Examination of consistency of QRPA approach to double-beta decay

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1. Brief review of neutrino mass and ββ decays
2. Gamow-Teller transition strength
3. Self-check of applicability of QRPA
4. Result of my calculation
5. Summary

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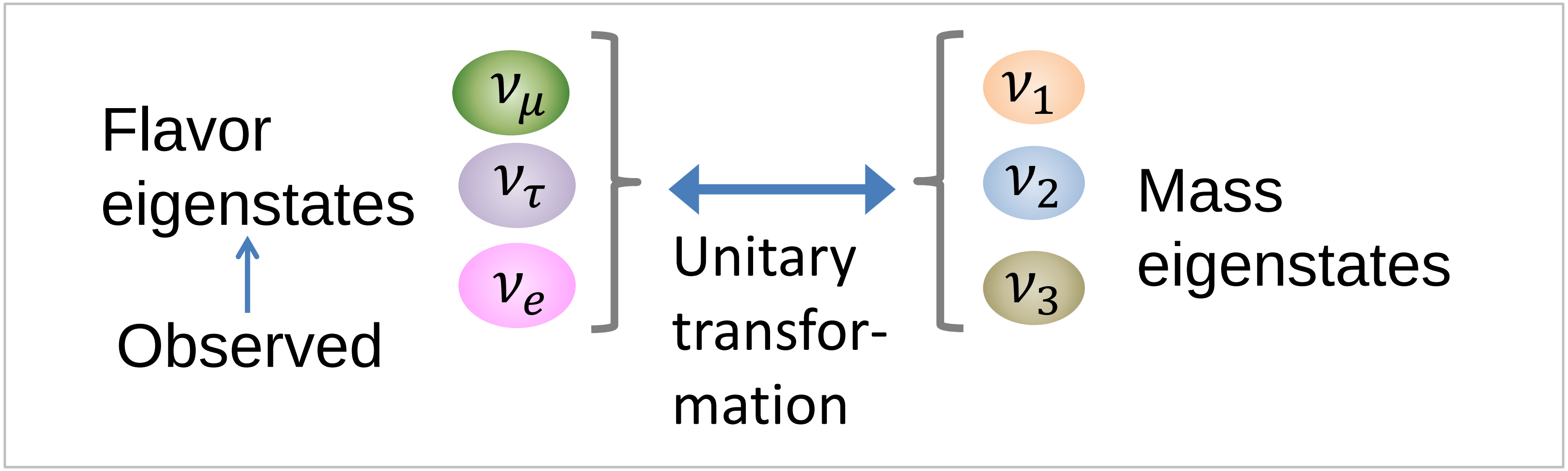
Three generations of neutrino



Neutrino oscillation:
The flavor composition of neutrino changes in flight.

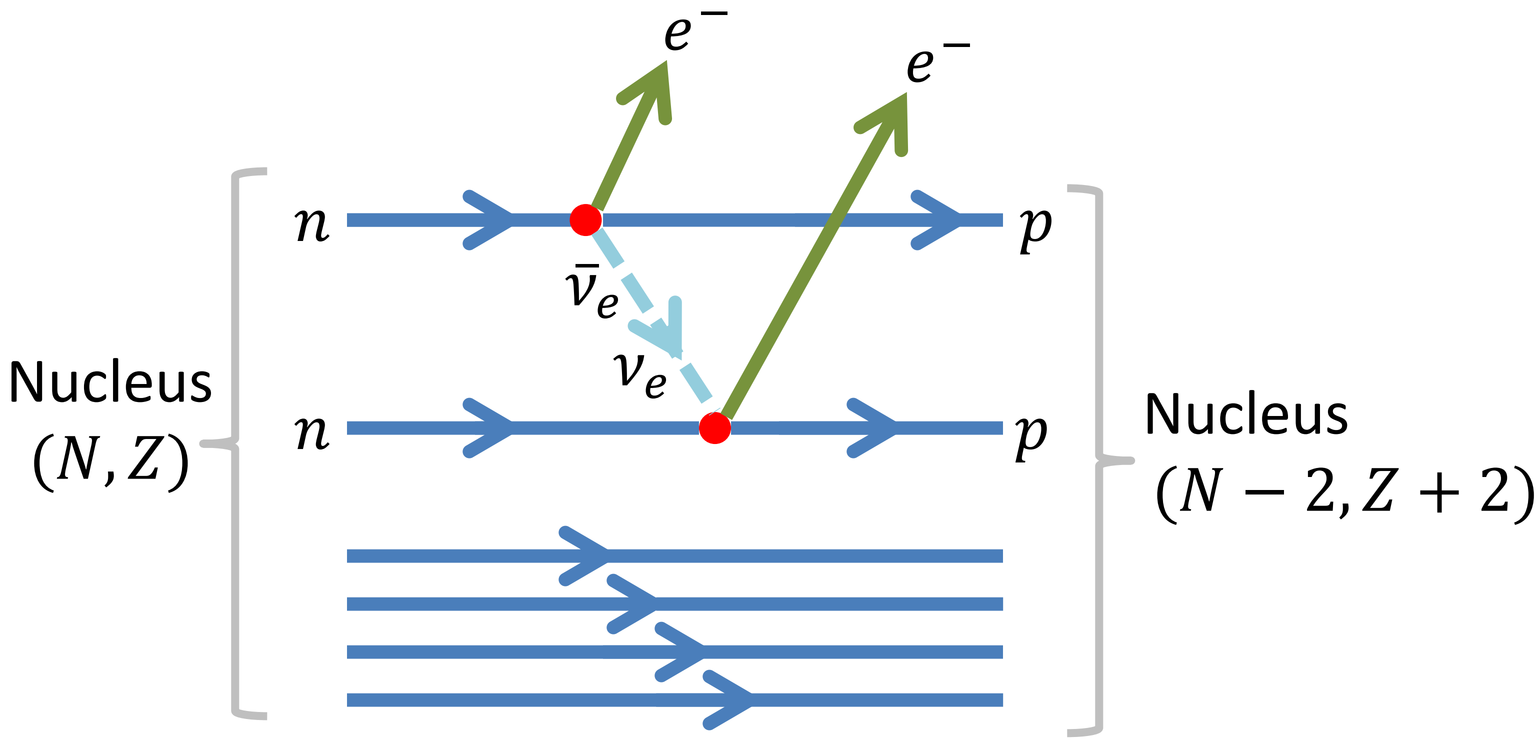
Probability of flavor change $\alpha \rightarrow \beta$

