

Nuclear structure and dynamics from chiral forces

Nuclear Theory in the Supercomputing Era – 2018
(NTSE-2018)

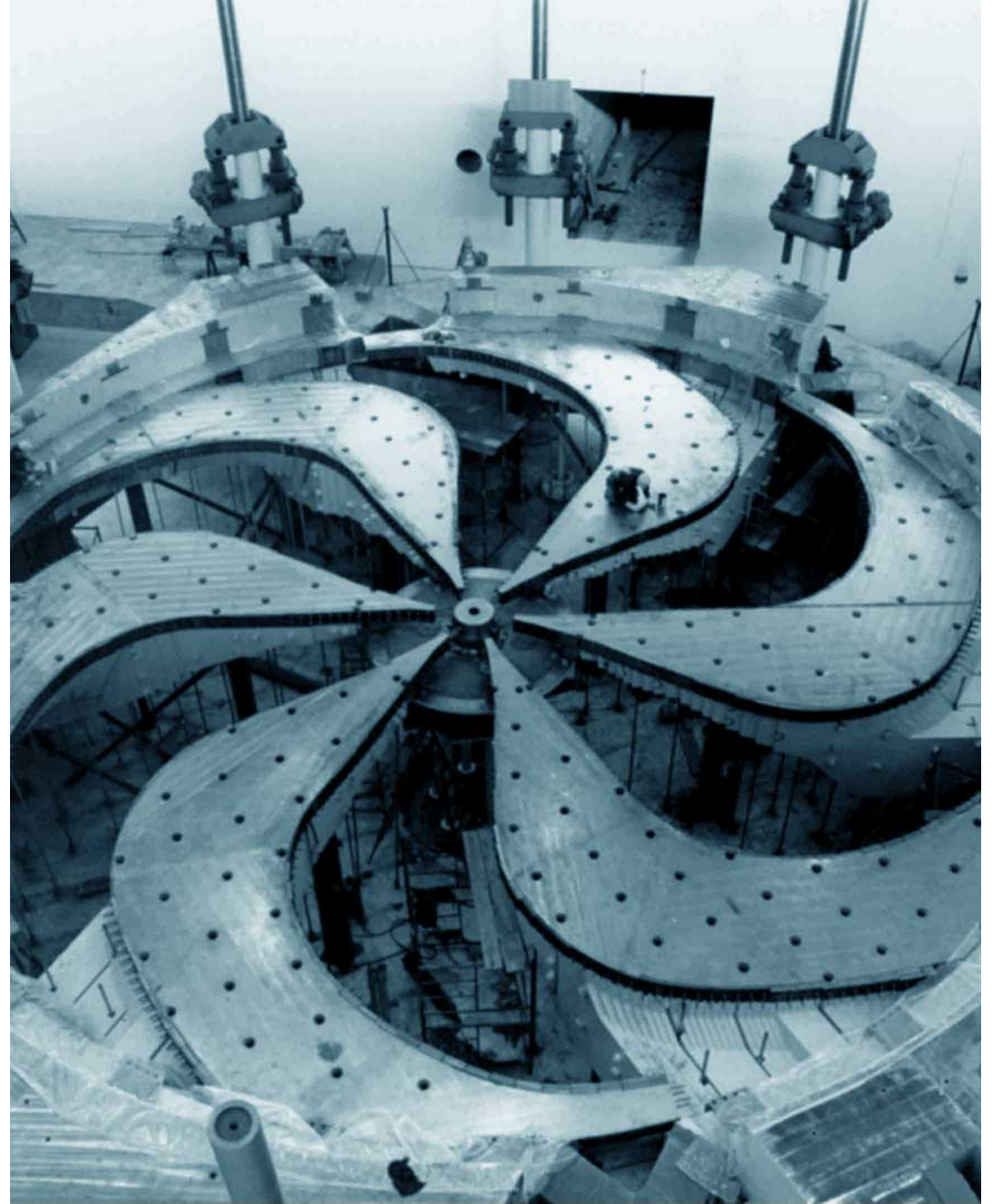
IBS Headquarters, Daejeon, Korea
29 October – 2 November 2018

Petr Navratil

TRIUMF

Collaborators: S. Quaglioni (LLNL), G. Hupin (Orsay),
M. Vorabbi, A. Calci (TRIUMF), P. Gysbers (UBC/TRIUMF),
M. Gennari (U Waterloo)

2018-10-31



Outline

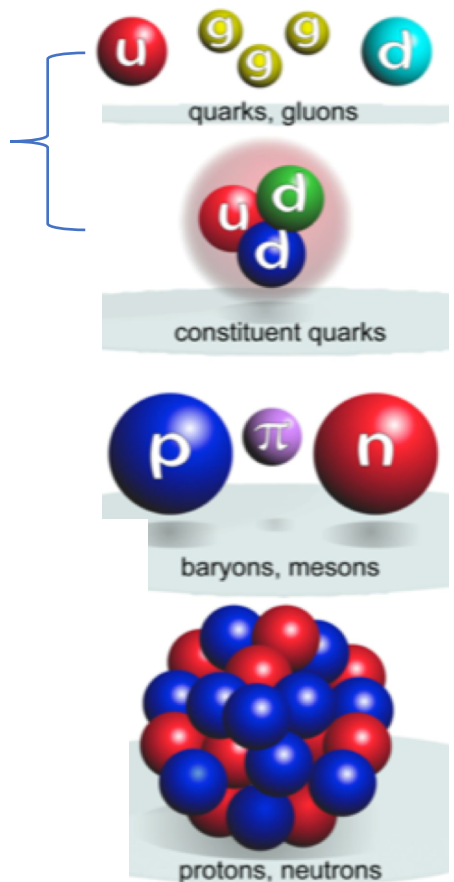
- Nuclear structure and reactions from first principles
- New chiral NN $N^4\text{LO} + 3\text{N}$
 - Beta decays of light nuclei in NCSM
 - Microscopic optical potentials from NCSM densities
- No-Core Shell Model with Continuum (NCSMC)
- $\text{N}-^4\text{He}$ scattering and polarized $\text{D}+\text{T}$ fusion
- Structure of ^7He
- ^{12}N , $^{11}\text{C}(\text{p},\text{p})$ scattering and $^{11}\text{C}(\text{p},\gamma)^{12}\text{N}$ capture
 - Support of approved TRIUMF TUDA experiment

First principles or *ab initio* nuclear theory

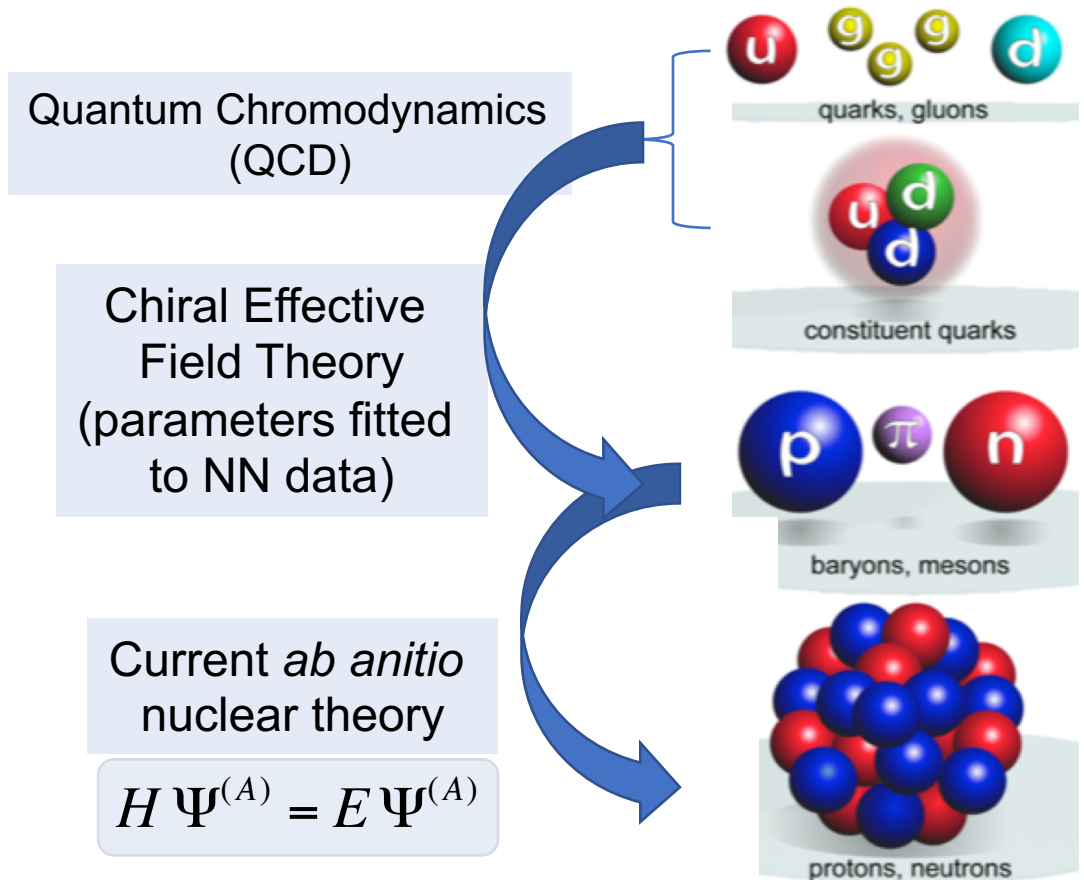
Quantum Chromodynamics
(QCD)



Genuine *Ab Initio*



First principles or *ab initio* nuclear theory – what we do at present

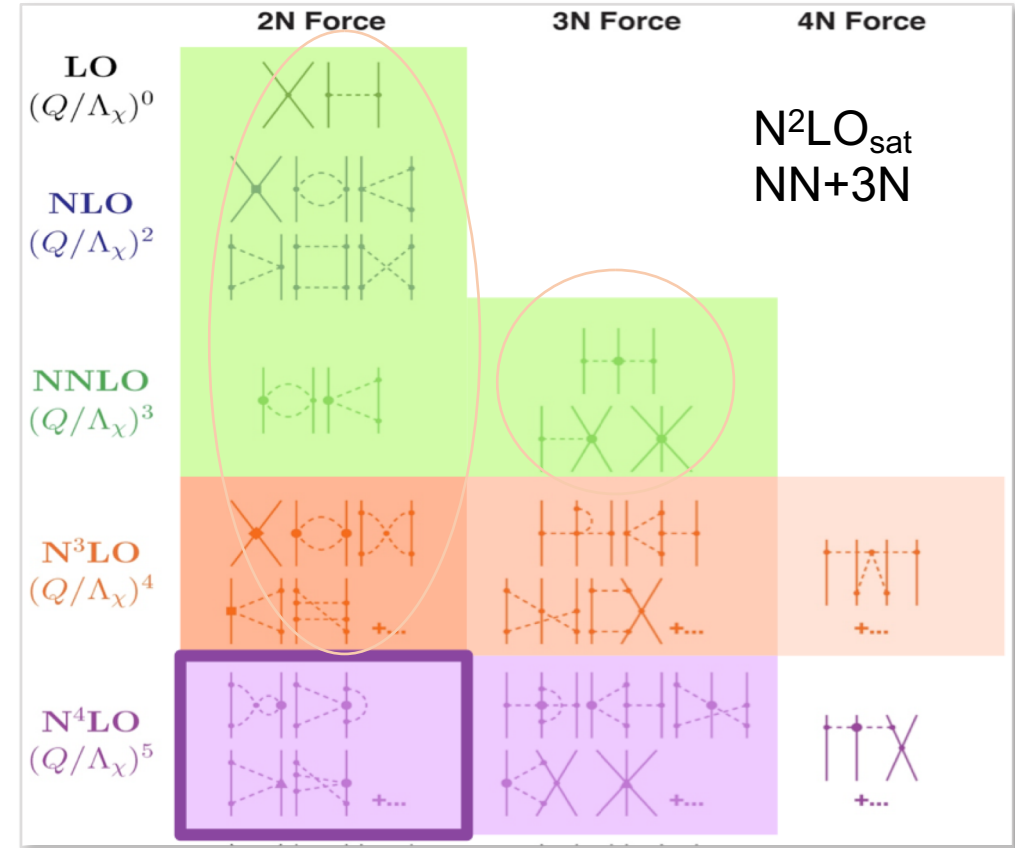


- *Ab initio*
 - ✧ Degrees of freedom: Nucleons
 - ✧ All nucleons are active
 - ✧ Exact Pauli principle
 - ✧ Realistic inter-nucleon interactions
 - ✧ Accurate description of NN (and 3N) data
 - ✧ Controllable approximations

Chiral Effective Field Theory

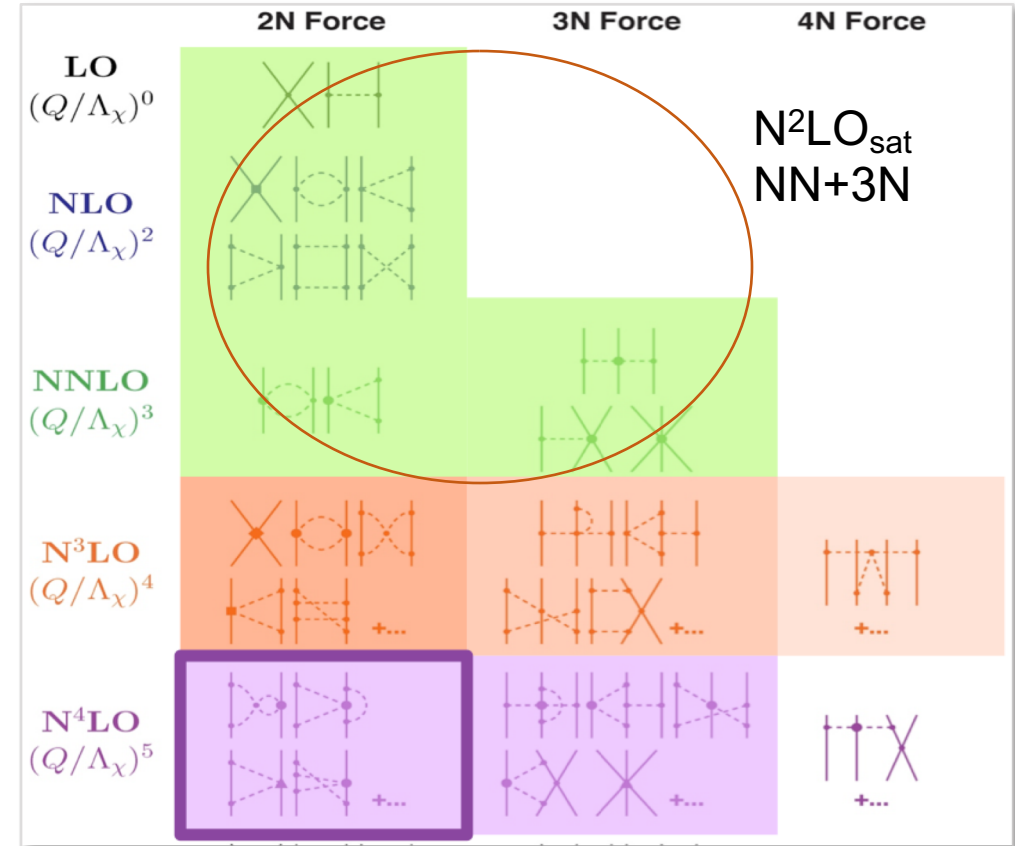
- Inter-nucleon forces from chiral effective field theory
 - Based on the symmetries of QCD
 - Chiral symmetry of QCD ($m_u \approx m_d \approx 0$), spontaneously broken with pion as the Goldstone boson
 - Degrees of freedom: nucleons + pions
 - Systematic low-momentum expansion to a given order (Q/Λ_χ)
 - Hierarchy
 - Consistency
 - Low energy constants (LEC)
 - Fitted to data
 - Can be calculated by lattice QCD

