Description of Continuum States within No-Core Shell Model. Single-State HORSE Method

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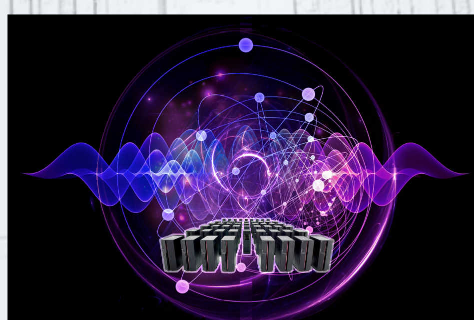

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Motivation

- There are many ab initio approaches for describing bound states of light nuclei (for example, No Core Shell Model). In these approaches most states are in the continuum.
- Can we extract information about resonances and nonresonant scattering from shell model results?

Motivation

- There are many ab initio approaches for describing bound states of light nuclei (for example, NCSM).
- Can we extract information about resonances and nonresonant scattering from shell model results?
- We suggest an approach for describing scattering states based on shell model calculations of nuclei: NCSM Single-State HORSE method.
- We suggest an interpretation of shell model results lying above decaying thresholds

NCSM Single-State HORSE method:

No-Core Shell Model

+

Harmonic Oscillator Representation of Scattering Equation (Single-State version)

+

Low-energy phase shift parametrization

NCSM Single-State HORSE method:

No-Core Shell Model

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Harmonic Oscillator Representation of Scattering Equation (Single-State version)

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Low-energy phase shift parametrization

Application:

The resonance and non-resonance **Nα** scattering within NCSM Single-State HORSE method with realistic NN-interactions JISP16 and Daejeon16

No-Core Shell Model

- ✓ *Modern version of Nuclear Shell Model*

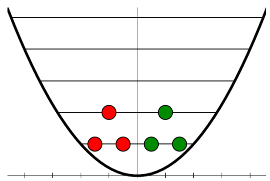
B. R. Barrett, P. Navratil, J. P. Vary, **Progr. Part. Nucl. Phys., 69, 131 (2013).**

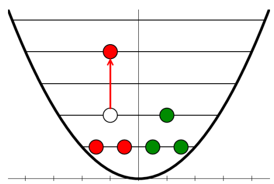
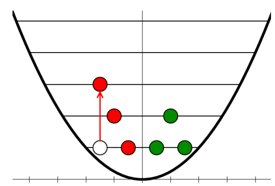
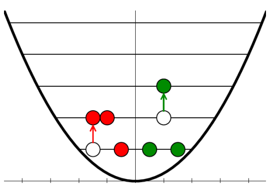
No-Core Shell Model

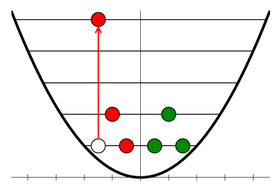
✓ *Modern version of Nuclear Shell Model*

✓ *All nucleons are spectroscopically active*

^{6}Li :

$0\hbar\Omega$ 

$+\ 2\hbar\Omega$  +  +  + ⋯

$+\ 4\hbar\Omega$ ⋯ +  + ⋯

$+\ \cdots\ +\ N_{max}\hbar\Omega$

N_{max} – *maximum number of excitation quanta*

$\hbar\Omega$ – *parameter of oscillator function*

No-Core Shell Model

- ✓ *Modern version of Nuclear Shell Model*
- ✓ *All nucleons are spectroscopically active*
- ✓ *Basis: many-body oscillator functions*

Example: ^{14}F, model space $N_{max} = 8$
contains **1 990 061 078** basis functions

No-Core Shell Model

- ✓ *Modern version of Nuclear Shell Model*
- ✓ *All nucleons are spectroscopically active*
- ✓ *Basis: many-body oscillator functions*
- ✓ *Only NN (+ NNN) interaction as input*

Below we use realistic NN interactions:

JISP16 A. M. Shirokov, J. P. Vary, A. I. Mazur and T. A. Weber, **Phys. Lett. B 644, 33 (2007)**.

a **Fortran code** generating the JISP16 matrix elements is available at **http://lib.dr.iastate.edu/energy_datasets/2/.**

Daejeon16 A. M. Shirokov, I. J. Shin, Y. Kim, M. Sosonkina, P. Maris and J. P. Vary, **Phys. Lett. B 761, 87 (2016).**

a **Fortran code** generating the Daejeon16 matrix elements is available at **http://lib.dr.iastate.edu/energy_datasets/1/**

HORSE

Harmonic Oscillator Representation of Scattering Equations

is the effective method to describe two-body scattering

H=T+V

$$H^l u_l(E,r) = E u_l(E,r)$$

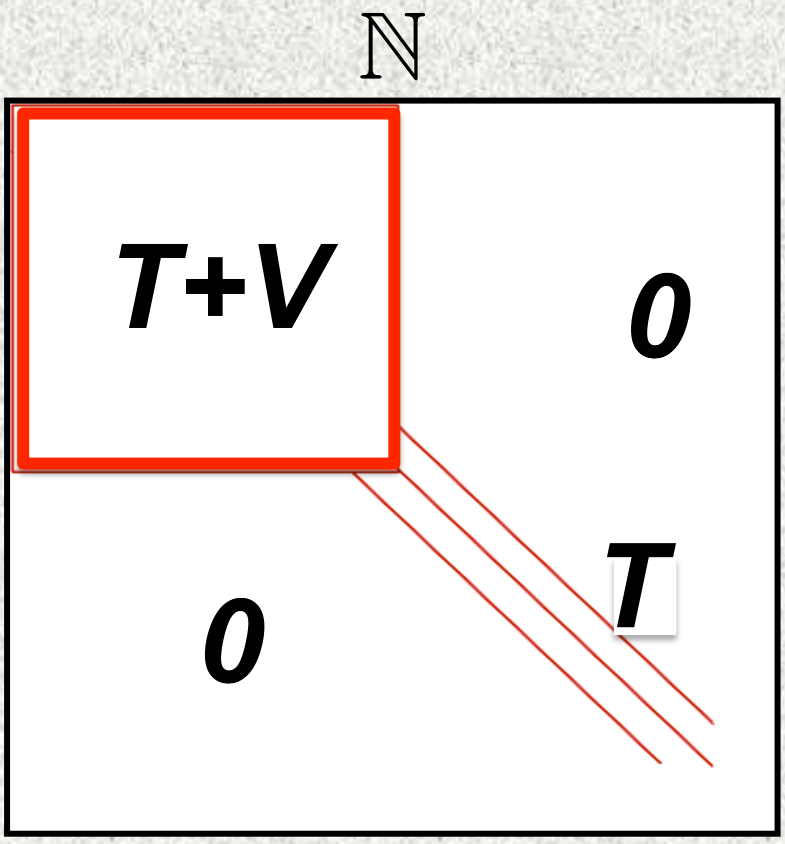
$$u_l(E,r) = \sum_{N=N_0}^{\infty} a_{Nl}(E) R_{Nl}(r)$$

$$\sum_{N'=N_0}^{\infty} \left(H^l_{N,N'} - \delta_{N,N'}\right) a_{N'l}(E) = 0$$

$$R_{Nl}(r) - \text{Osc. functions}$$

$$N = 2n + l$$

HORSE



Parameters $\mathbb{N}, \ \hbar\Omega$

- ✓ *Potential matrix elements $V_{NN'}$ decrease when N,N' grows while kinetic energy matrix elements T_{NN}, T_{NN+2} increase (~ N).*
  So potential matrix V may be truncated.

- ✓ *Tridiagonal kinetic matrix T is not truncated*

This is an
exactly solvable problem !

