Basis Light-Front Quantization Approach to Nucleon

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2^{nd} November 2018

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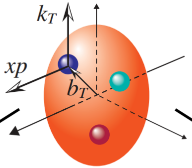
- ☐ Basis Light-Front Quantization (BLFQ) approach to nucleon
  - ✓ Form Factors
  - ✓ Parton distribution Functions (PDFs)
  - ✓ Generalized parton distributions (GPDs)
- ☐ Conclusions

What could we learn about nucleon structure?

Momentum Space

TMDs

Confined motion



Coordinate Space

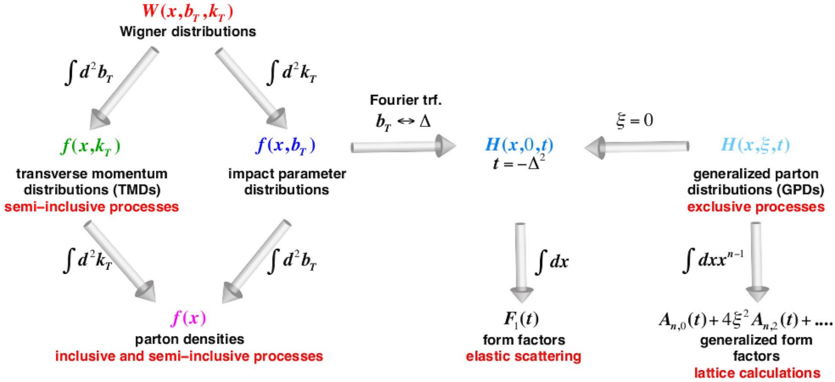
GPDs

Spatial distribution

$\int d^2b_T$ → $f(x,k_T)$

$\int d^2k_T$ → $f(x,b_T)$

Two-scales observables



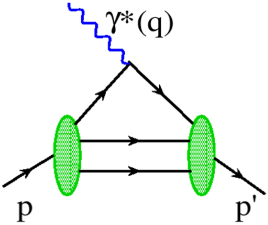
Nucleon Form Factors

- Elastic electron scattering established the extended nature of the proton, proton radius: 0.77 fm.

[*R. Hofstadter, Nobel Prize 1961*]

- The electromagnetic form factor can be probed through elastic scattering

$$\langle N(p')|J^\mu(0)|N(p)\rangle = \bar{u}(p')\left[\gamma^\mu \underbrace{F_1(q^2)} + \frac{i\sigma^{\mu\nu}}{2m_N} q_\nu \underbrace{F_2(q^2)}\right] u(p)$$



The Fourier transform of the form factors provide the (*charge and magnetization distribution*).

Charge density →

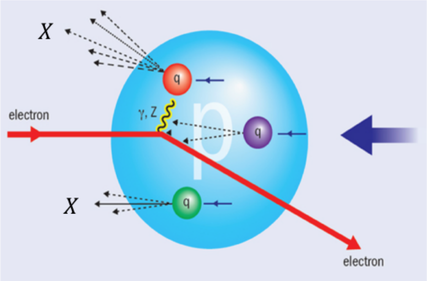
$$\rho(\vec{b}_\perp) = \int \frac{dQ}{2\pi} Q\, J_0(Qb_\perp)\, F_1(Q^2)$$

- FFs do not provide dynamical information (*angular momentum !!*)

Parton distribution functions (PDFs)

➢ Deep Inelastic Scattering (SLAC 1968)

$$e(p) + h(P) = e'(p') + X(P')$$

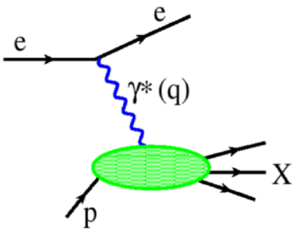


✧ **Localized probe:**

$$Q^2 = -(p - p')^2 \gg 1\ \text{fm}^{-2}$$

$$\Rightarrow \frac{1}{Q} \ll 1\ \text{fm}$$

Discovery of spin ½ quarks and partonic structure



➢ **Parton distribution functions** (PDFs) are extracted from **DIS** processes.

PDFs encode the *distribution of longitudinal momentum and polarization* carried by the constituents

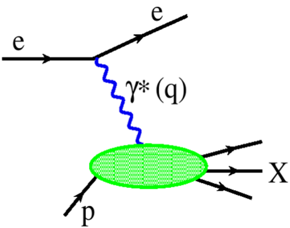
Parton distribution functions (PDFs)

- **Deep Inelastic Scattering** (DIS) discovered the existence of quasi-free point-like objects (quarks) inside the nucleon.

  [*Friedman, Kendall, Taylor Nobel Prize 1990*]

- Parton distribution functions (PDFs) are extracted from **DIS** processes.

$$\underbrace{q(x)} = \frac{p^+}{4\pi}\int dy^- e^{ixp^+y^-} \times \langle p|\underbrace{\bar{\psi}_q(0)\mathcal{O}\psi_q(y)}|p\rangle\Big|_{y^+=\vec{y}_\perp=0}$$



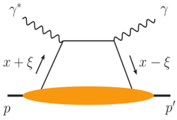
PDFs encode the distribution of longitudinal momentum and polarization carried by the constituents

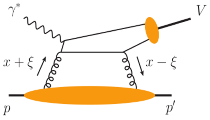
- Missing information: The PDFs provide no knowledge of spatial locations of parton !!