$\Lambda_\chi \sim 1 \text{ GeV}$:
Chiral symmetry breaking scale



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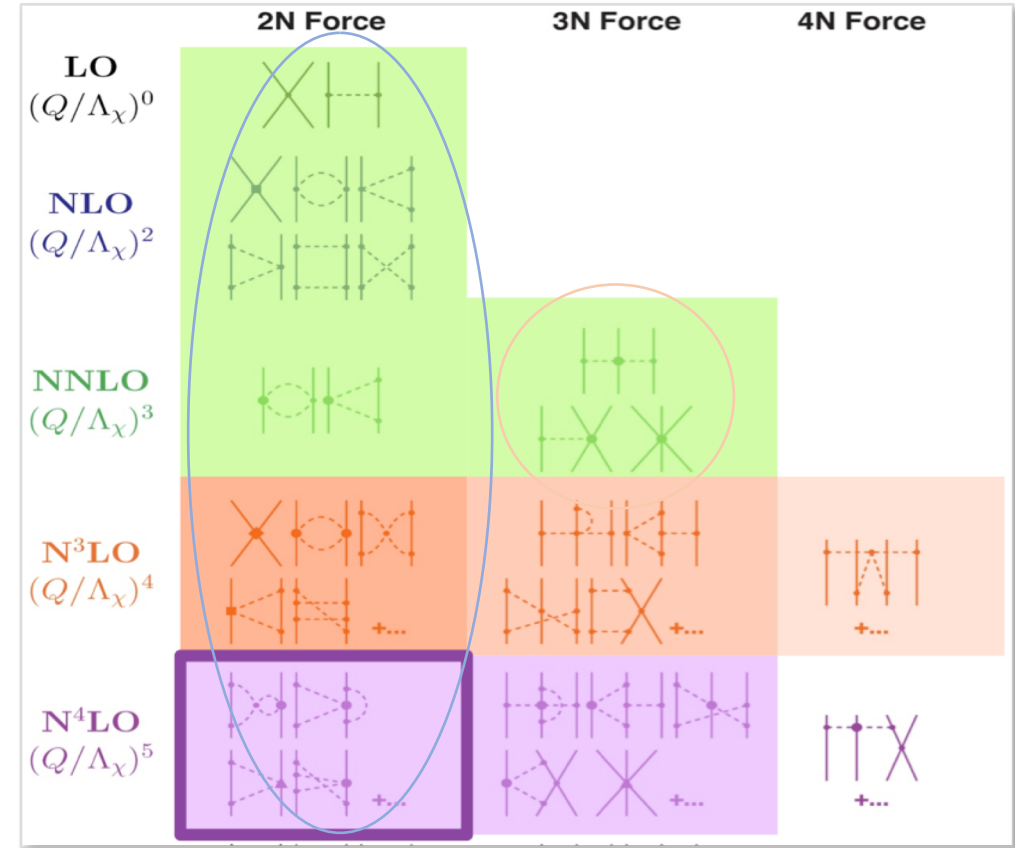


$\Lambda_\chi \sim 1$ GeV :
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Chiral Effective Field Theory

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$\Lambda_\chi \sim 1 \text{ GeV}$:
Chiral symmetry breaking scale



N⁴LO500 NN
+ N²LO 3N

Currents in chiral EFT

- Meson-exchange current

PHYSICAL REVIEW C **67**, 055206 (2003)

Parameter-free effective field theory calculation for the solar proton-fusion and hep processes

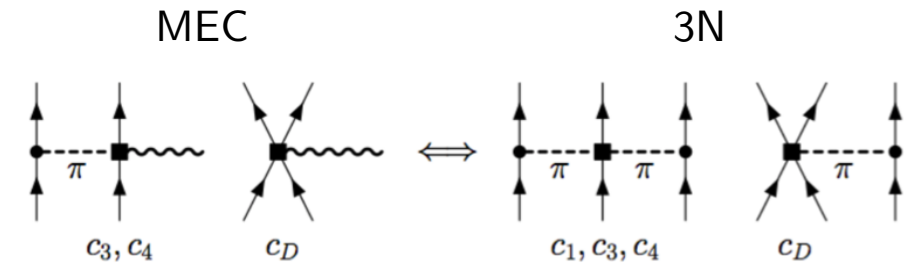
T.-S. Park,^{1,2,3} L. E. Marcucci,^{4,5} R. Schiavilla,^{6,7} M. Viviani,^{5,4} A. Kievsky,^{5,4} S. Rosati,^{5,4} K. Kubodera,^{1,2}
D.-P. Min,⁸ and M. Rho^{1,9}

- weak axial current
 - one-body: LO - Gamow-Teller

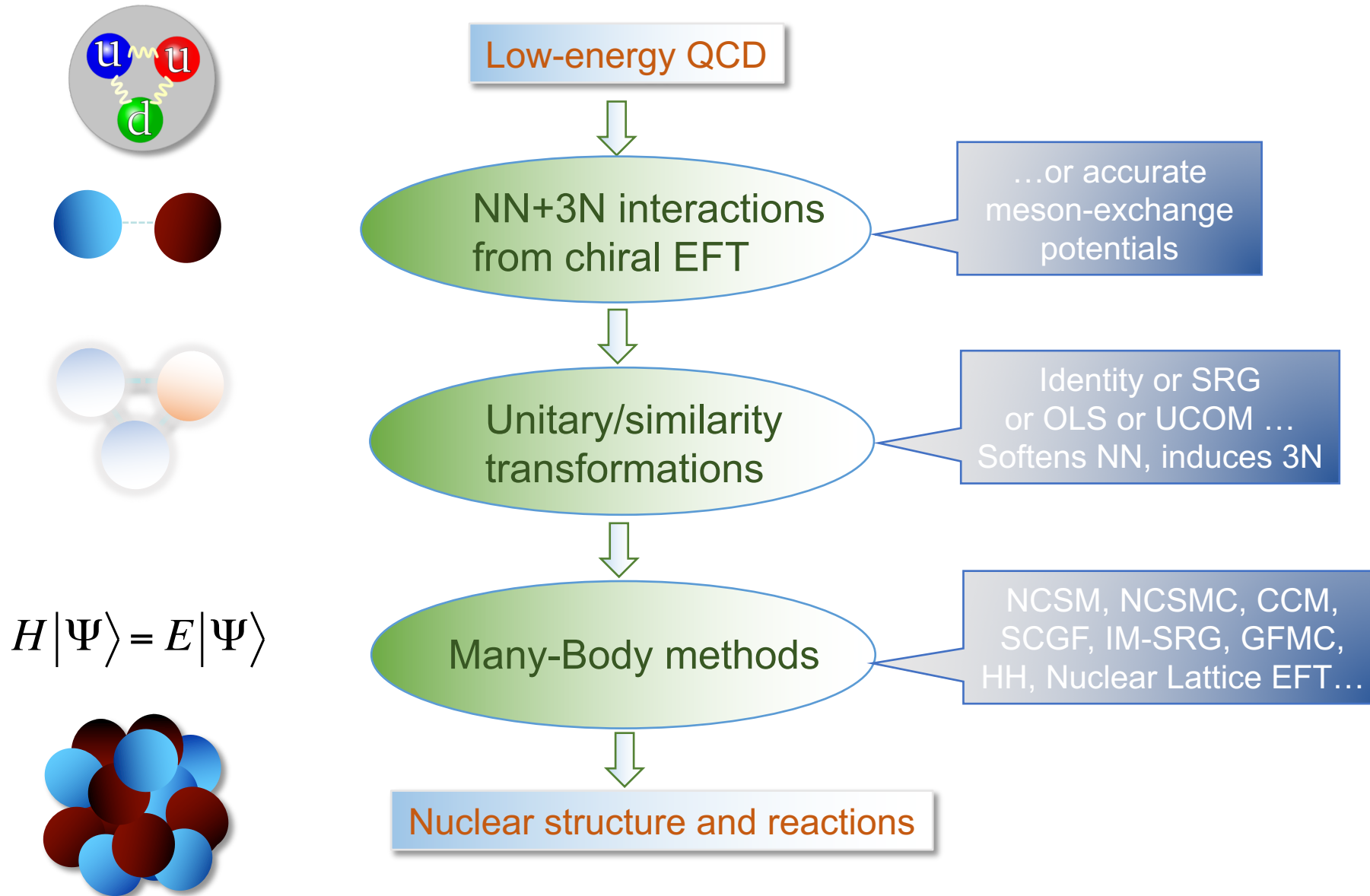
$$A_l = -g_A \tau_l^- e^{-iq \cdot r_l} \left[\boldsymbol{\sigma}_l + \frac{2(\bar{\mathbf{p}}_l \boldsymbol{\sigma}_l \cdot \bar{\mathbf{p}}_l - \boldsymbol{\sigma}_l \bar{\mathbf{p}}_l^2) + i\mathbf{q} \times \bar{\mathbf{p}}_l}{4m_N^2} \right]$$

- two-body: MEC

$$A_{12} = \frac{g_A}{2m_N f_\pi^2} \frac{1}{m_\pi^2 + k^2} \left[-\frac{i}{2} \tau_\times^- \mathbf{p} (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) \cdot \mathbf{k} \right. \\ \left. + 4\hat{c}_3 \mathbf{k} \mathbf{k} \cdot (\tau_1^- \boldsymbol{\sigma}_1 + \tau_2^- \boldsymbol{\sigma}_2) + \left(\hat{c}_4 + \frac{1}{4} \right) \tau_\times^- \mathbf{k} \times [\boldsymbol{\sigma}_\times \times \mathbf{k}] \right] \\ + \frac{g_A}{m_N f_\pi^2} [2\hat{d}_1 (\tau_1^- \boldsymbol{\sigma}_1 + \tau_2^- \boldsymbol{\sigma}_2) + \hat{d}_2 \tau_\times^a \boldsymbol{\sigma}_\times],$$



From QCD to nuclei

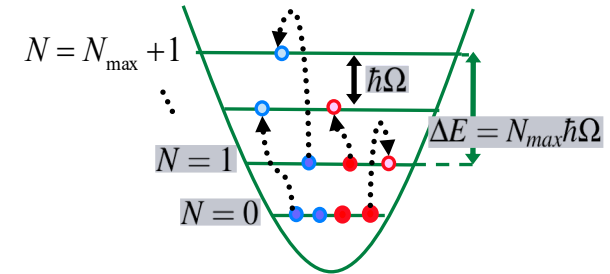


Conceptually simplest *ab initio* method: No-Core Shell Model (NCSM)



NCSM

- Basis expansion method
 - Harmonic oscillator (HO) basis truncated in a particular way (N_{\max})
 - Why HO basis?
 - Lowest filled HO shells match magic numbers of light nuclei (2, 8, 20 – ^4He , ^{16}O , ^{40}Ca)
 - Equivalent description in relative-coordinate and Slater determinant basis
- Short- and medium range correlations
- Bound-states, narrow resonances



$${}^{(A)} \Psi^A = \sum_{N=0}^{N_{\max}} \sum_i c_{Ni} \Phi_{Ni}^{HO}(\vec{\eta}_1, \vec{\eta}_2, \dots, \vec{\eta}_{A-1})$$

$${}^{(A)} \Psi_{SD}^A = \sum_{N=0}^{N_{\max}} \sum_j c_{Nj}^{SD} \Phi_{SDNj}^{HO}(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_A) = \Psi^A \varphi_{000}(\vec{R}_{CM})$$

Progress in Particle and Nuclear Physics 69 (2013) 131–181

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journal homepage: www.elsevier.com/locate/ppnp

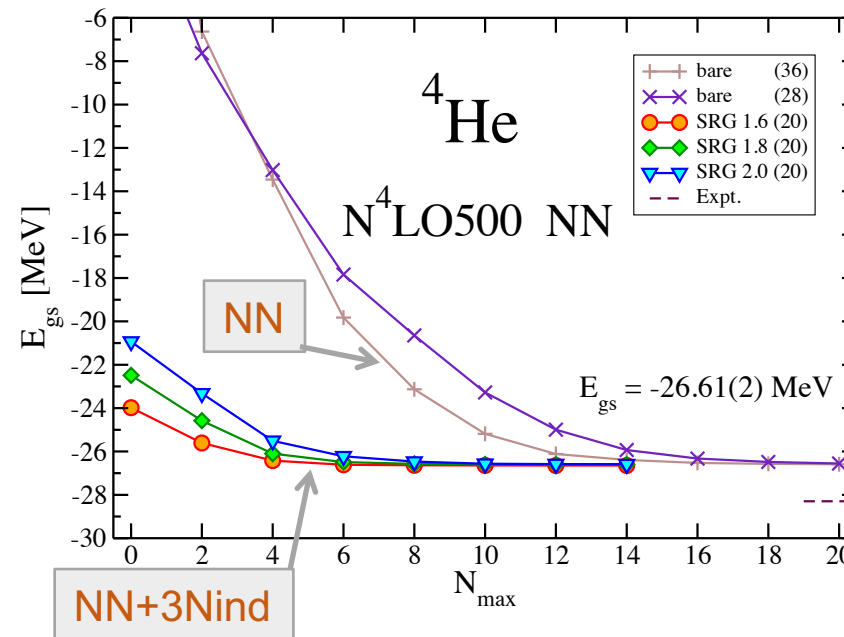
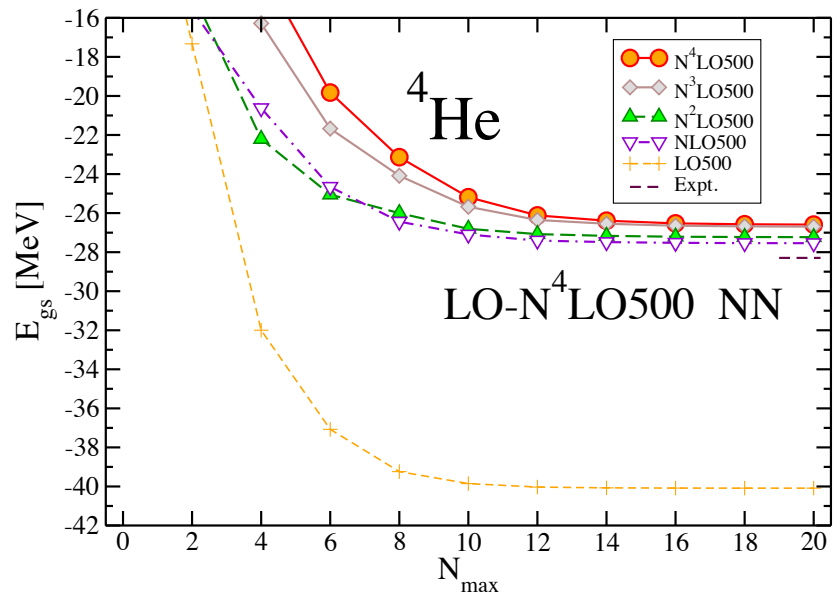
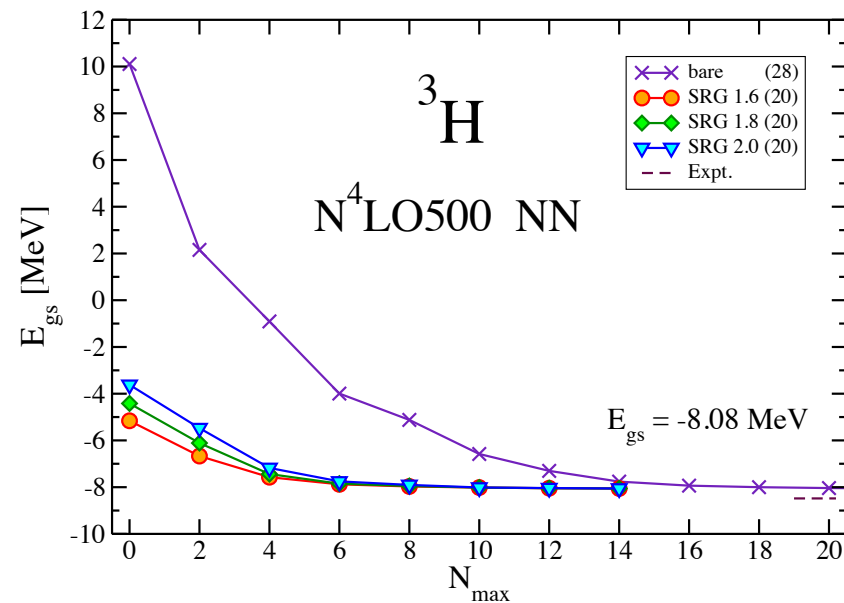
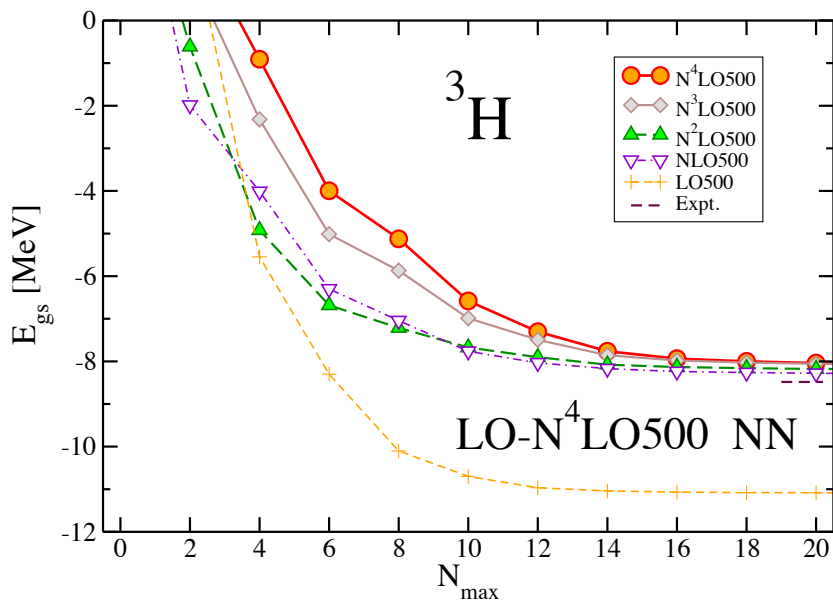


