Algebraic Versions of Resonating Group and Orthogonality Condition Models as Fundamentals of Theoretical Approaches to the Description of Radiative Capture

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Abstract

Radiative capture reactions being an important class of nuclear fusion processes attract a significant interest, in particular, for nuclear astrophysics. Their cross sections at low energies are strongly suppressed by the Coulomb barrier and therefore are not available for reliable experimental measurements. As a result, there is a strong need in theoretical approaches to the studies of the radiative capture reactions cross sections.

In this work, the basic ideas of the algebraic versions of the resonating group and orthogonality condition models are presented. Microscopic approaches to the radiative capture reactions based on the algebraic version of resonating group model and semimicroscopic one combining the algebraic versions of resonating group and orthogonality condition models, are reviewed. An applicability of these approaches is demonstrated. Perspectives of their further applications are discussed.

Keywords: *Algebraic version of resonating group model; algebraic version of orthogonality condition model; microscopic approach; radiative capture reactions; low energies; cross section; astrophysical S-factor; nuclear astrophysics*

1 Introduction

Cross sections of a number of nuclear reactions at low sub-Coulomb energies are desired for numerous fundamental studies and advanced applications. For example, radiative capture cross sections at low energies are required for the studies of stellar processes, nucleosynthesis in the Universe, etc. [1–5]. These cross sections are strongly suppressed by the Coulomb barrier and therefore are not available for reliable experimental measurements. As a rule, cross section extrapolations to low energies also

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turn out to be unreliable. As a result, theoretical predictions based on microscopic approaches are expected to be the most justified and promising way to obtain the cross sections in the energy region inaccessible for experiment. These microscopic approaches should be able to describe a dynamics of all nucleons of a nuclear system considered with a complete account of the Pauli exclusion principle and a rigorous treatment of the center-of-mass motion. From the mathematical viewpoint, it means that the wave functions should be fully antisymmetrized and translationally invariant and have an explicit dependence on space and spin-isospin coordinates of all the nucleons of the system.

At the present moment, there is a number of approaches to the description of radiative capture reactions. In particular, there are various two-body approaches utilizing either the direct capture model [6, 7] or the potential cluster model [8–12]. There are also hybrid approaches which use either the variational Monte Carlo method [13, 14] or the no-core shell model [15–17] for bound states and the potential cluster model for continuum. Finally, there are fully microscopic approaches based either on the resonating group model (RGM) [18–21] or on the fermionic molecular dynamics [22], as well as on the no-core shell model with continuum [23].

In the present work, two microscopic approaches and a semimicroscopic approach to the radiative capture reactions [24–32] are briefly reviewed. One of these microscopic approaches [24–29] is based on the single-scale algebraic version of RGM (AVRGM) [33, 34] while the other [32] relies on the multiscale AVRGM. The semimicroscopic approach [30, 31] combines the single-scale AVRGM with the algebraic version of the orthogonality condition model (AVOCM) [35–39]. An applicability to the radiative capture processes and capabilities of these approaches have been demonstrated in the studies of mirror $^3\mathrm{H}(\alpha,\gamma)^7\mathrm{Li}$ and $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ reactions important for nuclear astrophysics. Both these reactions are responsible for the ^{7}Li production during the Big Bang nucleosynthesis. Their cross sections at low energies are necessary for calculating the ^{7}Li abundance required to resolve a number of problems concerning the Big Bang nucleosynthesis and to get a general understanding the primordial nucleosynthesis. Moreover, the latter reaction is a starting point of the second and the third chains of the *pp* cycle of hydrogen burning in stars. The cross section of this reaction at low energies is necessary for the studies of processes in the solar core and for the solar model verification.

2 Brief description of AVRGM and AVOCM. Generating functions method

In the framework of the single-channel RGM, the total wave function of a two-cluster nuclear system is expressed as an antisymmetrized product of internal wave functions $\phi^{(1)}$, $\phi^{(2)}$ of the clusters and the wave function f of their relative motion [40,41]:

$$\Psi = \hat{A}\{\phi^{(1)}\phi^{(2)}f\}. \tag{1}$$

The translationally-invariant oscillator shell-model wave functions of the lowest states compatible with the Pauli principle are conventionally adopted as the internal wave functions of the clusters. The relative motion wave function is unknown and should be found by solving the integro-differential equation of RGM.