$$\left| \sum_{i=1,2,3} U_{\alpha i}^{*} U_{\beta i} e^{-\frac{i m_i^2 L}{2E}} \right|^2 \neq 0, \text{for } \alpha \neq \beta$$

m_i : eigen mass
L: flight distance
E: energy

$\Longrightarrow$ Neutrino is massive.

Neutrinoless double-β decay



If the neutrino is a Majorana particle ($\nu_e = \bar{\nu}_e$), this decay occurs.

Principle to determine effective neutrino mass

$$\langle m_\nu \rangle = \left| \sum_{i=1,2,3} U_{ei}^2 m_i \right|$$

$$1/T_{0\nu}(\mathrm{g.s.} \rightarrow \mathrm{g.s.}) = g_A^4 \left| M^{(0\nu)} \right|^2 G_{01} \left(\frac{\langle m_\nu \rangle}{m_e} \right)^2$$

Half-life

Nuclear matrix element

Phase -space factor

Effective ν mass

Experimental measurement

Theoretical calculation

Nuclear matrix element

$$M^{(0\nu)} = \sum_b \sum_{pp'} \sum_{nn'} \langle pp'|V(r_{12},E_b)|nn'\rangle \langle 0_f^+|c_{p'}^\dagger c_{n'}|b\rangle\langle b|c_p^\dagger c_n|0_i^+\rangle$$

Final state,
ground state of
nucleus(N-2,Z+2)

Intermediate state,
nucleus (N-1,Z+1)

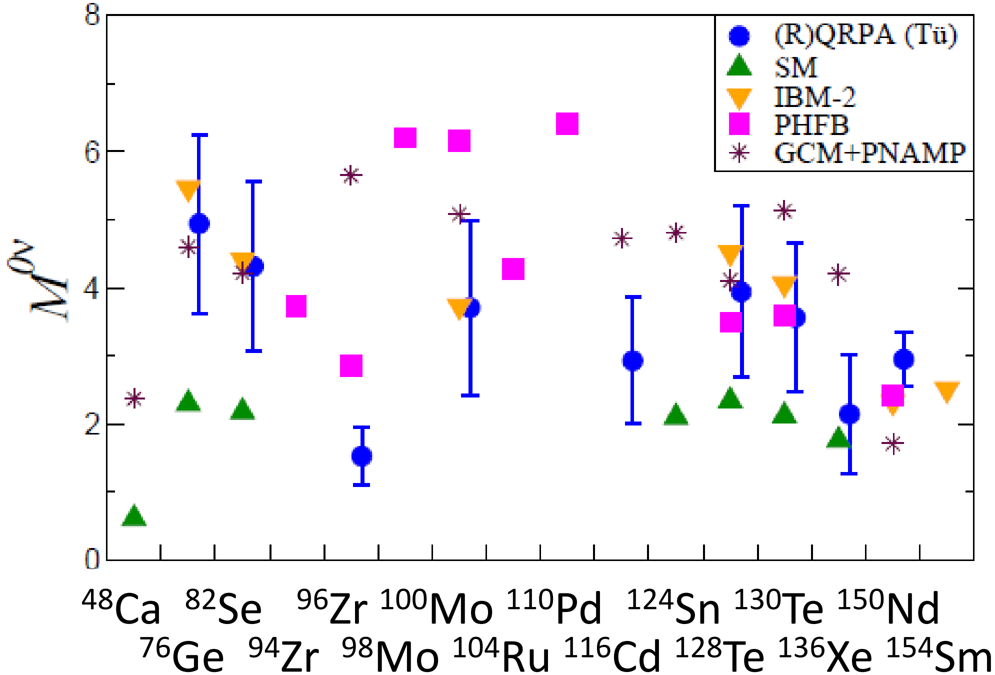
Initial state,
ground state of
nucleus (N,Z)