Review

Ab initio no core shell model

Bruce R. Barrett^a, Petr Navrátil^b, James P. Vary^{c,*}

^3H and ^4He with chiral EFT interactions up to N^4LO



${}^3\text{H} \rightarrow {}^3\text{He} \beta$ decay

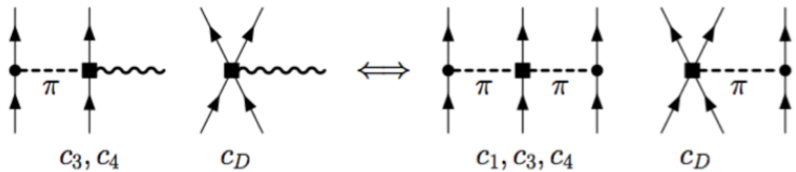
$$\hat{O} = GT^{(1)} + MEC^{(2)} \rightarrow \hat{O}_\alpha = GT^{(1)} + GT_\alpha^{(2)} + MEC_\alpha^{(2)} + \dots$$

Operator:

Gamow-Teller (1-body) + chiral meson exchange current (2-body)
Park (2003)

Potential: "N⁴LO NN"

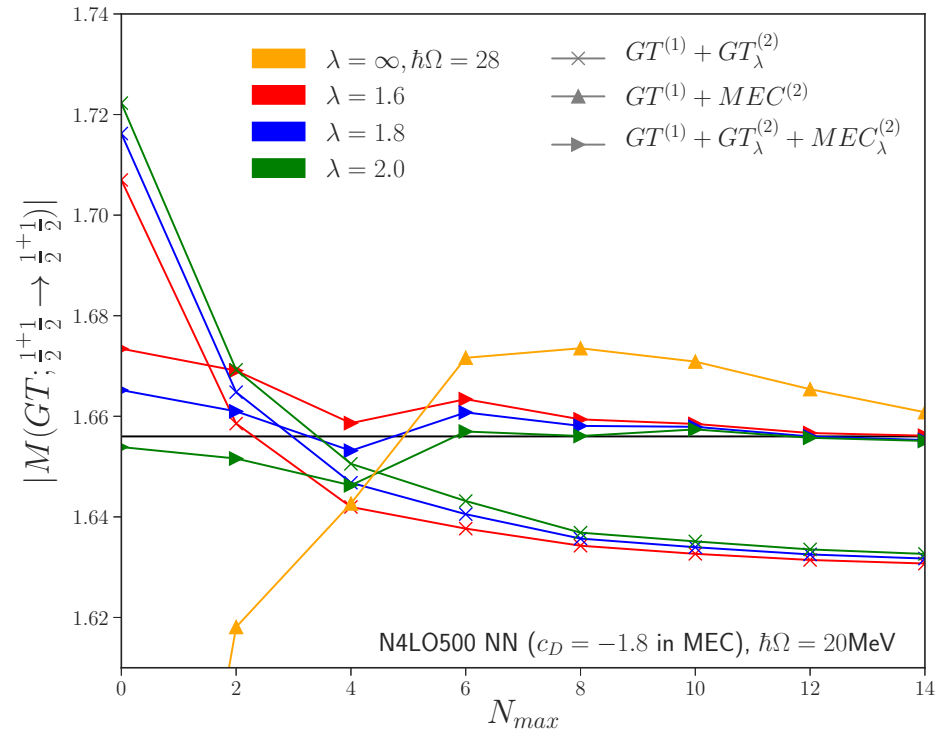
- chiral NN @ N⁴LO, Machleidt PRC96 (2017), 500MeV cutoff
- LEC $c_D = -1.8$ determined



Original EM 2003 N³LO NN $c_D = +0.8$
(3N repulsive)

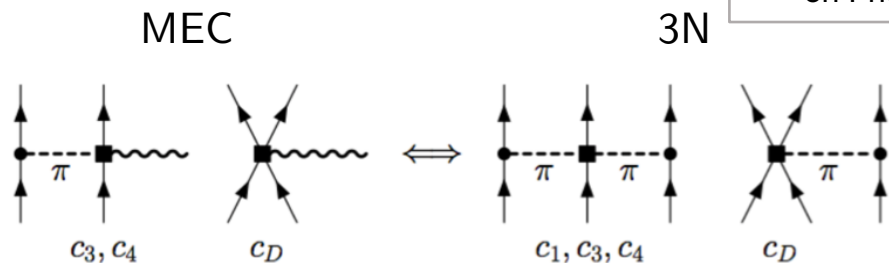


Determination of the c_D parameter
relevant to chiral 3N force $c_D = -1.8$
(3N attractive)



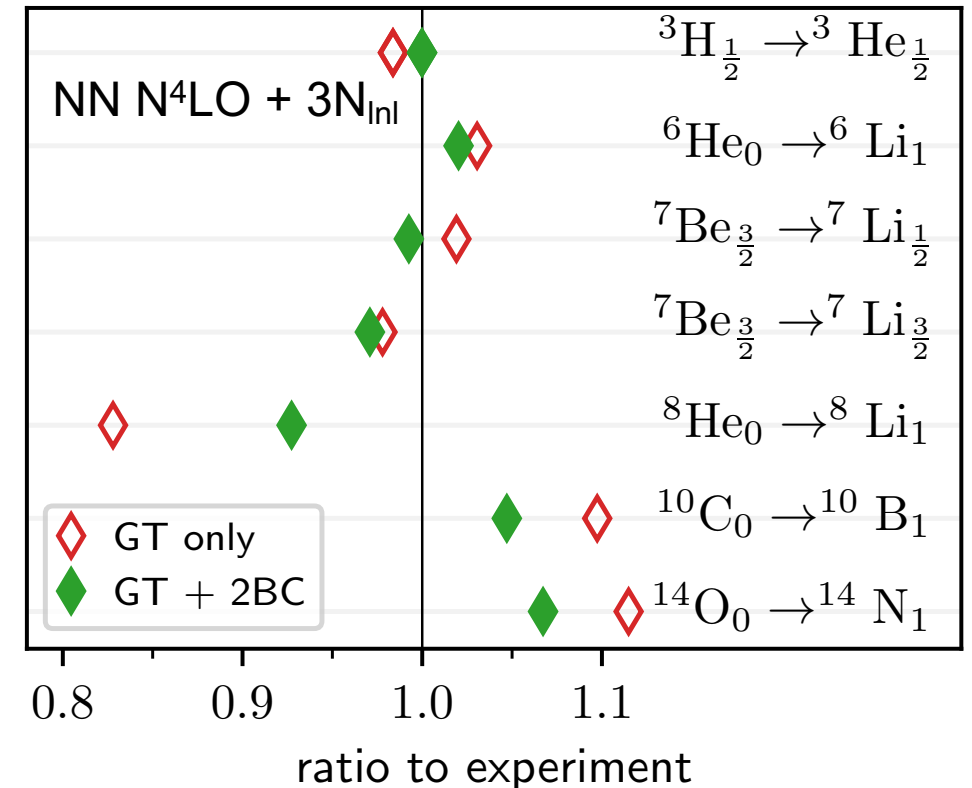
Applications to β decays in p-shell nuclei and beyond

- Does inclusion of the MEC explain g_A quenching?
- In light nuclei correlations present in *ab initio* (NCSM) wave functions explain almost all of the quenching compared to the standard shell model
 - MEC inclusion overall improves agreement with experiment
- The effect of the MEC inclusion is greater in heavier nuclei
- SRG evolved matrix elements used in coupled-cluster and IM-SRG calculations (up to ^{100}Sn)



See talk
by Gaute Hagen
on Friday

Hollow symbols – GT
Filled symbols – GT+MEC
Both Hamiltonian and operators SRG evolved
Hamiltonian and current consistent parameters



Microscopic optical potentials derived from *ab initio* translationally invariant nonlocal one-body densities

Michael Gennari*

University of Waterloo, 200 University Avenue West Waterloo, Ontario N2L 3G1, Canada
and TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia V6T 2A3, Canada

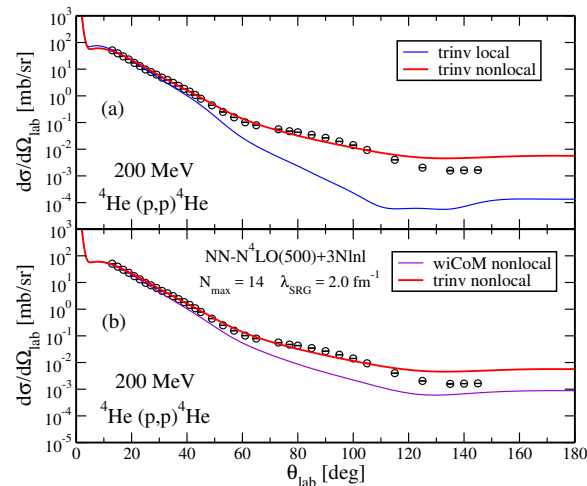
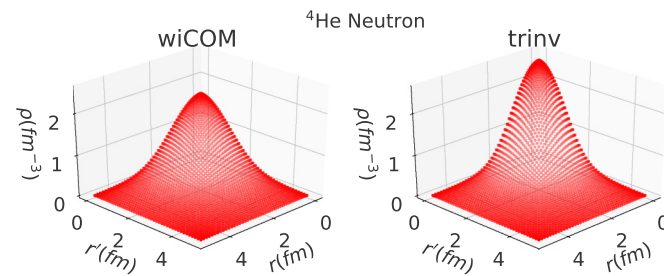
Matteo Vorabbi,[†] Angelo Calci, and Petr Navrátil[‡]

TRIUMF, 4004 Wesbrook Mall, Vancouver, British Columbia V6T 2A3, Canada

14

Microscopic optical potentials from NCSM densities

- Translationally-invariant non-local densities from NCSM calculations with chiral NN $N^4\text{LO} + 3\text{N } N^2\text{LO}$ interactions
- High-energy proton-nucleus scattering with microscopic optical potentials from chiral $N^4\text{LO}$ NN interaction and NCSM densities



Microscopic optical potentials from NCSM densities

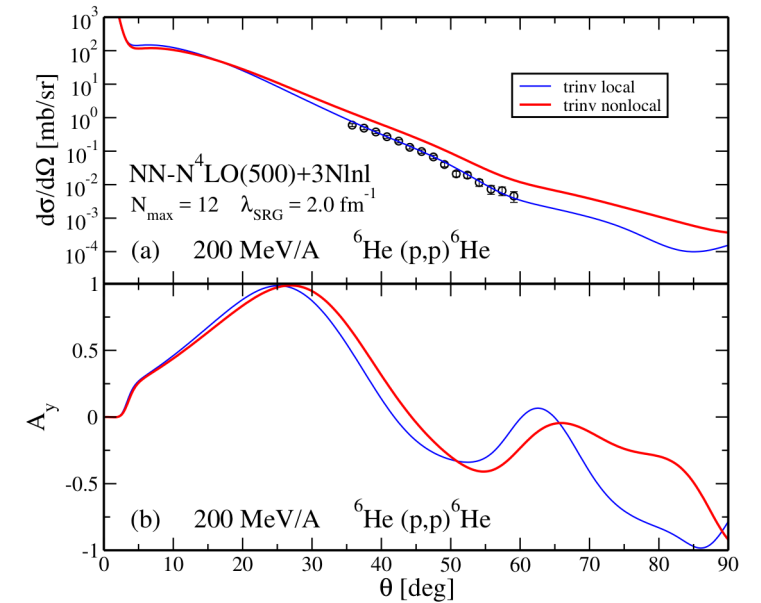
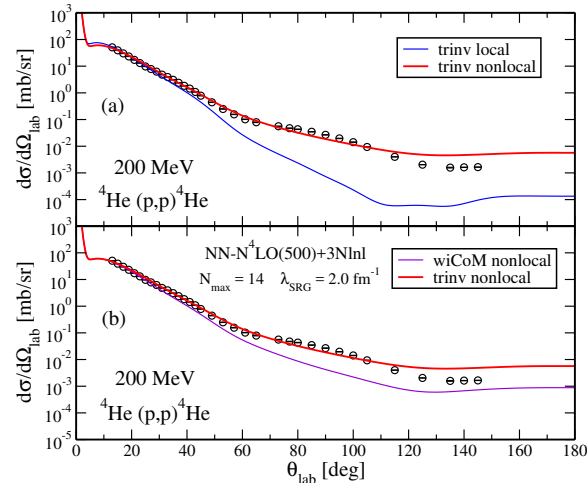
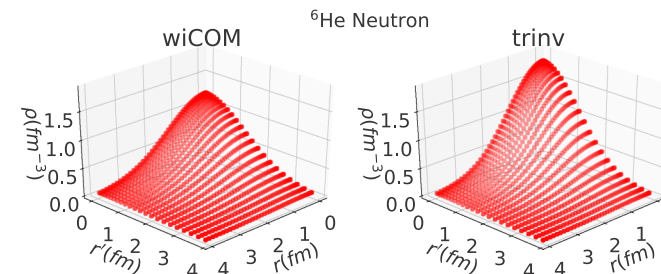
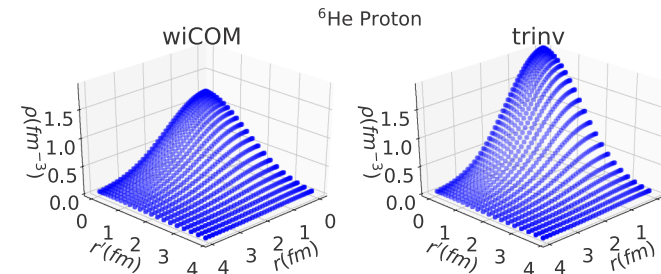
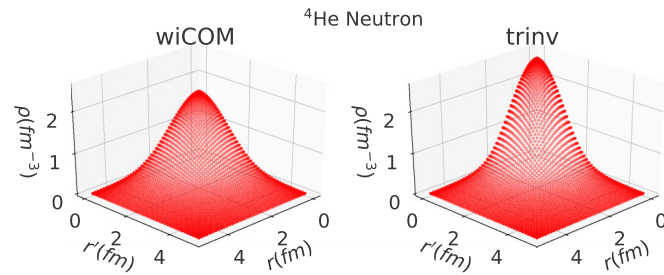
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15