The main idea of AVRGM is to expand the relative motion wave function in series of the oscillator basis functions:

$$f_{\nu lm}(\vec{q}) = N_{\nu l}\,\overline{q}^{\,l}\,L^{(l+1/2)}_{(\nu-l)/2}(\overline{q}^2)\,\exp(-\overline{q}^2/2)\,Y_{lm}(\vec{n}_{\vec{q}}),$$
$$\overline{q} = q/r_0, \quad N_{\nu l} = (-1)^{(\nu-l)/2}\sqrt{\frac{2\Gamma((\nu-l+2)/2)}{r_0^3\,\Gamma((\nu+l+3)/2)}}, \tag{2}$$

where $\vec{q}$ is the Jacobi vector characterizing the relative distance between the clusters; r_0 is the oscillator radius; ν is the oscillator quanta; l and m are the orbital momentum and its projection respectively; Γ is the gamma-function; $L_n^{(\beta)}$ is the generalized Laguerre polynomial; Y_{lm} is the spherical harmonic. As a result, the total wave function can be written as an expansion,

$$\Psi = \sum_{J^\pi M l s \nu} C_{J^\pi M l s \nu}\,\Psi_{J^\pi M l s \nu}, \tag{3}$$

over the so-called AVRGM basis,

$$\Psi_{J^\pi M l s \nu} = N_{J^\pi l s \nu}\,\hat{A}\left\{\sum_{m+\sigma=M} C^{JM}_{lm\,s\sigma}[\phi^{(1)}_{s_1}\phi^{(2)}_{s_2}]_{s\sigma}\,f_{\nu lm}(\vec{q})\right\}. \tag{4}$$

Here J and M are the total angular momentum and its projection respectively; π is the parity of the system; s_1 and s_2 are the cluster spins coupled to the channel spin s with projection σ; $C^{JM}_{lm\,s\sigma}$ is the Clebsch–Gordan coefficient; $N_{J^\pi l s \nu}$ is the normalization factor; $C_{J^\pi M l s \nu}$ are unknown expansion coefficients satisfying an infinite set of homogeneous linear algebraic equations [33, 42],

$$\begin{cases} \sum\limits_{s=|s_1-s_2|}^{s_1+s_2}\ \sum\limits_{l=|J-s|}^{J+s}\ \sum\limits_{\nu=\nu_0}^{\infty}\Big(\big\langle J^\pi M\tilde{l}\tilde{s}\tilde{\nu}\big|\hat{H}\big|J^\pi M l s\nu\big\rangle - E\,\delta_{\tilde{s}s}\,\delta_{\tilde{l}l}\,\delta_{\tilde{\nu}\nu}\Big)C_{J^\pi M l s\nu} = 0, \\ \tilde{s} = |s_1-s_2|,\dots,s_1+s_2, \qquad \tilde{l} = |J-\tilde{s}|,\dots,J+\tilde{s}, \qquad \tilde{\nu} = \nu_0,\,\nu_0+2,\dots \end{cases} \tag{5}$$

Here $\hat{H}$ and E are the Hamiltonian and the total energy of the system respectively, ν_0 is the minimal oscillator quanta compatible with the Pauli principle. It should be noted that the summations over ν in Eqs. (3) and (4) as well as in other expressions below are performed with a step of 2 since $\nu = 2n_r + l$, where $n_r = 0, 1, 2, \dots$ is the radial quantum number.

In the case of the discrete spectrum, we can use instead of Eq. (5) a reduced finite set of algebraic equations,

$$\begin{cases} \sum\limits_{s=|s_1-s_2|}^{s_1+s_2}\ \sum\limits_{l=|J-s|}^{J+s}\ \sum\limits_{\nu=\nu_0}^{\nu_{\max}}\Big(\big\langle J^\pi M\tilde{l}\tilde{s}\tilde{\nu}\big|\hat{H}\big|J^\pi M l s\nu\big\rangle - E\,\delta_{\tilde{s}s}\,\delta_{\tilde{l}l}\,\delta_{\tilde{\nu}\nu}\Big)C^{(\mathrm{D})}_{J^\pi M l s\nu} = 0, \\ \tilde{s} = |s_1-s_2|,\dots,s_1+s_2, \quad \tilde{l} = |J-\tilde{s}|,\dots,J+\tilde{s}, \quad \tilde{\nu} = \nu_0,\nu_0+2,\dots,\nu_{\max}, \end{cases} \tag{6}$$

where $\nu_{\max}$ should be sufficiently large depending on a desired accuracy. In the case of the continuum, the AVRGM equations (5) should be rewritten as a set of inhomogeneous linear algebraic equations,

