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INSTITUTE OF NUCLEAR & PARTICLE PHYSICS

Nucleon-Nucleus Elastic Scattering using *ab initio* Folding Potentials based on NCSM Nonlocal One-Body Densities

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NERSC National Energy Research Scientific Computing Center

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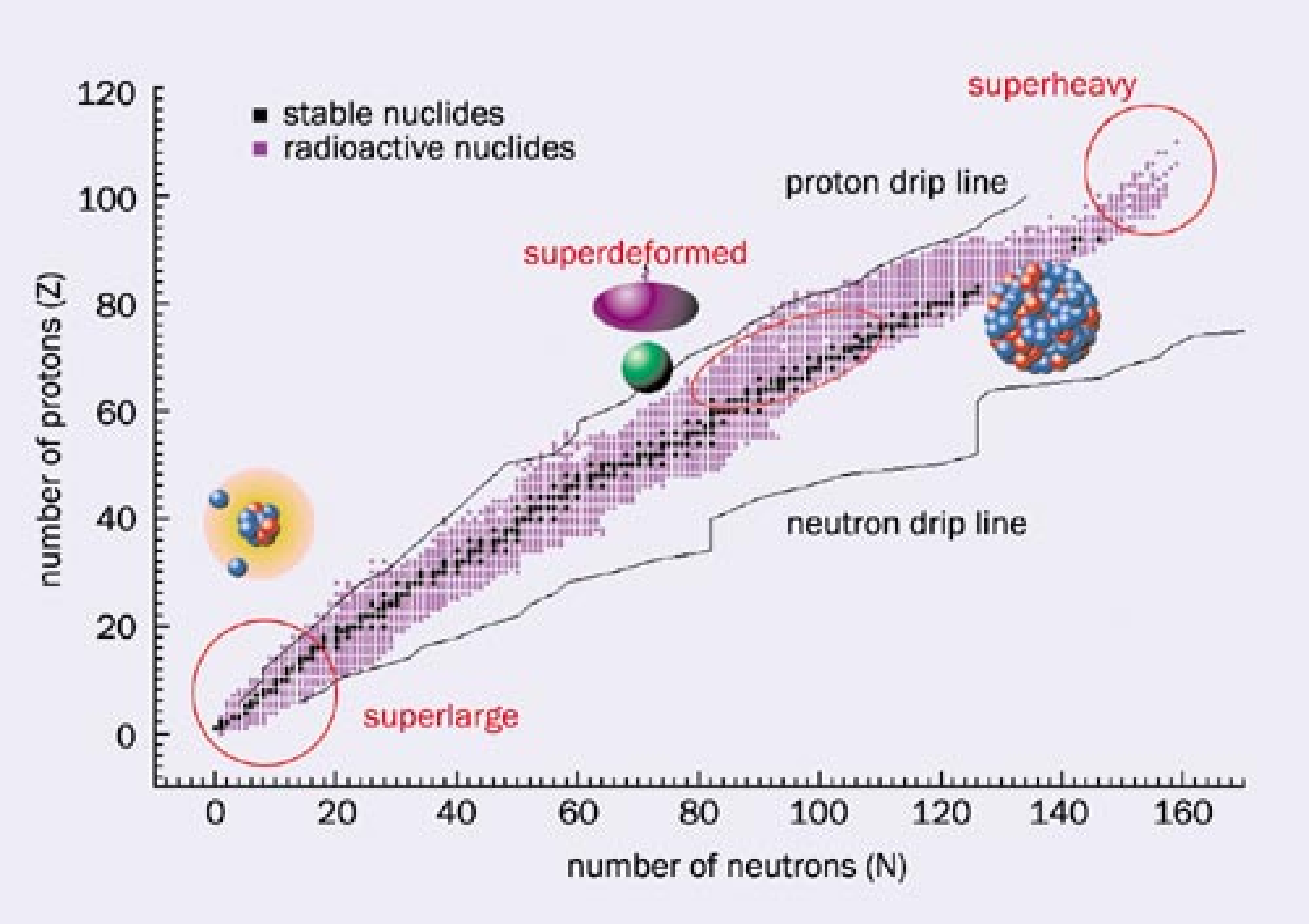
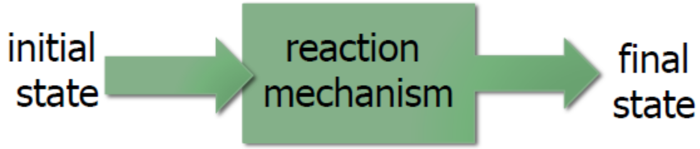


stable nuclides
radioactive nuclides
superheavy
proton drip line
superdeformed
neutron drip line
superlarge
number of protons (Z)
number of neutrons (N)
120
100
80
60
40
20
0
0
20
40
60
80
100
120
140
160

Exotic Nuclei are usually short lived:

Have to be studied with reactions in inverse kinematics

e.g. direct reaction:

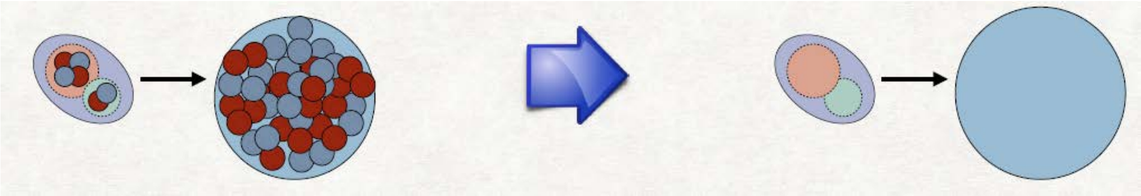


Challenge:

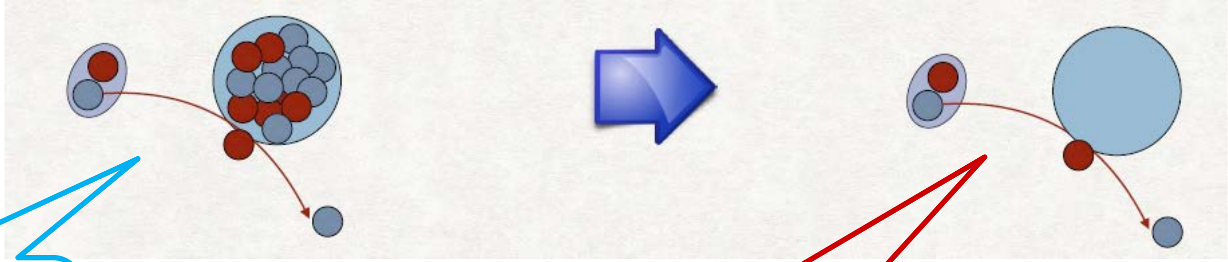
- **In the continuum, theory can solve the few-body problem exactly.**
- **Reaction theories need to map onto the many-body problem!**

Few-Body Ansatz in Nuclear Reactions

Cluster structure in nuclei:



Single particle motion of the "last" nucleon in a nucleus near the dripline



Many-body problem

Few-body problem

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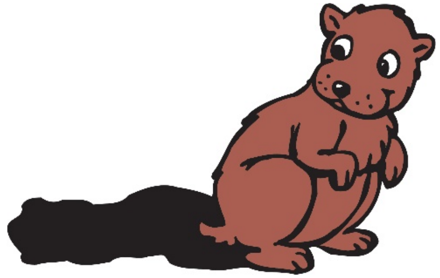
Example (d,p) Reactions:

Reduce Many-Body to Few-Body Problem

Solve few-body problem

“Shadow” ?



Challenges & Opportunities

Hamiltonian for effective few-body poblem:

$$H = H_0 + V_{np} + V_{nA} + V_{pA}$$

- **Nucleon-nucleon interaction believed to be well known:**
  today: chiral interactions, ‘high precision’ potentials

- **Effective proton (neutron) interactions:**
  - purely phenomenological optical potentials fitted to data
  - optical potentials with theoretical guidance
  - microscopic optical potentials

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Isolate relevant degrees of freedom

Formally: separate Hilbert space into P and Q space, and calculate in P space

Projection on P space requires introducing **effective interactions** between the degrees of freedom that are treated explicitly (Feshbach, Annals Phys. 5 (1958) 357-390)

Effective Interactions: non-local and energy dependent

Isolate relevant degrees of freedom

Formally: separate Hilbert space into P and Q space, and calculate in P space

Projection on P space requires introducing **effective interactions** between the degrees of freedom that are treated explicitly (Feshbach, Annals Phys. 5 (1958) 357-390)

Effective Interactions: non-local and energy dependent

History: **Phenomenological optical potentials**

Either fitted to a large global data set OR to a restricted data set

Most general form of optical potential

- $\Sigma_i [V_{A,Z,N,E}(r) + i\, W_{A,Z,N,E}(r)]\ \mathrm{Operator}_{(i)}$
- **Functions are of Woods-Saxon type**

Have central and spin orbit term

Fit cross sections, angular distributions polarizations, for a set of nuclei (lightest usually ^{12}C).

No connection to microscopic theory

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Today: effective interaction from ab initio methods

Start from many-body Hamiltonian with 2 and 3 body forces

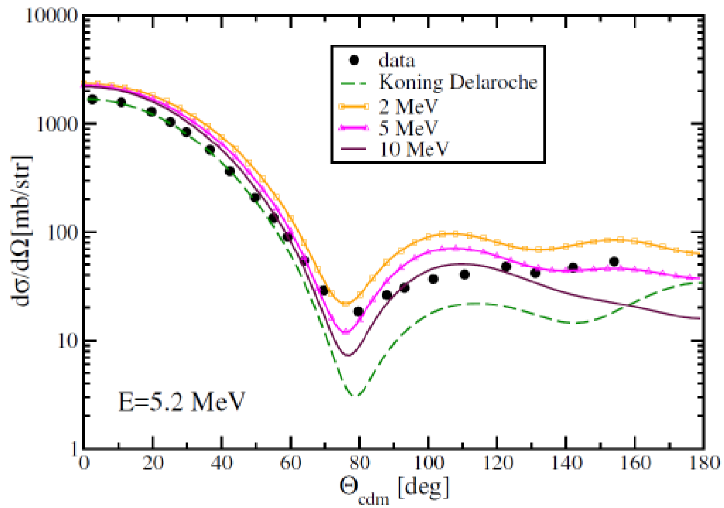
Theoretical foundations laid by Feshbach and Watson in the 1950s

Feshbach:

