Nuclear energy density functional by KIDS

Chang Ho Hyun
Daegu University, Korea

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- **KIDS EDF**

Korea: (taiji: source, beginning of the world)

IBS: Panagiota Papakonstantinou, Yeunhwan Lim

Daegu Univ.: Chang Ho Hyun

Sungkyunkwan Univ.: Tae-Sun Park

- **Application to nuclei**

Panagiota Papakonstantinou (RISP/IBS, Korea)

Hana Gil (Kyungpook Nat'l Univ., Korea)

Yongseok Oh (Kyungpook Nat'l Univ., Korea)

- **Application to astrophysics**

Chang-Hwan Lee (Pusan Nat'l Univ., Korea)

Kyujin Kwak (UNIST, Korea)

Young-Min Kim (UNIST, Korea)

Outline

I. KIDS EDF

II. Results for nuclear matter

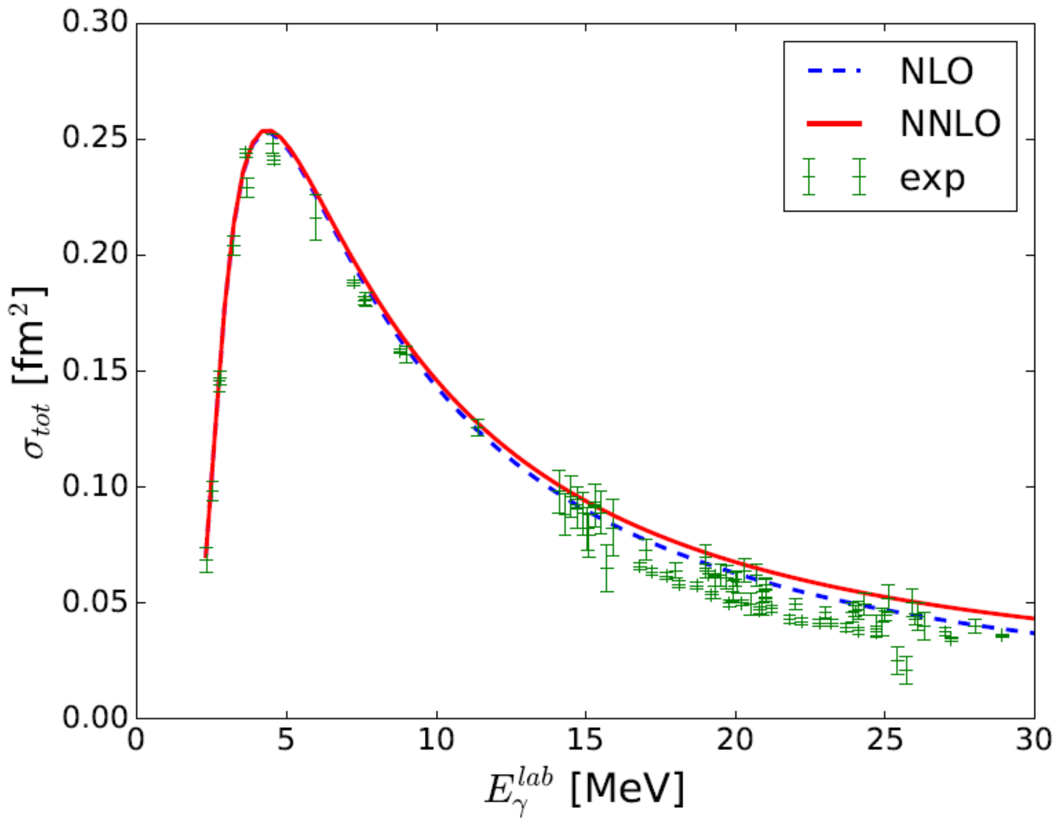
III. Results for nuclei

IV. Summary

I. KIDS EDF

- Pionless EFT for the total cross section of np → dγ at BBN energies

Pionless EFT: Valid for $p < m_\pi$ ($p \sim m_\pi$, $E_\gamma \sim 10$ MeV)



Y.-H. Song, S.-i. Ando, CHH, PRC 96 (2017)

- Pionless EFT for dilute neutron many-body system
  - Energy per particle of neutron gas with a pionless EFT

H.-W. Hammer, R.J.Furnstahl, NPA 678 (2000)

$$\mathcal{E}_1 = \rho\,(g-1)\,\frac{k_{\rm F}^2}{2M}\,\frac{2}{3\pi}\,k_{\rm F}a_s\ , \qquad \mathcal{E}_2 = \rho\,(g-1)\,\frac{k_{\rm F}^2}{2M}\,(k_{\rm F}a_s)^2\,\frac{4}{35\pi^2}\,(11-2\ln 2)$$

$$\mathcal{E}_3 = \rho\,\frac{k_{\rm F}^2}{2M}\Big[(g-1)\frac{1}{10\pi}\,(k_{\rm F}a_s)^2\,k_{\rm F}r_s + (g+1)\,\frac{1}{5\pi}\,(k_{\rm F}a_p)^3$$
$$+(g-1)\{(0.07550\pm 0.00003)+(g-3)\,(0.05741\pm 0.00002)\}\,(k_{\rm F}a_s)^3\Big]\,,$$