The decay operator used in my calculation is

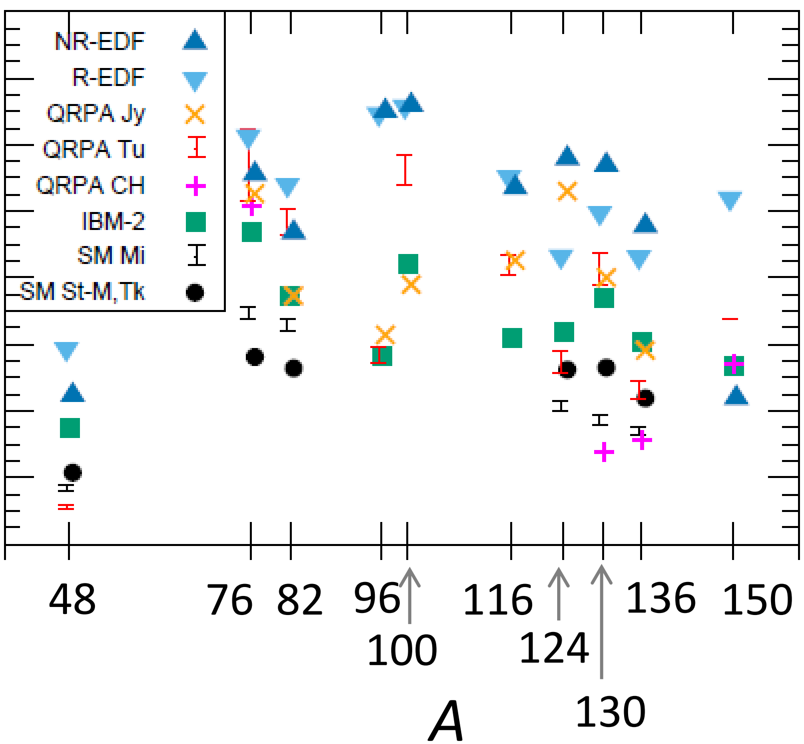
$$V(r_{12},E_b) \cong h_+(r_{12})\{-\boldsymbol{\sigma}(1)\cdot\boldsymbol{\sigma}(2) + g_V^2/g_A^2\}\,\tau^+(1)\tau^+(2)$$

Double-Gamow-Teller + Double-Fermi

Status of 0νββ nuclear matrix element $M^{0\nu}$



A. Feassler, J. Phys.: Conf. Ser. **337**, 012065 (2012)



J. Engel and J. Menéndez, Rep. Prog. Phys. 80 (2017) 046301

Check of transition density using Gamow-Teller transition

Gamow-Teller transition matrix element

$$\langle F|\boldsymbol{\sigma\tau}|I\rangle = \sum_{np} \langle p|\boldsymbol{\sigma\tau}|n\rangle \, \langle F|c_p^\dagger c_n|I\rangle$$