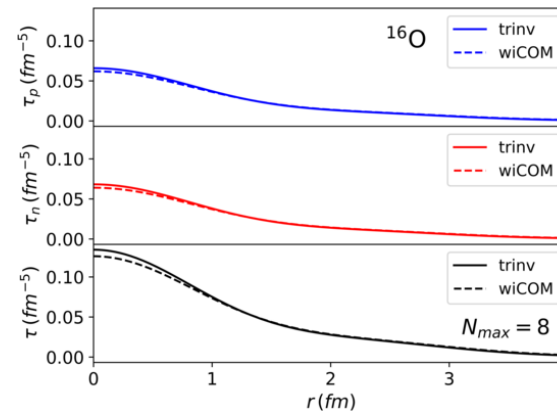
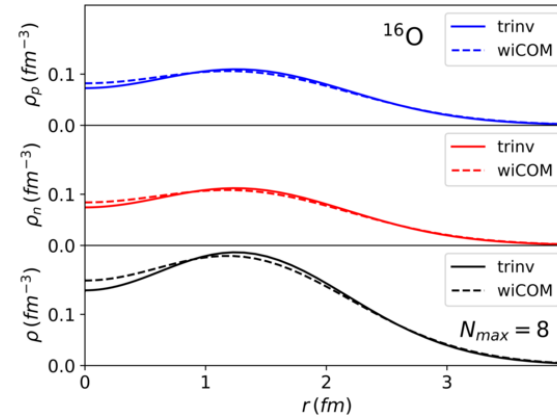
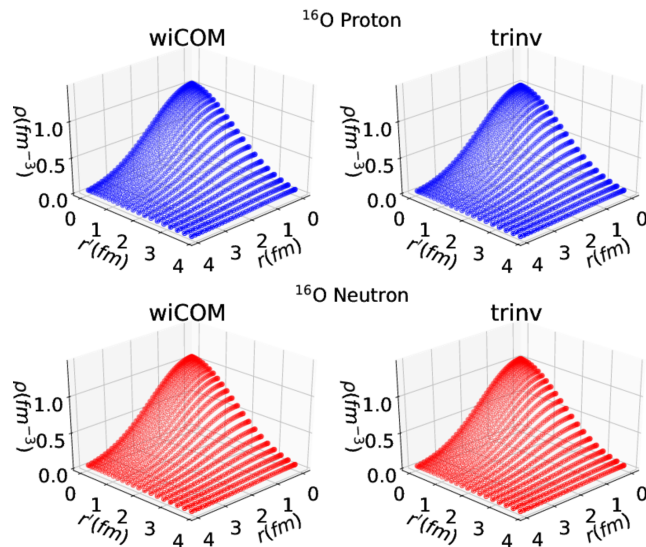
- Translationally-invariant non-local densities from NCSM calculations with chiral NN N⁴LO + 3N N²LO interactions
- High-energy proton-nucleus scattering with microscopic optical potentials from chiral N⁴LO NN interaction and NCSM densities



Nuclear kinetic density from NCSM wave functions

- DFT calculations include kinetic density
 - Might contain center-of-mass contamination
- Can be calculated for light nuclei in NCSM
 - Translationally invariant

$$\tau_{\mathcal{N}}(\vec{r}) = \left[\vec{\nabla} \cdot \vec{\nabla}' \rho_{\mathcal{N}}(\vec{r}, \vec{r}') \right]_{\vec{r}=\vec{r}'}$$

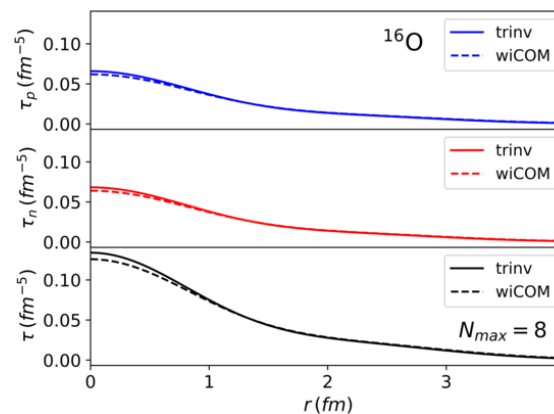
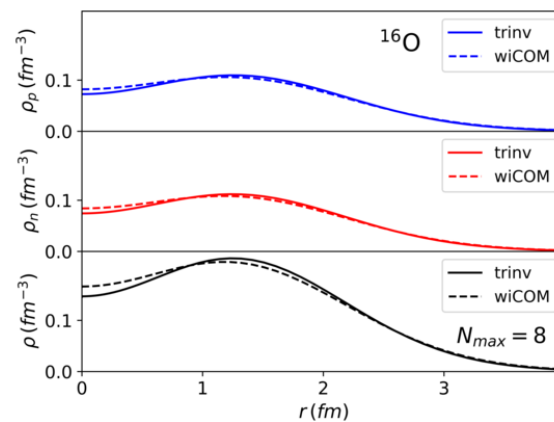
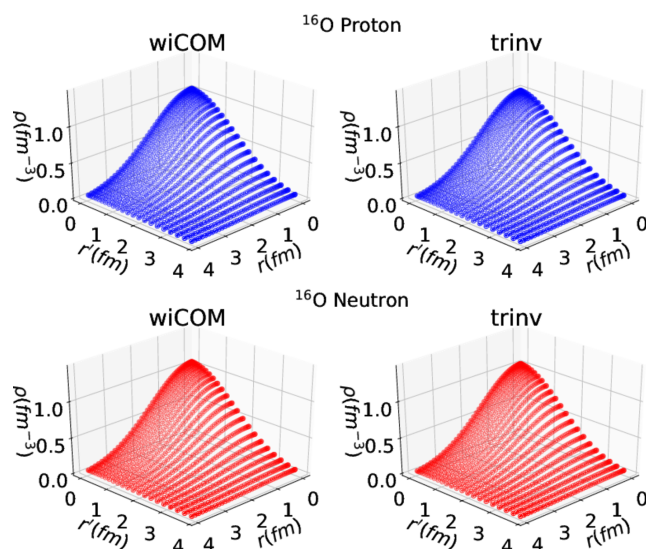


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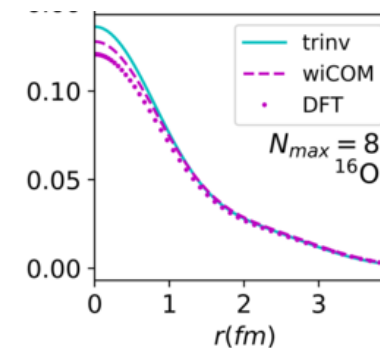
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$$\tau_{DFT}(\vec{r}) = \left(1 - \frac{1}{A} \right) \tau_{wiCOM}(\vec{r})$$

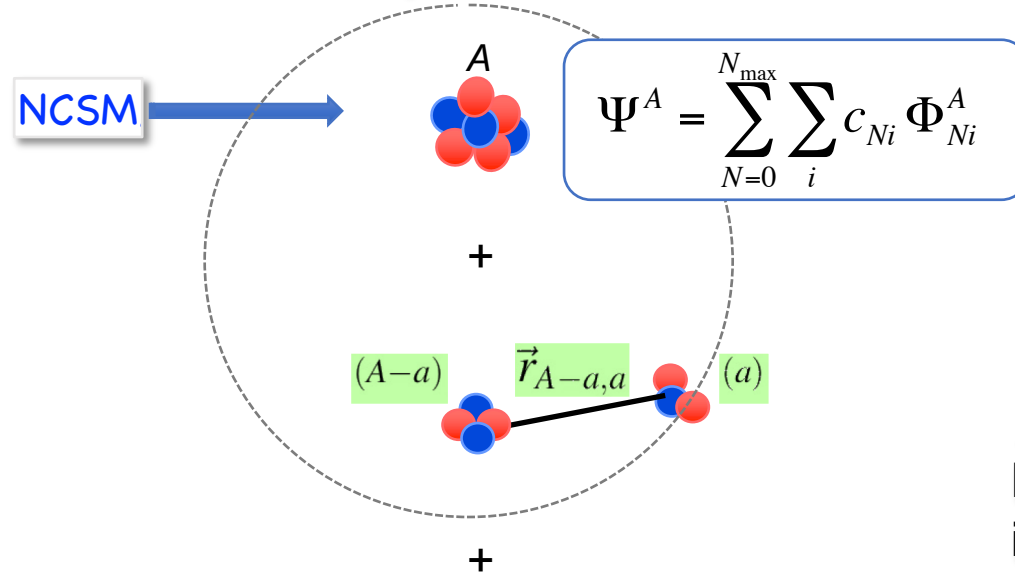


Nucleus	N_{max}	$\langle T_{int} \rangle$	$\langle T_{wiCOM} \rangle$	$\langle T_{DFT} \rangle$
^4He	14	51.91	66.91	50.18
^6He	12	78.26	93.26	77.72
^8He	10	116.30	131.30	114.89
^{12}C	8 IT	219.84	234.84	215.27
^{16}O	8 IT	301.69	316.69	296.90

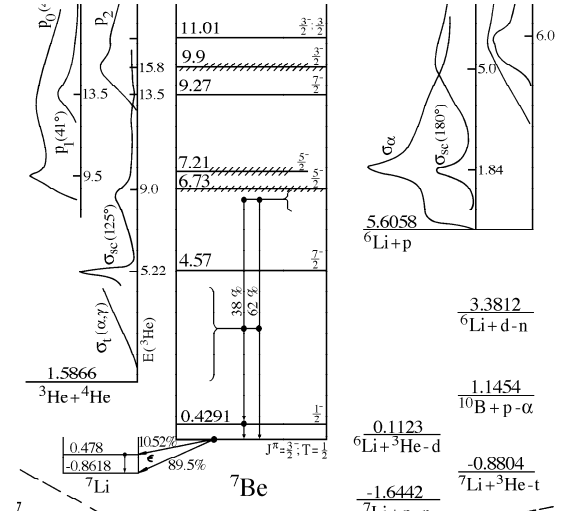


Extending no-core shell model beyond bound states

Include more many nucleon correlations...



...using the Resonating Group Method (RGM) ideas

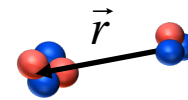


Unified approach to bound & continuum states; to nuclear structure & reactions

- No-core shell model (NCSM)
 - A -nucleon wave function expansion in the harmonic-oscillator (HO) basis
 - short- and medium range correlations
 - Bound-states, narrow resonances
- NCSM with Resonating Group Method (NCSM/RGM)
 - cluster expansion, clusters described by NCSM
 - proper asymptotic behavior
 - long-range correlations
- Most efficient: *ab initio* no-core shell model with continuum (NCSMC)



NCSM



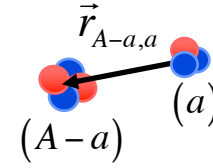
NCSM/RGM

NCSMC

$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| \begin{matrix} (A) \\ \text{Nucleus} \\ \lambda \end{matrix} \right\rangle + \sum_{\nu} \int d\vec{r} \gamma_{\nu}(\vec{r}) \hat{A}_{\nu} \left| \begin{matrix} (A-a) & (a) \\ \text{Cluster 1} & \text{Cluster 2} \\ \nu \end{matrix} \right\rangle$$

Unknowns

S. Baroni, P. Navratil, and S. Quaglioni,
PRL **110**, 022505 (2013); PRC **87**, 034326 (2013).



Binary cluster basis

- Working in partial waves ($\nu \equiv \{A-a \alpha_1 I_1^{\pi_1} T_1; a \alpha_2 I_2^{\pi_2} T_2; s\ell\}$)

$$|\psi^{J^{\pi T}}\rangle = \sum_{\nu} \hat{A}_{\nu} \left[\underbrace{\left(|A-a \alpha_1 I_1^{\pi_1} T_1\rangle \right)}_{\text{Target}} \underbrace{\left(|a \alpha_2 I_2^{\pi_2} T_2\rangle \right)}_{\text{Projectile}} \right]^{(sT)} Y_{\ell}(\hat{r}_{A-a,a}) \left]^{(J^{\pi T})} \frac{g_{\nu}^{J^{\pi T}}(r_{A-a,a})}{r_{A-a,a}}$$

- Introduce a dummy variable \vec{r} with the help of the delta function

$$|\psi^{J^{\pi T}}\rangle = \sum_{\nu} \int \frac{g_{\nu}^{J^{\pi T}}(r)}{r} \hat{A}_{\nu} \left[\left(|A-a \alpha_1 I_1^{\pi_1} T_1\rangle \right) \left(|a \alpha_2 I_2^{\pi_2} T_2\rangle \right) \right]^{(sT)} Y_{\ell}(\hat{r}) \left]^{(J^{\pi T})} \delta(\vec{r} - \vec{r}_{A-a,a}) r^2 dr d\hat{r}$$

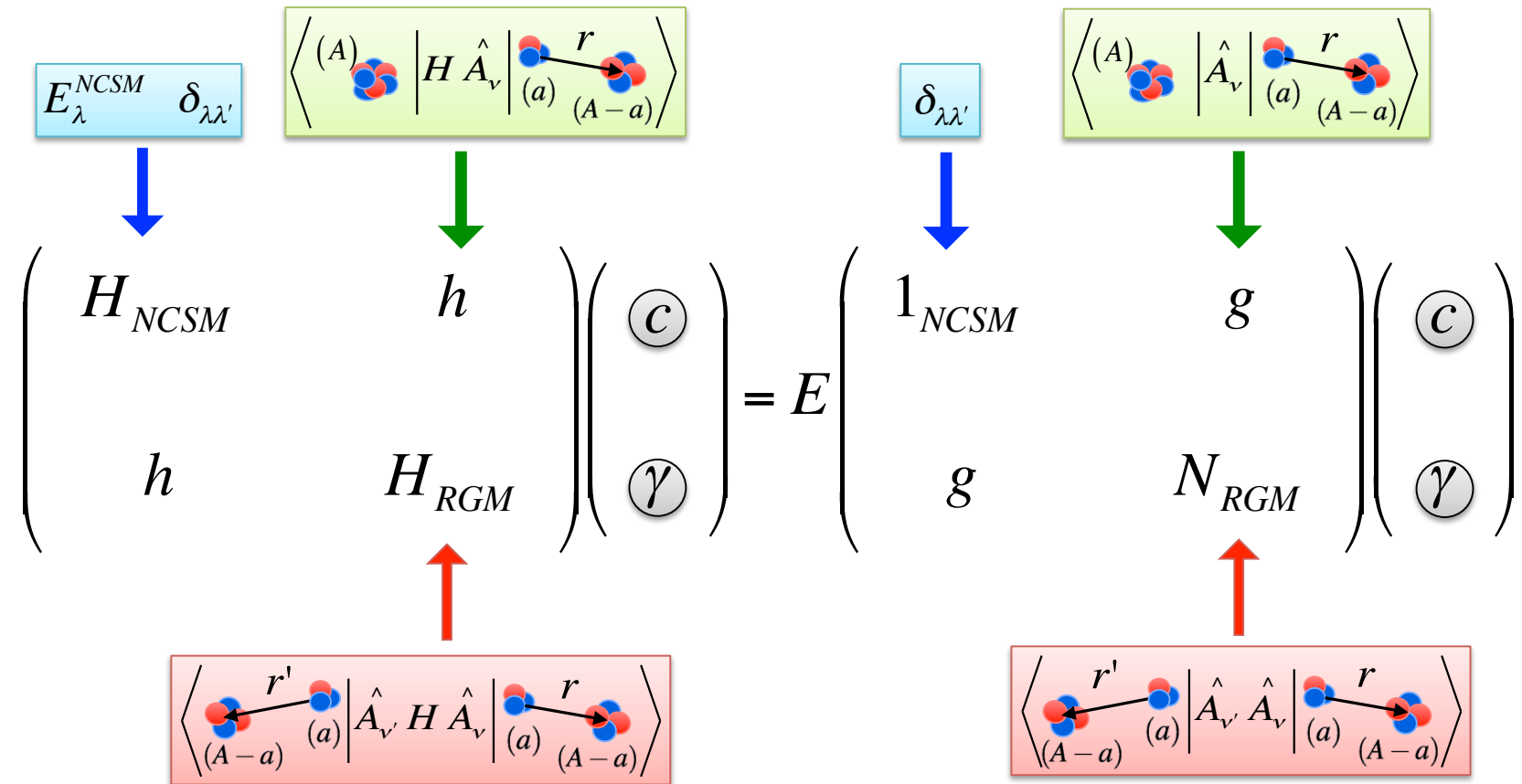
- Allows to bring the wave function of the relative motion in front of the antisymmetrizer

$$\sum_{\nu} \int d\vec{r} \gamma_{\nu}(\vec{r}) \hat{A}_{\nu} \left| \begin{array}{c} \vec{r} \\ (A-a) \quad (a) \end{array}, \nu \right\rangle$$

