Interplay of single-particle and cluster degrees of freedom in atomic nuclei

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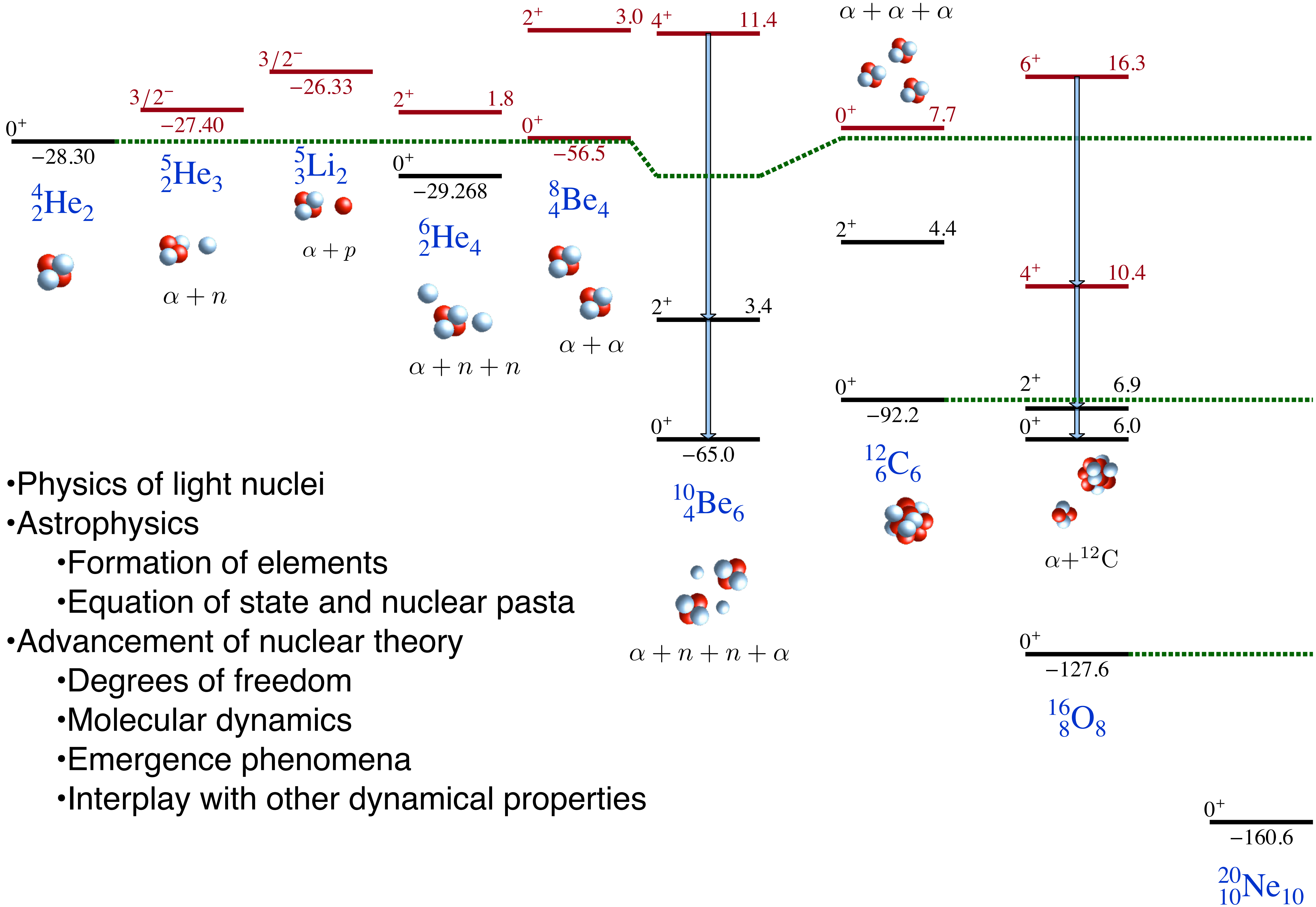
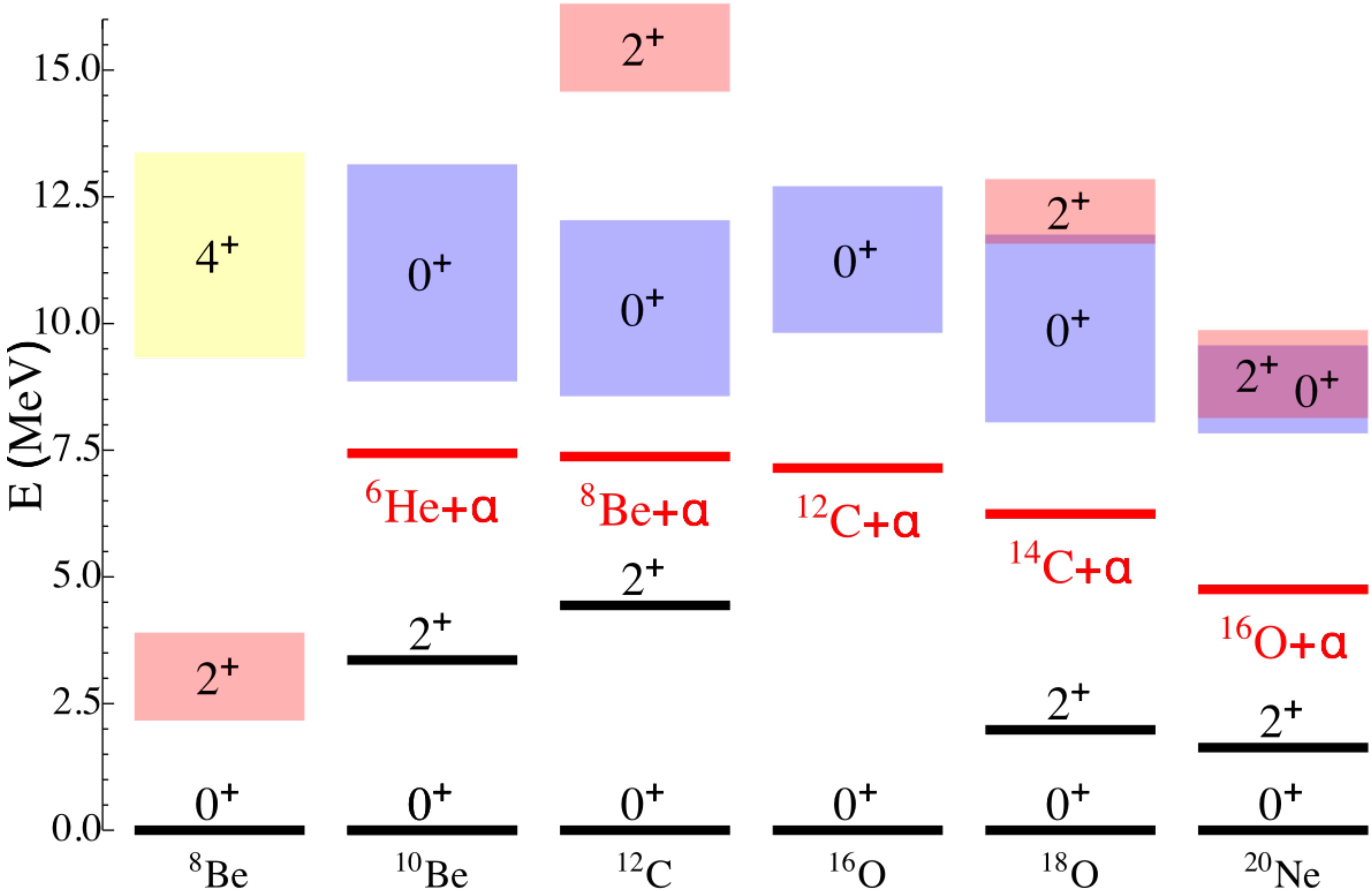
Clustering in light nuclei

Connecting bound state calculations with scattering and reactions



- Physics of light nuclei
- Astrophysics
    - Formation of elements
    - Equation of state and nuclear pasta
- Advancement of nuclear theory
    - Degrees of freedom
    - Molecular dynamics
    - Emergence phenomena
    - Interplay with other dynamical properties

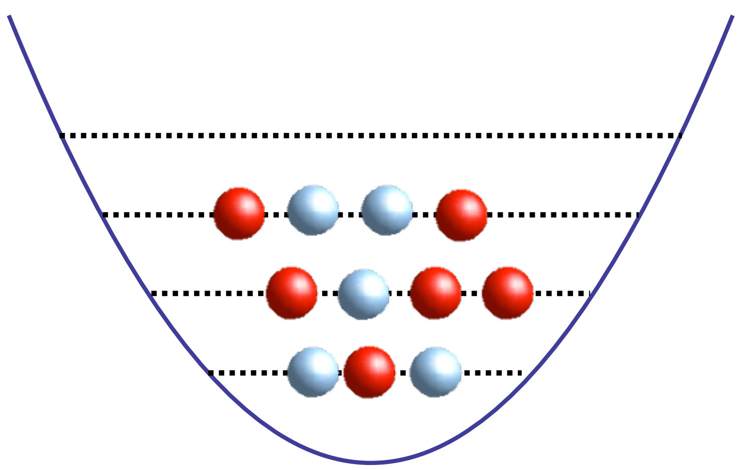
Clustering and continuum



Configuration interaction approach and clustering

Traditional shell model configuration m-scheme

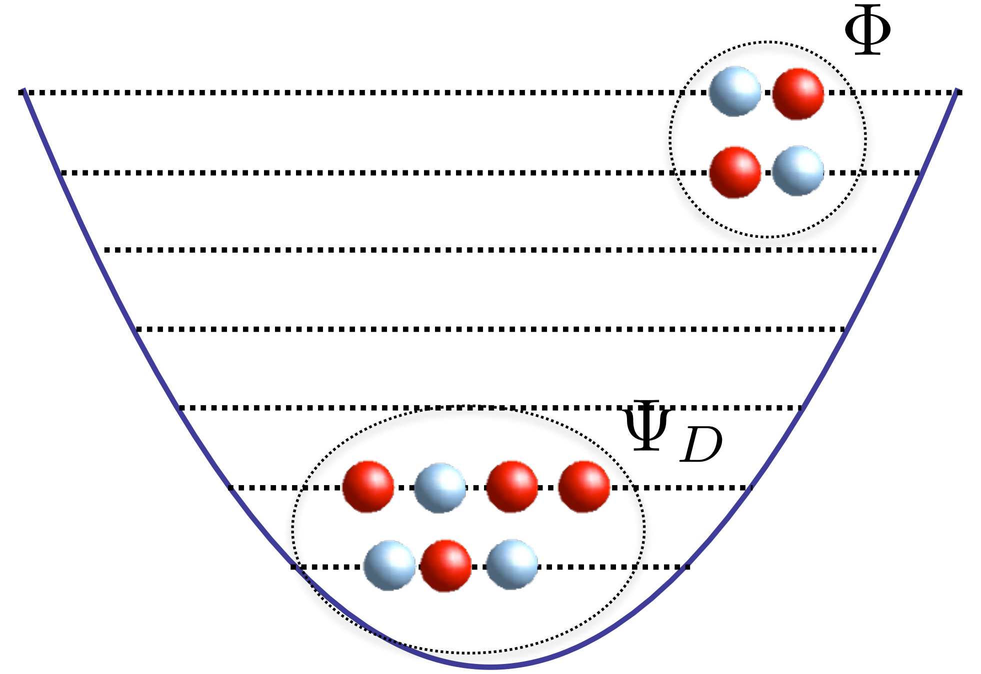
$$|\Psi\rangle = \Psi^\dagger|0\rangle \sim a_1^\dagger a_2^\dagger \ldots a_A^\dagger|0\rangle$$



+

Cluster configuration SU(3)-symmetry basis

$$|\text{channel}\rangle \sim |\Phi\Psi_D\rangle \equiv \Phi^\dagger\Psi_D^\dagger|0\rangle$$