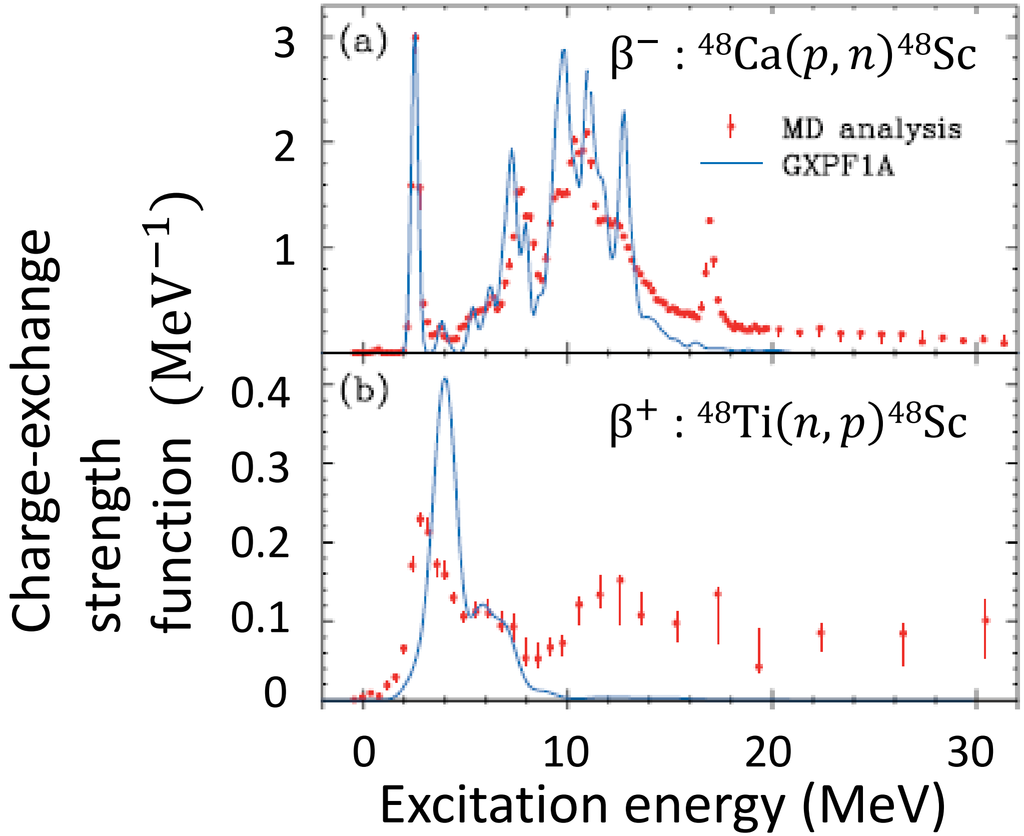
Proton Neutron

Initial state of nucleus

Final state of nucleus

0vββ nuclear matrix element

$$M^{(0\nu)} = \sum_b \sum_{pp'} \sum_{nn'} \langle pp'|V(r_{12}, E_b)|nn'\rangle \langle 0_{\mathrm{f}}^+|c_{p'}^\dagger c_{n'}|b\rangle\langle b|c_p^\dagger c_n\ |0_{\mathrm{i}}^+\rangle$$



Isolated points with error bars: exp. data by K. Yako et al. PRL **103**, 012503 (2009)

Solid line: shell-model calculation by Horoi et al.

My idea

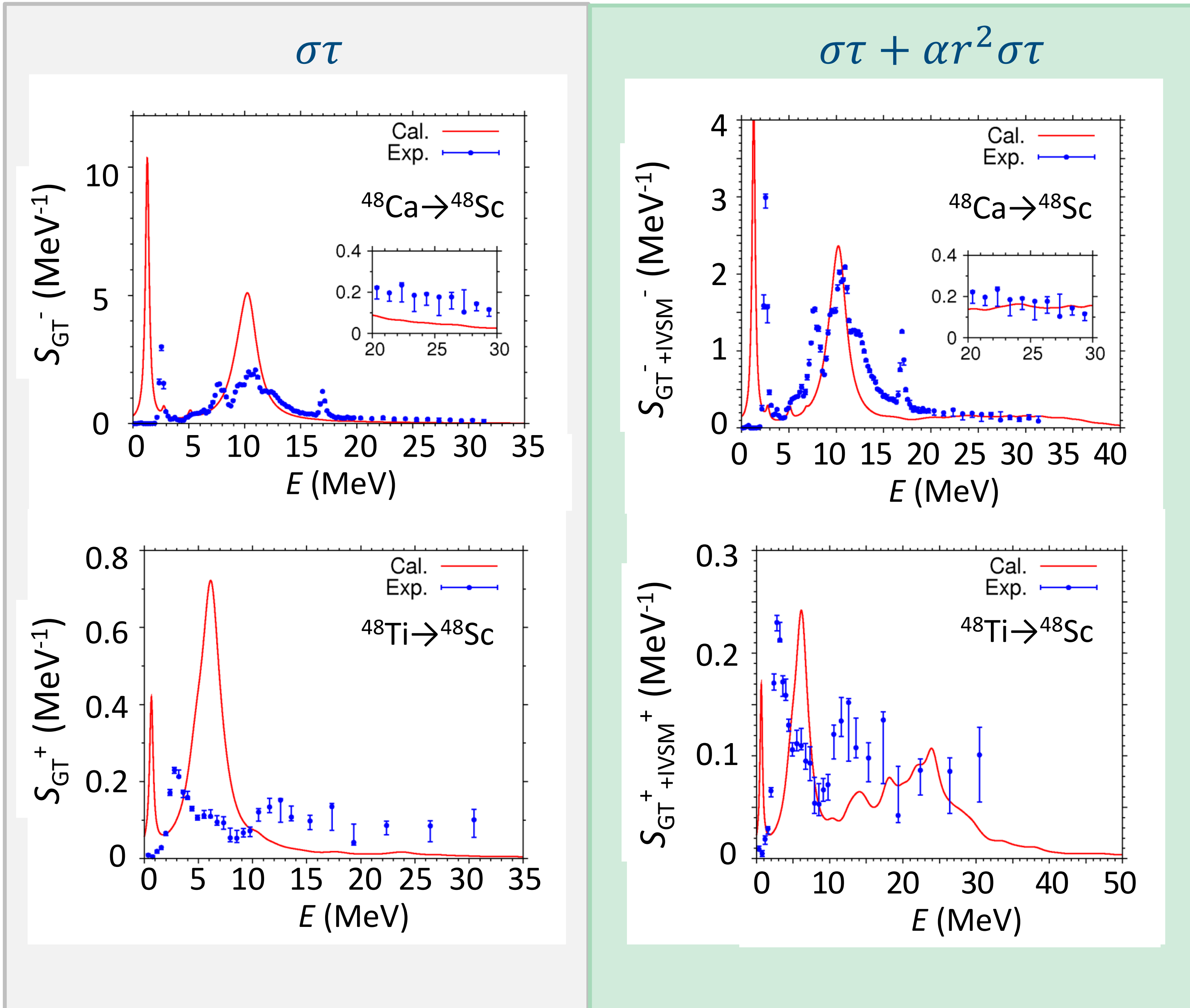
The transition operator is
$$\sigma\tau + \alpha r^2 \sigma\tau.$$

α: phenomenological constant determined so as to reproduce the long low strength distribution in the high-energy region