$$\begin{cases} \sum\limits_{s}\sum\limits_{l}\ \sum\limits_{\nu=\nu_0}^{\nu_{\mathrm{as}}-2}\Big(\big\langle J^\pi M\tilde{l}\tilde{s}\tilde{\nu}\big|\hat{H}\big|J^\pi M l s\nu\big\rangle - E\,\delta_{\tilde{s}s}\,\delta_{\tilde{l}l}\,\delta_{\tilde{\nu}\nu}\Big)C^{(\mathrm{C})}_{J^\pi M l s\nu} = F_{J^\pi M\tilde{l}\tilde{s}\tilde{\nu}}\,, \\ \tilde{s} = |s_1-s_2|,\dots,s_1+s_2, \quad \tilde{l} = |J-\tilde{s}|,\dots,J+\tilde{s}, \quad \tilde{\nu} = \nu_0,\nu_0+2,\dots,\nu_{\mathrm{as}}\,. \end{cases} \tag{7}$$

The expansion coefficients $C^{(\mathrm{C})}_{J^\pi Mls\nu}$ are replaced starting from $\nu = \nu_{\mathrm{as}}$ by their asymptotic expressions $C^{(\mathrm{as})}_{J^\pi Mls\nu}$ [34, 43] entering the right-hand side (inhomogeneity) of Eq. (7):

$$F_{J^\pi M\tilde{l}\tilde{s}\tilde{\nu}} = -\sum_s \sum_l \sum_{\nu=\nu_{\mathrm{as}}}^{\nu'_{\max}} \big\langle J^\pi M\tilde{l}\tilde{s}\tilde{\nu}\big|\hat{H}\big|J^\pi Mls\nu\big\rangle C^{(\mathrm{as})}_{J^\pi Mls\nu}\,. \tag{8}$$

Although the AVRGM and the RGM are similar from the physical viewpoint, their numerical realizations differ essentially: the AVRGM requires to find solutions of linear algebraic equations while within the conventional single-channel RGM one has to solve a more complicated integro-differential equation. In the case of the multichannel RGM, one faces a problem of solving a set of integro-differential equations [40, 41].

One of the main problems of the AVRGM realization is a calculation of the Hamiltonian matrix elements between the antisymmetrized AVRGM basis functions (4). This problem can be resolved using an elegant technique of the generating functions method [33, 42, 44, 45]. The basic idea of this method is to utilize the generating function of the harmonic oscillator functions:

$$\begin{aligned} f_{\nu lm}(\vec{q}) &= A_{\nu l}\frac{\partial^\nu}{\partial R^\nu}\int \exp(-q^2/2r_0^2 + \vec{q}\vec{R}/r_0 - R^2/4)\, Y_{lm}(\vec{n}_{\vec{R}})\, d\vec{n}_{\vec{R}}\Big|_{R=0}, \\ A_{\nu l} &= \frac{2^{\nu-1/2}}{(\pi r_0)^{3/2}\,\nu!}\sqrt{\Gamma\big((\nu-l+2)/2\big)\,\Gamma\big((\nu+l+3)/2\big)}. \end{aligned} \tag{9}$$

With the help of Eq. (9) one can easily derive the generating functions for the AVRGM basis. The calculations are essentially simplified by constructing Slater determinants of the generating functions for the initial and final states,

$$\begin{aligned} \big|\vec{R}\big\rangle &= \frac{1}{\sqrt{A!}}\sum_{\{j_1,\,j_2,\ldots,\,j_A\}} (-1)^{P(\{j_1,\,j_2,\ldots,\,j_A\})}\,\varphi_{j_1}(1)\,\varphi_{j_2}(2)\ldots\varphi_{j_A}(A), \\ \big|\vec{Q}\big\rangle &= \frac{1}{\sqrt{A!}}\sum_{\{j_1,\,j_2,\ldots,\,j_A\}} (-1)^{P(\{j_1,\,j_2,\ldots,\,j_A\})}\,\phi_{j_1}(1)\,\phi_{j_2}(2)\ldots\phi_{j_A}(A), \end{aligned} \tag{10}$$

or sums of these Slater determinants. In Eq. (10), $P(\{j_1, j_2, \ldots, j_A\})$ is the parity of the permutation $\{j_1, j_2, \ldots, j_A\}$ of indices $\{1, 2, \ldots, A\}$. Moreover, expressing matrix elements $\big\langle J^{\pi_f}_f M_f l_f s_f \nu_f\big|V\big|J^{\pi_i}_i M_i l_i s_i \nu_i\big\rangle$ of some operator V in the AVRGM basis through its generating matrix elements $\big\langle \vec{Q}, s_f\,\sigma_f\big|V\big|\vec{R}, s_i\,\sigma_i\big\rangle$,

