Algebraic approach to the isovector pairing problem

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Introduction

In the LST-coupling scheme of a spherical shell model, let $\{a^{\dagger}_{\omega,\sigma,\tau},\ a_{\omega,\sigma,\tau}\}$ be a set of (valence) nucleon creation and annihilation operators, where $\omega = 1,\ 2,\ \cdots,\ \Omega \equiv \sum_{l}(2l+1)$ is the spatial indices, σ and τ are spin and isospin index, respectively. It is well known that the total number of many-particle product states $\mathcal{N}_t$ provided by $\{\Pi^{\dagger}_{\phi_k}|0\rangle = \prod_{i=1}^{k} a^{\dagger}_{\omega_i,\sigma_i,\tau_i}|0\rangle\}$, where $|0\rangle$ is the (valence) nucleon vacuum state, and $\phi_k \equiv \{\omega_1, \sigma_1, \tau_1, \cdots, \omega_k, \sigma_k, \tau_k\}$ stands for all sub-indices involved, is given by

$$\mathcal{N}_t = \sum_{k=0}^{4\Omega} \frac{(4\Omega)!}{k!(4\Omega-k)!} = 2^{4\Omega}.$$

The set of operators $\{Q_{\phi_k,\phi'_{k'}} = \Pi^{\dagger}_{\phi_k}\Pi_{\phi'_{k'}},\ 1 \leq k, k' \leq 4\Omega\}$ generates the unitary group $U(2^{4\Omega})$. The set of the many-particle product (Fock) states $\{\Pi^{\dagger}_{\phi_1}|0\rangle, \cdots, \Pi^{\dagger}_{\phi_{4\Omega}}|0\rangle\}$ spans the fundamental irrep $[1, 0, \cdots, 0]$ of $U(2^{4\Omega})$.

Introduction

A subset of $\{\Pi^\dagger_{\phi_k}, \Pi_{\phi_k}\}$ with $k = 1, 2$ and $H_{\phi\phi'} = \Pi^\dagger_{\phi_1}\Pi_{\phi'_1}$ generate the $O(8\Omega + 1)$ group. And $U(2^{4\Omega}) \supset O(8\Omega + 1)$ with the branching rule $[1, 0, \cdots, 0] \downarrow (\frac{1}{2}, \cdots, \frac{1}{2})$, where $(\frac{1}{2}, \cdots, \frac{1}{2})$ with 4Ω components to be $\frac{1}{2}$ is a spinor representation of $O(8\Omega + 1)$. The largest nontrivial subgroup of $O(8\Omega + 1)$ is $O(8\Omega)$ generated by $\{\Pi^\dagger_{\phi_2}, \Pi_{\phi_2}, H_{\phi\phi'}\}$ with the branching rule:

$$\begin{array}{l} O(8\Omega+1) \downarrow \quad O(8\Omega) \\ (\frac{1}{2}, \cdots, \frac{1}{2}) \downarrow (\frac{1}{2}, \cdots, \frac{1}{2}, \frac{1}{2}) \oplus (\frac{1}{2}, \cdots, \frac{1}{2}, -\frac{1}{2}). \end{array}$$

$$O(8\Omega) \begin{array}{l} \nearrow [O(8) \supset U(4) \supset SU_S(2) \otimes SU_T(2)] \otimes [O(\Omega) \supset O_L(3)] \\ \rightarrow O_T(5) \otimes (U(2\Omega) \supset Sp(2\Omega) \supset O_J(3)) \\ \searrow U(4\Omega) \supset [U(4) \supset SU_S(2) \otimes SU_T(2)] \otimes [U(\Omega) \supset O(\Omega) \supset O_L(3)]. \end{array}$$

[M. Moshinsky and C. Quesne, PLB 29, 482 (1969);
B. R. Judd and J. P. Elliott, *Topics in Atomic and Nuclear Theory*, 1970;
V. K. B. Kota, J. A. Castilho Alcarás, NPA **764**, 181 (2006)]

