Isospin-symmetry breaking correction to superallowed $0^+ \to 0^+$ β-decay

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Tests of the Standard Model symmetries

- **Conserved Vector Current (CVC)** hypothesis $\Rightarrow$ vector coupling constant G_V

$$\boxed{ft^{0^+\to 0^+} = \frac{K}{|M_F^0|^2 G_V^2}}$$

$$K = 2\pi^3 \ln 2\, \hbar^7 c^6/(m_e c^2)^5\,, \qquad |M_F^0| = \sqrt{T(T+1) - T_{zi}T_{zf}}$$

$ft^{0^+\to 0^+}$ *: Sherr, Gerhart, 1953; CVC : Feynman, Gell-Mann, 1958*

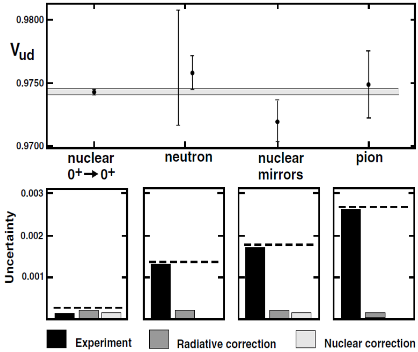
- **Unitarity of the Cabibbo-Kobayashi-Maskawa (CKM)** quark-mixing matrix
  Cabibbo, 1963; Kobayashi, Maskawa, 1973

$$|V_{ud}| = G_V/G_F\,, \qquad G_F/(\hbar c)^3 = 1.1663787(6)\times 10^{-4}\ \mathrm{GeV}^{-2}$$

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 \stackrel{?}{=} 1$$

V_{ud} determination and error budget



Towner, Hardy, RPP73, 2010

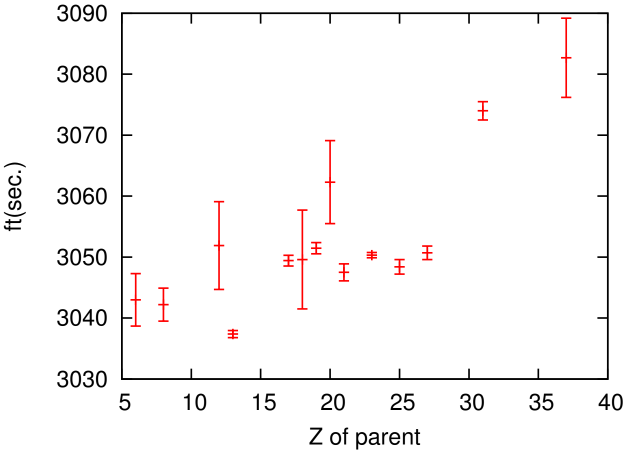
- $0^+ \to 0^+$ (nuclear matrix element)
- neutron decay (GT/F branching ratio)
- Mirror decays between $T = 1/2$ (GT/F branching ratio, nuclear matrix element)
- pion decay (very weak branching ratio 10^{-8})

$ft^{0^+\to 0^+}$-values from the experiment (Q, $t_{1/2}$, BR)

14 best known $T=1$ emitters ($ft^{0^+\to 0^+}$-value known with a precision $\lesssim 0.4\%$):

^{10}C, ^{14}O, ^{22}Mg, ^{26m}Al, ^{34}Cl, ^{34}Ar, ^{38m}K, ^{38}Ca, ^{42}Sc, ^{46}V, ^{50}Mn, ^{54}Co, ^{62}Ga, ^{74}Rb

J.C. Hardy, I.S. Towner, PRC91, 025501 (2015)



Absolute Ft value

Theoretical corrections and the Ft value

$$Ft^{0^+\to 0^+} \equiv ft^{0^+\to 0^+}(1+\delta'_R)(1+\delta_{NS}-\delta_C) = \frac{K}{|M_F^0|^2 G_V^2(1+\Delta_R)}$$

- Radiative corrections

$$\Delta_R^V = (2.361 \pm 0.038)\%$$
$$\delta'_R \sim (1.50 \pm \sim 0.12)\%$$
$$|\delta_{NS}| \lesssim 0.3\%$$