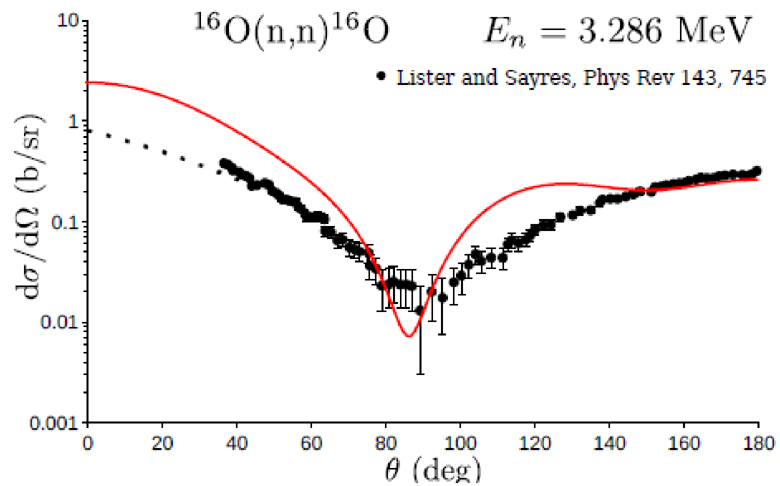
→ effective nA interaction via Green's function from solution of many body problem using basis function expansion, e.g. SCGF, CCGF (current truncation to singles and doubles)

energy ~ 10 MeV

$^{40}Ca(n,n)^{40}Ca$



Rotureau, Danielewicz, Hagen, Jansen, Nunes
arXiv: 1808.04535 and PRC 95, 024315 (2017)



Idini, Barbieri, Navratil
J.Phys.Conf. 981. 012005 (2018)
Acta Phys. Polon. B48, 273 (2017)

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Today: effective interaction from ab initio methods

Start from many-body Hamiltonian with 2 and 3 body forces

Theoretical foundations laid by Feshbach and Watson in the 1950s

Watson:

→ Multiple scattering expansion, e.g. spectator expansion (current truncation to 2 active particles)

Spectator Expansion:

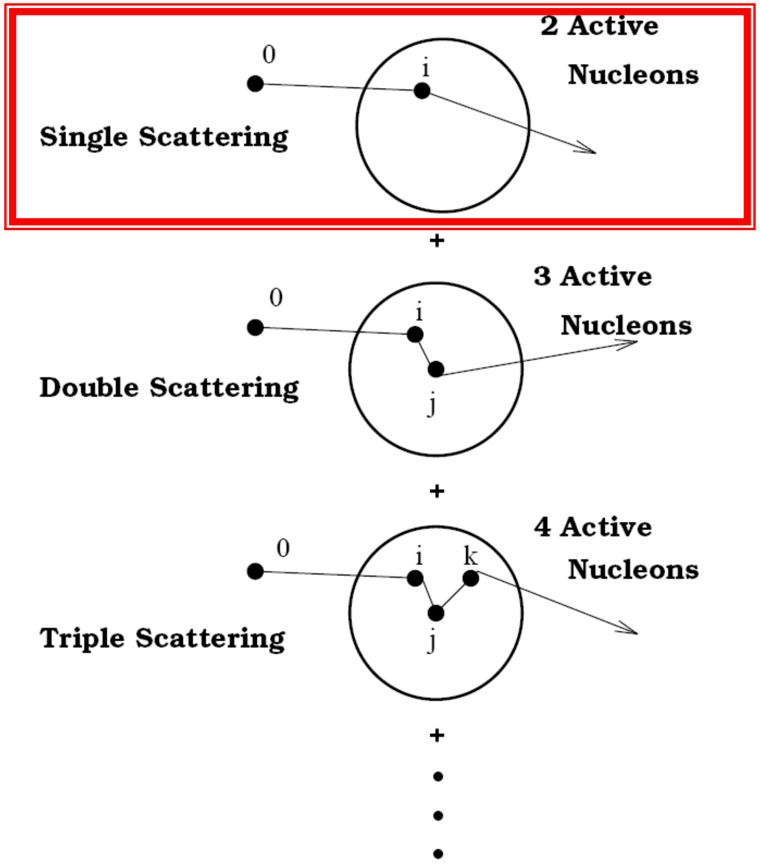
Formulated for scattering amplitude:

Siciliano, Thaler (1977)

Picklesimer, Thaler (1981)

Expansion in:

- particles active in the reaction
- Antisymmetrized in active particles



Elastic Scattering

- In- and Out-States have the target in ground state Φ_0
- Projector on ground state $\mathbf{P} = |\Phi_0\rangle\langle\Phi_0|$
  - With $\mathbf{1=P+Q}$ and $\mathbf{[P,G_0]=0}$
- For elastic scattering one needs: $\mathbf{P\,T\,P = P\,U\,P + P\,U\,P\,G_0(E)\,P\,T\,P}$

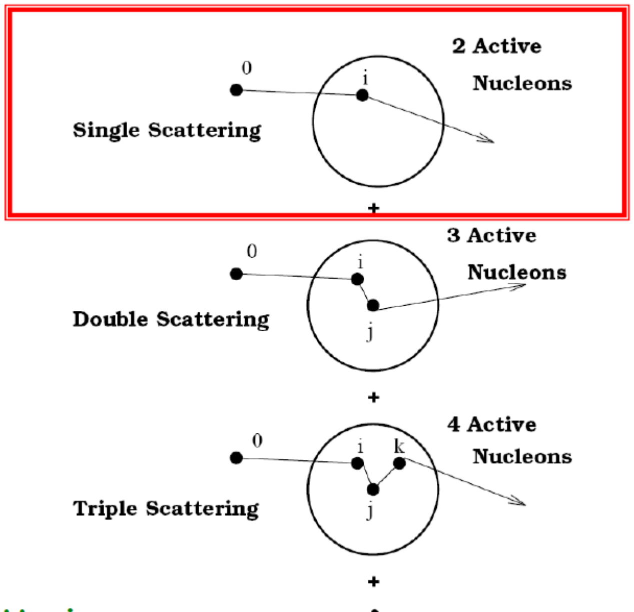
$$\mathbf{T = U + U\,G_0(E)\,P\,T}$$

$$\mathbf{U = V + V\,G_0(E)\,Q\,U}$$

$\Leftarrow$ **effective (optical) potential**

Up to here exact

Spectator Expansion of U :



1st order: single scattering: $\mathbf{U^{(1)} \approx \Sigma^{A}_{i=0}\,\tau_{0i}}$

Chinn, Elster, Thaler, PRC 47, 2242 (1993)

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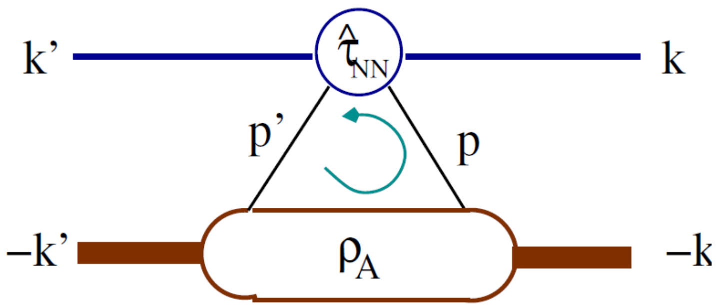
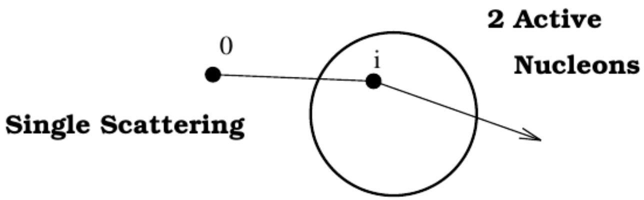
Computing the first order folding potential $U^{(1)} \approx \Sigma^{A}_{i=0}\, \tau_{0i}$





NN scattering amplitudes

Nuclear one-body density

$$U_{el}(\mathbf{q},\mathbf{K}) = \sum_{i=n,p} \int d\mathbf{P}\ \eta(\mathbf{q},\mathbf{K},\mathbf{P})\ \hat{\tau}_{0i}\left(\mathbf{q}, \frac{1}{2}(\frac{A+1}{A}\mathbf{K}-\mathbf{P}), \mathcal{E}\right) \rho_i(\mathbf{P}-\frac{A-1}{2A}\mathbf{q}, \mathbf{P}+\frac{A-1}{2A}\mathbf{q})$$

$$\mathbf{K} \equiv \frac{\mathbf{k}+\mathbf{k}'}{2} \qquad \mathbf{P} \equiv \frac{\mathbf{k}_i+\mathbf{k}'_i}{2} + \frac{\mathbf{K}}{A}$$

$$\mathrm{q} \equiv \mathrm{k}' - \mathrm{k}$$

Same NN Interaction can now be used for NN t-matrix and one-body density matrix

Effective Potential is non-local and depends on energy

Implementation designed for energies ≥ 100 MeV

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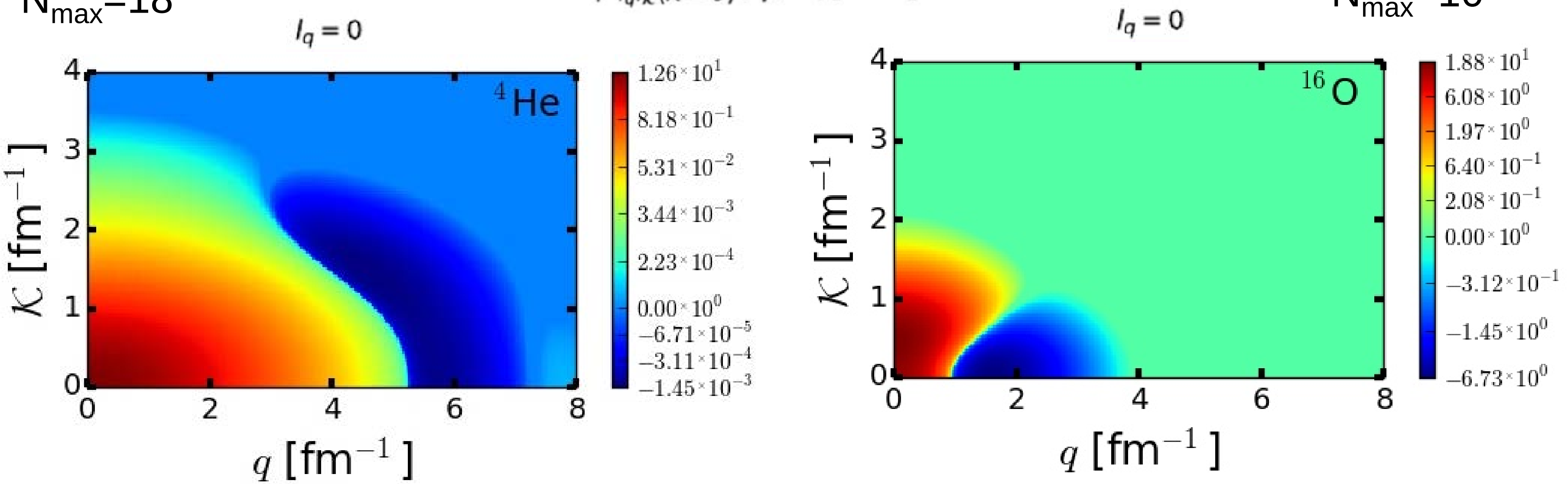
Nonlocal one-body densities from NCSM

translationally invariant (NNLO$_{opt}$, proton distribution, ħω=20 MeV)