Generalized parton distribution functions (GPDs)

- GPDs appear in the *exclusive processes* like deeply virtual Compton scattering (DVCS) or vector meson productions.

$$\int \frac{dz^-}{8\pi} e^{ixP^+z^-/2} \langle P' | \bar{\Psi}(0)\gamma^+\Psi(z^-)|P\rangle|_{z^+=0,z^\perp=0}$$
$$= \frac{1}{2\bar{P}^+}\left(H^q(x,\zeta,t)\bar{u}(P')\gamma^+ u(P) \right.$$
$$\left. + E^q(x,\zeta,t)\bar{u}(P')\frac{i\sigma^{+j}(-\Delta_j)}{2M}u(P)\right),$$





GPDs provide information about the *3D spatial structure of nucleon* as well as *spin and angular momentum* of the constituents

- ***Longitudinal momentum fraction*** $x = \frac{k^+}{P^+}$
- ***Longitudinal momentum transfer -->***

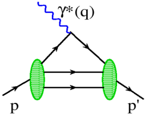
  skewness $\xi = \frac{\Delta^+}{P^+}$
- ***Square of total mom transfer***

  $t = \Delta^2 = (P' - P)^2$

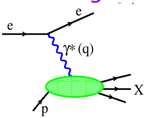
- Many activities are going on (COMPASS, HERMES, ZEUS, JLAB etc.) to gain insight into GPDs

Form Factors Vs PDFs Vs GPDs

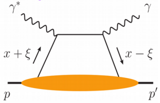
Elastic Scattering



Deep Inelastic Scattering



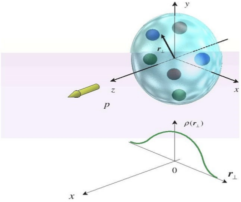
Deeply virtual Compton Scattering

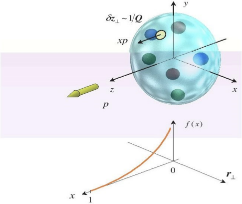


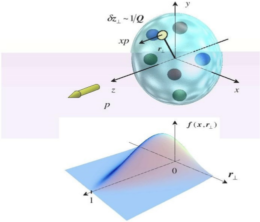
Established extended nature of nucleon

discovered the existence (quarks) inside the nucleon

provides 3D spatial structure of the nucleon

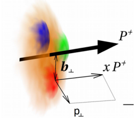


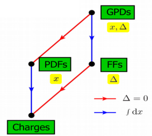




charge and magnetization distribution

longitudinal momentum distribution





Basis Light-Front Quantization (BLFQ)

J. Vary *et. al.*, PRC 81 (2010)

BLFQ: approach for solving quantum field theory

- **Nonperturbative:**
  for systems with strong interaction
- **First-principles:**
  effective Hamiltonian as input/ direct access to wavefunction of bound states
- **Light-front dynamics:**
  spectrum and light-front Fock-state wavefunctions are obtained from

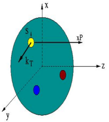
$$H_{LF}|\psi\rangle = M^2|\psi\rangle$$

$$H_{LF} \equiv P_\mu P^\mu = P^+P^- - \mathbf{P}_\perp^2$$

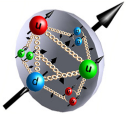
$$P^\pm = P^0 \pm P^3$$

LF wavefunctions

→ **Proton 3D imaging**



→ **Proton spin**



FF | GPD | TMD...

General Procedure for BLFQ discussed by X. Zhao

✓ Construct the basis state: $|\alpha\rangle$

✓ Derive/write the Light-Front Hamiltonian: P^-

✓ Calculate Hamiltonian matrix elements: $\langle\alpha'|P^-|\alpha\rangle$

✓ Diagonalize the Hamiltonian: $P^-|\beta\rangle = P^-_\beta|\beta\rangle$

✓ Evalute the observables $\mathcal{O} \equiv \langle\beta|\hat{\mathcal{O}}|\beta\rangle$

Previous application (QCD)

- In heavy quarkonium: decay constant, elastic form factor, radii, radiative transitions, distribution amplitude, GPDs

—Y Li, G Chen, X Zhao, P Maris, J Vary, L Adhikari, M Li, A El-Hady (2016 - 2018)

Previous application (QED)

- electron anomalous magnetic moments
- wave function, spectroscopy of positronium system
- GPDs of the electron and positronium

—X. Zhao, P. Wiecki, Y. Li, H. Honkanen, D. Chakrabarti, P. Maris, J. P. Vary, S. J. Brodsky (2013 - 2018)

Basis construction

- Example: the basis state of proton
  - Fock's space expansion

$$|\,N\,\rangle_{\text{baryon}} = a|\,qqq\,\rangle + b|\,qqqg\,\rangle + c|\,qqqq\bar{q}\,\rangle + \cdots .$$

  - For each Fock particle
    - ✓ For each quark: n_q, m_q, k_q, $\lambda_q = (\frac{1}{2}, -\frac{1}{2})$
    - ✓ For each gluon: n_g, m_g, k_g, $\lambda_g = (1, -1)$
  - For the first Fock sector:

$$|\,qqq\,\rangle = |\,n_{q_1}, m_{q_1}, k_{q_1}, \lambda_{q_1}\,\rangle \otimes |\,n_{q_2}, m_{q_2}, k_{q_2}, \lambda_{q_2}\,\rangle \otimes |\,n_{q_3}, m_{q_3}, k_{q_3}, \lambda_{q_3}\,\rangle.$$

- Truncation of the basis
  - Fock sector truncation
  - For each Fock sector:
    - ✓ "K_{max}" truncation in the longitudinal direction: $\sum_i k_i = K_{max}$
    - ✓ "N_{max}" in the transverse direction: $\sum_i (2n_i + |m_i| + 1) \leq N_{max}$

