Calculation of ground-state energy for light nuclei with the Strutinsky's method

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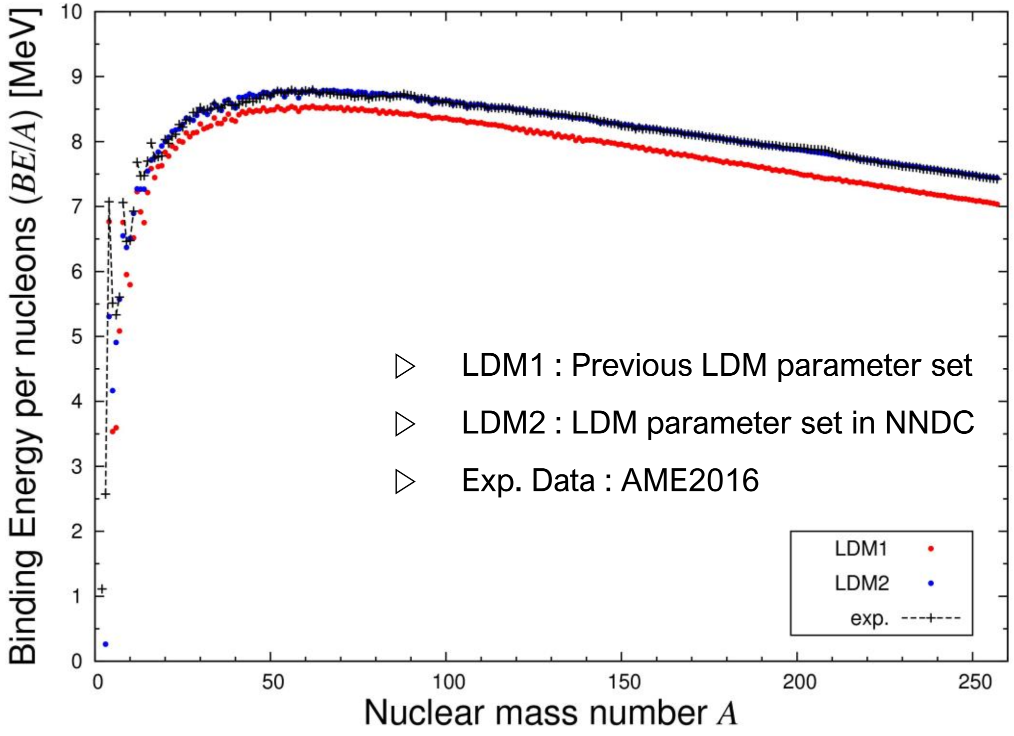
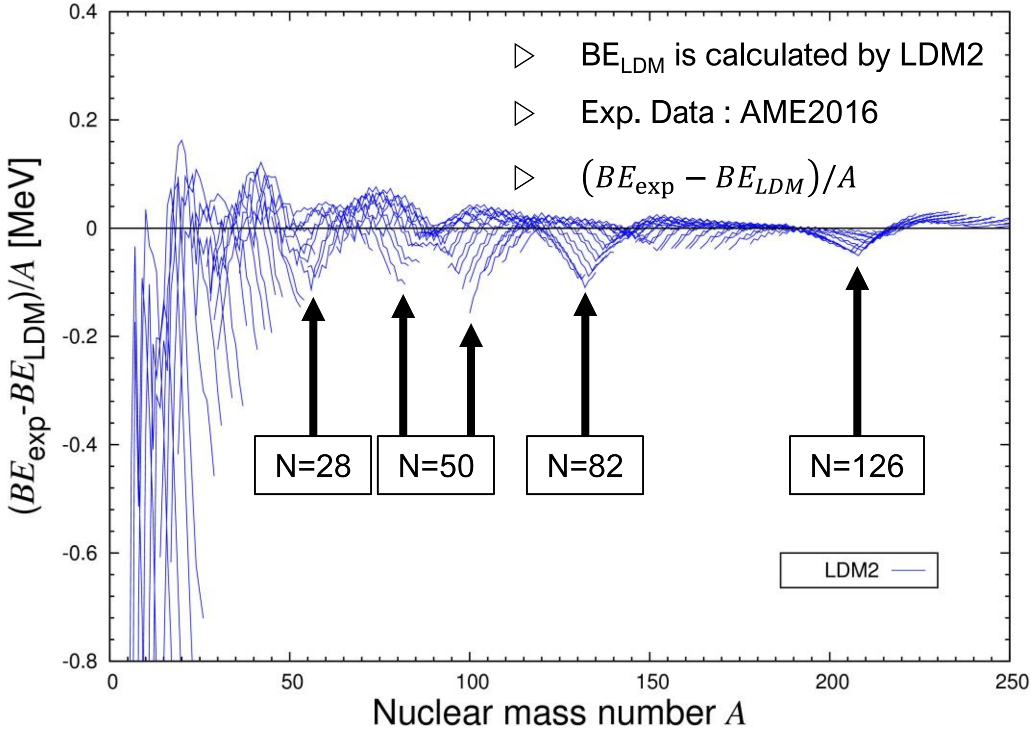
Introduction