Energy per particle expanded in powers of $(k_{\rm F}\ a_{s,\,p})$

$a_{s,\,p}$: scattering length in free space (~ 20 fm ~1/(10 MeV))

Expansion scheme in dense nuclear matter

- Momentum scale: k_F (270 MeV for saturated symmetric matter)
- k_F / m_ρ : less than 1 even at $\rho = 8\rho_0$ (because $k_F \propto \rho^{1/3}$)
- Expand the energy density in powers of k_F / m_ρ

Rules

- Rule1: Expand EDF for **homogeneous matter** in powers of k_F
- Rule2: Fit the parameters to the well-known nuclear matter properties
- Rule3: Keeping the parameters unchanged, apply the model to nuclei

II. Results for nuclear matter

- KIDS Ansatz

$$\mathcal{E}(\rho,\delta) = \mathcal{T}(\rho,\delta) + \sum_{i=0} c_i(\delta)\rho^{1+i/3}, \qquad \rho = \rho_n + \rho_p$$

$$c_i(\delta) = \alpha_i + \delta^2\beta_i, \qquad \delta \equiv (\rho_n - \rho_p)/\rho$$

- Fitting
  - $c_0(0)$, $c_1(0)$, $c_2(0)$: ρ_0, E/A, K_0 (assume $c_3(0) = 0$)
  - $c_i(1)$: APR PNM EoS (14 data in $\rho = 0.02 - 0.96$ fm^{-3})

Fitting result

$$\chi^2(\delta) = \sum_j \exp\{-\beta\rho_j/\varrho_0\} \left(\frac{\mathcal{E}(\rho_j) - D_j}{\mathcal{T}(\rho_j)} \right)^2 \; ; \quad \beta \geq 0$$

- **Fitting to APR PNM EoS**

	c0	c0 , c1	c0 - c2 (P3)	c0 - c3 (P4)	c0 - c5 (P5)
χ^2	0.071632	0.001566	0.000529	0.000138	0.000115

- Fitting improves with more terms: Natural
- Improvement saturates at P5 (5 terms for PNM)
- **Double check: Fitting to QMC PNM EoS** (J. Carlson et al., Rev, Mod. Phys. 87 (2015))

	P4	P5	P6
χ^2	1.5×10^{-6}	1.3×10^{-6}	1.3×10^{-6}

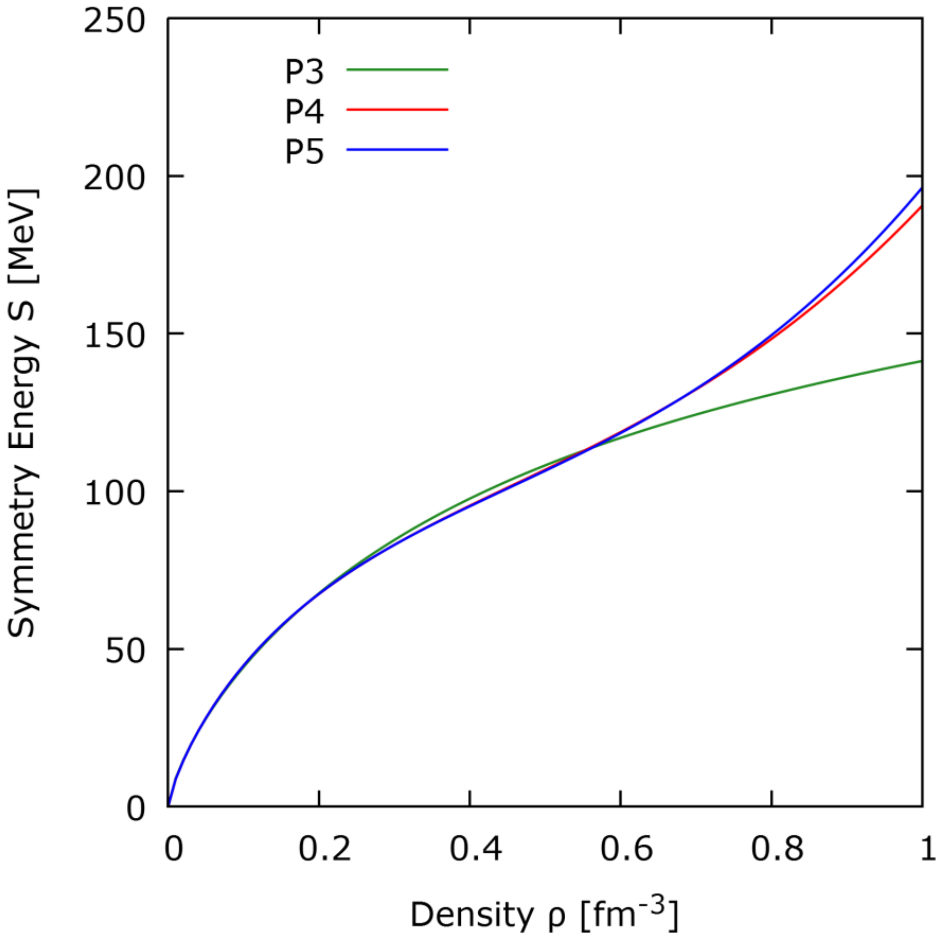
- **Different combination of terms: ex) Fitting with two terms**

