Ab initio description of resonances of light nuclei

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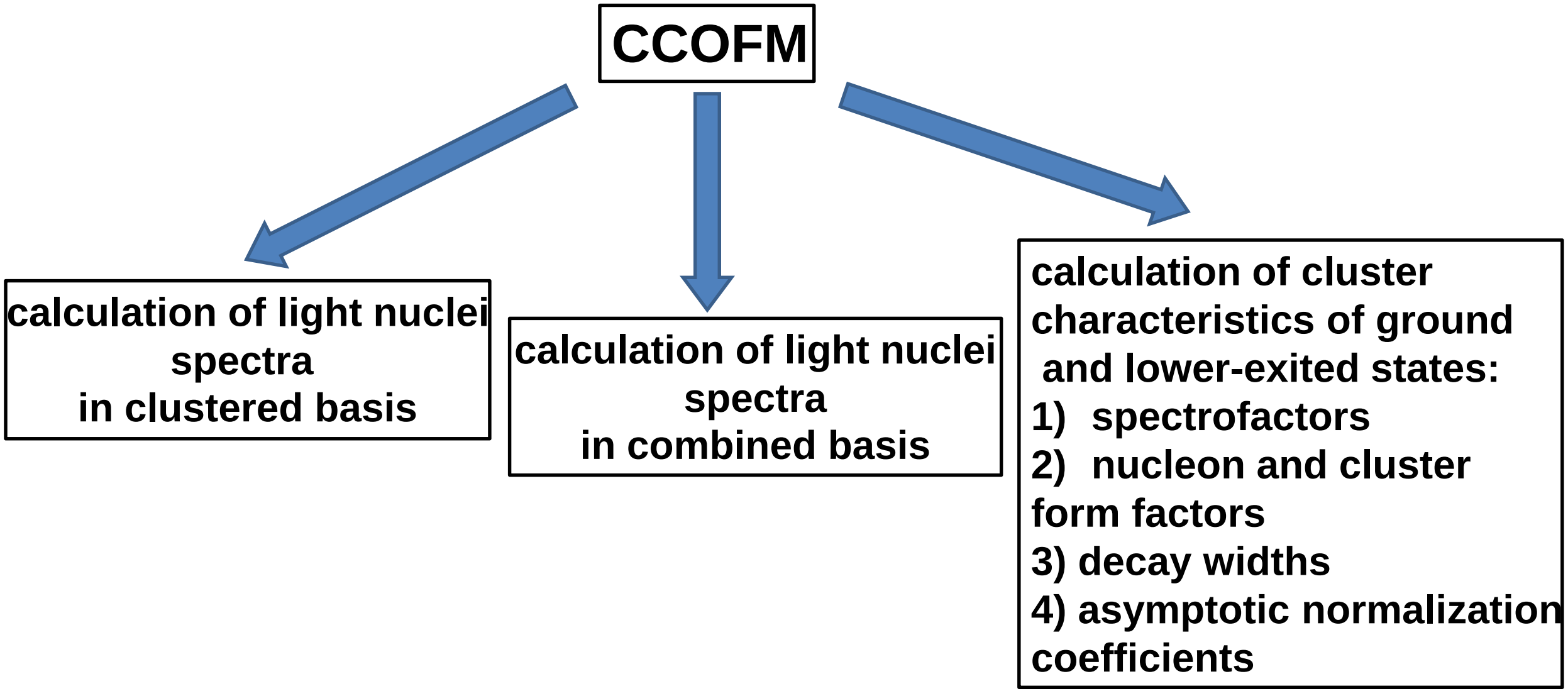
THE USE OF CCOFM MODEL

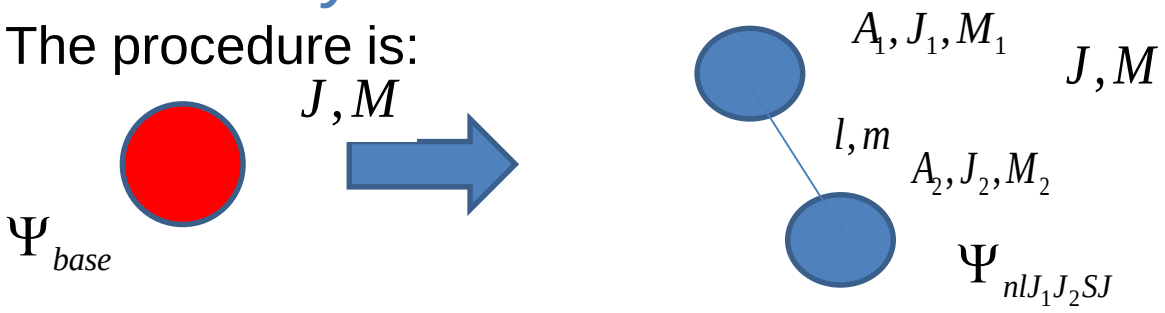
CCOFM(cluster channel orthogonal function model) could be used for ab initio calculations of various nuclei characteristics.



Decay widths of resonances

The procedure is:

1)



2) Basis functions are choosen to be $\Psi_{A}^{nlJ_1J_2SJ} = A\{\left\{\Psi_{A_1}^{J_1M_1}\Psi_{A_2}^{J_2M_2}\right\}_{SM_S}\varphi_{nlm}(\rho)\}_{JM_J}$,

3) Orthogonalization of the WF for each channel is done

$$\Psi_{A}^{i} = \frac{1}{\sqrt{\varepsilon_i}} A\{\left\{\Psi_{A_1}^{J_1M_1}\Psi_{A_2}^{J_2M_2}\right\}_{SM_S}\varphi_{ilm}(\rho)\}_{JM_J}, \quad \varphi_{ilm}(\rho) = \sum_n B_n^i \varphi_{nlm}(\rho)$$

4) Projection of the initial wave function Ψ_{base} on a certain channel (calculation of the spectroscopic amplitude) is

$$C_{MDC}^{nl} = \left\langle \Psi_{base} \middle| \Psi_{A}^{nlJ_1J_2SJ} \right\rangle$$

5) Determination of the cluster form factor is

$$F_l(\rho) = \sum_n A_n^l \varphi_{nl}(\rho), \; A_n^l = \sum_{i,n'} \left(\varepsilon_i\right)^{-1/2} C_{MDC}^{n'l} B_{nl}^i B_{n'l}^i$$

6) Calculations of narrow neutron resonances

In case of narrow neutron resonances $n_l(\mathrm{r}) >> \mathrm{j}_l(\mathrm{r})$