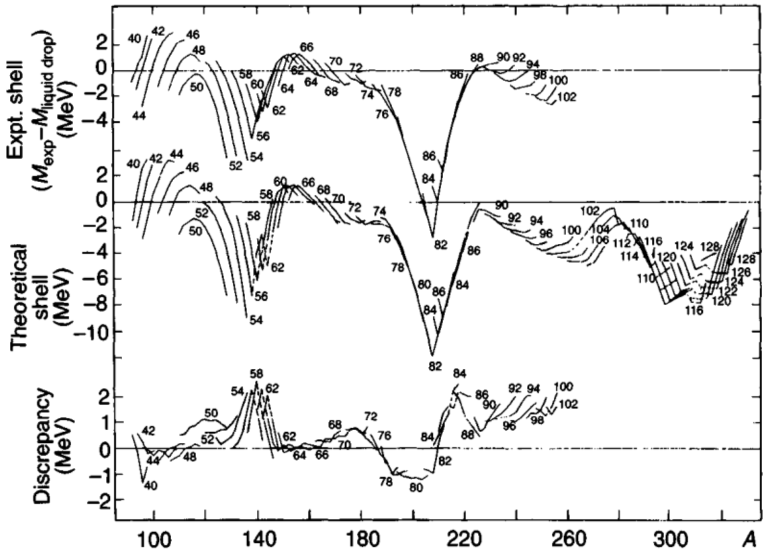
- The binding energy per nucleons (BE/A)
  - Tendency of BE/A which is come from experimental data can be reproduced by liquid drop model
  - Many LDMs and parameter sets are existed to explain nuclear mass
  - LDM can't explain shell closure (magic numbers) : Difference value between experimental data and LDM



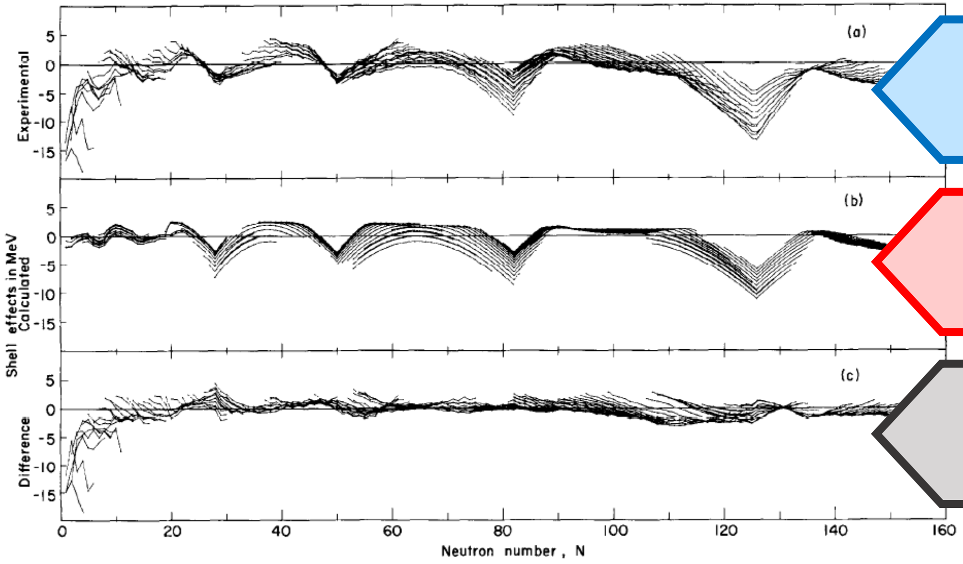


Introduction

- Shell effects in total binding energy



✓ S.G. Nilsson & I. Ragnarsson (1995)



✓ W.D. Myers (1976)

