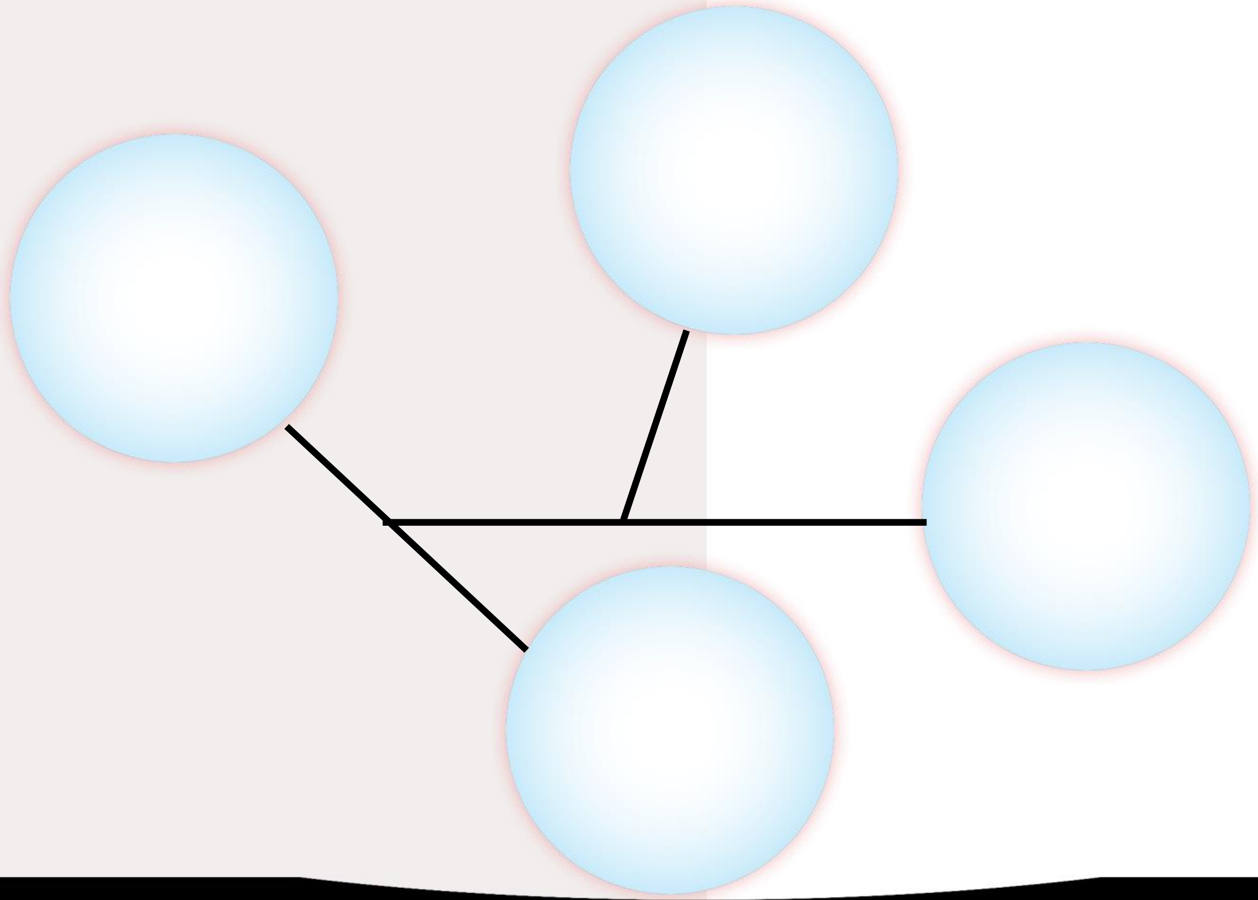
On the solution of the Faddeev-Yakubovsky equations for five-nucleon system



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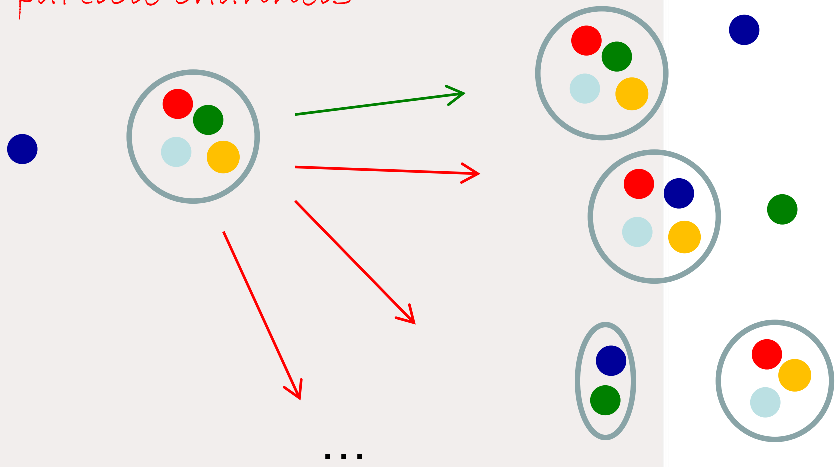
Contents

- 5-body Faddeev-Yakubovsky equations
- Applications
  - n-^{4}He scattering
  - Resonances in ^{5}H

Non-relativistic Collisions

- ✓ Schrodinger/Lippmann-Schwinger eq. does not provide formal tools to determine an unique (physical) solution for the multiparticle scattering problems involving coupled particle channels



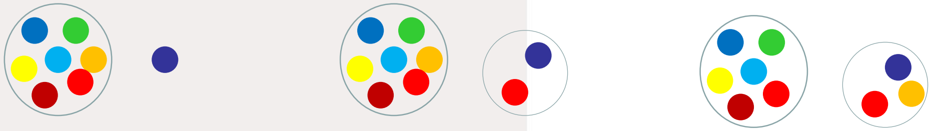
- ✓ Mathematically rigorous framework for a 3-body system was provided by Faddeev. Formalism has been generalized by Yakubovsky to an arbitrary number of particles. Faddeev-Yakubovsky equations are equivalent to Schrodinger's equation but provide additional constrains.

L. D. Faddeev, Zh. Eksp. Teor. Fiz. 39, 1459 (1960). [Sov. Phys. JETP 12, 1014(1961)].
O. A. Yakubovsky, Sov. J. Nucl. Phys. 5, 937 (1967).

Properties of the rigorous scattering eq.

- Should separate all possible scattering channels to incorporate proper asymptotes! Number of binary channels increases $\sim 2^N$

$$\Psi_N = \sum_{perm} \Psi_{(N-1)(1)} + \sum_{perm} \Psi_{(N-2)(2)} + \sum_{perm} \Psi_{(N-3)(3)} + \cdots$$

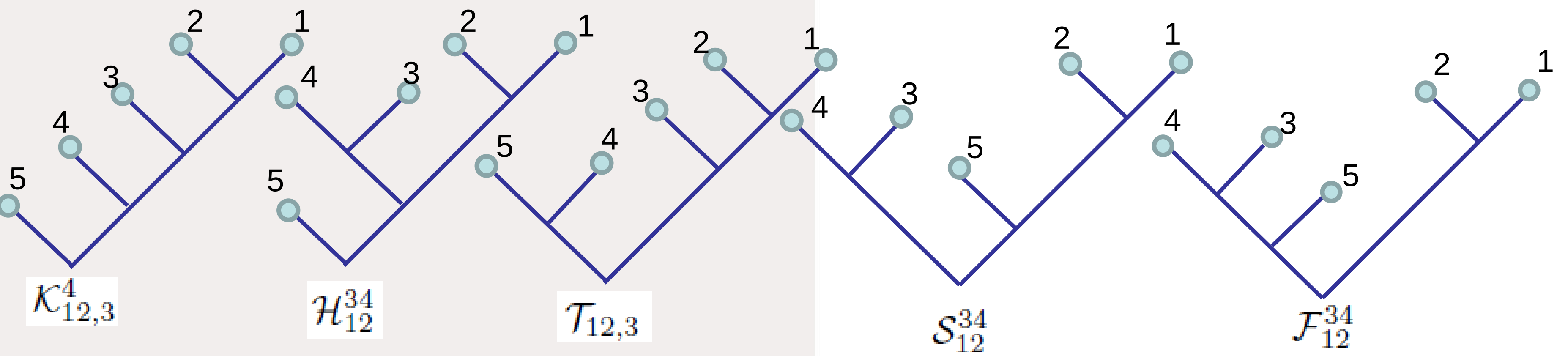


- Should be systematically reducible to smaller subsystems, in order to built proper asymptotic solutions and to be consistent to its subsystems (tree-like structure)