Basis construction: quantum numbers

- Longitudinal direction: plane-wave basis
  ✓ discrete longitudinal momentum (labeled by k): $p^+ = \frac{2\pi}{L} k$
- Transverse: ✓ 2D harmonic oscillator basis (labeled by $n,\ m$)

$$\phi^b_{n,m}(p_\perp) = \frac{1}{b\sqrt{\pi}}\sqrt{\frac{n!}{(n+|m|)!}}\, e^{-\frac{p^2}{2b^2}}\, e^{-im\phi}\left(\frac{p}{b}\right)^{|m|} L_n^{|m|}\left(\frac{p^2}{b^2}\right) \begin{cases} b \equiv \sqrt{M\Omega} \\ p = \sqrt{p_1^2 + p_2^2} \end{cases}$$

- For the leading Fock sector:

$$|\, qqq\,\rangle = |\, n_{q_1}, m_{q_1}, k_{q_1}, \lambda_{q_1}\,\rangle \otimes |\, n_{q_2}, m_{q_2}, k_{q_2}, \lambda_{q_2}\,\rangle \otimes |\, n_{q_3}, m_{q_3}, k_{q_3}, \lambda_{q_3}\,\rangle.$$

☐ Truncation of the basis

- Fock sector truncation
- For each Fock sector:
  ✓ "K_{max}" truncation in the longitudinal direction: $\sum_i k_i = K_{max}$
  ✓ "N_{max}" in the transverse direction: $\sum_i (2n_i + |m_i| + 1) \le N_{max}$

Effective Hamiltonian

$$H_{eff} = \underbrace{\sum_a \frac{k_{a\perp}^2 + m_a^2}{x_a}}_{\text{LF Kinetic energy}} + \underbrace{\frac{1}{2}\sum_{a,b} V_{ab}^{(CON)}}_{\text{Confinement}} + \underbrace{\frac{1}{2}\sum_{a,b} V_{ab}^{(OGE)}}_{\text{One gluon exchange}}$$

- Light-Front kinetic energy
- Confinement in transverse direction $\Rightarrow$
  $V_{ab}^{(SW)} = \kappa_T^4 x_a x_b (r_{a\perp} - r_{b\perp})^2$ inspired by Light-Front holography

—Brodsky, Teramond (2006)

- Longitudinal confinement $\Rightarrow V_{ab}^{(L)} = \frac{\kappa_L^4}{(m_a+m_b)^2} \partial_{x_a}(x_a x_b \partial_{x_b})$
  ✓ reduce to harmonic oscillator potential at non-relativistic limit

—Y Li, X Zhao, P Maris, J Vary (2016)

- $V_{ab}^{(OGE)} = f \frac{4\pi\alpha_s(Q_{ab}^2)}{Q_{ab}^2} \bar{u}_{s'_a}(k'_a)\gamma^\mu u_{s_a}(k_a)\bar{u}_{s'_b}(k'_b)\gamma^\nu u_{s_b}(k_b) g_{\mu\nu}$
  ✓ introduce short distance physics with spin structure
  ✓ provides the P-wave WFs, essential to generate the Pauli-FF

Effective Light-front Hamiltonian

$$P^{-}_{baryon} = H_{K.E.} + H_{trans} + H_{longi} + H_{OGE}$$

$$H_{K.E.} = \sum_i \frac{p_i^2 + m_q^2}{p_i^+} \qquad m_u = 0.3GeV, m_d = 0.301GeV$$

$$H_{trans} \sim \kappa_T^4 b^4 \zeta^2$$ -- Brodsky, Teramond arXiv: 1203.4025

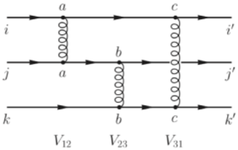
$$H_{longi} = -\sum_{ij} \kappa_L^4 b^4 \partial_{x_i}\left(x_i x_j \partial_{x_j}\right)$$ ---Y Li, X Zhao , P Maris , J Vary PLB 758(2016)

$$H_{OGE} = -\frac{C_F 4\pi\alpha_s(Q^2)}{Q^2} \sum_{i,j(i<j)} \bar{u}_{s_i'}(k_i')\gamma^\mu u_{s_i}(k_i)\bar{u}_{s_j'}(k_j')\gamma_\mu u_{s_j}(k_j)$$

Infrared cut off : $m_g = 0.01\ GeV, C_F = -\frac{2}{3}$

$$|P_{baryon}\rangle = \boxed{|qqq\rangle} + |qqqg\rangle + |qqq\ q\bar{q}\rangle + \cdots\cdots$$

Although we truncate to the leading Fock sector, we can solve the baryon system with multi-particle (at least three particle).



Wave-function production

- Calculate the Hamiltonian matrix elements:

$$H_{eff}^{\alpha'\alpha} = \langle\alpha'|H|\alpha\rangle$$

$\alpha'|$ & $|\alpha\rangle$ are the basis state of BLFQ, such as $|qqq\rangle$.

- Diagonalize H_{eff} and obtain its eigen spectrum

$$H_{eff}|\beta\rangle = H_{eff}^{\beta}|\beta\rangle$$

✓ $|\beta\rangle$ is the physical state and eigenstate of Hamiltonian. In case of proton $|\beta\rangle = |P_{proton}\rangle$.