	c0, c1	c0, c2	c0,c3	c1, c2	c1, c3	c2, c3
χ^2	0.001566	0.000719	0.003220	0.010973	0.023312	0.050970

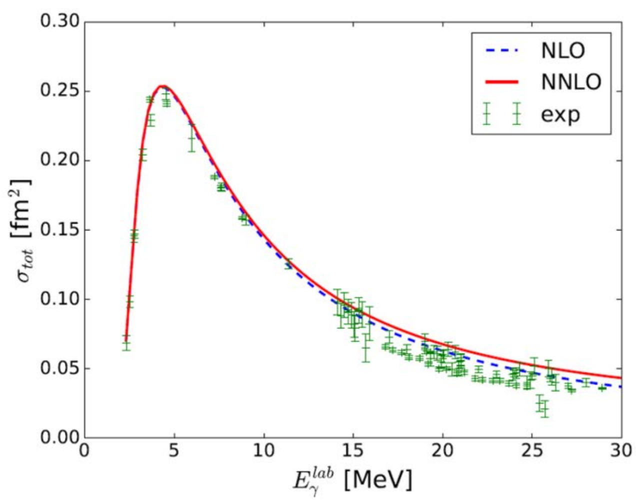
- Absence of lowest order term (c0) makes the fitting worse.
- High order terms make χ^2 larger.
- Best fitting result obtained when we increase order from the lowest order.

Convergence

- Symmetry energy up to P5



- Around 0.25 fm^{-3}, P3 deviates from P4 but agrees with each other to 0.6 fm^{-3}.
- To 0.8 fm^{-3}, P4 and P5 coincide, and behave very similar to ρ ~ 1 f m^{-3}.

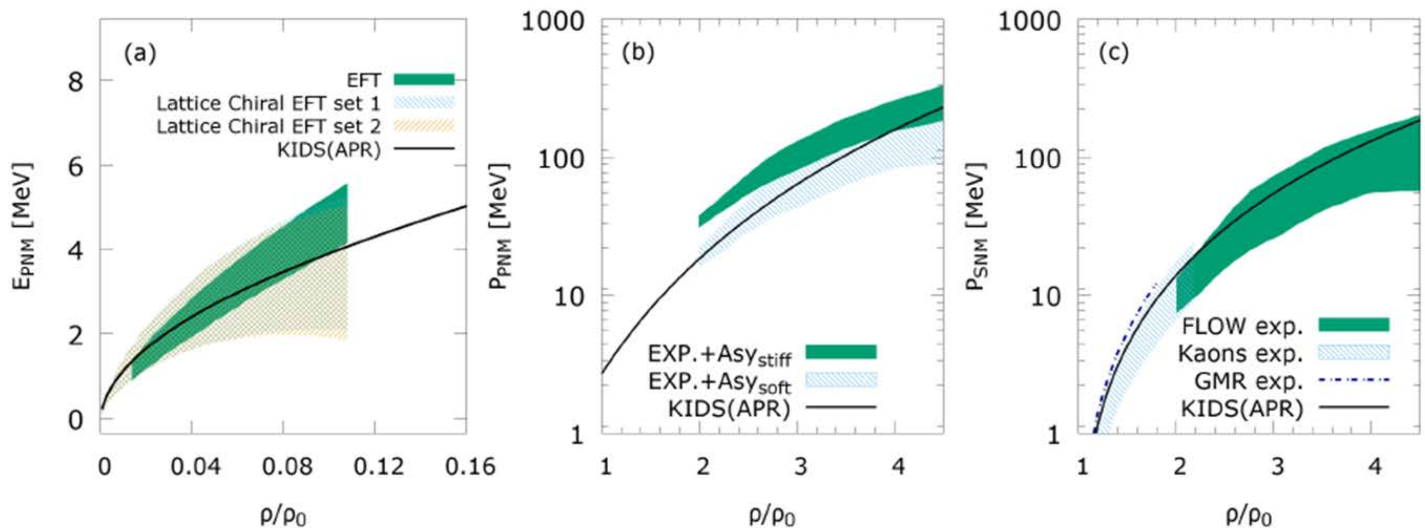


Nuclear matter properties: P4 parameters

Dutra et al., PRC 85 (2012)