HORSE

Phase shift:

$$\tan\delta_\ell(E) = -\frac{S_{\mathbb{N}\ell}(E) - G_{\mathbb{N}\mathbb{N}}(E)T^{\ell}_{\mathbb{N},\mathbb{N}+2}S_{\mathbb{N}+2,\ell}(E)}{C_{\mathbb{N}\ell}(E) - G_{\mathbb{N}\mathbb{N}}(E)T^{\ell}_{\mathbb{N},\mathbb{N}+2}C_{\mathbb{N}+2,\ell}(E)}$$

$$G_{NN'}(E) = -\sum_{\nu=0}^{\mathcal{N}-1}\frac{\langle\nu|N\ell\rangle\,\langle N'\ell|\nu\rangle}{E_\nu - E}$$

All information about interaction

$$\sum_{N'=N_0,N_0+2,\ldots,\mathbb{N}} H^{\ell}_{NN'}\langle N'\ell|\nu\rangle = E_\nu\langle N\ell|\nu\rangle$$

Oscillator free solutions

$$S_{N\ell}(E) = \sqrt{\frac{\pi n!}{\Gamma(n+\ell+\frac{3}{2})}}\left(\frac{2E}{\hbar\Omega}\right)^{\frac{\ell+1}{2}}\exp\left(-\frac{E}{\hbar\Omega}\right)L_n^{\ell+\frac{1}{2}}\left(\frac{2E}{\hbar\Omega}\right)$$

$$C_{N\ell}(E) = \sqrt{\frac{\pi n!}{\Gamma(n+\ell+\frac{3}{2})}}\frac{(-1)^n}{\Gamma(-\ell+\frac{1}{2})}\left(\frac{2E}{\hbar\Omega}\right)^{-\frac{\ell}{2}}\exp\left(-\frac{E}{\hbar\Omega}\right)\times$$

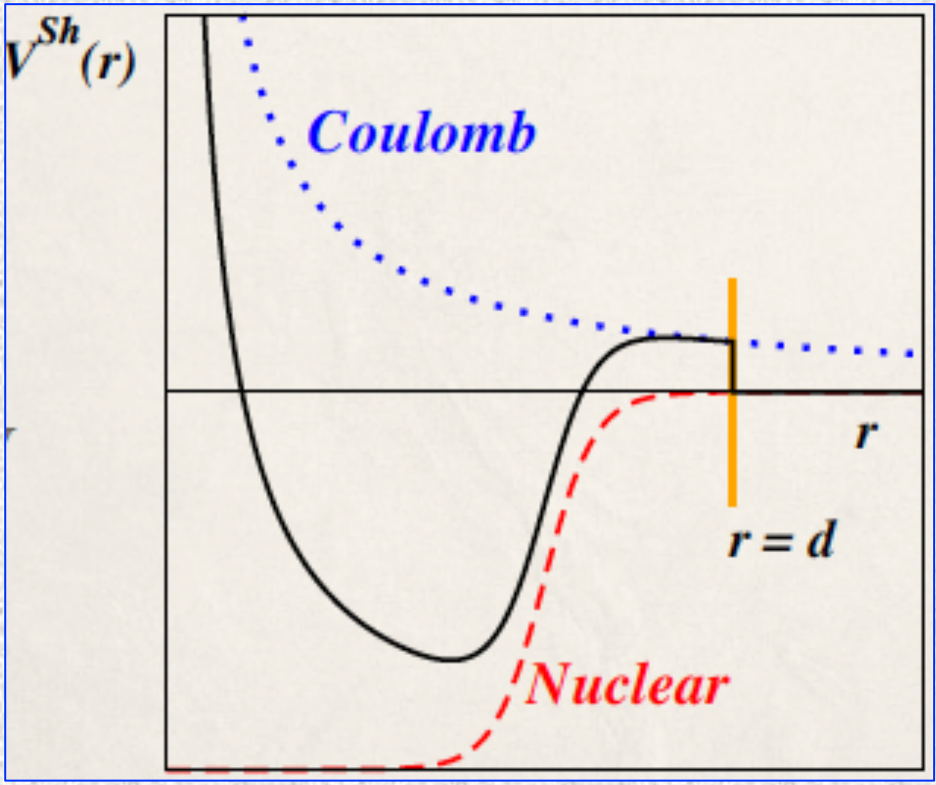
$$\times\,{}_1F_1\left(-n-\ell-\frac{1}{2},-\ell+\frac{1}{2};\frac{2E}{\hbar\Omega}\right)$$

HORSE: *charged particles*

J. M. Bang, A. I. Mazur, A. M. Shirokov, Yu. F. Smirnov, S. A. Zaytsev,
Ann. Phys. (N.Y.) 280, 299 (2000)

Auxiliary short range potential

$$V^{Sh} = \begin{cases} V^{Nucl} + V^{Coul}, & r \le d \\ 0, & r > d \end{cases}$$



Optimal value of d is determined by classical turning point of basic state with oscillator quantum number $(\mathbb{N}+2)$

$$d = b_0 = 2r_0\sqrt{\mathbb{N}+7/4}$$

$$r_0 = \sqrt{\frac{\hbar}{\mu\Omega}}$$ *is the oscillator radius*

HORSE: *charged particles*

Phase shift δ^{sh} can be calculated within HORSE

$$\tan\delta^{sh} = -\frac{S_{\mathbb{N},l} - G_{\mathbb{N}\mathbb{N}} T^{l}_{\mathbb{N},\mathbb{N}+2} S_{\mathbb{N}+2,l}}{C_{\mathbb{N},l} - G_{\mathbb{N}\mathbb{N}} T^{l}_{\mathbb{N},\mathbb{N}+2} C_{\mathbb{N}+2,l}}$$

Matching with Coulomb asymptotic at $r = b_0$

$$\tan\delta_l = -\frac{W_{b_0}(j_l, F_l) - W_{b_0}(n_l, F_l)\tan\delta_l^{Sh}}{W_{b_0}(j_l, G_l) - W_{b_0}(n_l, G_l)\tan\delta_l^{Sh}}$$

Quasi-Wronskians

$$W_{b_0}(j_l, F_l) = \left(\frac{dj_l(kr)}{dr} F_l(\eta, kr) - j_l(kr)\frac{dF_l(\eta, kr)}{dr}\right)\bigg|_{r=b_0}$$

$j_l(kr), \quad n_l(kr)$ — ***Spherical Bessel and Neumann functions;***

$F_l(\eta,kr), \quad G_l(\eta,kr)$ — ***Coulomb wave functions;***

$\eta = \dfrac{\mu Z_1 Z_2 e^2}{k}$ — ***Sommerfeld parameter.***

Single-State HORSE

HORSE *requires* ***ALL*** *shell model states:*

$$\tan\delta_\ell(E) = -\frac{S_{\mathbb{N}\ell}(E) - G_{\mathbb{N}\mathbb{N}}(E)T^{\ell}_{\mathbb{N},\mathbb{N}+2}S_{\mathbb{N}+2,\ell}(E)}{C_{\mathbb{N}\ell}(E) - G_{\mathbb{N}\mathbb{N}}(E)T^{\ell}_{\mathbb{N},\mathbb{N}+2}C_{\mathbb{N}+2,\ell}(E)}$$

$$G_{NN'}(E) = -\sum_{\nu=0}^{\mathcal{N}-1}\frac{\langle\nu|N\ell\rangle\langle N'\ell|\nu\rangle}{E_\nu - E}$$

So it is impossible for modern large-scale calculation in NCSM!

Single-State HORSE

A. M. Shirokov, A. I. Mazur, I. A. Mazur and J. P. Vary,
Phys. Rev. C 94, 064320 (2016); Phys. Part. Nucl. 48, 84 (2017).

But in the case $E = E_\nu$

$$\tan \delta_\ell(E_\nu) = -\frac{S_{\mathbb{N}+2,\ell}(E_\nu)}{C_{\mathbb{N}+2,\ell}(E_\nu)}$$

Now we get rid not only of the need to sum over a huge number of eigenstates but also from the shell model wave function component defining the desired channel.

Hence this equation can be used for scattering channels of any type.

By $\mathbb{N},\ \hbar\Omega$ *variation, we get* $\delta_\ell(E)$ *in some energy interval!*

Single-State HORSE: charged particles.

In the case $E = E_\nu$ *phase shift*

$$\tan\delta_l(E_\nu) = -\frac{W_{b_0}(n_l, F_l)S_{\mathbb{N}+2,l}(E_\nu) + W_{b_0}(j_l, F_l)C_{\mathbb{N}+2,l}(E_\nu)}{W_{b_0}(n_l, G_l)S_{\mathbb{N}+2,l}(E_\nu) + W_{b_0}(j_l, G_l)C_{\mathbb{N}+2,l}(E_\nu)}$$

also depends only on the energy E_ν *.*

Low-energy phase shift parametrization

An important feature of the Single-State HORSE method is the correct description a phase shift in the low-energy region.