$BE_{\exp} - BE_{LDM}$

Shell effect

Theoretical shell calculation

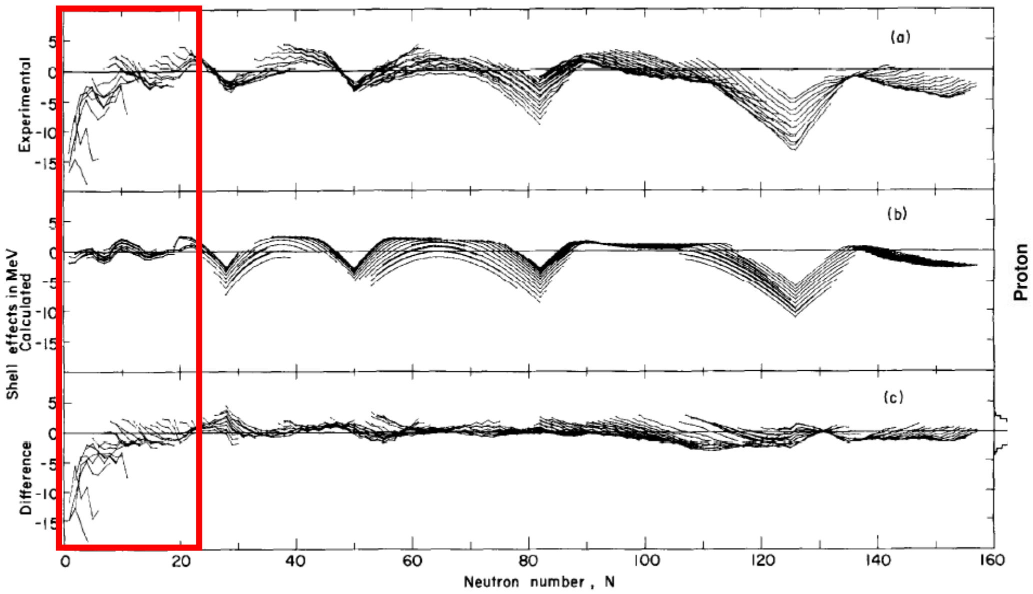
Discrepancy

Model and Experimental

- Occurrence of Shell effects due to shell closure (magic number)
- **Strutinsky's method** : Macroscopic-microscopic approach to shell effect
- Shell correction method for shell effect is developed by V.M. Strutinsky

Motivation

- $\left(BE_{\mathrm{exp}} - BE_{LDM}\right)/A$ value in region of light nuclei



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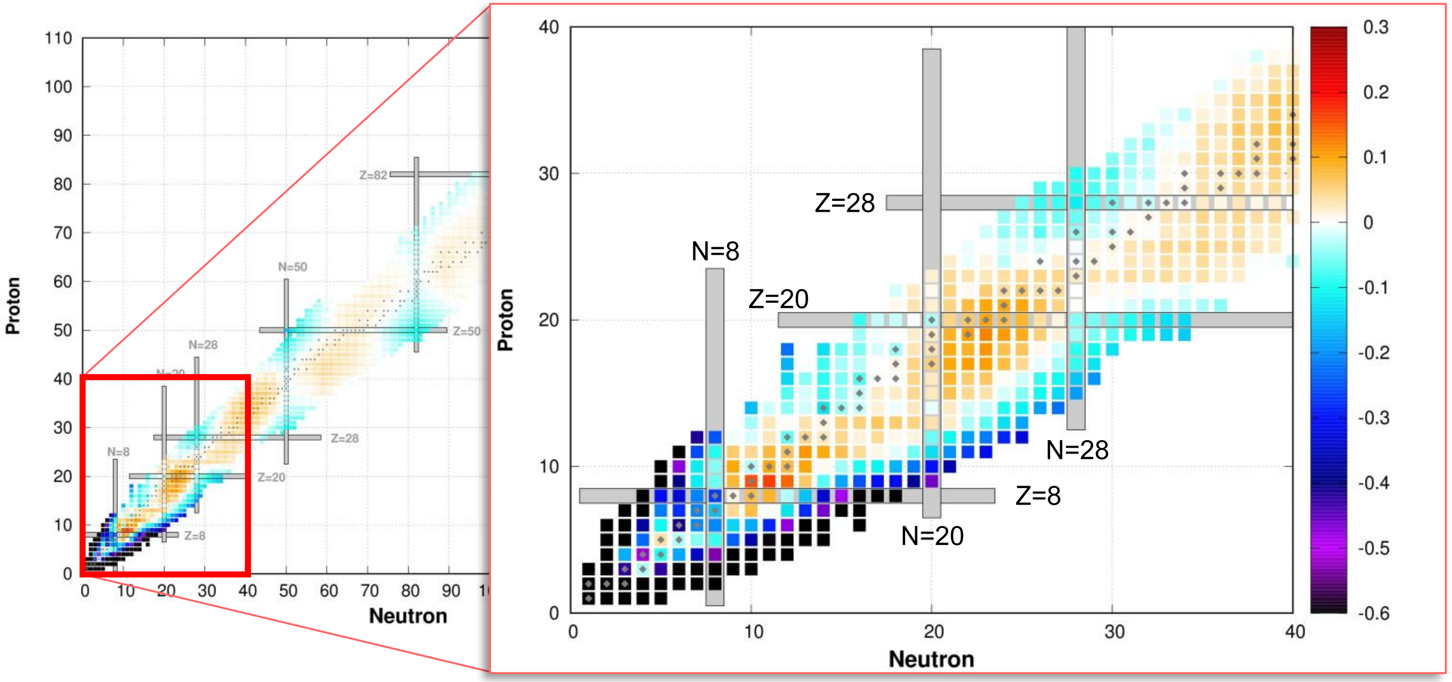


- Theoretical difficulties for light nuclei
  - Plateau condition in light nuclei
  - Any other effects in nuclear structure

- Microscopic researches of our group
  - Deformed potential
  - Pairing with BCS
  - Weakly-bound
  - Deformed BCS

Formalism

- Main concept of Strutinsky's method
  - A combination of macroscopic (LDM) and microscopic (SM) approaches
  - The total binding energy (TBE) is calculated by *liquid drop model* part and *shell correction* part

$$E_{\mathrm{TBE}} = E_{\mathrm{LDM}} + E_{\mathrm{osc}}$$

- The macroscopic term (E_{LDM}) : spherical Liquid Drop Model
  - Semi-empirical mass formula

$$E_{\mathrm{LDM}} = a_{vol}A - a_{surf}A^{2/3} - a_{sym}(N-Z)^2/2A - a_C Z^2/A^{1/3} - \delta(A)$$

