Nuclear mass table
in deformed relativistic continuum
Hartree-Bogoliubov theory

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Continuum and pairing correlations play a critical role in exotic nuclei.

- Relativistic continuum Hartree-Bogoliubov (RCHB) theory
  - treat pairing correlations in the presence of the continuum properly
  - RCHB theory is used to explore nuclear mass, especially nucleon drip lines by assuming spherical symmetry
    - Calculations for $8 \leq Z \leq 120$ isotopes was completed$^{1}$

- Deformed RHB (DRHB) theory in continuum
  - assume axial symmetry
  - Investigate deformation effects on the neutron drip line

Introduction

- Covariant density functional theory
  - Finite-range meson-exchange
  - Zero-range point-coupling

- Our purpose
  - effect of deformation on position of neutron drip line with DRHB theory
  - Ar isotopes as examples
Lagrangian density of the point-coupling model\(^1\)

\[ \mathcal{L} = \mathcal{L}^{\text{free}} + \mathcal{L}^{4f} + \mathcal{L}^{\text{hot}} + \mathcal{L}^{\text{der}} + \mathcal{L}^{\text{em}} \]

where

\[ \mathcal{L}^{\text{free}} = \bar{\psi} (i\gamma_\mu \partial^\mu - m) \psi \]
\[ \mathcal{L}^{4f} = -\frac{1}{2} \alpha_S (\bar{\psi} \psi)(\bar{\psi} \psi) - \frac{1}{2} \alpha_V (\bar{\psi} \gamma_\mu \psi)(\bar{\psi} \gamma^\mu \psi) - \frac{1}{2} \alpha_{TV} (\bar{\psi} i\tau_\mu \gamma_\mu \psi)(\bar{\psi} i\tau^\mu \gamma^\mu \psi) \]
\[ \mathcal{L}^{\text{hot}} = -\frac{1}{3} \beta_S (\bar{\psi} \psi)^3 - \frac{1}{4} \gamma_S (\bar{\psi} \psi)^4 - \frac{1}{4} \gamma_V [(\bar{\psi} \gamma_\mu \psi)(\bar{\psi} \gamma^\mu \psi)]^2 \]
\[ \mathcal{L}^{\text{der}} = -\frac{1}{2} \delta_S \partial_\nu (\bar{\psi} \psi) \partial^\nu (\bar{\psi} \psi) - \frac{1}{2} \delta_V \partial_\nu (\bar{\psi} \gamma_\mu \psi) \partial^\nu (\bar{\psi} \gamma^\mu \psi) \]
\[ \phantom{\mathcal{L}^{\text{der}}} - \frac{1}{2} \delta_{TV} \partial_\nu (\bar{\psi} i\tau_\mu \gamma_\mu \psi) \partial^\nu (\bar{\psi} i\tau^\mu \gamma^\mu \psi) \]
\[ \mathcal{L}^{\text{em}} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - e \frac{1 - \tau_3}{2} \bar{\psi} \gamma^\mu \psi A_\mu \]

\[1\] Zhao, Li, Yao, Meng, PRC 82, 054319 (2010)
Relativistic continuum Hartree-Bogoliubov theory

- Relativistic Hartree-Bogoliubov (RHB) equation

\[
\begin{pmatrix}
  h_D - \lambda & \Delta \\
  -\Delta^* & -h_D^* + \lambda
\end{pmatrix}
\begin{pmatrix}
  U_k \\
  V_k
\end{pmatrix} = E_k
\begin{pmatrix}
  U_k \\
  V_k
\end{pmatrix}
\]

- Dirac Hamiltonian

\[
h_D = \alpha \cdot p + \beta (M + S(r)) + V(r)
\]

where scalar and vector potentials

\[
S(r) = \alpha_S \rho_S + \beta_S \rho_S^2 + \gamma_S \rho_S^3 + \delta_S \Delta \rho_S
\]

\[
V(r) = \alpha_V \rho_V + \gamma_V \rho_V^3 + \delta_V \Delta \rho_V + eA_0 + \alpha_{TV} \tau_3 \rho_{TV} + \delta_{TV} \tau_3 \Delta TV
\]

with local densities

\[
\rho_S = \sum_{k>0} \bar{V}_k(r)V_k(r), \quad \rho_V = \sum_{k>0} V_k^+(r)V_k(r), \quad \rho_{TV} = \sum_{k>0} V_k^+(r)\tau_3 V_k(r)
\]

The point-coupling constants of PC-PK1 set\(^1\).