Here we use the K-parametrization

Neutral particles ($\eta = 0$)

$$K_l^{(i)}(E) = k^{2l+1} \cot \delta_l(E_0^{(i)})$$

Coulomb-modified effective radius function

$$K_l(k^2) = k^{2l+1}\left(c_{l\eta}\right)^{-1}\left\{\frac{2\pi\eta}{\exp(2\pi\eta)-1}\left[ctg\ \delta_l(k)-i\right]+2\eta H(\eta)\right\}$$

$$H(\eta) = \Psi(i\eta) + (2i\eta)^{-1} - \ln(i\eta)$$

Parametrization of the K-matrix by means of a Padé-approximant

$$K_l(k^2) = \frac{w_0 + w_1 k^2 + w_2 k^4}{1 + v_1 k^2}$$

Fitting procedure

The final values for w_0, w_1, w_2, v_1 *are determined by minimizing the functional*

$$\Xi = \sqrt{\frac{1}{d}\sum_{i=1}^{d}\left(E_0^{(i)} - \varepsilon^{(i)}\right)^2}$$

Here $E_0^{(i)} \quad (i = 1, 2, ..., d)$ *are NCSM results with selected* $N^{(i)}, \ \hbar\Omega^{(i)}$

and $\varepsilon_0^{(i)}$ *are solutions of the transcendental equation*

$$\frac{w_0 + w_1 k^2 + w_2 k^4}{1 + v_1 k^2} = k^{2l+1}(c_{l\eta})^{-1}\left\{-\frac{2\pi\eta}{\exp(2\pi\eta) - 1}\times\right.$$

$$\left.\times\left[\frac{S_{\mathbb{N}+2,l}(k)\, W_b(n_l, G_l) + C_{\mathbb{N}+2,l}(k)\, W_b(j_l, G_l)}{S_{\mathbb{N}+2,l}(k)\, W_b(n_l, F_l) + C_{\mathbb{N}+2,l}(k)\, W_b(j_l, F_l)} + i\right] + 2\eta H(\eta)\right\}.$$

with fixed parameters w_0, w_1, w_2, v_1 *for the same* $N^{(i)}, \ \hbar\Omega^{(i)}$

Resonance energy and width

The renormalized Coulomb-nuclear scattering amplitude:

$$f_l = \frac{k^{2l}}{K_l(k^2) - 2\eta k^{2l+1} H(\eta) \left(c_{l\eta}\right)^{-1}}$$

We obtain resonance energies E_r and widths Γ by a numerical location of the S -matrix poles which coincide with the poles of scattering amplitude, that is by solving the equation

$$K_l(k^2) - 2\eta k^{2l+1} H(\eta) \left(c_{l\eta}\right)^{-1} = 0$$

$$E_r = Re(E_p), \;\; \Gamma/2 = Im(E_p)$$

Neutral particles

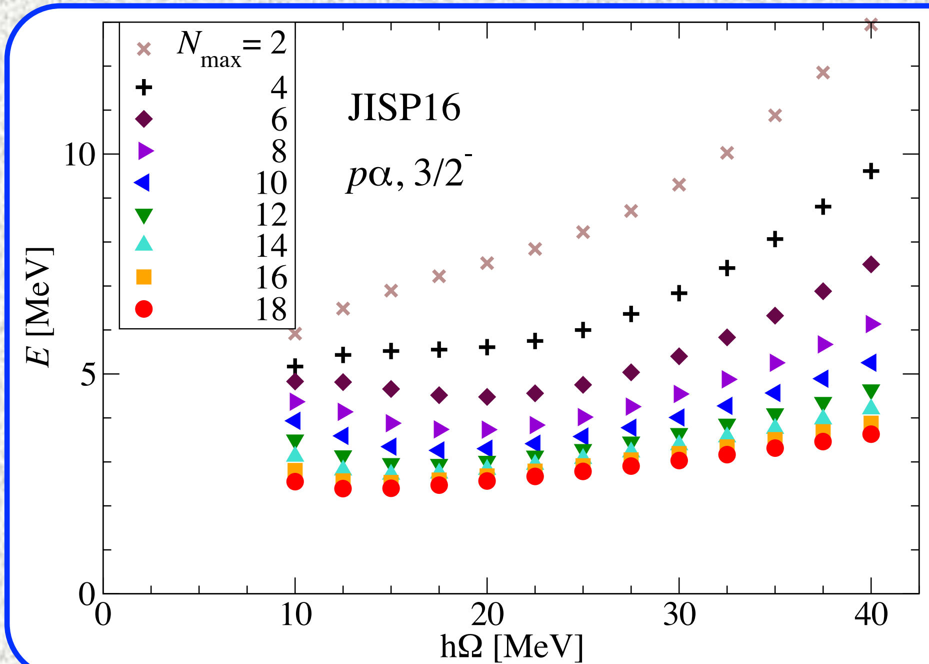
$$K_l(k^2) - i\, k^{2l+1} = 0$$

NCSM Single-State HORSE method: how it works

^{5}Li (3/2$^-$)

NCSM with JISP16

(Energies $E_\nu(\hbar\Omega)$ are measured from the proton-a threshold)



NCSM Single-State HORSE method

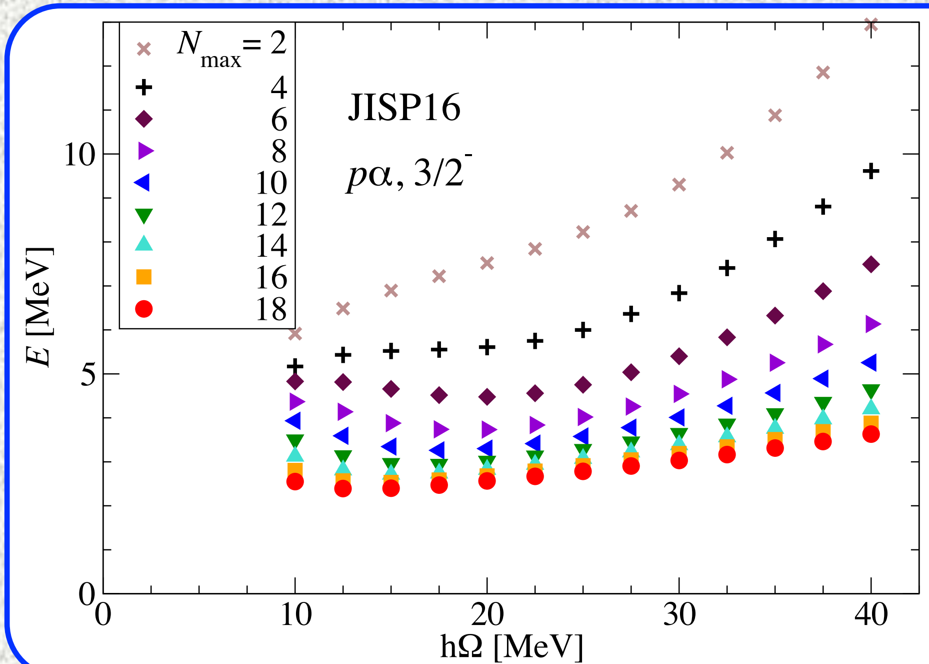
^{5}Li (3/2$^-$)

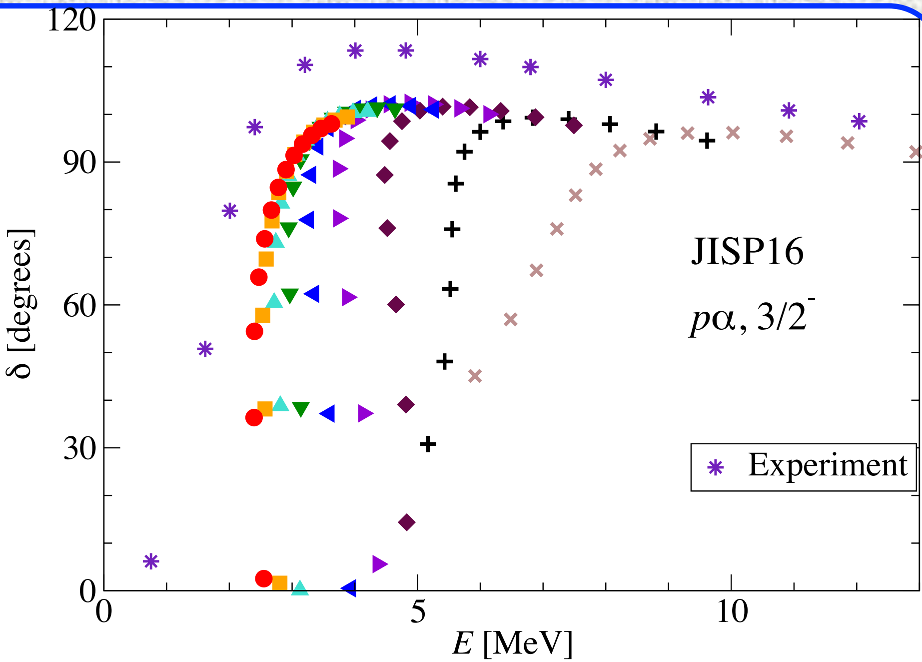
NCSM with JISP16

Single-State HORSE phase shifts

$$\tan\delta_l(E_\nu) = -\frac{W_{b_0}(n_l, F_l)S_{\mathbb{N}+2,l}(E_\nu) + W_{b_0}(j_l, F_l)C_{\mathbb{N}+2,l}(E_\nu)}{W_{b_0}(n_l, G_l)S_{\mathbb{N}+2,l}(E_\nu) + W_{b_0}(j_l, G_l)C_{\mathbb{N}+2,l}(E_\nu)}$$