✓ Bethe-Weizsäcker mass formula are referred by P. Ring and P. Schuck, The Nuclear Many-Body Problem (2004)

a_{vol} [MeV]	a_{surf} [MeV]	a_{sym} [MeV]	a_C [MeV]	$\delta(A)$ [MeV]		
				even-even	odd-even	odd-odd
15.74063	17.61628	23.42742	0.71544	$+12.59898 \cdot A^{-3/4}$	0	$-12.59898 \cdot A^{-3/4}$

✓ National Nuclear Data Center, Chart of Nuclides Description (http://www.nndc.bnl.gov/chart/help/index.jsp)

Formalism

- Single particle energy level with the deformed Woods-Saxon

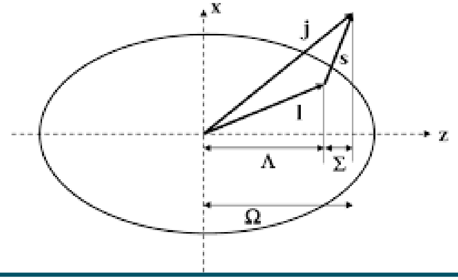
  - The deformed Woods-Saxon potential

$$V_{WS}(r,\theta,\beta_2) = -V_0/\{1+\exp[R(r,\theta,\beta_2)/a]\}$$

$$R(r,\theta,\beta_2) = r - R_0[1+\beta_2 Y_{20}(\theta)]$$

  - Nilsson basis for axial-symmetric potential

$$H\left|\psi_{N,n_z,\Lambda,\Omega^\pi}\right\rangle = E\left|\psi_{N,n_z,\Lambda,\Omega^\pi}\right\rangle$$



- Ω : j projection onto symmetry-axis
- Λ : l projection onto symmetry-axis
- Σ : spin
- $\Omega \equiv \Lambda + \Sigma$
- $N = n_\perp + n_z$, $n_\perp = n_x + n_y$

  - The equilibrium deformation β_{eq}

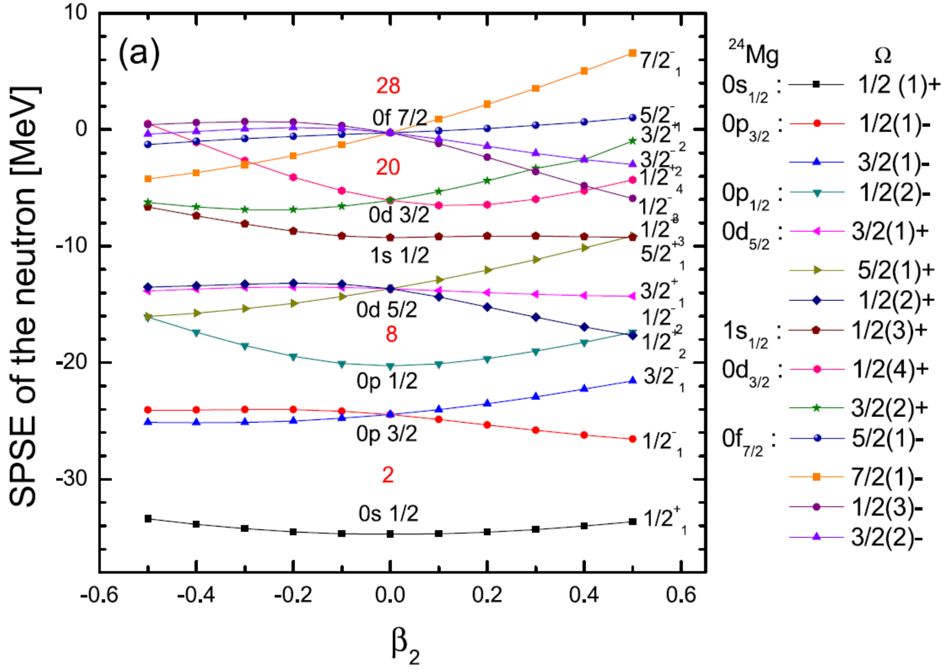
$$\left(\frac{\partial \mathfrak{H}}{\partial \beta}\right)_{\beta_{\mathrm{eq}}} = \left[\frac{\partial(\sum_i H_i)}{\partial \beta}\right]_{\beta_{\mathrm{eq}}} = 0$$

  - Total Hamiltonian for deformed Woods-Saxon

$$H = T + V_{WS} + V_{SO} + V_{Coulomb}$$

$$V_{SO} = -\lambda(\hbar/2mc)^2[\nabla V_{WS}(r,\theta,\beta_2)](\vec{\sigma}\times\vec{p})$$

