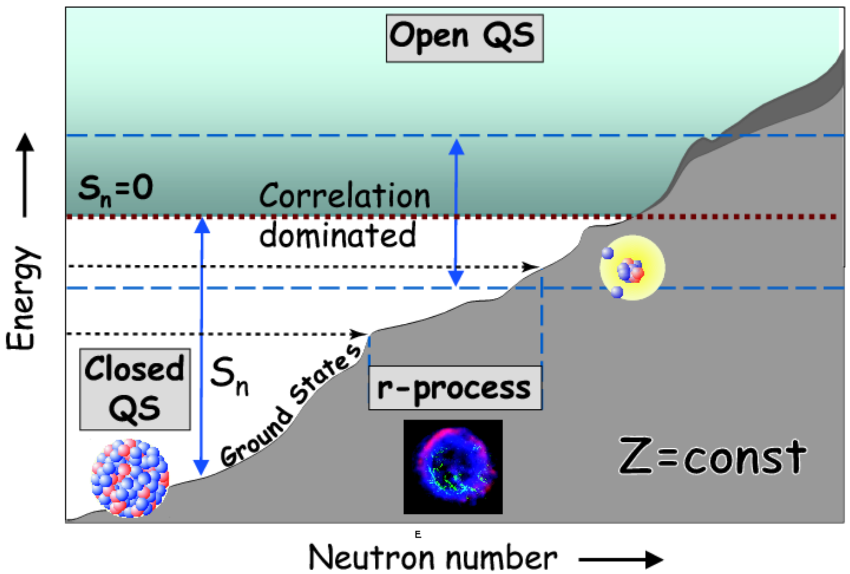
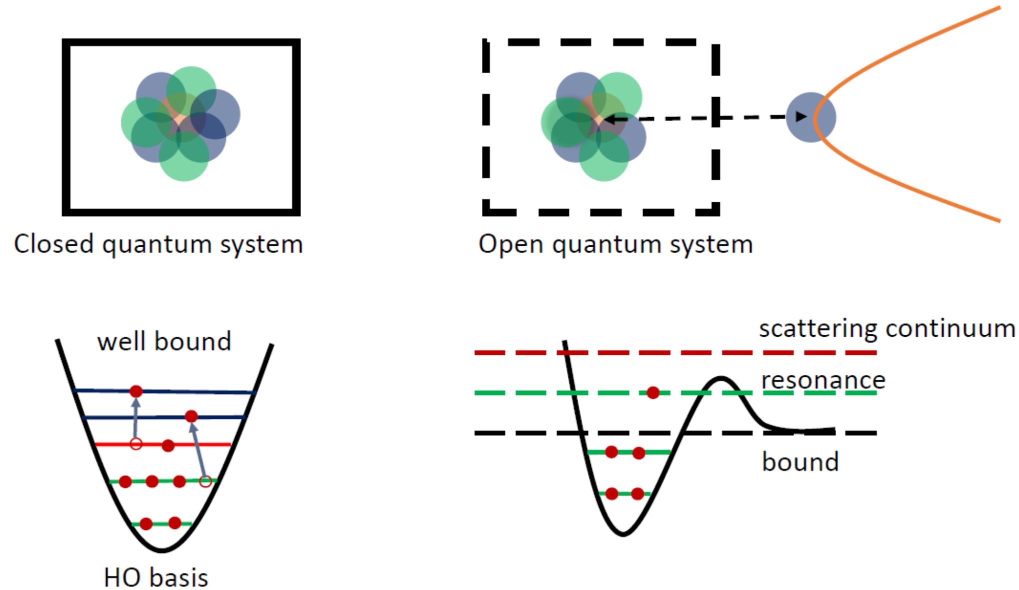
北京大学物理学院
School of Physics, Peking University

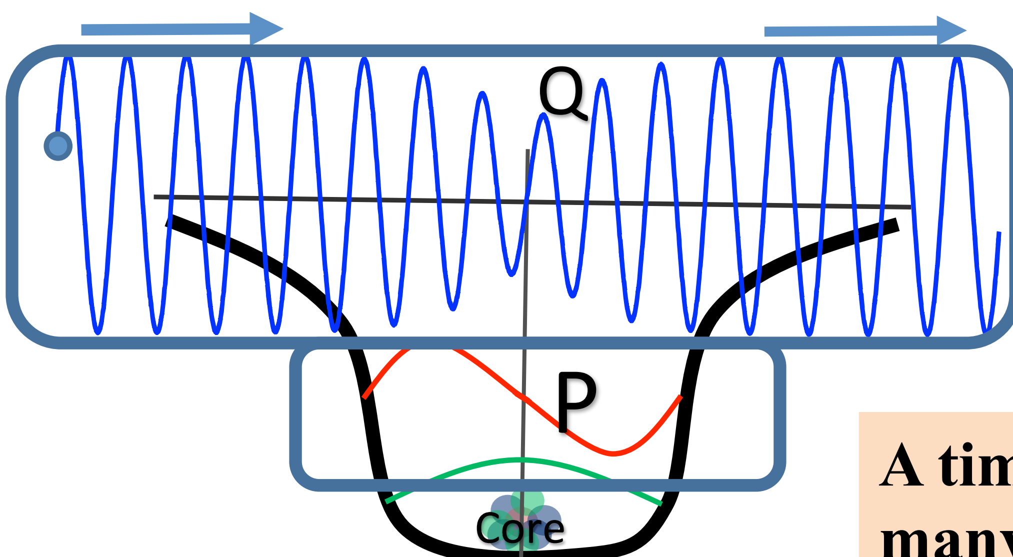
Nuclear resonances

Furong Xu

- CD-Bonn Gamow Shell Model for the long chain of calcium isotopes
- N^2LO_opt Gamow IM-SRG + EOM for neutron-rich carbon isotopes





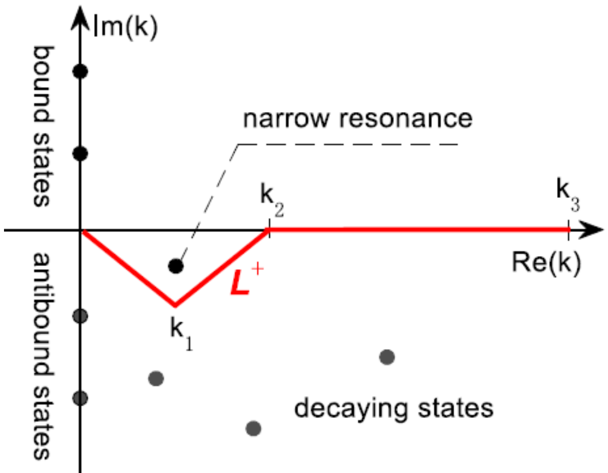


A time-dependent many-body problem

How to treat time-dependent quantum problems?

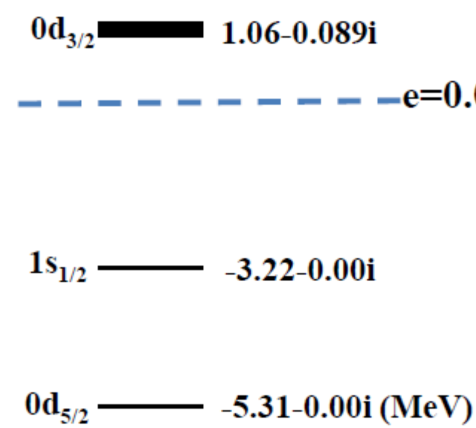
T. Berggren, Nucl. Phys. A109 (1968) 265