N_{max}=18 $\qquad \rho_{l_q l_\mathcal{K}(K=0)}(q,\mathcal{K})[fm^3]$ $\qquad$ N_{max}=10



$$\vec{q} = \vec{p'} - \vec{p}$$
$$\vec{\mathcal{K}} = \frac{1}{2}(\vec{p'} + \vec{p})$$

Burrows, Elster, Popa, Launey, Nogga, Maris, PRC 97, 024325 (2018)

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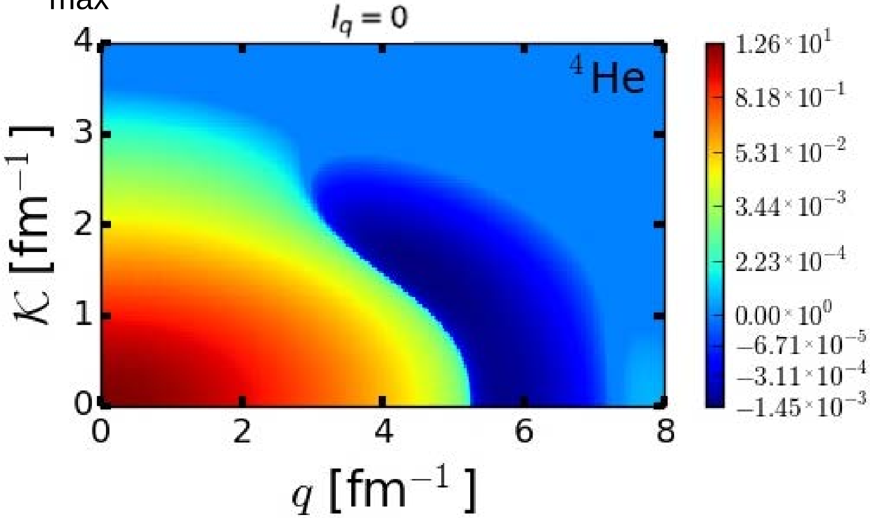
$\rho_{l_q l_{\mathcal{K}}(K=0)}(q,\mathcal{K})[fm^3]$

N$_{max}$=18



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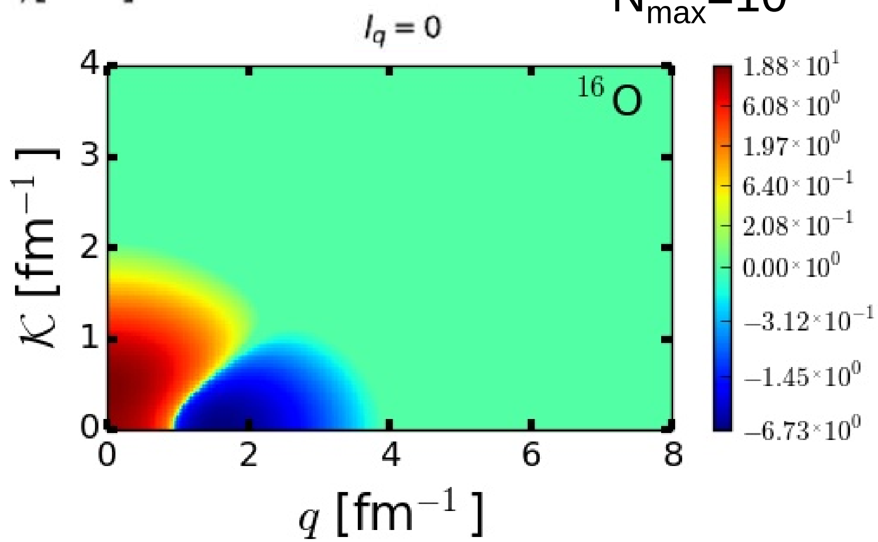

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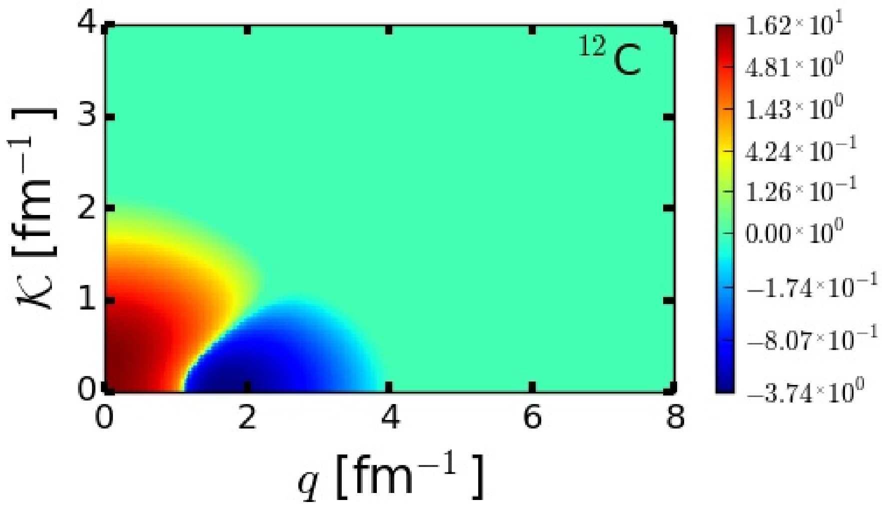

N$_{max}$=10

NN amplitude: $f_{NN}(k'k;E) = C \langle k' | t_{NN}(E) | k \rangle$

Variables (E,k',k,φ) ⇒ (E, q, K, θ)

with q = k' - k
K = ½ (k' + k)

NN t-matrix in Wolfenstein representation:

Ground state with intrinsic spin 0

Projectile "0" : plane wave basis
Struck nucleon "i" : target basis

$$\overline{\mathrm{M}}(\mathbf{q},\mathbf{K}_{NN},\mathcal{E}) = A(\mathbf{q},\mathbf{K}_{NN},\mathcal{E})\,\mathbf{1} + i \cdot C(\mathbf{q},\mathbf{K}_{NN},\mathcal{E})\,(\boldsymbol{\sigma}^{(0)} \otimes \mathbf{1} + \mathbf{1} \otimes \boldsymbol{\sigma}^{(i)}) \cdot \hat{\mathrm{n}}_{NN}$$
$$+ M(\mathbf{q},\mathbf{K}_{NN},\mathcal{E})(\boldsymbol{\sigma}^{(0)} \cdot \hat{\mathrm{n}}_{NN}) \otimes (\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{n}}_{NN})$$
$$+ (G(\mathbf{q},\mathbf{K}_{NN},\mathcal{E}) - H(\mathbf{q},\mathbf{K}_{NN},\mathcal{E}))(\boldsymbol{\sigma}^{(0)} \cdot \hat{\mathrm{q}}) \otimes (\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{q}})$$
$$+ (G(\mathbf{q},\mathbf{K}_{NN},\mathcal{E}) + H(\mathbf{q},\mathbf{K}_{NN},\mathcal{E}))(\boldsymbol{\sigma}^{(0)} \cdot \hat{\mathrm{K}}_{NN}) \otimes (\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{K}}_{NN})$$

– –

$$+ D(\mathbf{q},\mathbf{K}_{NN},\mathcal{E}) \left[(\boldsymbol{\sigma}^{(0)} \cdot \hat{\mathrm{q}}) \otimes (\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{K}}_{NN}) + (\boldsymbol{\sigma}^{(0)} \cdot \hat{\mathrm{K}}_{NN}) \otimes (\boldsymbol{\sigma}^{(i)} \cdot \hat{\mathrm{q}})\right]$$

Off-shell

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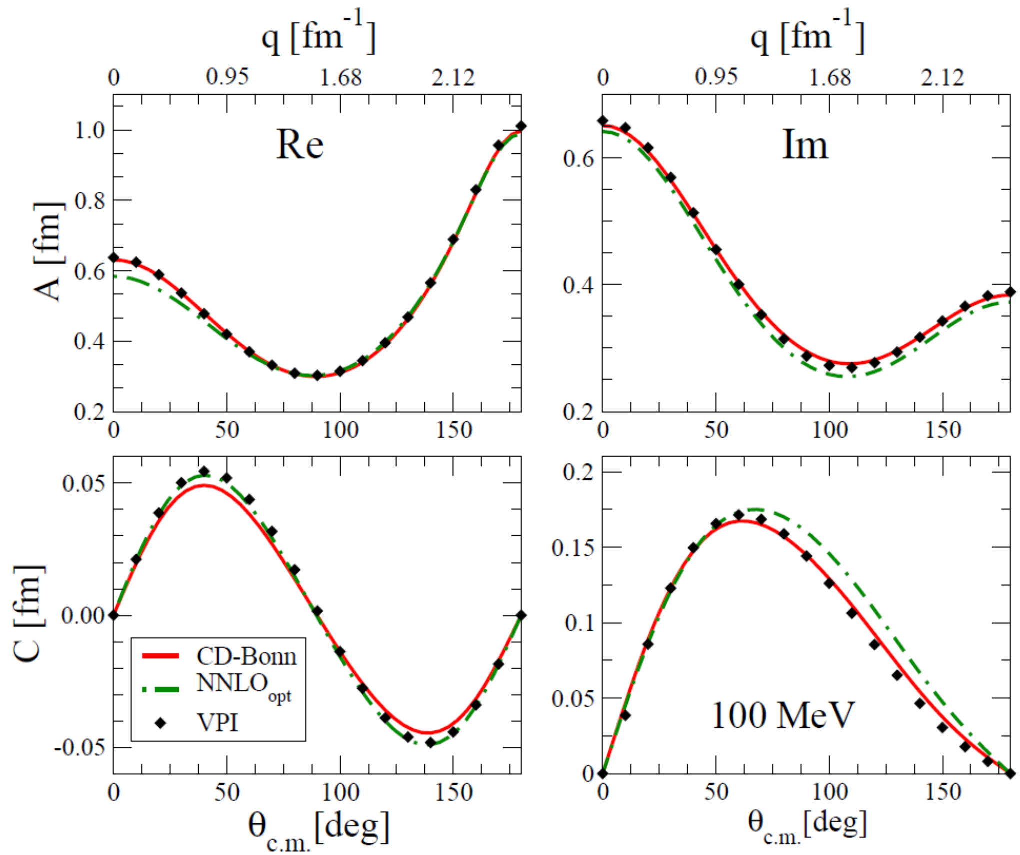
NNLO$_{opt}$
fitted to
E_{lab}=125 MeV