  - Nilsson diagram about ^{24}Mg nucleus



Formalism

- The microscopic term : Shell correction energy (Shell oscillation energy)
  - Shell correction energy is consisted with a discrete shell energy and a smoothed shell energy

$$E_{osc} = E_{shell} - \widetilde{E_{shell}}$$

- A discrete shell energy
  - The discrete level density

$$g(\epsilon) = 2\sum_i \delta(\epsilon - \epsilon_i)$$

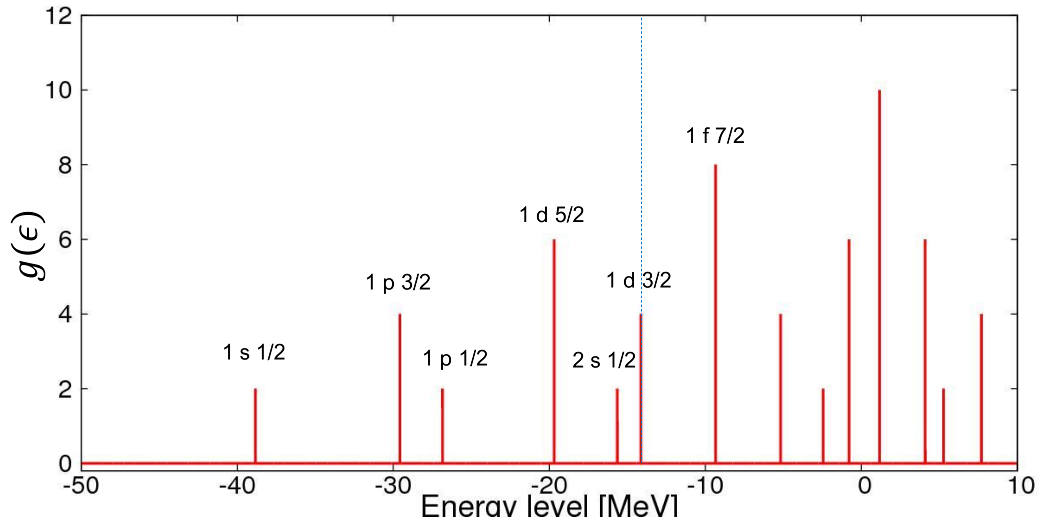
  - Nuclear mass number with the level density

$$A = \int_{-\infty}^{\lambda} g(\epsilon)\, d\epsilon$$

  - Calculation of discrete shell energy

$$E_{shell} = \int_{-\infty}^{\lambda} \epsilon g(\epsilon) d\epsilon = 2\sum_i^{N/2, Z/2} \epsilon_i$$

- An Example of level density
  - The deformed Woods-Saxon potential, $\beta_2 = 0.0$
  - The neutron level density for ^{40}Ca



Formalism

- The smoothed shell energy for deformed Woods-Saxon potential

  - The smoothed level density

$$\tilde{g}(\epsilon) = \frac{2}{\gamma}\int_{-\infty}^{+\infty}\left[\sum_i \delta(\epsilon-\epsilon_i)\right] w\left(\frac{\epsilon-\epsilon_i}{\gamma}\right) L_M^{1/2}\left(\frac{\epsilon-\epsilon_i}{\gamma}\right) d\epsilon$$

  - A weight function : Gaussian

$$w\left(\frac{\epsilon-\epsilon_i}{\gamma}\right) = \frac{1}{\sqrt{\pi}} e^{-\left(\frac{\epsilon-\epsilon_i}{\gamma}\right)^2}$$

  - A generalized Laguerre polynomial

$$L_M^{1/2}\left(\frac{\epsilon-\epsilon_i}{\gamma}\right) = \sum_{n=0}^{M} \frac{H_{2n}\left(\frac{\epsilon-\epsilon_i}{\gamma}\right) H_{2n}(0)}{2^{2n}\cdot(2n)!}$$

  - Number of mass and the chemical potential $\tilde{\lambda}$

$$A = \int_{-\infty}^{\tilde{\lambda}} \tilde{g}(\epsilon)\, d\epsilon$$

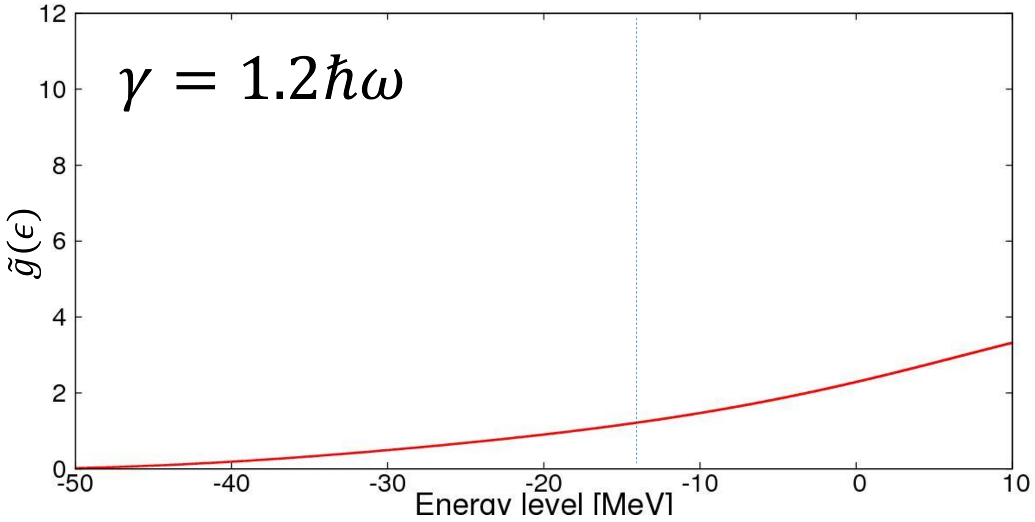
$\tilde{\lambda}$ is determined by iteration procedure for γ