Coupled NCSMC equations

$$H \Psi^{(A)} = E \Psi^{(A)}$$

$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| \begin{matrix} (A) \\ \text{cluster} \end{matrix}, \lambda \right\rangle + \sum_{\nu} \int d\vec{r} \gamma_{\nu}(\vec{r}) \hat{A}_{\nu} \left| \begin{matrix} (A-a) & (a) \\ \text{cluster} & \text{cluster} \end{matrix}, \nu \right\rangle$$

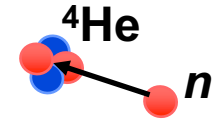


Solved by Microscopic R-matrix theory on a Lagrange mesh – efficient for **coupled channels**

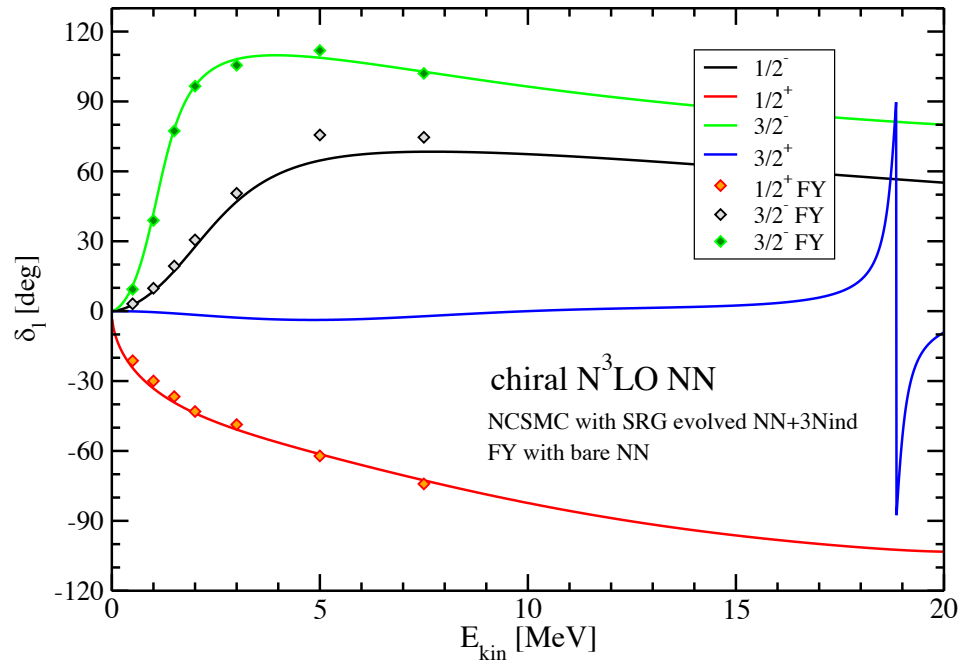
Computational aspects of NCSMC

- NN matrix elements
 - Choice of various potentials – chiral N³LO, LO-N⁴LO...
 - SRG (also for one and two-body operators)
 - Transformation to s.p. basis (up to N_{12max}~30)
 - ncsmv2b - Fortran90 with OpenMP
- 3N matrix elements
 - Choice of various NN (chiral N³LO, LO-N⁴LO) and 3N (N²LO with local or local/non-local regulators, N⁴LO contacts) potentials
 - SRG (also for one, two- and three-body operators)
 - Transformation to s.p. basis (up to N_{123max}~17)
 - manyeff, v3trans (op3trans) – Fortran90 with OpenMP
- NCSM diagonalization
 - NN, NN+3N or NN+3N(NO2b) interactions
 - Calculations of N_{max} sequence (0(1),2(3),...Nmax)
 - Importance truncation
 - ncsd – Lanczosh algorithm, bit operations, hashing, partial or full storing of non-zero matrix elements, Fortran 90 with MPI, ~12,000 MPI tasks
- Transition densities and/or RGM and coupling kernels
 - One- and two-body transition densities for wave functions of the same or different nuclei (three- and four-body for A=3,4 nuclei)
 - Coordinate space local and nonlocal translationally invariant one-body densities
 - RGM and coupling kernel calculation for the NCSMC (including the 3N interaction)
 - Normal ordering of 3N interaction
 - trdens – bit operations, hashing, Fortran90 with MPI, ~8,000 MPI tasks
- NCSMC calculation
 - RGM and coupling kernels either input or calculated from densities
 - Solves NCSMC coupled equations, calculates the S-matrix and scattering/reaction observables
 - ncsmc – Fortran90 with OpenMP and MPI

n-⁴He scattering within NCSMC



n-⁴He scattering phase-shifts for chiral NN



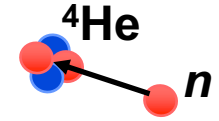
FY: Faddeev-Yakubovsky method - Rimantas Lazauskas

Invited Comment

Unified *ab initio* approaches to nuclear structure and reactions

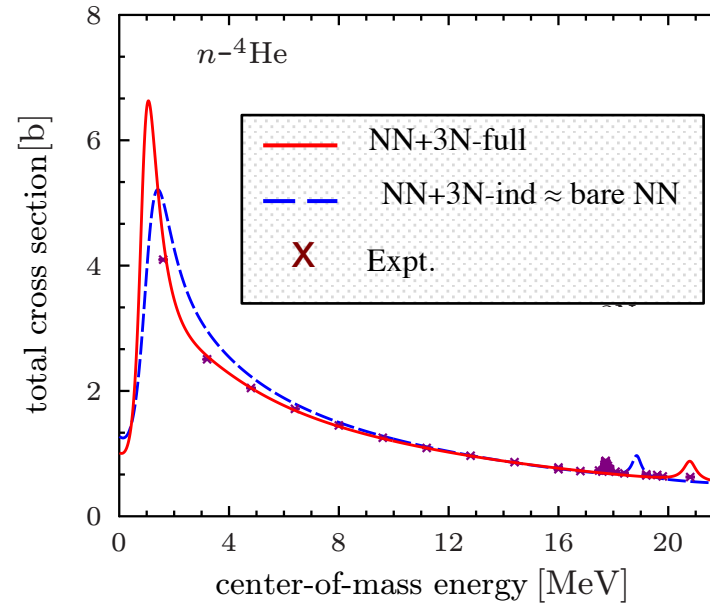
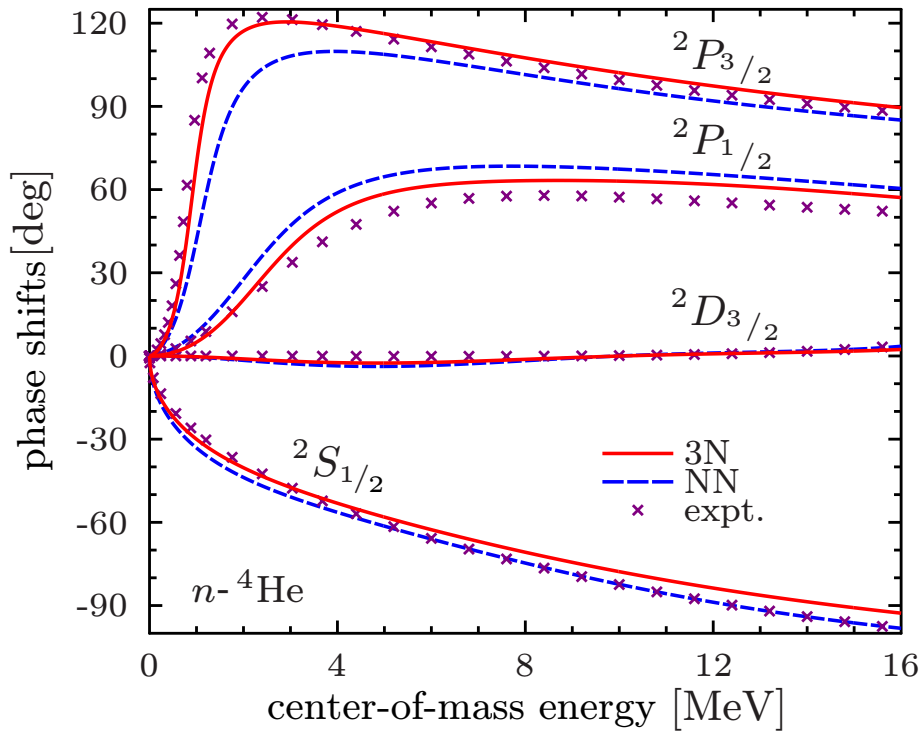
Petr Navrátil¹, Sofia Quaglioni², Guillaume Hupin^{3,4}, Carolina Romero-Redondo⁵ and Angelo Calci

n - ^4He scattering within NCSMC



n - ^4He scattering phase-shifts for chiral NN and NN+3N500 potential

Total n - ^4He cross section with NN and NN+3N potentials

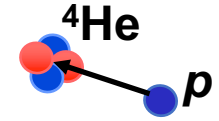


3N force enhances $1/2^- \leftrightarrow 3/2^-$ splitting: Essential at low energies!

PHYSICAL REVIEW C **88**, 054622 (2013)
Ab initio many-body calculations of nucleon- ^4He scattering with three-nucleon forces
 Guillaume Hupin,^{1,*} Joachim Langhammer,^{2,†} Petr Navrátil,^{3,‡} Sofia Quaglioni,^{1,§} Angelo Calci,^{2,||} and Robert Roth^{2,¶}

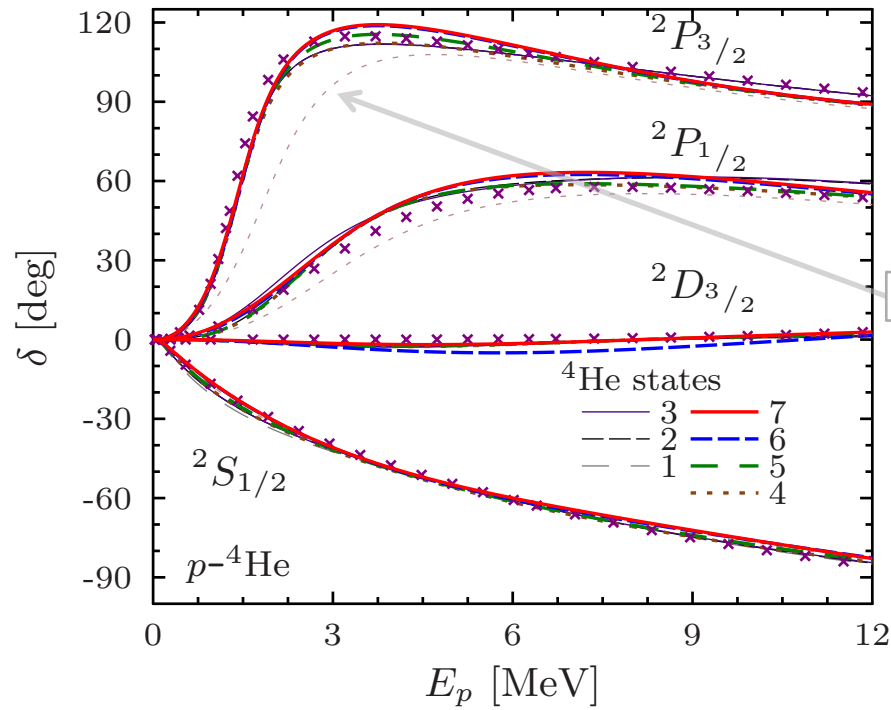
IOP Publishing | Royal Swedish Academy of Sciences
 Phys. Scr. 00 (2016), 000000 (37pp) Physica Scripta
 Invited Comment
Unified *ab initio* approaches to nuclear structure and reactions
 Petr Navrátil¹, Sofia Quaglioni², Guillaume Hupin^{1,4},
 Carolina Romero-Redondo⁵ and Angelo Calci¹

p - ^4He scattering within NCSMC

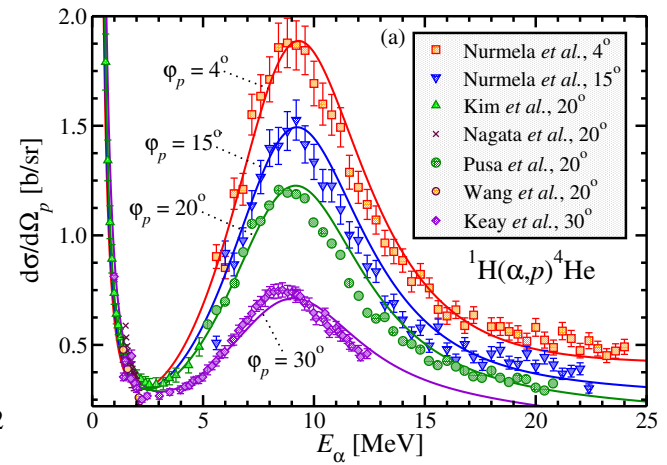
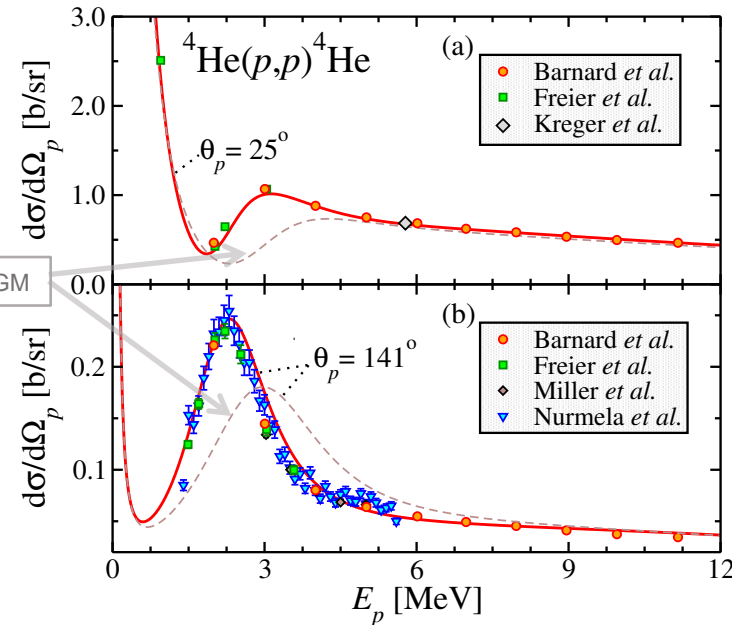


p - ^4He scattering phase-shifts for NN+3N500 potential:
Convergence

Differential p - ^4He cross section with NN+3N potentials



NCSM/RGM

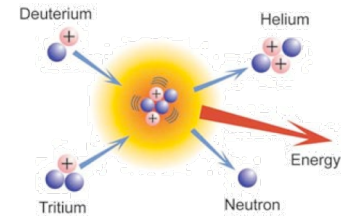
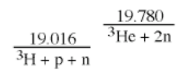
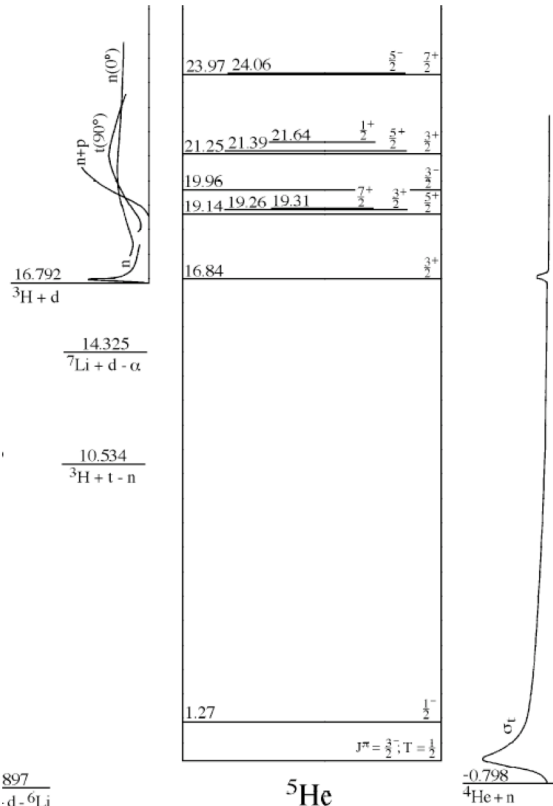