It means that $P_l(r) = (\mathrm{G}_l^2(\mathrm{r}) + \mathrm{F}_l^2(\mathrm{r}))^{-1} \approx (\mathrm{kr})^{-2}(n_l(r))^{-2}$

For the matching point R_{point} of ab initio WF with 2-body solution the following equation is used $\frac{F'_l(r)}{F_l(r)} = \frac{n'_l(r)}{n_l(r)}$

Then partial width of neutron resonance is $\Gamma = \frac{\hbar^2}{\mu k}\left(\frac{F_l(R_{point})}{n_l(R_{oint})}\right)^2$

7) Calculations of broad neutron resonances

In that case approximation $n_l(\mathrm{r}) >> \mathrm{j}_l(\mathrm{r})$ couldn't be used

Partial width is calculated by the use of R-matrix theory $\Gamma_n = 2k(F_l^2 + G_l^2)^{-1}_{r=R}\gamma^2$

8) Calculations of cluster resonances

For cluster channels Coulomb functions should be used instead of Bessel and Neiman functions

9) Calculations of asymptotic normalization coefficients

$$ANC = \frac{rF_l(r)}{W_{-\eta,l+1/2}(2kr)}$$

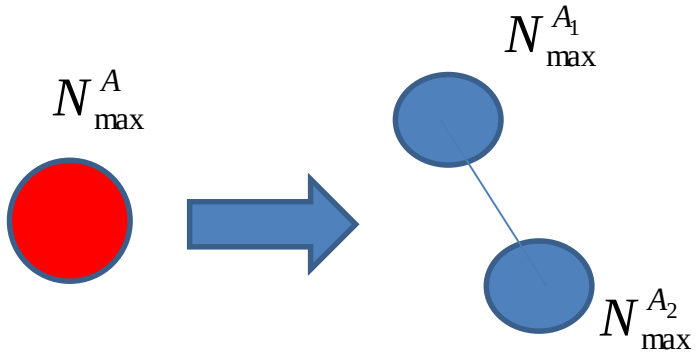
Calculations

For ab initio calculations realistic NN-potential **Daejeon16** (A.M. Shirokov, I.J. Shin, Y. Kim et al, PLB **761**, 87 (2016)) was used.

Ab initio calculations were carried out for low-lying resonances of ^{5}He as ^{4}He+n system and for ground and lower-exited states of ^{7}Li as ^{4}He+^{3}H system.

To check up the stability of the calculations, the truncation levels N^{A}_{max} of each subsystem and oscillator parameter hw were varied.

For NCSM calculation well-known shell model codes Antoine and Bigstick were used. For calculation of clustered terms of the discussed basis was created a code based on the algorithm described above.



Total binding energies of discussed nuclei

Using NCSM model ground states of ^{7}Li and ^{5}He were carried out with truncation level of N^{max} = 13(17). The subsystems ^{4}He, ^{3}H were calculated with truncation level up to N^{max} = 4.

Table 1. Total binding energies of ^{7}Li, ^{5}He, ^{4}He, ^{3}H

	hw=12.5	*hw=15*	*hw=17.5*	*hw=20*	*hw=22.5*	*hw=25*	*Exp.*
^{5}He	27.335	27.279	27.209	-----	------	--------	27.41
^{7}Li	------	------	------	38.901	38.607	38.192	39.245
^{4}He	26.778	27.784	28.102	28.158	28.126	28.051	28.296
%(0s1/2^{4})	0.891	0.9373	0.9694	0.9848	0.9861	0.97724	-----
^{3}H	------	------	------	7.976	7.802	7.598	8.482
%(0s1/2^{3})	------	------	------	0.9740	0.9628	0.9489	-----

At the used truncation levels ground states of ^{7}Li and ^{5}He and subsystems ^{4}He, ^{3}H are described well.

Calculation of the decay width for 3/2- state ^{5}He

For ab initio calculation of 3/2- state Daejeon16 potential was used with different values of oscillation parameter hw and truncation level Nmax. In table 2 total binding energy of this state among as spectroscopic factor (SF), as a value of amount of clustering, is shown.

Table 2. Total binding energies and spectroscopic factors of 3/2- state of ^{5}He

	exp.	Nmax=13 hw= 12.5	Nmax=13, hw= 15	Nmax=13, hw= 17.5	Nmax=15, hw= 12.5	Nmax=15, hw= 15	Nmax=15, hw= 17.5	Nmax=17, hw= 12.5	Nmax=17, hw= 15	Nmax=17, hw= 17.5
E_{tot}	27.406	27.171	27.126	27.042	27.268	27.214	27.137	27.335	27.279	27.209
SF	----	0.9155	0.951983	0.964979	0.91499	0.9521	0.9653	0.9148	0.9523	0.9658

The calculated result demonstrate convergence of total binding energy and stability of the value of spectroscopic factor.

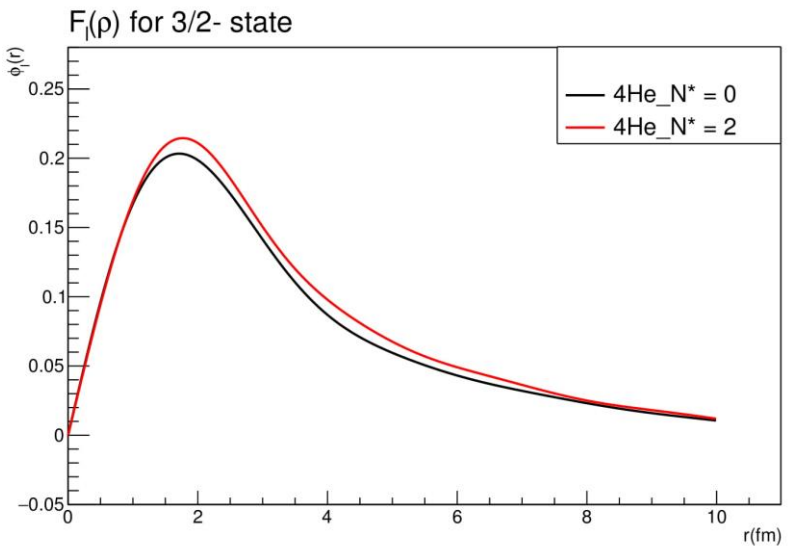
For ab initio calculations of resonance decay width the oscillator amplitudes of neutron form factor are needed. These values were calculated using ^{4}He with truncation level N_{He} = 0,2,4. For illustration of the method, the amplitude values for Nmax = 15 and hw = 15 MeV are given in Table 3.

Table 3. Values of oscillator amplitudes of neutron form factor for 3/2- ^{5}He state.