$$\big\langle J^{\pi_f}_f M_f l_f s_f \nu_f\big|V\big|J^{\pi_i}_i M_i l_i s_i \nu_i\big\rangle = \frac{1}{\kappa_{\nu_f l_f s_f}\,\kappa_{\nu_i l_i s_i}\,\nu_f!\,\nu_i!}\,\frac{\partial^{\nu_f}}{\partial Q^{\nu_f}}\,\frac{\partial^{\nu_i}}{\partial R^{\nu_i}}\,I_{i\to f}(Q,R)\Big|_{Q=R=0}, \tag{11a}$$

$$\begin{aligned} I_{i\to f}(Q,R) = &\sum_{m_f\sigma_f m_i\sigma_i} C^{J_f M_f}_{l_f m_f\; s_f\sigma_f}\, C^{J_i M_i}_{l_i m_i\; s_i\sigma_i} \\ &\times \iint Y^*_{l_f m_f}(\vec{n}_{\vec{Q}})\,\big\langle \vec{Q}, s_f\sigma_f\big|V\big|\vec{R}, s_i\sigma_i\big\rangle\, Y_{l_i m_i}(\vec{n}_{\vec{R}})\, d\vec{n}_{\vec{Q}}\, d\vec{n}_{\vec{R}}, \end{aligned} \tag{11b}$$

$$\kappa^2_{\nu ls} = \frac{1}{(\nu!)^2}\frac{\partial^\nu}{\partial Q^\nu}\frac{\partial^\nu}{\partial R^\nu}\iint Y^*_{lm}(\vec{n}_{\vec{Q}})\,\langle \vec{Q}, s\sigma | \vec{R}, s\sigma\rangle\, Y_{lm}(\vec{n}_{\vec{R}})\, d\vec{n}_{\vec{Q}}\, d\vec{n}_{\vec{R}}\Big|_{Q=R=0}, \quad (11c)$$

one can additionally simplify the calculations by making use of the so-called recurrence technique [42] suitable for numerical realization.

If the single-particle states φ_j and ϕ_k entering the Slater determinants in Eq. (10) satisfy the orthogonality condition

$$\langle \phi_k | \varphi_j \rangle \sim \delta_{kj}, \quad (12)$$

matrix elements in the basis of Slater determinants of an operator V which is a sum of two-particle operators V_{kj}, $V = \sum\limits_{k>j}^{A} V_{kj}$, can be written as

$$\langle \vec{Q}|V|\vec{R}\rangle = \sum_{k>j}^{A}\Big(\langle \phi_k(1)|\langle \phi_j(2)|V_{12}|\varphi_j(2)\rangle|\varphi_k(1)\rangle - \langle \phi_k(1)|\langle \phi_j(2)|V_{12}|\varphi_k(2)\rangle|\varphi_j(1)\rangle\Big)\prod_{n\neq k,j}\langle \phi_n|\varphi_n\rangle. \quad (13)$$

In this expression, the terms of the type

$$\langle \phi_k(1)|\langle \phi_j(2)|V_{12}|\varphi_j(2)\rangle|\varphi_k(1)\rangle\prod_{n\neq k,j}\langle \phi_n|\varphi_n\rangle \quad (14)$$

are referred to as direct ones while the terms of the type

$$\langle \phi_k(1)|\langle \phi_j(2)|V_{12}|\varphi_k(2)\rangle|\varphi_j(1)\rangle\prod_{n\neq k,j}\langle \phi_n|\varphi_n\rangle \quad (15)$$

are referred to as exchange ones. If the operator V is an operator describing the interaction in the system, the terms (15) with the indices k and j corresponding to single-nucleon states belonging to different clusters are responsible for the exchange effects in the cluster-cluster interaction. The neglect of these exchange terms in the interaction matrix elements simplifies significantly the calculations and leads to the AVOCM [35–39].

It should be emphasized that the potential cluster model widely used in literature neglects all exchange terms in the matrix elements of the Hamiltonian and all other operators describing the reactions. From this point of view, the AVOCM is a better approximation since within this model the exchange terms associated with permutations of indexes related to nucleons belonging to different clusters are only neglected in the matrix elements of the interaction operator. Matrix elements of the kinetic energy and electromagnetic operators as well as the overlaps are calculated precisely.