	K_0 (MeV)	$-Q_0$ (MeV)	J (MeV)	L (MeV)	K_τ (MeV)	$S(\rho_0/2)/J$	$3P_{PNM}/(L\,\rho_0)$
KIDS	240.00*	372.65	32.75	49.10	-377.06	0.667	1.03
Exp./Emp.	200-260	200-1200	30-35	40-76	-760,-372	0.57-0.86	0.90-1.10

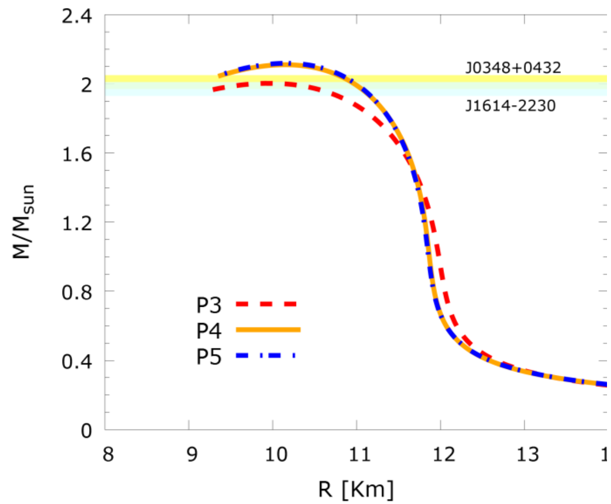
*: Input data

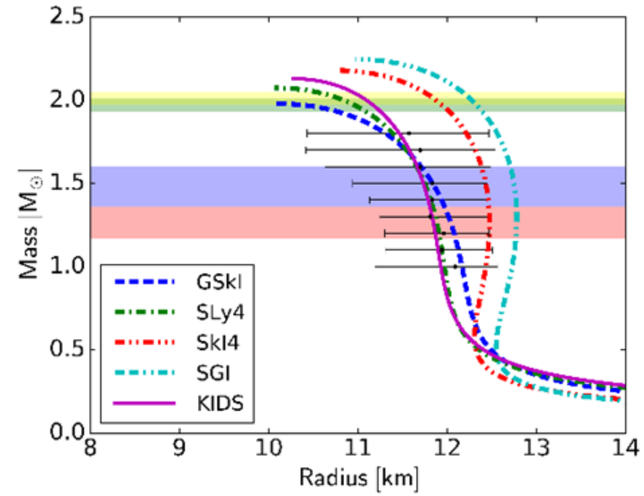


Lattice chiral EFT: E .Epelbaum et al., EPJA40 (2009)

Neutron star

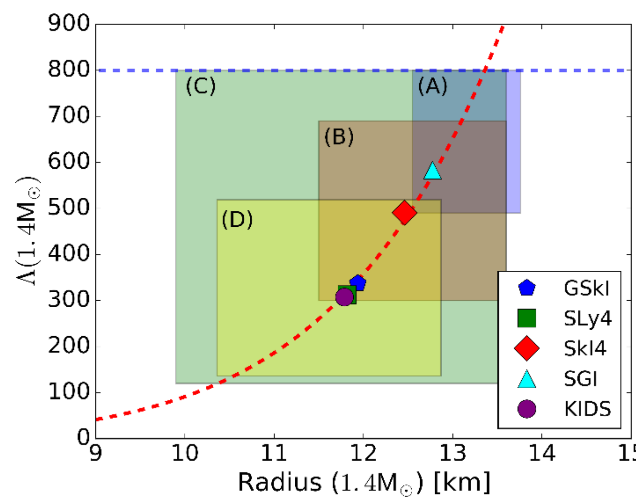
- Mass-radius





- Tidal deformability

	GSkI	SLy4	SkI4	SGI	KIDS
$R_{1.4M_\odot}$ [km]	11.94	11.82	12.46	12.77	11.79
$k_2(1.4M_\odot)$	0.079	0.077	0.092	0.097	0.076
$\lambda(1.4M_\odot)$ $[10^{36}\text{g cm}^2\text{s}^2]$	1.906	1.770	2.772	3.292	1.737
$\Lambda(1.4M_\odot)$	337.2	312.9	490.9	583.0	307.5



More recent analysis (B. P. Abbott et al. arXiv:180511581v1 [gr-qc]): $\Lambda(1.4M_\odot) = 190^{+390}_{-120}$

Red dot curve: $\Lambda \propto R^{7.5}$ (E. Annala et al., PRL120(2018))