  - Calculation of smoothed shell energy

$$\widetilde{E_{shell}} = \int_{-\infty}^{\tilde{\lambda}} \epsilon\tilde{g}(\epsilon) d\epsilon$$

- An Example of smoothed level density

  - The deformed Woods-Saxon potential, $\beta_2 = 0.0$
  - The smoothed level density for ^{40}Ca with γ



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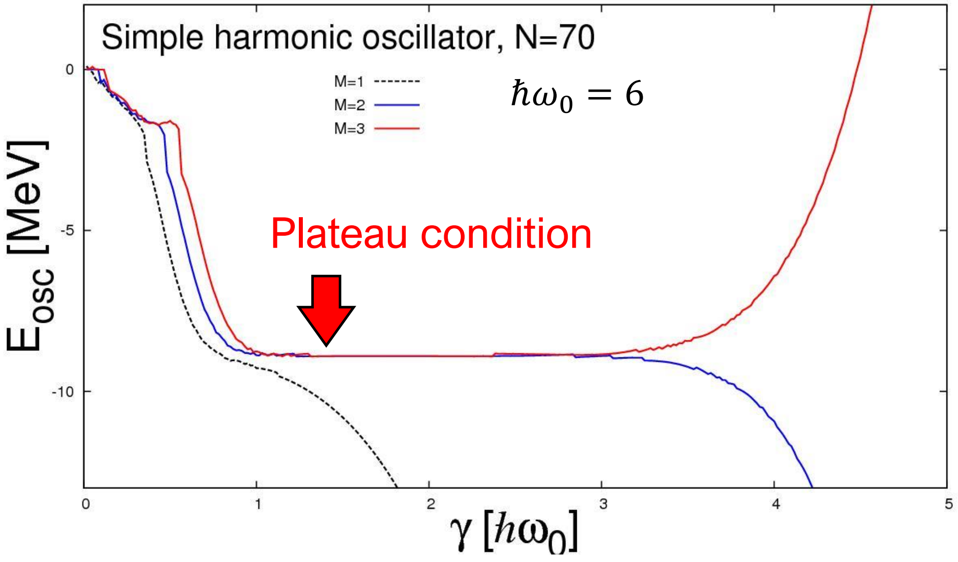
$$\widetilde{E_{shell}} = \int_{-\infty}^{\tilde{\lambda}} \epsilon\tilde{g}(\epsilon) d\epsilon$$

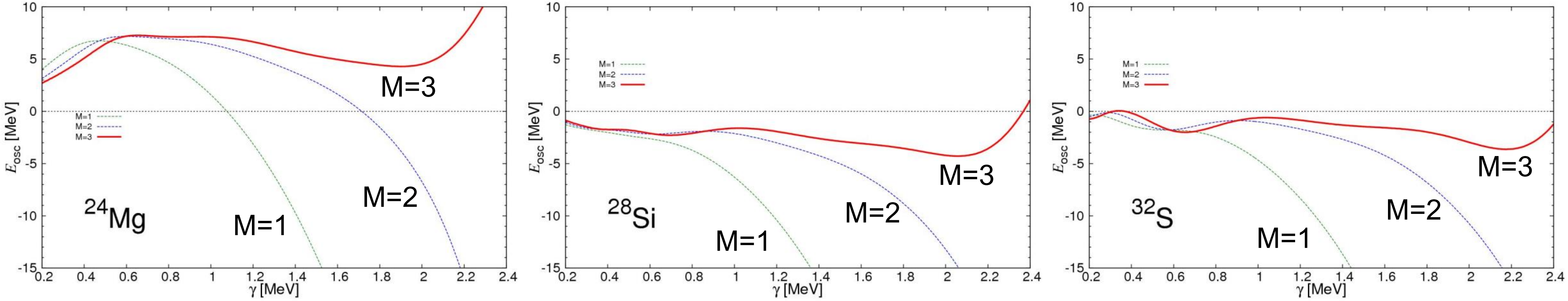
- Plateau condition

$$\frac{\partial\widetilde{E_{shell}}}{\partial\gamma_{\text{plateau}}} = 0\,, \qquad \frac{\partial E_{osc}}{\partial\gamma_{\text{plateau}}} = 0$$