3 Approaches to description of radiative capture. The ^{3}H$(\alpha,\gamma)^7$Li and ^{3}He$(\alpha,\gamma)^7$Be reactions

At low energies, the mirror $^3\mathrm{H}(\alpha,\gamma)^7\mathrm{Li}$ and $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ reactions proceed mainly via the electric dipole ($E1$) transitions with formation of ^{7}Li and ^{7}Be nuclei in their

ground and first excited states. The respective cross sections are denoted σ_0 and σ_1 and the total cross section is their sum, $\sigma = \sigma_0 + \sigma_1$. Similarly, σ_0 and σ_1 are the sums of respectively three and two partial cross sections. An expression for these partial cross sections was derived [25,27,29] using the Fermi golden rule, the long-wavelength limit [46], and an expansion of the initial and final state wave functions in the series of the AVRGM basis functions (4):

$$\sigma_{i\to f}(E_{\text{c.m.}}) = \frac{8\pi}{9\hbar(2l_i+1)}\left(\frac{E_\gamma}{\hbar c}\right)^3 \times \left|\sum_{\nu_i,\nu_f} C^{(\text{D})}_{J_f^{\pi_f} l_f s\nu_f} \big\langle J_f^{\pi_f} l_f s\nu_f \big|\big| M_1^{\text{E}} \big|\big| J_i^{\pi_i} l_i s\nu_i \big\rangle C^{(\text{C})}_{J_i^{\pi_i} l_i s\nu_i}\right|^2. \quad (16)$$

Here $E_{\text{c.m.}}$ is the relative motion energy of the colliding clusters (nuclei) in the center-of-mass system, E_γ is the energy of the emitted photon, and M_1^{E} is the electric dipole operator. For the considered $E1$ transitions, a pair of the initial quantum numbers (J_i, l_i) in Eq. (16) can take the values of $(1/2, 0)$, $(3/2, 2)$ and $(5/2, 2)$ for the capture to the ground state $[(J_f, l_f) = (3/2, 1)]$, and the values of $(1/2, 0)$ and $(3/2, 2)$ for the capture to the first excited state $[(J_f, l_f) = (1/2, 1)]$.

We use three approaches in the present work to calculate the discrete $C^{(\text{D})}_{J_f^{\pi_f} l_f s\nu_f}$ and continuous $C^{(\text{C})}_{J_i^{\pi_i} l_i s\nu_i}$ spectrum wave function expansion coefficients in the AVRGM basis and hence the partial cross sections of the considered reactions. The first approach [24–29] hereafter referred to as a conventional AVRGM, is based on the single-scale AVRGM. In the framework of this microscopic approach, we utilize an unified AVRGM basis with a single oscillator radius playing a role of scale parameter. The second approach [30, 31] combining the single-scale AVRGM and AVOCM, is hereafter referred to as a combined AVRGM + AVOCM. The AVOCM is utilized in this semimicroscopic approach to simplify the calculation of the expansion coefficients for the continuum wave functions, all the rest calculations are performed using the single-scale AVRGM. The third approach [32] is based on the multiscale AVRGM. This is a more advanced approach utilizing the AVRGM bases with different oscillator radii to expand the discrete and continuous spectrum wave functions. It is the principle feature of this microscopic approach which we refer to as a multiscale or generalized AVRGM.

The radiative capture cross section drops down exponentially with the energy decrease at low sub-Coulomb energies. As a result, this cross section is conventionally expressed through the astrophysical S-factor $S(E_{\text{c.m.}})$,

$$\sigma(E_{\text{c.m.}}) = \frac{1}{E_{\text{c.m.}}} \exp\big(-\sqrt{E_{\text{G}}/E_{\text{c.m.}}}\,\big)\, S(E_{\text{c.m.}}), \quad (17)$$

where E_{G} is the Gamow energy. The astrophysical S-factor has a smoother behavior than the cross section and therefore is more suitable for analysis at low energies.