Complex-momentum space: **bound, resonance and continuum**

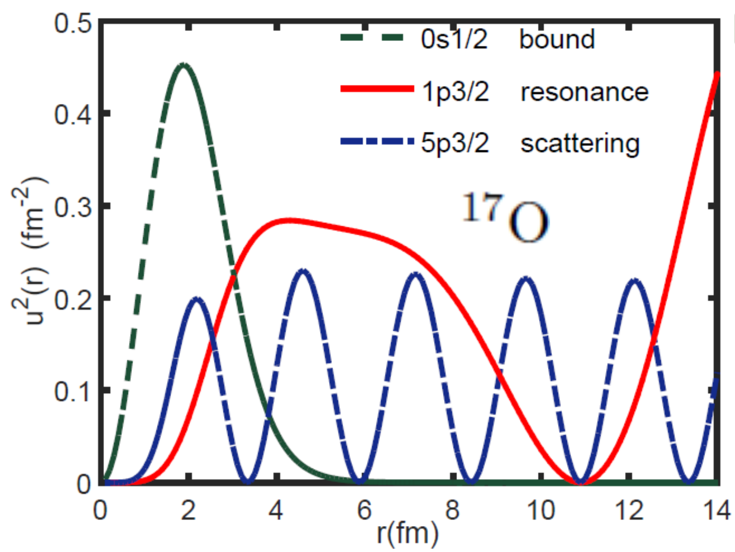


$$e = \frac{\hbar^2 k^2}{2m} = e_n - i\frac{\gamma_n}{2}$$

Woods-Saxon in complex-*k* space



WS SPE's



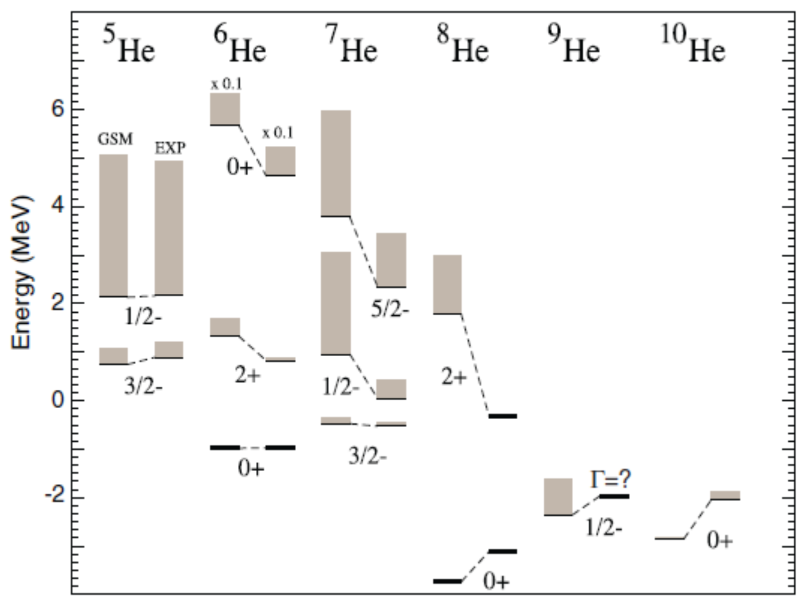
Gamow shell model for weakly bound or unbound nuclei

$$H = H_{onebody}(WS) + V_{J,T}(\vec{r}_1, \vec{r}_2)$$

$$V_{J,T}(\mathbf{r}_1, \mathbf{r}_2) =$$
$$V_0(J,T) \cdot \exp\left[- \left(\frac{\mathbf{r}_1 - \mathbf{r}_2}{\mu} \right)^2 \right] \cdot \delta(|\mathbf{r}_1| + |\mathbf{r}_2| - 2 \cdot R_0)$$

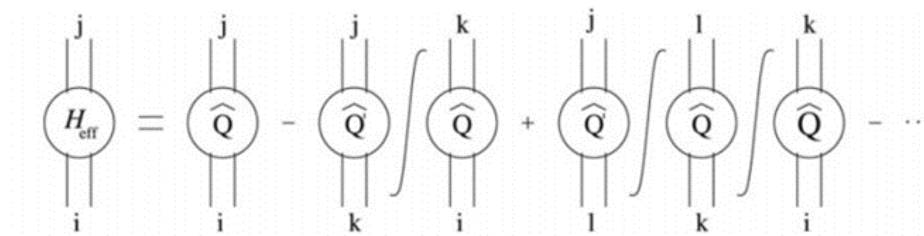
$$H = \sum_{i=1}^{A} \frac{p_i^2}{2m} + \sum_{i<j=1}^{A} v_{ij}^{NN} - \frac{\boldsymbol{P}^2}{2Am}$$

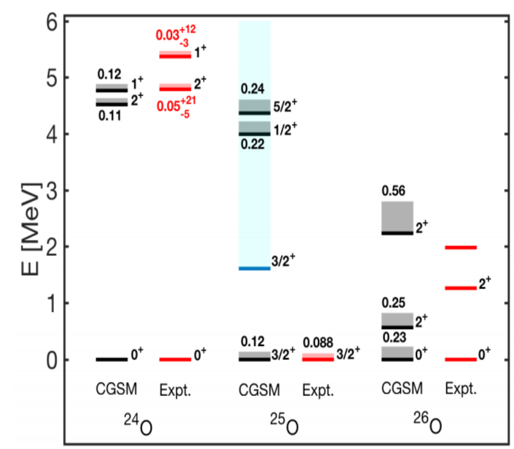
CD-Bonn MBPT

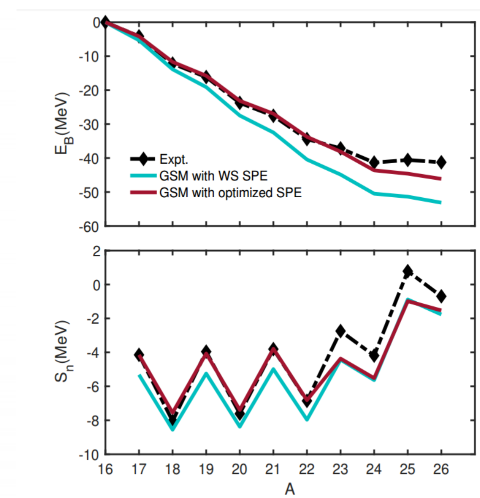


Michel, Nazarewicz, Płoszajczak, Vertse,

Phys. G: Nucl. Part. Phys. 36 (2009) 013101







Z.H. Sun, Q. Wu, Z.H. Zhao, B.S. Hu, S.J, Dai,

F.R. Xu, Phys. Lett. B 769 (2017) 227

$$H_{\rm int} = \sum_{i=1}^{A} \frac{p_i^2}{2m} + \sum_{i<j} V(|\vec{r}_i - \vec{r}_j|) - \frac{P^2}{2Am} \qquad \vec{P} = \sum_{i=1}^{A} \vec{p}_i$$

Interaction MBPT in Gamow space

Bare forces: Strong repulsion, slow convergence

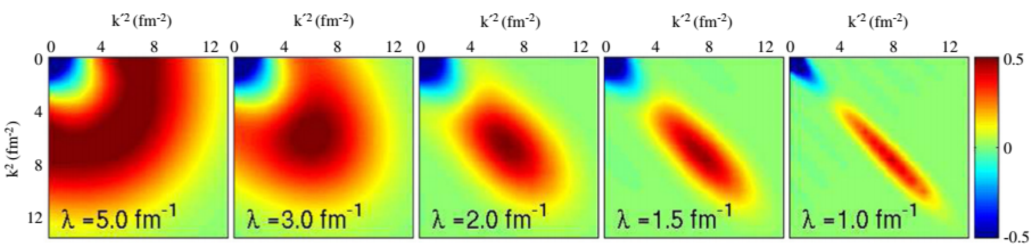
V↓low k **or SRG**

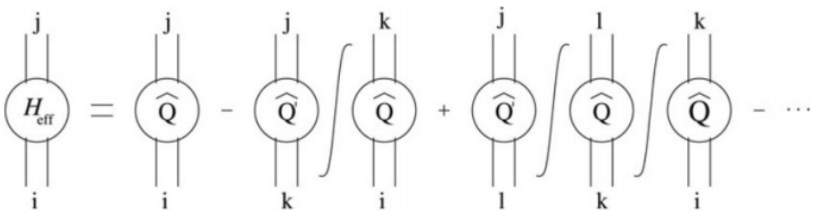
To remove hard core, but still keep good descriptions of NN scattering phase shifts