→ max. momentum transfer ≈ 2.45 fm^{-1}

Wolfenstein Amplitudes A and C



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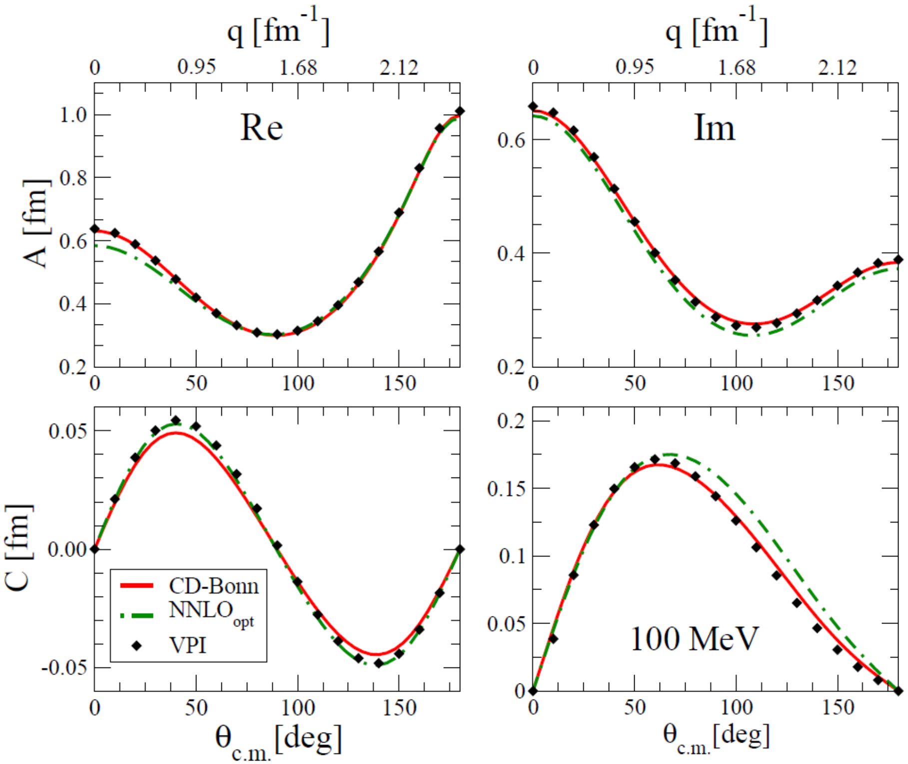

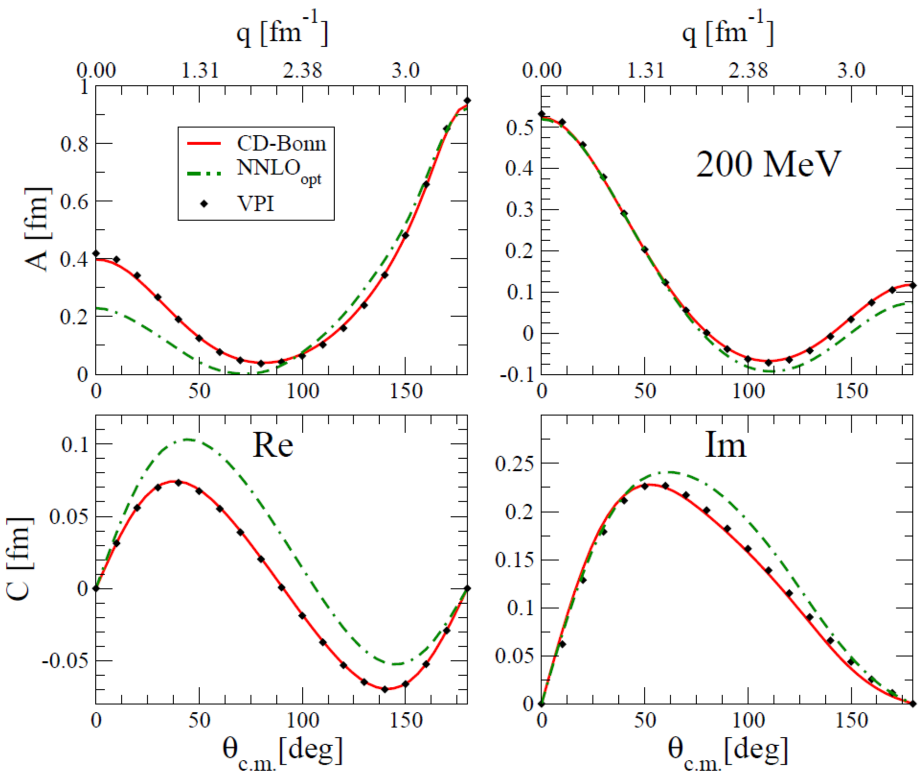

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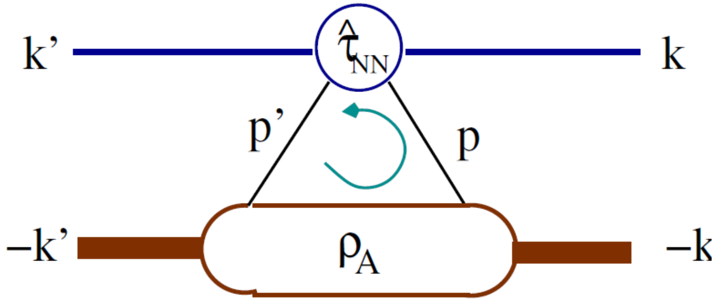
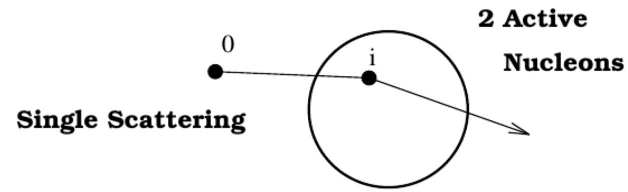
Computing the first order folding potential $U^{(1)} \approx \Sigma^{A}_{i=0}\, \tau_{0i}$





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$$\mathrm{q} \equiv \mathrm{k}' - \mathrm{k}$$

Same NN Interaction can now be used for NN t-matrix and one-body density matrix

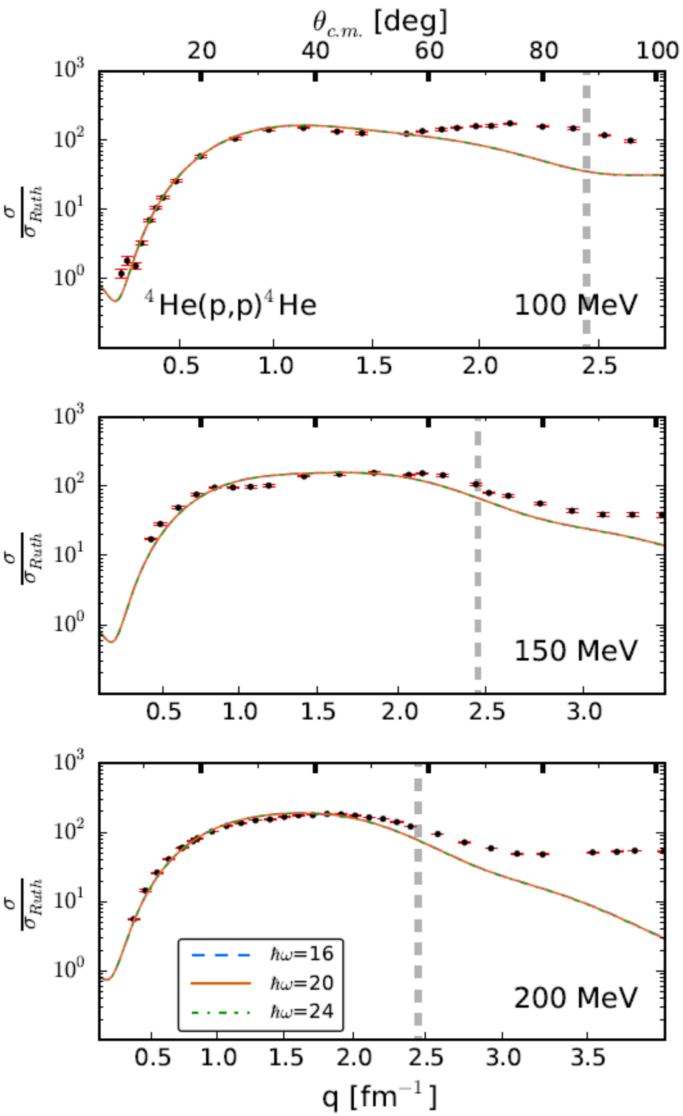
Effective Potential is non-local and depends on energy

^{4}He

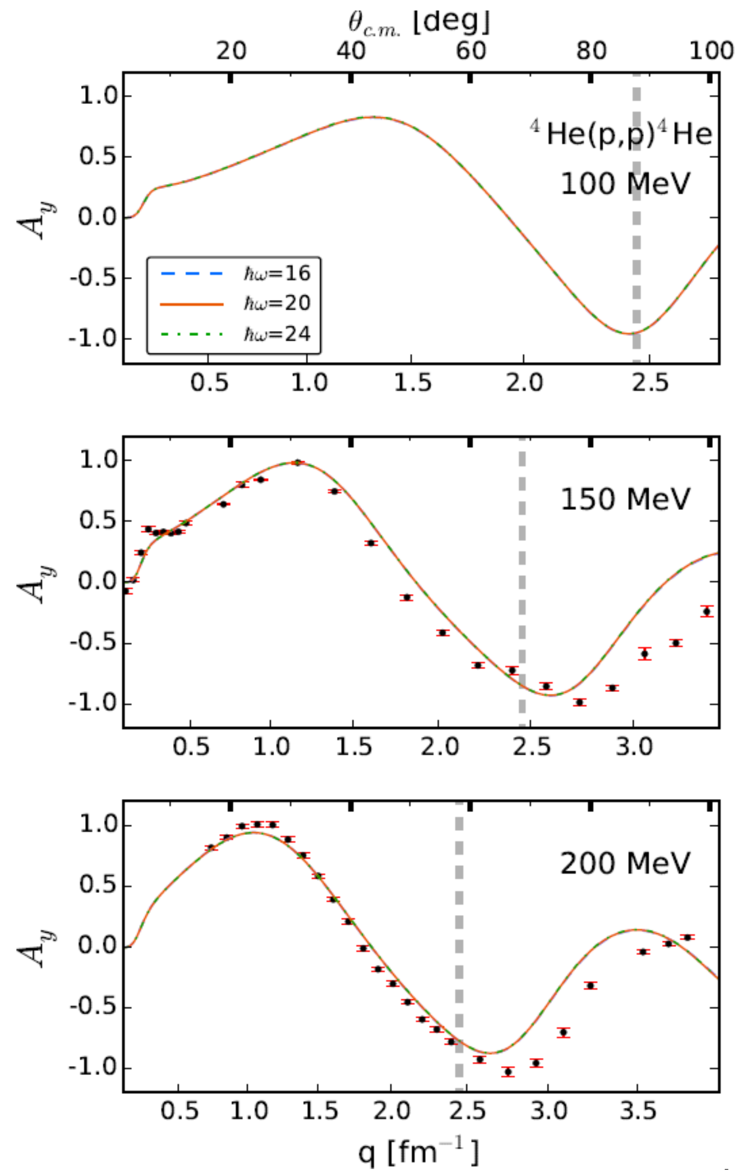
$$\vec{q}_{nn} = \vec{q}_{nA} = \vec{q}$$