Canonical and non-canonical bases of $O_T(5)$

The generators of $O_T(5)$ can be expressed by linear combinations of a set of operators $\{E_{ij}\}$ $(1 \le i, j \le 4)$ satisfying

$$[E_{ij},\ E_{lk}] = \delta_{jl} E_{ik} - \delta_{ik} E_{lj}, \quad (E_{ij})^{\dagger} = E_{ji}.$$

In the $SU_{\Lambda}(2) \otimes SU_I(2)$ basis, the generators of $O_T(5)$ may be expressed as

$$\nu_+ = E_{12}, \nu_- = E_{21}, \nu_0 = \frac{1}{2}(E_{11} - E_{22}),$$

$$\tau_+ = E_{34},\ \tau_- = E_{43},\ \tau_0 = \frac{1}{2}(E_{33} - E_{44}),$$

$$U_{\frac{1}{2}\frac{1}{2}} = \sqrt{\frac{1}{2}}(E_{14} + E_{32}), U_{\frac{1}{2}-\frac{1}{2}} = \sqrt{\frac{1}{2}}(E_{42} - E_{13}),$$
$$U_{-\frac{1}{2}\frac{1}{2}} = \sqrt{\frac{1}{2}}(E_{24} - E_{31}),\ U_{-\frac{1}{2}-\frac{1}{2}} = -\sqrt{\frac{1}{2}}(E_{41} + E_{23}).$$

[K. T. Hecht, NP **63**, 214 (1965);
R.T. Sharp and S.C. Pieper, JMP **9**, 663 (1968);
N. Kemmer, D.L. Pursey, and S.A. Williams, JMP **9**, 1224 (1968).]

Canonical and non-canonical bases of $O_T(5)$

After a linear transformation, the generators of $O_T(5)$ in the $O_T(5) \supset O_T(3) \times O_{\mathcal{N}}(2)$ basis may be expressed as

$$
\begin{aligned}
A_1^\dagger &= E_{12} = \nu_+, \quad A_{-1}^\dagger = E_{34} = \tau_+, \\
A_1 &= E_{21} = \nu_-, \quad A_{-1} = E_{43} = \tau_-, \\
A_0^\dagger &= \sqrt{\tfrac{1}{2}}(E_{14} + E_{32}) = U_{\frac{1}{2}\frac{1}{2}}, \quad A_0 = \sqrt{\tfrac{1}{2}}(E_{41} + E_{23}) = -U_{-\frac{1}{2}-\frac{1}{2}}, \\
T_+ &= E_{13} - E_{42} = -\sqrt{2}U_{\frac{1}{2}-\frac{1}{2}}, \quad T_- = E_{31} - E_{24} = -\sqrt{2}U_{-\frac{1}{2}\frac{1}{2}}, \\
T_0 &= \tfrac{1}{2}(E_{11} - E_{22} - E_{33} + E_{44}) = \nu_0 - \tau_0, \\
\hat{\mathcal{N}} &= \tfrac{1}{2}(E_{11} - E_{22} + E_{33} - E_{44}) = \nu_0 + \tau_0,
\end{aligned}
$$

where $\{T_+, T_-, T_0\}$ generate the subgroup $O_T(3)$, and $\hat{\mathcal{N}}$ generates the $O_{\mathcal{N}}(2)$. $\hat{\mathcal{N}} = \frac{\hat{n}}{2} - \Omega$, where $\Omega = \sum_j (j + 1/2)$, in which the sum runs over all single-particle orbits considered, and $\hat{n}$ is the total number operator of valence nucleons. [K. T. Hecht, Nucl. Phys. 63, 214 (1965);
R.P. Hemenger and K.T. Hecht, NPA 145, 468 (1970);
K. Ahmed and R.T. Sharp, JMP 11, 1112 (1970);
K. T. Hecht, NPA **493**, 29 (1989)]