$|\Psi\rangle$ + $\Phi^\dagger|\Psi_D\rangle$

Translational invariance and Center of Mass (CM)

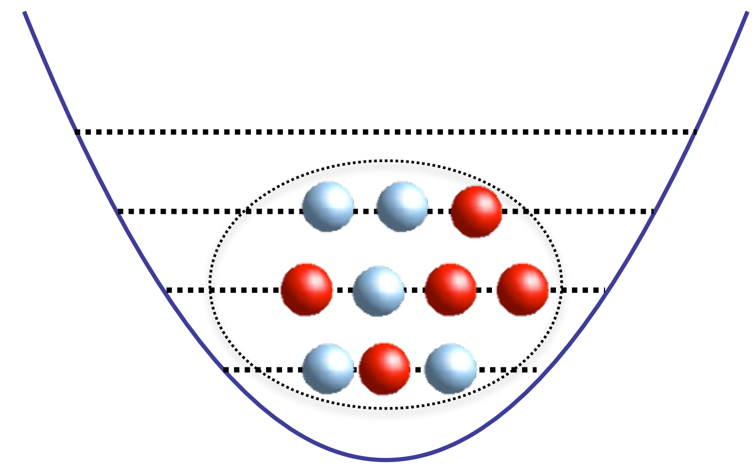
Shell model, Glockner-Lawson procedure

$$\Psi = \phi_{000}(\mathbf{R})\Psi'$$

SM state

Center-of-mass vibration

Intrinsic state



Controlling CM

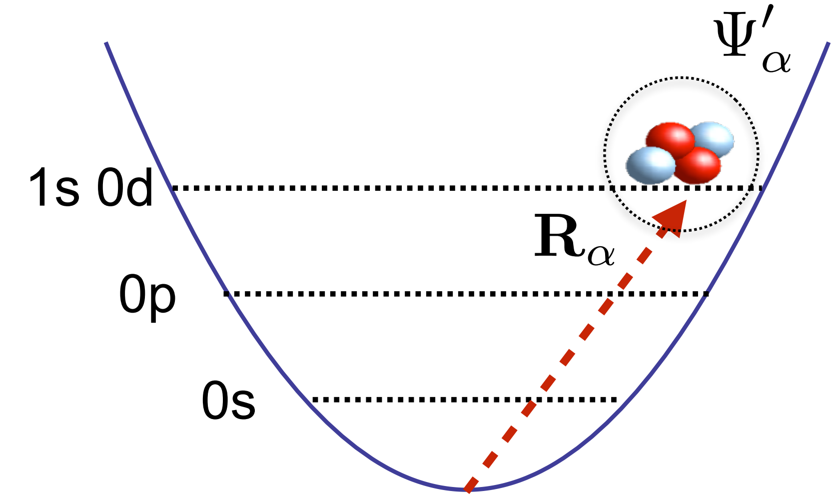
$$D_\mu = \sqrt{\frac{4\pi}{3}} R_\mu$$

Control only CM quanta

$$R_\mu = \sqrt{\frac{\hbar}{2Am\omega}} (\mathcal{B}_\mu^\dagger + \mathcal{B}_\mu)$$

Center-of-Mass boosts

$$\Psi_{n\ell m} = \phi_{n\ell m}(\mathbf{R})\,\Psi'$$



$\boldsymbol{\mathcal{B}}^\dagger$ and $\boldsymbol{\mathcal{B}}$ CM quanta creation and annihilation (vectors)

$$\Psi_{n+1\ell m} \propto \boldsymbol{\mathcal{B}}^\dagger \cdot \boldsymbol{\mathcal{B}}^\dagger \Psi_{n\ell m}$$

$\boldsymbol{\mathcal{B}}^\dagger \times \boldsymbol{\mathcal{B}}$ CM angular momentum operator

$$R_\mu = \sqrt{\frac{\hbar}{2Am\omega}}(\mathcal{B}^\dagger_\mu + \mathcal{B}_\mu)$$

K Kravvaris and A. Volya, Journal of Phys, Conf. Proc. 863, 012016 (2017)

Center-of-Mass boosts

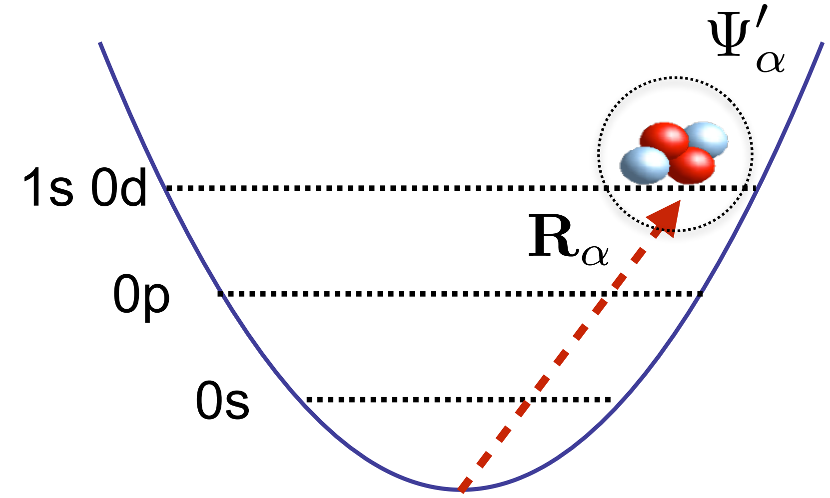
$$\Psi_{n\ell m} = \phi_{n\ell m}(\mathbf{R})\,\Psi'$$

$\mathcal{B}^\dagger$ and $\mathcal{B}$ CM quanta creation and annihilation (vectors)

$$\Psi_{n+1\ell m} \propto \mathcal{B}^\dagger \cdot \mathcal{B}^\dagger \Psi_{n\ell m}$$

$\mathcal{B}^\dagger \times \mathcal{B}$ CM angular momentum operator

$$\Psi_\alpha = \phi_{n\ell m}(\mathbf{R})\,\Psi'_\alpha = \sum_\eta X^\eta_{n\ell}\,\Phi^\eta_{(n,0):\ell m}$$



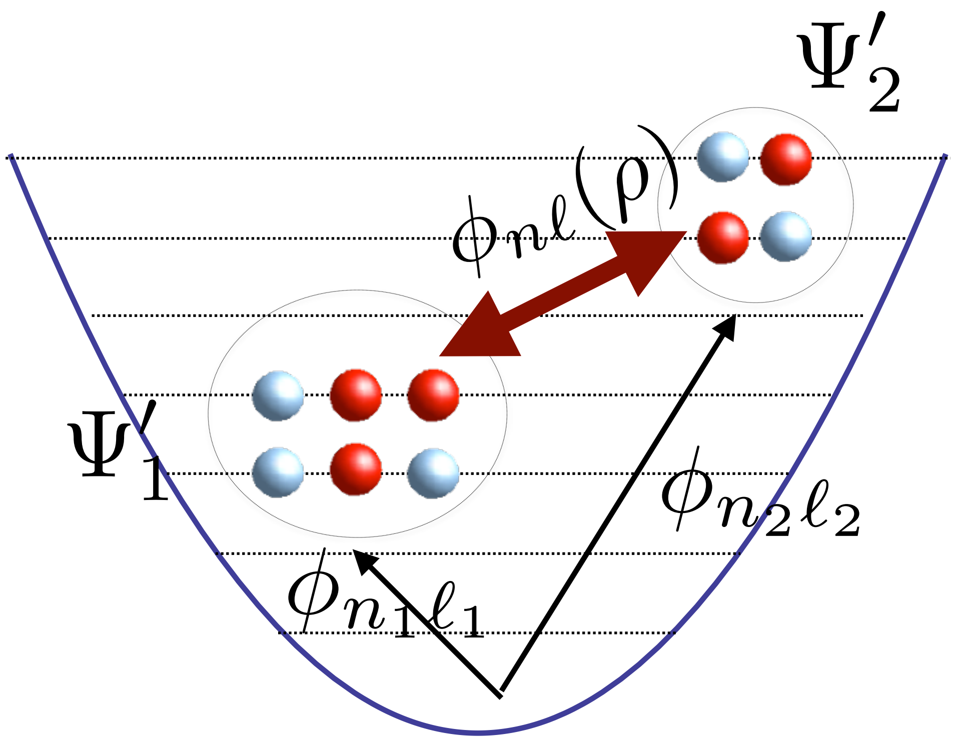
Configuration	$N_{\max} = 0$	$N_{\max} = 4$
$(sd)^4$	0.038	0.035
$(p)(sd)^2(pf)$	0.308	0.282
$(p)^2(pf)^2$	0.103	0.094
$(p)^2(sd)(sdg)$	0.154	0.141
$(p)(sd)(sdg)(pfh)$	0.000	0.005
$(p)(sd)(pf)(sdg)$	0.000	0.009

Select configuration content of NCSM wave functions for ^{4}He with $\Omega = 20$ MeV boosted by 8 quanta (L = 0).

K Kravvaris and A. Volya, Journal of Phys, Conf. Proc. 863, 012016 (2017)

Clustering reaction basis channel

(basis states for clustering)



$$\Psi = \phi_{000}(\mathbf{R})\Psi'$$

Boost

$$\Psi_{n\ell m} = \phi_{n\ell m}(\mathbf{R})\,\Psi'$$

CM-Recouple

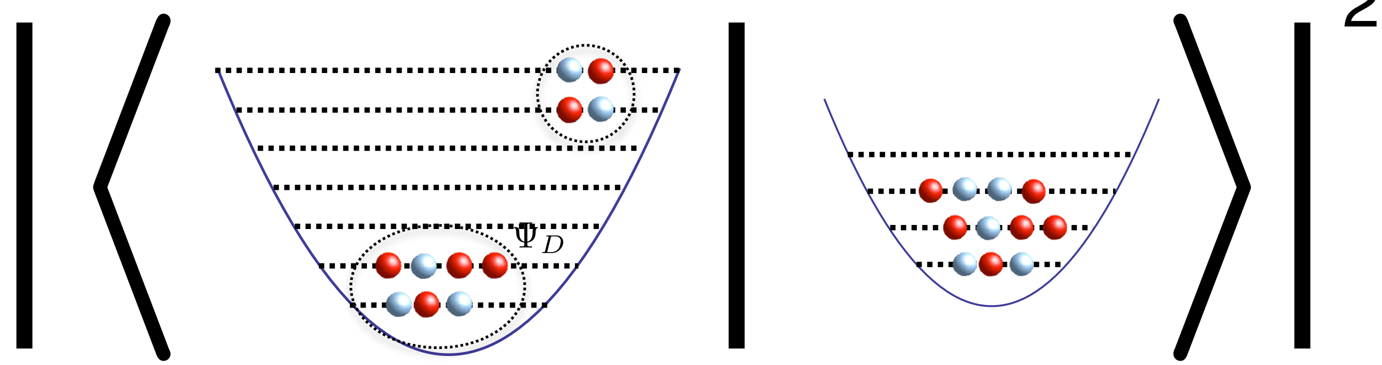
$$\Phi_{n\ell m} = \mathcal{A}\left\{\phi_{000}(\mathbf{R})\phi_{n\ell m}(\boldsymbol{\rho})\Psi'^{(1)}\Psi'^{(2)}\right\}$$

$$\Phi^{\dagger}_{n\ell} = \sum_{\substack{n_1\ell_1\\ n_2\ell_2}} \mathcal{M}^{n\ell 00;\ell}_{n_1\ell_1 n_2\ell_2}\left[\Psi^{\dagger}_{n_1\ell_1 m_1} \times \Psi^{\dagger}_{n_2\ell_2 m_2}\right]_{\ell}$$

Cluster Spectroscopic Characteristics

Traditional spectroscopic factor

$$S_{n,\ell} \equiv \left| \langle \Psi^{(A)} | \Phi_{n\ell} \rangle \right|^2 =$$



$$\mathcal{N}^{(\ell)}_{nn'} = \langle \Phi_{n\ell} | \Phi_{n'\ell} \rangle$$

Norm kernel

$$|\Psi^{(\ell,\mathrm{ocm})}_{\nu}\rangle = \sum_n \left(\frac{1}{\sqrt{\mathcal{N}^{(\ell)}}} \right)_{\nu n} |\Phi_{n\ell}\rangle$$