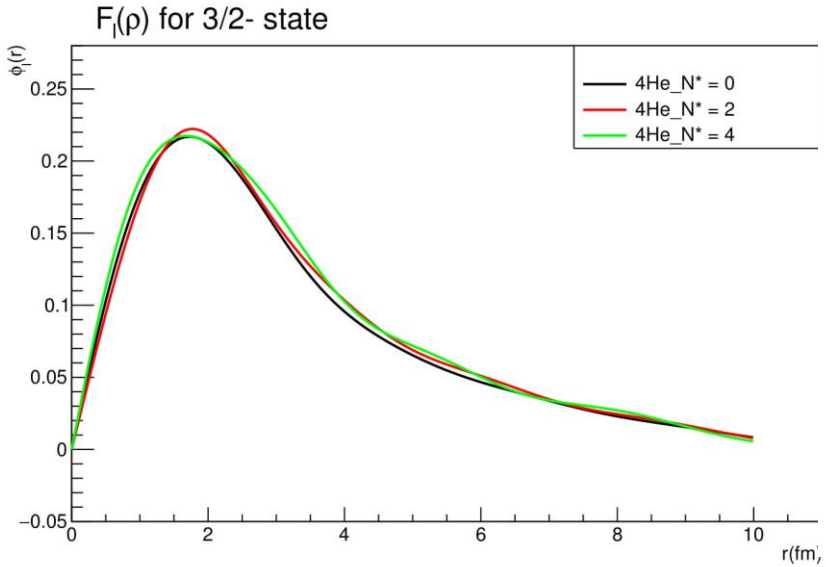
	n=0	n=1	n=2	n=3	n=4	n=5	n=6	n=7
N_{He} = 0	0.792	-0.154	0.277	-0.135	0.112	-0.067	0.038	-0.016
N_{He} = 2	0.859	-0.198	0.305	-0.148	0.123	-0.073	0.040	
N_{He} = 4	0.888	-0.207	0.302	-0.153	0.127	-0.075		

Neutron form factors of 3/2- state ^{5}He

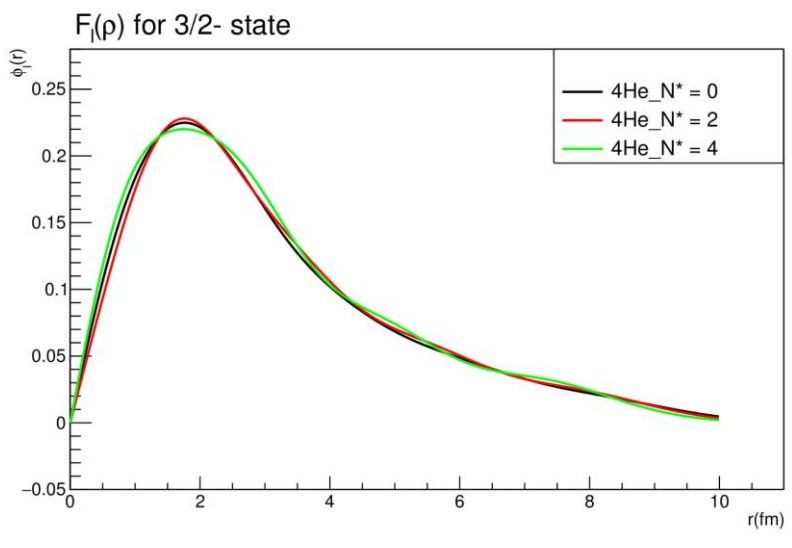
hw = 12.5



hw = 15



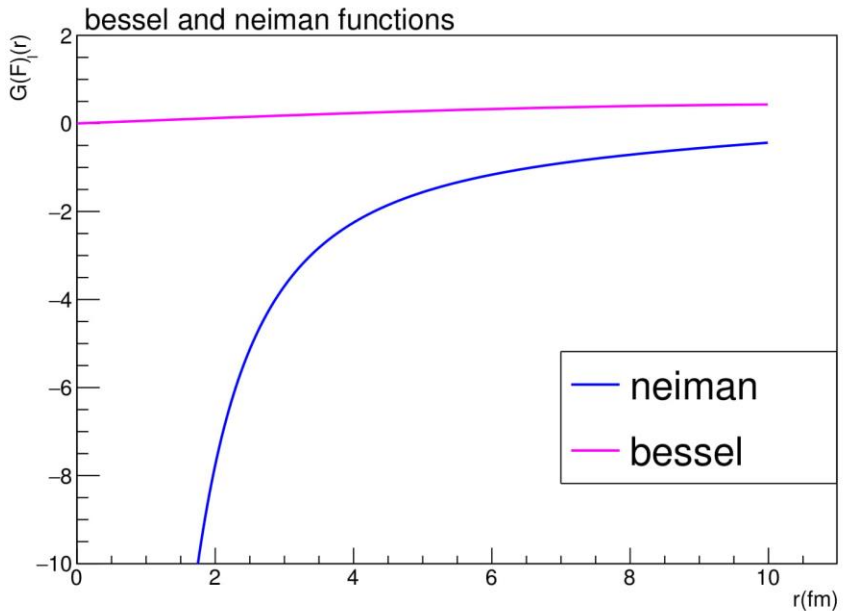
hw = 17.5



The function of neutron form factor show light dependence on the value of oscillator parameter hw.