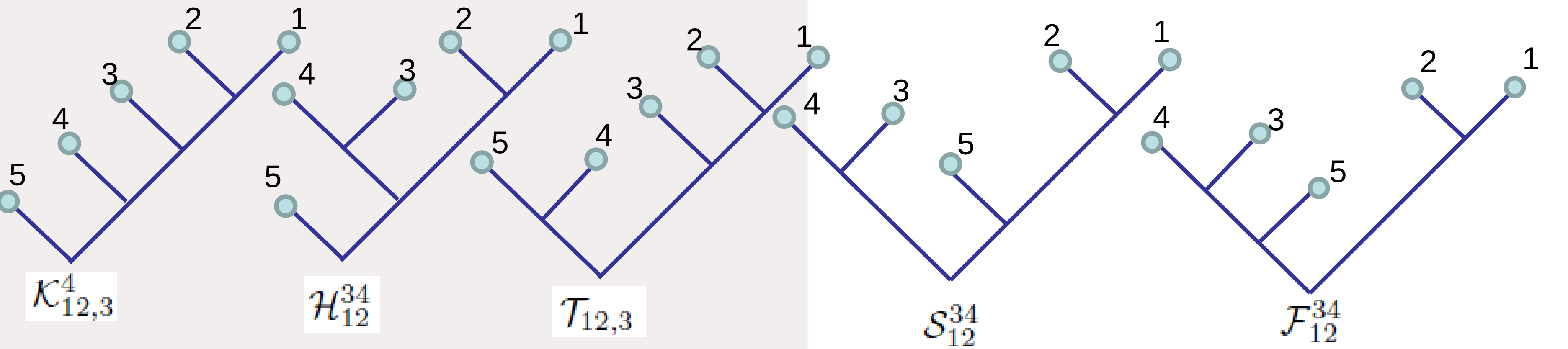
$$\Psi_{(N-i)(i)} = \left(\Psi_{N-i} \bigcup \Psi_i \right)$$

- These constrains are imposed at the price of additional equations

5-body Faddeev-Yakubovski eq



$$
\begin{aligned}
(E-H_0-V_{12})\,\mathcal{K}^{4}_{12,3} &= V_{12}\,(\mathcal{K}^{4}_{13,2}+\mathcal{K}^{4}_{23,1}+\mathcal{K}^{5}_{13,4}+\mathcal{K}^{5}_{23,4}+\mathcal{K}^{2}_{13,4}+\mathcal{K}^{1}_{23,4}\\
&\quad+\mathcal{T}_{13,4}+\mathcal{T}_{23,4}\\
&\quad+\mathcal{H}^{24}_{13}+\mathcal{H}^{14}_{23}+\mathcal{S}^{24}_{13}+\mathcal{S}^{14}_{23}+\mathcal{F}^{24}_{13}+\mathcal{F}^{14}_{23})\\
(E-H_0-V_{12})\,\mathcal{H}^{34}_{12} &= V_{12}\,(\mathcal{H}^{12}_{34}+\mathcal{K}^{2}_{34,1}+\mathcal{K}^{1}_{34,2}+\mathcal{K}^{5}_{34,1}+\mathcal{K}^{5}_{34,2}\\
&\quad+\mathcal{T}_{34,1}+\mathcal{T}_{34,2})\\
(E-H_0-V_{12})\,\mathcal{T}_{12,3} &= V_{12}\,(\mathcal{T}_{13,2}+\mathcal{T}_{23,1}\\
&\quad+\mathcal{H}^{45}_{13}+\mathcal{H}^{45}_{23}+\mathcal{S}^{45}_{13}+\mathcal{S}^{45}_{23}+\mathcal{F}^{45}_{13}+\mathcal{F}^{45}_{23})\\
(E-H_0-V_{12})\,\mathcal{S}^{34}_{12} &= V_{12}\,(\mathcal{F}^{12}_{34}+\mathcal{S}^{15}_{34}+\mathcal{S}^{25}_{34}\\
&\quad+\mathcal{F}^{15}_{34}+\mathcal{F}^{25}_{34}\\
&\quad+\mathcal{H}^{15}_{34}+\mathcal{H}^{25}_{34})\\
(E-H_0-V_{12})\,\mathcal{F}^{34}_{12} &= V_{12}\,(\mathcal{S}^{12}_{34}+\mathcal{K}^{1}_{34,5}+\mathcal{K}^{2}_{34,5}+\mathcal{T}_{34,5})
\end{aligned}
$$



Merits / price:

- ✓ Simple handling of symmetries
- ✓ Boundary conditions for binary channels
- ✓ Easy & systematic reduction to subsystems

- ✓ Special treatment for Coulomb and 3-body forces is required

- ✓ Overcomplexity

Problem	Number eq. (identical particles)	Number eq. (different particles)
A=2	1	1
A=3	1	3
A=4	2	18
A=5	5	180
A=6	15	2700
A=N	nint(	

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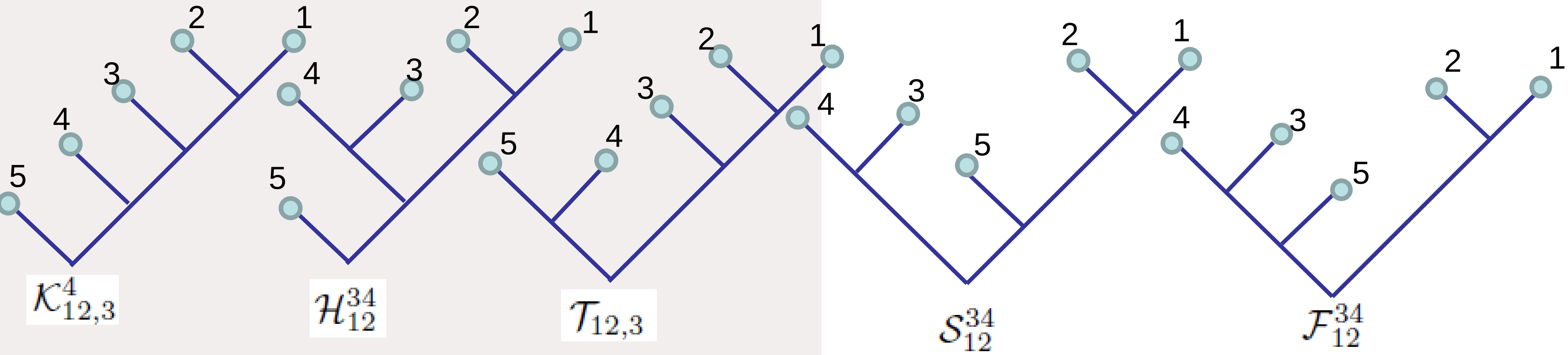
5-body Faddeev-Yakubovski eq



$$\mathcal{K}^{4}_{12,3}(\vec{x},\vec{y},\vec{z},\vec{w},S,L,T)=\sum_{\alpha_K=(l_{..},s_{..},t_{..})}\frac{f_{\alpha_K}(x,y,z,w)}{xyzw}\left[\left\{(l_x l_y)_{l_{xy}}(l_z l_w)_{l_{zw}}\right\}_L\{\ldots\}_S\right]_{JM}\{\ldots\}_T$$

NUMERICAL SOLUTION

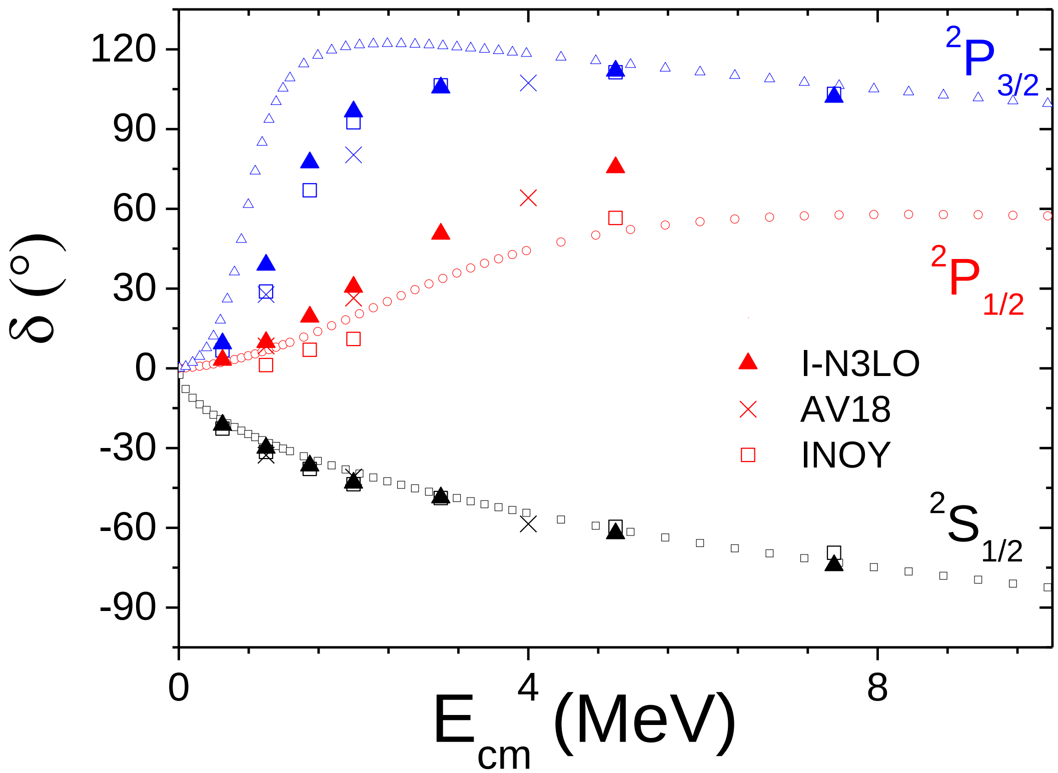
*R.L., PhD Thesis, Université Joseph Fourier, Grenoble (2003).