Basis vectors of $O_T(5) \supset O_T(3) \otimes O_{\mathcal{N}}(2)$

A basis vector of $O_T(5) \supset SU_\Lambda(2) \otimes SU_I(2)$ is also an eigenstate of T_0 and $\hat{\mathcal{N}}$ with eigenvalues

$$M_T = m_\Lambda - m_I, \quad \mathcal{N} = m_\Lambda + m_I. \tag{1}$$

For a given irrep (v_1, v_2) of $O(5)$, all possible basis vectors of $O(5) \supset SU_\Lambda(2) \otimes SU_I(2) \supset U_\Lambda(1) \otimes U_I(1)$ restricted by the conditions (1) form a complete set for the fixed M_T and $\mathcal{N}$. Thus, the basis vectors of $O(5) \supset O_T(3) \otimes O_{\mathcal{N}}(2)$ can be expanded in terms of them with the restriction on the quantum numbers

$$m_\Lambda = \frac{1}{2}(\mathcal{N} + M_T), \quad m_I = \frac{1}{2}(\mathcal{N} - M_T).$$

Basis vectors of $O_T(5) \supset O_T(3) \otimes O_{\mathcal{N}}(2)$

For fixed $\mathcal{N}$ and $M_T \geq 0$, the following basis vectors are all possible within the $O(5)$ irrep (v_1, v_2):

$$\left| \begin{array}{cc} \multicolumn{2}{c}{(v_1, v_2)} \\ \Lambda, & I \\ \frac{1}{2}(\mathcal{N} + M_T), & \frac{1}{2}(\mathcal{N} - M_T) \end{array} \right\rangle \tag{2}$$

with the restrictions:

$$\frac{1}{2}|\mathcal{N} + M_T| \leq \Lambda \leq \frac{1}{2}(v_1 + v_2), \quad \frac{1}{2}|\mathcal{N} - M_T| \leq I \leq \frac{1}{2}(v_1 - v_2). \tag{3}$$

Basis vectors of $O_T(5) \supset O_T(3) \otimes O_{\mathcal{N}}(2)$

Hence, the basis vectors of $O(5) \supset O_T(3) \otimes O_{\mathcal{N}}(2)$ may be expanded in terms of $O(5) \supset SU_\Lambda(2) \otimes SU_I(2)$ as

$$\left| \begin{array}{c} (v_1, v_2) \\ \zeta \;\; T = M_T, \mathcal{N} \end{array} \right\rangle = \sum_{q=0}^{v_1-v_2} \sum_{p=\mathrm{Max}[0,\; q-v_1+v_2+|\mathcal{N}-T|]}^{\mathrm{Min}[v_1+v_2-q-|\mathcal{N}+T|,\; 2v2]} c^{(\zeta)}_{p,q} \times$$

$$\left| \begin{array}{cc} & (v_1, v_2) \\ \Lambda = \frac{1}{2}(v_1+v_2-p-q), & I = \frac{1}{2}(v_1-v_2+p-q) \\ \frac{1}{2}(\mathcal{N}+T), & \frac{1}{2}(\mathcal{N}-T) \end{array} \right\rangle,$$

which must satisfy

$$-\sqrt{\frac{1}{2}}T_+ \left| \begin{array}{c} (v_1, v_2) \\ \zeta \;\; T = M_T, \mathcal{N} \end{array} \right\rangle = U_{\frac{1}{2}-\frac{1}{2}} \left| \begin{array}{c} (v_1, v_2) \\ \zeta \;\; T = M_T, \mathcal{N} \end{array} \right\rangle = 0.$$

which leads to the following four-term relation to determine the expansion coefficients $\{c^{(\zeta)}_{p,q}\}$:[NPA **974** (2018) 86]