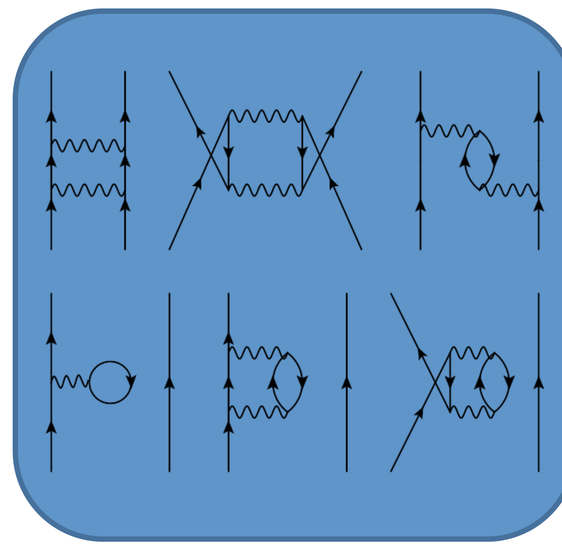
$$\langle \alpha_P | \bar{H}_{\rm eff} | \alpha_{P'} \rangle = \sum_{\alpha_{P''}} \sum_{\alpha_{P'''}} \sum_{kk'k'' \in \mathcal{K}} \langle \alpha_P | \tilde{k}'' \rangle \langle \tilde{k}'' | \alpha_{P''} \rangle \langle \alpha_{P''} | \tilde{k} \rangle E_k \langle \tilde{k} | \alpha_{P'''} \rangle \langle \alpha_{P'''} | \tilde{k}' \rangle \langle \tilde{k}' | \alpha_{P'} \rangle$$

$$\frac{dH_\lambda}{d\lambda} = -\frac{4}{\lambda^5}[[T_{\rm rel}, H_\lambda], H_\lambda]$$

Q





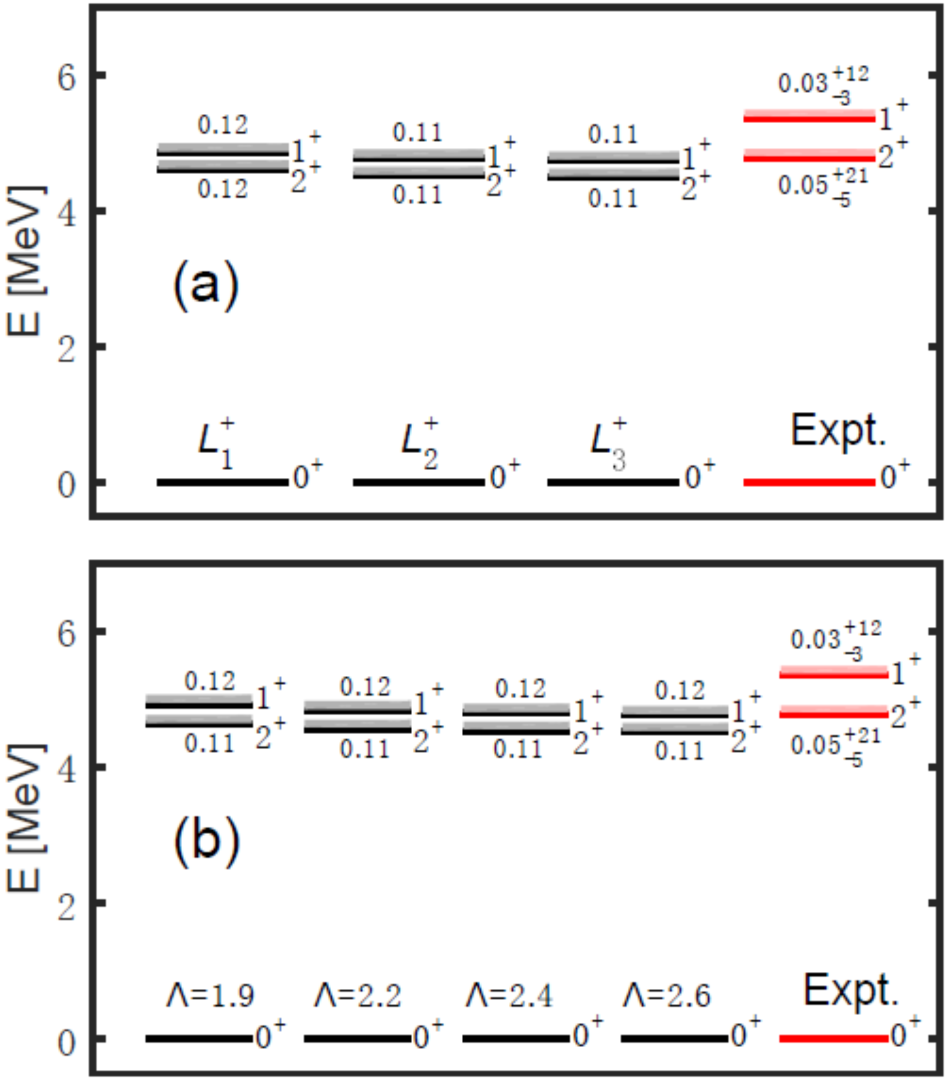


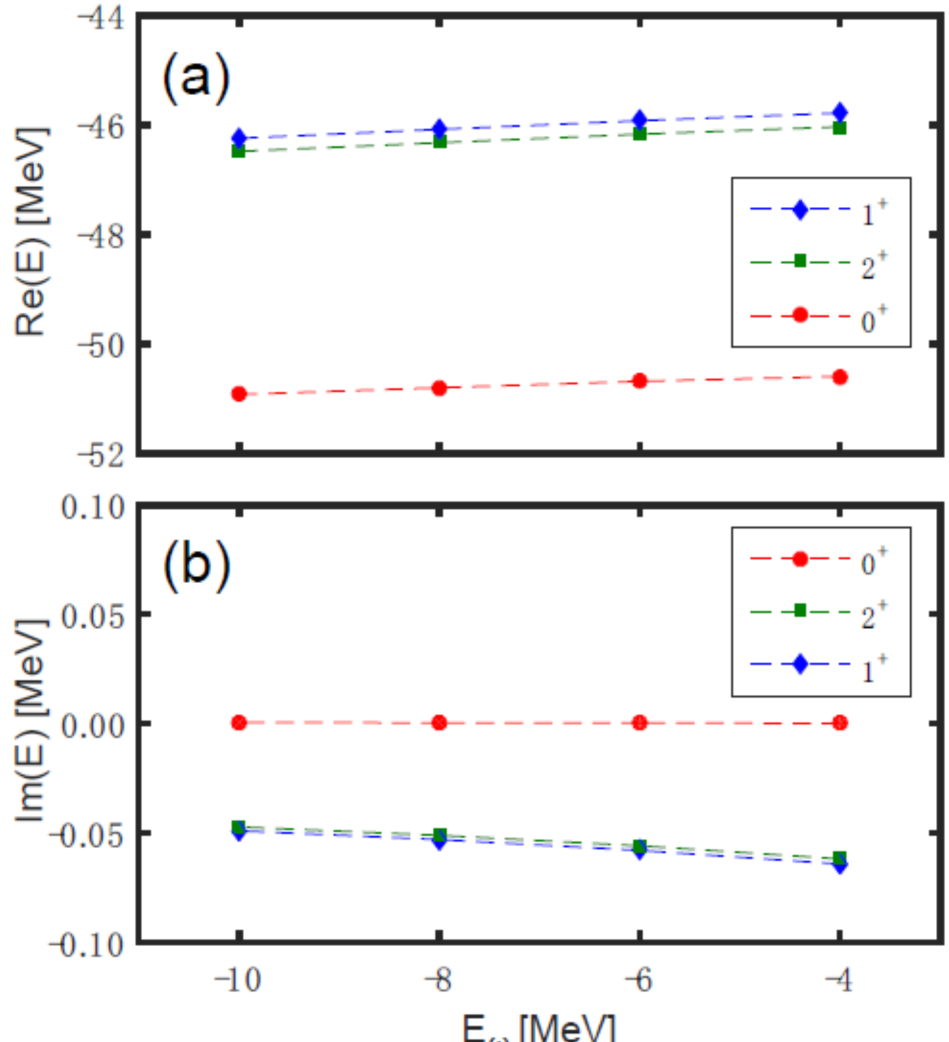
$$\hat{Q}(E) = PVP + PVQ\frac{1}{E-H}QVP,$$