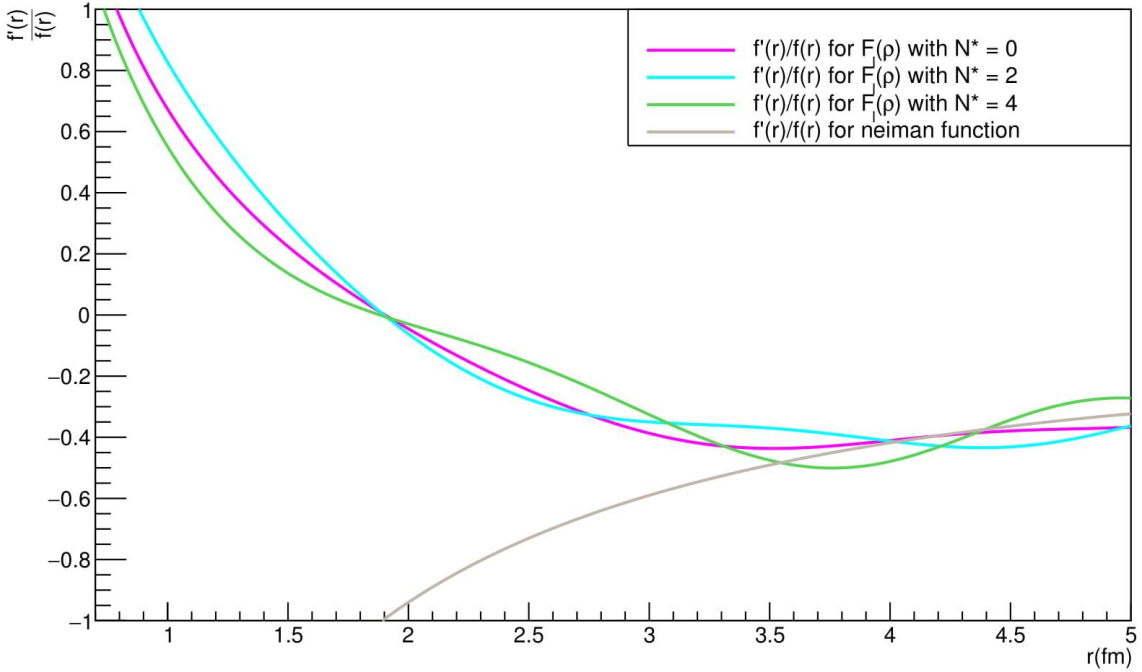
For the value of neutron resonance energy(0.890 MeV) in significant area for r coordinate $n_l(\mathrm{r}) >> \mathrm{j}_l(\mathrm{r})$

In this case $P_l(r) = (\mathrm{G}_l^2(\mathrm{r}) + \mathrm{F}_l^2(\mathrm{r}))^{-1} \approx (\mathrm{kr})^{-2}(n_l(r))^{-2}$



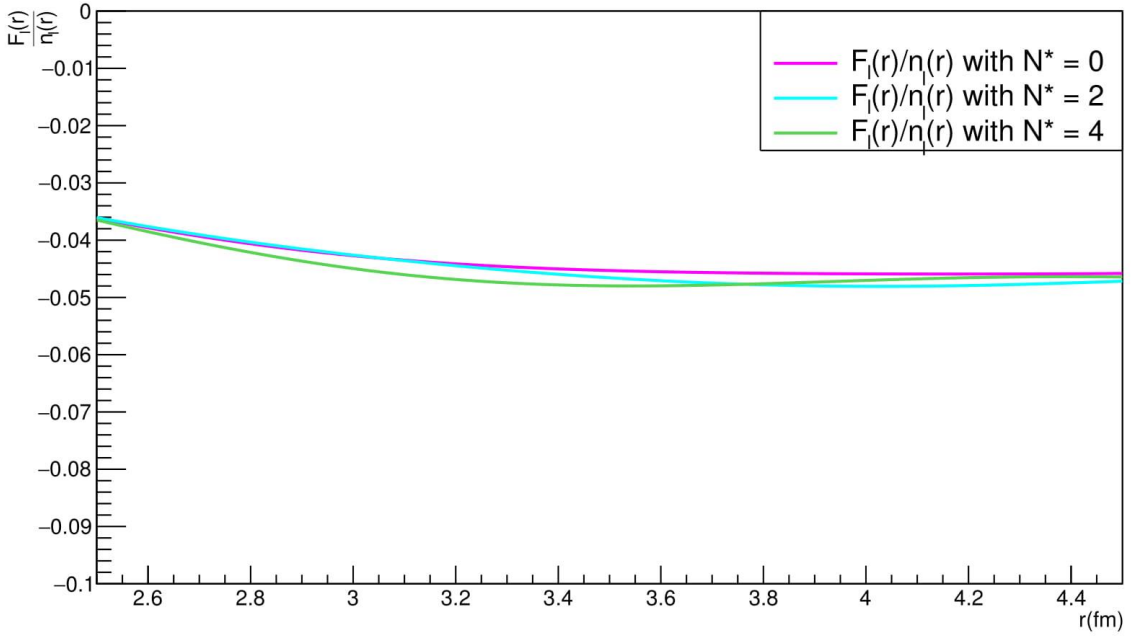
For the matching point R_{point} of ab initio WF with 2-body solution the previosly mentioned equation is used in case of ^{4}He with truncation level N_{He} = 0,2,4

$$\frac{F'_l(r)}{F_l(r)} = \frac{n'_l(r)}{n_l(r)} \Rightarrow \Gamma_{3/2-} = \frac{\hbar^2}{\mu k}\left(\frac{F_l(R_{point})}{n_l(R_{oint})}\right)^2$$



Logarithmic derivatives of neutron form factors and Neiman function, hw = 17.5 MeV

Consequently the asymptotic region is achieved in ab initio calculations



Results of the calculation of the decay width for 3/2- ^{5}He state

Calculations of 3/2- ^{5}He state were carried out in the wide range for the truncation levels of ^{5}He (Nmax) and ^{4}He (N_{He}) and oscillator parameter hw. Results are shown in the table 4. Experimental result of this value is equal to 600 keV.

Table 4. Decay width for 3/2- ^{5}He state

	Nmax=13, hw= 12.5	Nmax=13, hw= 15	Nmax=13, hw= 17.5	Nmax=15, hw= 12.5	Nmax=15, hw= 15	Nmax=15, hw= 17.5	Nmax=17, hw= 12.5	Nmax=17, hw= 15	Nmax=17, hw= 17.5
$N_{He} = 0$	469 keV	548 keV	628 keV	443 keV	530 keV	597 keV	427 keV	578 keV	577 keV
$N_{He} = 2$	545 keV	588 keV	625 keV	545 keV	590 keV	615 keV	515 keV	587 keV	600 keV
$N_{He} = 4$	642 keV	678 keV	642 keV	563 keV	629 keV	630 keV	------	611 keV	620 keV

Theoretical calculations show good agreement with experiment and stability when changing the parameters of calculations.

Calculation of ground and lower exited states of ^{7}Li

Using Bigstick code total binding energies of ground and lower exited states of ^{7}Li were calculated.