$$
\begin{aligned}
&c^{(\zeta)}_{p,q+1}(-1)^{2\mathcal{N}-2q+2v_1}\left[\frac{(1+q)(2v_1-q+2)(v_1+v_2-q+1)(v_1+v_2-p-q+T+\mathcal{N}+1)(v_1-v_2+T-\mathcal{N}+p-q+1)(v_1-v_2-q)}{(v_1+v_2-p-q)(v_1-v_2+p-q)}\right]^{\frac{1}{2}}\\
&+c^{(\zeta)}_{p+1,q}(-1)^{v_1+v_2+\mathcal{N}-p-q+T}\left[\frac{(1+p)(2v_2-p)(v_1+v_2-p+1)(v_1+v_2+T+\mathcal{N}-p-q+1)(v_1-v_2+p+2)(v_1-v_2-T+\mathcal{N}+p-q+1)}{(v_1+v_2-p-q)(v_1-v_2+p-q+2)}\right]^{\frac{1}{2}}\\
&+c^{(\zeta)}_{p-1,q}(-1)^{v_1-v_2+\mathcal{N}+p-q-T}\left[\frac{p(2v_2-p+1)(v_1+v_2-p+2)(v_1+v_2-T-\mathcal{N}-p-q+1)(v_1-v_2+p+1)(v_1-v_2+T-\mathcal{N}+p-q+1)}{(v_1+v_2-p-q+2)(v_1-v_2+p-q)}\right]^{\frac{1}{2}}\\
&+c^{(\zeta)}_{p,q-1}\left[\frac{q(2v_1-q+3)(v_1+v_2-q+2)(v_1+v_2-T-\mathcal{N}-p-q+1)(v_1-v_2-T+\mathcal{N}+p-q+1)(v_1-v_2-q+1)}{(v_1+v_2-p-q+2)(v_1-v_2+p-q+2)}\right]^{\frac{1}{2}}=0.
\end{aligned}
$$

$$
\mathbf{P}((v_1, v_2), \mathcal{N}, T)\mathbf{c}^{(\zeta)} = \Lambda \mathbf{c}^{(\zeta)},
$$

$$\mathbf{P}((v_1, v_2), \mathcal{N}, T)\mathbf{c}^{(\zeta)} = \Lambda \mathbf{c}^{(\zeta)}, \quad (4)$$

Once the matrix $\mathbf{P}((v_1, v_2), \mathcal{N}, T)$ is constructed, it can be verified that the number of $\Lambda = 0$ solutions of Eq. (4) equals exactly to the number of rows of $\mathbf{P}((v_1, v_2), \mathcal{N}, T)$ with all entries zero.

The eigenvectors $\mathbf{c}^{(\zeta)}$ belong to the null space of $\mathbf{P}((v_1, v_2), \mathcal{N}, T)$.

Since there are many ways currently available to find null-space vectors of a matrix, to find solutions of Eq. (4) with $\Lambda = 0$ becomes practically easy.

Non-orthogonality

The projection matrix $\mathbf{P}((v_1, v_2), \mathcal{N}, T)$ is nonsymmetric. The basis vectors are also non-orthogonal with respect to the multiplicity label ζ.

Basis vectors of $O_T(5) \supset O_T(3) \otimes O_{\mathcal{N}}(2)$

Computing time and memory requirements.

It can be observed tat the maximal number of terms occurs in $T = \mathcal{N} = 0$ case. In such extreme case, the upper bound of the number of terms involved in the expansion can be estimated by

$$d(\mathcal{N}=0, T=0) \leq \sum_{q=0}^{v_1-v_2} \sum_{p=\mathrm{Max}[0,\ q-v_1+v_2]}^{\mathrm{Min}[v_1+v_2-q,\ 2v_2]} 1 = (1+v_1-v_2)(2v_2+1), \quad (5)$$

which shows that $\mathrm{Max}[d(\mathcal{N}, T)] \leq d(\mathcal{N}=0, T=0)$ increases with v_1 linearly and with v_2 quadratically.