III. Results for nuclei

- Skyrme type force: reverse transformation of KIDS EDF

$$v_{i,j}(\mathbf{k},\mathbf{k}') = (t_0 + y_0 P_\sigma)\delta(\mathbf{r}_i - \mathbf{r}_j) + \frac{1}{6}\sum_{n=1}^{3}(t_{3n} + y_{3n}P_\sigma)\rho^{n/3}\delta(\mathbf{r}_i - \mathbf{r}_j)$$
$$+ \frac{1}{2}(t_1 + y_1 P_\sigma)[\delta(\mathbf{r}_i - \mathbf{r}_j)\mathbf{k}^2 + \mathbf{k}'^2\delta(\mathbf{r}_i - \mathbf{r}_j)] + (t_2 + y_2 P_\sigma)\mathbf{k}' \cdot \delta(\mathbf{r}_i - \mathbf{r}_j)\mathbf{k}$$
$$+ iW_0\,\mathbf{k}' \times \delta(\mathbf{r}_i - \mathbf{r}_j)\,\mathbf{k} \cdot (\boldsymbol{\sigma}_i - \boldsymbol{\sigma}_j),$$

- Transformation of coefficients

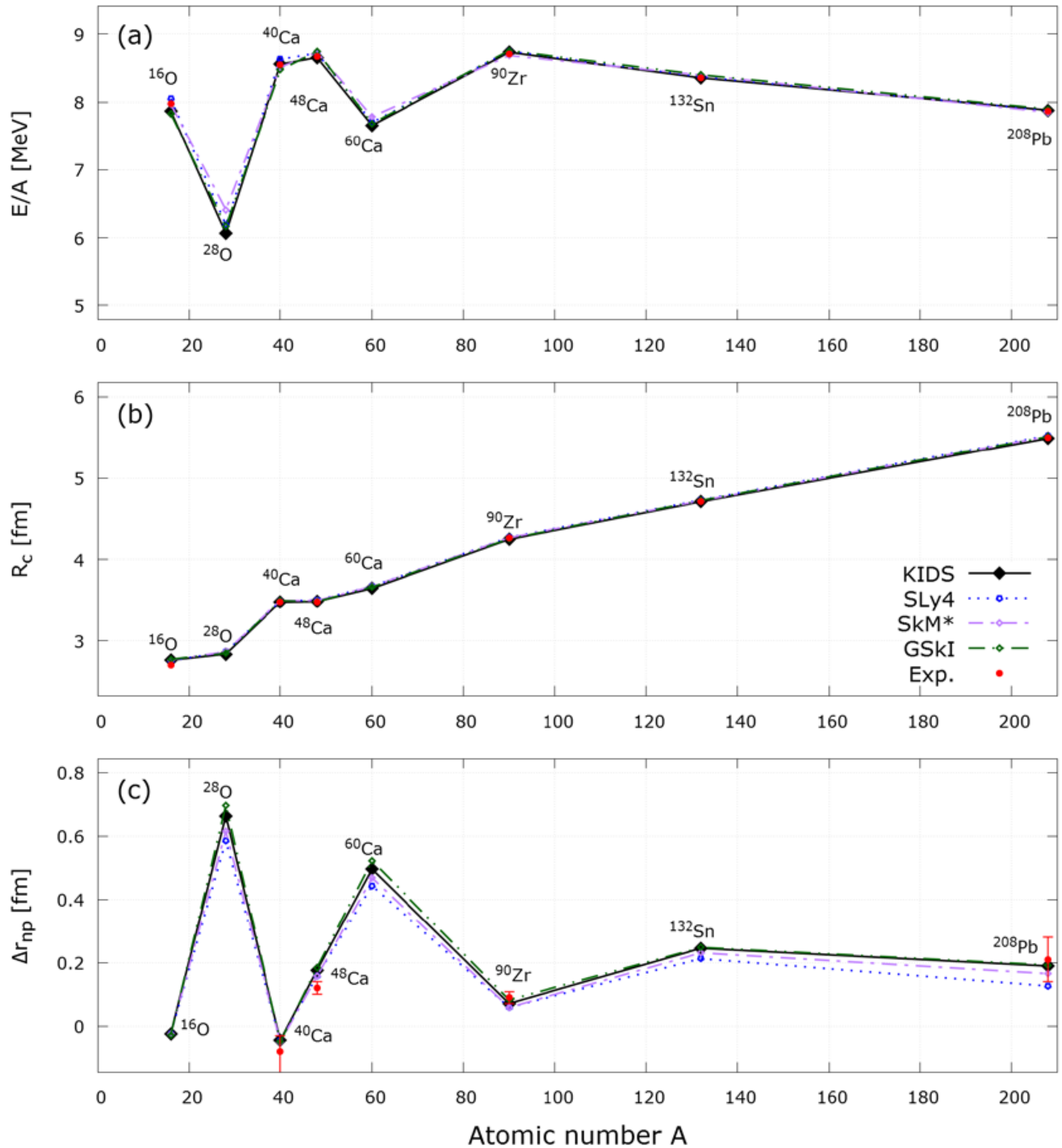
$$t_0 = \frac{8}{3}c_0(0)\,, \quad y_0 = \frac{8}{3}c_0(0) - 4c_0(1),$$
$$t_{3n} = 16c_n(0)\,, \quad y_{3n} = 16c_n(0) - 24c_n(1), \quad (n \neq 2)$$
$$t_{32} = 16c_2(0) - \frac{3}{5}\left(\frac{3}{2}\pi^2\right)^{2/3}\theta_s \equiv 16c_2(0)(1-k)$$
$$y_{32} = 16c_2(0) - 24c_2(1) + \frac{3}{5}(3\pi^2)^{2/3}\left(3\theta_\mu - \frac{\theta_s}{2^{2/3}}\right) \equiv [16c_2(0) - 24c_2(1)](1-k')$$