- PW decomposition of the components K,H,T,S,F
- Radial parts expanded using Lagrange-mesh method

D. Baye, Physics Reports 565 (2015) 1

- Resulting linear algebra problem solved using iterative methods
- Observables extracted using integral relations

n-^{4}He scattering



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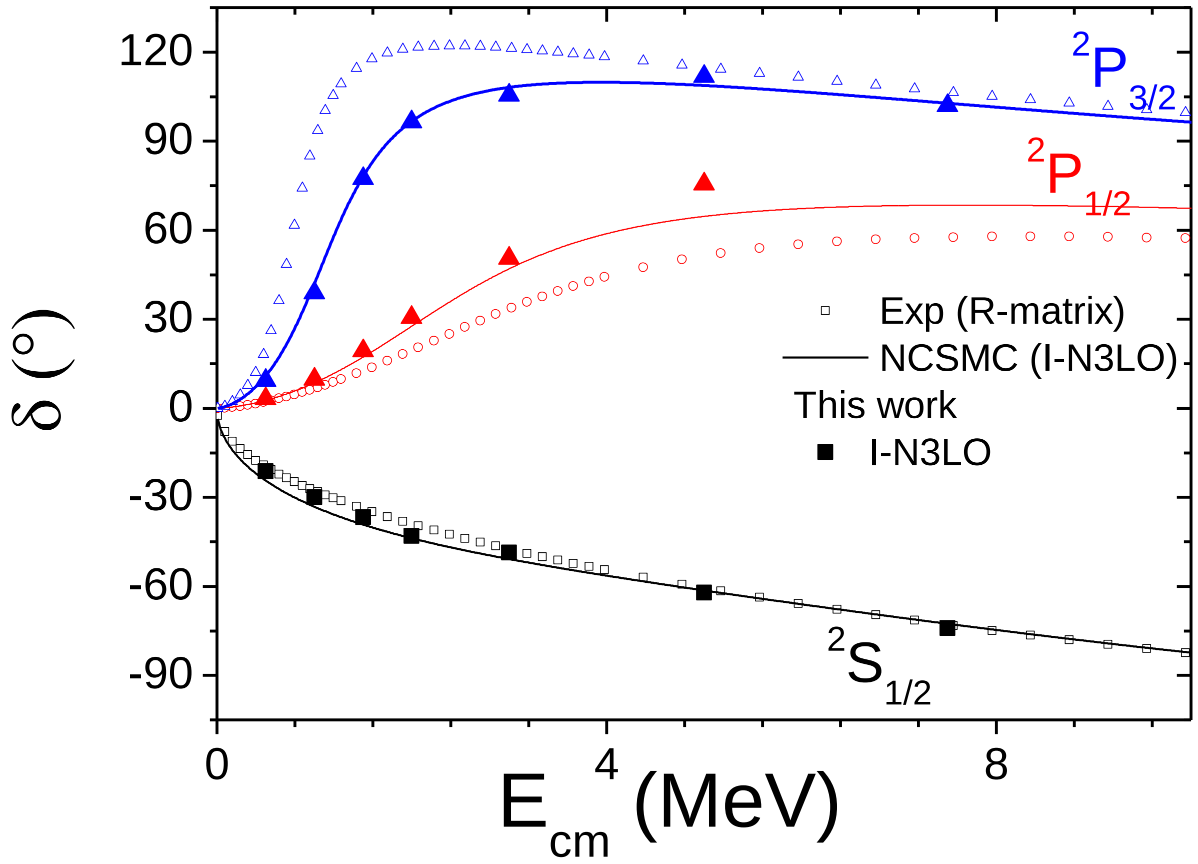
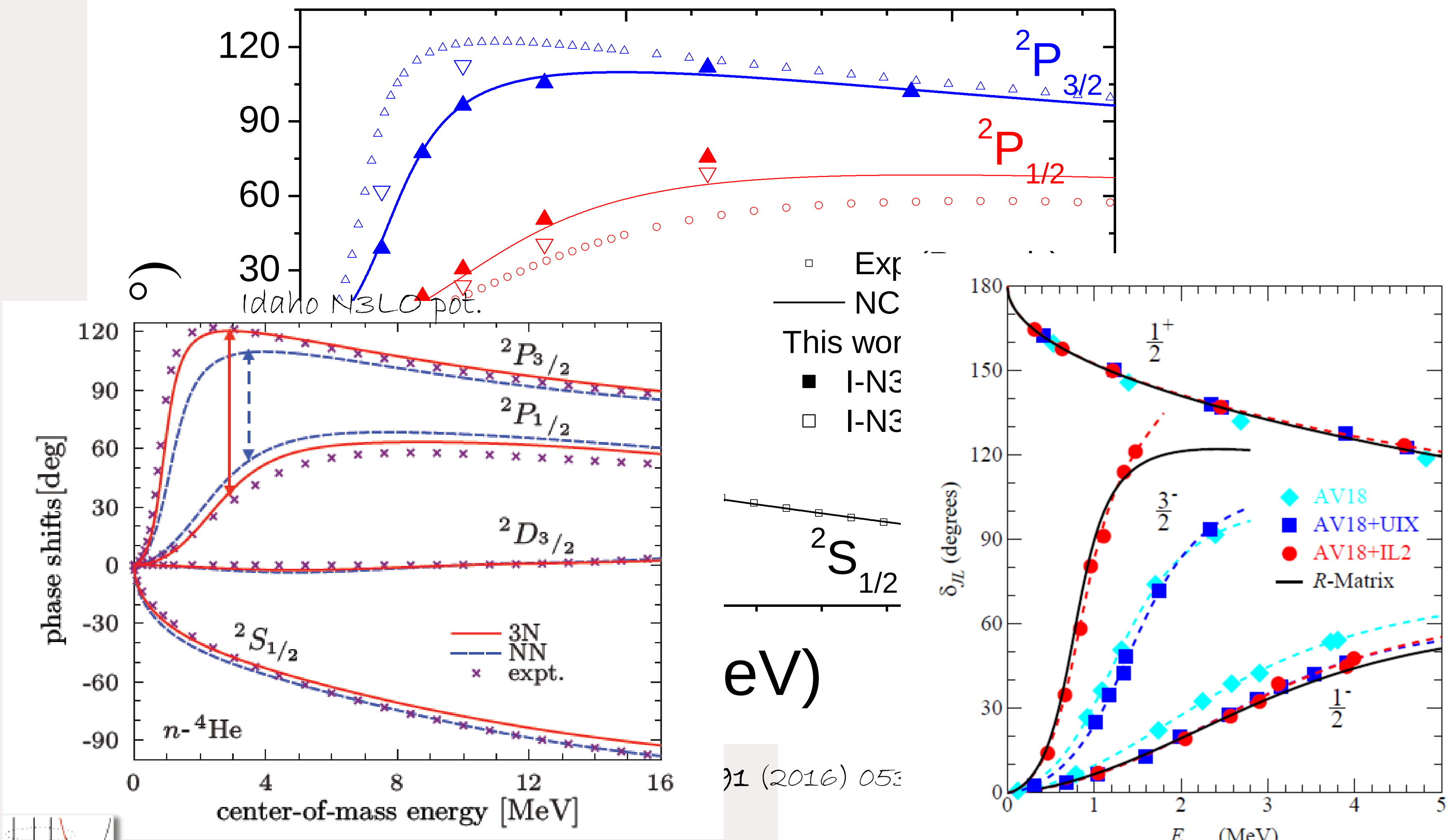
n-^{4}He scattering