- Evaluate observables:

$$\mathcal{O} = \langle\beta|\hat{\mathcal{O}}|\beta\rangle$$

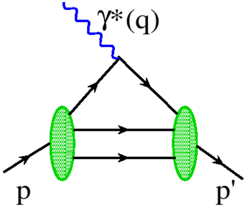
Form Factor in BLFQ

work in progress

- EM form factors in light-front (with $q^+ = 0$),

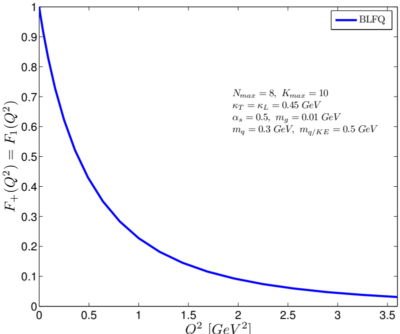
$$\checkmark\ \langle P+q;\Lambda|\frac{J^+(0)}{2P^+}|P;\Lambda\rangle = F_1(q^2)$$

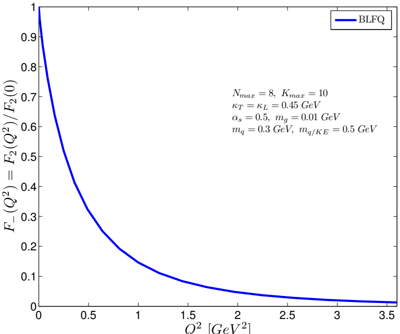
$$\langle P+q;\Lambda|\frac{J^+(0)}{2P^+}|P;-\Lambda\rangle = -(q^1 - iq^2)\frac{F_2(q^2)}{2M}$$



very preliminary results

$(Q^2 = -q^2)$





Form Factor in BLFQ

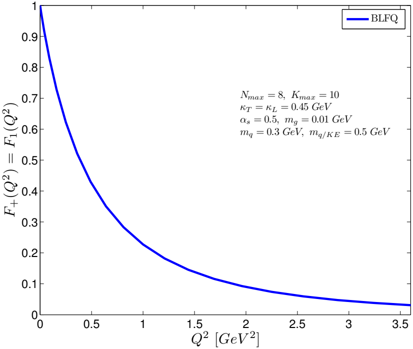
work in progress

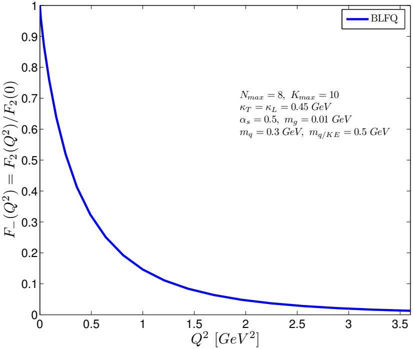
In terms of overlap of light-front WFs,

$$F_1(q^2) \sim \sum_{\lambda_i} \int [dx_i \; d^2\mathbf{k}_{\perp i}] \; \Psi^{\Lambda *}_{\lambda_i}(x_i, \mathbf{k}'_{\perp i}) \Psi^{\Lambda}_{\lambda_i}(x_i, \mathbf{k}_{\perp i})$$

$$F_2(q^2) \sim \sum_{\lambda_i} \int [dx_i \; d^2\mathbf{k}_{\perp i}] \Psi^{\Lambda *}_{\lambda_i}(x_i, \mathbf{k}'_{\perp i}) \Psi^{-\Lambda}_{\lambda_i}(x_i, \mathbf{k}_{\perp i})$$

very preliminary results $(Q^2 = -q^2)$





Up and down quark form factor in proton

work in progress

- Flavor form factors:

$$F_1^q = n_q F_+(Q^2) \qquad F_2^q = \kappa_q F_-(Q^2)$$

- Normalization constants:

$$n_d = 1, \quad n_u = 2 \qquad \text{quark numbers}$$

$$\kappa_d = -2.033, \quad \kappa_u = 1.673 \qquad \text{anomalous magnetic moments}$$

Up and down quark form factor in proton

work in progress

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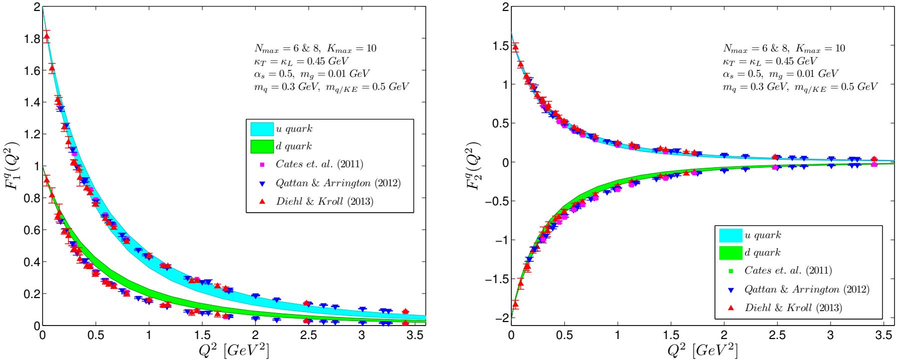

Ratios of Flavor FFs

work in progress

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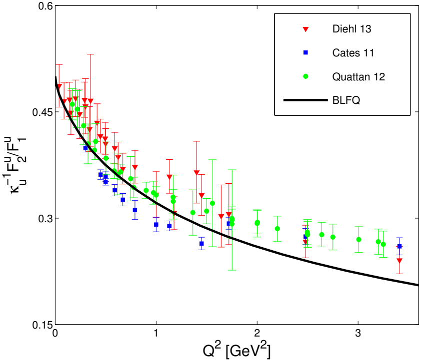
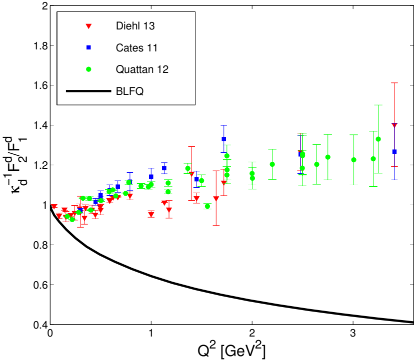
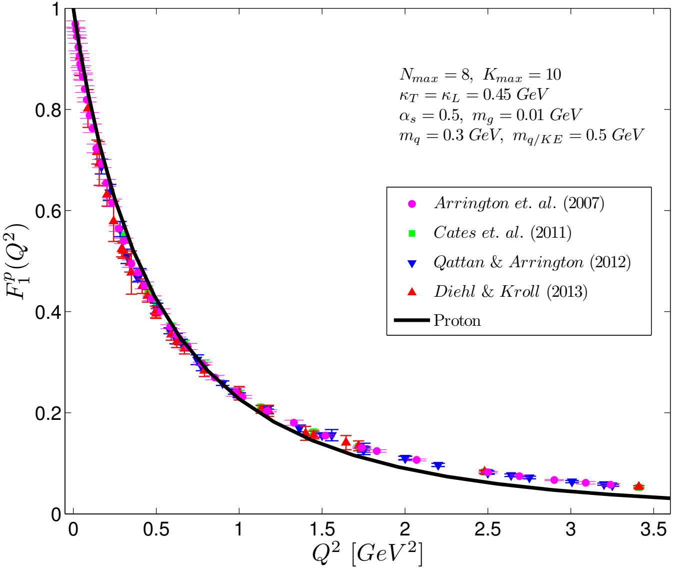