The astrophysical S-factors for the considered reactions calculated within the conventional AVRGM are presented in Figs. 1 and 2 by solid curves. The dashed curves in these figures are the results obtained within the combined AVRGM + AVOCM. All calculations are performed with the oscillator radius $r_0 = 1.22$ fm and the adjustable intensity of the central Majorana force $g_{\text{c}} = 1.035$ which is a parameter of

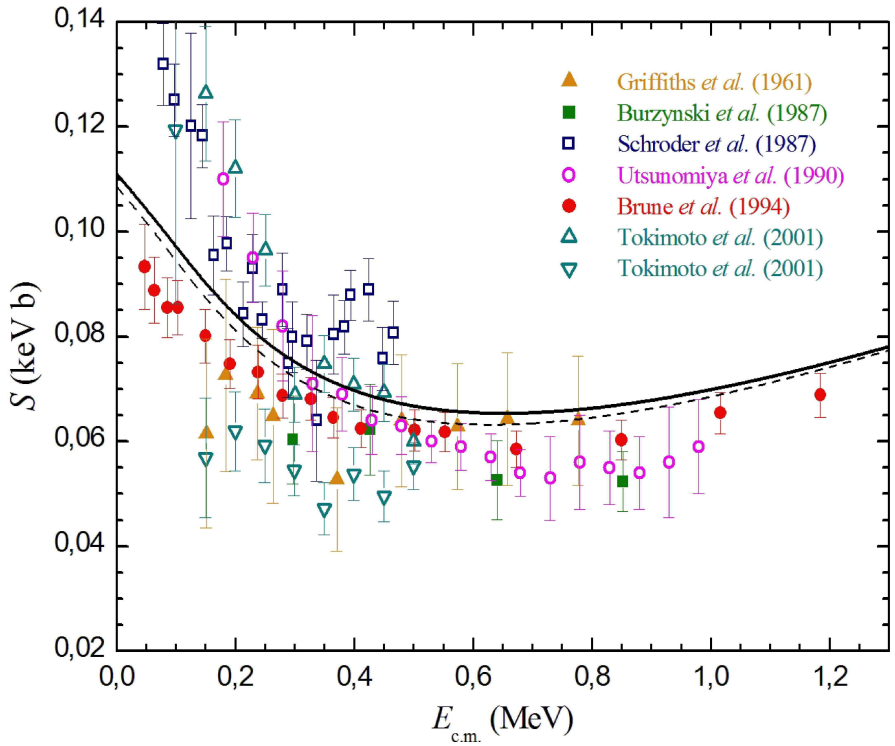

Figure 1: Astrophysical S-factor for the $^3\mathrm{H}(\alpha,\gamma)^7\mathrm{Li}$ reaction. Solid curve — conventional AVRGM; dashed curve — combined AVRGM + AVOCM; symbols — experimental data from Refs. [48–53].

the effective modified Hasegawa–Nagata NN potential [47] used to describe the internucleon interaction. The obtained cross sections are seen to be in an agreement with experimental data of Refs. [48–53] and [54–64] for the $^3\mathrm{H}(\alpha,\gamma)^7\mathrm{Li}$ and $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ reactions respectively. The results for the phase shifts in the entrance channels of these reactions also agree with experimental findings of Refs. [65–71].

It should be noted that the conventional AVRGM generates observables that differ only slightly from those obtained in the framework of the combined AVRGM+AVOCM (see Figs. 1 and 2). Therefore the exchange effects do not affect essentially these reactions in the low energy region. This fact can be used to develop approximate approaches to the description of reactions in heavier systems where the neglect of the exchange terms becomes necessary due to an avalanche-like growth of the calculation complexity with the number of nucleons.

We present in Figs. 3 and 4 the astrophysical S-factors for the $^3\mathrm{H}(\alpha,\gamma)^7\mathrm{Li}$ and $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ reactions calculated within the generalized AVRGM. The results are seen to agree well with the experimental data of Refs. [48–64]. The continuum wave functions for the $^4\mathrm{He}$ + $^3\mathrm{H}$ and $^4\mathrm{He}$ + $^3\mathrm{He}$ systems are expanded over the AVRGM basis with the oscillator radius $r_{01} = 1.386$ fm which results in the α particle binding energy of $E_\alpha = 28.296$ MeV consistent with experiment [72]. The wave functions of the ground and first excited states of $^7\mathrm{Li}$ are expanded over the AVRGM bases with the oscillator radii $r_{020} = 1.303$ fm and $r_{021} = 1.282$ fm respectively; in the case