NCSMC: P. Navratil et al., Physica Scripta **91** (2016) 053002



P. Navratil et al., Physica Scripta 91 (2016) 053002

K.M. Nollett et. al., Phys.Rev.Lett.99:022502,2007

Case of « little interest »: S-wave

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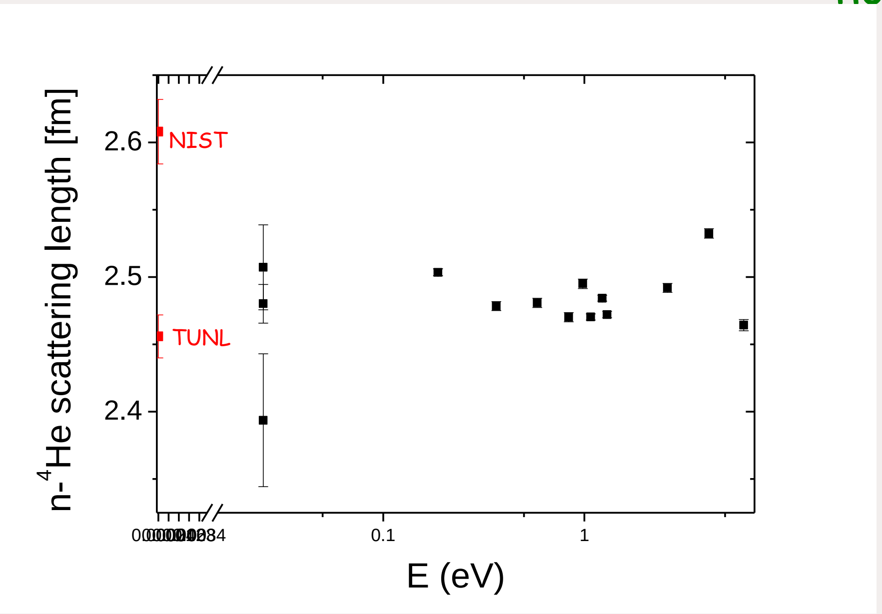
Case of « little interest »: S-wave

Experimental n-^{4}He scattering length …

nothing should be as easy to measure…



NIST (Neutron News 3, 1992)

	Coh a (fm)	Inc b (fm)
^{1}H	-3.7406(11) -3.79406(11)	25.274(9)
^{2}H	6.671(4)	4.04(3)
^{3}H	4.792(27)	-1.04(17)
^{3}He	5.74(7)-1.483(2)*i*	-2.5(6)+2.568(3)*i*
^{4}He	3.26(3)	

TUNL: D.R. Tilley et al., Nucl. Phys. **A708** (2002) 3

NIST: https://www.ncnr.nist.gov

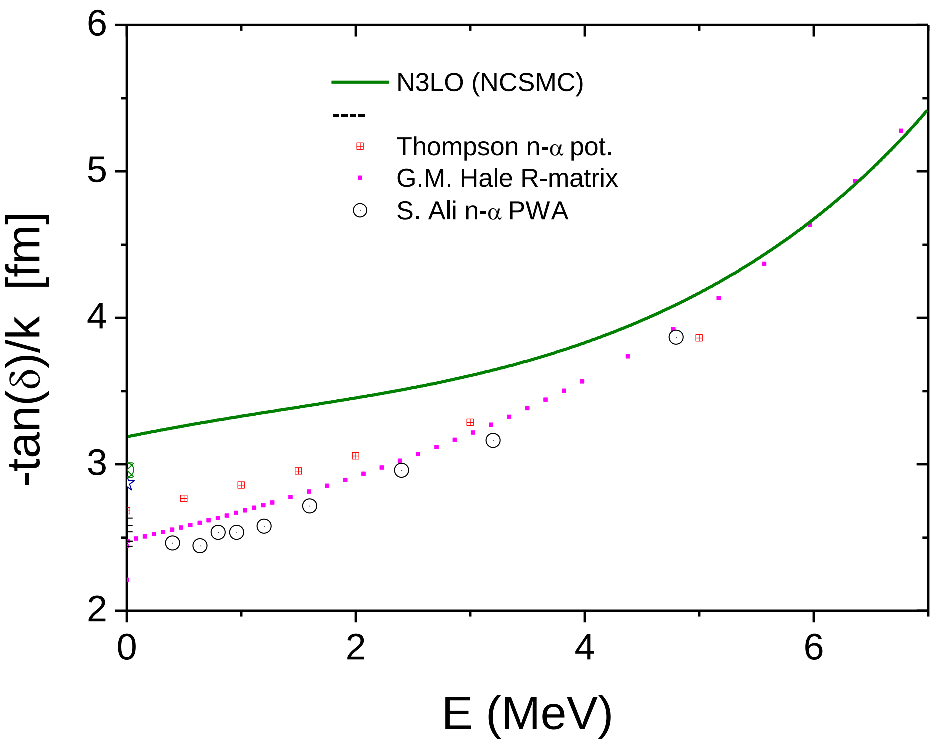
Experimental data:

D.C.Rorer et al., Nucl. Phys. **A 133** (1969) 410

S.F.Mughabghab, Atlas of Neutron Resonances (2006)

R.Genin et al., Journal de Physique **24** (1963) 21

Case of « little interest »: S-wave



TUNL: D.R. Tilley et al., Nucl. Phys. **A708** (2002) 3
NIST: https://www.ncnr.nist.gov
S. Ali PSA: S. Ali et al., Rev. Mod. Phys. **57** (1985) 923
Bang-Gignoux pot: J. Bang, C. Gignoux, Nucl. Phys. A 313 (1979) 119
NCSMC: P. Navratil et al., Physica Scripta **91** (2016) 053002
GFMC: K.M. Nollett, PRL **99**, 022502 (2007)

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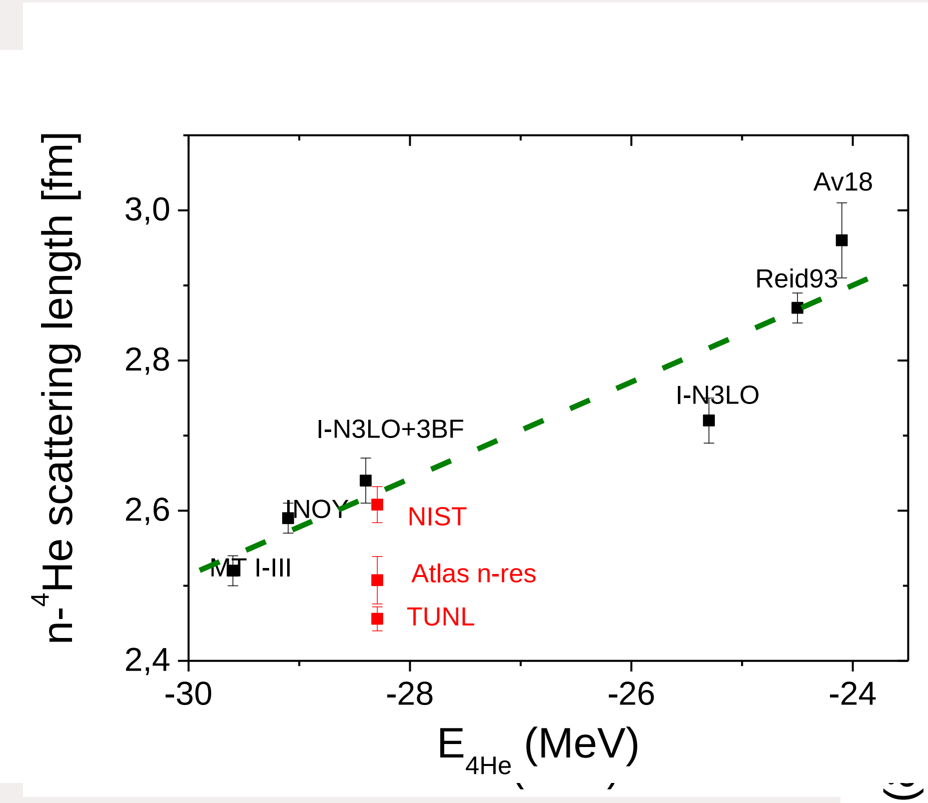
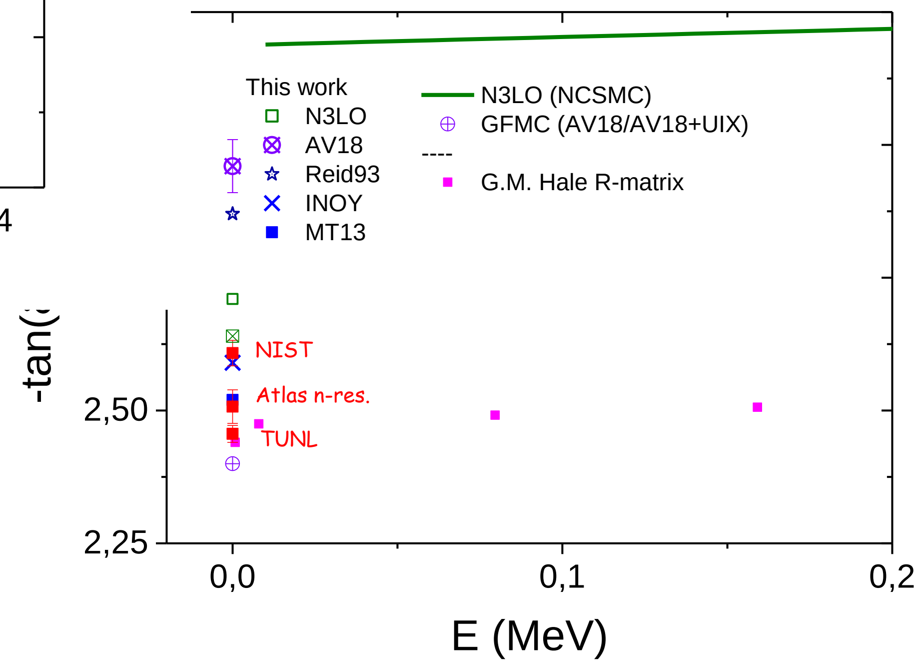
Case of « little interest »: S-wave





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NIST: https://www.ncnr.nist.gov
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NCSMC: P. Navratil et al., Physica Scripta 91 (2016) 05300
GFMC: K.M. Nollett, PRL 99, 022502 (2007)

^{5}H resonances?