Orthonormalized basis channels

$$S^{(\mathrm{ocm})}_{\ell} \equiv \sum_{\nu} \left| \langle \Psi^{(A)} | \Psi^{(\ell,\mathrm{ocm})}_{\nu} \rangle \right|^2$$

Sum of all new SF from all parent states to a given final state equals to the number of channels

R. Id Betan and W. Nazarewicz Phys. Rev. C 86, 034338 (2012)
S. G. Kadmenskya, S. D. Kurgalina, and Yu. M. Tchuvil'sky Phys. Part. Nucl., 38, 699–742 (2007).
R. Lovas et al. Phys. Rep. 294, No. 5 (1998) 265 – 362.
T. Fliessbach and H. J. Mang, Nucl. Phys. A **263**, 75–85 (1976).
H. Feschbach et al. Ann. Phys. 41 (1967) 230 – 286
A. Volya and Y. M. Tchuvil'sky, Phys.Rev.C 91, 044319 (2015);

Resonating Group Method (RGM) Spectroscopic Factors

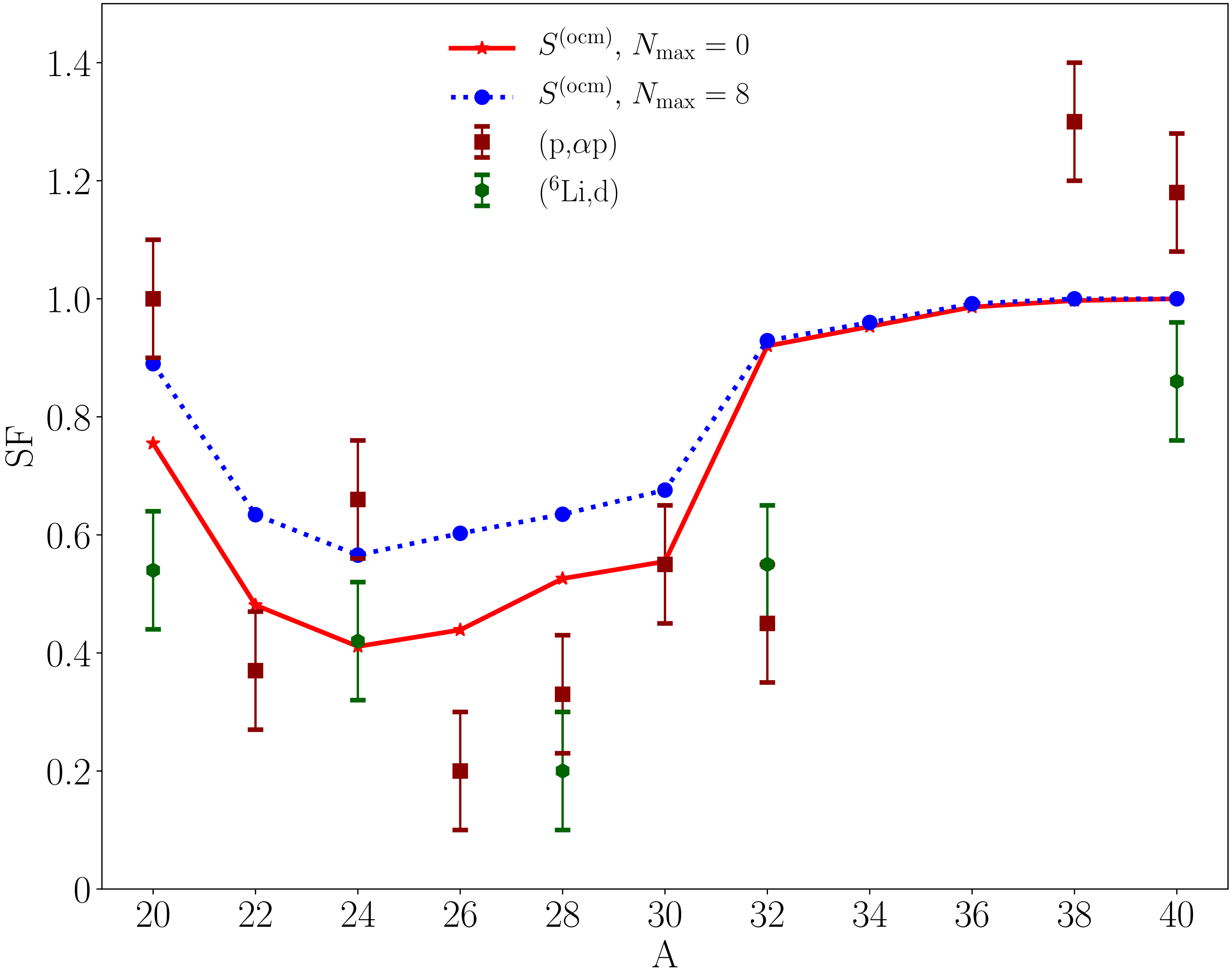
$$|\Psi^{(\ell,\mathrm{rgm})}\rangle = \sum_n \chi_n |\Phi_{n\ell}\rangle$$

$$\sum_{n'} \mathcal{H}_{nn'}\chi_{n'} = E \sum_{n'} \mathcal{N}_{nn'}\chi_{n'}$$

$$\mathcal{H}_{nn'} = \langle \Psi_{n\ell}|H|\Psi_{n'\ell}\rangle, \mathcal{N}_{nn'} = \langle \Psi_{n\ell}|\Psi_{n'\ell}\rangle$$

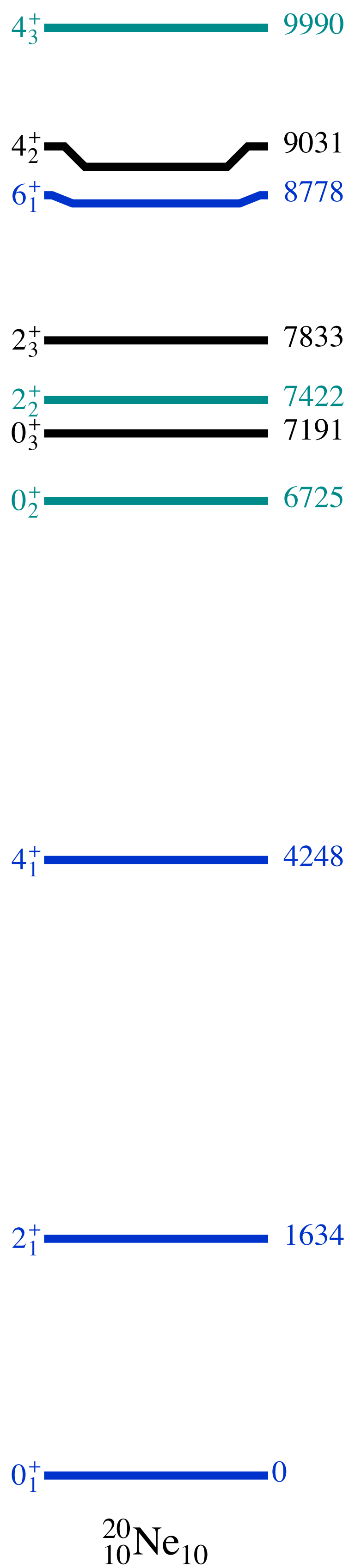
$$S^{(\mathrm{rgm})}_{\beta,\ell} \equiv \left|\langle \Psi^{(A)}|\Psi^{(\ell,\mathrm{rgm})}_{\beta}\rangle\right|^2$$

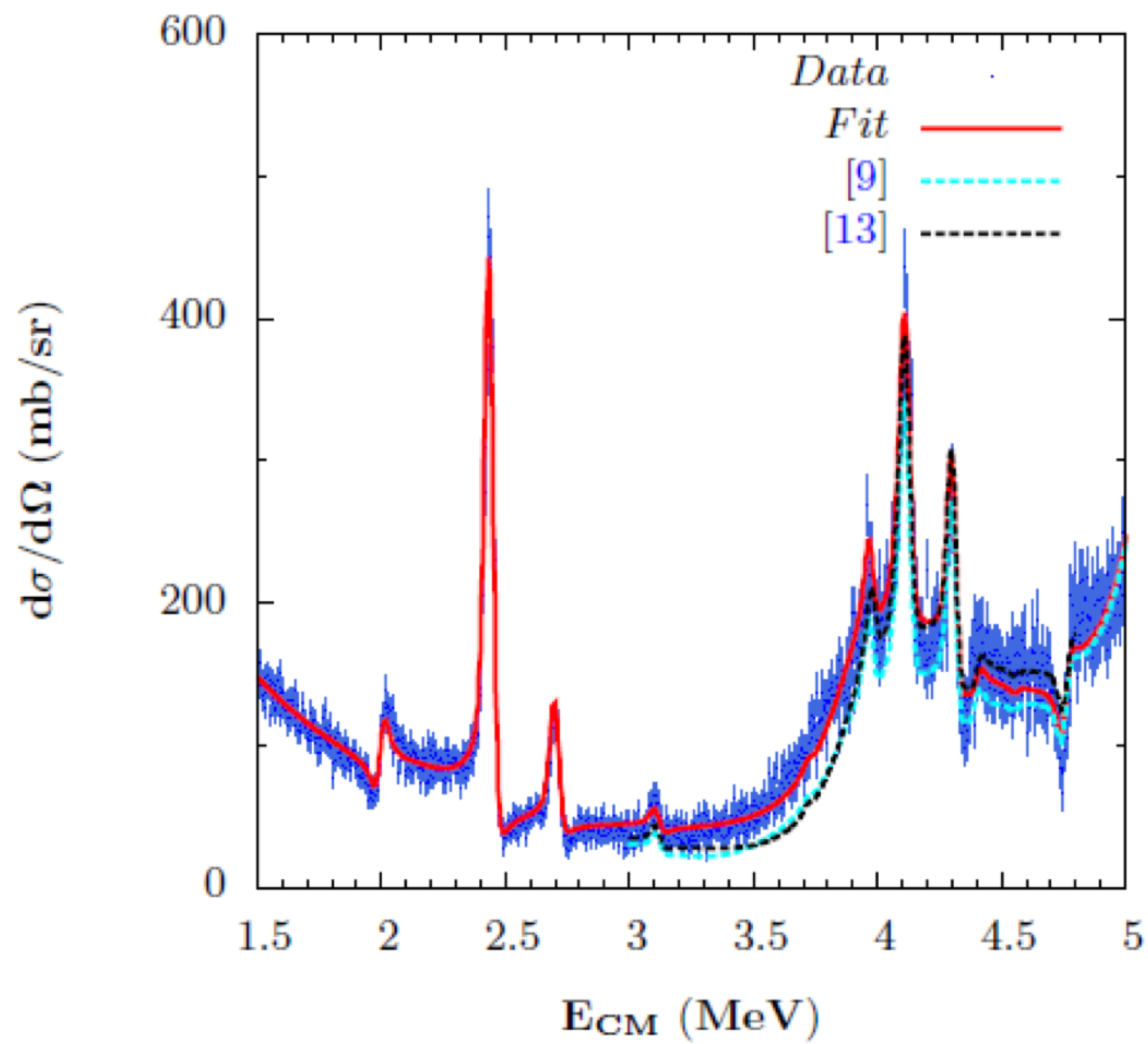
SD nuclei, cluster spectroscopic characteristics



[1] T.A. Carey, P.G. Roos, N.S. Chant, A. Nadsen, H.L. Chen, Phys. Rev. C 23,576(R) (1981)
[2] N. Anantaraman et al. Phys. Rev. Lett. 35, 1131 (1975)