A. Sirlin, W.J. Marciano, R. Zucchini; W. Jaus, G. Rasche

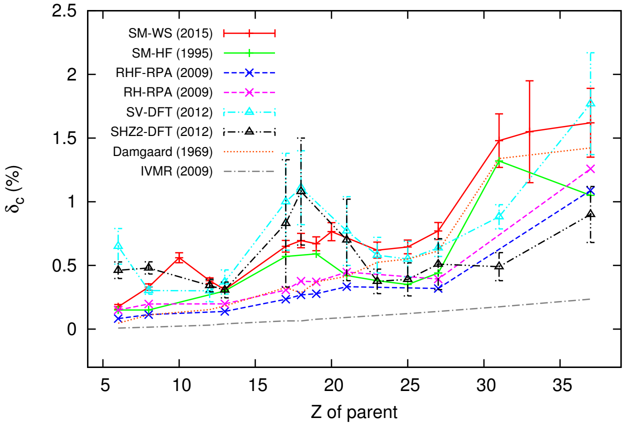
- Nuclear-structure correction

$$|M_F|^2 = |M_F^0|^2(1-\delta_C)$$
$$|M_F^0|^2 = T(T+1) - T_{zi}T_{zf}$$
$$\delta_C \approx 0.1 - 2.0\%$$

δ_C — large ambiguities from various theoretical models

J.C. Hardy, I.S. Towner, PRC91, 025501 (2015)

Present status of δ_C from various models



- Shell Model + WS (*I.S. Towner, J.C. Hardy*)
- Shell Model + HF (*W.E. Ormand, B.A. Brown*)
- JT-projected DFT (*W. Satula et al*)
- RHF-RPA and RH-RPA (*H. Liang et al*)
- Damgaard Model (*J. Damgaard*)
- Isovector Monopole Resonance (*N. Auerbach*)

The CVC test

To test the ability of a model to produce a mutually consistent set of $\mathcal{F}t$ values
Towner, Hardy, PRC82 (2010)

$$\delta_C = 1 + \delta_{NS} - \frac{\overline{\mathcal{F}t}}{ft(1+\delta'_R)}$$

Only SM-WS calculation of Towner, Hardy (2015) has a non-zero (17%) CL

$$\overline{\mathcal{F}t} = 3027.72 \pm 0.72\,\mathrm{s}$$

$|V_{ud}|$ and CKM

$$|V_{ud}| = 0.97417(21)$$

$$|V_{us}| = 0.2253(14) \quad \mathrm{(PDG14)}$$

$$|V_{ub}| = (4.15 \pm 0.49) \times 10^{-3} \quad \mathrm{(PDG14)}$$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.99978(55)$$

Nuclear $0^+ \to 0^+$ β-decay and the Standard Model

Remarks

- Consistency with the CVC does not contrain the absolute *Ft* value !
- New experimental measuremants or theoretical calculations may arrive.

New calculation of Δ_R^V

$$\Delta_R^V = 0.02467(22)$$

C.-Y. Seng, M. Gorchtein, H.H. Patel, M. Ramsey-Musolf, arXiv:1807.10197

(Current value: $\Delta_R^V = 0.02368(38)$ from *W.J. Marciano, A. Sirlin, PRL96, 2006*)

$$|V_{ud}| = 0.97366(15)$$

$$\boxed{|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985(5)}$$

Shell-model study of δ_C

Existing work within the shell model

- Shell-model + WS from I.S. Towner, J.C. Hardy (2002, 2008 with update in 2015)

  Phys. Rev. C91, 025501 (2015) and refs. therein

- Shell-model + HF from W.E. Ormand, B.A. Brown (1995)

  PRL62 (1989); PRC52, 2455 (1995)

The aim of the present work:

- Larger model spaces
- New effective interactions
- Revision of constraints on single-particle potentials
- WS & HF radial wave functions