TABLE I. Summary of experimental results for ^{5}H. Resonance energies are given relative to ^{3}H + 2n.

Reference	Reaction	Detected	E_R (MeV)	Γ (MeV)	E_{beam} (A MeV)
[17]	^{3}H$(t,p)^5$H	p	≈ 1.8	≈ 1.5	7.42
[18]	^{6}He$(p,2p)^5$H	$2p$	1.7 ± 0.3	1.9 ± 0.4	36
[19]	^{3}H$(t,p)^5$H	t,p,n	1.8 ± 0.1	< 0.5	19.2
[21]	^{3}H$(t,p)^5$H	t,p,n	≈ 2	–	19.2
[22]	^{3}H$(t,p)^5$H	t,p,n	≈ 2	≈ 1.3	19.2
[24]	^{6}He$(^{12}$C$,X+2n)^5$H	$t,2n$	≈ 3	≈ 6	240
[25]	^{6}He$(d,^3$He$)^5$H	^{3}He$,t$	1.8 ± 0.1	< 0.6	22
[26]	^{6}He$(d,^3$He$)^5$H	^{3}He$,t$	1.8 ± 0.2	1.3 ± 0.5	22
[27]	^{6}He$(d,^3$He$)^5$H	^{3}He$,t$	1.7 ± 0.3	≈ 2.5	22
[28]	^{9}Be$(\pi^-,pt)^5$H	p,t	5.2 ± 0.3	5.5 ± 0.5	$E_\pi < 30$ MeV
[28]	^{9}Be$(\pi^-,dd)^5$H	p,t	6.1 ± 0.4	4.5 ± 1.2	$E_\pi < 30$ MeV

[17] P. G. Young, Richard H. Stokes, and Gera
Rev. 173, 949 (1968).
[18] A. A. Korsheninnikov, M. S. Golovkov,
Rodin, A. S. Fomichev, S. I. Sidorchuk, S.
Chelnokov, V. A. Gorshkov, D. D. Bogdan
Ter-Akopian et al., Phys. Rev. Lett. 87, 09
[19] M. S. Golovkov, Yu. Ts. Oganessian, D.
Fomichev, A. M. Rodin, S. I. Sidorchuk,
Stepantsov, G. M. Ter-Akopian, R. Wolski
566, 70 (2003).
[21] M. S. Golovkov, L. V. Grigorenko, A.
Krupko, Yu. Ts. Oganessian, A. M. Rod
R. S. Slepnev, S. V. Stepantsov, G. M. Ter-
Rev. Lett. 93, 262501 (2004).
[22] M. S. Golovkov, L. V. Grigorenko, A.
Krupko, Yu. Ts. Oganessian, A. M. Rod
R. S. Slepnev, S. V. Stepantsov, G. M. Ter-
Rev. C 72, 064612 (2005).

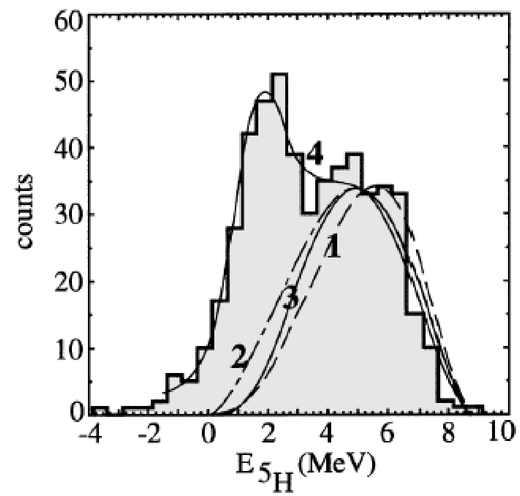

FIG. 3. Spectrum of ^{5}H from the reaction $p(^6$He$, ppt)$. Curves show calculations explained in the text.

ov, H. Simon, T. Aumann, M. J. G.
mling, H. Geissel, M. Hellstrom, B.
Lett. 91, 162504 (2003).
Bogdanov, A. S. Fomichev, M. S.
essian, A. M. Rodin, R. S. Slepnev,
Ter-Akopian, R. Wolski et al., Nucl.
).
Golovkov, A. S. Fomichev, A. M.
S. Slepnev, G. M. Ter-Akopian, M. L.
cov, Yu. Ts. Oganessian et al., Nucl.
S. Fomichev, M. S. Golovkov, L. V.
o, Yu. Ts. Oganessian, A. M. Rodin,
nev, S. V. Stepantsov et al., Eur. Phys.
r, D. V. Aleshkin, B. A. Chernyshev,
Morokhov, V. A. Pechkurov, N. O.
sky, and M. V. Telkushev, Eur. Phys.

TABLE II. Summary of some theoretical results for ^{5}H. Resonance energies are given relative to ^{3}H + 2n.

Reference	Method	E_R (MeV)	Γ (MeV)
[7]	Cluster, model with source	2–3	4–6
[23]	Three-body cluster	2.5–3	3–4
[31,35]	Cluster, J-matrix, resonating group model	1.39	1.60
[36]	Cluster, complex scaling adiabatic expansion	1.57	1.53
[32]	Cluster, generator coordinate method	≈ 3	≈ 1–4
[33]	Cluster, complex scaling	1.59	2.48
[34]	Cluster, analytic coupling in continuum constant	1.9 ± 0.2	0.6 ± 0.2

[7] L. V. Grigorenko, N. K. Timofeyuk, and M. V. Zhukov, Eur. Phys. J. A **19**, 187 (2004).
[31] A. V. Nesterov, F. Arickx, J. Broeckhove, and V. S. Vasilevsky, Phys. Part. Nucl. **41**, 716 (2010).
[32] P. Descouvemont and A. Kharbach, Phys. Rev. C **63**, 027001 (2001).
[33] K. Arai, Phys. Rev. C **68**, 034303 (2003).
[34] A. Adachour and P. Descouvemont, Nucl. Phys. A **813**, 252 (2008).
[35] J. Broeckhove, F. Arickx, P. Hellinckx, V. S. Vasilevsky, and A. V. Nesterov, J. Phys. G **34**, 1955 (2007).
[36] R. de Diego, E. Garrido, D. V. Fedorov, and A. S. Jensen, Nucl. Phys. A **786**, 71 (2007).

Approximate dynamics of ^{3}H, no full 5-body dynamics

How to handle resonances?

- Direct methods with Kapur-Peierls bc ..

complicated 3-body boundary condition

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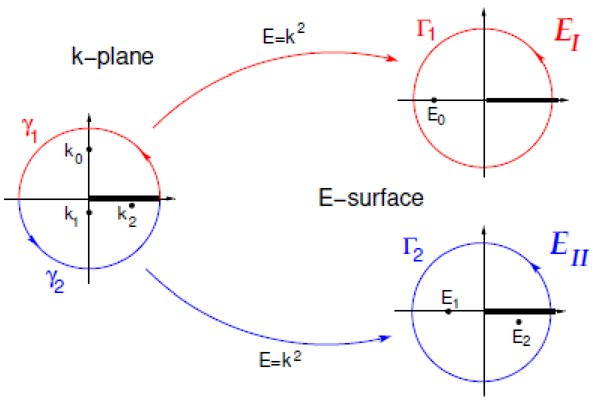
How to handle resonances?