$$q \approx 480 \text{ MeV} = 2.45 \text{ fm}^{-1}$$

Nmax=18



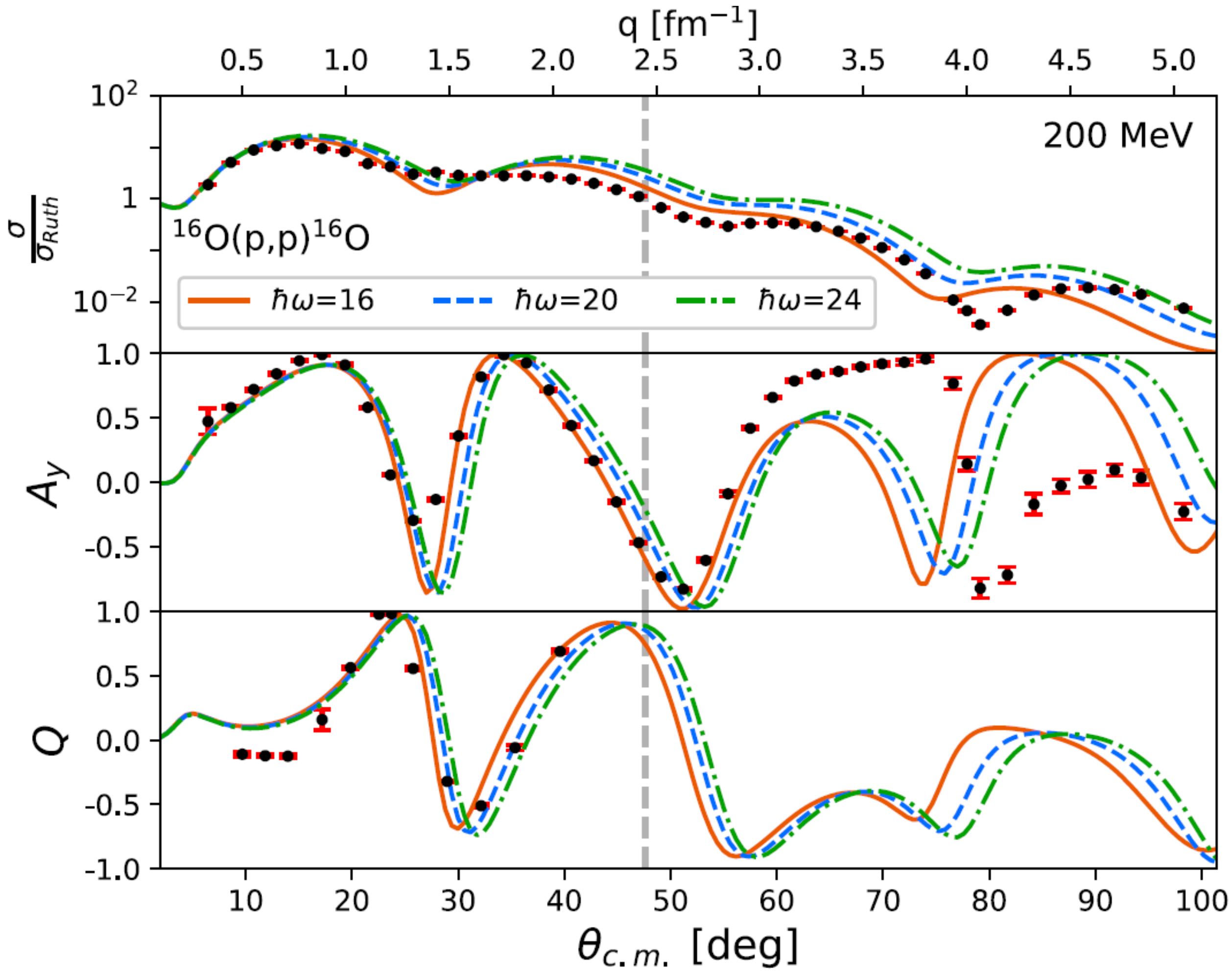
NNLO$_{opt}$ fitted up to Elab=125 MeV



Burrows, Elster, Weppner, Launey, Maris, Nogga, Popa
arXiv:1810.06442

^{16}O

N_{max}=10



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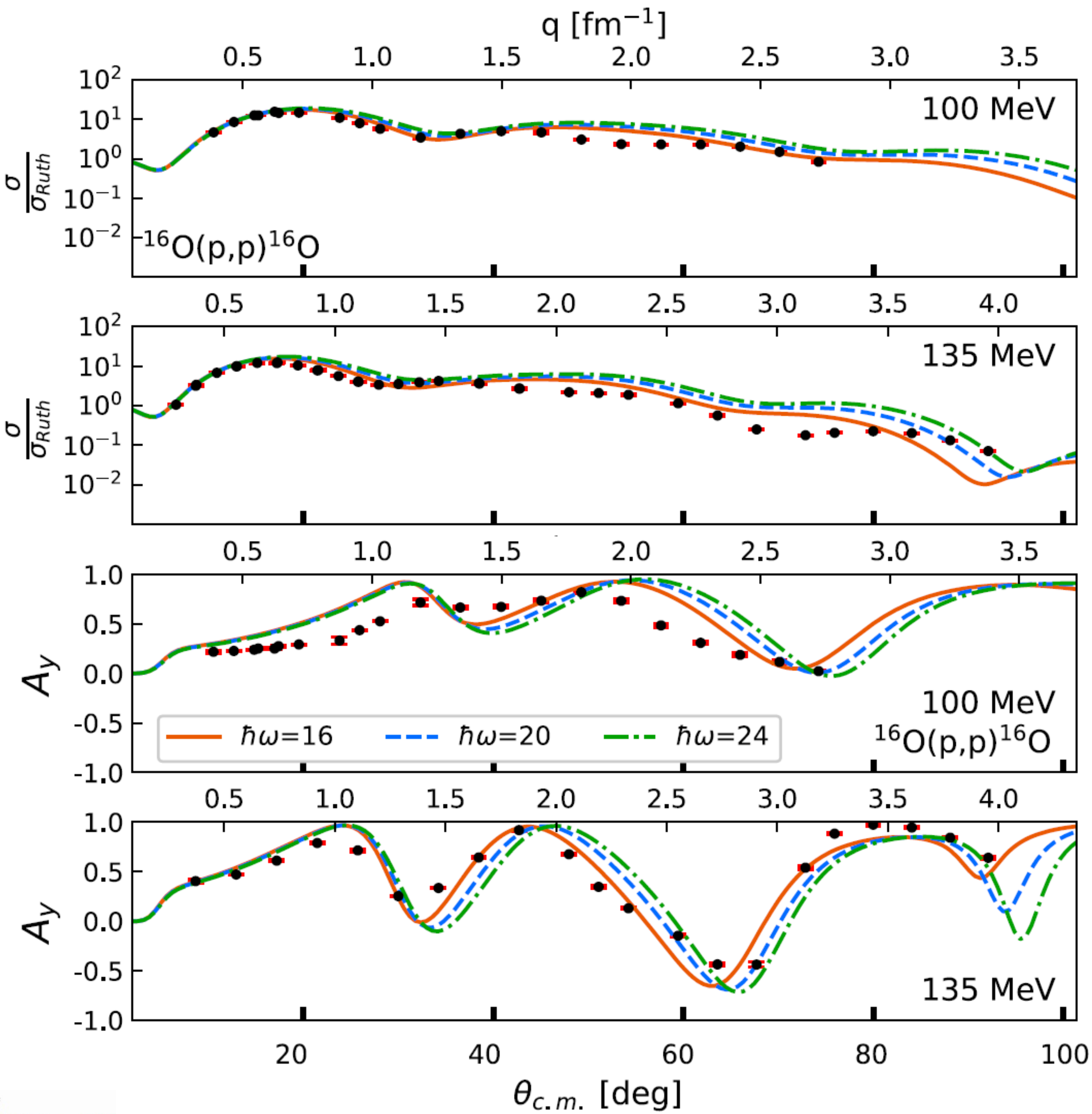
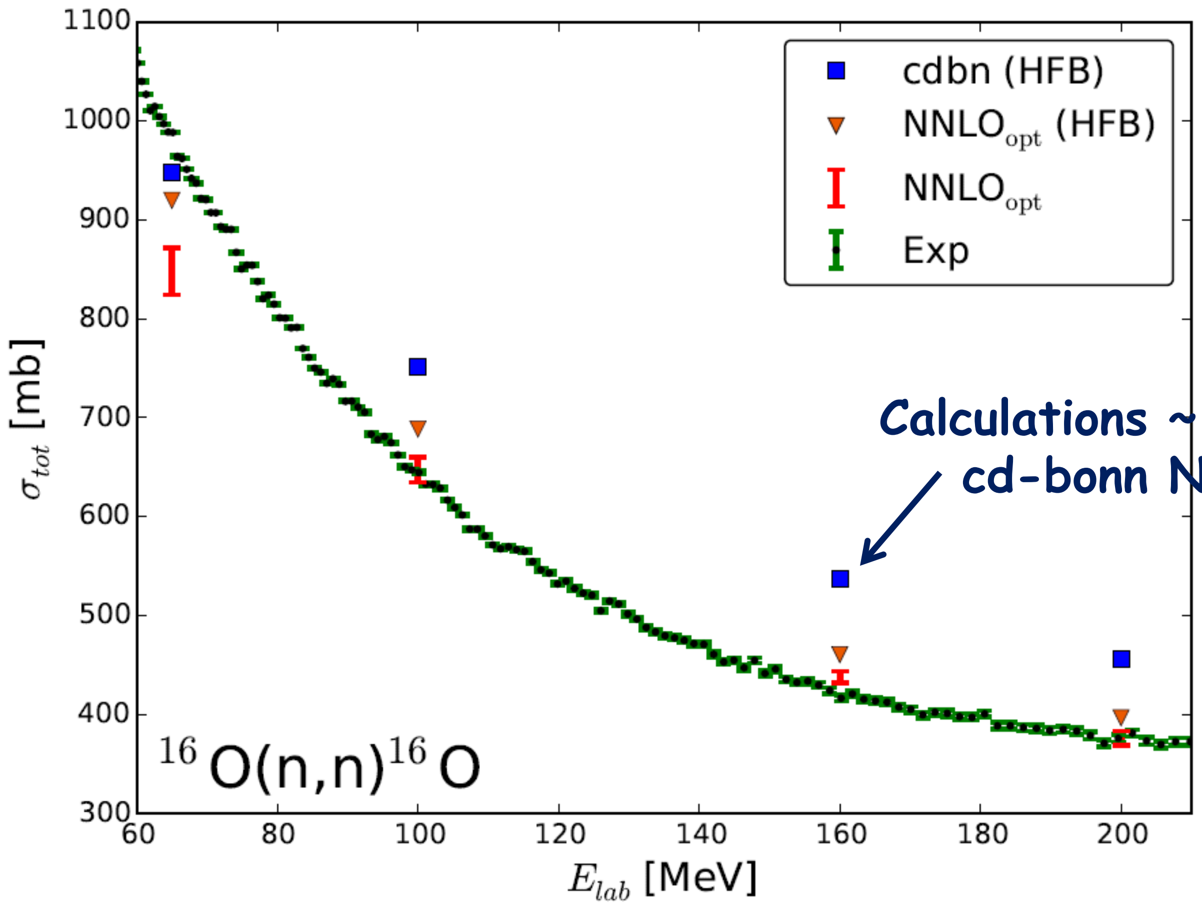
^{16}O