<table>
<thead>
<tr>
<th>Coupling constant</th>
<th>Value</th>
<th>Dimension</th>
</tr>
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<tr>
<td>( \alpha_s )</td>
<td>(-3.96291 \times 10^{-4} )</td>
<td>MeV(^{-2})</td>
</tr>
<tr>
<td>( \beta_s )</td>
<td>(8.6653 \times 10^{-11} )</td>
<td>MeV(^{-5})</td>
</tr>
<tr>
<td>( \gamma_s )</td>
<td>(-3.80724 \times 10^{-17} )</td>
<td>MeV(^{-8})</td>
</tr>
<tr>
<td>( \delta_s )</td>
<td>(-1.09108 \times 10^{-10} )</td>
<td>MeV(^{-4})</td>
</tr>
<tr>
<td>( \alpha_V )</td>
<td>(2.6904 \times 10^{-4} )</td>
<td>MeV(^{-2})</td>
</tr>
<tr>
<td>( \gamma_V )</td>
<td>(-3.64219 \times 10^{-18} )</td>
<td>MeV(^{-8})</td>
</tr>
<tr>
<td>( \delta_V )</td>
<td>(-4.32619 \times 10^{-10} )</td>
<td>MeV(^{-4})</td>
</tr>
<tr>
<td>( \alpha_{TV} )</td>
<td>(2.95018 \times 10^{-5} )</td>
<td>MeV(^{-2})</td>
</tr>
<tr>
<td>( \delta_{TV} )</td>
<td>(-4.11112 \times 10^{-10} )</td>
<td>MeV(^{-4})</td>
</tr>
</tbody>
</table>

fitted to observables of 60 selected spherical nuclei, including the binding energies, charge radii, and empirical pairing gaps.

[1] Zhao, Li, Yao, Meng, PRC 82, 054319 (2010)
Relativistic continuum Hartree-Bogoliubov theory

- Relativistic Hartree-Bogoliubov equation\(^1\)

\[
\begin{pmatrix}
 h_D - \lambda & \Delta \\
 -\Delta^* & -h_D^* + \lambda
\end{pmatrix}
\begin{pmatrix}
 U_k \\
 V_k
\end{pmatrix}
= E_k
\begin{pmatrix}
 U_k \\
 V_k
\end{pmatrix}
\]

- Pairing potential

with a density-dependent delta pairing force

\[
V_{pp}(r, r') = \frac{V_0}{2} (1 - P^\sigma) \delta(r - r') (1 - \frac{\rho(r)}{\rho_{sat}})
\]

with the saturation density \(\rho_{sat} = 0.152 \text{ fm}^{-3}\),
and the pairing force strength \(V_0 = -380.0 \text{ MeV} \cdot \text{fm}^3\)

For axially deformed nuclei, potential $S(\mathbf{r})$ and $V(\mathbf{r})$ and densities are expanded in terms of the Legendre polynomials:

$$f(\mathbf{r}) = \sum_{\lambda} f_{\lambda}(r) P_{\lambda}(\cos \theta), \lambda = 0, 2, 4, ...$$

with

$$f_{\lambda}(r) = \frac{2\lambda + 1}{4\pi} \int d\Omega f(\mathbf{r}) P_{\lambda}(\Omega).$$
Numerical details: box size, mesh size

- Numerical parameters in calculation
  - box size, mesh size
Numerical details: energy and angular momentum cutoff

- Numerical parameters in calculation
  - energy cutoff, angular momentum cutoff

![Graphs showing energy and angular momentum cutoff](image-url)
Density-dependent delta pairing force

\[ V^{pp}(\mathbf{r}, \mathbf{r}') = \frac{V_0}{2} (1 - P^\sigma) \delta(\mathbf{r} - \mathbf{r}') (1 - \frac{\rho(\mathbf{r})}{\rho_{sat}}) \]

with the saturation density \( \rho_{sat} = 0.152 \text{ fm}^{-3} \), and the pairing force strength \( V_0 = -380.0 \text{ MeV} \cdot \text{fm}^3 \).
Comparison of binding energies between experimental values\(^1\) and calculations by using RCHB and DRHB theory + PC-PK1. Both models reproduce well the experiment.

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Comparison of Fermi energy and two neutron separation energy between calculations by using RCHB and DRHB theory + PC-PK1

In the case of Ar isotopes, the neutron drip-line nucleus from $^{62}\text{Ar}$ in the RCHB to $^{70}\text{Ar}$ in the DRHB calculations
05 Results

- Compare with FRDM (finite range droplet model)\(^1\), the neutron drip-line nucleus predicted by RCHB\(^2\) theory has more neutrons.

<table>
<thead>
<tr>
<th>Element (Z)</th>
<th>More N</th>
<th>Element (Z)</th>
<th>More N</th>
</tr>
</thead>
<tbody>
<tr>
<td>O (8)</td>
<td>2</td>
<td>Ca (20)</td>
<td>10</td>
</tr>
<tr>
<td>Ne (10)</td>
<td>10</td>
<td>Mo (42)</td>
<td>12</td>
</tr>
<tr>
<td>Mg (12)</td>
<td>6</td>
<td>Ru (44)</td>
<td>10</td>
</tr>
<tr>
<td>Si (14)</td>
<td>6</td>
<td>Pd (46)</td>
<td>13</td>
</tr>
<tr>
<td>S (16)</td>
<td>6</td>
<td>Cd (48)</td>
<td>6</td>
</tr>
</tbody>
</table>