Shell-model calculation of δ_C

Fermi β-decay matrix element

$$H|\Psi_p\rangle = E_p|\Psi_p\rangle\,, \quad |\Psi_p\rangle = \sum_k c_{kp}|\Phi_k\rangle$$

$$\boxed{M_F = \sum_\alpha \langle \Psi_f|a^\dagger_{\alpha_n} a_{\alpha_p}|\Psi_i\rangle \langle \alpha_n|t_+|\alpha_p\rangle}$$

$$\langle \Psi_f|a^\dagger_{\alpha_n} a_{\alpha_p}|\Psi_i\rangle = \frac{\langle \omega_f J_f|a^\dagger_{\alpha_n} a_{\alpha_p}|\omega_i J_i\rangle}{\sqrt{2J_f+1}} \equiv \rho_\alpha$$

$$\langle \alpha_n|t_+|\alpha_p\rangle = \int_0^\infty R_{\alpha_n}(r) R_{\alpha_p}(r) r^2 dr \equiv \Omega_\alpha$$

Exact isospin operator: $n_{\alpha_n} \neq n_{\alpha_p}$

G.A. Miller, A. Schwenk, PRC78 (2008); PRC80 (2009)

Nuclear-structure correction δ_C

Isospin-symmetry limit

$$M_F^0 = \sum_\alpha \rho_\alpha^T \Omega_\alpha^T = \sqrt{T(T+1) - T_{zi} T_{zf}}\,, \qquad \Omega_\alpha^T = 1$$

Realistic model (Coulomb and charge-symmetry breaking effective nuclear forces)

$$M_F = \sum_\alpha \rho_\alpha \Omega_\alpha$$

$$|M_F|^2 \approx |M_F^0|^2 \Big[1 - \underbrace{\frac{2}{M_F^0} \sum_\alpha \left(\rho_\alpha^T - \rho_\alpha \right)}_{\delta_{IM}} - \underbrace{\frac{2}{M_F^0} \sum_\alpha \rho_\alpha^T (1 - \Omega_\alpha)}_{\delta_{RO}} \Big],$$

$$\delta_C = \delta_{IM} + \delta_{RO}$$

- δ_{IM} is the *isospin-mixing* part
- δ_{RO} is the *radial-overlap* part

Shell-model calculations

^{22}Mg, ^{26m}Al, ^{26}Si, ^{30}S, ^{34}Cl, ^{34}Ar, ^{38m}K, ^{38}Ca, ^{46}V, ^{50}Mn, ^{54}Co, ^{62}Ga, ^{66}As

Model spaces and effective interactions (+ charge-dependence)

- *sd*-shell: USD (Wildenthal, 1984) and USDA/USDB (B.A.Brown, W.A. Richter, 2006)
- *pf*-shell: KB3G (A. Poves et al, 2004) and GXPF1A (M. Honma et al, 2004).
- $pf_{5/2}g_{9/2}$: JUN45 (M. Honma et al, 2009) and RG (F. Nowacki et al, 1996).

`NuShellX@MSU` shell-model code (W.D.M. Rae, B.A. Brown).

Isospin-symmetry breaking corrections

- δ_{IM}: isospin-nonconserving Hamiltonian
- δ_{RO}: spherical WS or HF radial wave functions

Isospin-mixing correction δ_{IM}

Y.H. Lam, N. S., E. Caurier, PRC87 (2013).

Isospin-mixing correction δ_{IM}

- We start with an isospin-symmetry invariant shell-model Hamiltonian

$$\hat{H}\Psi_{TT_z} \equiv (\hat{H}_0 + \hat{V})\Psi_{TT_z} = E_T \Psi_{TT_z}, \quad \Psi_{TT_z} = \sum_k a_{T\,k} \Phi_{TT_z\,k}$$

Y.H. Lam, N. S., E. Caurier, PRC87 (2013).

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- We consider an isospin-symmetry non-conserving term

$$\hat{V}_{INC} = \underbrace{\lambda_C \hat{V}_C}_{Coulomb} + \underbrace{\lambda_1 \hat{V}^{(1)}}_{CSB} + \underbrace{\lambda_2 \hat{V}^{(2)}}_{CIB} + \underbrace{\hat{H}_0^{IV}}_{\sum_\alpha(\varepsilon^p_\alpha - \varepsilon^n_\alpha)}$$

Y.H. Lam, N. S., E. Caurier, PRC87 (2013).