PHYSICAL REVIEW C **90**, 061601(R) (2014)
Predictive theory for elastic scattering and recoil of protons from ^4He
 Guillaume Hupin,^{1,*} Sofia Quaglioni,^{1,†} and Petr Navrátil^{2,‡}

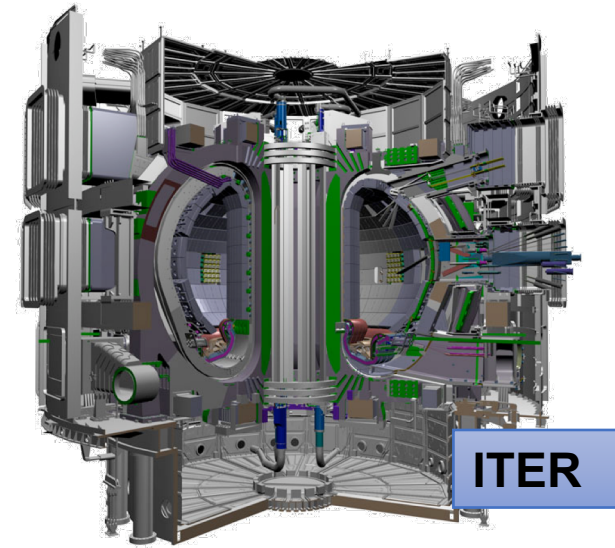
Predictive power in the 3/2- resonance region:
Applications to material science

Deuterium-Tritium fusion

- The $d+^3\text{H}\rightarrow n+^4\text{He}$ reaction
 - The most promising for the production of fusion energy in the near future
 - Used to achieve inertial-confinement (laser-induced) fusion at NIF, and magnetic-confinement fusion at ITER
 - With its mirror reaction, $^3\text{He}(d,p)^4\text{He}$, important for Big Bang nucleosynthesis



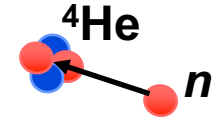
Resonance at $E_{cm} = 48$ keV ($E_d = 105$ keV) in the $J=3/2^+$ channel
 Cross section at the peak: 4.88 b
17.64 MeV energy released:
14.1 MeV neutron and 3.5 MeV alpha



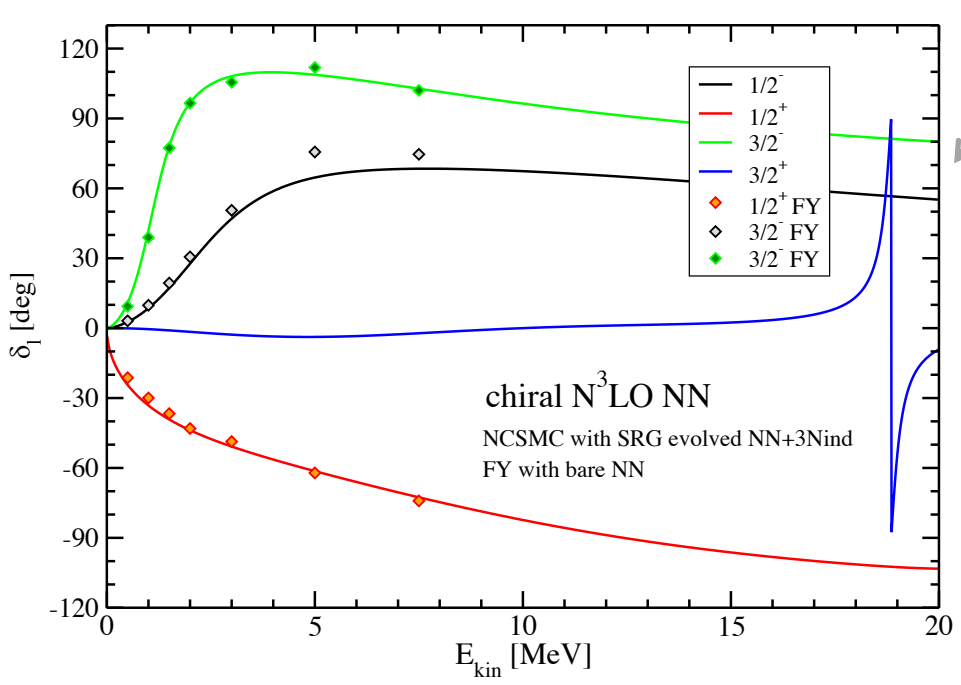
ITER

897
d - ^6Li

n - ^4He scattering and $^3\text{H}+d$ fusion within NCSMC

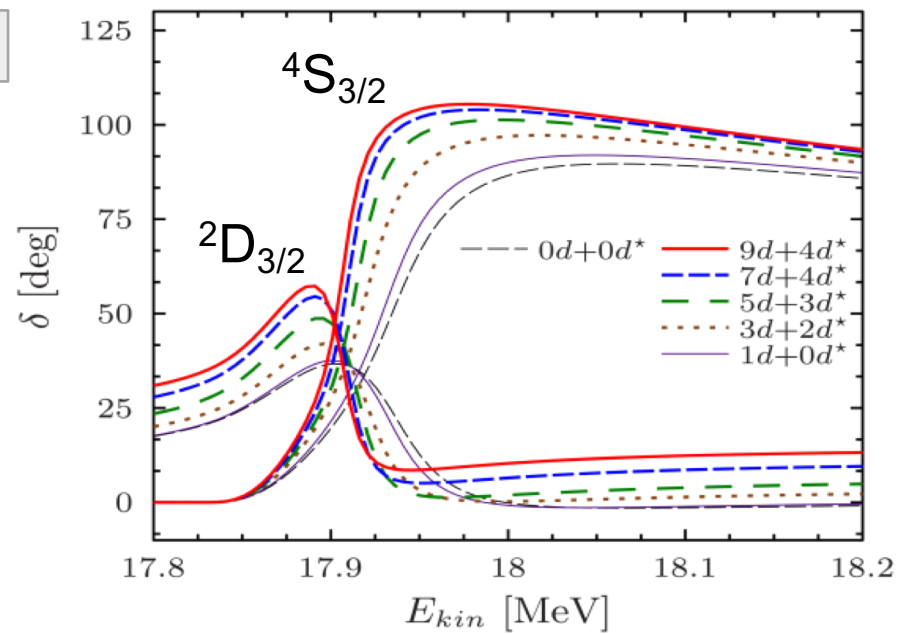


n - ^4He scattering phase-shifts for chiral NN



$^4\text{He}+n$

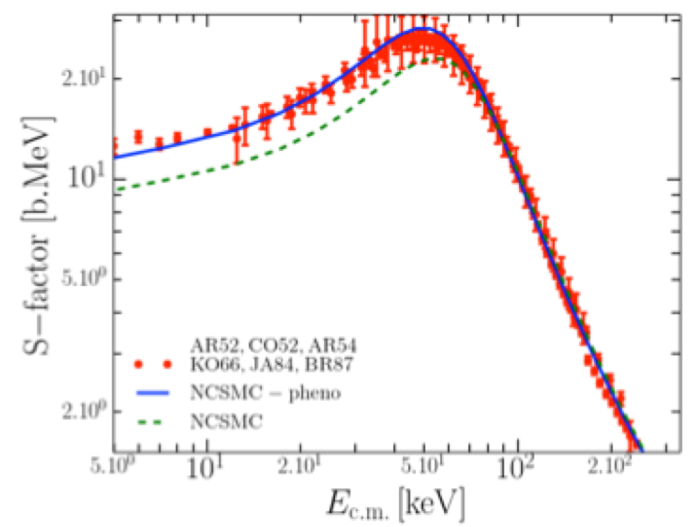
$^4\text{He}+n \rightarrow ^3\text{H}+d$



FY: Faddeev-Yakubovsky method - Rimantas Lazauskas

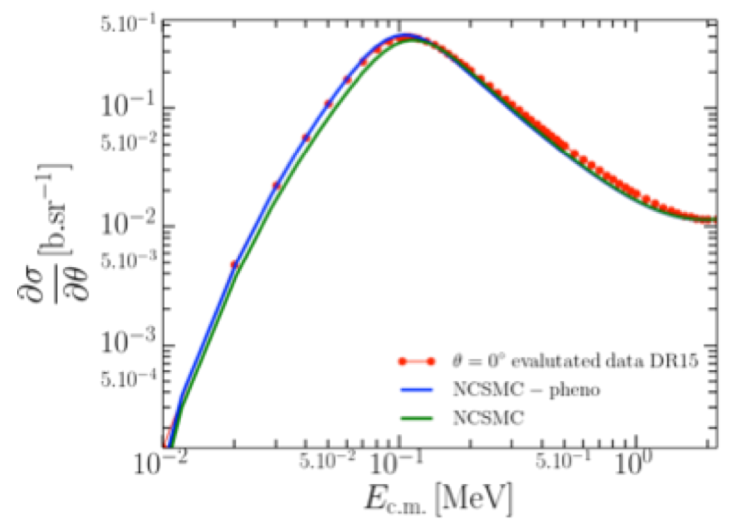
The d - ^3H fusion takes place through a transition of $d+^3\text{H}$ is S -wave to $n+^4\text{He}$ in D -wave: Importance of the **tensor** and **3N** force

${}^3\text{H}(d,n){}^4\text{He}$ with chiral NN+3N500 interaction



$$S(E) = E\sigma(E) \exp[2\pi\eta(E)]$$

$$\eta(E) = Z_{A-a}Z_a e^2 / \hbar v_{A-a,a}$$

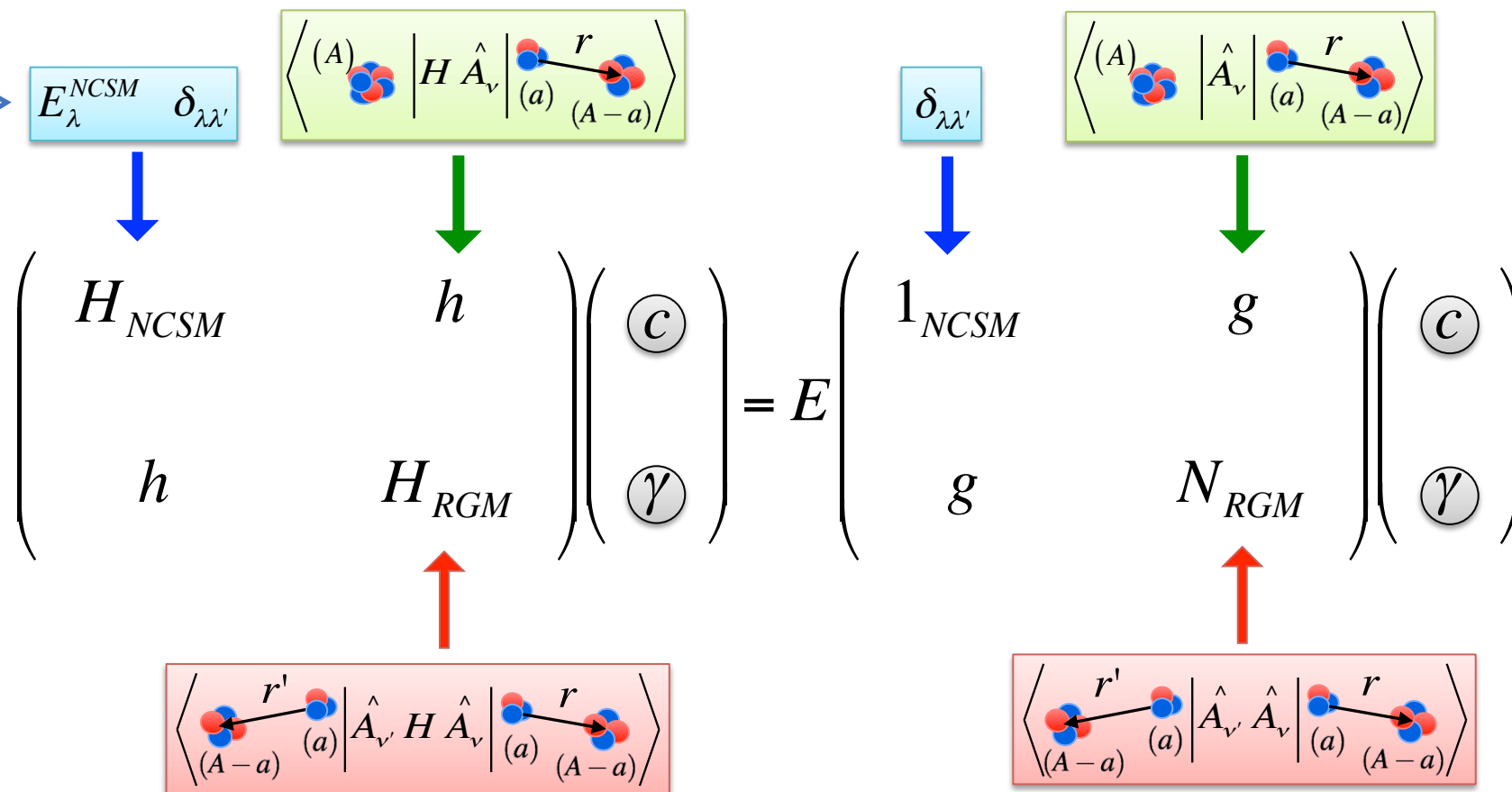


NCSMC phenomenology

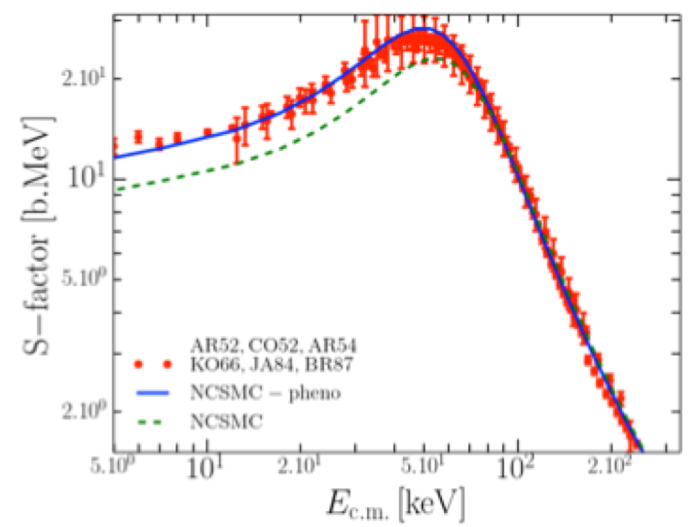
$$H \Psi^{(A)} = E \Psi^{(A)}$$

$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| \begin{array}{c} (A) \\ \text{[orbital diagram]} \end{array}, \lambda \right\rangle + \sum_{\nu} \int d\vec{r} \gamma_{\nu}(\vec{r}) \hat{A}_{\nu} \left| \begin{array}{c} (A-a) \\ \text{[orbital diagram]} \end{array}, \nu \right\rangle$$