$$N_{max} + N_0 = \mathbb{N}$$



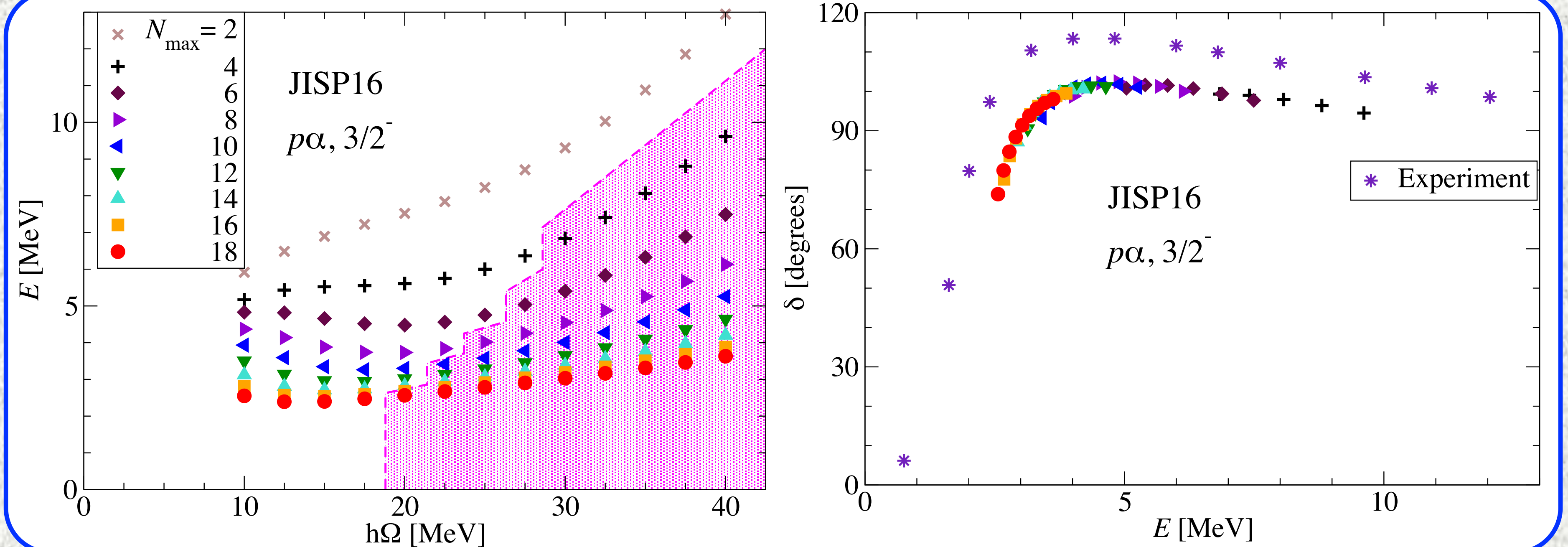


NCSM Single-State HORSE method

^{5}Li (3/2$^-$)
NCSM (JISP16)

Single-State HORSE phase shifts

+ the selection of the input data



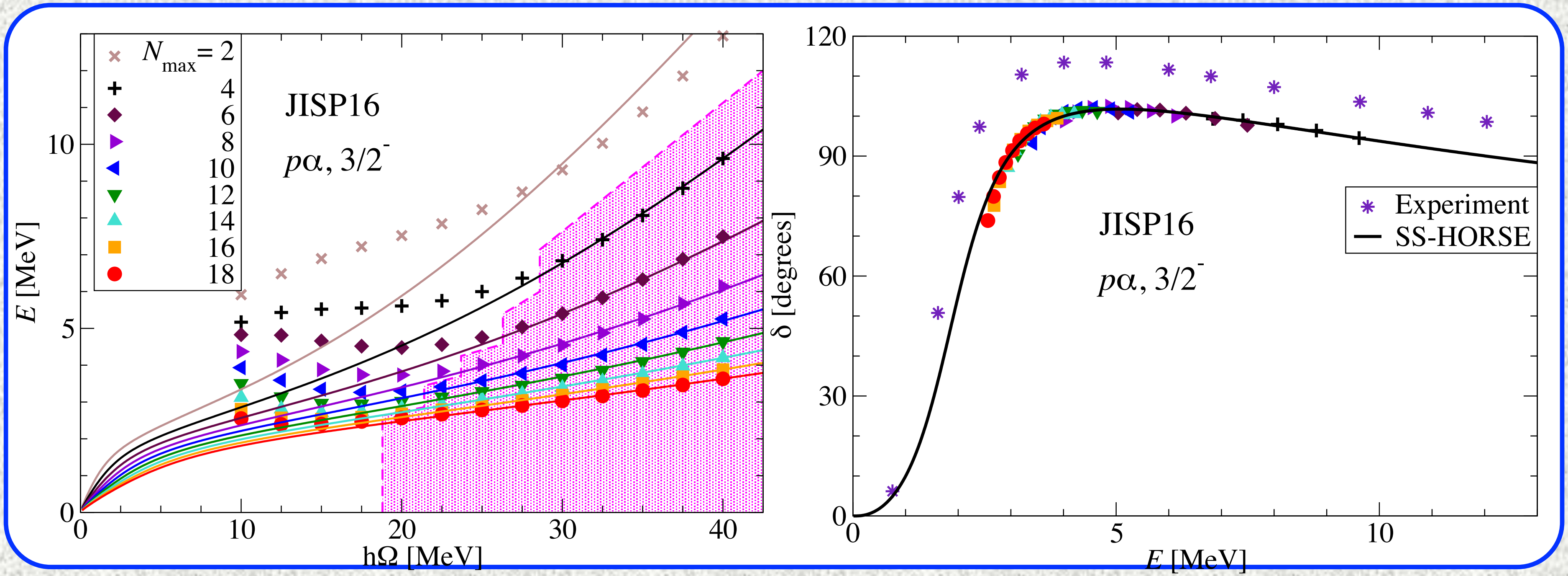
NCSM Single-State HORSE method

^{5}Li (3/2$^-$)
NCSM (JISP16)

Single-State HORSE phase shifts

+ the selection of the input data
+ phase shift parametrization

$$K_l(k^2) = \frac{w_0 + w_1\, k^2 + w_2\, k^4}{1 + v_1\, k^2}$$

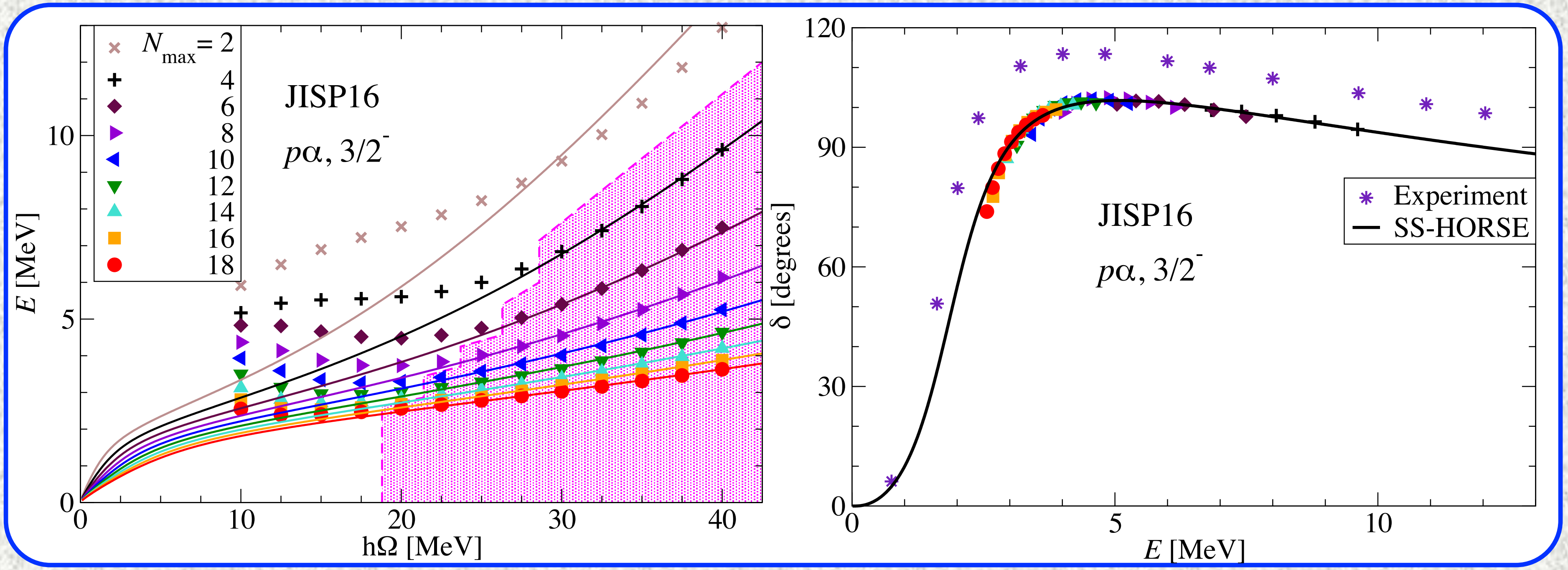


NCSM Single-State HORSE method

^{5}Li (3/2$^-$)
NCSM (JISP16)

Single-State HORSE phase shifts

+ the selection of the input data
+ phase shift parametrization



S-matrix poles
$E_p = E_r - i\,\Gamma/2$

$$K_l(k^2) - 2\eta k^{2l+1} H(\eta)\,(c_{l\eta})^{-1} = 0$$

Nα-scattering. Results.