Matrix representations of $O_T(5) \supset O_T(3) \otimes O_{\mathcal{N}}(2)$

Using the Wigner-Eckart theorem, we have

$$\left\langle \begin{matrix} (v_1, v_2) \\ \zeta' \ T' \ M'_T, \mathcal{N}' \end{matrix} \middle| \mathcal{A}^+_\mu \middle| \begin{matrix} (v_1, v_2) \\ \zeta \ T \ M_T, \mathcal{N} \end{matrix} \right\rangle = \delta_{\mathcal{N}',\mathcal{N}+1} \langle TM_T, 1\mu | T' \ M'_T \rangle \times \left\langle \begin{matrix} (v_1, v_2) \\ \zeta' \ T', \mathcal{N}+1 \end{matrix} \middle\| \mathcal{A}^+ \middle\| \begin{matrix} (v_1, v_2) \\ \zeta \ T, \mathcal{N} \end{matrix} \right\rangle .$$

Example

$$\left\langle \begin{matrix} (v_1, v_2) \\ \zeta' \ T+1, \mathcal{N}+1 \end{matrix} \middle\| \mathcal{A}^+ \middle\| \begin{matrix} (v_1, v_2) \\ \zeta \ T, \mathcal{N} \end{matrix} \right\rangle = -\frac{1}{2} \sum_{q,p} \tilde{c}^{(\zeta')}_{p,q}(\mathcal{N}+1, T+1) \times$$

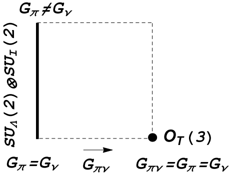
$$\tilde{c}^{(\zeta)}_{p,q}(\mathcal{N}, T) \sqrt{(v_1 + v_2 - p - q - \mathcal{N} - T)(v_1 + v_2 - p - q + \mathcal{N} + T + 2)}.$$

Applications to the isovector pairing model

The isovector pairing interaction Hamiltonian may be written as

$$\hat{H} = \sum_j \epsilon_j n_j - G_\pi A_{+1}^\dagger A_{+1} - G_{\pi\nu} A_0^\dagger A_0 - G_\nu A_{-1}^\dagger A_{-1}, \qquad (6)$$

which can be diagonalized in the $\bigotimes_{i=1}^{p} O_i(5)$ subspace, where p is the number of j-orbits.



The parameter rectangle of the isovector pairing Hamiltonian.

$SU_\Lambda(2) \otimes SU_I(2)$: Richardson-Gaudin;

$O_T(3)$: F. Pan, and J. P. Draayer, PRC **66**, 044314 (2002); J. Dukelsky et al, PRL **96**, 072503 (2006)

Extended model within the seniority-zero symmetric subspace

Let $|\rho\rangle$ be the orthonormalized basis vectors of $O(5) \supset O_T(3) \otimes O_{\mathcal{N}}(2)$, $\rho \equiv \{(\omega_1, \omega_2)\, \beta\, \mathcal{N}\; T\; M_T;\; \eta\}$, where $(\omega_1, \omega_2) = (\Omega - v/2,\, t)$ is an irrep of $O(5)$ from the Kronecker product of p copies of $O(5)$ irreps

$$\otimes_{i=1}^{p}(\omega_{1,i},\, \omega_{2,i}) \text{ of } O_1(5) \otimes \cdots \otimes O_p(5) \downarrow O(5)\ ,$$

$\Omega = \sum_{i=1}^{p} \Omega_i = \sum_i (j_i + 1/2)$, v is the total seniority number, t is the reduced isospin of unpaired nucleons, β is the branching-multiplicity label needed in the $O(5) \downarrow O_T(3) \otimes O_{\mathcal{N}}(2)$ reduction, T and M_T are quantum number of total isospin and that of its projection, respectively, $\mathcal{N} = \Omega - N/2$ with N being the total number of nucleons, and η stands for a set of other quantum numbers related to the total angular momentum.
$\{|\rho\rangle\}$ is a complete set of basis vectors needed in the $O(5) \supset O_T(3) \otimes O_{\mathcal{N}}(2)$ basis.