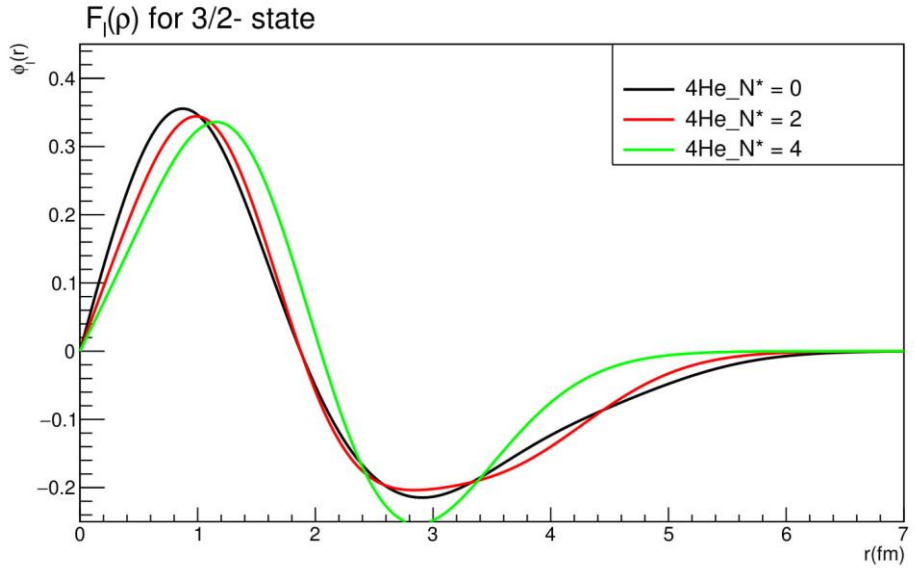
Table 5. The spectrum of lowest energy levels of 7Li

	exp.(E)	Γ(exp.)	hw=20(E)	SF	hw=22.5(E)	SF	hw=25(E)	SF
3/2-	39.245	-----	38.901	0.857	38.607	0.855	38.192	0.851
1/2-	38.788	73 fs	38.021	0.832	37.667	0.828	37.177	0.823
7/2-	34.615	93 keV	34.203	0.722	33.922	0.716	33.538	0.708
5/2-	32.565	880 keV	30.404	0.419	29.871	0.467	29.185	0.506
5/2-	31.786	89 keV	31.280	0.340	30.865	0.286	30.338	0.239
7/2-	29.575	400 keV	28.018	0.141	27.438	0.143	26.719	0.148

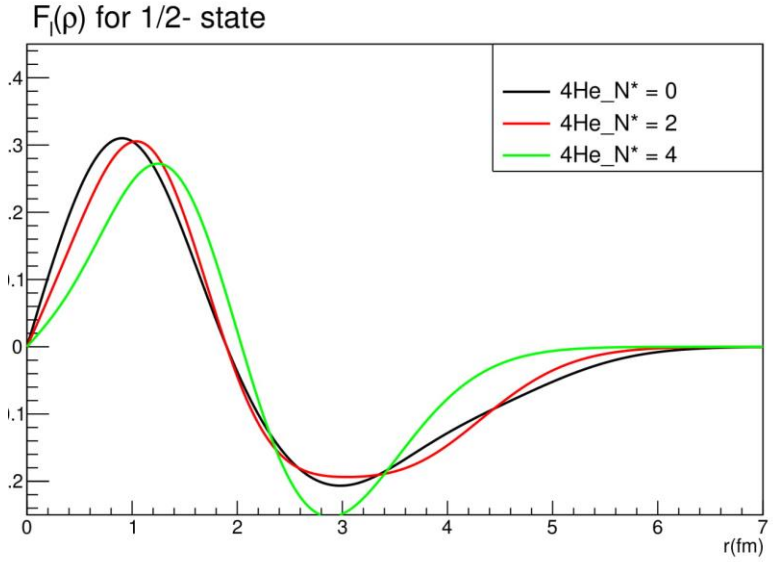
The WF of these states were calculated using VNIIA computing cluster. The spectrum of these states show rather good agreement with experiment, so these WF could be used for decay width calculations.