Within NCSM with interactions JISP16 and Daejeon16 we calculated:

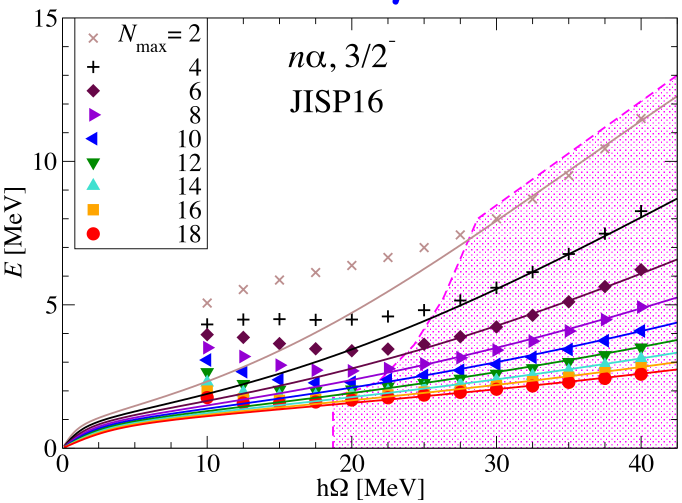
- *the lowest energy* E_0 *of the nuclei* ^{5}**He** *u* ^{5}**Li** *for states* $3/2^-,\ 1/2^-,\ 1/2^+$ *(in model spaces* $N_{max} \leq 18$ *with* $10 \leq \hbar\Omega \leq 40$*)*
- *the lowest energy ground state of the nucleus* ^{4}**He**

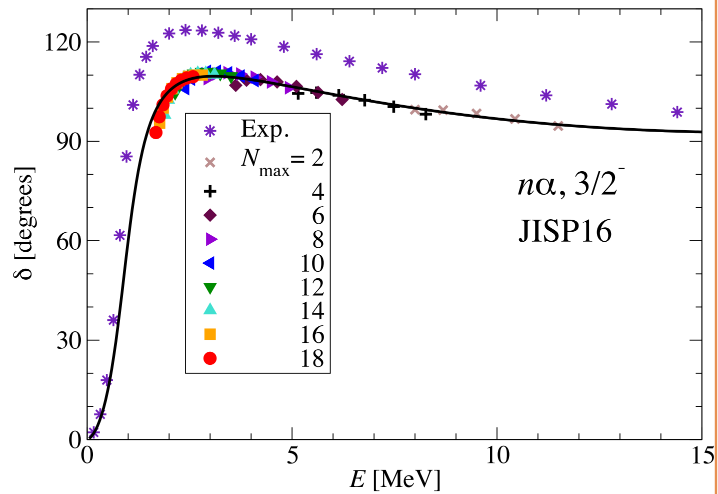
Single-State HORSE:

- *Energies were measured from the threshold* $N\alpha$
- *The total number of oscillator quanta of NCSM is identified with the number of quanta of relative* $N\alpha$ *motion in HORSE:* $N_{max} + 1 = \mathbb{N}$

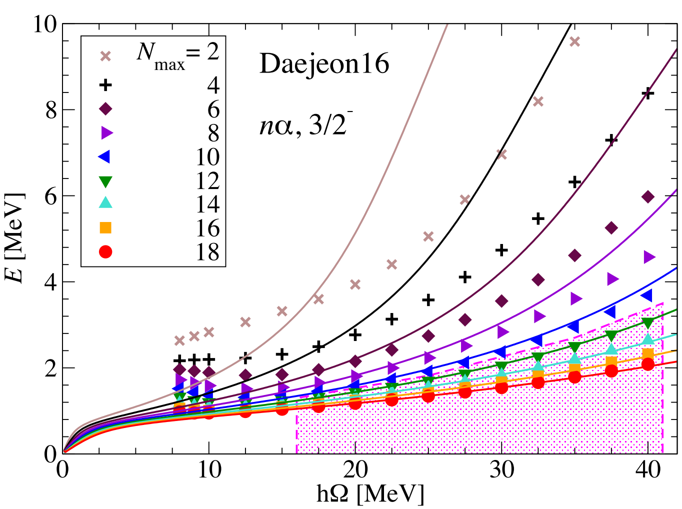
nα scattering: resonant state 3/2⁻

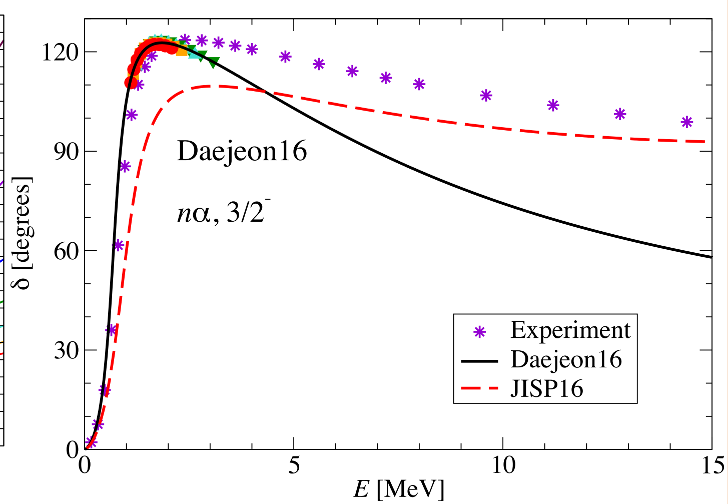
JISP16 (Phys. Rev. C 94, 0624320, 2016)