Extended model within the seniority-zero symmetric subspace

$$\hat{H}_{\rm SP} = -G \sum_{\mu} A^{\dagger}_{\mu} A_{\mu} = \sum_{\rho} E^{\rho} |\rho\rangle\langle\rho|,$$

which may be projected into the seniority-zero (symmetric) subspace:

$$\tilde{H}_{\rm SP} = P_{v=0}\, \hat{H}_{\rm SP}\, P_{v=0},$$

where

$$P_{v=0} = \sum_{\mathcal{N}\, T\, M_T} |(\Omega,\, 0)\, \mathcal{N}\ T\ M_T\rangle\langle(\Omega,\, 0)\, \mathcal{N}\ T\ M_T|$$

is a projection operator. The above Hamiltonian can be expressed as

$$\tilde{H}_{\rm SP} = \sum_{n\, T\, M_T} E^{(\Omega,\,0)\, n\, T} \sum_{\rho_1,\cdots,\rho_p,\tilde{\rho}_1,\cdots,\tilde{\rho}_p} F^{n\, T\, M_T}_{\rho_1,\cdots,\rho_p} \times$$

$$F^{n\, T\, M_T}_{\tilde{\rho}_1,\cdots,\tilde{\rho}_p} \prod_{i=1}^{p} K^{-1}_{n_i\, T_i}\, Z^{(n_i\, 0)}_{T_i\, M_{T,\, i}} [\mathbf{A}^{\dagger}(j_i)] \prod_{i'=1}^{p} K^{-1}_{\tilde{n}_{i'}\, \tilde{T}_{i'}}\, Z^{(\tilde{n}_{i'}\, 0)}_{\tilde{T}_{i'}\, \tilde{M}_{T,\, i'}} [\mathbf{A}(j_{i'})].$$

Extended model within the seniority-zero symmetric subspace

$$Z^{(n\,0)}_{TM_T}[\mathbf{A}^\dagger] = \left[\frac{2^{T+M_T}(2T+1)!!(T+M_T)!(T-M_T)!T!}{(n-T)!!(n+T+1)!!(2T)!}\right]^{\frac{1}{2}} \times$$

$$\left(2\,A^\dagger_1\,A^\dagger_{-1} - A^{\dagger\,2}_0\right)^{\frac{n-T}{2}} \sum_{x=\mathrm{Max}[0,M_T]}^{[(T+M_T)/2]} \frac{A^{\dagger\,y}_1 A^{\dagger\;T+M_T-2y}_0 A^{\dagger\;y-M_T}_{-1}}{2^y(y-M_T)!y!(T+M_T-2y)!},$$

$$K^{-1}_{n\,T} = \left[\frac{2^{\frac{1}{2}(n-T)}(\Omega-(n+T)/2)!(2\Omega+1-n+T)!!}{\Omega_i!(2\Omega+1)!!}\right]^{\frac{1}{2}}, \qquad (7)$$

[K. T. Hecht, NPA **493**, 29 (1989)] and

$$F^{n\,T\,M_T}_{\rho_1,\cdots,\rho_p} = \langle\rho_1,\cdots,\rho_p|(\Omega,\,0)\,\mathcal{N}\;T\;M_T\rangle \qquad (8)$$

is the $O(5) \supset (O_T(3) \supset O_T(2)) \otimes O_{\mathcal{N}}(2)$ multi-coupling coefficient.

Extended model within the seniority-zero symmetric subspace

$$F^{n\,T\,M_T}_{\rho_1,\cdots,\rho_p} = \frac{K^{-1}_{n\,T}}{\prod_{i=1}^{p} K^{-1}_{n_i\,T_i}} \left\langle \begin{array}{ccc} (n_1,\,0) & \cdots & (n_p,\,0) \\ T_1\,M_{T,\,1} & \cdots & T_p\,M_{T,\,p} \end{array} \middle| \begin{array}{c} (n,\,0) \\ T,\,M_T \end{array} \right\rangle =$$

$$\frac{K^{-1}_{n\,T}}{\prod_{i=1}^{p} K^{-1}_{n_i\,T_i}} \langle 0| \prod_{i=1}^{p} Z^{(n_i\,0)}_{T_i\,M_{T,\,i}}[\mathbf{b}(j_i)] Z^{(n\,0)}_{T\,M_T}[\mathbf{b}^\dagger]|0\rangle,$$

$$= \frac{K^{-1}_{n\,T}}{\prod_{i=1}^{p} K^{-1}_{n_i\,T_i}} \langle 0| \prod_{i=1}^{p} Z^{(n_i\,0)}_{T_i\,M_{T,\,i}}[\mathbf{b}] Z^{(n\,0)}_{T\,M_T}[\mathbf{b}^\dagger]|0\rangle,$$