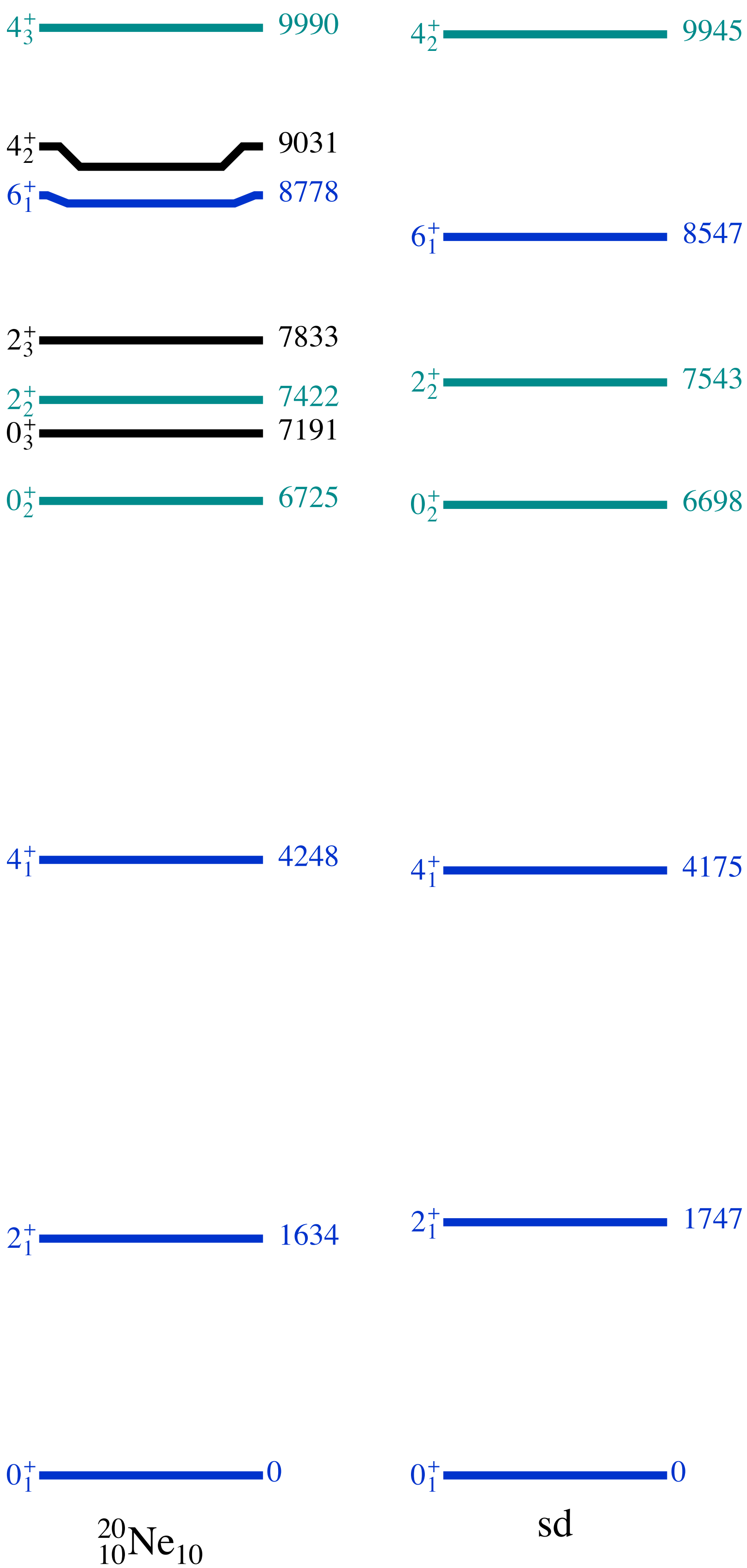
Clustering in ^{20}Ne



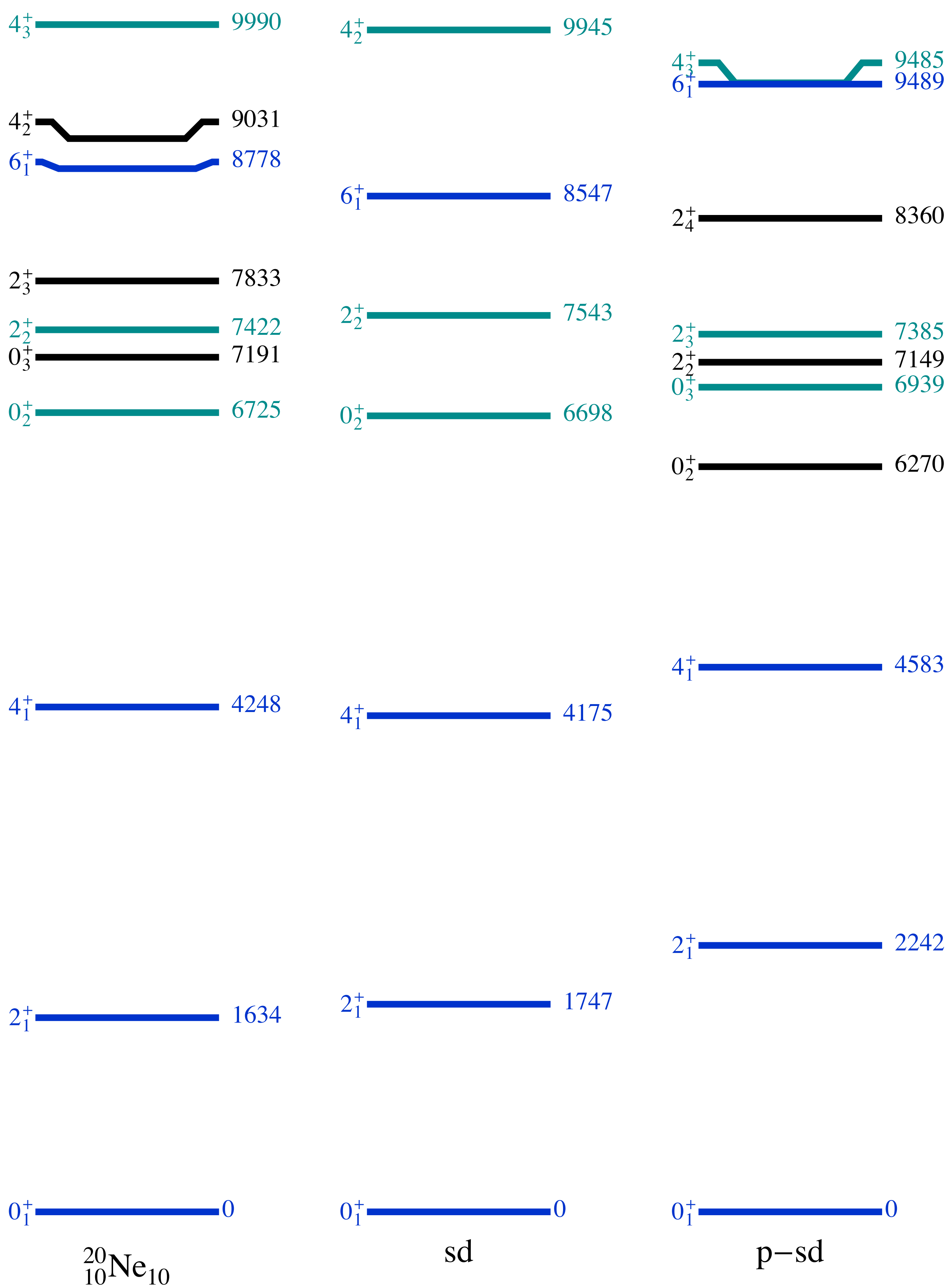


D.K. Nauruzbayev et al., (2017)