Daejeon16

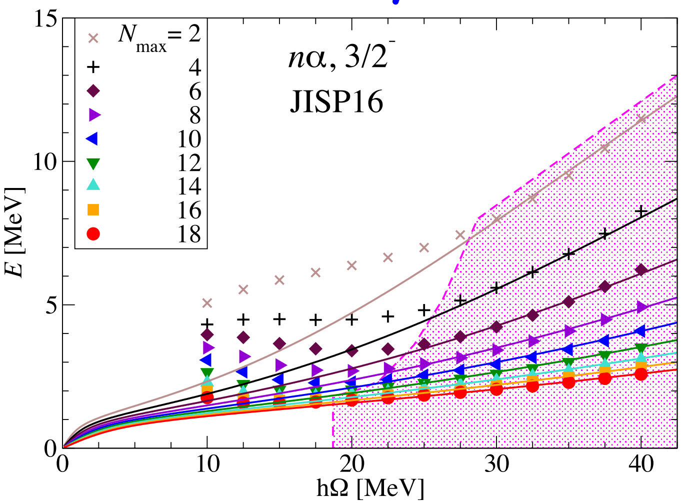


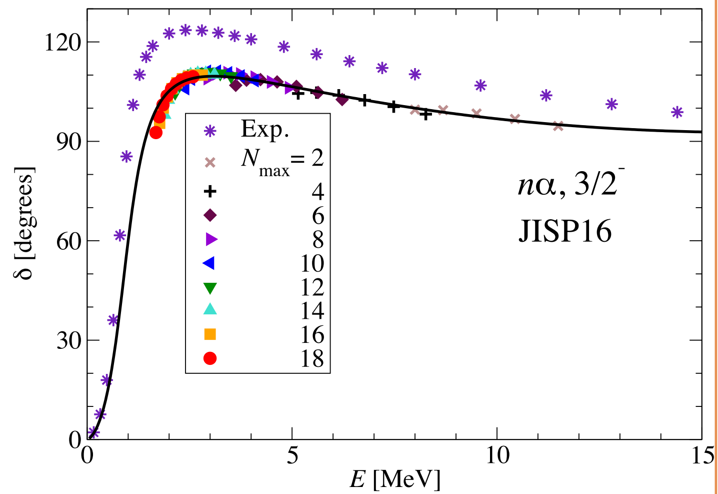


	E_r MeV	Γ MeV
JISP	0.89	0.99
Dj16	0.68	0.52
Exp.	0.80	0.65

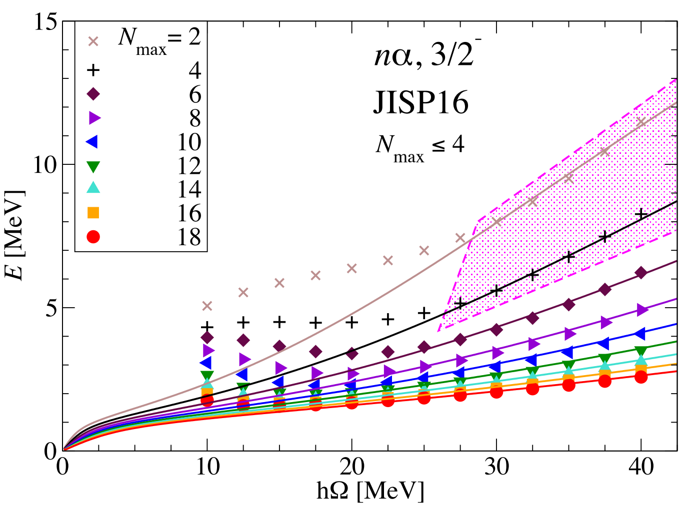
nα scattering: resonant state 3/2⁻

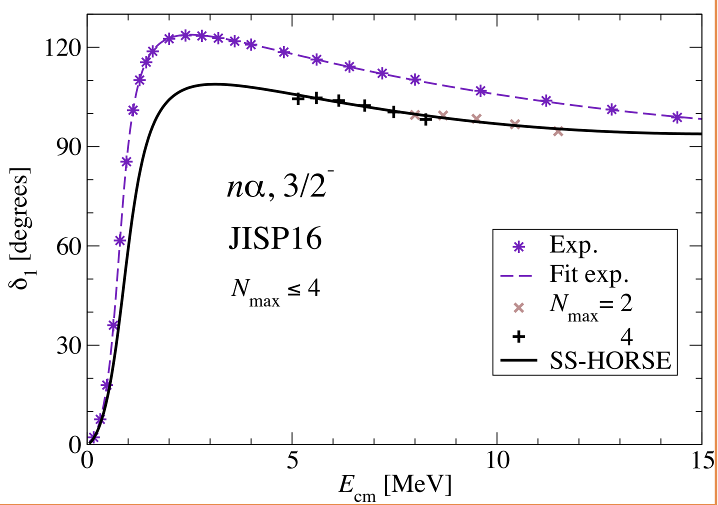
JISP16 (Phys. Rev. C 94, 0624320, 2016)





JISP16 (N_{max} = 2,4 only)

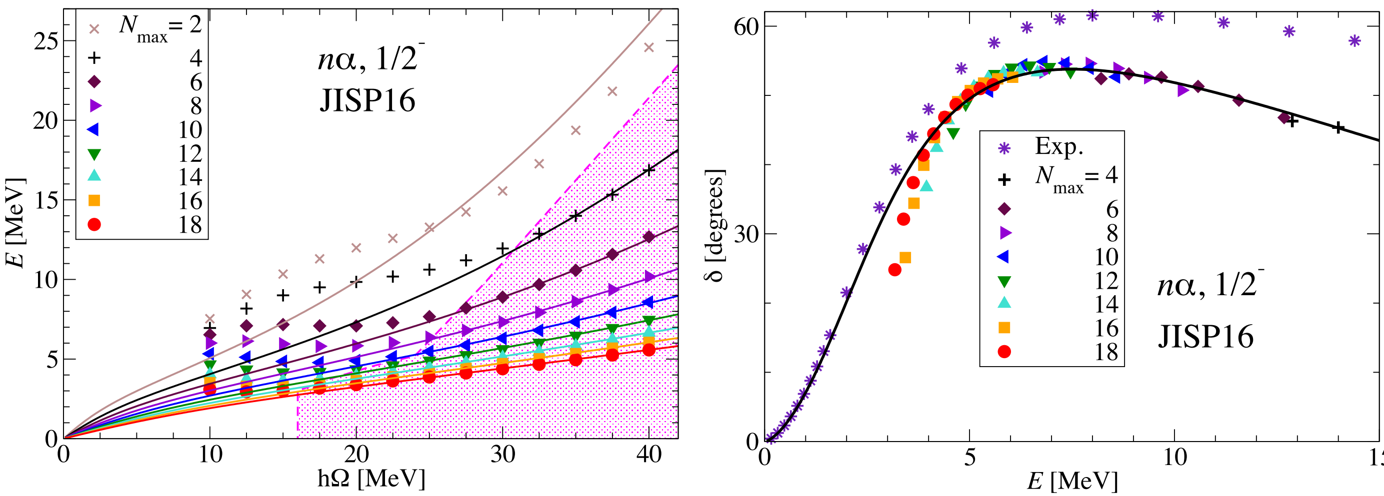




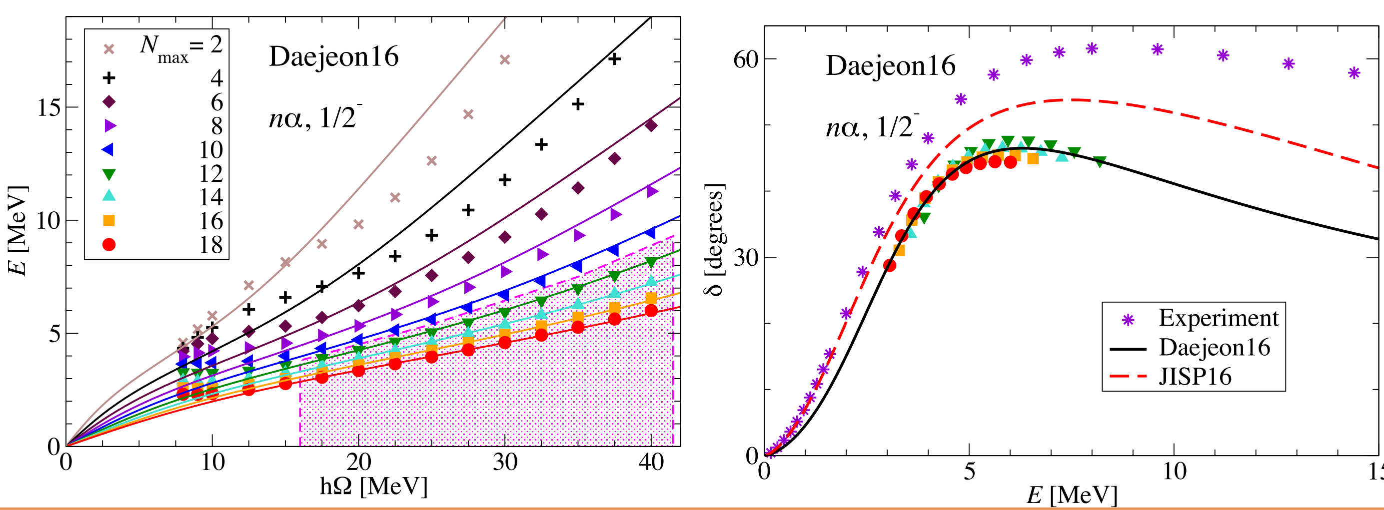
	E_r MeV	Γ MeV
JISP	0.89	0.99
JISP (2-4)	0.89	1.01
Exp.	0.80	0.65

nα scattering: resonant state $1/2^-$

JISP16 (Phys. Rev. C 94, 0624320, 2016)