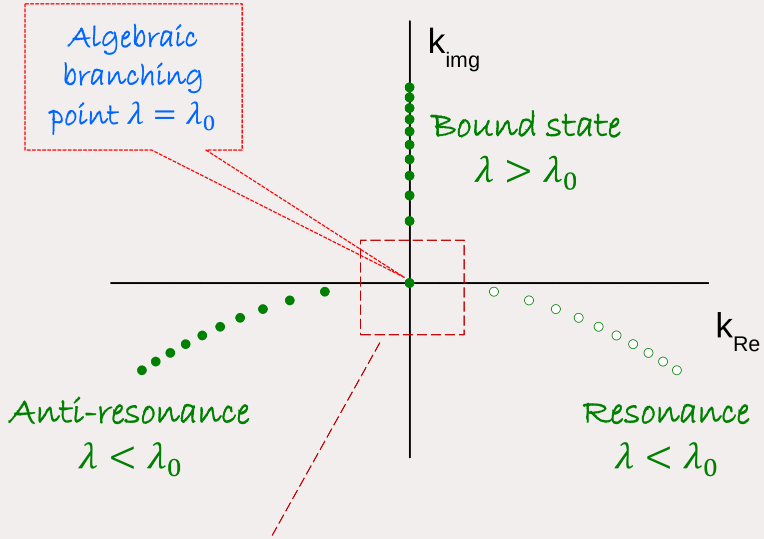
- ACCC : Analytic continuation in the coupling constant method (V.I. Kukulin et al., « Theory of resonances », Kluwer AP 1989)
- « Dirty » smooth exterior complex scaling method

Extrapolation is a flourishing business

- In QM scattering problem the « Energy manifold » E is **not an axis**, but a two-dimensional Riemann sheet!
- When moving from bound state region (axis!) to continuum one should take into account the proper analytical behavior: **branching points + non-trivial dependence in $k(\lambda V)$**



Trajectory of the S-matrix pole with a coupling constant λ with $k(\lambda_0) = 0$



Analytic behavior in the vicinity of the branching point

$$k(\lambda \to \lambda_0) \sim i\sqrt{\lambda - \lambda_0}$$

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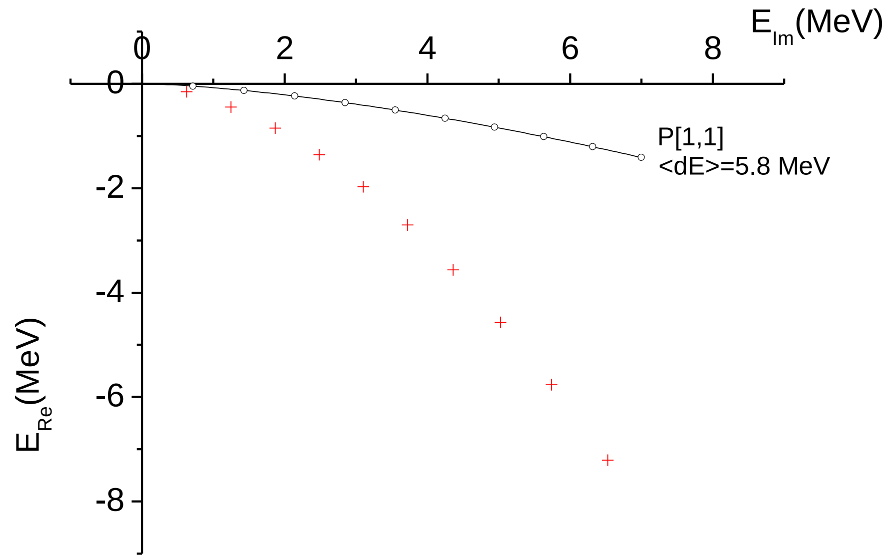
ACCC method (V.I. Kukulin et al., « Theory of resonances », Kluwer AP 1989)

- Add artificial binding potential to Hamiltonian $\lambda V(r)$
- Calculate several binding energies of the system $E_i(\lambda_i)$
- Determine accurately λ_0 such that $E(\lambda_0)=0$!
- Extrapolate $E_{res}=E(\lambda=0)$ using $E_i(\lambda_i)$ and λ_0 values, knowing that

$$\sqrt{E_{res}(\lambda \to \lambda_0)} \sim i\sqrt{\lambda - \lambda_0}$$



Binding Energy input
(0,200) MeV / 30 points

Padé extrapolation is used:

$$\sqrt{E_{res}(\lambda \to \lambda_0)} = iP[n,m]\left(\sqrt{\lambda - \lambda_0}\right)$$

$$P[n,m](q) = \frac{a_1 q + a_1 q^2 + .. + a_n q^n}{1 + b_1 q + .. + b_m q^m}$$

2b-example

$$V(l=1) = \frac{1.8}{r}\left(1438.72e^{-3.11r} - 626.885e^{-1.55r}\right)$$

$$\lambda V(r) = \lambda r^2 e^{-r^2/3.5}$$

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ACCC method

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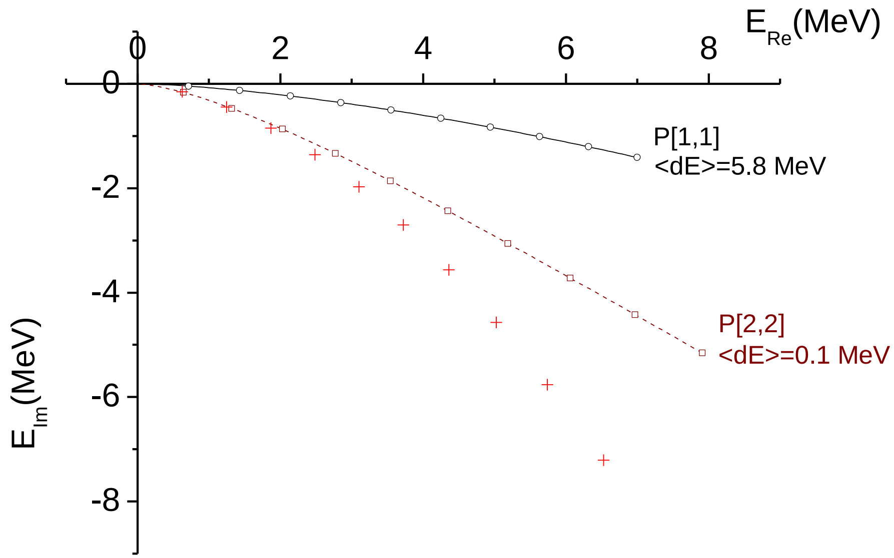

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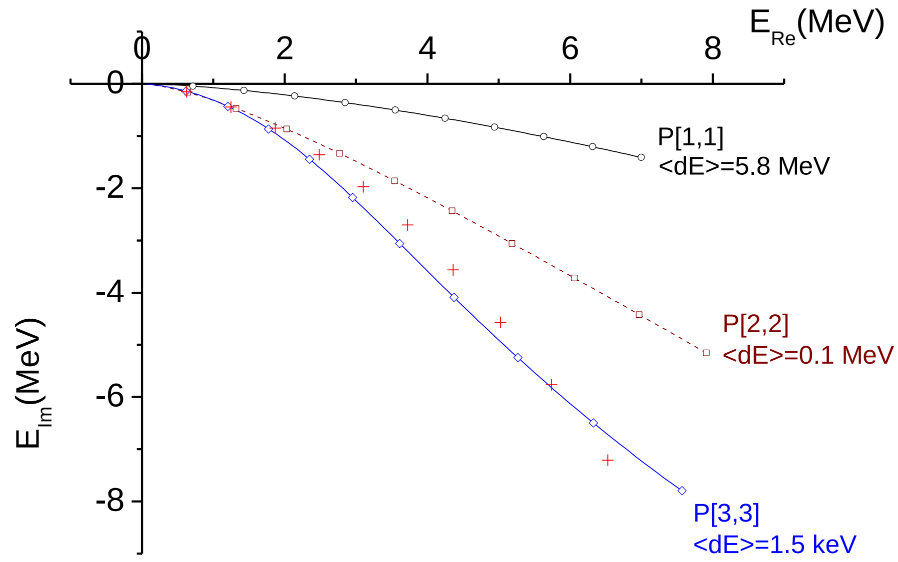

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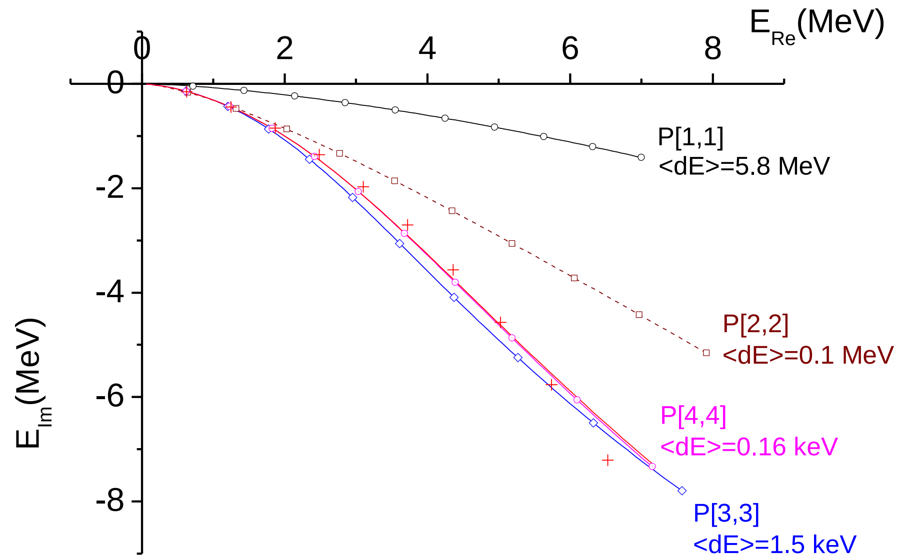