Dirac Form Factor for proton in BLFQ

work in progress

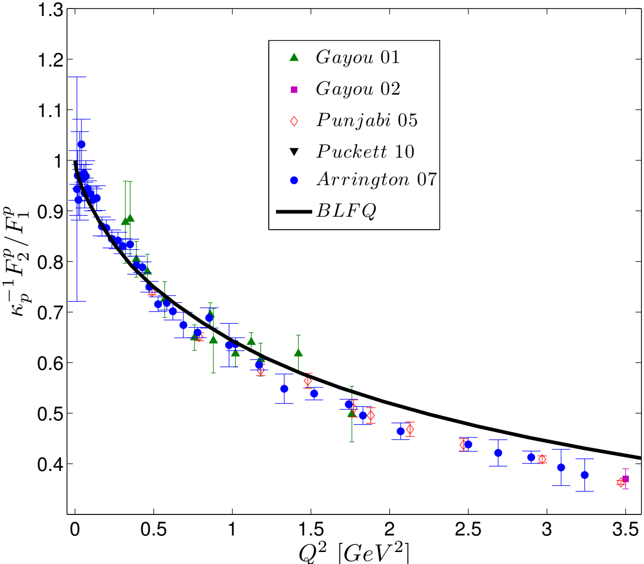
Flavor decomposition: $F_i^{p/n} = e_{u/d}F_i^u + e_{d/u}F_i^d$ — Cates $et.\ al.$ PRL 106



Ratio of proton form factors in BLFQ

work in progress

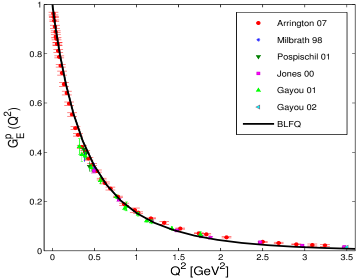
Flavor decomposition: $F_i^{p/n} = e_{u/d}F_i^u + e_{d/u}F_i^d$ — Cates $et.\ al.$ PRL 106

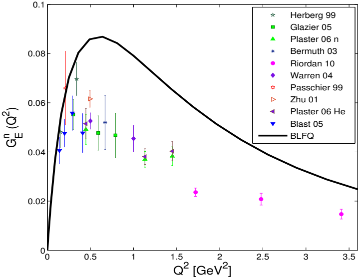


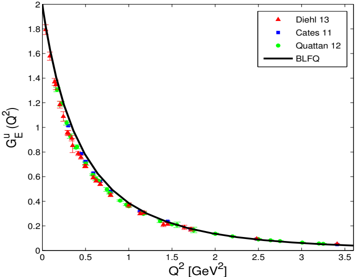
Sachs Form Factors in BLFQ

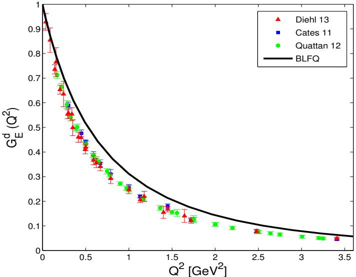
work in progress

Electric Sach's FF $\Rightarrow G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M_n^2} F_2(Q^2)$





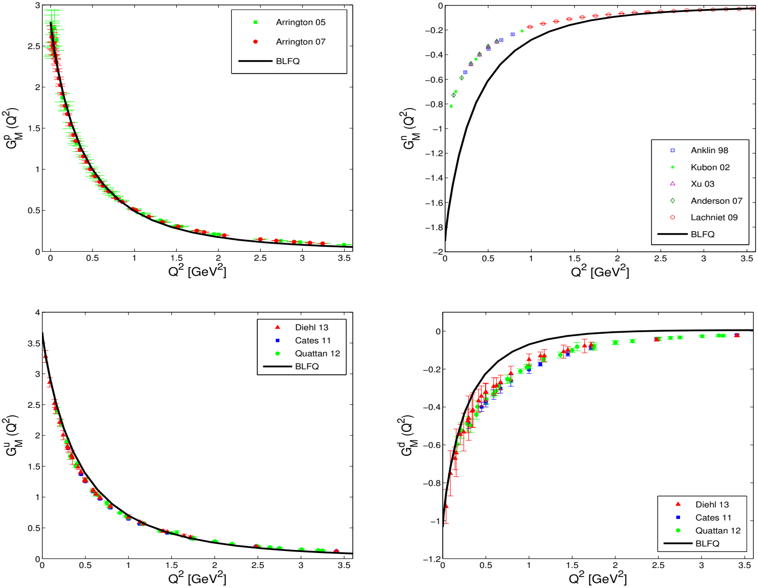




Sach's Form Factors in BLFQ

work in progress

Magnetic Sach's FF $\Rightarrow G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$