$\alpha = -0.03\ \mathrm{fm}^{-2}$, $^{48}Ca \rightarrow {}^{48}Sc$
$-0.0253\ \mathrm{fm}^{-2}$, $^{48}Ti \rightarrow {}^{48}Sc$

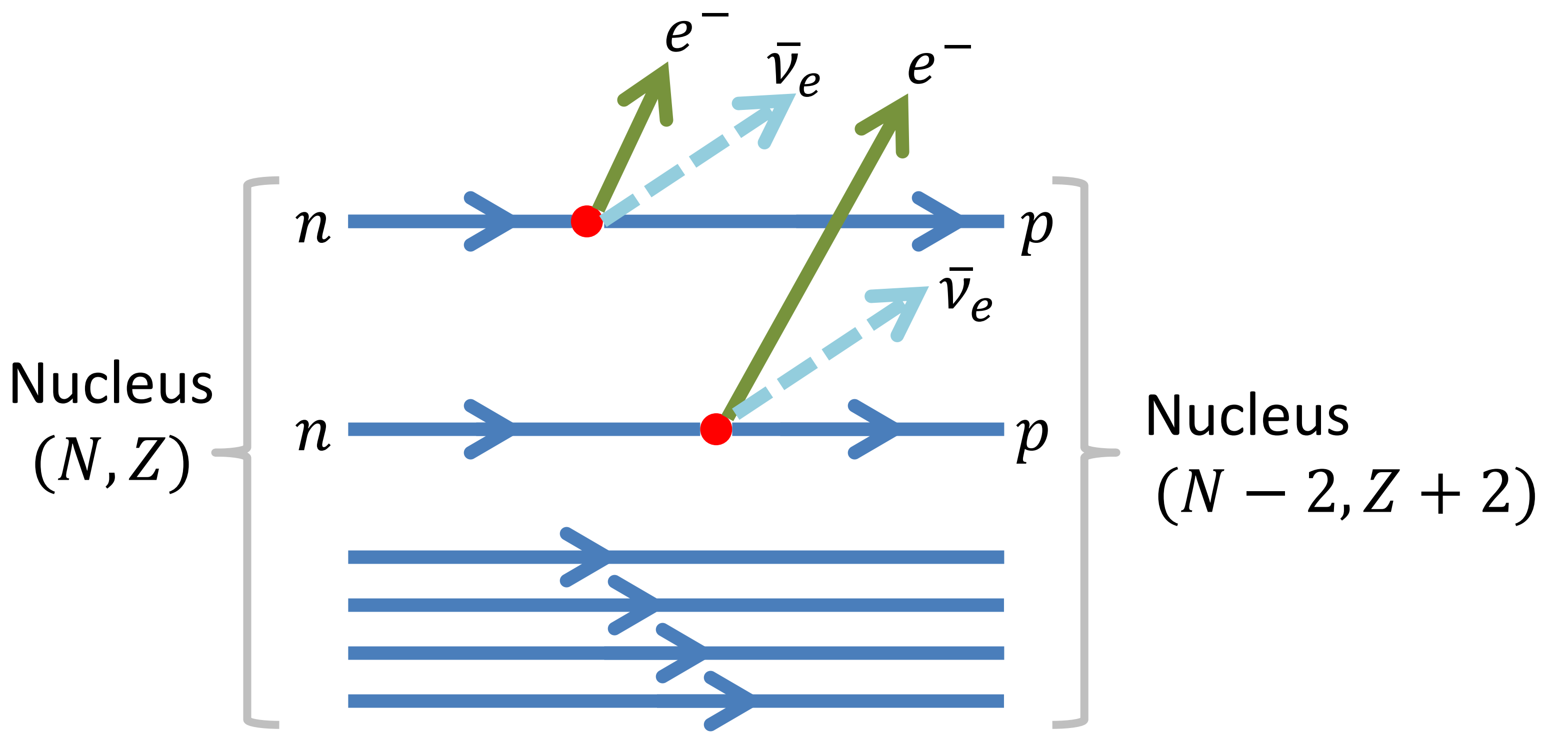
The excitation by $r^2\sigma\tau$ is called the **isovector spin monopole** mode.

Interaction : Skyrme SkM* and contact volume pairing interactions



στ
στ + αr²στ
Cal.
Exp.
48Ca→48Sc
SGT- (MeV-1)
E (MeV)
Cal.
Exp.
48Ti→48Sc
SGT+ (MeV-1)
E (MeV)
Cal.
Exp.
48Ca→48Sc
SGT-+IVSM- (MeV-1)
E (MeV)
Cal.
Exp.
48Ti→48Sc
SGT++IVSM+ (MeV-1)
E (MeV)

Self-check of applicability of QRPA using two-neutrino double-β decay



The intermediate states are virtual states.