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Complex scaling:

$$r \longrightarrow re^{i\theta}$$

$$e^{i|k|r} \longrightarrow e^{i|k|re^{i\theta}} = e^{i|k|r\cos\theta} e^{-|k|r\sin\theta}$$

exp. bound if 0<θ<π

Numerical complications when transforming short ranged potentials $V(r) \to V(re^{i\theta})$

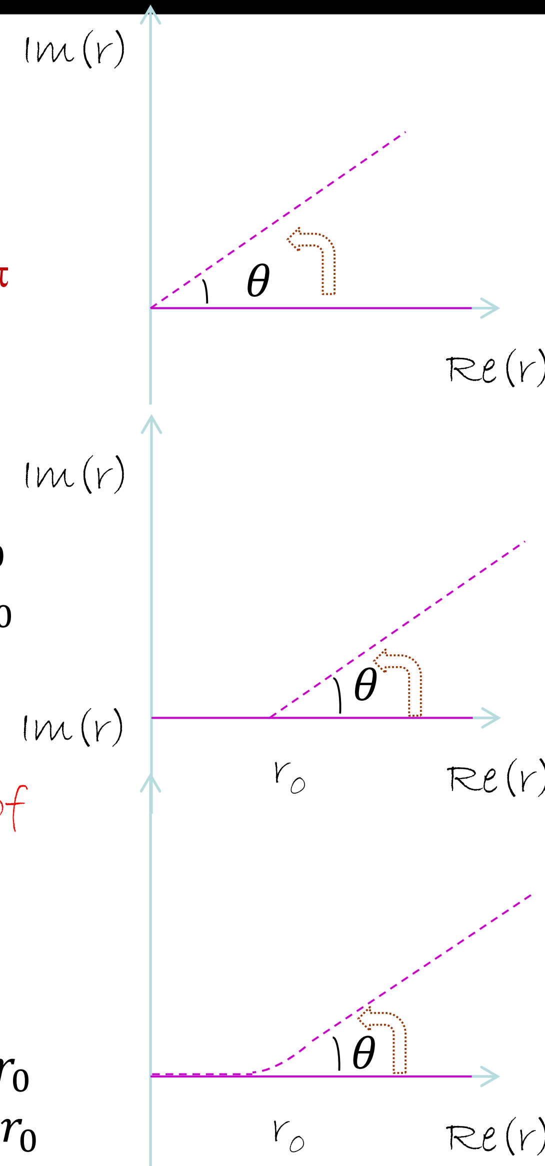
Exterior complex scaling:

$$\begin{cases} r \to r & r < r_0 \\ r \to r_0 + (r - r_0)e^{i\theta} & r > r_0 \end{cases}$$

Numerical instabilities due to non-analyticity (sharpness) of the transformation

Smooth exterior complex scaling:

$$\begin{cases} r \to r & r \ll r_0 \\ r \to r_0 + (r - r_0)e^{i\theta} & r \gg r_0 \end{cases}$$



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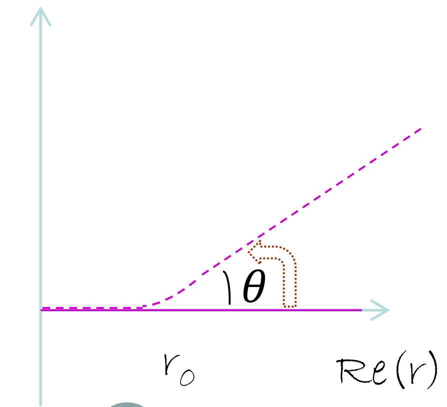
Exterior complex scaling method

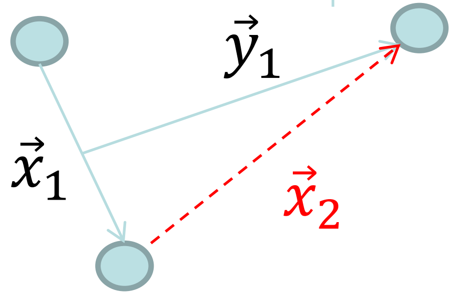
Smooth exterior complex scaling:

$$\begin{cases} r \to r & r \ll r_0 \\ r \to r_0 + (r - r_0)e^{i\theta} & r \gg r_0 \end{cases} \quad \text{i.e.} \quad r \to f(r, \mathrm{r}_0)$$



But… in conflict with PW business for A>2

$$\begin{cases} x_1 \to f(x_1, \mathrm{r}_0) \\ y_1 \to f(y_1, \mathrm{r}_0) \end{cases} \qquad x_2(\vec{x}_1, \vec{y}_1) \to ????$$



« Dirty » smooth exterior complex scaling method:

- Choose sharp transformation function, which almost does not affect r in r<r_0
- Fix r_0 beyond the physical interaction region
- Ignore inconsisitencies in transformation between different Jacobi bases

In the end, only kinetic energy operator is transformed.

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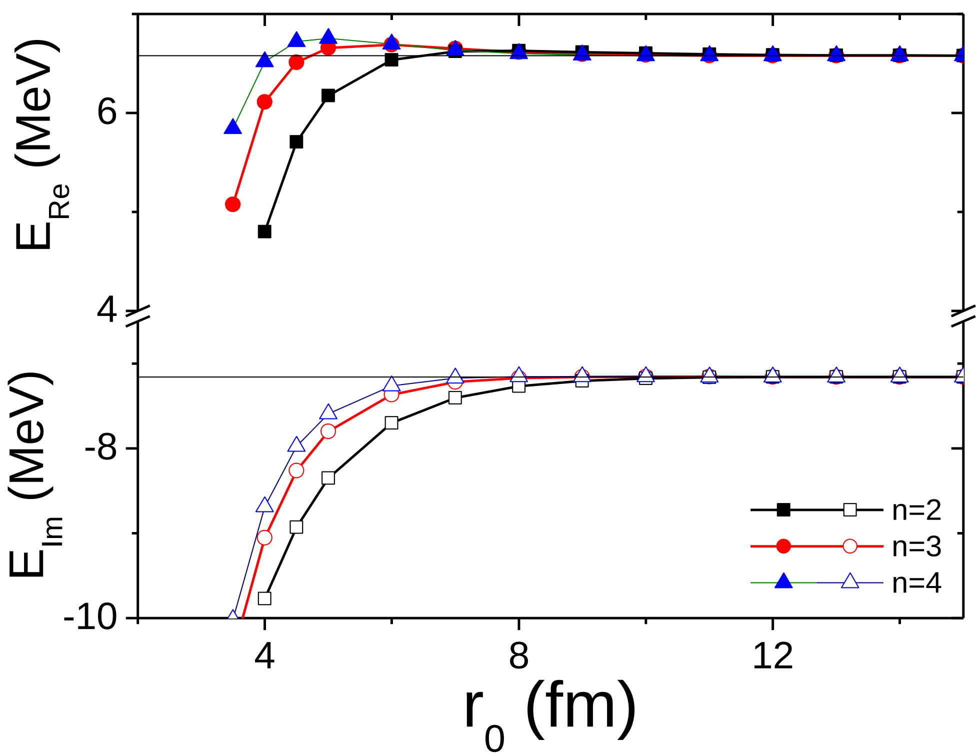
"Dirty" exterior complex scaling method (DEXCSM):



2b-example

$$V(l=1)=\frac{1.8}{r}\left(1438.72e^{-3.11r}-626.885e^{-1.55r}\right)$$

$$r\rightarrow(1-f(r))r+f(r)re^{i\theta}\qquad f(r)=\exp\left(-\left(\frac{r_0}{r}\right)^n\right)$$