Electromagnetic radii

$$\langle r_E^2 \rangle^N = -6 \frac{dG_E^N(Q^2)}{dQ^2}\Big|_{Q^2=0},$$

$$\langle r_M^2 \rangle^N = -\frac{6}{G_M^N(0)} \frac{dG_M^N(Q^2)}{dQ^2}\Big|_{Q^2=0}$$

The Sachs form factors are defined as

$$G_E^N(Q^2) = F_1^N(Q^2) - \frac{Q^2}{4M_N^2} F_2^N(Q^2),$$

$$G_M^N(Q^2) = F_1^N(Q^2) + F_2^N(Q^2).$$

Quantity	BLFQ	Data from PDG
r_E^p (fm)	0.804	0.877 ± 0.005
r_M^p (fm)	0.917	0.777 ± 0.016
$\langle r_E^2 \rangle^n$ (fm^2)	-0.1214	-0.1161 ± 0.0022
r_M^n (fm)	1.007	$0.862^{+0.009}_{-0.008}$

Generalized parton distributions (GPDs)

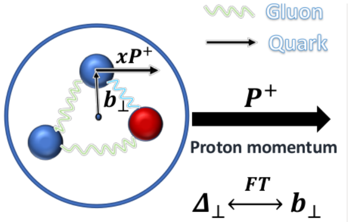
- Off-forward matrix element $\Rightarrow$ no probabilistic interpretation !! [in momentum space]
- In forward limit : GPDs $\Rightarrow$ PDFs.
- First moments of GPDs are related to the Form Factors.

- GPDs[$\xi = 0$] in impact parameter space $\Rightarrow$ distribution of parton in transverse position space

$$\mathcal{X}(x,b) = \frac{1}{(2\pi)^2}\int d^2\Delta e^{-i\Delta^{\perp}\cdot b^{\perp}}\mathcal{X}(x,t).$$

➢ **GPDs** appear in **DVCS** processes.

- GPDs are functions of three variables :
    - *Longitudinal momentum fraction* $x = \frac{k^+}{P^+}$
    - *Longitudinal momentum transfer -->* *skewness* $\xi = \frac{\Delta^+}{P^+} = 0$
    - *Square of total mom transfer* $t = \Delta^2 = (P' - P)^2$



where the $b_\perp$ is transverse position of parton

Form Factors Vs. GPDs

operator	forward matrix elem.	off-forward matrix elem.	position space
$\bar{q}\gamma^+ q$	Q	$F(t)$	$\rho(\vec{r})$
$\int \frac{dx^- e^{ixp^+x^-}}{4\pi} \bar{q}\left(\frac{-x^-}{2}\right)\gamma^+ q\left(\frac{x^-}{2}\right)$	$q(x)$	$H(x,0,t)$	$q(x,\mathbf{b}_\perp)$

$q(x,\mathbf{b}_\perp)$ = impact parameter dependent PDF

Nucleon GPDs:

✓For unpolarized nucleon:

$$\int \frac{dx^-}{4\pi} e^{ixP^+x^-} \langle p+\Delta, \uparrow | \bar{\psi}_q(0) \gamma^+ \psi_q(x^-) | p, \uparrow \rangle = H(x, t = \Delta^2)$$

$$\int \frac{dx^-}{4\pi} e^{ixP^+x^-} \langle p+\Delta, \uparrow | \bar{\psi}_q(0) \gamma^+ \psi_q(x^-) | p, \downarrow \rangle = -\frac{\Delta_x - i\Delta_y}{2M} E(x, t = \Delta^2)$$

Nucleon GPDs:

✓For unpolarized nucleon:

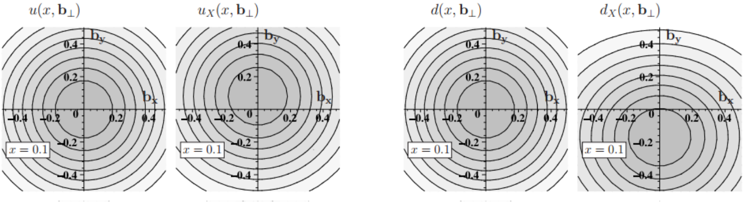
$$\int \frac{dx^-}{4\pi} e^{ixP^+x^-} \langle p+\Delta,\uparrow|\bar{\psi}_q(0)\gamma^+\psi_q(x^-)|p,\uparrow\rangle = H(x,t=\Delta^2)$$

$$\int \frac{dx^-}{4\pi} e^{ixP^+x^-} \langle p+\Delta,\uparrow|\bar{\psi}_q(0)\gamma^+\psi_q(x^-)|p,\downarrow\rangle = -\frac{\Delta_x - i\Delta_y}{2M} E(x,t=\Delta^2)$$

✓For transversely polarized nucleon: $|X\rangle \equiv |p,\uparrow\rangle + |p,\downarrow\rangle$: $\Rightarrow$

unpolarized quark distribution for this state:

$$q_X(x,b_\perp) = \mathcal{H}(x,b_\perp) - \frac{1}{2M}\frac{\partial}{\partial b_y}\int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\Delta^\perp\cdot b^\perp} E(x,t=\Delta^2)$$



Burkardt, Int J mod Phys. A18, 173 (2003)