Asymptotic normalization coefficients of 3/2-, 1/2- states

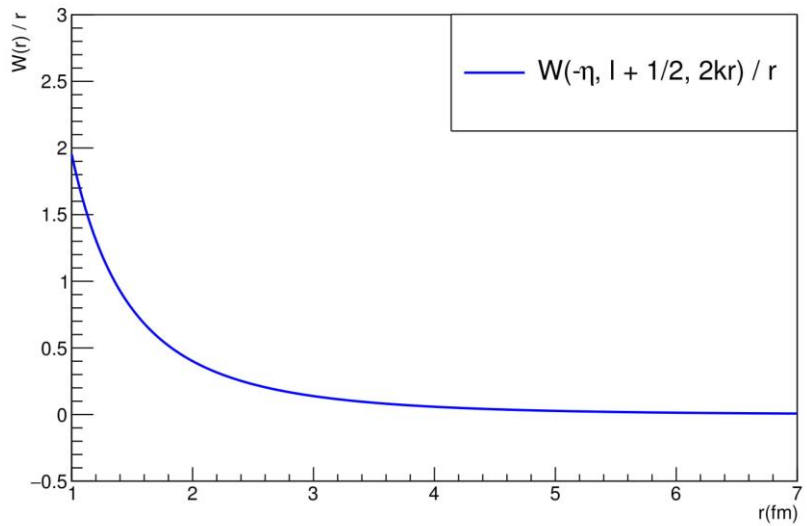
Form factor of 3/2- state



Form factor of 1/2- state



2-body solution – Whittaker function



2-body solution – Whittaker function

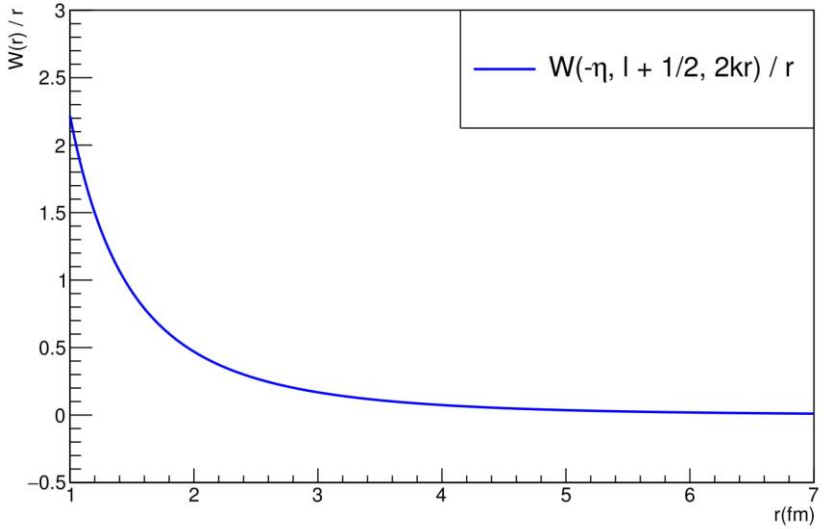
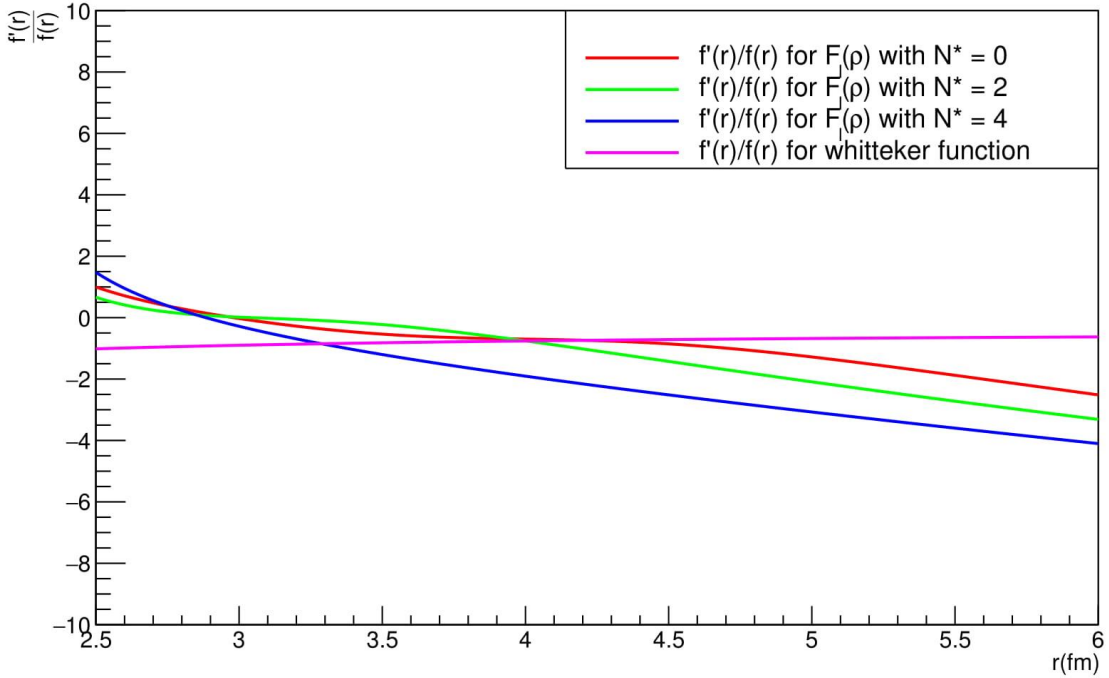
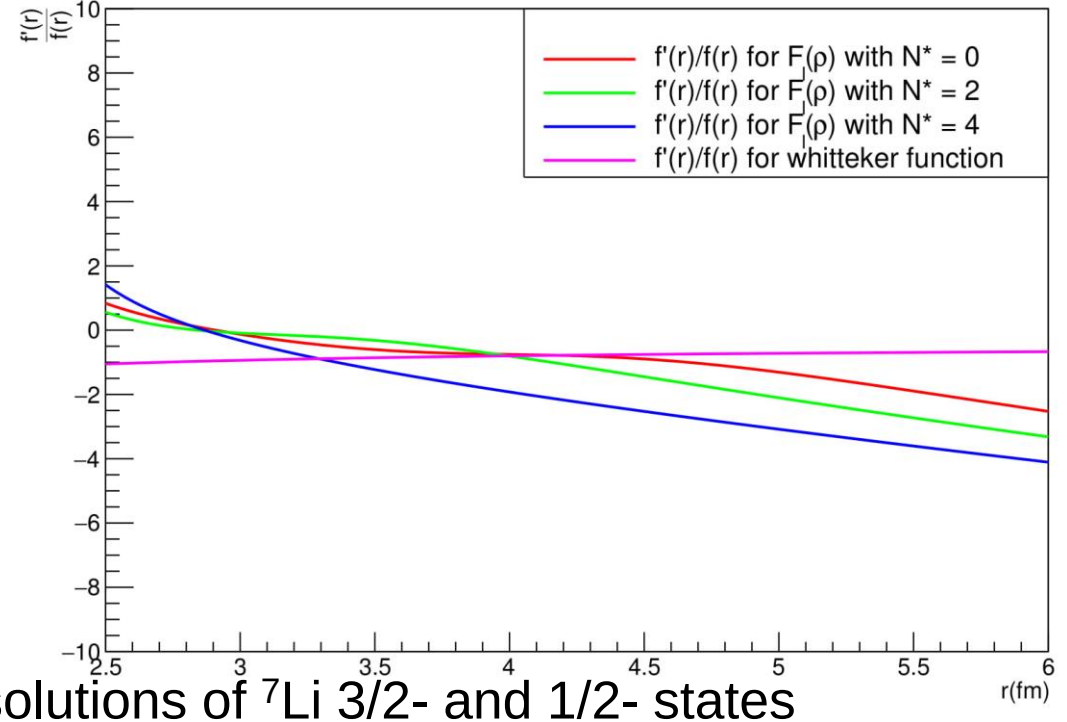


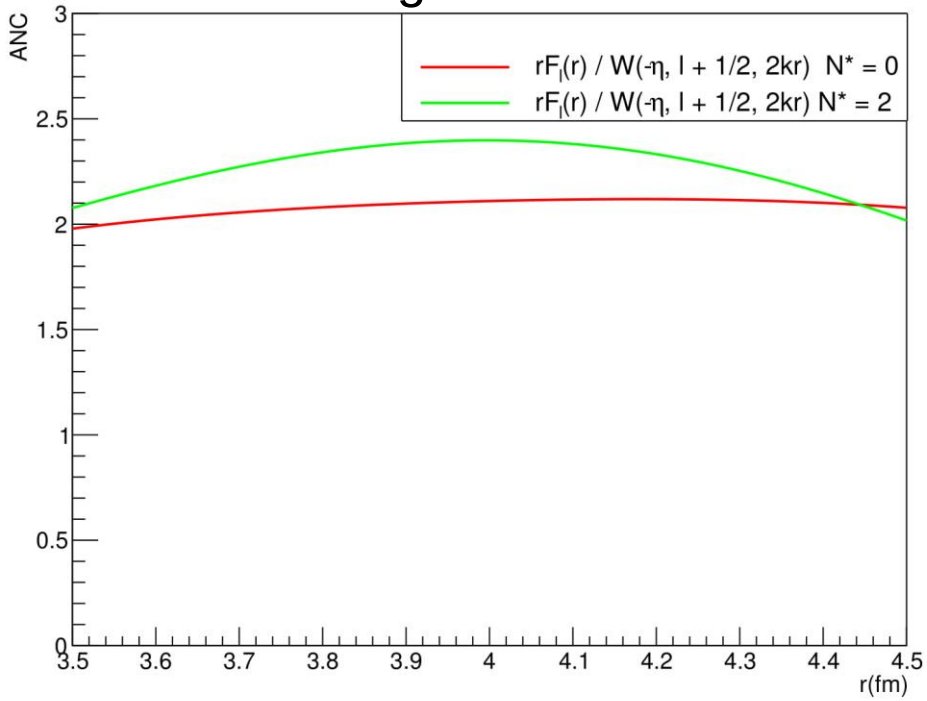
Table 6. The summed quantum number for ^{7}Li