Extended model within the seniority-zero symmetric subspace

A spherical mean-field plus the extended pairing Hamiltonian

$$\hat{H}_{\rm ext} = \sum_{i=1}^{p} \epsilon_{j_i} \hat{N}_{j_i} + \tilde{H}_{\rm SP}, \qquad (9)$$

where ϵ_{j_i} $(i = 1, 2, \cdots, p)$ are single-particle energies generated from any mean-field, is exactly solvable within the seniority-zero symmetric subspace, namely, with $v_i = 0 \; \forall \; i$. The eigenstate may be written as

$$|\zeta_{n\,T}, \mathcal{N}\; T\; M_T\rangle = \sum_{\rho_1,\cdots,\rho_p} \frac{F^{n\,T\,M_T}_{\rho_1,\cdots,\rho_p}}{2\sum_{i=1}^{p} \epsilon_{j_i} n_i - E^{(\zeta_n\,T)}_{n\;T}} \times$$

$$\prod_{i'=1}^{p} K^{-1}_{n_{i'}\,T_{i'}}\, Z^{(n_{i'}\,0)}_{T_{i'}\,M_{T,i'}} [\mathbf{A}^\dagger(j_{i'})]|0\rangle, \qquad (10)$$

Extended model within the seniority-zero symmetric subspace

The eigenvalue $E_{n\,T}^{(\zeta_n\,T)}$ is determined by

$$1 - \frac{G_{\rm ext}}{2}\left(n(2\Omega + 3 - n) - T(T+1)\right) \times \sum_{\rho_1,\cdots,\rho_p} \frac{\left(F_{\rho_1,\cdots,\rho_p}^{n\,T\,M_T}\right)^2}{2\sum_{i=1}^{p}\epsilon_{j_i} n_i - E_{n\,T}^{(\zeta_n\,T)}} = 0 \quad \text{for} \quad n \neq 0. \tag{11}$$

and $E_{0\,0}^{(\zeta_{0,0}=1)} = 0$ for $n = 0$.

Extended model within the seniority-zero symmetric subspace

Example

We use this model to estimate np-pairing contribution in even-even $N \sim Z$ nuclei suitably to be described in the f_5pg_9-shell outside the ^{56}Ni core with the single-particle energies given in [M. Honma et al, PRC **80**, 064323 (2009)]. For even-even $N \sim Z$ nuclei, the average np-interaction energy defined as [J.-Y. Zhang, R. F. Casten, and D. S. Brenner, PLB **227**,1 (1989)]:

$$\begin{aligned} \delta V_{\rm pn}^{\rm ee}(A = Z + N) \equiv \delta V_{\rm pn}^{\rm ee}(Z,\, N) = \\ \tfrac{1}{4}\left(B(Z,\, N) + B(Z-2,\, N-2) - \right. \\ \left. B(Z,\, N-2) - B(Z-2,\, N)\right), \end{aligned} \tag{12}$$

where $B(Z,\, N)$ is the binding energy of the even-even nucleus.

Extended model within the seniority-zero symmetric subspace

Example

Since ^{56}Ni is taken to be the core, the binding energy of a nucleus considered is defined as

$$B(28+N_\pi,\,28+N_\nu)=B(28,\,28)+E_{\rm C}(28,\,28)-E_{\rm C}(28+N_\pi,\,28+N_\nu)-E_{\rm sym}(28+N_\pi,\,28+N_\nu)+(N_\pi+N_\nu)E_0-E^{(1)}_{(N_\pi+N_\nu)/2},$$