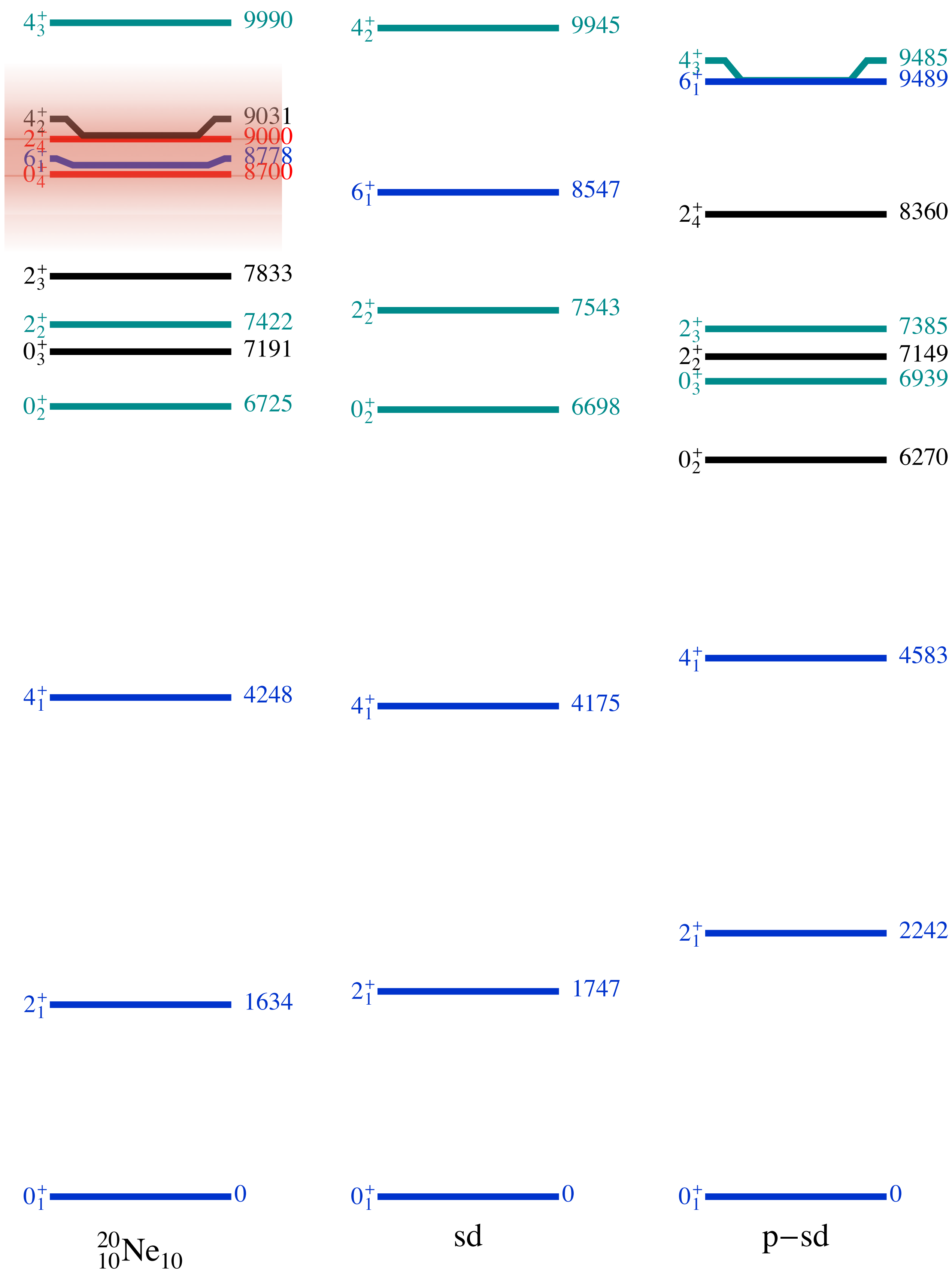
Clustering in ^{20}Ne



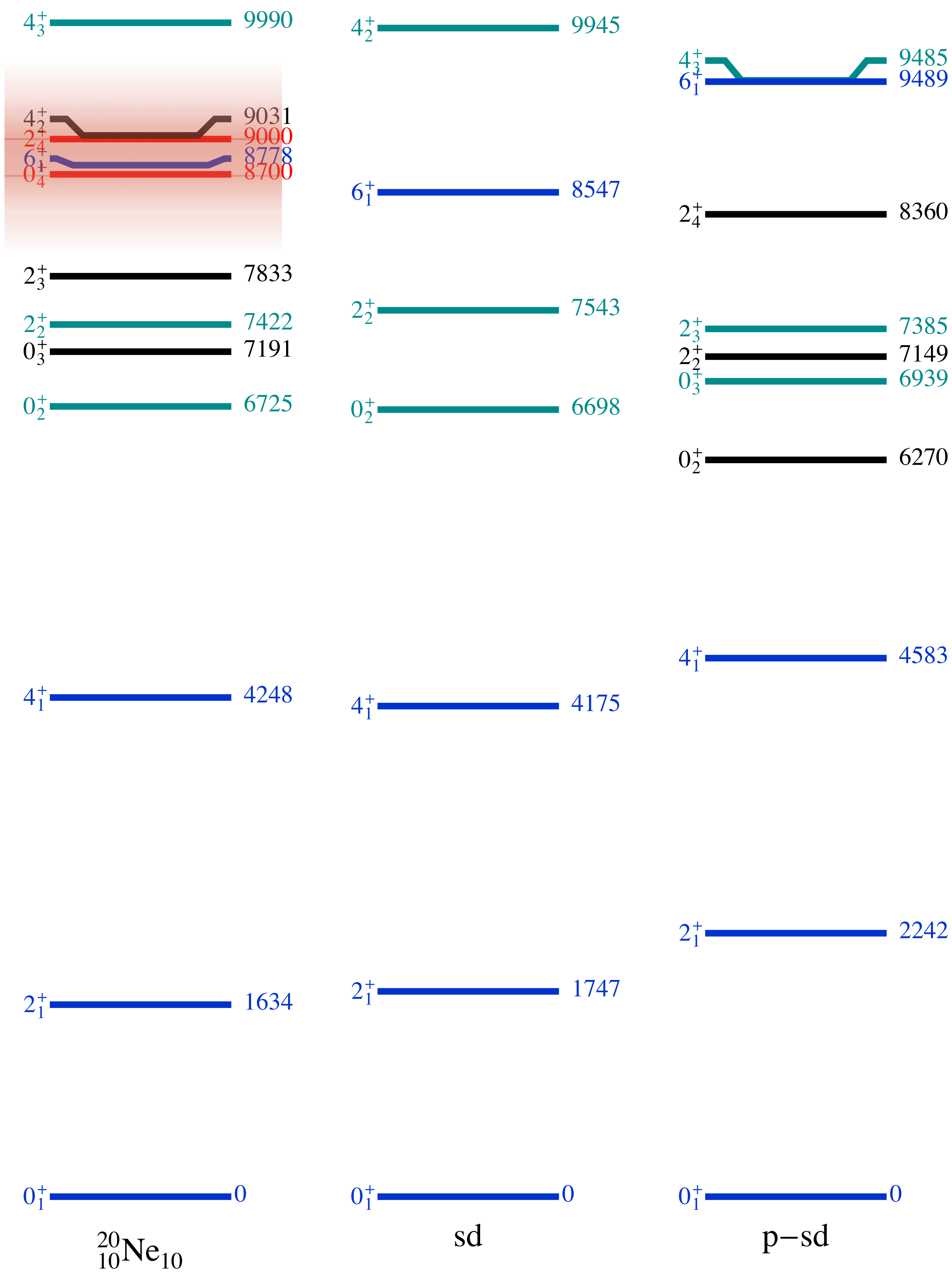
Clustering in ^{20}Ne



Clustering in ^{20}Ne



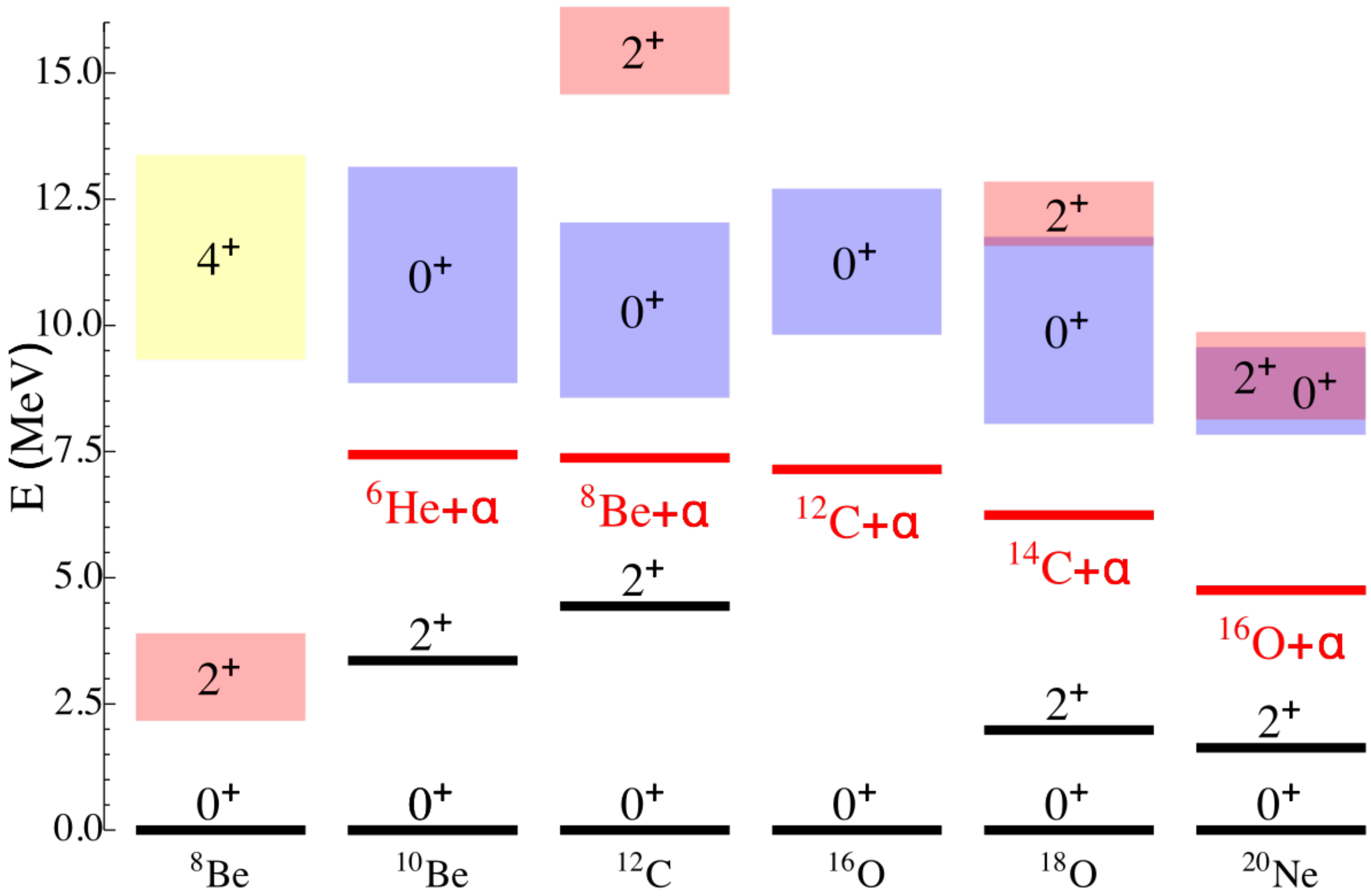
Clustering in ^{20}Ne



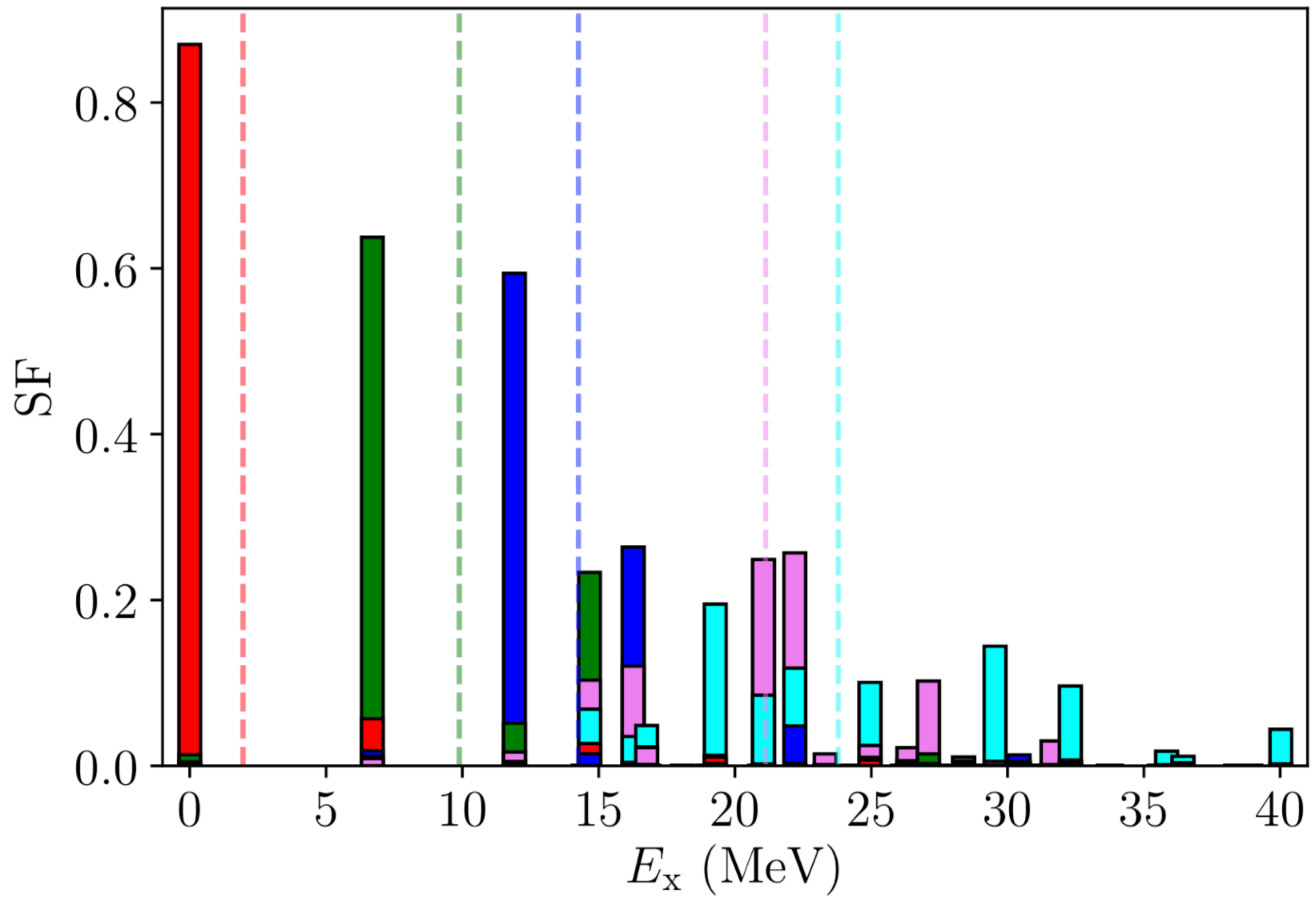
J	E MeV	Γ width	SF ex	SF th.
0+	0	0		0.73
2+	1.63	0		0.67
4+	4.25	0		0.62
0+	6.73	19	0.47	0.46
0+	7.19	3.4	0.02	0.10
2+	7.42	15	0.19	0.12
2+	7.83	2	0.01	0.09
0+	8.7	800	0.3	
6+	8.78	0.11	0.5	0.51
2+	9.00	800	0.86	

D.K. Nauruzbayev et al., Phys. Rev. C **96**, 014322 (2017)

Searching for clustering states



Searching for clustering strength



Distribution of dynamic spectroscopic factors for ^{20}Ne → ^{16}O(g.s.) + α. The dashed lines correspond to the RGM energies for each decay channel.