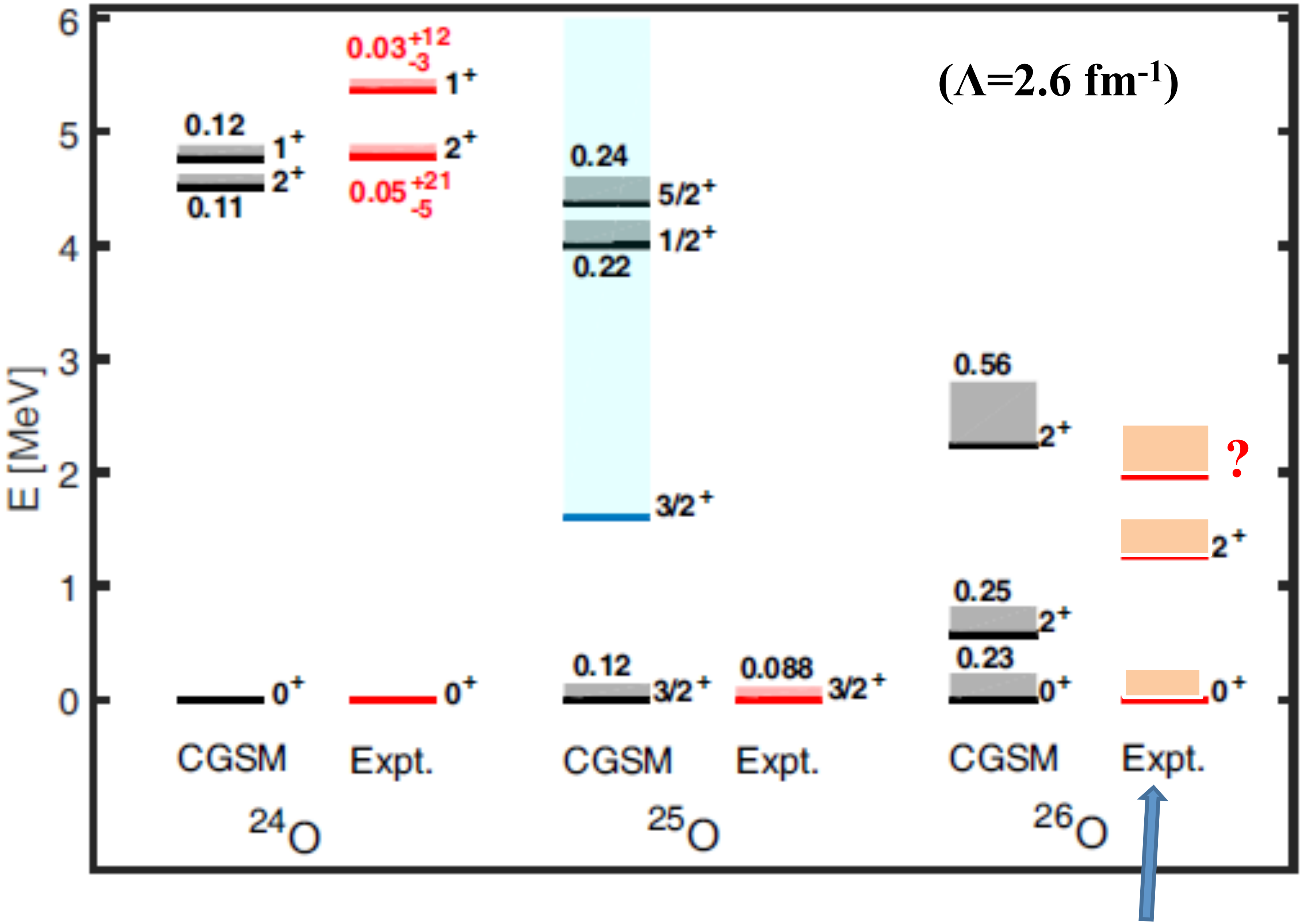
Non-degenerate extended Kuo-Krenciglowa folded-diagram method (EKK) by Takayanagi, NPA 852, 61 (2011);

Convergences of spectroscopic calculations

^{24}O

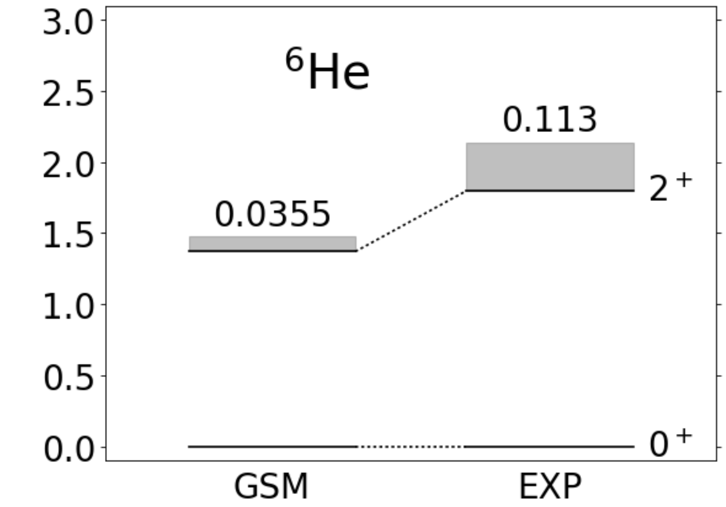


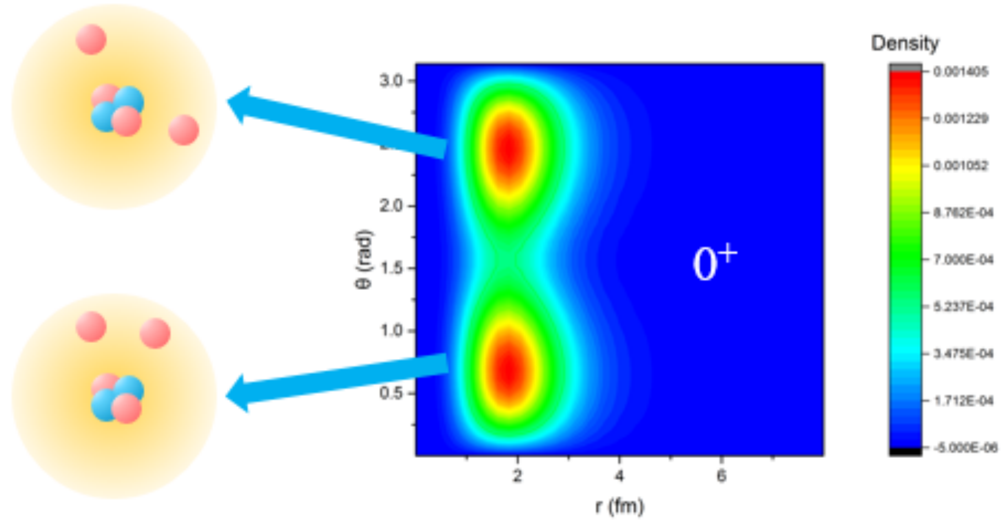


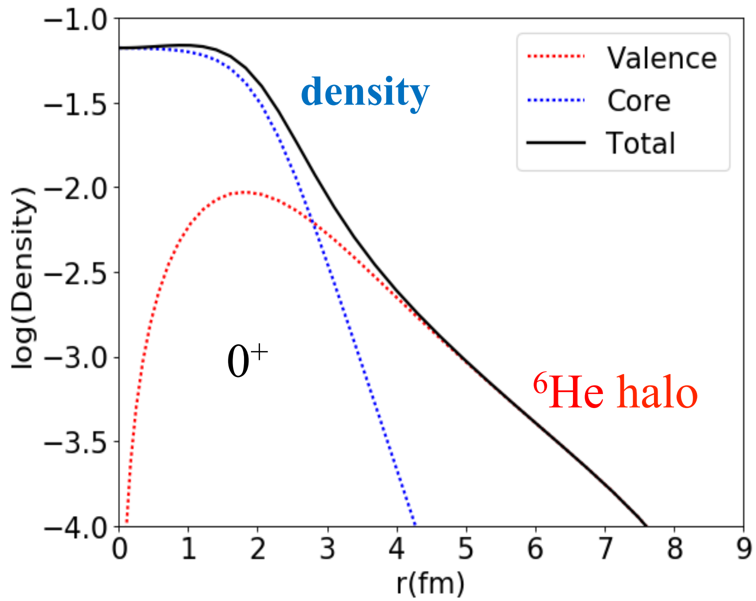


Y. Kondo *et al*., PRL 116, 102503 (2016)

^{6}He halo







^{6}He correlated density distribution

$$\rho(r,\theta) = \langle\Psi|\delta(r_1 - r)\delta(r_2 - r)\delta(\theta_{12} - \theta)|\Psi\rangle$$