- The neutron number of the most neutron-rich even-even nuclei predicted to be bound in the RCHB and DRHB theory, in comparing with the calculations without pairing correlations Preliminary

<table>
<thead>
<tr>
<th>Element (Z)</th>
<th>Neutron number</th>
<th>Element (Z)</th>
<th>Neutron number</th>
</tr>
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<tbody>
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<td></td>
<td>No pairing</td>
<td>RCHB</td>
<td>DRHB</td>
</tr>
<tr>
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<td>20</td>
</tr>
<tr>
<td>Ne (10)</td>
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<td>32</td>
<td>30</td>
</tr>
<tr>
<td>Mg (12)</td>
<td>34</td>
<td>34</td>
<td>34</td>
</tr>
<tr>
<td>Si (14)</td>
<td>34</td>
<td>38</td>
<td>38</td>
</tr>
<tr>
<td>S (16)</td>
<td>40</td>
<td>40</td>
<td>40</td>
</tr>
<tr>
<td>Ar (18)</td>
<td>40</td>
<td>44</td>
<td>52</td>
</tr>
</tbody>
</table>

Ca (20) 40 60 58
Mo (42) 112 112 78
Ru (44) 112 112 114
Pd (46) 112 118 118
Cd (48) 112 126 124

Preliminary

### 06 Mass table with DRHB theory

- Example of mass table with DRHB theory
  - Z=18 (Ar) isotopes

<table>
<thead>
<tr>
<th>A</th>
<th>N</th>
<th>$E_b^{\text{cal}}$ (MeV)</th>
<th>$E_b^{\text{exp}}$ (MeV)</th>
<th>$E_b^{\text{cal}}/A$ (MeV)</th>
<th>$E_b^{\text{exp}}/A$ (MeV)</th>
<th>$S_{2n}$ (MeV)</th>
<th>$\lambda_n$ (MeV)</th>
<th>$\lambda_p$ (MeV)</th>
<th>$R_m$ (fm)</th>
<th>$R_n$ (fm)</th>
<th>$R_p$ (fm)</th>
<th>$R_c^{\text{cal}}$ (fm)</th>
<th>$R_c^{\text{exp}}$ (fm)</th>
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</thead>
<tbody>
<tr>
<td>32</td>
<td>14</td>
<td>245.13</td>
<td>246.40</td>
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<td>7.70</td>
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<td>-1.55</td>
<td>3.150</td>
<td>2.996</td>
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<td>8.52</td>
<td>23.92</td>
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<td>-5.64</td>
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<td>3.245</td>
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</tr>
</tbody>
</table>
We studied on deformation effect on the position of neutron drip line with DRHB + PC-PK1 calculation in continuum.

We compared the neutron number of the even-even nuclei predicted to be bound in the RCHB and DRHB theory, taking Ar isotopes as an example.

We can find that the deformation would affect the position of neutron drip line.

Future work
- Additional calculations for other isotopes to study on deformation effects of neutron drip line position
- Study on other characteristic of deformed nuclei such as decoupling shape of halo and core densities
Thank you
Pairing energies for argon isotopic chain

(a) Neutron pairing energy (MeV) vs. Neutron number

(b) Proton pairing energy (MeV) vs. Neutron number

- RCHB
- DRHBc
00 Mass table calculated by RCHB theory

- Mass table calculated by RCHB theory

The limits of the nuclear landscape explored by the relativistic continuum Hartree–Bogoliubov theory


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Table 1

Ground-state properties of nuclei calculated by RCHB theory with PC-FR1, in comparison with the available data of masses and charge radii. In addition, the data labeled with underline means the nucleus is unbound.

<table>
<thead>
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<th>A</th>
<th>N</th>
<th>$E^{ca}$ (MeV)</th>
<th>$E^{rm}$/A (MeV)</th>
<th>$E^{em}$/A (MeV)</th>
<th>$S_{2n}$ (MeV)</th>
<th>$S_{2p}$ (MeV)</th>
<th>$S_{p}$ (MeV)</th>
<th>$\lambda_{n}$ (MeV)</th>
<th>$\lambda_{p}$ (MeV)</th>
<th>$R_{n}$ (fm)</th>
<th>$R_{p}$ (fm)</th>
<th>$R_{ei}$ (fm)</th>
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<th>$J^{(P)}$</th>
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<td>3.576</td>
<td>2.790</td>
<td>2.903</td>
<td>0#</td>
<td>0#</td>
<td></td>
</tr>
</tbody>
</table>

σ 2.55
Why adopt density functional PC-PK1?

A crucial test for covariant density functional theory against new and accurate mass measurement for 53 neutron-rich isotopes from Sn to Pa

- For 12 even-even nuclei, the theory agrees the data within about 600 keV.
- For 25 odd-$A$ and 16 add-add nuclei, the rms values given by PC-PK1 are still within 1MeV.