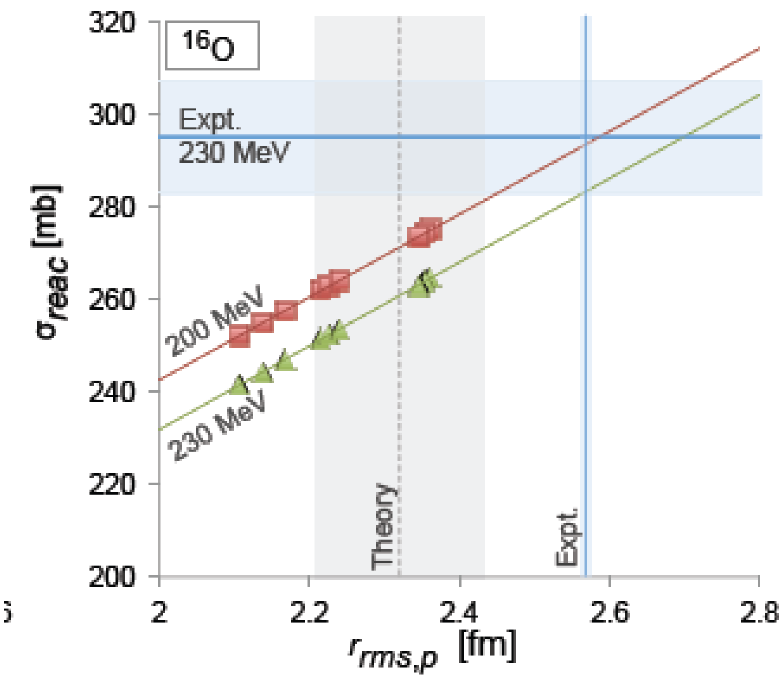
q [fm^{-1}]
0.5
1.0
1.5
2.0
2.5
3.0
3.5
100 MeV
^{16}O(p,p)^{16}O
$\frac{\sigma}{\sigma_{Ruth}}$
135 MeV
4.0
A_y
$\hbar\omega$=16
$\hbar\omega$=20
$\hbar\omega$=24
20
40
60
80
100
$\theta_{c.m.}$ [deg]

Total cross section for neutron scattering

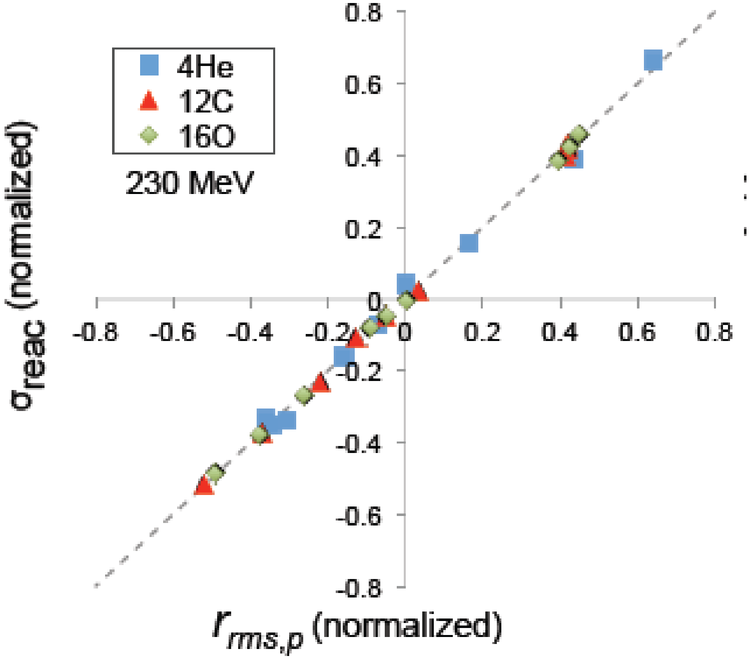


Calculations ~1995 using cd-bonn NN t-matrix

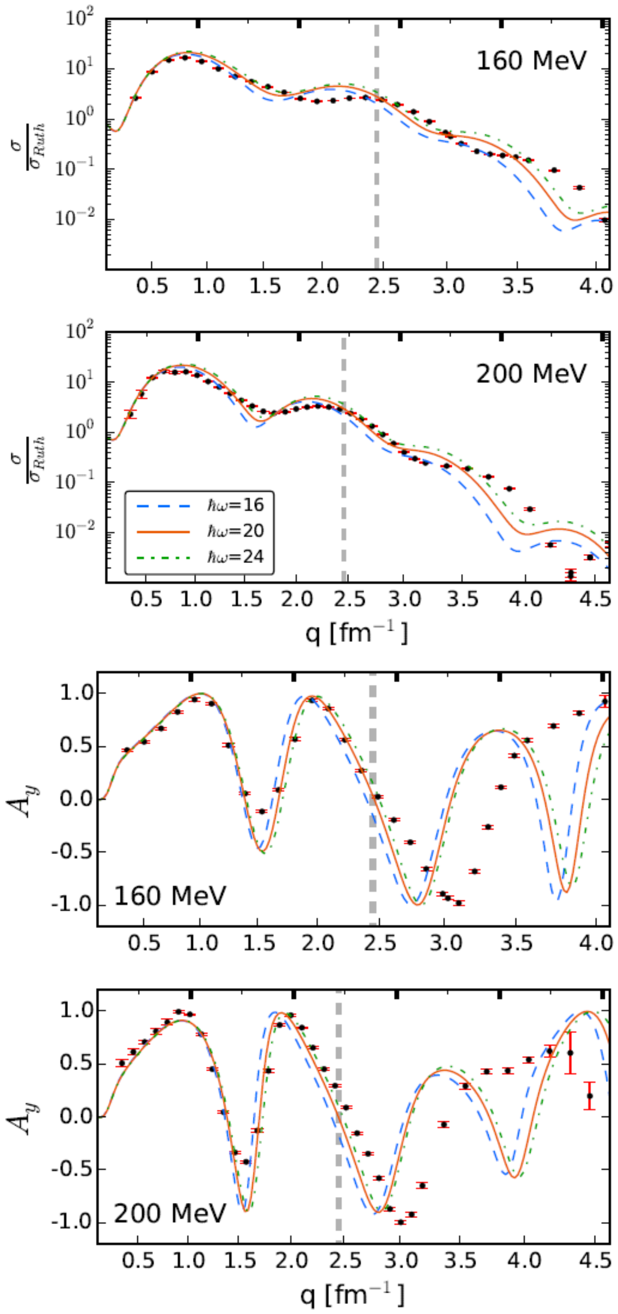
Reaction cross section and point proton radius



Reaction cross section and point proton radius

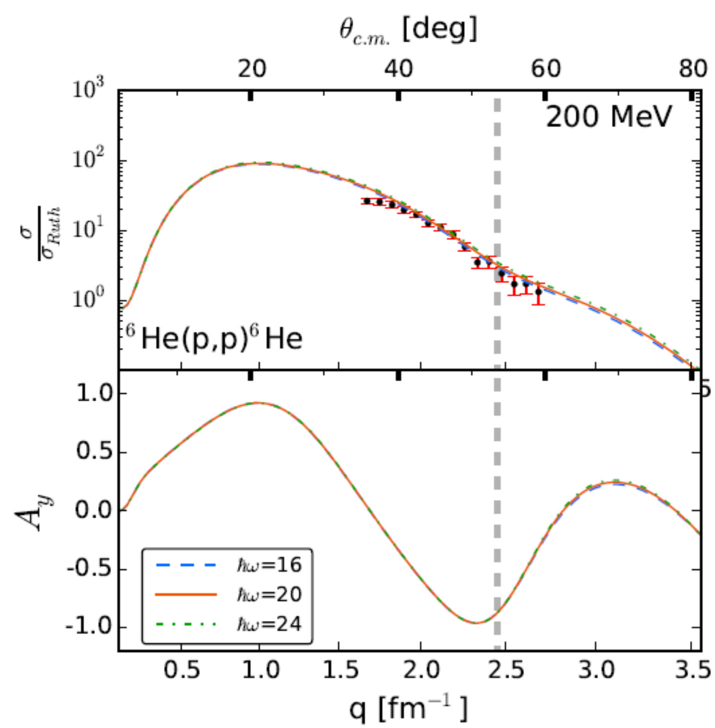


$^{12}C(p,p)^{12}C$



Note:

Implementation of first order term
(past, present, all groups)
only exact for nuclear states with intrinsic spin 0
(≡ spin-flip of struck target nucleon neglected)



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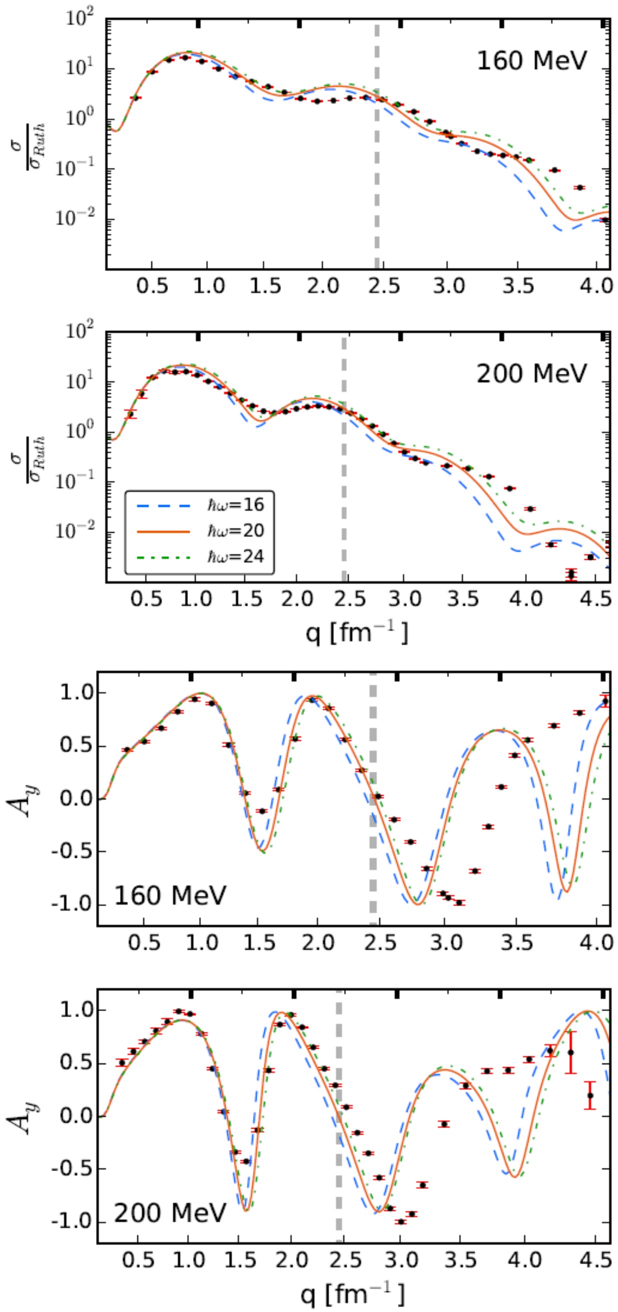