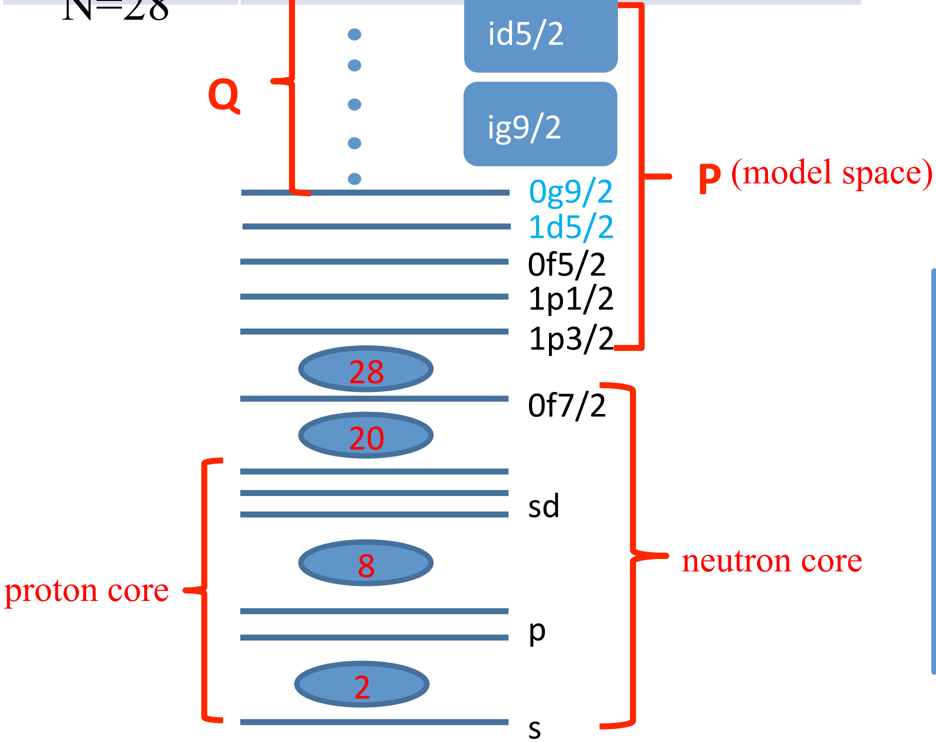
Core Gamow Shell Model (CGSM)

Single-particle energies in ^{48}Ca

parity(+ , −) : V_0= −54.60, r_0=1.15, d=0.63, V_{ls}=16.20

orbits	Expt. (MeV)	Wood-Saxon(MeV)
0g9/2		2.37 − i0.009
1d5/2		2.00 − i0.863
0f5/2	−1.153	−1.15
1p1/2	−3.123	−3.05
1p3/2	−5.146	−5.26

N=28



Witek talked on the Ca chain on Tue.

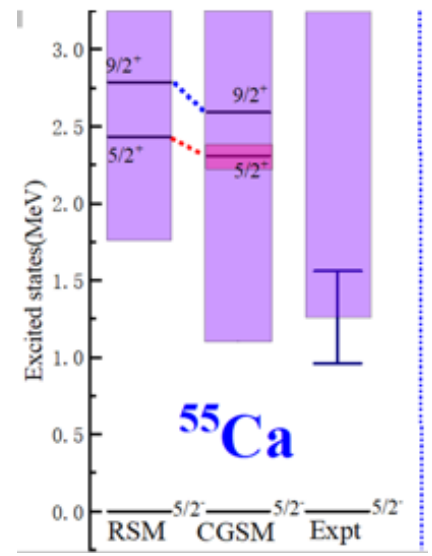
Single particle energies in ^{54}Ca

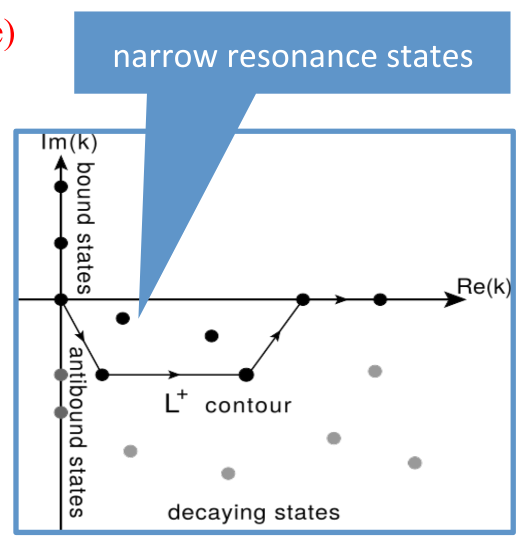
Parity + : V_0= −54.40, r_0=1.15, d=0.63, V_{ls}=10.50

Parity − : V_0= −50.30, r_0=1.15, d=0.63, V_{ls}=16.00

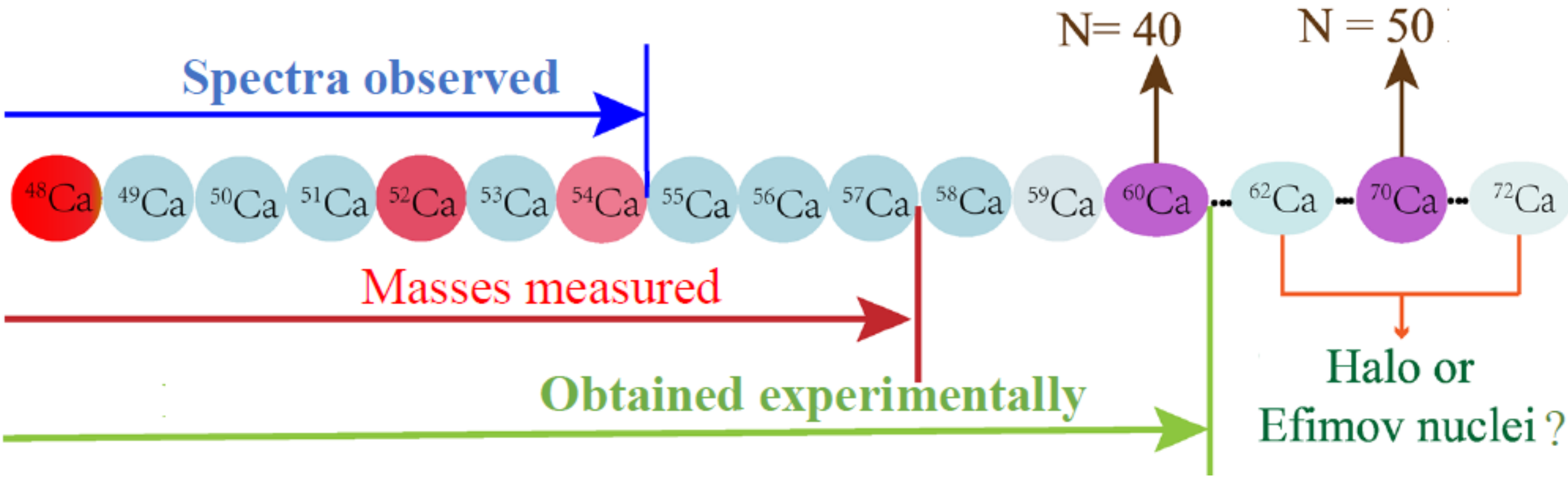
orbits	Experiment(MeV)	Wood-Saxon(MeV)
0g9/2		1.569 − i0.002
1d5/2		1.426 − i0.405
0f5/2	−(1.26±0.3) (AME2016)	−1.1388