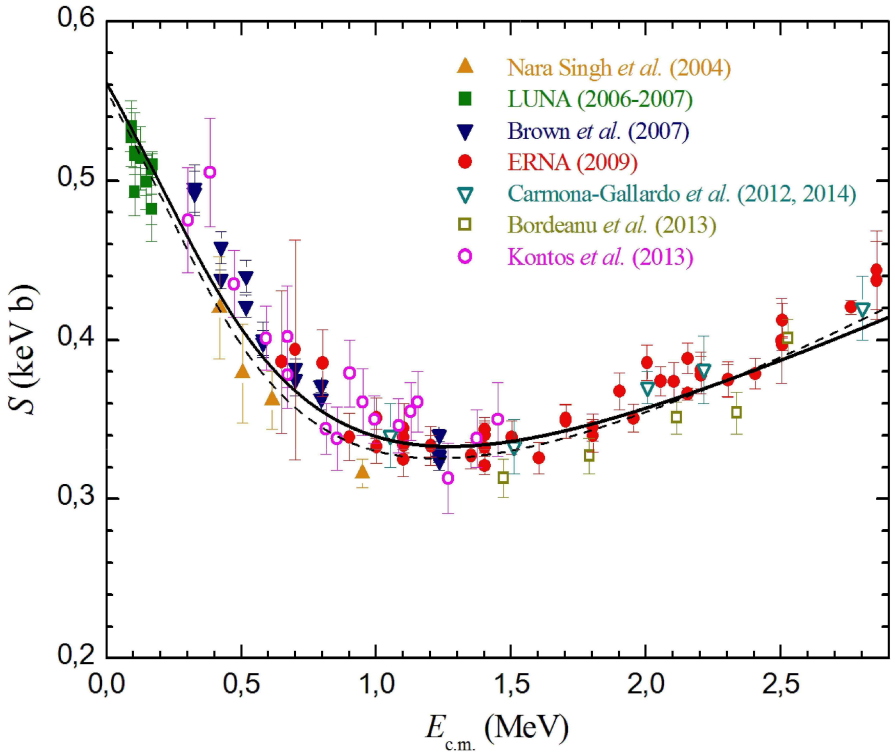

Figure 2: Astrophysical S-factor for the ^{3}He$(\alpha,\gamma)^7$Be reaction. Symbols — experimental data from Refs. [54–64]; see Fig. 1 for other details.

of ^{7}Be the respective oscillator radii are $r_{020} = 1.3068$ fm and $r_{021} = 1.4205$ fm — these values are tuned to reproduce the experimental breakup thresholds [73] of the ground ($\varepsilon_0 = 2.467$ MeV) and the first excited ($\varepsilon_1 = 1.989$ MeV) states of ^{7}Li with respect to the ^{4}He + ^{3}H channel and the ^{7}Be thresholds ($\varepsilon_0 = 1.586$ MeV, $\varepsilon_1 = 1.157$ MeV) with respect to the ^{4}He + ^{3}He cnannel. The intensity of the central Majorana force $g_c = 0.977$ is set to describe the ^{4}He + ^{3}H and ^{4}He + ^{3}He phase shifts extracted from the experiments in Refs. [65–71].

Thus the generalized AVRGM provides reasonable energy dependences of the astrophysical S-factors of the ^{3}H$(\alpha,\gamma)^7$Li and ^{3}He$(\alpha,\gamma)^7$Be reactions as well as of the scattering phase shifts in the entrance channels of these reactions together with the description of the α-particle binding energy and of the breakup thresholds in ^{7}Li and ^{7}Be nuclei. This is a significant advantage of the generalized AVRGM as compared to the conventional AVRGM and combined AVRGM + AVOCM which underestimate [25, 27–29] the α-particle binding energy and the ^{7}Li and ^{7}Be breakup thresholds.

4 Conclusions

The main points of the present work are the following:

