^4LO predictions agree with those obtained using the AV18 interaction. The picture is more complicated for the JISP16 model. For some observables like the differential cross section or the deuteron tensor analyzing power T_{21}, the JISP16 results, in general, follow the data and predictions based on the remaining potentials. However, for other observables like the deuteron vector analyzing power iT_{11}, even at the lower energy, an essential discrepancy between the JISP16 results and predictions based on other applied interactions exists. The description of the three-nucleon scattering data obtained with the JISP16 model is not as good as the description of energy levels in nuclei observed for this force.

Comparing the AV18 + Urbana IX results with those based on the two-body JISP16 force only, we cannot conclude that the JISP16 results are closer to the predictions based on the two- and three-body potentials than the predictions obtained with other models of the nucleon-nucleon interaction.

We have found that the observed discrepancies originate from the off-shell behavior of the t-matrix operator for different P-waves derived from the JISP16 force. This in turn leads to the conclusion that in the future models of nuclear forces derived within the inverse scattering methods, the polarization observables should also be included into the set of observables used to fix parameters of the potential. Such an investigation is planned.

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