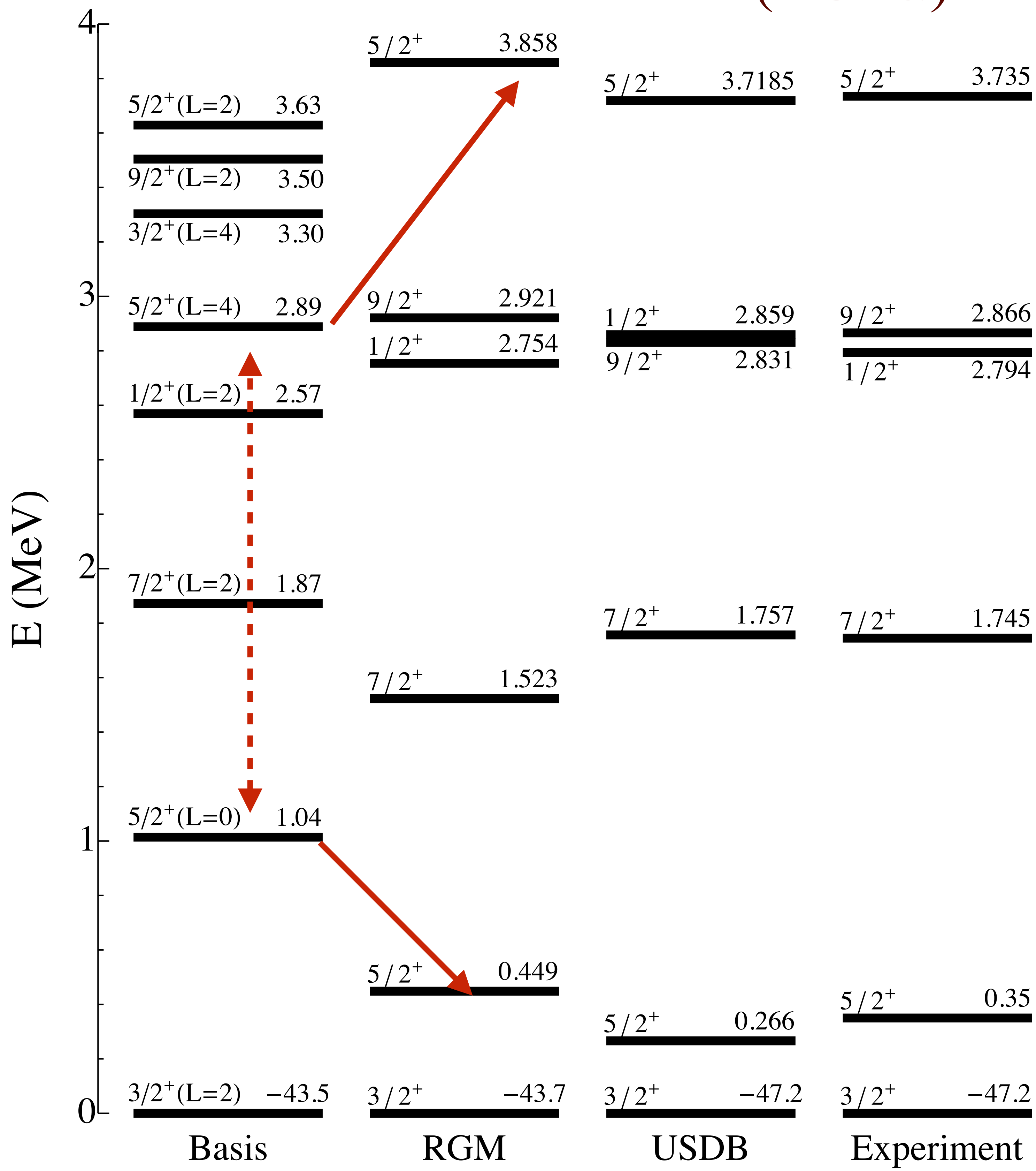
Effective sd-space operator

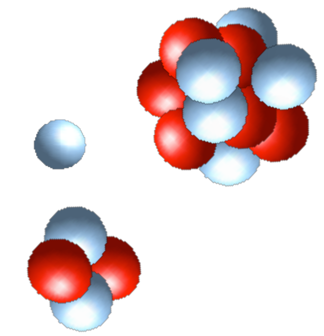
- sd-valence space optimal hw=14 MeV (common for all).
- alpha JISP16, N_{max}=8. (only 70% s^4).
- Example l=0, only 4 possible J=L=S=T=0 operators possible.
- Basis reaction channels with n=0,1..4 are contributing

n	X^2	(8,0)	(4,2)	(0,4)	(2,0)
4	0.02848	1.0	0.0	0.0	0.0
3	0.00697	0.561658	0.438338	0.0	0.0
2	0.00169	0.549804	0.0451847	0.3363	0.0636439
1	0.00018	0.0693304	0.735878	0.0134005	0.147418
0	0.00011	0.0693304	0.261291	0.0990471	0.0384533

Molecular orbits ^{21}Ne

$(^{16}O+\alpha)+n$



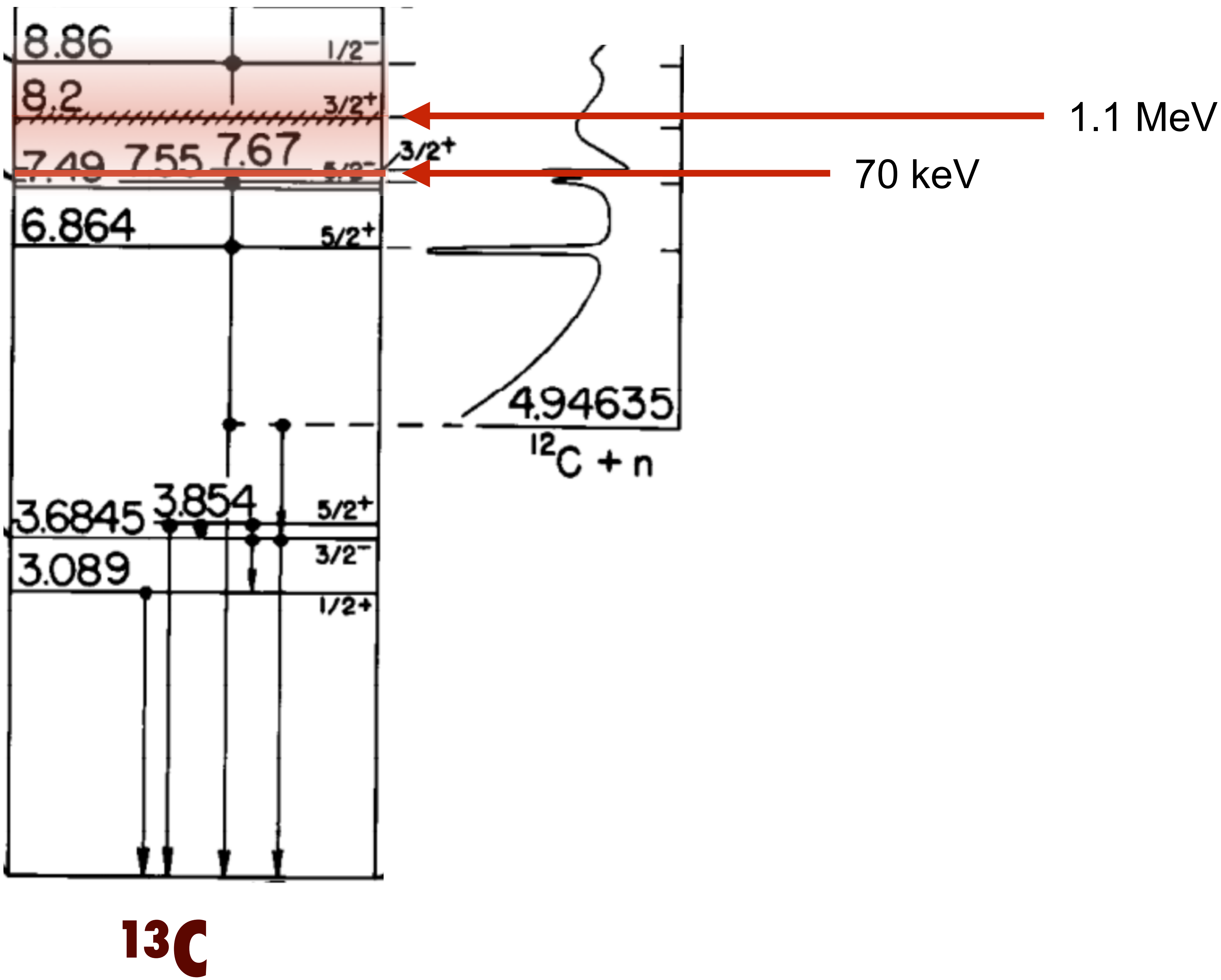


Weak-Coupling Behavior

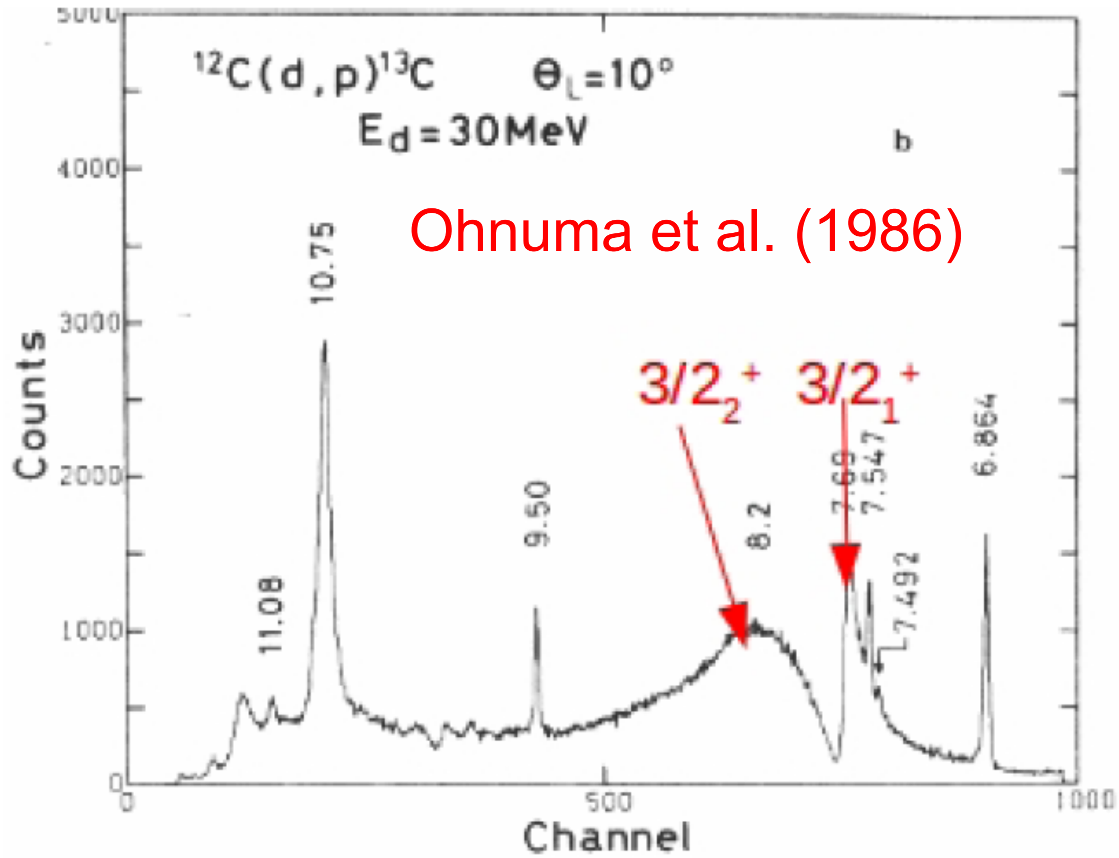
$\mathcal{S}^{(\exp)}$	$3/2^+$	$5/2^+$	$7/2^+$	$9/2^+, 1/2^+$
$\ell = 0$		1.04 ± 0.41		
$\ell = 2$	1.0 ± 0.05	$\ldots$	0.91 ± 0.08	0.9 ± 0.05
$\ell = 4$	0.42 ± 0.04	0.32 ± 0.18	0.23 ± 0.04	0.29 ± 0.03
$\mathcal{S}^{(\mathrm{rgm})}$				
$\ell = 0$		0.78		
$\ell = 2$	1.0	0.02	0.9	0.81
$\ell = 4$	0.18	0.44	0.14	0.33