1. Theoretical approaches to the description of radiative capture reactions based

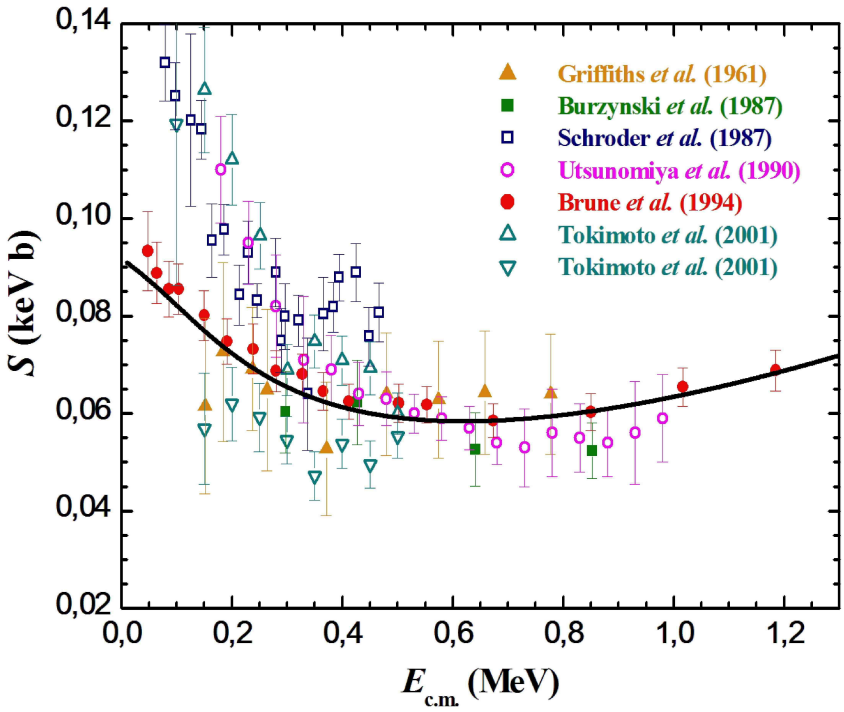

Figure 3: Astrophysical S-factor for the $^3\mathrm{H}(\alpha,\gamma)^7\mathrm{Li}$ reaction. Solid curve — generalized AVRGM.

on AVRGM and AVOCM have been reviewed.

2. The results of the mirror $^3\mathrm{H}(\alpha,\gamma)^7\mathrm{Li}$ and $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ reaction studies within these approaches have been discussed.

3. Abilities of these approaches to describe simultaneously the astrophysical S-factors and scattering phase shifts in the entrance channels of the radiative capture reactions have been demonstrated.

4. We revealed an advantage of the generalized AVRGM over other reviewed approaches that is a capability of an unified description of the astrophysical S-factors of the mirror $^3\mathrm{H}(\alpha,\gamma)^7\mathrm{Li}$ and $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ reactions and of the scattering phase shifts in the $^4\mathrm{He}$ + $^3\mathrm{H}$ and $^4\mathrm{He}$ + $^3\mathrm{He}$ systems along with the breakup thresholds in the $^7\mathrm{Li}$ and $^7\mathrm{Be}$ nuclei.

5. An insignificance of the exchange terms in the matrix elements of interaction operator in the entrance channels of the reactions at the considered energies has been shown. This feature is useful for realization of approximate approaches to the description of reactions with heavier nuclei.

Acknowledgments

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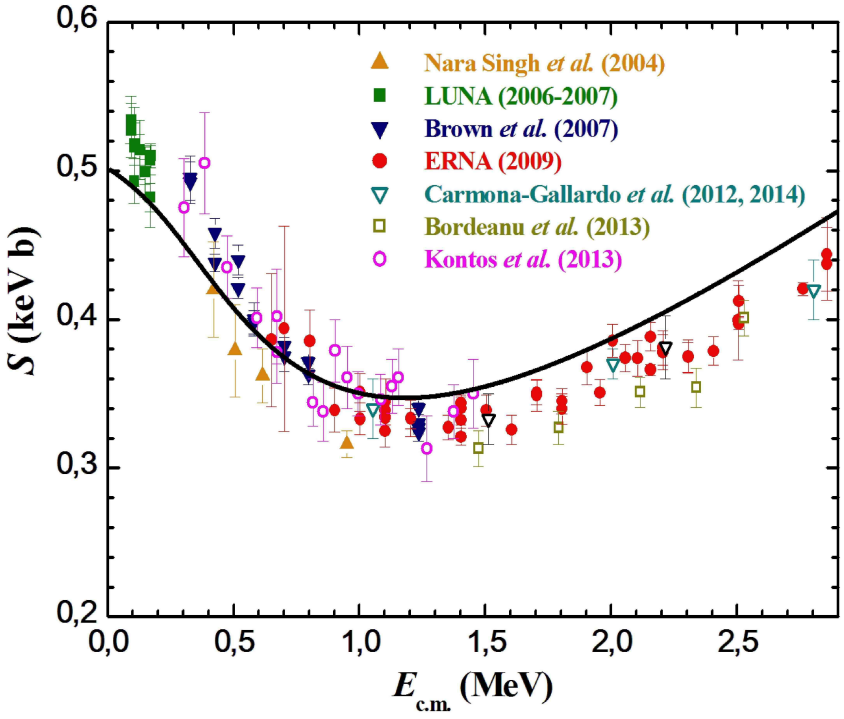

Figure 4: Astrophysical S-factor for the ^{3}He$(\alpha,\gamma)^7$Be reaction. Solid curve — generalized AVRGM.

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