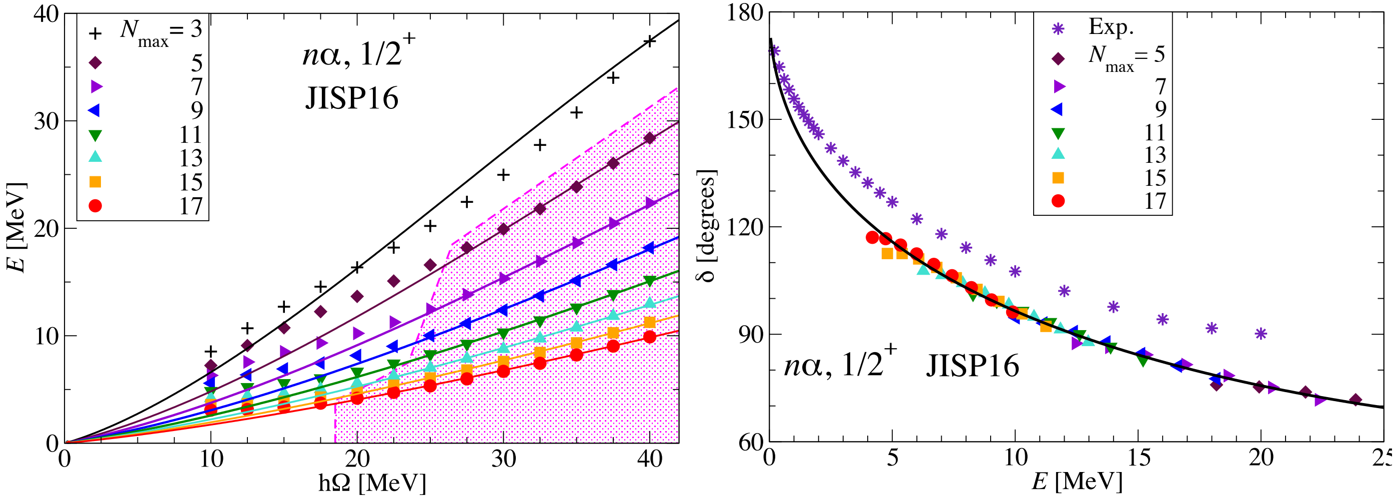
Daejeon16



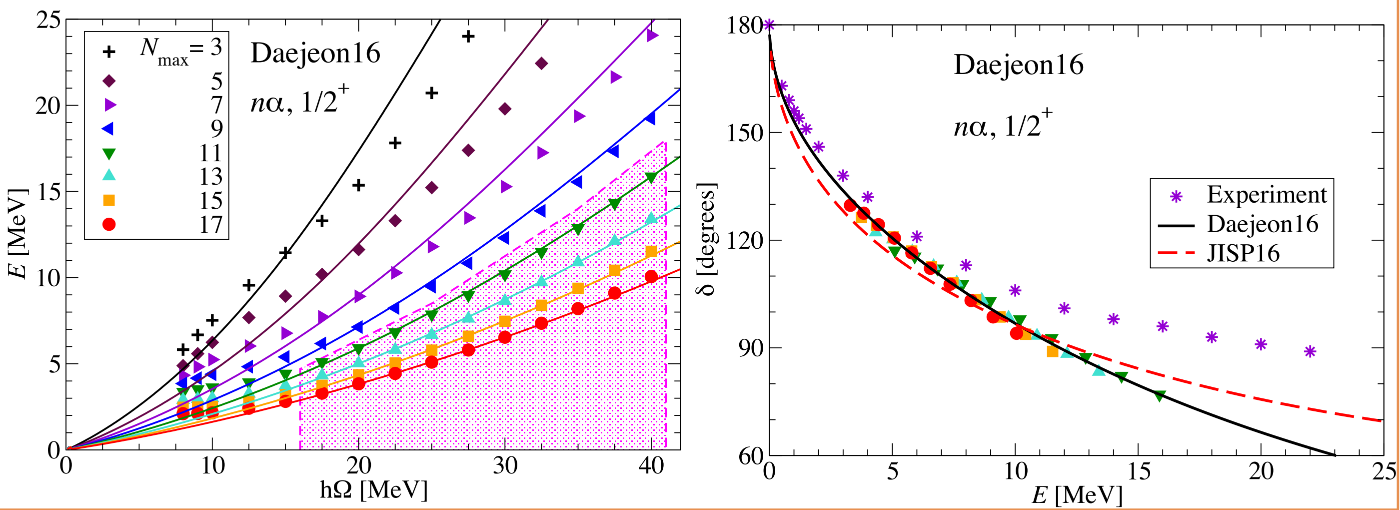
	E_r MeV	Γ MeV
JISP	1.86	5.46
Dj16	2.22	5.13
Exp.	2.07	5.57

Non-resonant nα scattering: 1/2⁺

JISP16 (Phys. Rev. C 94, 0624320, 2016)