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- Within perturbation theory:

$$\langle \Psi_{TT_z} | \hat{V}_{INC} | \Psi_{TT_z} \rangle = E^{(0)}(\alpha, T) + E^{(1)}(\alpha, T) T_z + E^{(2)}(\alpha, T) \left[3T_z^2 - T(T+1) \right]$$

Y.H. Lam, N. S., E. Caurier, PRC87 (2013).

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- Fit to experimental coefficients of the Isobaric Mass Multiplet Equation (IMME):

$$M(\alpha,T,T_z) = a(\alpha,T) + b(\alpha,T)T_z + c(\alpha,T)T_z^2\,,$$

Y.H. Lam, N. S., E. Caurier, PRC87 (2013).

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- Fit to experimental coefficients of the Isobaric Mass Multiplet Equation (IMME):

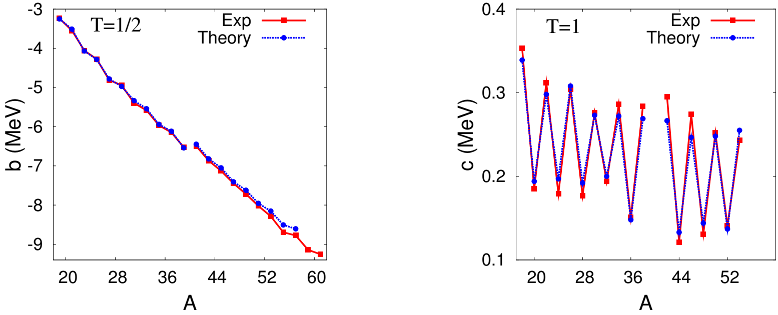
$$M(\alpha,T,T_z) = a(\alpha,T) + b(\alpha,T)T_z + c(\alpha,T)T_z^2\,,$$

- Diagonalization of the INC Hamiltonian $\hat{H}_{INC} = \hat{H} + \hat{V}_{INC}$

$$\hat{H}_{INC}\Psi = E\Psi$$

Y.H. Lam, N. S., E. Caurier, PRC87 (2013).

b and *c*-coefficients in *sd* and *pf* shell

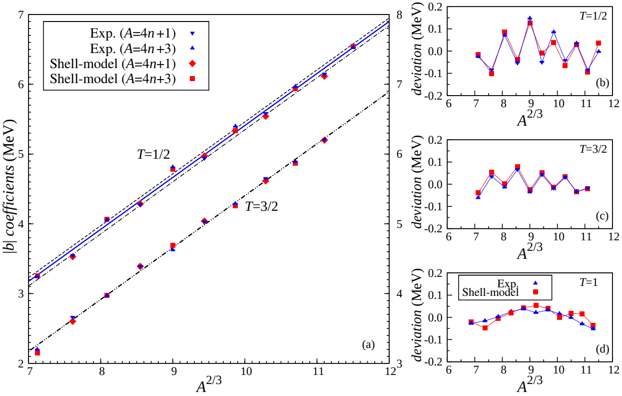


For *sd*-shell:

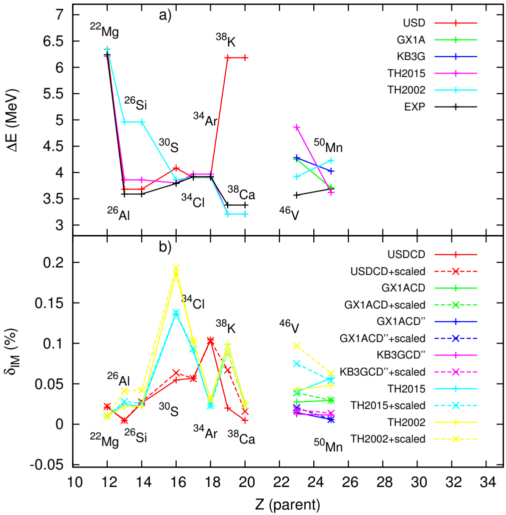
- *b* coefficients ($v_{pp} - v_{nn}$); 81 data points ($T = 1/2, 1, 3/2, 2$); rms $\approx$ 32 keV
- *c* coefficients ($v_{pp} + v_{nn} - 2v_{pn}^{T=1}$); 51 data points ($T_z = 1, 3/2, 2$); rms $\approx$ 10 keV