with

$$\theta_s \equiv 3t_1 + 5t_2 + 4y_2 = \frac{5}{3}(3\pi^2)^{-2/3}16c_2(0)k$$
$$\theta_\mu \equiv t_1 + 3t_2 - y_1 + 3y_2 = -\frac{5}{9}(3\pi^2)^{-2/3}[16c_2(0) - 24c_2(1)]k' + \frac{\theta_s}{3 \cdot 2^{2/3}}.$$

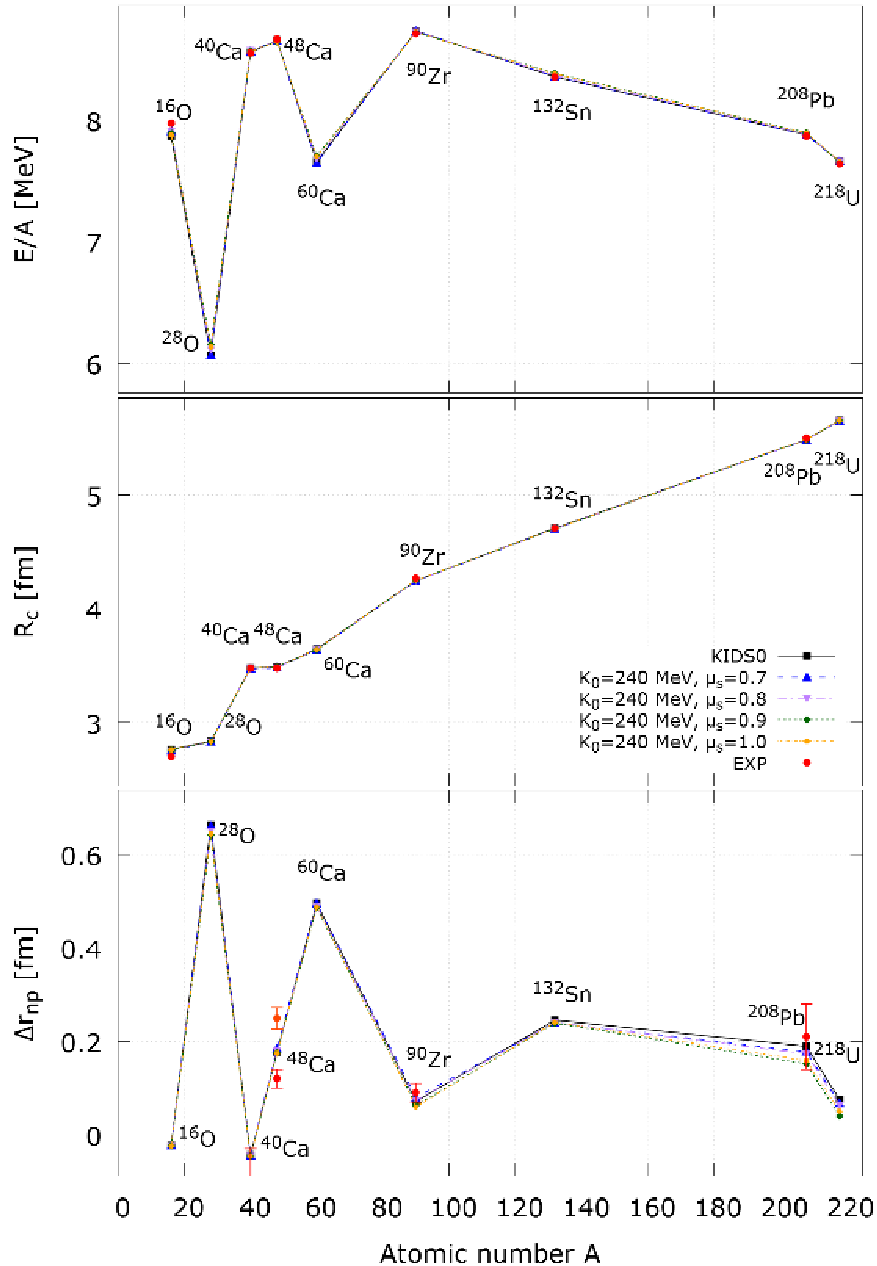
k, *k*’: fraction of gradient terms in the c2 term ($\rho^{2/3}$)