Caculation

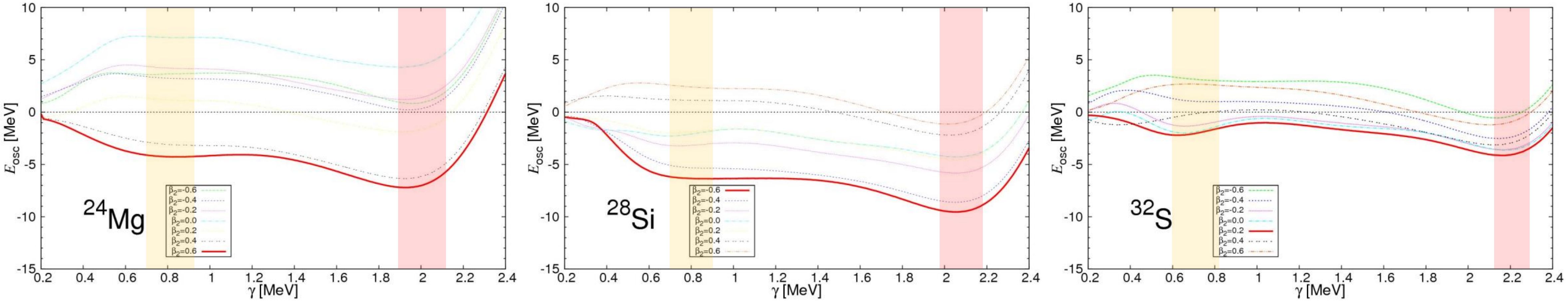
- Calculated nuclei : ^{24}Mg, ^{28}Si and ^{32}S
  - Light and stable nuclei
  - Observed quadrupole deformation β_2
  - N=Z nuclei
- Plateau condition by polynomial order M



  - Plateau conditions appeared in M=3 condition → Fixed order M=3 in our calculations

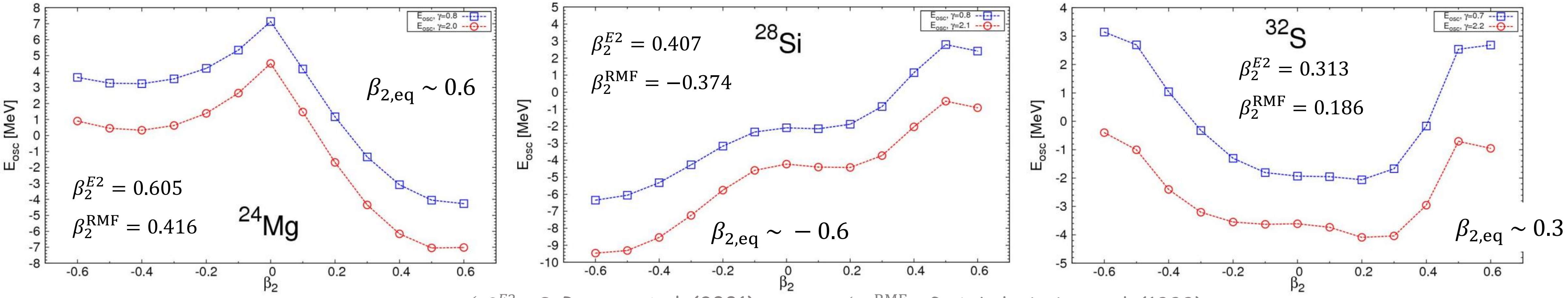
Calculation

- Plateau condition by quadrupole deformation parameter β_2



$\gamma \sim 0.8$ and $\gamma \sim 2.0$ $\gamma \sim 0.8$ and $\gamma \sim 2.1$ $\gamma \sim 0.7$ and $\gamma \sim 2.2$