Back to $^5H(J=1/2^+)$

- nn interaction described by the MT I-III potential
- auxilliary potential for ACCC

$$V_{5b}(\rho) = \lambda \rho^p exp(-\rho^2/\rho_0^2).$$

$$\rho^2 = x^2 + y^2 + z^2 + w^2 = 2\sum_{i=1}^{5} r_i^2$$

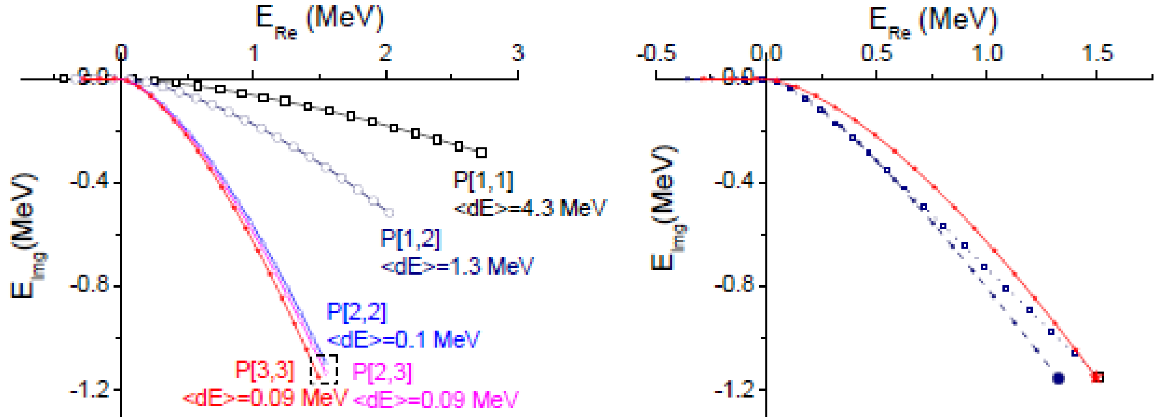


Fig. 3 Resonance trajectories for a $J^\pi = 1/2^+$ state of ^{5}H with respect to ^{3}H threshold. Each trajectory is split by points in 20 intervals of equal step in λ, starting at the position where ^{5}H nucleus is still weakly bound. The endpoint of the trajectory indicates extrapolated value for the bare NN interaction, corresponding $\lambda = 0$ case. In the left panel convergence of the results with respect to order of Padé expansion is presented; calculation is based on auxiliary potential defined in eq. (13) with ρ_0^2=78.4 fm^2 and $p = 0$. In the right panel converged results for three different external potentials are presented.

$E(^5H)-E(^3H)=1.4(1)-i1.2\ (1)$

$E(^5H)-E(^3H)=1.7(2)-i1.2\ (1)$

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Back to $^5H(J=1/2^+)$

$J=1/2^+$ ($L=0^+$, $S=1/2$)

ACCC:

$E(^5H)-E(^3H)=1.4(1)-i1.2(1)$

DEXCSM:

$E(^5H)-E(^3H)=1.7(2)-i1.2(1)$

$J=5/2^+$ ($L=2^+$, $S=1/2$)

DEXCSM:

$E(^5H)-E(^3H)=2.50(15)-i1.90(15)$

Negative parity states & ones with S=3/2 are much broader

To compare with ^4H resonances:

$E(^4H)-E(^3H)=$ $1.08(1)-i2.04(2)$ $(S=1, L=1^-)$

$0.88(3)-i2.20(4)$ $(S=0, L=1^-)$

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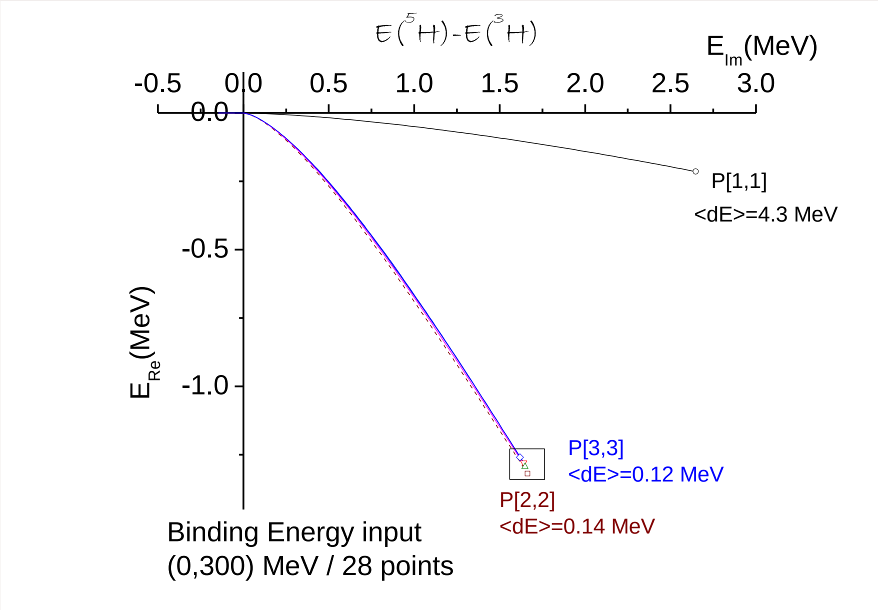
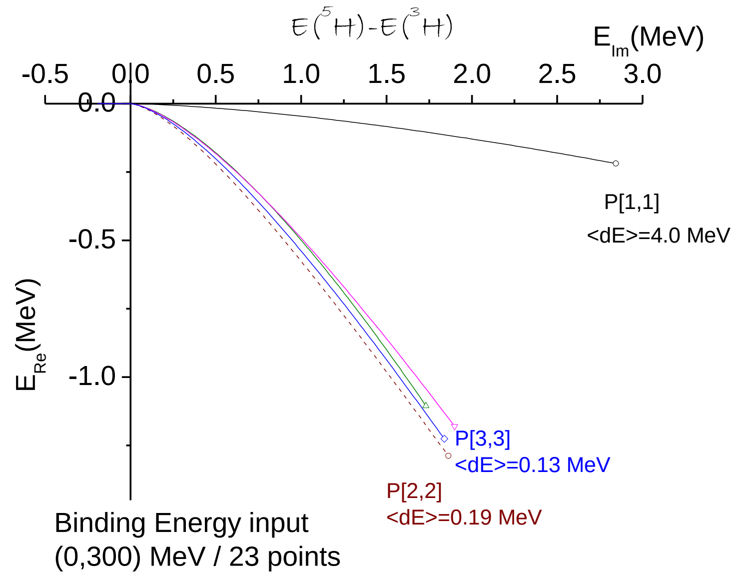
Back to $^5H(J=1/2^+)$

INOY Potential



I-N3LO Potential



$E(^5H)-E(^3H)=1.65(5)-i1.26(6)$

$E(^5H)-E(^3H)=1.8(1)-i1.15(15)$

DEXCSM: 1.8(1)-i1.2(1)

DEXCSM: 1.85(10)- i1.20(5)

To compare with ^4H resonances $J=2^-$:

$E(^4H)-E(^3H)=1.31(3)-2.08(2)$

$E(^4H)-E(^3H)=1.17(3)-1.99(3)$

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Conclusion

• FY eq. formalism remains reference in few-body scattering calculations. The first solution of 5-body FY equations is presented.

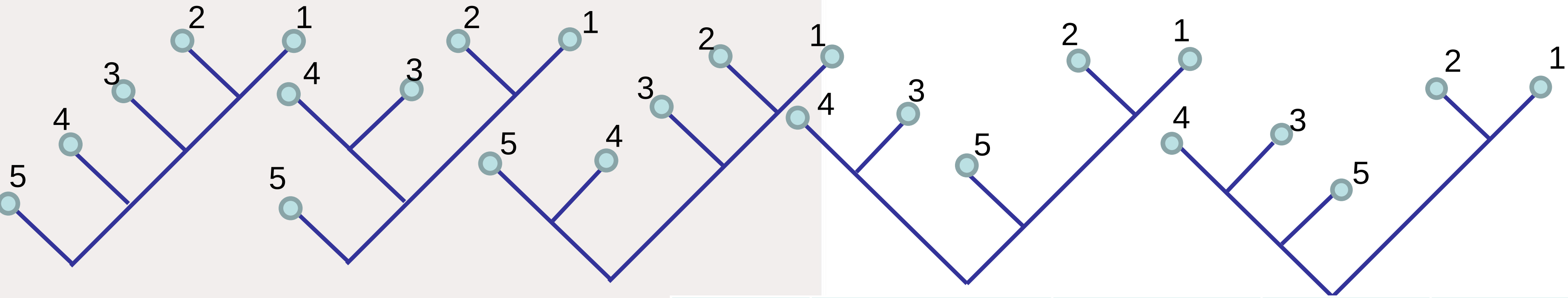
•n-^{4}He elasting scattering has been calculated using realistic NN+3N interactions

• A new technique (of « dirty smooth exterior complex scaling ») has been proposed to determine positions of relatively broad resonant states in fex-body systems

• Resonant states in ^{5}H system have been investigated, confirming presence of relatively broad but experimentally significant resonant states

Acknowledgements: The numerical calculations have been performed at IDRIS (CNRS, France). We thank the staff members of the IDRIS computer center for their constant help.

Numerical costs



Problem	Number eq. (ident particles)	Number eq. (diff. particles)	PW basis.	Radial disc.
2N	1	1	2	~N
3N	1	3	~100	$\sim N^2$
4N	2	18	$\sim 10^4$	$\sim N^3$
5N	5	180	$\sim 10^6$	$\sim N^4$

NUMERICAL SOLUTION

*R.L., PhD Thesis, Université Joseph Fourier, Grenoble (2003).

- PW decomposition of the components K,H,T,S,F
- Radial parts expanded using Lagrange-mesh method

D. Baye, Physics Reports 565 (2015) 1

- Resulting linear algebra problem solved using iterative methods
- Observables extracted using integral relations

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