E_{λ}^{NCSM} energies treated as adjustable parameters

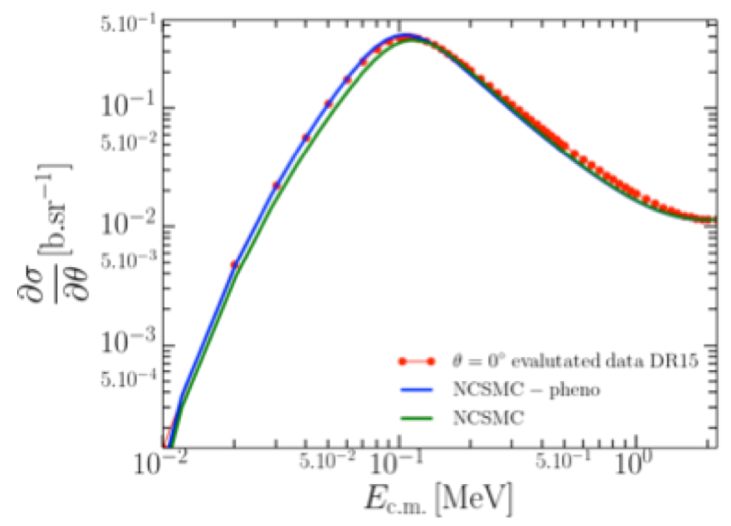


$^3\text{H}(d,n)^4\text{He}$ with chiral NN+3N500 interaction



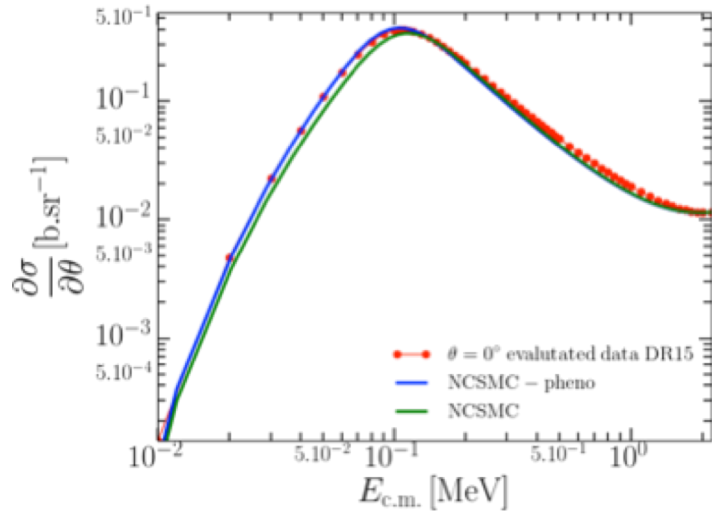
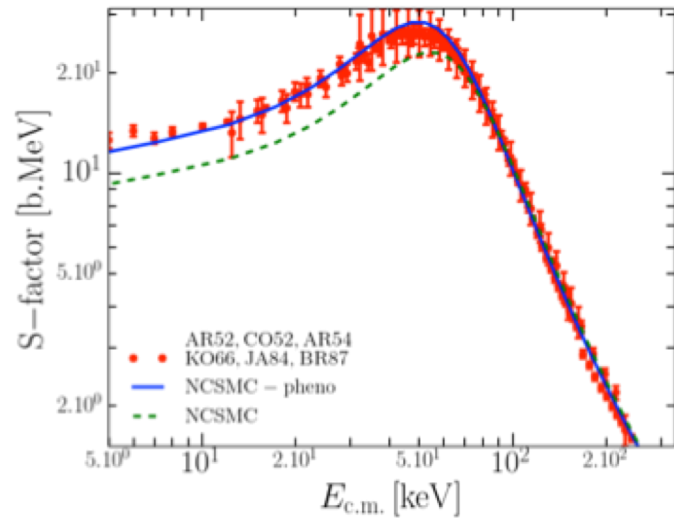
$$S(E) = E\sigma(E) \exp[2\pi\eta(E)]$$

$$\eta(E) = Z_{A-a}Z_a e^2 / \hbar v_{A-a,a}$$



$^3\text{H}(d,n)^4\text{He}$ with chiral NN+3N500 interaction

Polarized fusion

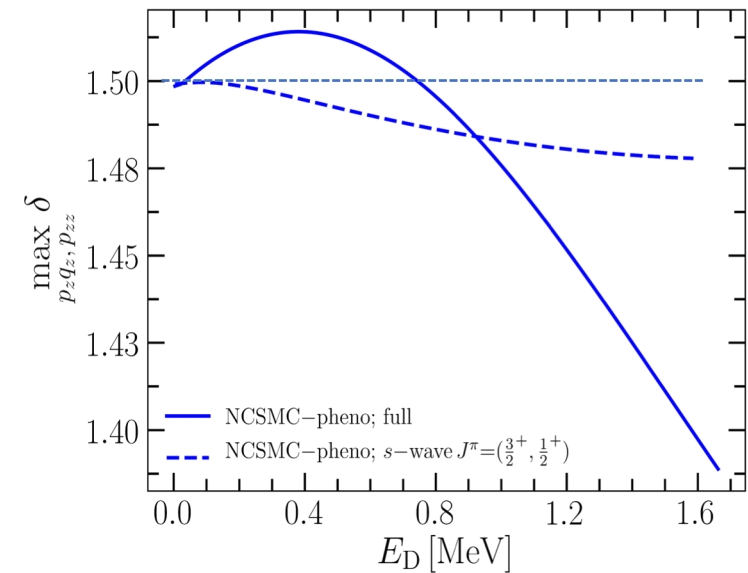
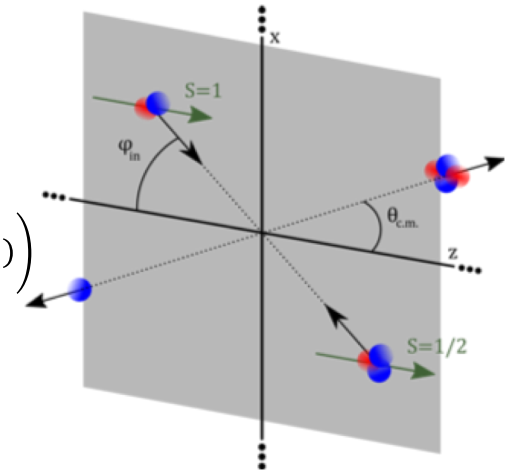
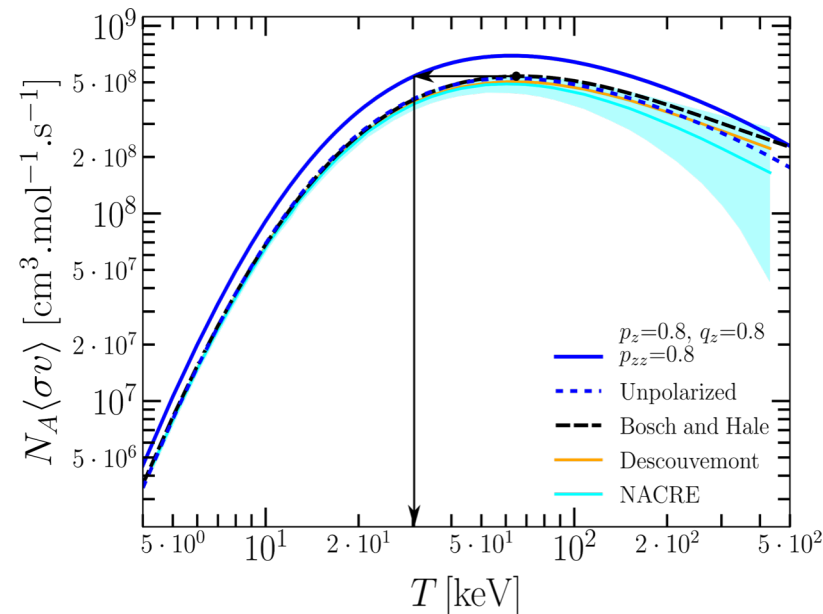


$$S(E) = E\sigma(E) \exp[2\pi\eta(E)]$$

$$\eta(E) = Z_{A-a}Z_a e^2 / \hbar v_{A-a,a}$$

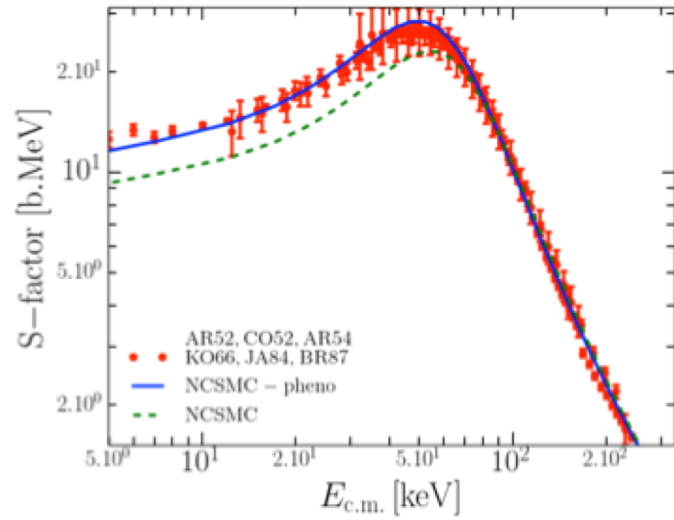
$$\frac{\partial\sigma_{pol}}{\partial\Omega_{c.m.}}(\theta_{c.m.}) = \frac{\partial\sigma_{unpol}}{\partial\Omega_{c.m.}}(\theta_{c.m.}) \left(1 + \frac{1}{2}p_{zz}A_{zz}^{(b)}(\theta_{c.m.}) + \frac{3}{2}p_z q_z C_{z,z}(\theta_{c.m.}) \right)$$

$$\langle\sigma v\rangle = \sqrt{\frac{8}{\pi\mu(k_b T)^3}} \int_0^\infty S(E) \exp\left(-\frac{E}{k_b T} - \sqrt{\frac{E_g}{E}}\right) dE,$$



$^3\text{H}(d,n)^4\text{He}$ with chiral NN+3N500 interaction

Polarized fusion

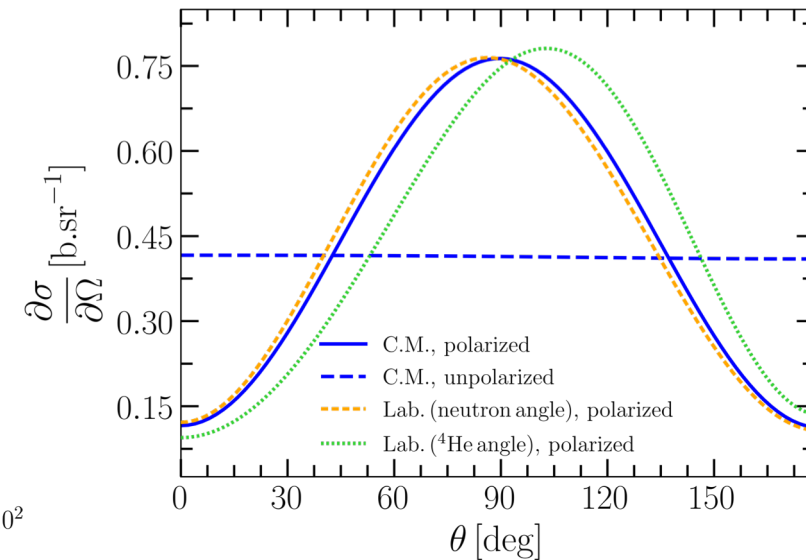
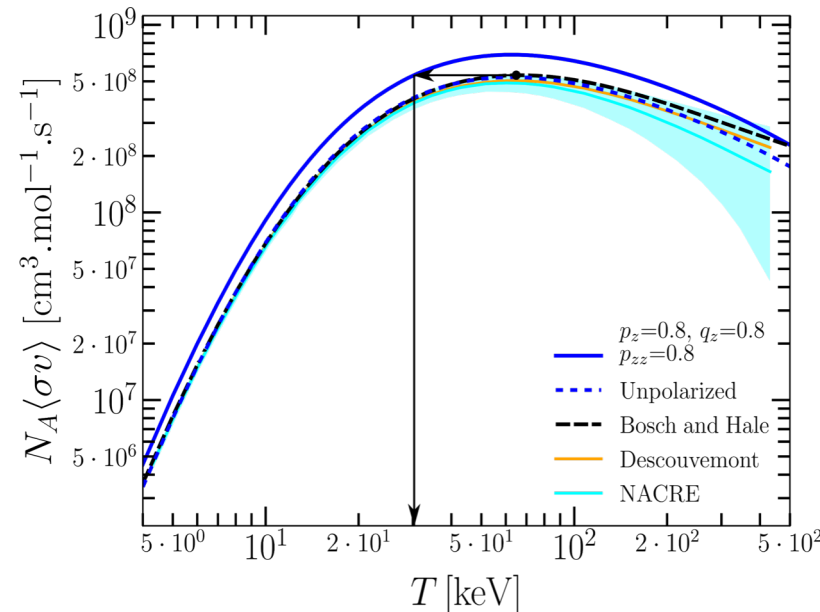
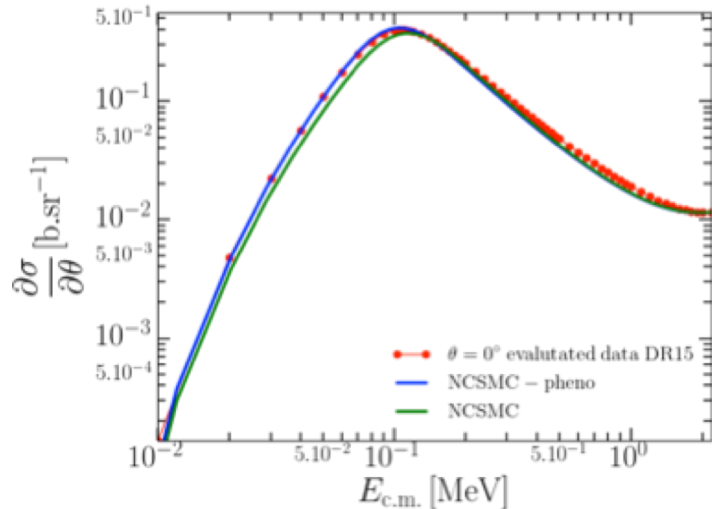
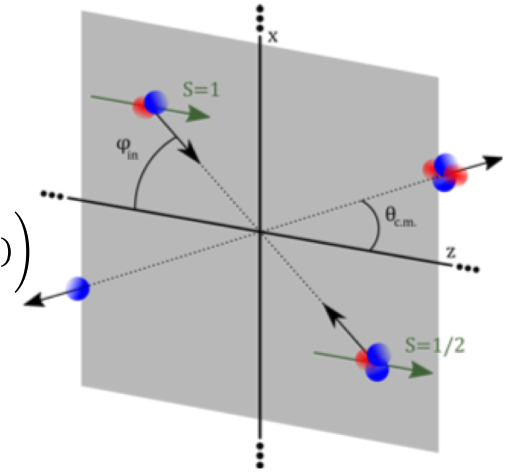


$$S(E) = E\sigma(E) \exp[2\pi\eta(E)]$$

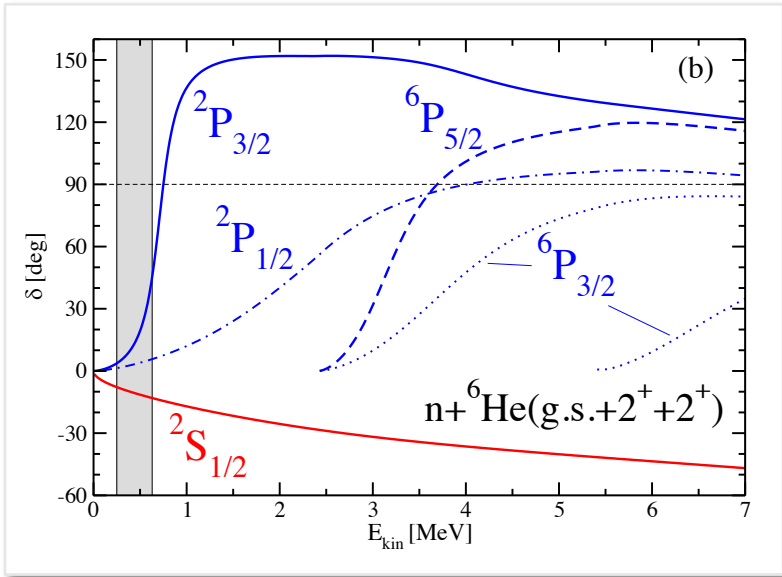
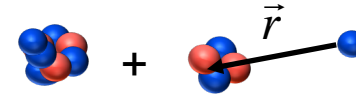
$$\eta(E) = Z_{A-a}Z_a e^2 / \hbar v_{A-a,a}$$

$$\frac{\partial\sigma_{pol}}{\partial\Omega_{c.m.}}(\theta_{c.m.}) = \frac{\partial\sigma_{unpol}}{\partial\Omega_{c.m.}}(\theta_{c.m.}) \left(1 + \frac{1}{2}p_{zz}A_{zz}^{(b)}(\theta_{c.m.}) + \frac{3}{2}p_z q_z C_{z,z}(\theta_{c.m.}) \right)$$

$$\langle\sigma v\rangle = \sqrt{\frac{8}{\pi\mu(k_b T)^3}} \int_0^\infty S(E) \exp\left(-\frac{E}{k_b T} - \sqrt{\frac{E_g}{E}}\right) dE,$$