	N_{tot}	N_{rel}	l_{rel}
4He_N*=0	13	13	1
4He_N*=2	13	9	1
4He_N*=4	13	5	1





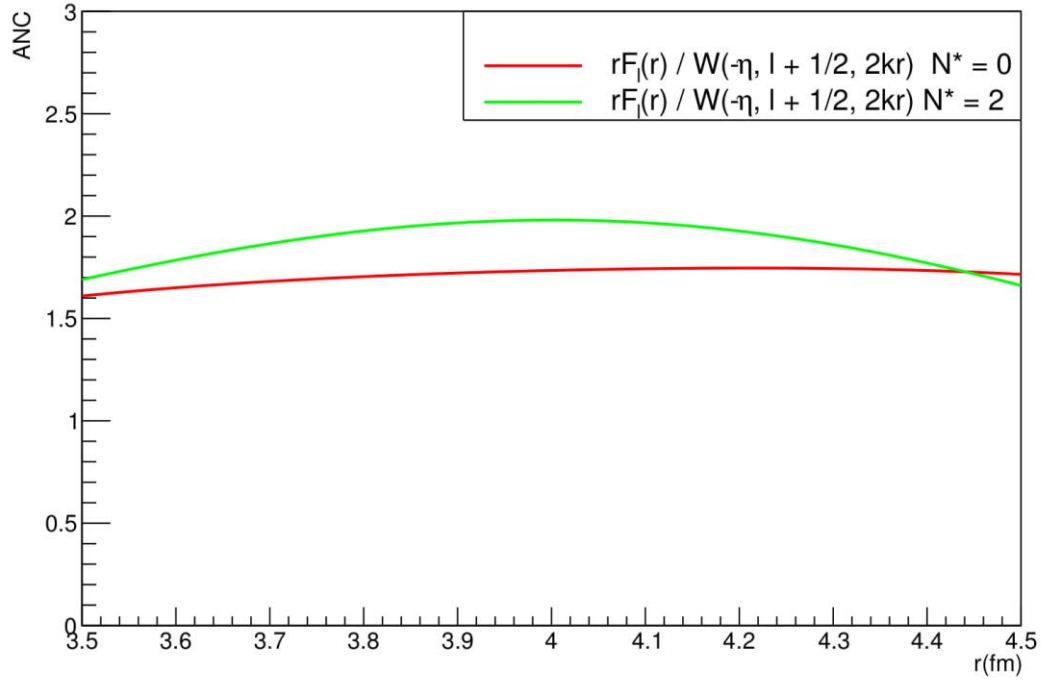
Logarithmic derivatives of ab initio and 2-body solutions of ^{7}Li 3/2- and 1/2- states



$$ANC = \frac{rF_l(r)}{W_{-\eta,l+1/2}(2kr)}$$

$$ANC_{3/2-} = 2.12\ fm^{-1/2}$$

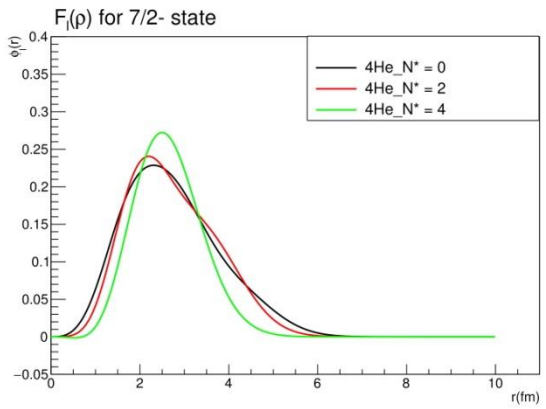
$$ANC_{1/2-} = 1.74\ fm^{-1/2}$$

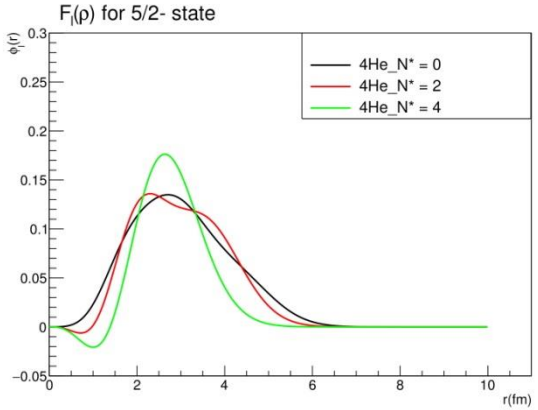


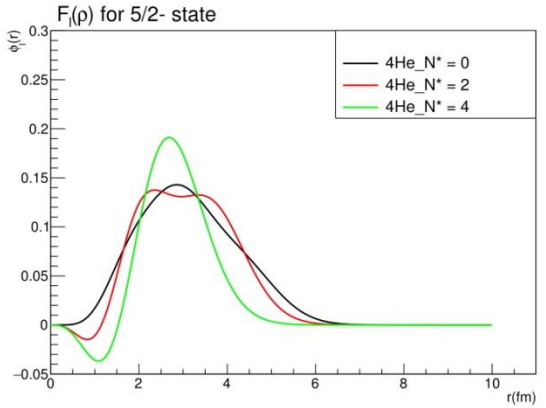
Partial decay widths of ^{4}He+^{3}H channel for 7/2-, 5/2- states

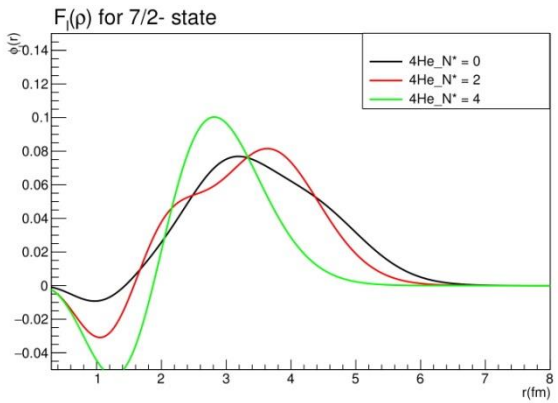
7/2- state 5/2- state 5/2- state 7/2- state