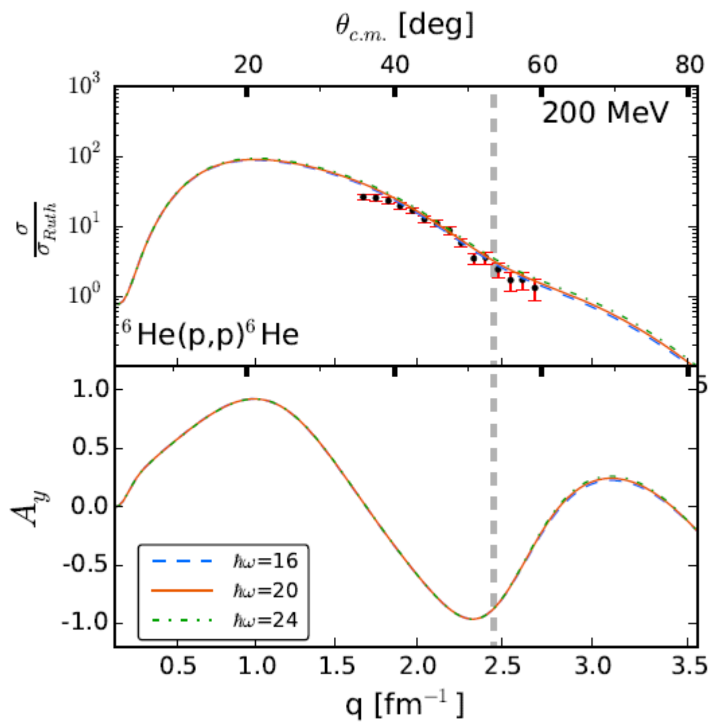
Note:

Implementation of first order term
(past, present, all groups)
only exact for nuclear states with intrinsic spin 0
(≡ spin-flip of struck target nucleon neglected)

^{12}C: spin-0 contribution ~60%

^{16}O: spin-0 contribution ~95%

^{6}He: spin-0 contribution ~ 80-85%



Central part of potential

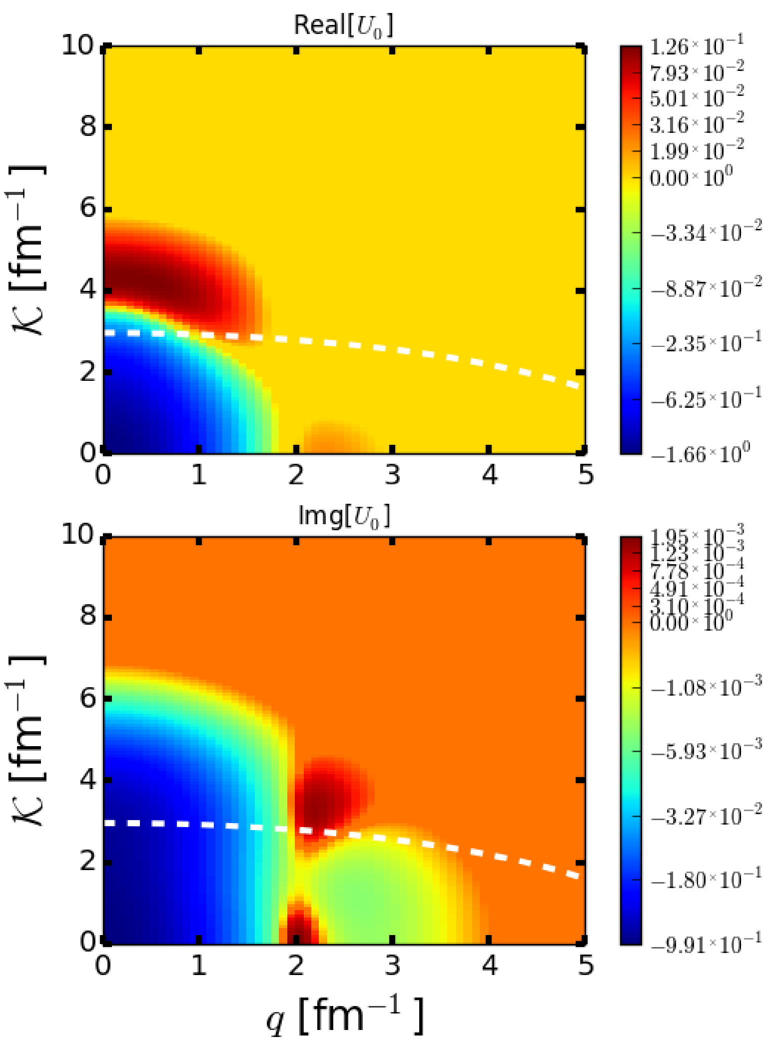
On-shell condition:

$$\left(\frac{A-1}{2A}\right)^2 q^2 + K^2 = k_0^2$$

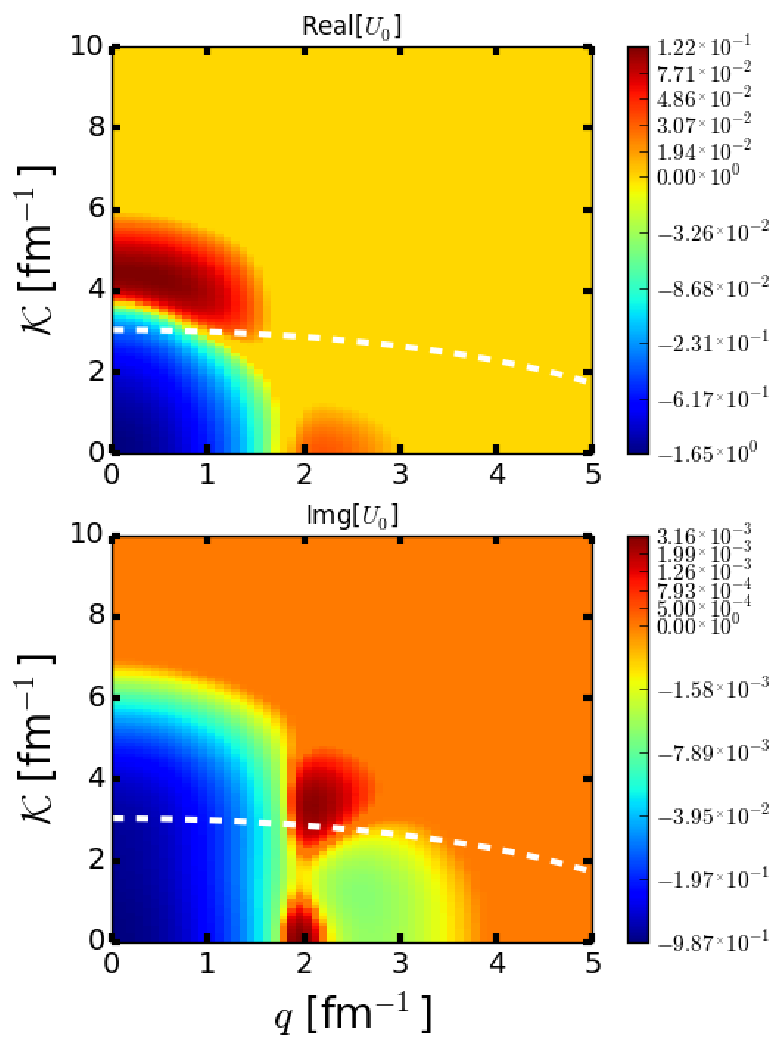
$$\mathbf{K} \equiv \frac{\mathbf{k}+\mathbf{k}'}{2}$$

$$\mathbf{q} \equiv \mathbf{k}' - \mathbf{k}$$

200 MeV, ^{12}C L=0(S-Wave)



200 MeV, ^{16}O L=0(S-Wave)



p+A and n+A effective interactions (‘optical potentials’)

- Renewed urgency in reaction theory community for microscopic input to e.g. (d,p) reaction models .
- Most likely complementary approaches needed for different energy regimes

- **Today:**
  Consistent approach to p+A effective interaction becomes possible.

- **In the multiple scattering approach first order term correction due to non-spin-0 components in ground state needs to be explored:**
  Important for exotic nuclei

- **Different structure approaches need to be explored in this context:**

  e.g. consistency of forces employed, heavier nuclei

- **Systematic approach to higher order corrections**
  (hard but needs to be attempted)

- **Similar formalism e.g. for (p,n) charge exchange reactions**

Central part of potential

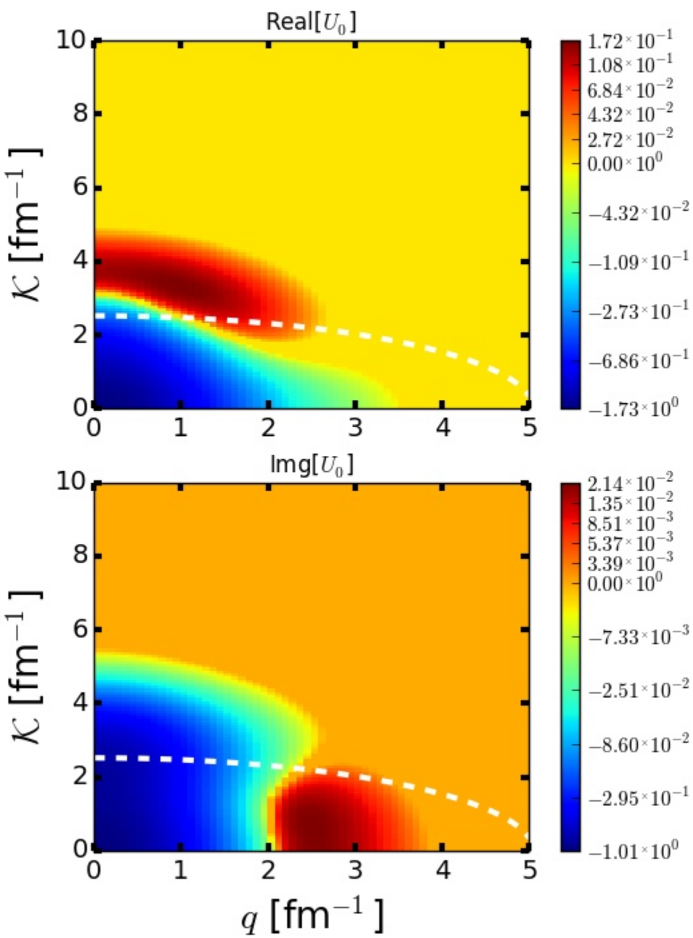
On-shell condition:

$$\left(\frac{A-1}{2A}\right)^2 q^2 + K^2 = k_0^2$$

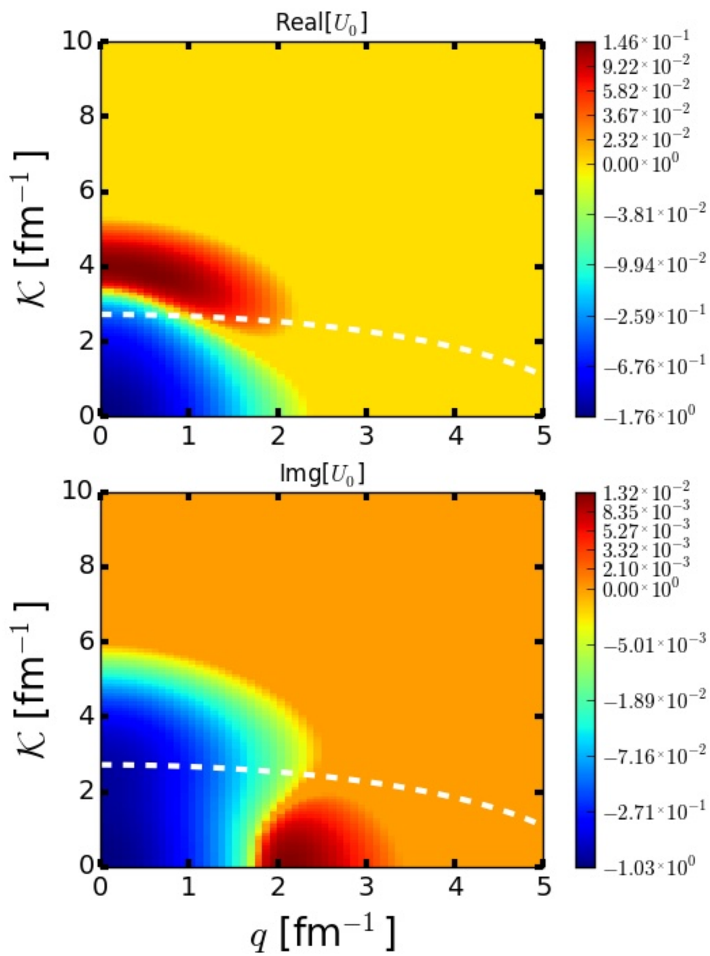
$$\mathbf{K} \equiv \frac{\mathbf{k} + \mathbf{k}'}{2}$$

$$\mathrm{q} \equiv \mathbf{k}' - \mathbf{k}$$

200 MeV, ^{4}He L=0(S-Wave)



200 MeV, ^{6}He L=0(S-Wave)



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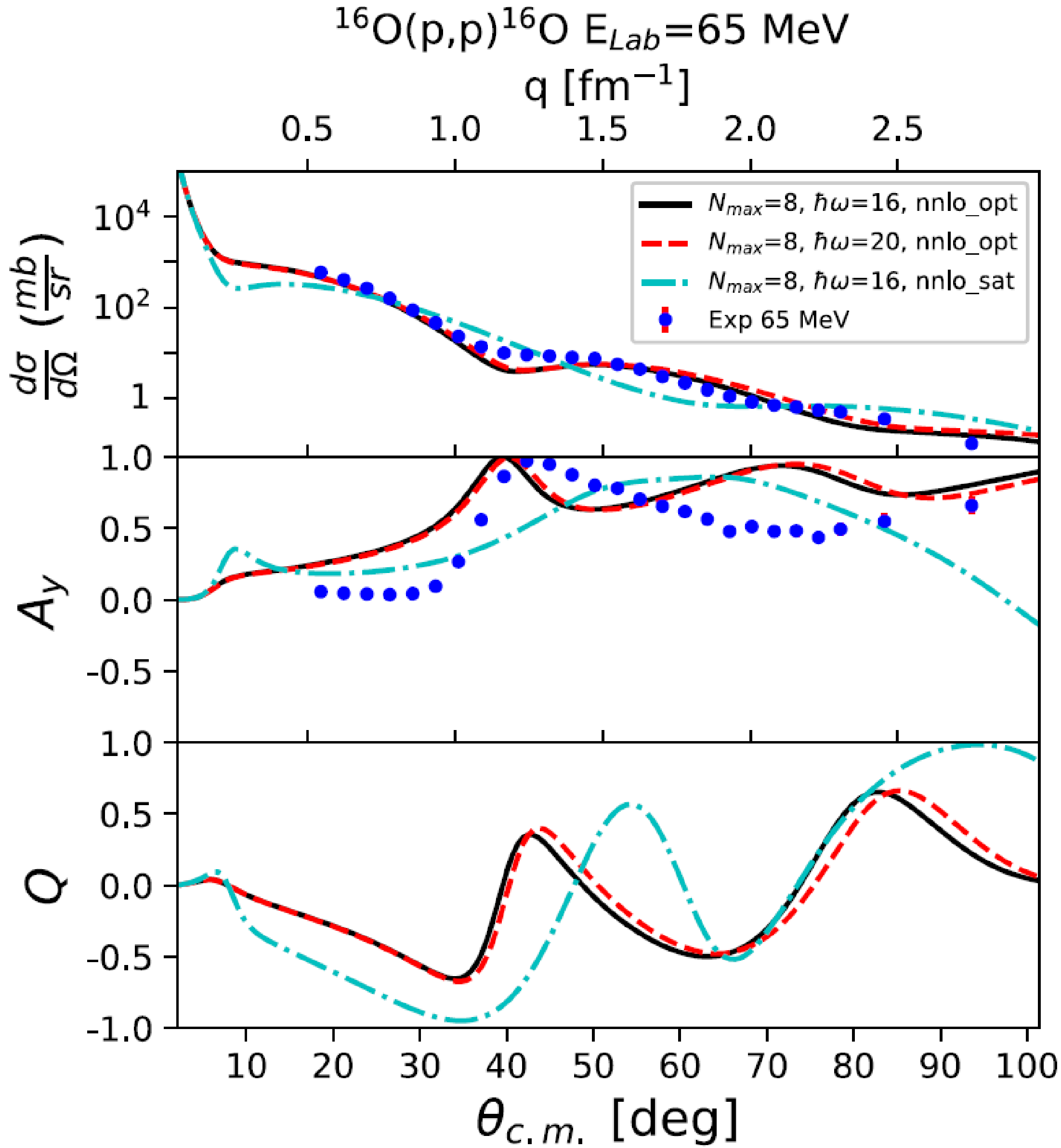

^{16}O(p,p)^{16}O E$_{Lab}$=65 MeV
q [fm^{-1}]
0.5
1.0
1.5
2.0
2.5
N_{max}=8, $\hbar\omega$=16, nnlo_opt
N_{max}=8, $\hbar\omega$=20, nnlo_opt
N_{max}=8, $\hbar\omega$=16, nnlo_sat
Exp 65 MeV
$\frac{d\sigma}{d\Omega}$ $(\frac{mb}{sr})$
10^4
10^2
1
A_y
1.0
0.5
0.0
-0.5
Q
-1.0
10 20 30 40 50 60 70 80 90 100
$\theta_{c.m.}$ [deg]

Previous calculations

Weppner, Elster, Hüber, PRC 57, 1378 (1998)

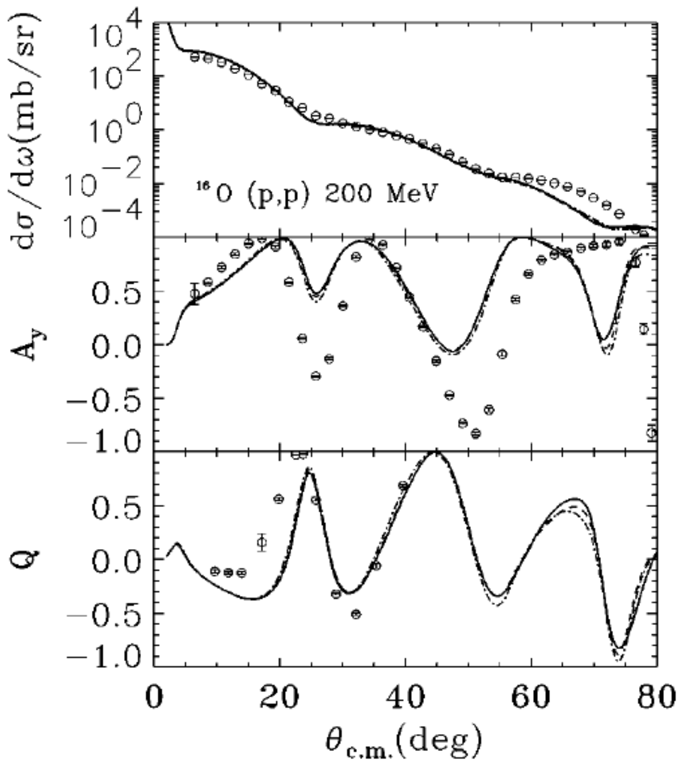

FIG. 1. The angular distribution of the differential cross section ($d\sigma/d\Omega$), analyzing power (A_y), and spin rotation function (Q) are shown for elastic proton scattering from ^{16}O at 200 MeV laboratory energy. The solid line represents the calculation performed with a first-order full-folding optical potential based on the DH density [14] and the CD-Bonn model [2]. The dashed line uses the NijmI model instead, the dash-dotted line the NijmII model [1]. The data are taken from Ref. [19].

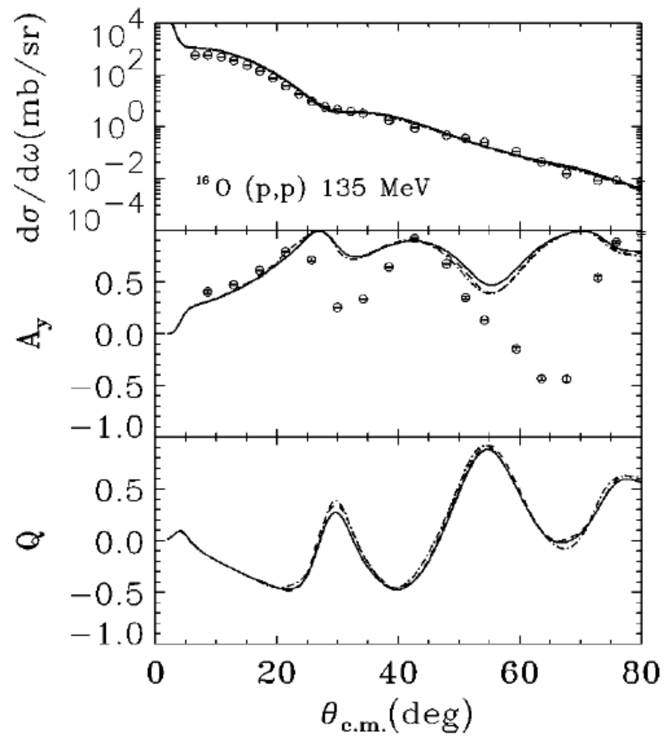

FIG. 5. Same as Fig. 1, except that the projectile energy is 135 MeV. The data are taken from Ref. [22].

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