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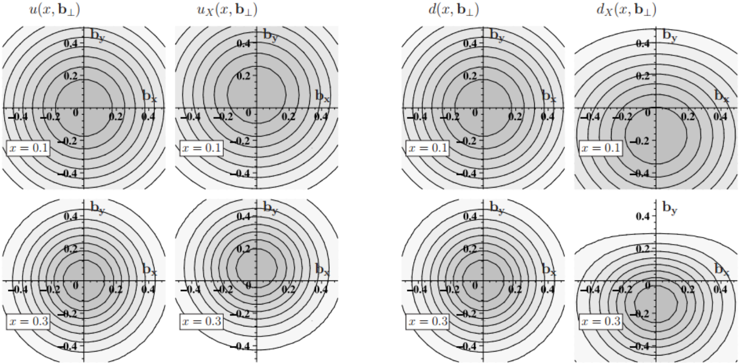


—Burkardt, Int J mod Phys. A18, 173 (2003)

Spin non-flip GPDs in BLFQ

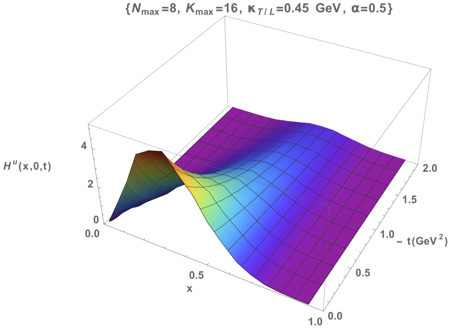
work in progress

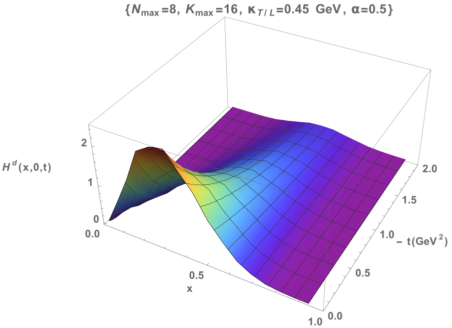
✓ Dirac form factor in light-front [with $q^+ = 0$],

$$F_1(-q^2) = \langle P+q; \Lambda | \frac{J^+(0)}{2P^+} | P; \Lambda \rangle; \qquad F_1^q(-q^2) = \int dx\ H^q(x, -q^2).$$

✓ In terms of overlap of light-front WFs:,

$$H^q(x, t=-q^2) \sim \sum_{\lambda_i} \int \left[dx_{i\neq 1}\ d^2\mathbf{k}_{\perp i}\right] \Psi^{\Lambda *}_{\lambda_i}(x_i, \mathbf{k}'_{\perp i}) \Psi^{\Lambda}_{\lambda_i}(x_i, \mathbf{k}_{\perp i})$$





Spin flip GPDs in BLFQ

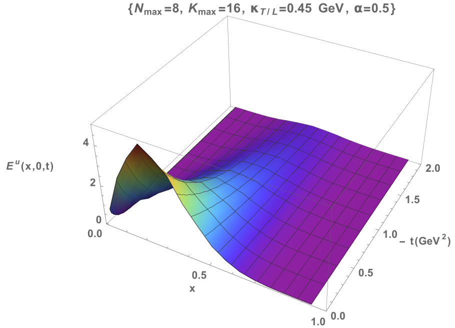
work in progress

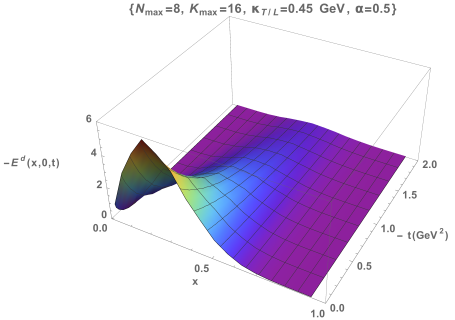
✓ Pauli form factor in light-front [with $q^+ = 0$],

$$F_2(-q^2) = \langle P+q;\Lambda|\frac{J^+(0)}{2P^+}|P;-\Lambda\rangle; \qquad F_2^q(-q^2) = \int dx\ E^q(x,-q^2).$$

✓ In terms of overlap of light-front WFs:,

$$E^q(x,t=-q^2) \sim \sum_{\lambda_i}\int \left[dx_{i\neq 1}\ d^2\mathbf{k}_{\perp i}\right]\ \Psi^{\Lambda *}_{\lambda_i}(x_i,\mathbf{k}'_{\perp i})\Psi^{-\Lambda}_{\lambda_i}(x_i,\mathbf{k}_{\perp i})$$

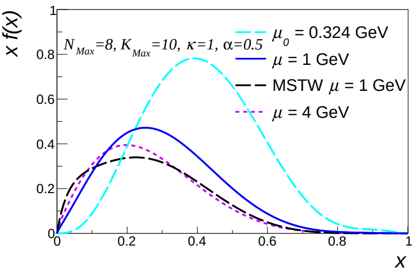


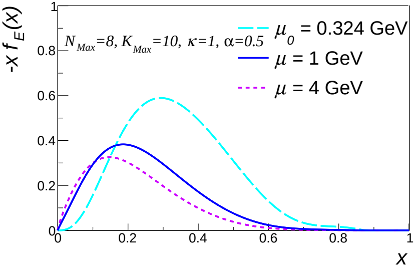


Parton distribution functions (PDFs)

PDFs is equal to the GPDs($t = 0$):

$f(x) = H(x, t = 0)$ $\qquad$ $f_E(x) = E(x, t = 0)$





We use the DGLAP equation to evolve the PDF. Qualitative behavior of PDF is almost same with the global fit MSTW(2008) PDF .