- Two parameter fitting
  - Assume $k=k'$
  - Equivalent to $y_1=y_2=0$
  - Parameters k, W_0 remaining
  - Fit to E/A and R_c of ^{40}Ca, ^{48}Ca, ^{208}Pb



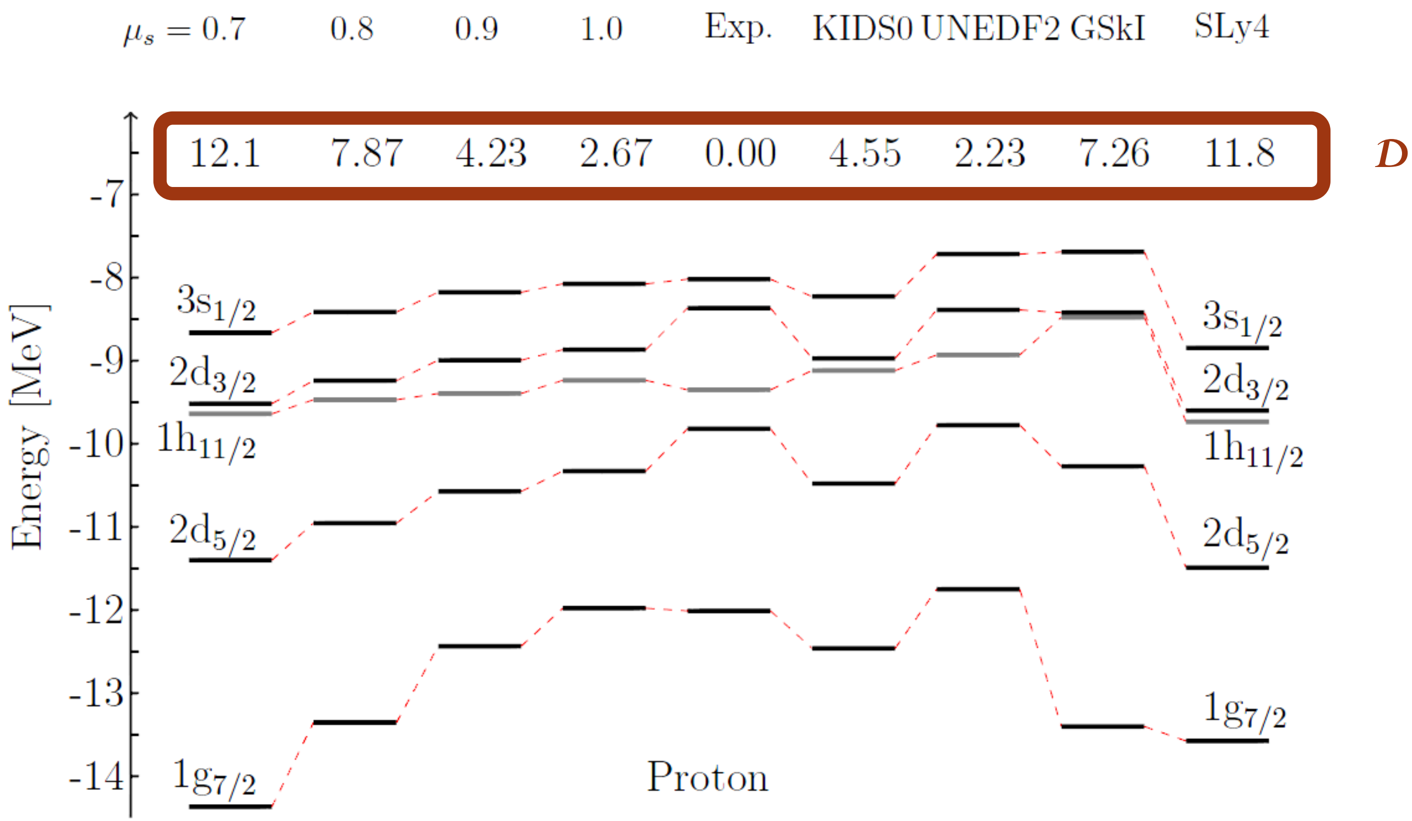
- Dependence on the effective mass
  - Assume non-zero to y_1, y_2values
  - Fit them to produce specific values of effective mass
  - Isoscalar effective mass

$$\mu_s^{-1} \equiv (m_{\rm IS}^*/m)^{-1} = 1 + \frac{m}{8\hbar^2}\,\rho\,\theta_s$$