Nuclear matrix element of 2vββ decay (GT component)

$$M_{\mathrm{GT}}^{(2\nu)} = 3\langle F_{\mathrm{exa}}|\tau^{-}\sigma_{K=0}\frac{m_e c^2}{H-\bar{M}}\tau^{-}\sigma_{K=0}|I_{\mathrm{exa}}\rangle,$$

$\bar{M} = (M_I + M_F)/2$

: mean value of masses of initial and final nuclei

If the QRPA is a good approximation, two equations are derived which are close to each other:

$$M_{\mathrm{GT}}^{(2\nu)} \cong 3 \sum_{a_I^{K=0}, a_F^{K=0}} \frac{m_e c^2}{E_{aI}^{K=0} - \bar{M}} \times \langle F|\tau^{-}\sigma_{K=0}\left|a_F^{K=0}\right\rangle\left\langle a_F^{K=0}\middle|a_I^{K=0}\right\rangle\left\langle a_I^{K=0}\right|\tau^{-}\sigma_{K=0}|I\rangle,$$

$E_{aI}^{K=0}$: intermediate-state energy obtained using the **initial** state

$\left|a_I^{K=0}\right\rangle$: pnQRPA state obtained from the **initial** state

$\left|a_F^{K=0}\right\rangle$: pnQRPA state obtained from the **final** state

If the QRPA is a good approximation, two equations are derived which are close to each other:

$$M_{\rm GT}^{(2\nu)} \cong 3 \sum_{a_I^{K=0}, a_F^{K=0}} \frac{m_e c^2}{E_{aI}^{K=0} - \bar{M}} \times \langle F | \tau^- \sigma_{K=0} \left| a_F^{K=0} \right\rangle \langle a_F^{K=0} | a_I^{K=0} \rangle \langle a_I^{K=0} | \tau^- \sigma_{K=0} | I \rangle,$$

$E_{aI}^{K=0}$: intermediate-state energy obtained using the **initial** state

$\left| a_I^{K=0} \right\rangle$: pnQRPA state obtained from the **initial** state

$\left| a_F^{K=0} \right\rangle$: pnQRPA state obtained from the **final** state

Another eq. $\cong$ the same equation except that $E_{aF}^{K=0}$ is used.

$^{48}Ca \rightarrow {}^{48}Ti$

Intermediate-state Energy	$M_{\mathrm{GT}}^{(2\nu)}$	$M_{\mathrm{F}}^{(2\nu)}$	$M^{(2\nu)}$
$E_{aI}^{K=0}$	**0.124**	**–0.0033**	**0.138**
$E_{aF}^{K=0}$	**0.112**	**–0.0052**	**0.133**

The QRPA is a good approximation for $^{48}Ca \rightarrow {}^{48}Ti$.

Note

$M_{\mathrm{F}}^{(2\nu)}$ is negligible because of the isospin symmetry;

the strength of the isovector pn pairing interaction
= the mean value of those of the pp and nn pairing interactions

Presentation of my result

- Nuclear matrix element $M^{(0\nu)}$

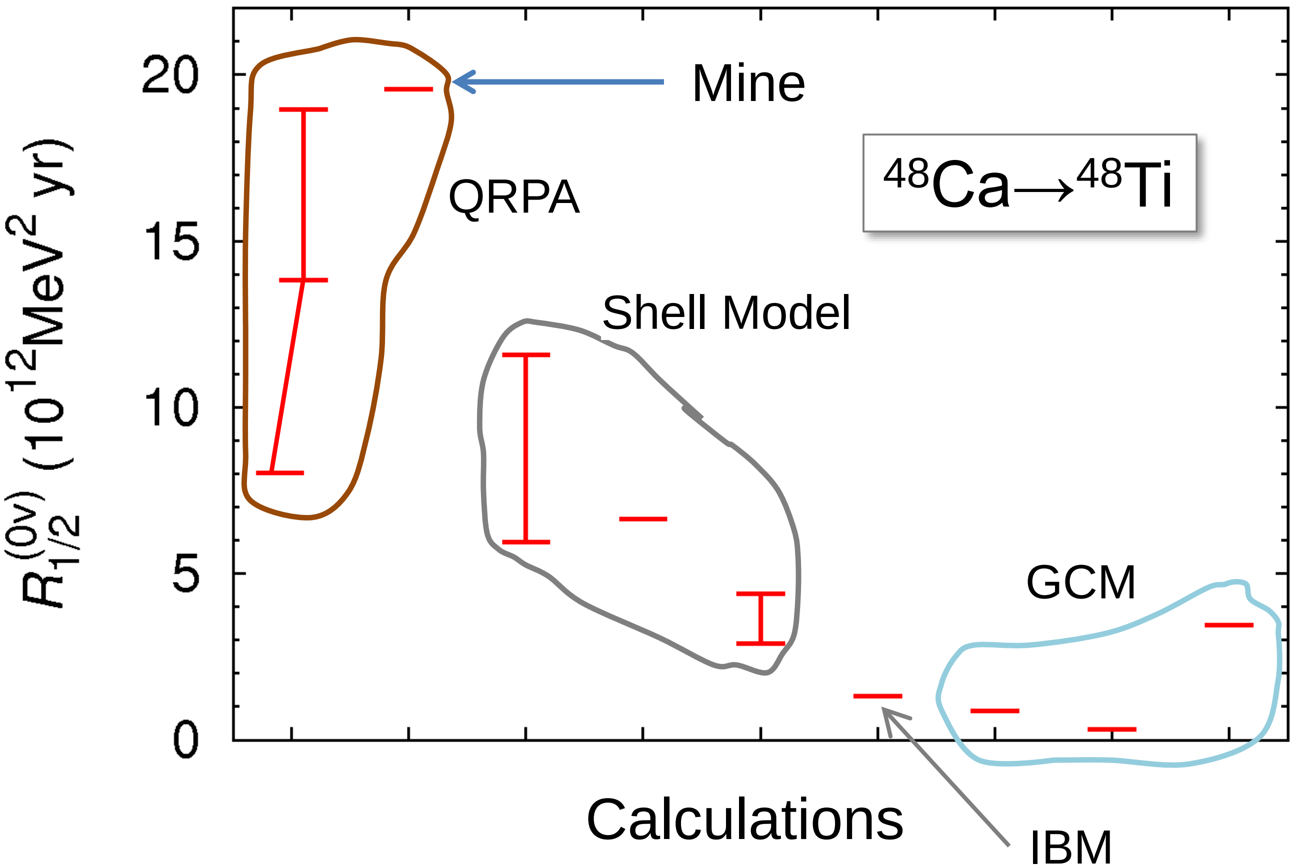
  If different effective g_A's are used, those $M^{(0\nu)}$'s are necessarily different. I use effective g_A ~0.5. Majority of effective g_A's are 1.0–1.27.