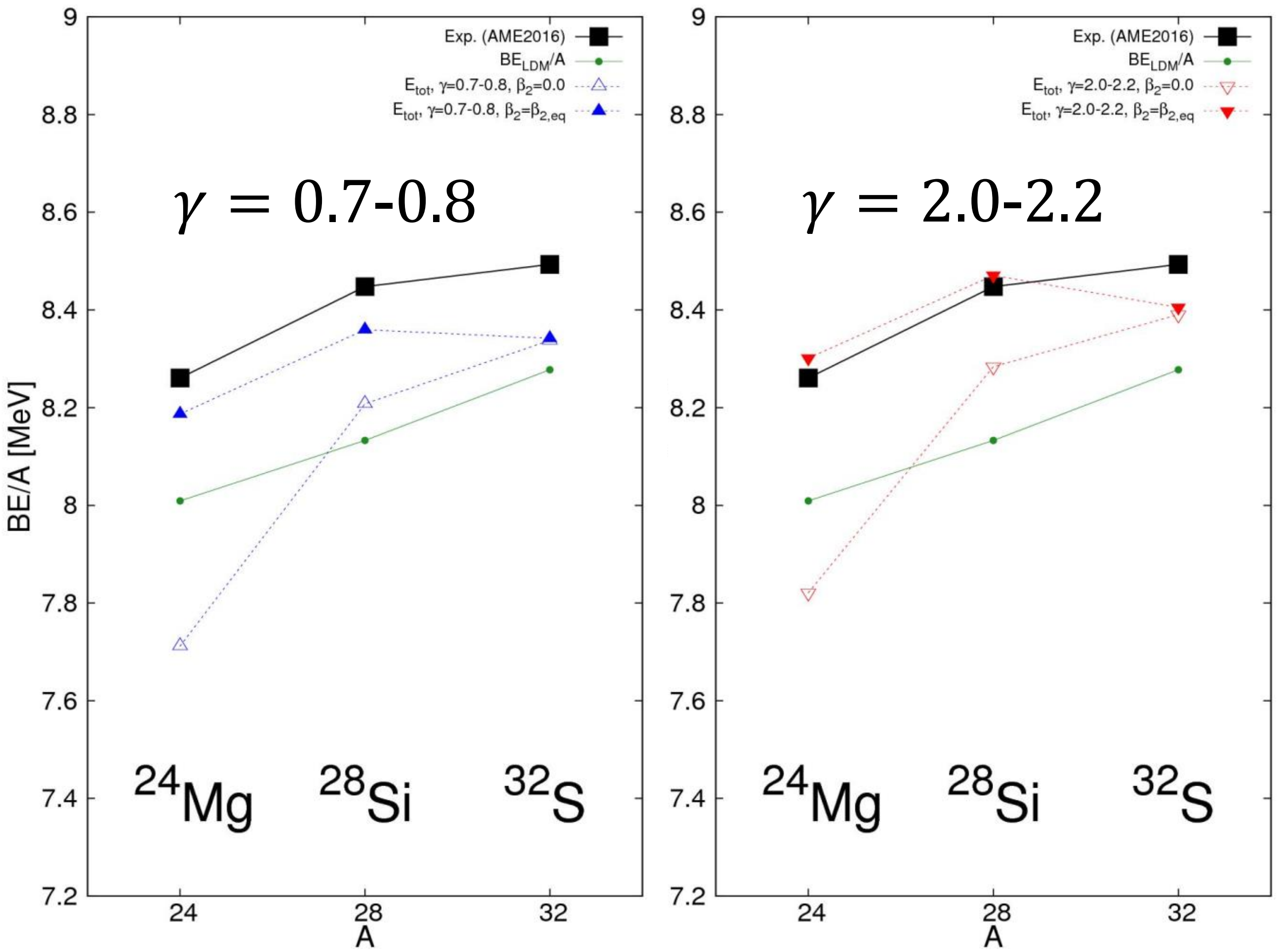
- Calculation results of shell correction energy E_{osc} by quadrupole deformation



✓ β_2^{E2} : S. Raman, et al. (2001) ✓ β_2^{RMF} : G. A. Lalazissis, et al. (1999)

Result

- Binding energy per nucleons



- To consider deformation
  - Improved shell oscillation energy
  - Exact deformation parameter is necessary
  - $\beta_4, \beta_6 \cdots$
- Weight parameter γ
  - γ = 2.0-2.2 is more predictable than 0.7-0.8
  - γ = 2.0-2.2 relate with continuum states
- Discussion
  - More iterative procedure and polynomial order
- Next step : to add pairing effects
  - $E_{tot} = E_{\mathrm{LDM}} + E_{shell} + E_{\mathrm{nn}} + E_{\mathrm{pp}} + E_{\mathrm{np}}$

Summary

- Strutinsky's method is macroscopic-macroscopic approach
- Total binding energy can be explained by Strutinsky's method
- To calculate shell correction energy accurately is important
  - Deformation for nuclear shape and considering equilibrium deformation
  - Plateau condition in the calculation of shell correction energy
  - We found more reasonable weight parameter for light nuclei ($\gamma \sim 2.0$)
- Future planes
  - Improvement our Strutinsky's method : Iterative process by γ and β
  - Pairing effects from microscopic calculation with Deformed BCS (nn, pp, np pairing)

Introduction

Nuclear masses and binding energy (BE)

- Definition between nuclear mass and total binding energy (TBE)

$$m(N,Z) = NM_{\mathrm{n}} + ZM_{\mathrm{H}} - B(N,Z)/c^2$$

- Experimental binding energy per nucleons : $BE_{\mathrm{exp}}/A = B(N,Z)/A$

$$B(N,Z)/(N+Z) = [NM_n + ZM_H - m(N,Z)]c^2/(N+Z)$$

- The semi-empirical mass formula (SEMF, Bethe-Weizsäcker formula, Liquid Drop Modle) for TBE

$$B_{\mathrm{LDM}}(N,Z) = a_{vol}A - a_{surf}A^{2/3} - \frac{1}{2}a_{sym}(N-Z)^2/A - a_C Z^2/A^{1/3} - \delta(A)$$

✓ Bethe-Weizsäcker mass formula are referred by P. Ring and P. Schuck, The Nuclear Many-Body Problem (2004)

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P. Ring & P. Schuck (2004)	15.68	18.56	28.1	0.717	$+34 \cdot A^{-3/4}$	0	$-34 \cdot A^{-3/4}$
NNDC	15.74063	17.61628	23.42742	0.71544	$+12.59898 \cdot A^{-3/4}$	0	$-12.59898 \cdot A^{-3/4}$

- Theoretical binding energy per nucleons : $BE_{\mathrm{LDM}}/A = B_{\mathrm{LDM}}(N,Z)/A$