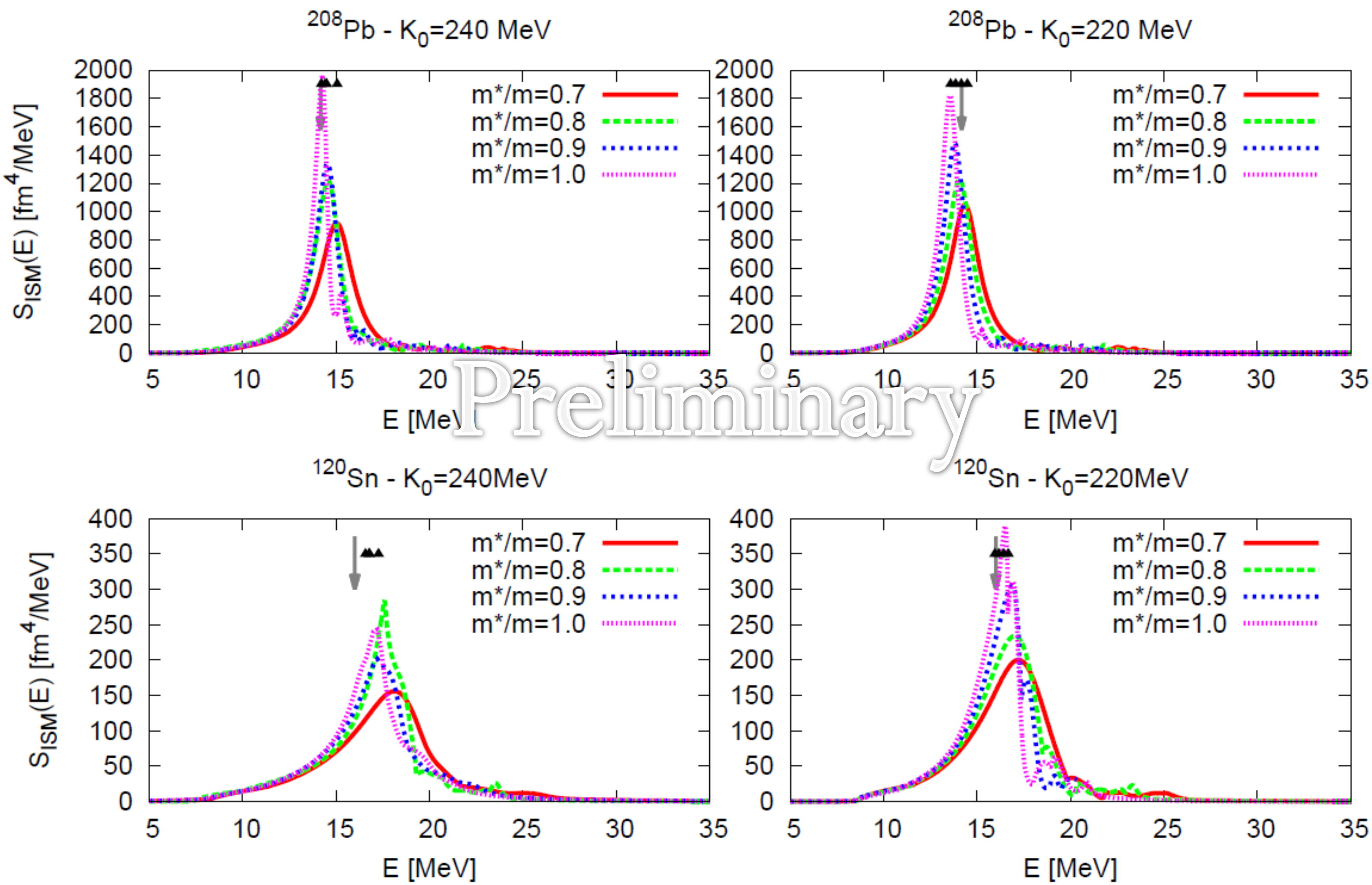


- Proton level scheme of 208Pb

$$D \equiv \frac{1}{N}\sum_{i=1}^{N}\left|\frac{E_i^{\mathrm{expt}} - E_i^{\mathrm{cal}}}{E_i^{\mathrm{expt}}}\right|$$



- Dynamic property: Isoscalar giant monopole resonance
  - Sn fluffiness: ISGMR energy of Sn isotope larger than exp. by about 1 MeV



IV. Summary

- Trial to link low energy EFT to nuclear matter and structures
- Novel EDF constructed with established rules

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- Minimal number of necessary terms identified
- Nuclear matter properties agree well with exp./emp. data
- Most updated data of neutron stars well reproduced
- Nuclear properties reproduced over wide range of mass number
- Effective mass controlled without altering bulk properties
- Possibility to reproduce dynamical properties of nuclei