Form factors of ^{4}He +^{3}H channel

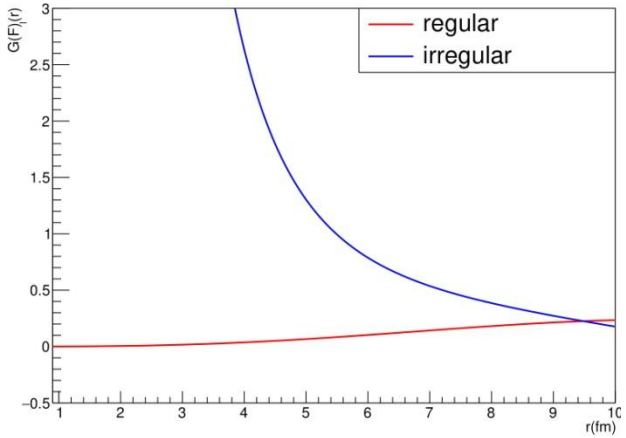


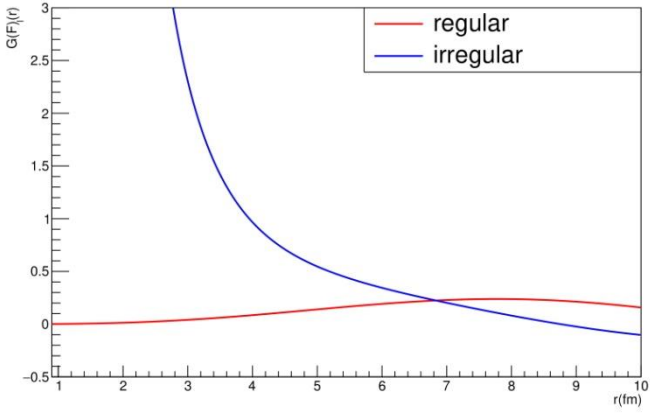


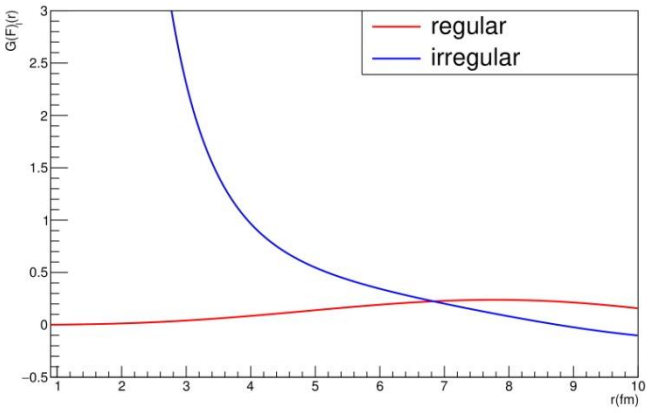


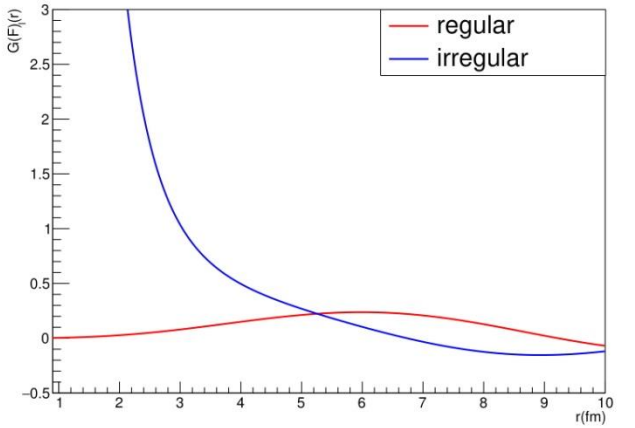


Coulomb functions



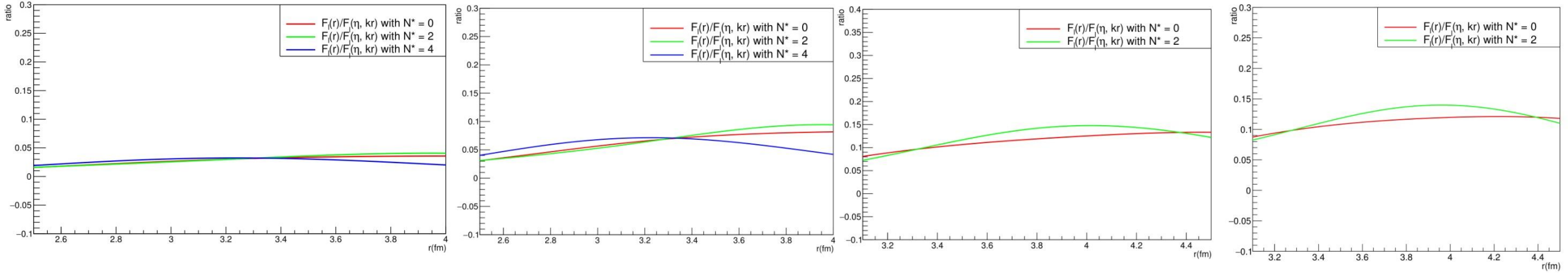






The matching points for all these states lie within limits of 3.2 – 4.0 fm

In all these cases approximation $F_l >> G_l$ could be used



The asymptotic region is achieved in ab initio calculations for channel WF with good relative motion description

Table 7. Calculated and experimental decay widths of 7/2- and 5/2- states of ^{4}He + ^{3}H channel				
	7/2-	5/2-	5/2-	7/2-
exp.(total)	98 keV	880 keV	89 keV	400 keV
hw=20	79 keV	589 keV	274 keV	560 keV
hw=22.5	71 keV	622 keV	223 keV	495 keV

The calculated decay width for most states show good agreement with experiment. The large difference for one of the 5/2- states could be explained by lack of oscillator basis size or inaccuracies of the nucleon-nucleon potential.

Conclusion

I. A method adapted for describing the characteristics of neutron and cluster resonances of light nuclei basing on ab initio nucleon-nucleon potentials has been developed.

II.Calculations of the 3/2- neutron resonance of ^{5}He show good agreement with experimental value and stability when changing the parameters of theoretical calculations.

III.Calculations of decay width of ^{4}He+^{3}H channel were performed for all narrow resonances of ^{7}Li. The results for the most states show good agreement with experiment. It was shown that these values could be another test for nucleon-nucleon potentials.

IV.The applicability of described method for ab initio calculation of asymptotic normalization coefficients was shown by the example of ground and lowest exited state of ^{7}Li.

THANK YOU FOR YOUR ATTENTION!