Staggering of *b*-coefficients of *sd*-shell nuclei

Y. H. Lam, N. S., E.Caurier, PRC87, 2013



Isospin-mixing correction δ_{IM}



L. Xayavong, Ph.D. thesis, U. de Bordeaux (2016)

$$\delta_{IM} \sim \frac{\langle 0_1^+ | V_{INC} | 0_2^+ \rangle^2}{\Delta E^2}$$

$$\delta_{IM} = \delta_{IM}^{th} \left(\frac{\Delta E^{th}}{\Delta E^{exp}} \right)^2$$

Strong dependence on V_{INC}

II. Radial overlap correction

Beyond the closure approximation

$$\delta_{RO} = \frac{2}{M_F^0} \sum_{\alpha} \langle \Psi_f | a^\dagger_{\alpha_n} a_{\alpha_p} | \Psi_i \rangle^T (1 - \Omega_\alpha)$$

$$\Downarrow$$

$$\delta_{RO} = \frac{2}{M_F^0} \sum_{\alpha,\pi} \langle \Psi_f | a^\dagger_{\alpha_n} | \pi \rangle^T \langle \pi | a_{\alpha_p} | \Psi_i \rangle^T (1 - \Omega^\pi_\alpha)$$

Two ingredients:

- *Spectroscopic amplitudes* (from the shell-model):

$$\langle \Psi_f | a^\dagger_{\alpha_n} | \pi \rangle = \frac{\langle \Psi_f || a^\dagger_{\alpha_n} || \pi \rangle}{\sqrt{2J_f + 1}}$$

- *Radial-overlap integrals* (from a realistic single-particle potential)

$$\Omega^\pi_\alpha = \int_0^\infty R^\pi_{\alpha_n}(r) R^\pi_{\alpha_p}(r) r^2 dr$$

Woods-Saxon potential

Parameterization

$$V_{WS}(r) = -V_0 f(r, r_0, a) - V_{so}\frac{1}{r}\frac{d}{dr}f(r, r_s, a_s)\vec{l}\cdot\vec{\sigma} + V_C(r)$$

- A. Bohr, B.R. Mottelson modified (BM$_m$) from *Nuclear Structure, Vol. I.*
- N. Schwierz, I. Wiedenhöver, A. Volya (SWV) from *nucl-th:0709.3525*

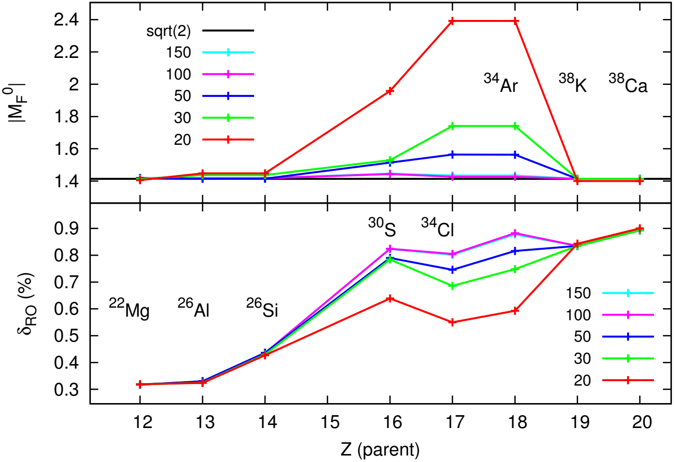
Adjustment of V_0, r_0, a

- $a = 0.662 \pm 0.010$ fm
- V_0 and r_0 are adjusted to reproduce experimental **nucleon separation energies** and **charge radii**

$$\psi(r) \to \exp\left(-\frac{\sqrt{2m|\epsilon|}r}{\hbar}\right)$$

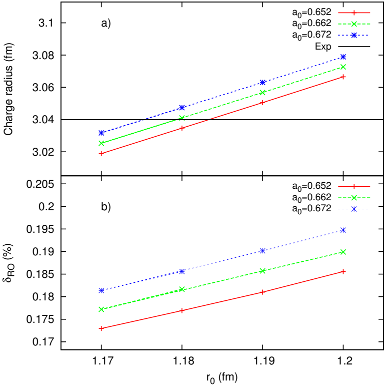
- $\langle r^2\rangle_{ch} = \frac{1}{Z}\sum_{\pi\alpha}\langle\alpha|r^2|\alpha\rangle^{\pi}|\langle\Psi_i||a^{\dagger}_{\alpha}||\pi\rangle|^2 + \frac{3}{2}\left(a_p^2 - b^2/A\right)$