N=34





Berggren basis

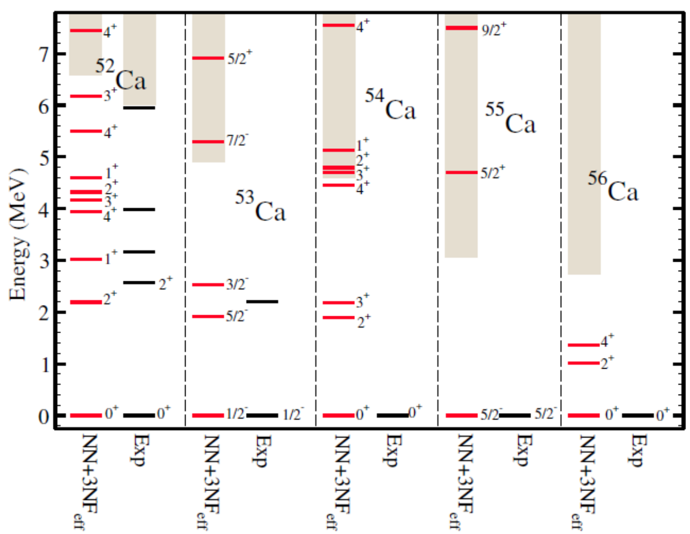


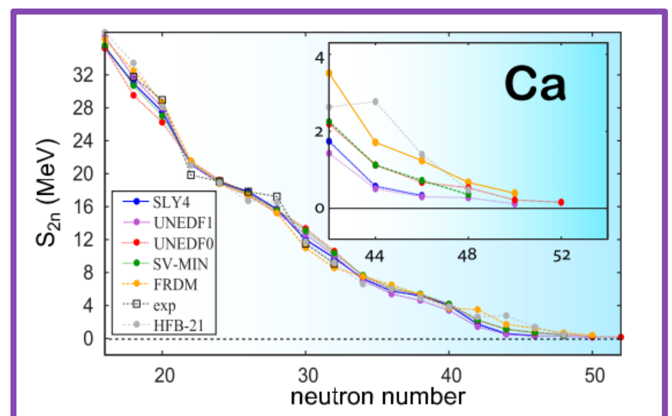
L. Coraggio *et al*., PRC 80, 044311 (2009), PRC 89, 024319 (2014): CD Bonn, MBPT

J.D. Holt *et al*., PRC 90, 024312 (2014): N3LO(NN)+N2LO(NNN), MBPT

H. Hergert *et al*., PRC 90, 041302(R) (2014): N3LO(NN)+N2LO(NNN), IM-SRG

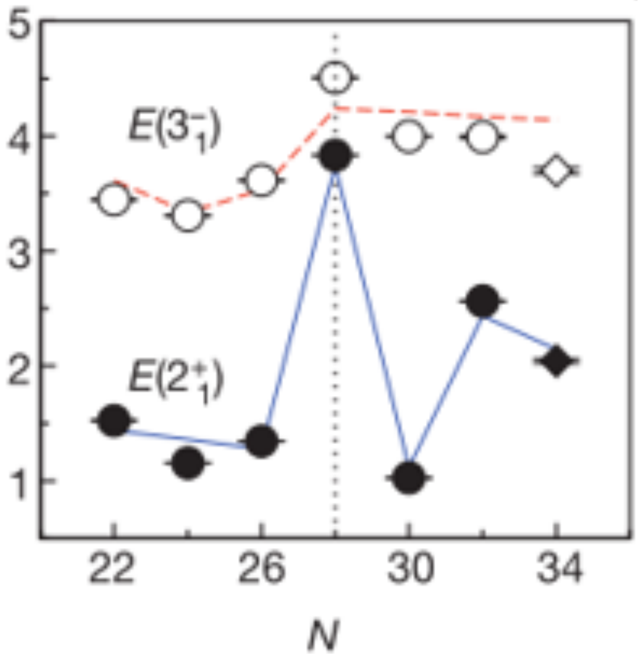
G. Hagen *et al*., PRL 109, 032502 (2012): NN+3NF$_{\mathrm{eff}}$, CC-continuum (closed shell ±1 or 2 particles)





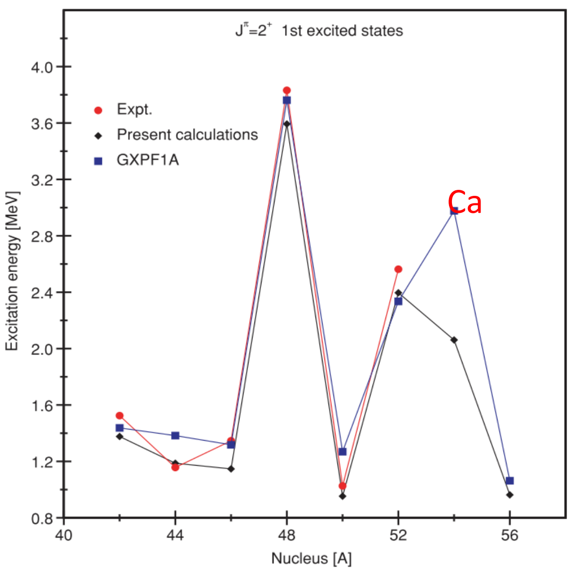
Mean-field calculations for ground states

C.Forssen *et al*., Phys. Scr. T 152, 014022 (2013)

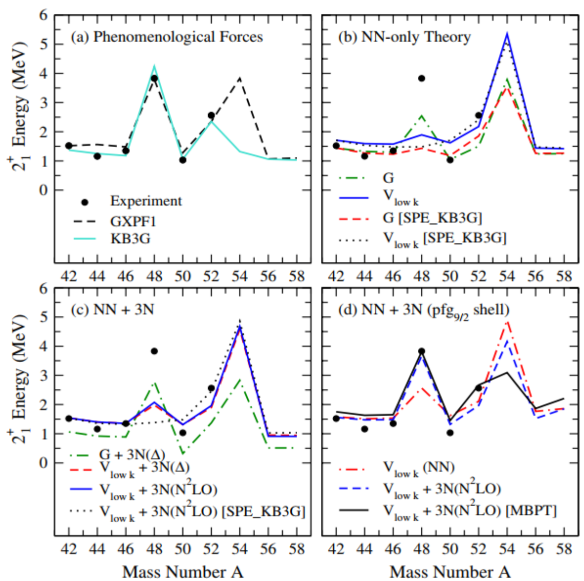


N=32, 34

Experiment at RIKEN: Steppenbeck *et al.*, Nature (2013)

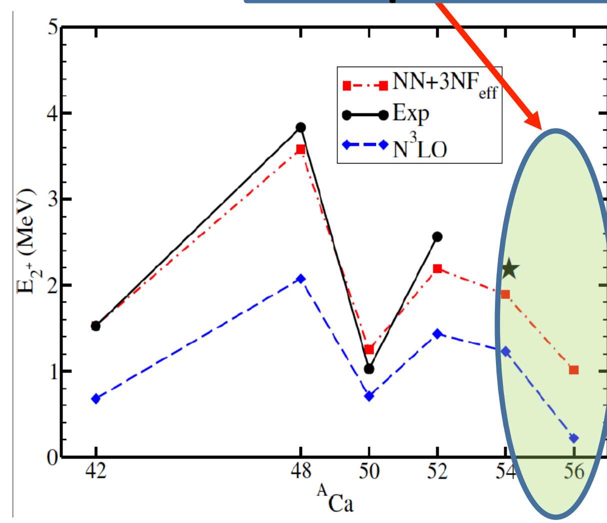


L.Caroggio *et al.*, PRC.80.044311(2009)
Q_box(Ca40,pf,HO,third order CDbonn)



J.D.Holt *et al.*, JPG/39/085111(2013)
Q_box(Ca40,pfg,HO,3rd order NN+3N)

Continuum is important



Hagen *et al.*, PRL. (2012)
Couple Cluster(continuum)

S. Michima N. Fukuda,[2]
T. Furuno, Y. Kubota,[2]
C. S. L . Sakai,[2]
Y. Shir . Yako,[1]

[3]Department 46556, USA
[5]I n
[6]Department 8580, Japan
[8]Facility 24, USA
[1] n