Note: In DGLAP, we use leading order running coupling constant:

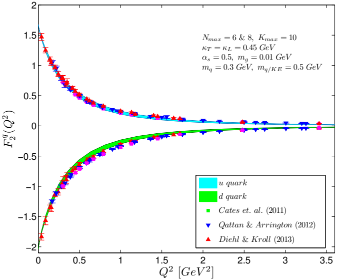
$$\alpha_s(Q^2) = \frac{4\pi}{9\ln(\frac{Q^2}{\Lambda^2_{\rm QCD}})}, \qquad \text{with} \quad \Lambda_{\rm QCD} = 226\ \text{MeV}$$

Conclusions & outlook

- We have discussed the very preliminary results of nucleon form factors, PDFs & GPDs in BLFQ approach.
- In the effective Hamiltonian, we have the kinetic energy & the confining potential in both the transverse and longitudinal direction and one gluon exchange with fixed coupling. Here, we consider only the leading Fock sector.
- BLFQ formalism provides promising results in order to understand the nucleon structure.

<u>Outlook:</u>

- Increase basis size
- Include the higher Fock component $|qqqg\rangle$.
- Investigate other nucleon properties..
- Investigate the structure of other baryons.



Thank You

QCD evolution of PDF

The Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation bridges PDFs between a final scale and a initial scale. The leading-order (LO) DGLAP equation with flavor number N_f reads:

$$\frac{\partial}{\partial\, ln\ \mu^2}\begin{pmatrix} f_q(x,\mu^2) \\ f_g(x,\mu^2) \end{pmatrix} = \frac{\alpha_s(\mu^2)}{2\pi}\int_x^1 \frac{dy}{y}\begin{pmatrix} P_{qq}(\frac{x}{y}) & P_{qg}(\frac{x}{y}) \\ P_{gq}(\frac{x}{y}) & P_{gg}(\frac{x}{y}) \end{pmatrix}\begin{pmatrix} f_q(y,\mu^2) \\ f_g(y,\mu^2) \end{pmatrix}, \tag{14}$$

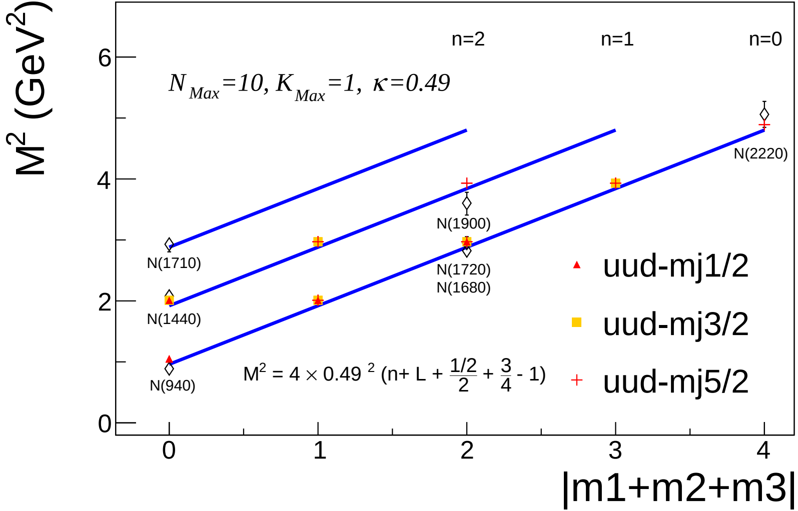
where the LO splitting functions are given by

$$\begin{aligned} P_{qq}(z) &= \frac{4}{3}\frac{1+z^2}{1-z} + 2\ \delta(1-z) - \frac{8}{3}\ \delta(1-z)\int_0^1 dz'\frac{1}{1-z'}, \\ P_{qg}(z) &= \frac{1}{2}\left[z^2 + (1-z)^2\right], \\ P_{gq}(z) &= \frac{4}{3}\frac{1+(1-z)^2}{z}, \\ P_{gg}(z) &= 6\left[\frac{1-z}{z} + z(1-z) + \frac{z}{1-z}\right] + \left(\frac{11}{2} - \frac{N_f}{3}\right)\delta(1-z) \\ &\quad - 6\ \delta(1-z)\int_0^1 dz'\frac{1}{1-z'}, \end{aligned} \tag{15}$$

and the LO running coupling constant is

$$\alpha_s(\mu^2) = \frac{12\pi}{(33-2N_f)\ ln\left(\mu^2/\Lambda^2_{N_f}\right)}. \tag{16}$$

Mass spectrum

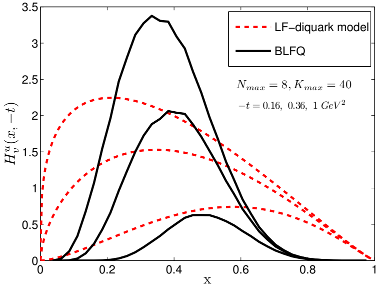


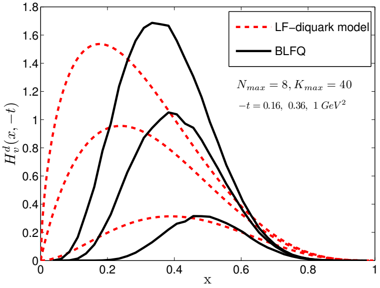
GPDs: BLFQ vs LF quark-diquark

✓ In terms of overlap of light-front WFs:, $H^q(x, -q^2)$:

$$H^q(x,-t) = \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \left[\psi^{+*}_{+q}(x,\mathbf{k}'_\perp)\psi^{+}_{+q}(x,\mathbf{k}_\perp) + \psi^{+*}_{-q}(x,\mathbf{k}'_\perp)\psi^{+}_{-q}(x,\mathbf{k}_\perp) \right]$$

where $\mathbf{k}'_\perp = \mathbf{k}_{\perp 1} - (1-x)\mathbf{q}_\perp; \quad t = -\mathbf{q}^2_\perp$





Effect of longitudinal confinement