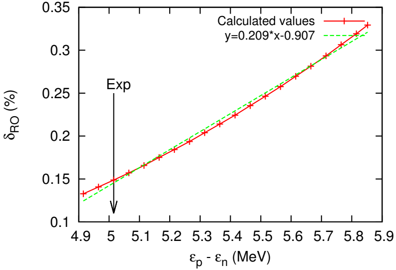
Convergence of M_F and δ_{RO}



- Convergence of δ_{RO} is faster than that of M_F.
- $N_\pi = 100$.

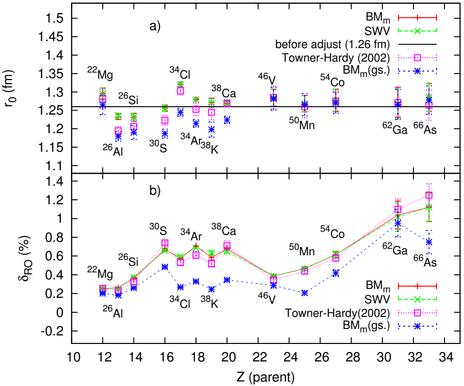
Sensitivity to WS potential parameters: ^{26}Al $\longrightarrow$ ^{26}Mg





Systematic errors (from radii and *a*) and statistical errors (use of various interactions)

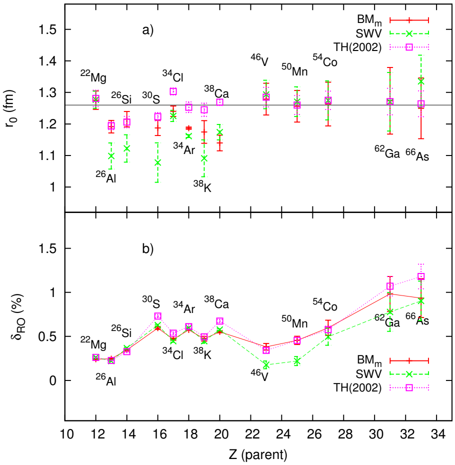
Results δ_{RO} from adjustement of V_0



- δ_{RO} increases when intermediate states are taken into account.
- Dependence of parameterization is removed.
- Uncertainty on δ_{RO} comes mainly from the experimental uncertainty on the charge radii.

L. Xayavong, Ph.D. thesis, U. of Bordeaux (2016); L. Xayavong, N. S., PRC97 (2018).

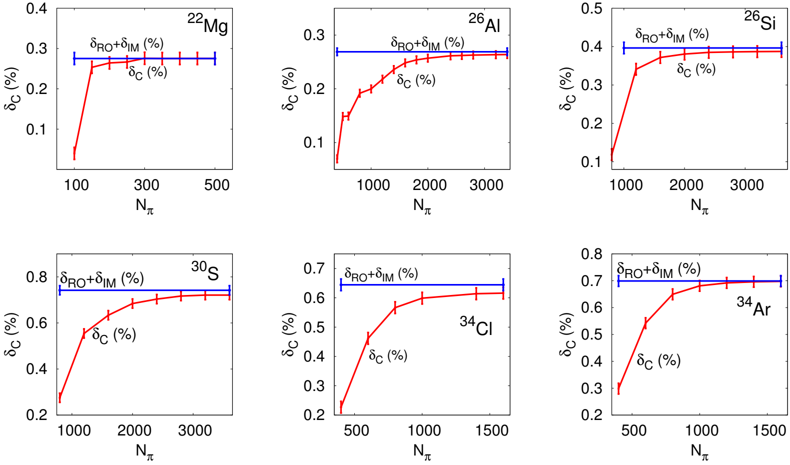
Results δ_{RO} from adjustment of V_g



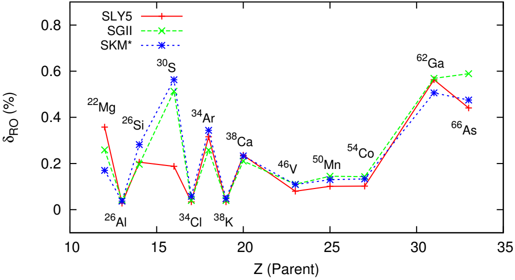
- Dependence of parameterization is observed.
- TH2002: average from the fits with V_0, V_g and V_h