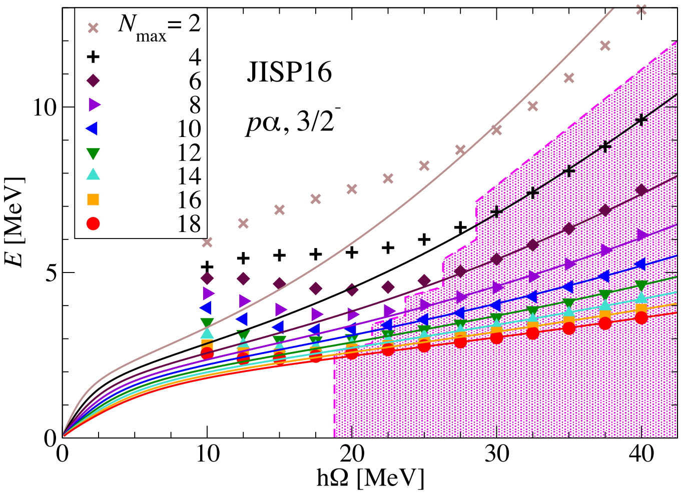


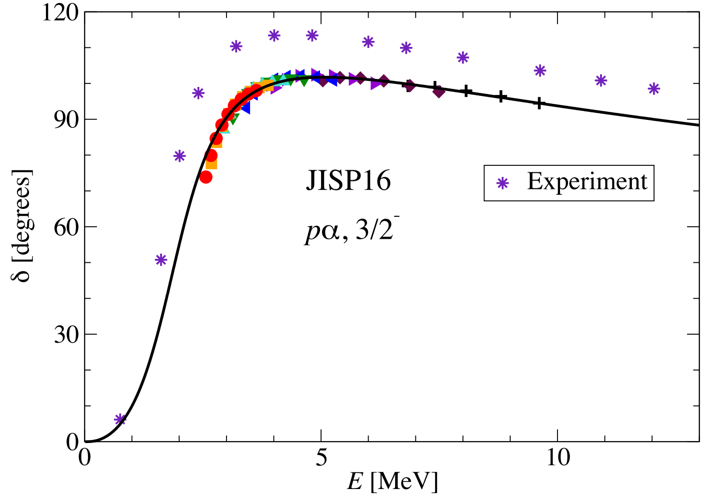
Daejeon16



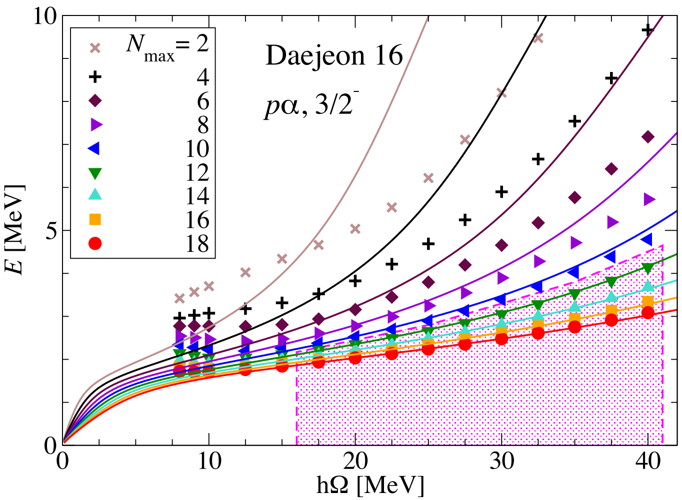
pα scattering: resonant state 3/2⁻

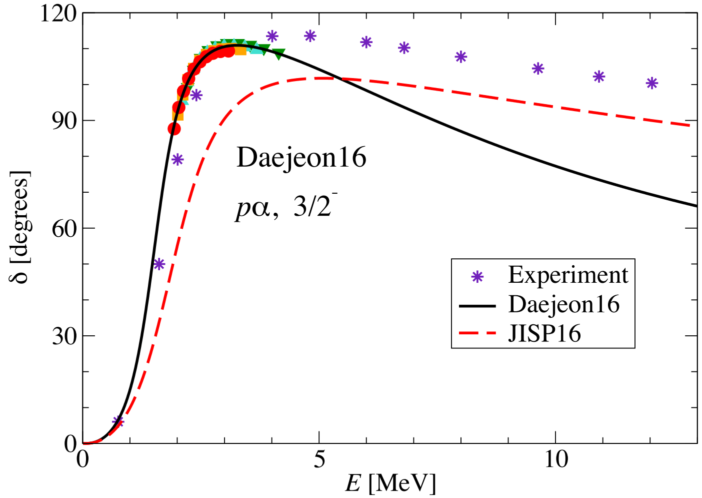
JISP16





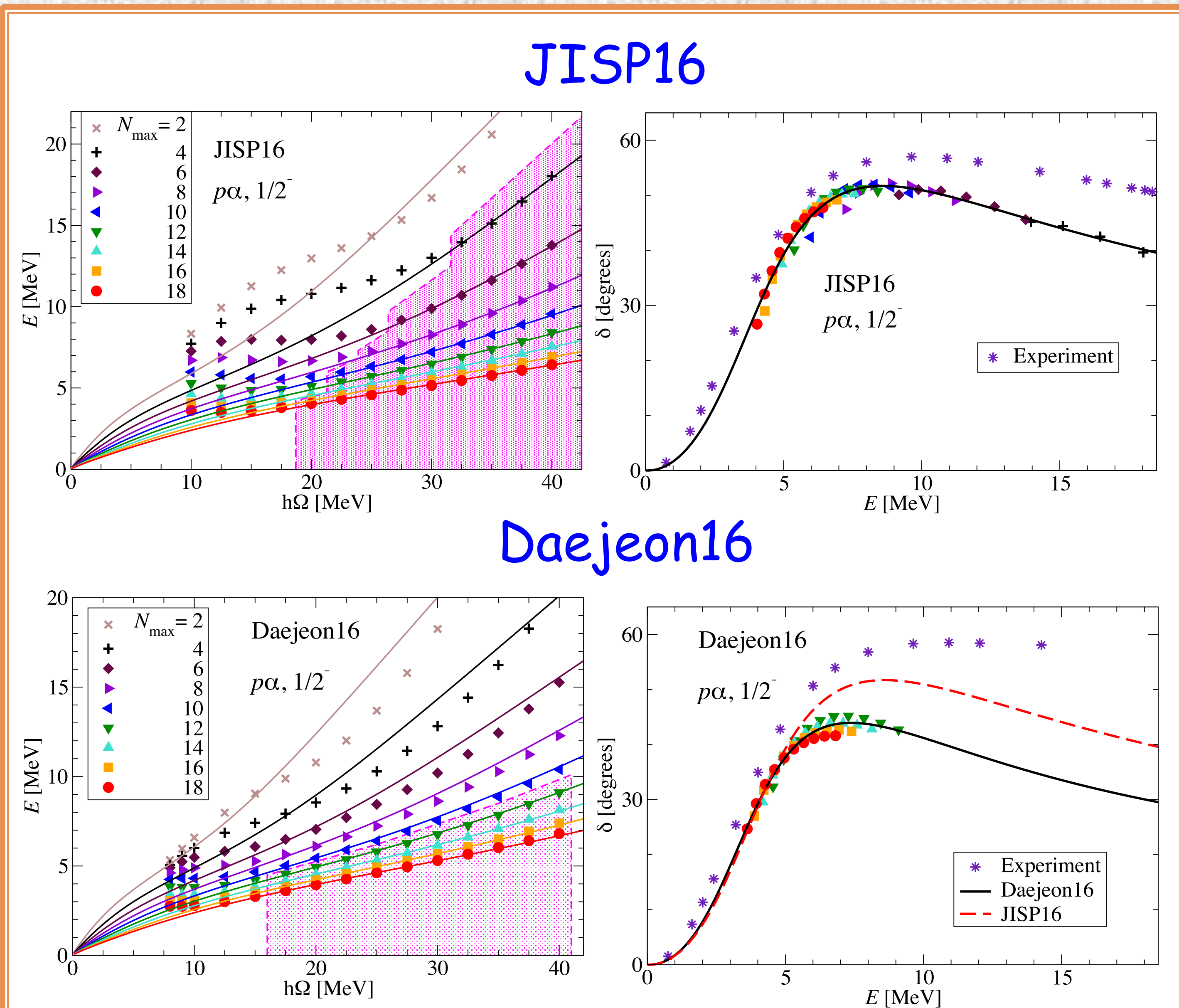
Daejeon16





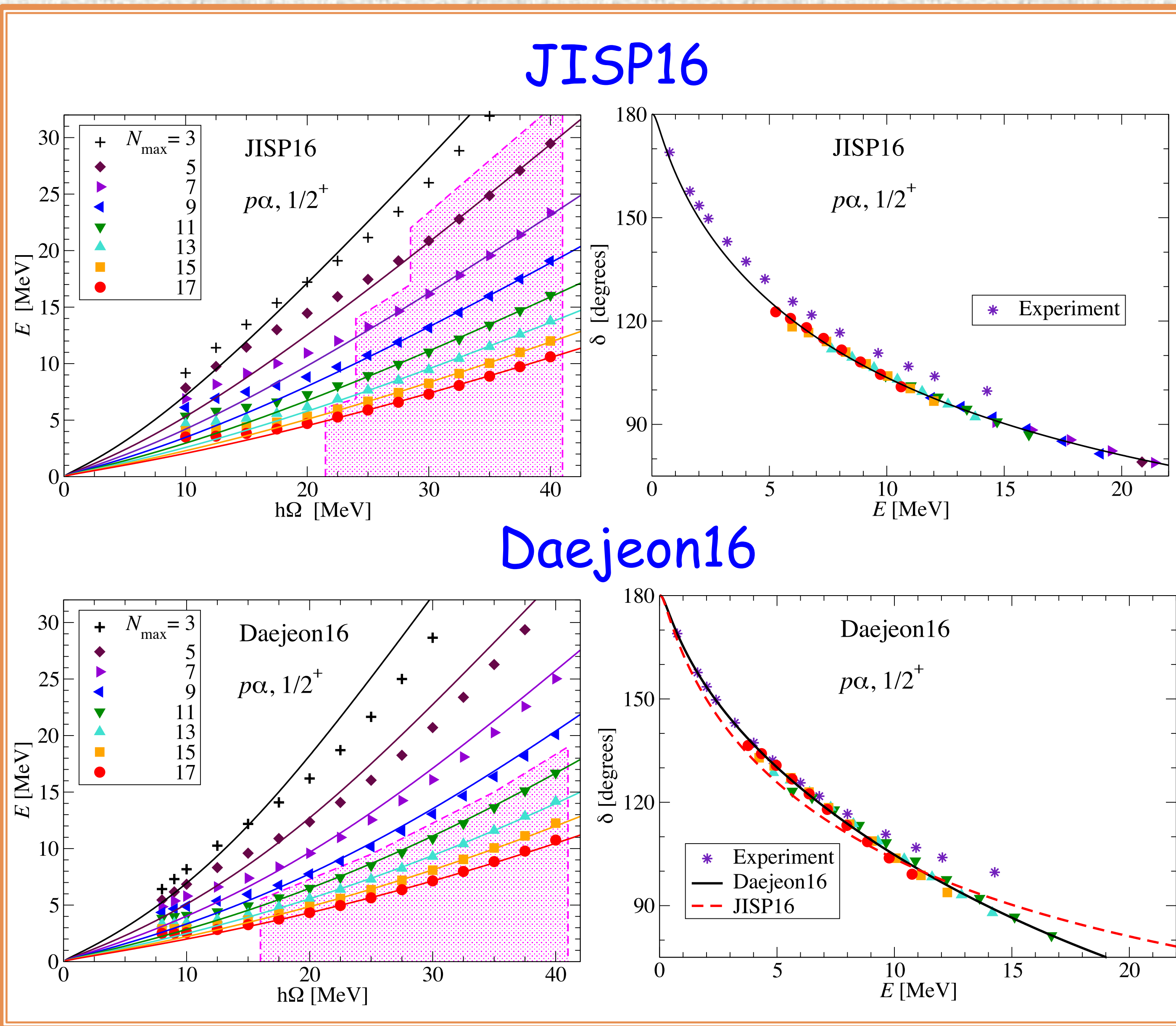
	E_r MeV	Γ MeV
JISP	1.81	1.78
Dj16	1.52	1.05
Exp.	1.69	1.23

pα scattering: resonant state 1/2⁻



	E_r MeV	Γ MeV
JISP	3.54	6.05
Dj16	3.18	5.63
Exp.	3.18	6.60

Non-resonant pα scattering: 1/2⁺



Energies and widths

	E_r MeV	Γ MeV	Ξ keV		E_r MeV	Γ MeV	Ξ keV	Δ MeV
	^{5}Li, 3/2$^-$				^{5}Li, 1/2$^-$			
Experim.	1.69	1.23			3.18	6.60		1.49
JISP16	1.81	1.78	44		3.54	6.05	66	1.73
Daejeon16	1.52	1.05	24		3.18	5.63	50	1.66
	^{5}He, 3/2$^-$				^{5}He, 1/2$^-$			
Experim.	0.80	0.65			2.07	5.57		1.27
JISP16	0.89	0.99	37		1.86	5.46	53	0.97
Daejeon16	0.68	0.52	22		2.22	5.13	48	1.54

$$\Delta = E_r^{(1/2-)} - E_r^{(3/2-)}$$

Conclusions

- ✓ Method *NCSM Single-State HORSE* for description scattering states, based on shell model calculations of nuclei, is suggested

- ✓ This method has been applied to Nα resonant and non-resonant scattering

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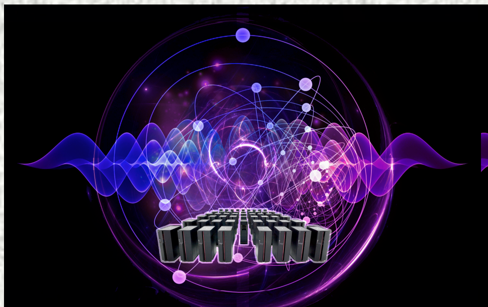
THANKS YOU for YOUR ATTENTION



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