- Reduced half-life $R_{1/2}^{(0\nu)}$

$$T_{1/2}^{(0\nu)} = \frac{R_{1/2}^{(0\nu)}}{\langle m_\nu \rangle^2}$$

Useful for comparison of calculations using different g_A.

$$R_{1/2}^{(0\nu)} = \left(G_{0\nu} g_A^4\right)^{-1} \left|M^{(0\nu)}\right|^{-2} m_e^2$$



For references, see J. T. PRC **97**, 034304 (2018).

If $\langle m_\nu \rangle$ is 10 meV,

$T_{1/2}^{0\nu} \cong$ 2x10^{29}yr (my prediction).

Cf. the age of the universe = (12–14)x10^{9} yr.

Summary

1. Review was presented on the status of the study of neutrino mass.
2. The data of Gamow-Teller strength was reproduced by a novel idea. The transition density of my calculation has been checked qualitatively.
3. A self-check was made on the applicability of the QRPA using the $2\nu\beta\beta$ nuclear matrix elements; the result is satisfactory.
4. The reduced half-life has been shown. If $\langle m_{\nu} \rangle$=10 meV, the half-life of ^{48}Ca to the 0vββ decay would be $\sim 10^{27} - 10^{29}$ yr (range by several independent calculations.).

References
J.T. PRC, **86**, 021301(R) (2012);
87, 024316 (2013);
91, 034318 (2015);
93, 024317 (2016);
97, 034304 (2018)

Supplement

Procedure to determine three parameters

1) The strength of the isovector pn pairing interaction is equal to the mean value of the pp and nn pairing interactions.

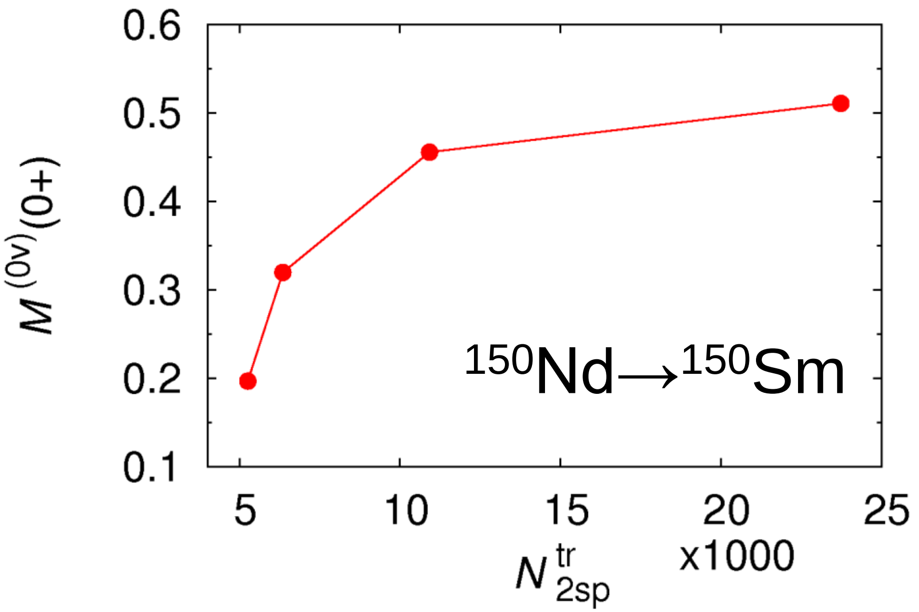
2) The strength of the isoscalar pairing interaction is determined so as to have the $0\nu\beta\beta$ GT nuclear matrix element by the double-β path equal to that by the two-particle-transfer path.