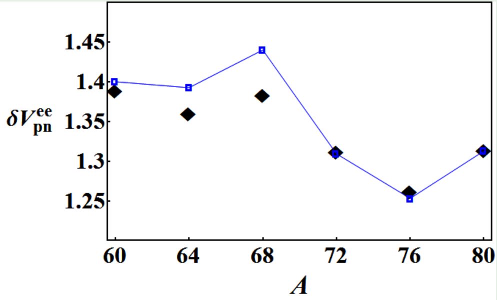
where $E_0=7.5$MeV, $E_{\rm C}(Z,\,N)=0.7173\,\frac{Z(Z-1)}{A^{1/3}}(1-Z^{-2/3})\,{\rm MeV}$ and

$$E_{\rm sym}(Z,\,N)=\frac{29.2876}{A}\,|N-Z|^2(1+\frac{2-|I|}{2+|I|\,A}-\frac{1.4492}{A^{1/3}})\,{\rm MeV}$$

with $I=|N-Z|/A$. [N. Wang, M. Liu, X. Wu, PRC 81, 059902 (2010)]

Extended model within the seniority-zero symmetric subspace

Example



$\delta V_{\rm pn}^{\rm ee}$ values (in MeV) derived from binding energies of even-even $N = Z$ and $N = Z \pm 2$ nuclei with mass number $A = 60 + 4k$ for $k = 0, 1, \cdots, 5$.

[F. Pan, X. Ding, K.D. Launey, L. Dai, J.P. Draayer, PLB 780(2018) 1]

Extended model within the seniority-zero symmetric subspace

Example

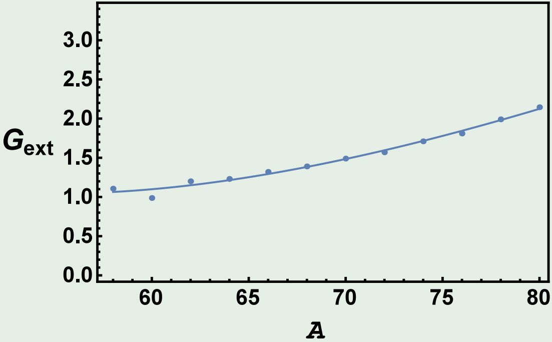


Figure: The extended isovector pairing interaction strength $G_{\rm ext}$ (in MeV) fitted by a quadratic function of the mass number A for $A =$ 58-80, from which we get $G_{\rm ext} = 4.262 - 0.1308A + 0.0013A^2$ MeV (solid line).

Summary

A simple and effective angular momentum projection to construct basis vectors of $O_T(5) \supset O_T(3) \otimes SO_{\mathcal{N}}(2)$ from the canonical basis vectors of $O_T(5) \supset SU_\Lambda(2) \otimes SU_I(2)$ is outlined.

The expansion coefficients can be obtained as components of the null-space vectors of a projection matrix, of which there are only four nonzero elements in each row.

Formulae for evaluating Matrix elements of $O_T(5)$ generators in the $O_T(5) \supset O_T(3) \otimes SO_{\mathcal{N}}(2)$ basis are explicitly given.

Summary

The null-space vectors are also non-orthogonal. The Gram-Schmidt orthonormalization is needed.

An extended pairing Hamiltonian that describes multi-pair interactions among isospin $T = 1$ and angular momentum $J = 0$ neutron-neutron, proton-proton, and neutron-proton pairs in a spherical mean field, such as the spherical shell model, is proposed based on the standard $T = 1$ pairing formalism.

As an example of the application, the average neutron-proton interaction in even-even $N \sim Z$ nuclei that can be suitably described in the f_5pg_9 shell is estimated in the present model, with a focus on the role of np-pairing correlations.

Summary

$O_T(5) \supset O_T(3) \otimes O_{\mathcal{N}}(2)$ basis may be adopted in diagonalizing the T=1 Hamiltonian with isospin symmetry breaking.

The new angular momentum projection method may be used to built basis vectors of the Wigner $U(4)$ group in $U(4) \supset SU_S(2) \otimes SU_T(2)$ basis by using the canonical $U(4)$ basis vectors.