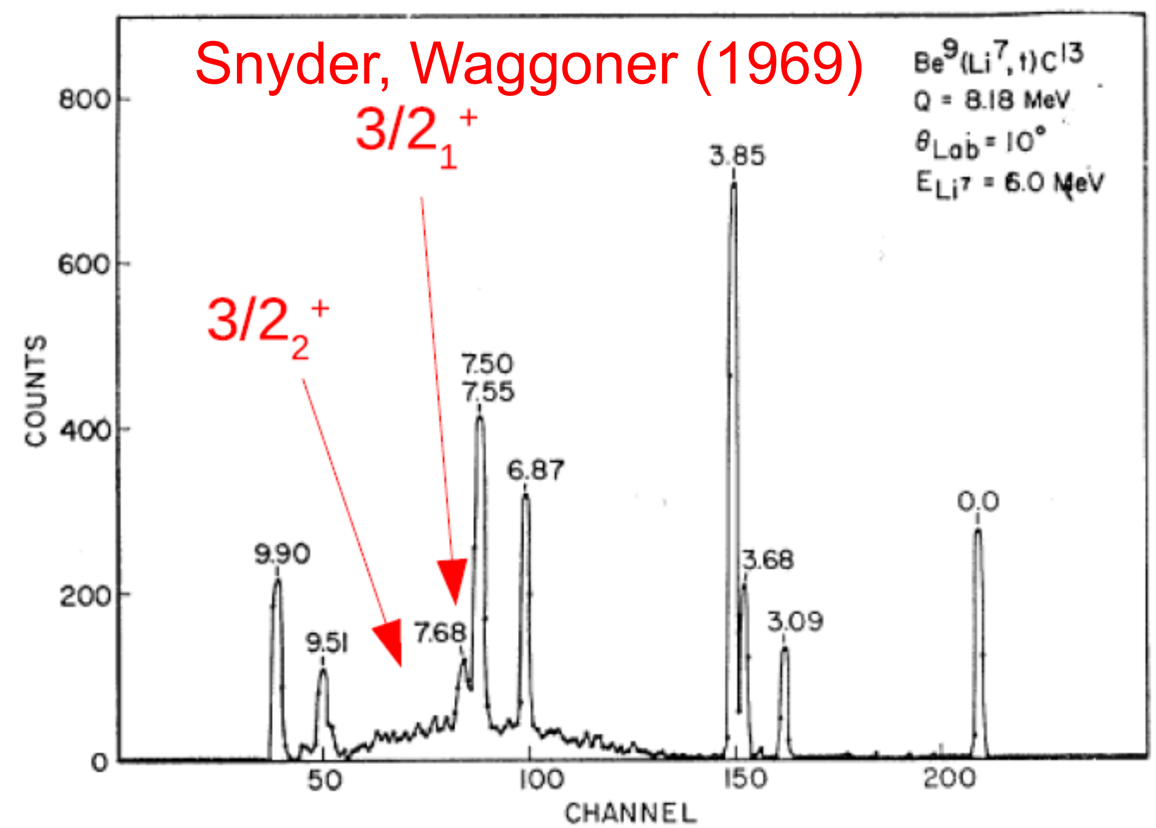
N. Anantaraman, J. P. Draayer, H. E. Gove, J. T¯oke, and H. T. Fortune. Alpha-particle stripping to ^{21}Ne. Phys.Rev. C18, 815 (1978); Phys.Lett. 74B, 199 (1978)

Clustering in ^{13}C



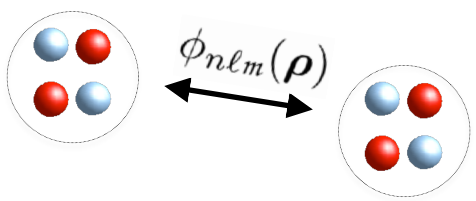
Clustering in ^{13}C

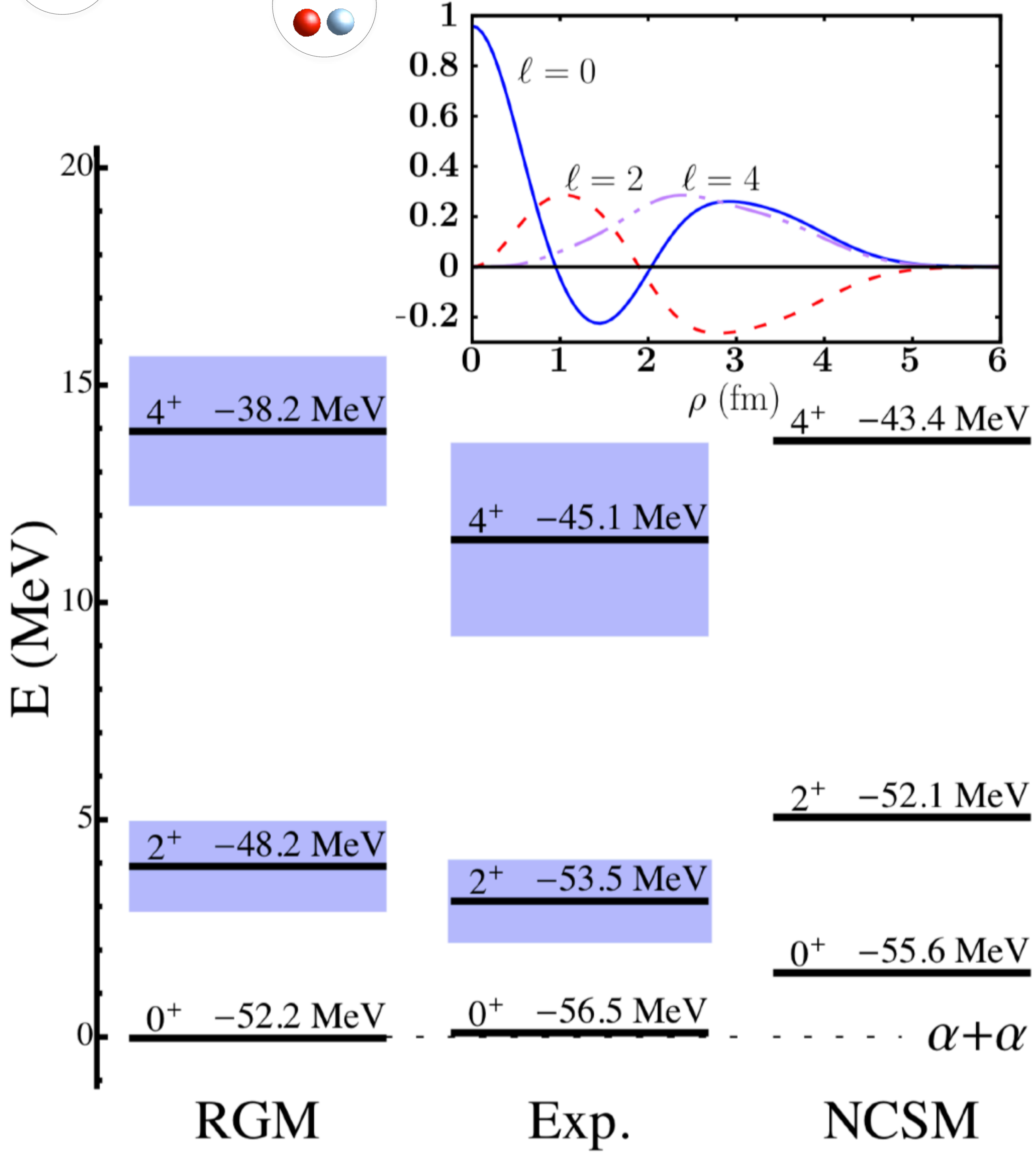




	Exp. Energy	Exp. Width	SM. Width	CSM Width	SM alpha SF	CSM alpha SF
3/2+ (1)	7.686(6)	0.070(5)	0.858	0.098	0.0256	0.0534
3/2+ (2)	8.2(1)	1.1(3)	0.342	1.031	0.0577	0.0366

Resonating group method ^{8}Be results



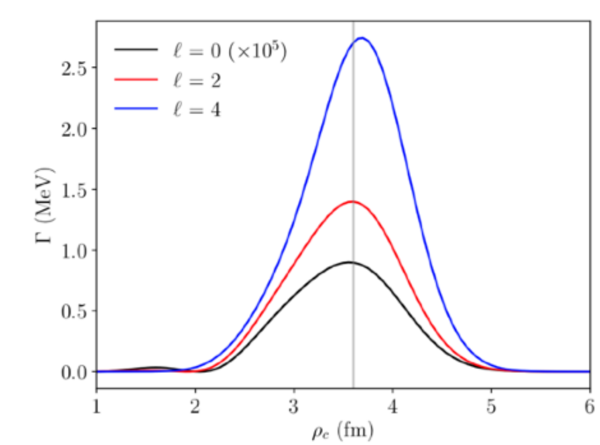


$$\hbar\Omega = 25 \text{ MeV}$$

ℓ	E_{ex}	Γ	$E_{\text{ex}}^{(\text{rgm})}$	$\Gamma^{(\text{rgm})}$	$\mathcal{S}^{(\text{rgm})}$
0	0.0	5.6*	0	8.9*	0.69
2	3.0	1.5	4.6	1.4	0.66
4	11.4	3.5	16.0	2.7	0.51

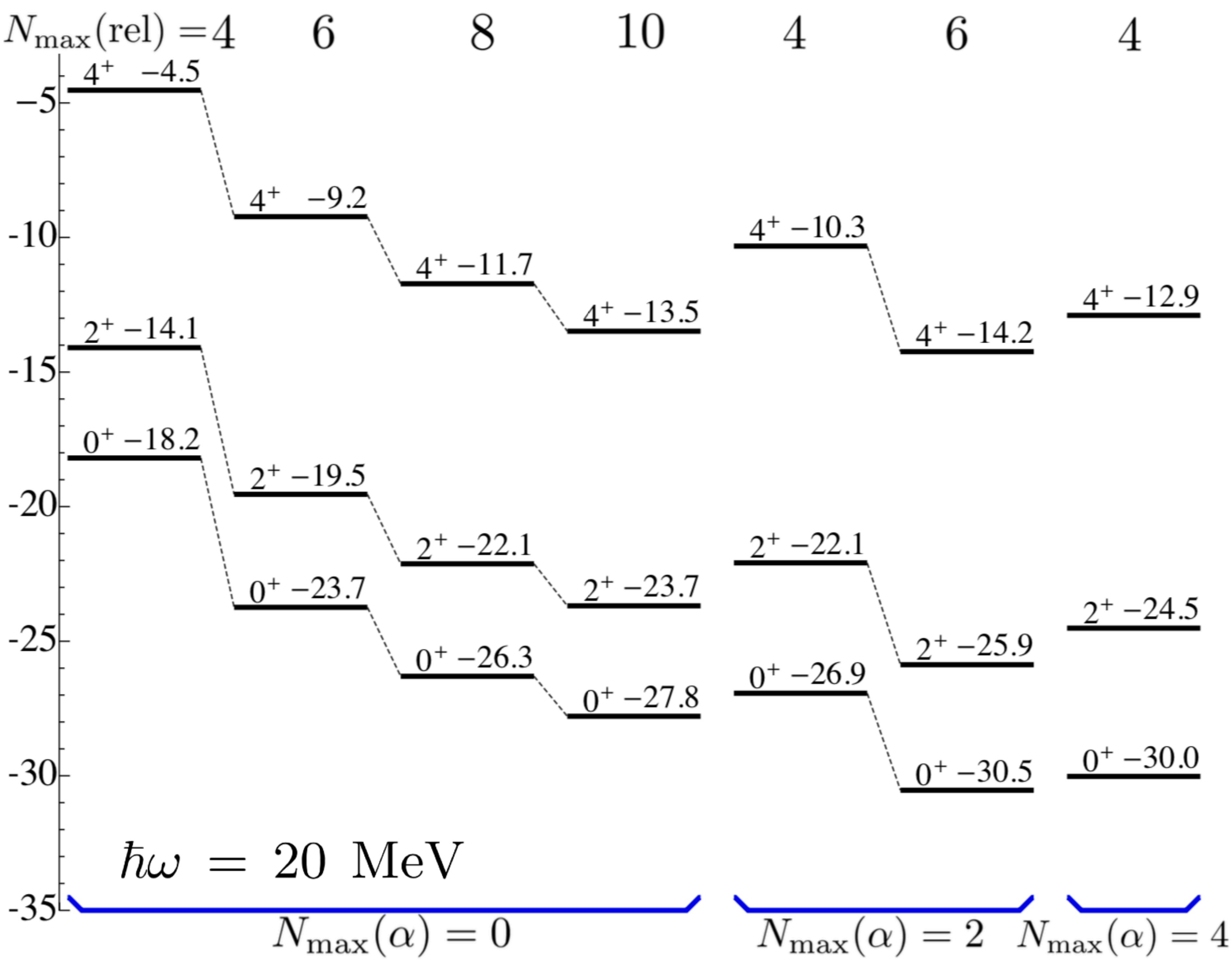
* Units of ev

$$\Gamma = 2P_L(\rho_c)|g(\rho_c)|^2$$



K Kravvaris and A. Volya, Phys.Rev.Lett, 119(6), 062501 (2017)