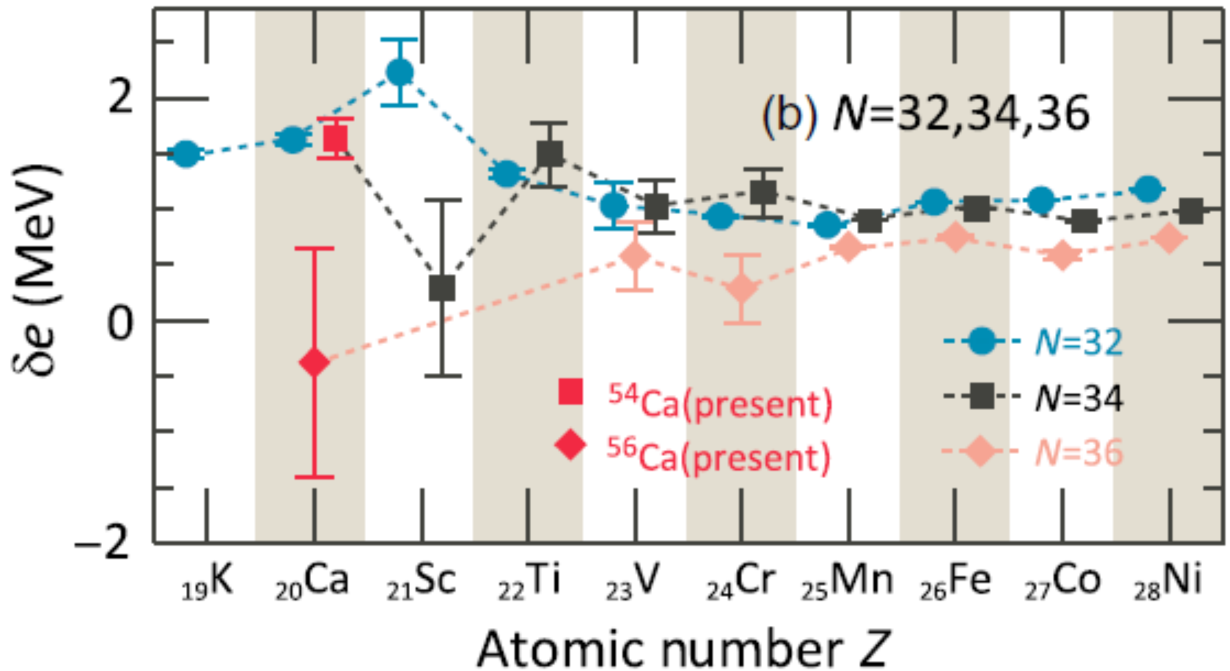
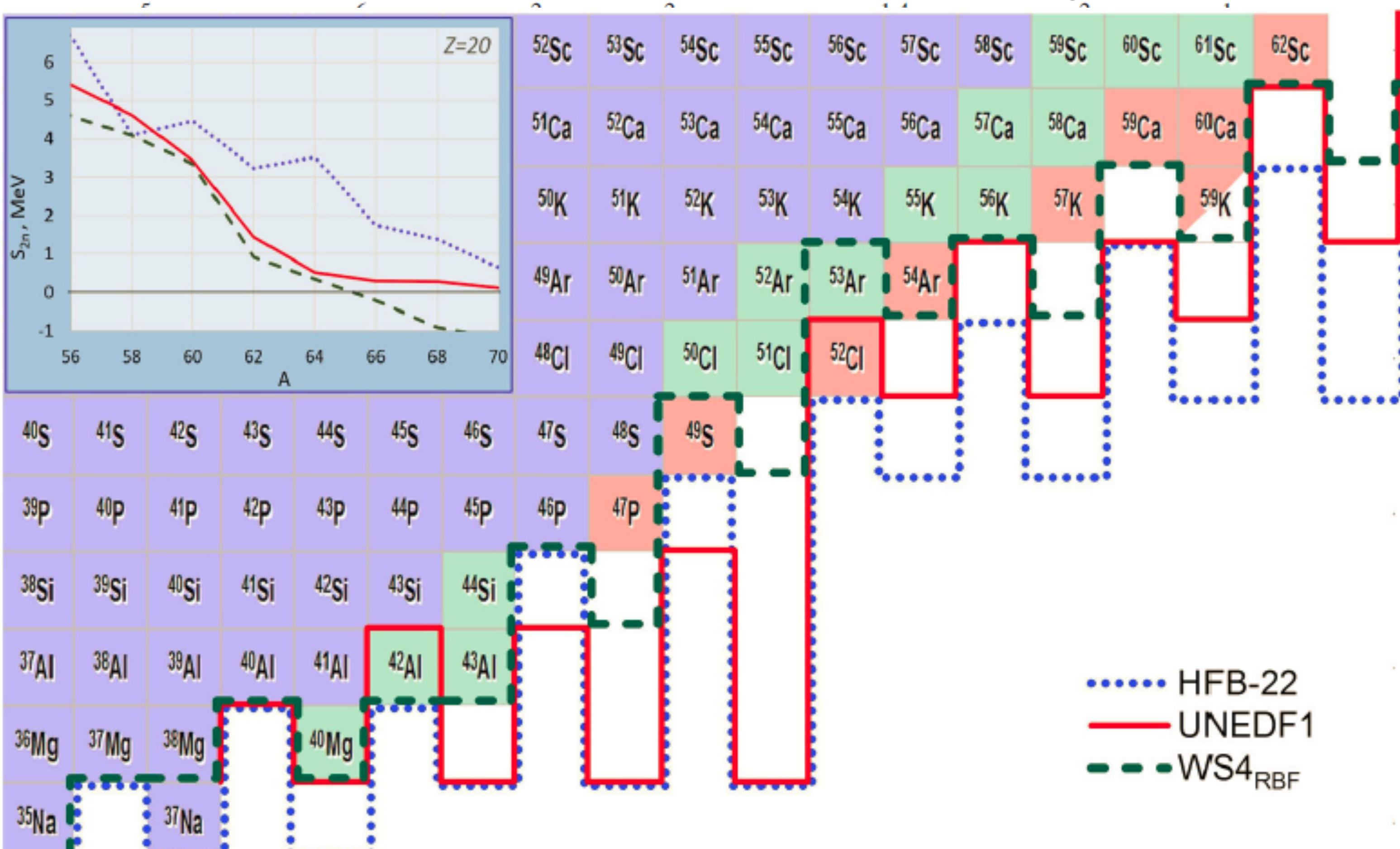


FIG. 4. Systematics of the empirical energy gaps (δe) of single-particle spectra. (a) Ca isotopes are shown with theoretical predictions. The present results are shown as red squares and the solid circles are literature values from the AME2016 database. The theoretical predictions are shown by lines with the same colors as those described in Fig. 3. (b) Isotonic chains at $N = 32$, 34, and 36 as a function of atomic number are shown. The circles, squares, and diamonds refer to $N = 32$, 34, and 36, respectively. The present results are shown as red symbols. The other values were obtained from AME2016 and Ref. [50].

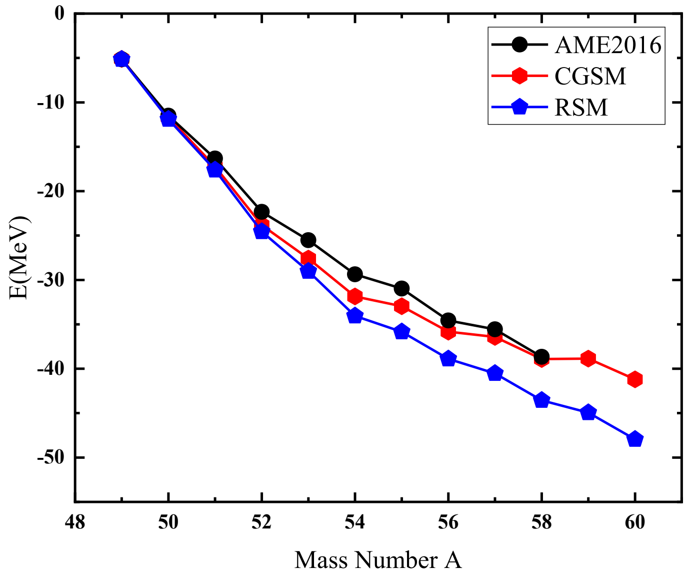
Editors' Suggestion

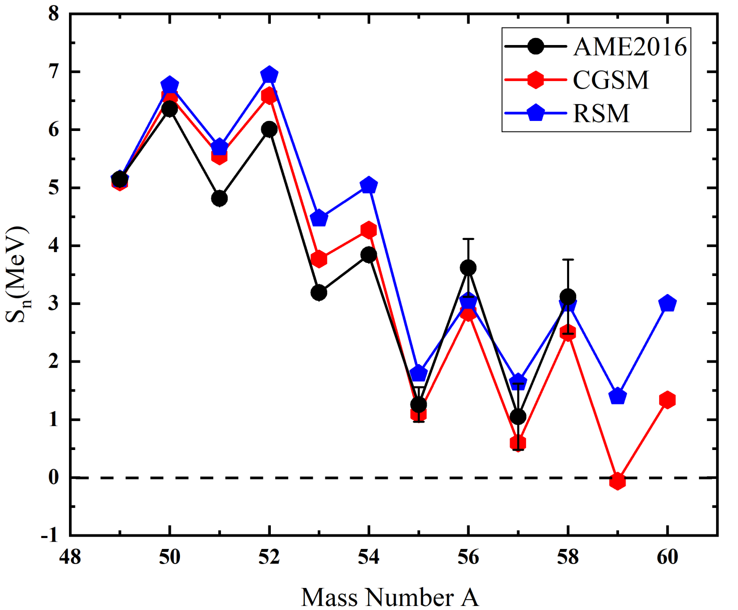
Discovery of ^{60}Ca and Implications For the Stability of ^{70}Ca