L. Xayavong, Ph.D. thesis, University of Bordeaux (2016).

Test of the separation ansatz for $\delta_C = \delta_{IM} + \delta_{RO}$



δ_{RO} from the shell model with Skyrme-HF wave functions



- SGII (*N. Van Giai, H. Sagawa, NPA371 (1981)*)
- SkM* (*J. Bartel et al, NPA386 (1982)*)
- Sly5 (*P. Chabanat et al, NPA635 (1998)*)

Slater approximation for the Coulomb exchange term is used.
Spherical HF code `Lenteur` (*K. Bennaceur, IPN Lyon*).

δ_{RO} from the shell model with Skyrme-HF wave functions

Optimization

- Energy-dependent *local equivalent potential*

  C.B.Dover, N. Van Giai, NPA190 (1972):

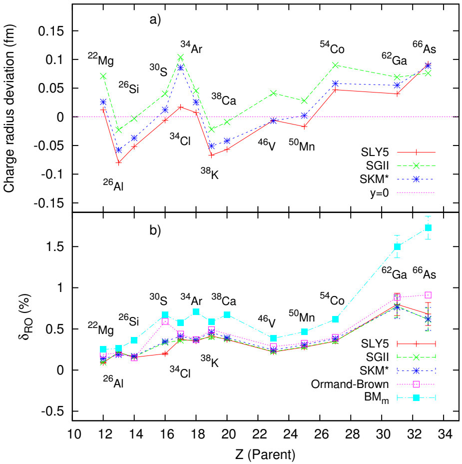
$$V^{LE}(r,\epsilon_\alpha) = V^0(r,\epsilon_\alpha) + V^{so}(r)\langle\vec{l}\cdot\vec{s}\rangle + V_C(r)$$

  for

$$R_\alpha(r) = \sqrt{\frac{m^*(r)}{m}}\,R_\alpha^{LE}(r).$$

- Adjustment of the central term $V^0(r,\epsilon_\alpha)$ by a scaling factor to match the experimental proton and neutron separation energies

δ_{RO} from an adjusted HF potential



δ_{RO} from different forces are consistent

L. Xayavong, N.S., M. Bender, K. Bennaceur, Acta Phys. Pol. B10, 285 (2017).

HF potential: comments

- Test of the Slater approximation for the exchange Coulomb term: valid
- Center-of-mass correction (two-body terms): not important
- Non-physical isospin symmetry breaking in $N \neq Z$ nuclei: negligible
- Difference between WS and HF results: from the interior region of the potential

Gogny-HF code `Ghost`, *K. Bennaceur* .

Ft values for 10 *sd* and *pf*-shell emitters ($\nu = 9$): preliminary results

$$\chi^2/\nu = \frac{1}{N-1}\sum_{i=1}^{N}\frac{\left(Ft_i - \overline{Ft}\right)^2}{\sigma_i^2}$$

If $\chi^2/\nu > 1$, then define a scaling factor $S = \sqrt{\chi^2/\nu}$ for σ

Model	$\overline{Ft}$	χ^2/ν
WS	3074.4(10)	2.7
WS-surf	3076.8(10)	2.3
HF	3078.8(11)	3.3
TH2002	3075.3(10)	2.4
TH2008	3073.0(8)	0.5

Summary and Perspectives

- New shell-model study of δ_C for some *sd* and *pf* shell $0^+ \to 0^+$ emitters.
- Calculations are under experimental constraints, the proposed procedure for δ_{RO} removes uncertainties due to different parameterizations. Results are consistent with previous studies and may lead to reconsideration of the average $\mathcal{F}t$ and larger uncertainties.
- Open problems to be addressed :
  - Exact isospin operator
  - Large model spaces
  - Inclusion of core orbitals
  - Extension of the studies to other emitters
- More work on the implementation of the HF wave functions.