NEW

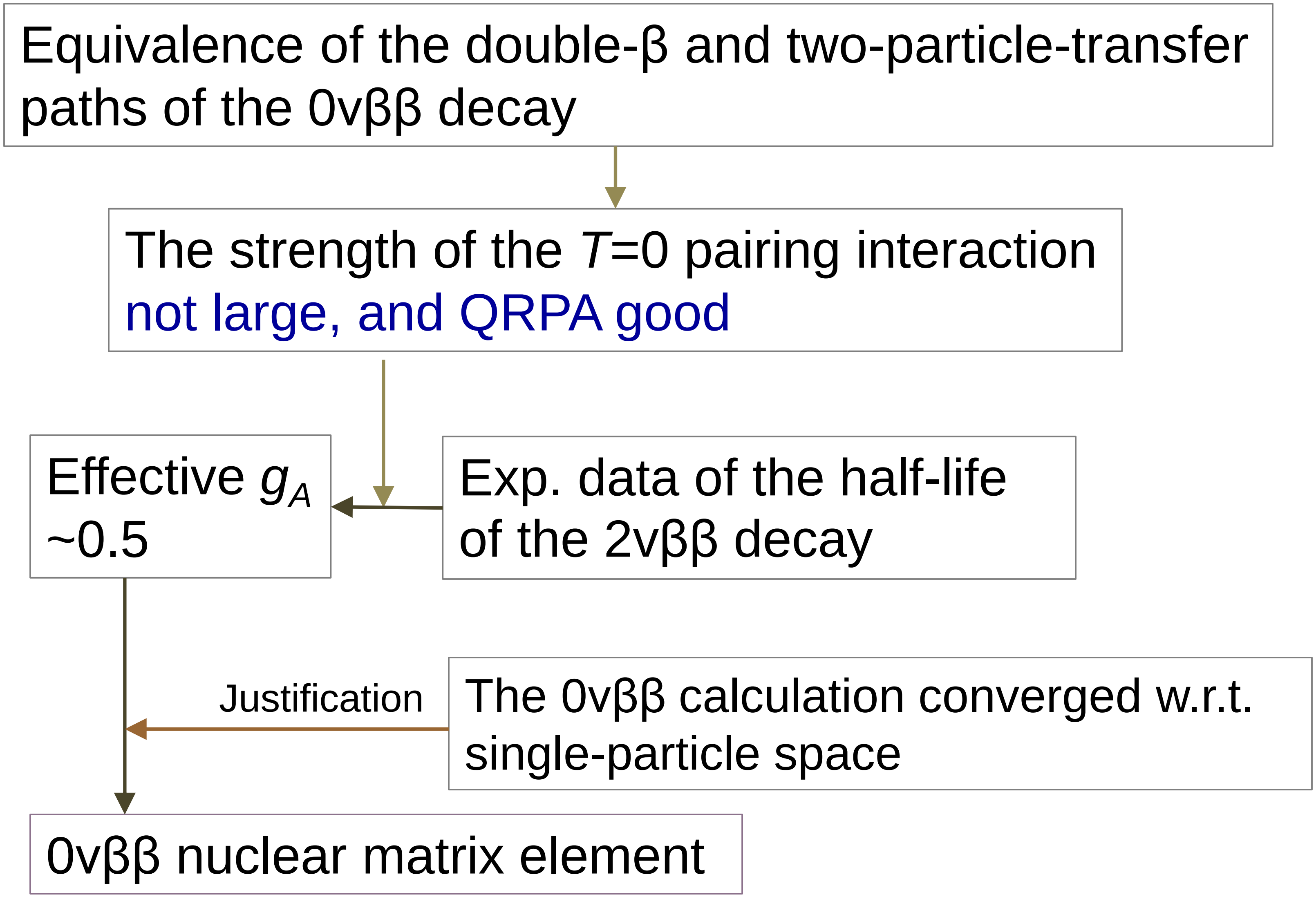
3) The effective axial-vector current coupling is determined so as to reproduce the exp. half-life of $2\nu\beta\beta$ decay.

Features of the procedure

1) Isospin symmetry
2) Identity under the closure approximation
3) The value of g_A is ~0.5. This coupling is applied to the 0vββ decay because my 0vββ calculation is converged w.r.t. the single-particle space.



Flow of calculation



Mechanism of improvement

Operator r^2 has two components: 0ħω jump and 2ħω jump.

The 0ħω-jump component can be represented by
$\langle r^2 \rangle_{\mathrm{s.p.}}$: expectation value of r^2 with the wave function of the single-particle involved in the charge change

This statement is correct if the isospin symmetry holds, and the charge change occurs between the single-particles sharing the good quantum numbers other than the charge.

↓

Applicable to the 0ħω transition of ^{48}Ca

Mechanism of improvement

Operator r^2 has two components: 0ħω jump and 2ħω jump.

The 0ħω-jump component can be represented by
$\langle r^2 \rangle_{\text{s.p.}}$: expectation value of r^2 with the wave function of the single-particle involved in the charge change

$$\sigma\tau + \alpha r^2 \sigma\tau = \underbrace{\sigma\tau + \alpha \langle r^2 \rangle_{\text{s.p.}} \sigma\tau}_{0\hbar\omega \text{ jump}} + \underbrace{\alpha (r^2 - \langle r^2 \rangle_{\text{s.p.}}) \sigma\tau}_{2\hbar\omega \text{ jump}}$$

0ħω jump → The low-energy region of the strength function

2ħω jump → The high-energy region of the strength function

$\alpha < 0$