RGM effects of truncation, ^{8}Be



Coupling with continuum

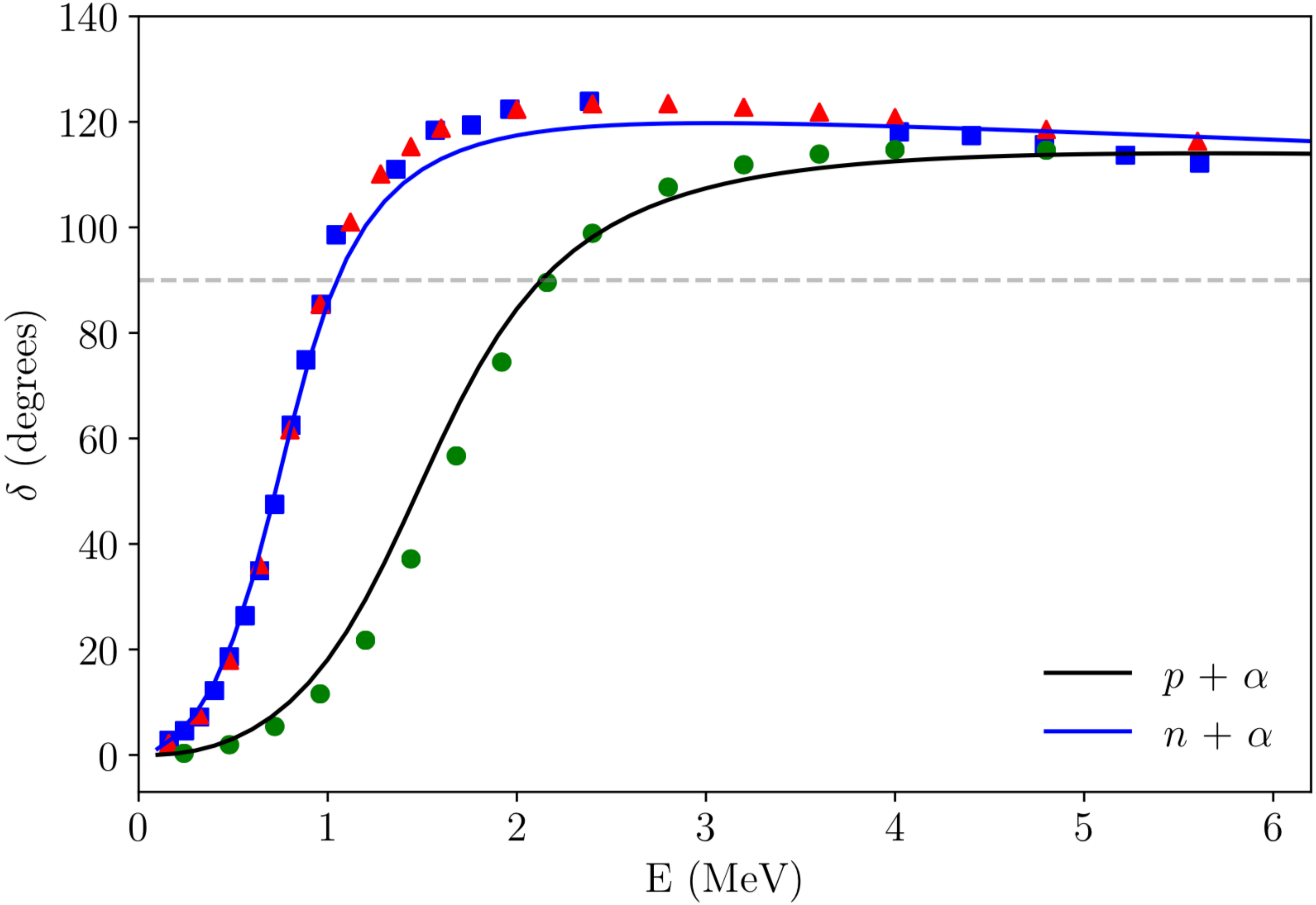
$$\sum_n \mathcal{H}^{(\ell)}_{nn'} \chi_{n'} = E \sum_n \mathcal{N}^{(\ell)}_{nn'} \chi_{n'}$$

$$\begin{pmatrix} \mathcal{H}_{00} & \dots & \mathcal{H}_{0n} & 0 & 0 & 0 \\ \vdots & \ddots & \vdots & 0 & \vdots & \vdots \\ \mathcal{H}_{n0} & \dots & \mathcal{H}_{nn} & T_{nn+1} & 0 & \vdots \\ 0 & 0 & T_{n+1n} & T_{n+1n+1} & T_{n+1n+2} & 0 \\ 0 & \dots & 0 & T_{n+2n+1} & T_{n+2n+2} & \ddots \\ 0 & \dots & \dots & 0 & \ddots & \ddots \end{pmatrix} \begin{pmatrix} \chi_0 \\ \vdots \\ \chi_n \\ x_{n+1} \\ x_{n+2} \\ \vdots \end{pmatrix} = \mathrm{E} \begin{pmatrix} \mathcal{N}_{00} & \dots & \mathcal{N}_{0n} & 0 & 0 & 0 \\ \vdots & \ddots & \vdots & 0 & \vdots & \vdots \\ \mathcal{N}_{n0} & \dots & \mathcal{N}_{nn} & 0 & 0 & \vdots \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \dots & 0 & 0 & 1 & 0 \\ 0 & \dots & \dots & 0 & 0 & \ddots \end{pmatrix} \begin{pmatrix} \chi_0 \\ \vdots \\ \chi_n \\ x_{n+1} \\ x_{n+2} \\ \vdots \end{pmatrix}$$

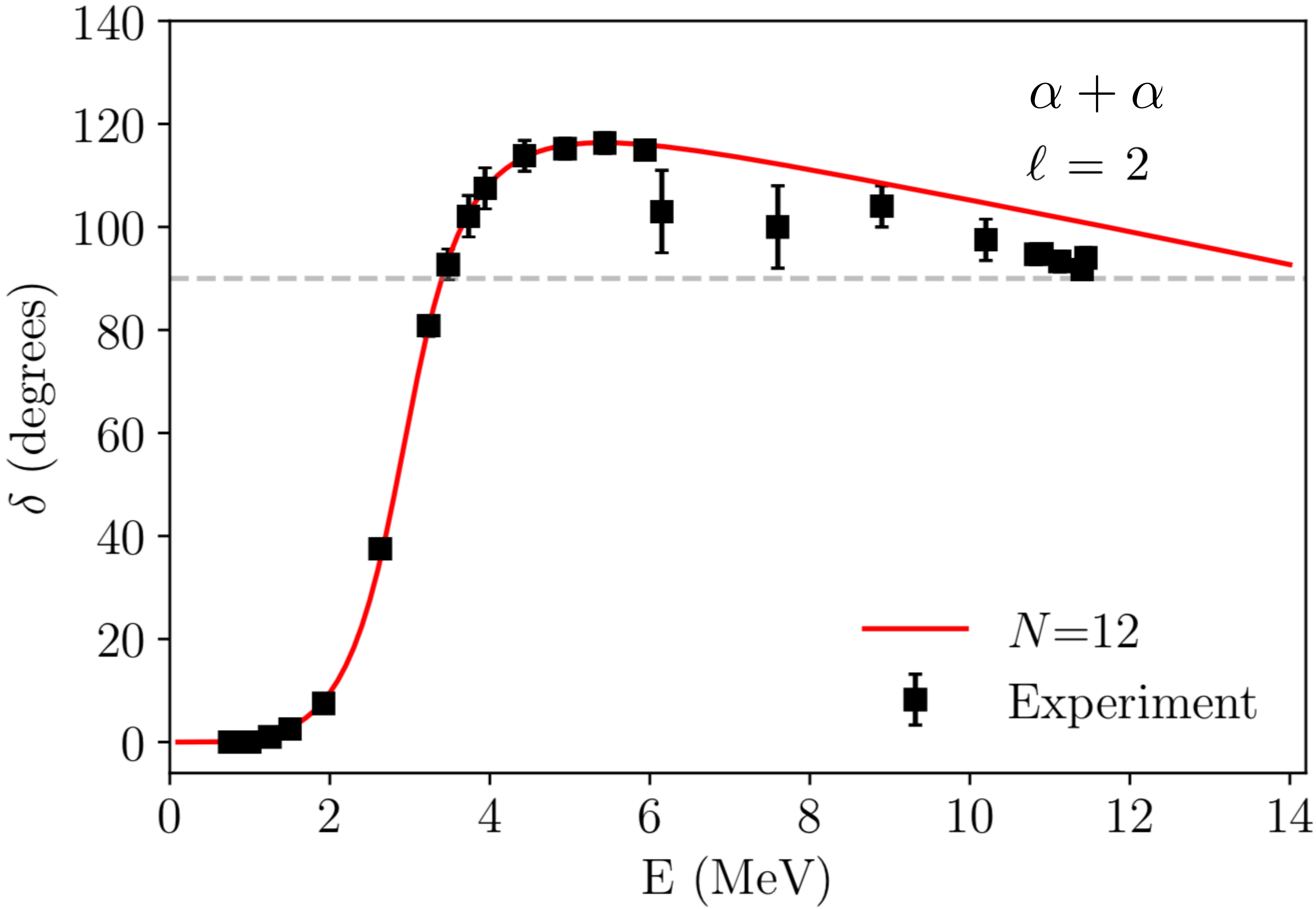
Asymptotic solution with phase shift

J-matrix (or HORSE) method: J. M. Bang, Annals of Physics 280, 299 (2000)
Experimental data: Phys. Rev. 168, 1114 (1968); Nucl. Phys. A287, 317 (1977)

nucleon+alpha scattering phase shifts



J-matrix (or HORSE) method: J. M. Bang, Annals of Physics 280, 299 (2000)
Experimental data: Phys. Rev. 168, 1114 (1968); Nucl. Phys. A287, 317 (1977)



Experiment: Rev. Mod. Phys. 41, 247 (1969)

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K. Kravvaris.

Yu. Tchuvil'sky, T Dytrych, A. Shirokov, J. Vary, G. V. Rogachev, V. Z. Goldberg.

Funding: U.S. DOE contract DE-SC0009883.

Publications:

K Kravvaris and A. Volya, Phys.Rev.Lett, 119(6), 062501 (2017); Journal of Phys 863, 012016 (2017)

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Phys. Rev. C 95, 055803

Resources: https://www.volya.net/ (see research, clustering)