NCSM with continuum: ${}^7\text{He} \leftrightarrow {}^6\text{He} + n$



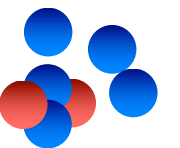
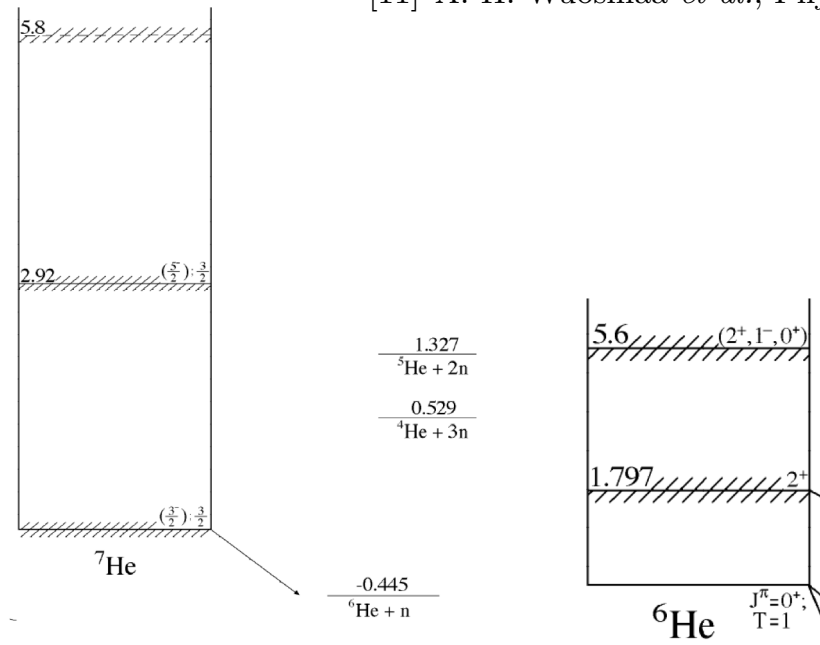
J^π	experiment			NCSMC	
	E_R	Γ	Ref.	E_R	Γ
$3/2^-$	0.430(3)	0.182(5)	[2]	0.71	0.30
$5/2^-$	3.35(10)	1.99(17)	[40]	3.13	1.07
$1/2^-$	3.03(10)	2	[11]	2.39	2.89
	3.53	10	[15]		
	1.0(1)	0.75(8)	[5]		

Experimental controversy:
Existence of low-lying $1/2^-$ state
... not seen in these calculations

[11] A. H. Wuosmaa *et al.*, Phys. Rev. C **72**, 061301 (2005).

$$\Gamma = \frac{2}{\partial\delta(E_{kin})/\partial E_{kin}} \Big|_{E_{kin}=E_R}$$

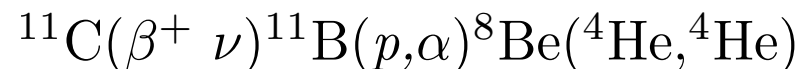
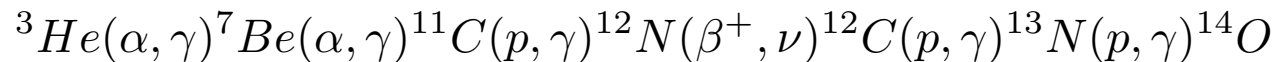
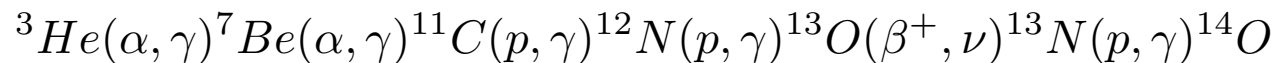
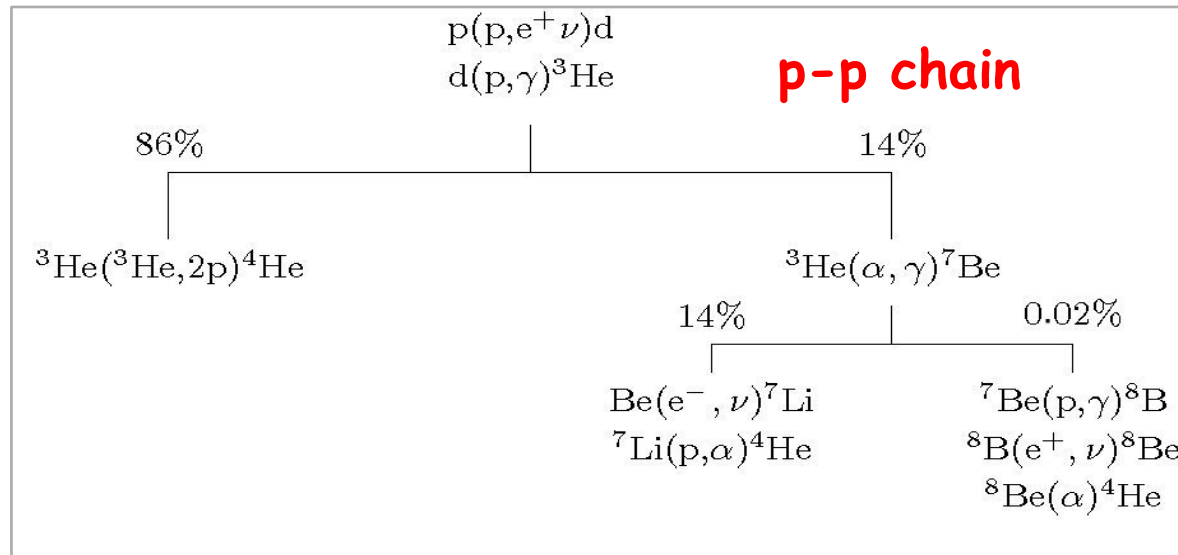
NCSMC
 with three ${}^6\text{He}$ states
 and ten ${}^7\text{He}$ eigenstates
 More **7-nucleon correlations**
 Fewer ${}^6\text{He}$ -core states needed



${}^7\text{He}$ unbound

p+¹¹C scattering and ¹¹C(p,γ)¹²N capture

- ¹¹C(p,γ)¹²N capture relevant in hot *p-p* chain: Link between pp chain and the CNO cycle - bypass of slow triple alpha capture ⁴He(αα,γ)¹²C

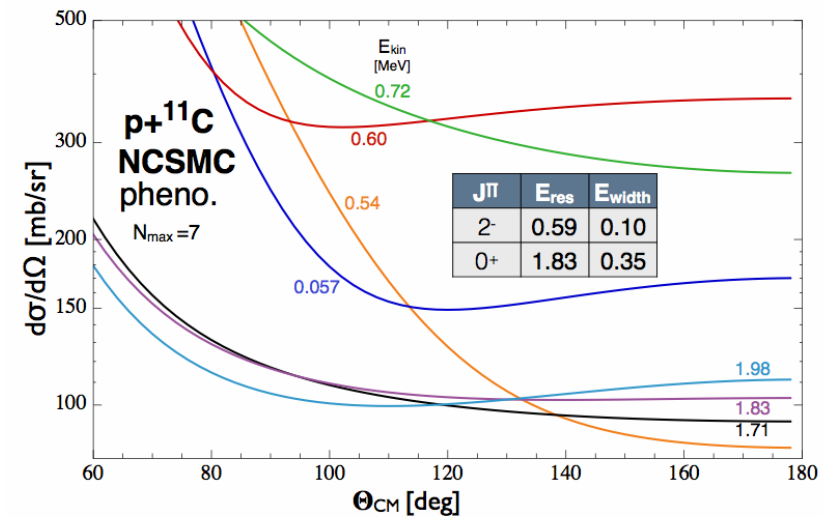
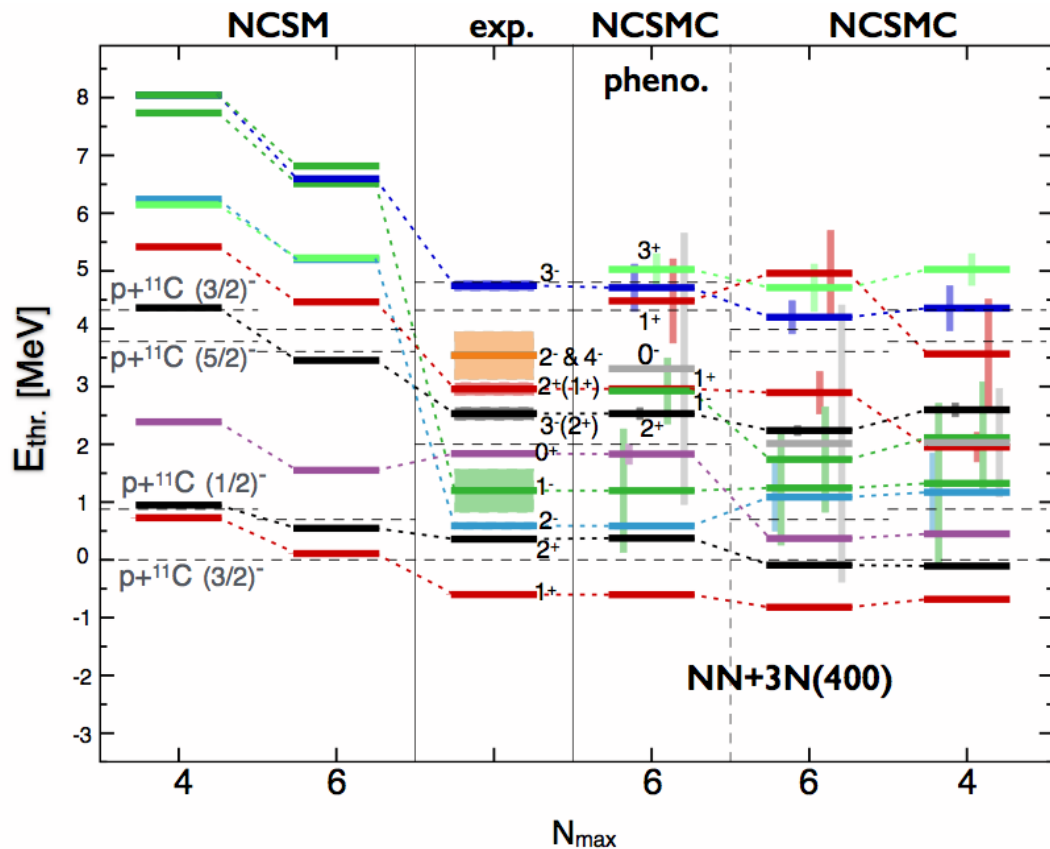


$p+^{11}\text{C}$ scattering and $^{11}\text{C}(p,\gamma)^{12}\text{N}$ capture

- Measurement of $^{11}\text{C}(p,p)$ resonance scattering planned at TRIUMF
 - TUDA facility
 - ^{11}C beam of sufficient intensity produced
 - Experiment approved with high priority
- NCSMC calculations of $^{11}\text{C}(p,p)$ with chiral NN+3N under way
- Obtained wave functions will be used to calculate $^{11}\text{C}(p,\gamma)^{12}\text{N}$ capture relevant for astrophysics

$p+^{11}\text{C}$ scattering and $^{11}\text{C}(p,\gamma)^{12}\text{N}$ capture

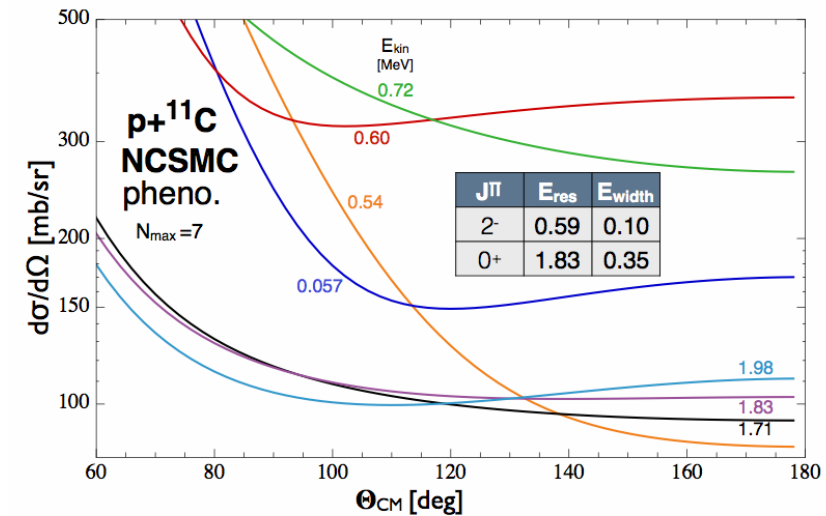
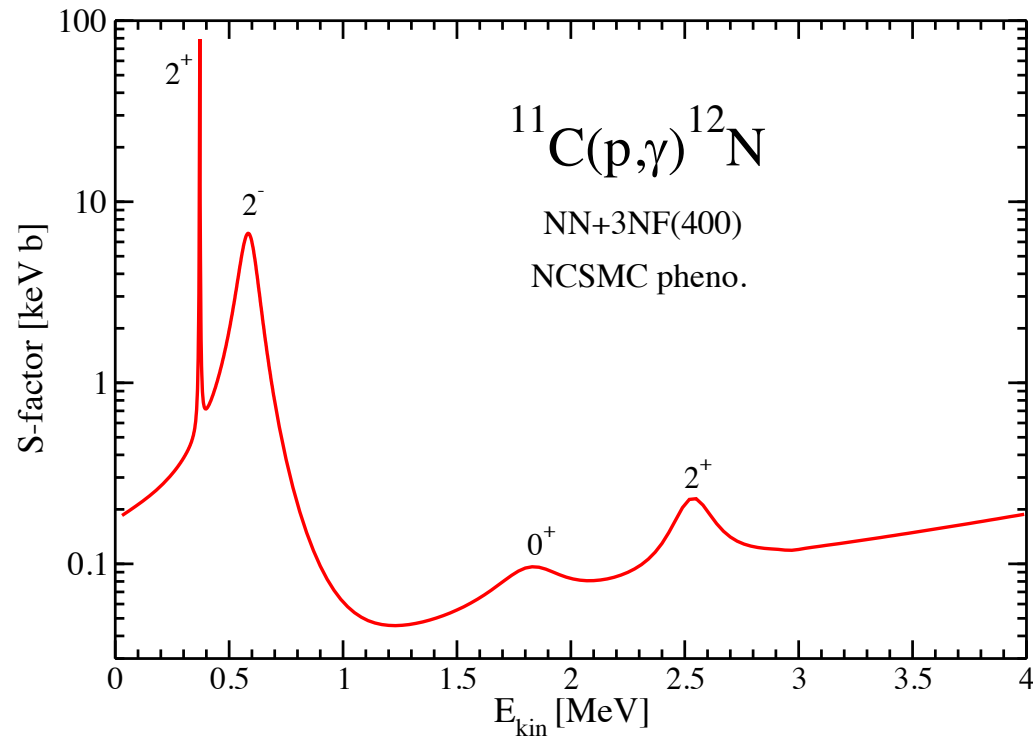
- NCSMC calculations of $^{11}\text{C}(p,p)$ with chiral NN+3N under way
 - ^{11}C : $3/2^-$, $1/2^-$, $5/2^-$, $3/2^-$ NCSM eigenstates
 - ^{12}N : ≥ 6 $\pi = +1$ and ≥ 4 $\pi = -1$ NCSM eigenstates



NCSMC calculations to be validated by measured cross sections and applied to calculate the $^{11}\text{C}(p,\gamma)^{12}\text{N}$ capture

$p+^{11}\text{C}$ scattering and $^{11}\text{C}(p,\gamma)^{12}\text{N}$ capture

- NCSMC calculations of $^{11}\text{C}(p,p)$ with chiral NN+3N under way
 - ^{11}C : $3/2^-$, $1/2^-$, $5/2^-$, $3/2^-$ NCSM eigenstates
 - ^{12}N : ≥ 6 $\pi = +1$ and ≥ 4 $\pi = -1$ NCSM eigenstates



NCSMC calculations to be validated by measured cross sections and applied to calculate the $^{11}\text{C}(p,\gamma)^{12}\text{N}$ capture

Conclusions

- *Ab initio* calculations of nuclear structure and reactions with predictive power becoming feasible beyond the lightest nuclei
- *Ab initio* structure calculations can even reach (selected) medium & medium-heavy mass nuclei
- These calculations make connections between the low-energy QCD, many-body systems, and nuclear astrophysics

Thank you!
Merci!
고맙습니다