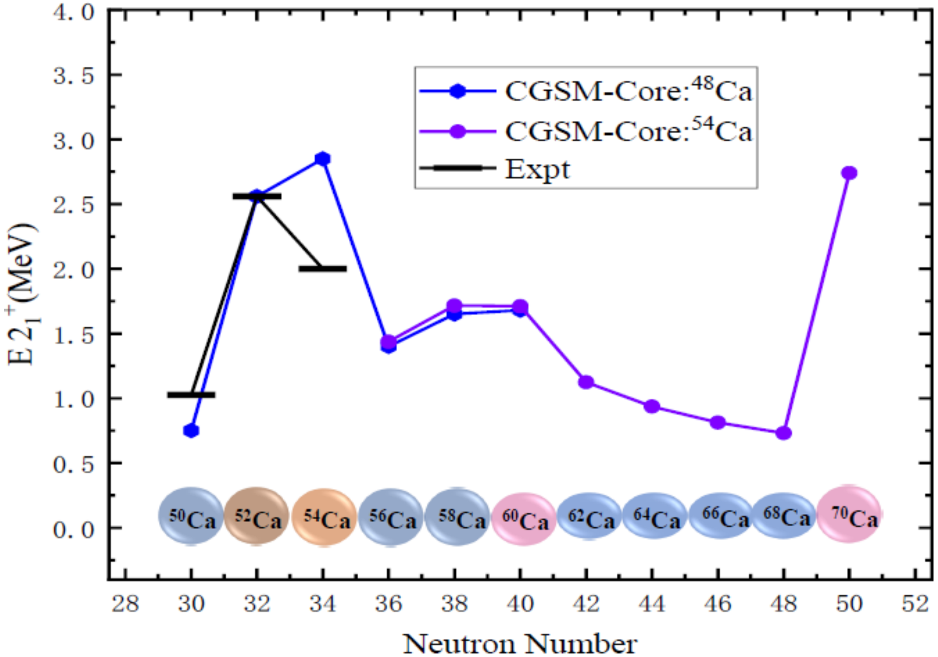
O. B. Tarasov,[1,2,3] D. S. Ahn,[2] D. Bazin,[1] N. Fukuda,[2] A. Gade,[1,4] M. Hausmann,[5] N. Inabe,[2] S. Ishikawa,[6]
N. Iwasa,[6] K. Kawata,[7] T. Komatsubara,[2] T. Kubo,[5] K. Kusaka,[2] D. J. Morrissey,[1,8] M. Ohtake,[2] H. Otsu,[2]

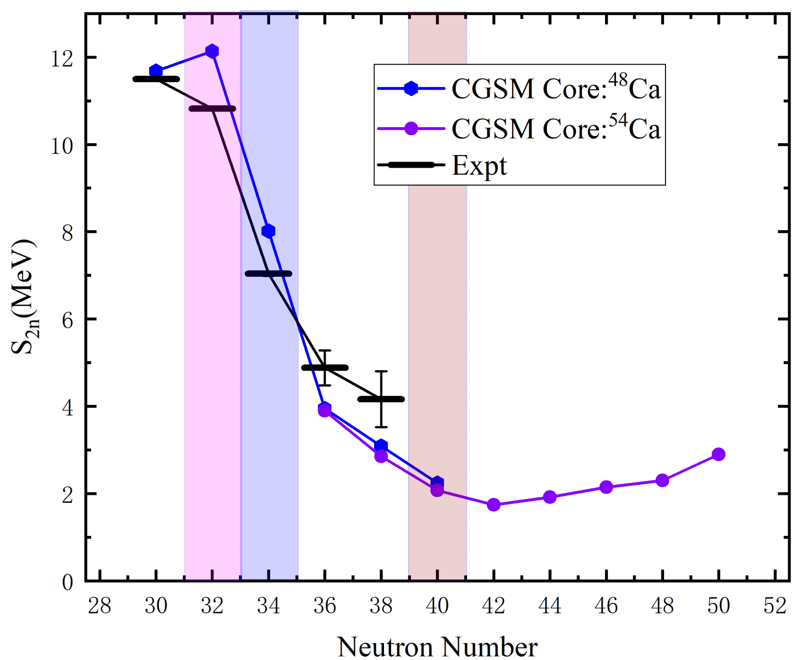


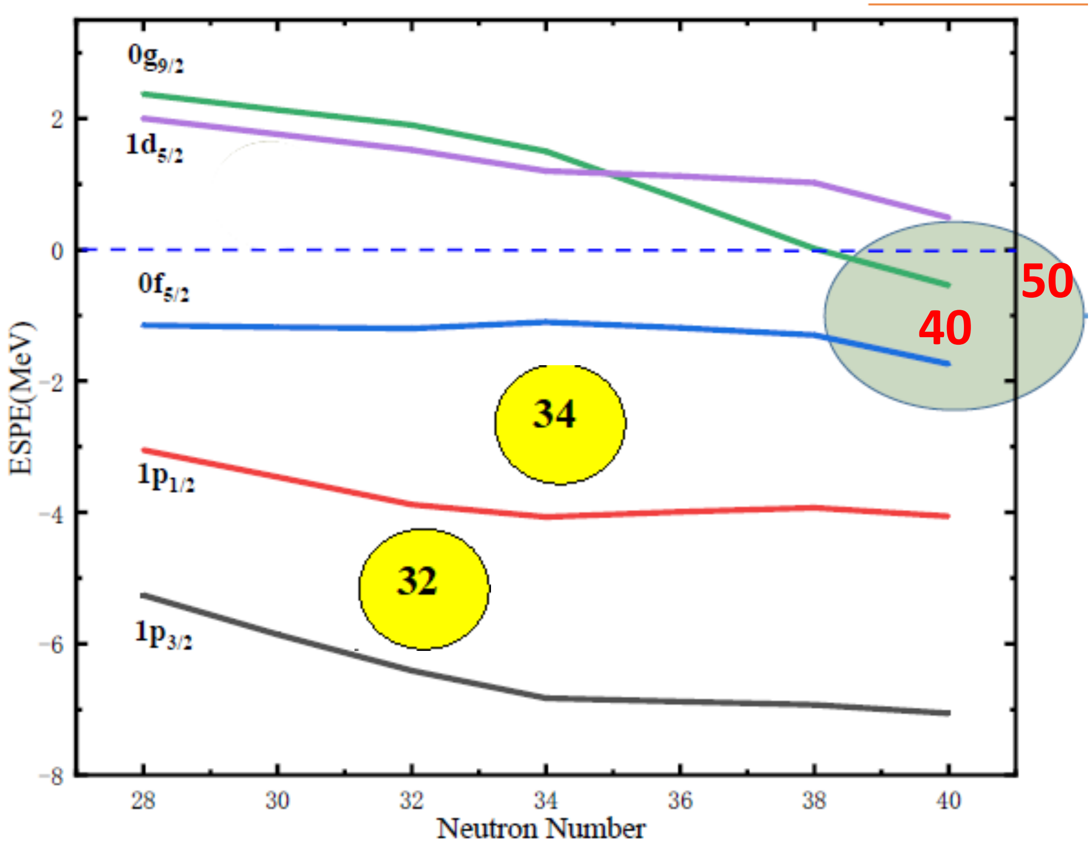
Our Core Gamow Shell Model (CGSM) with $V_{low\ k}$ CD Bonn for calcium isotopes











PHYSICAL REVIEW LETTERS **121,** 022501 (2018)

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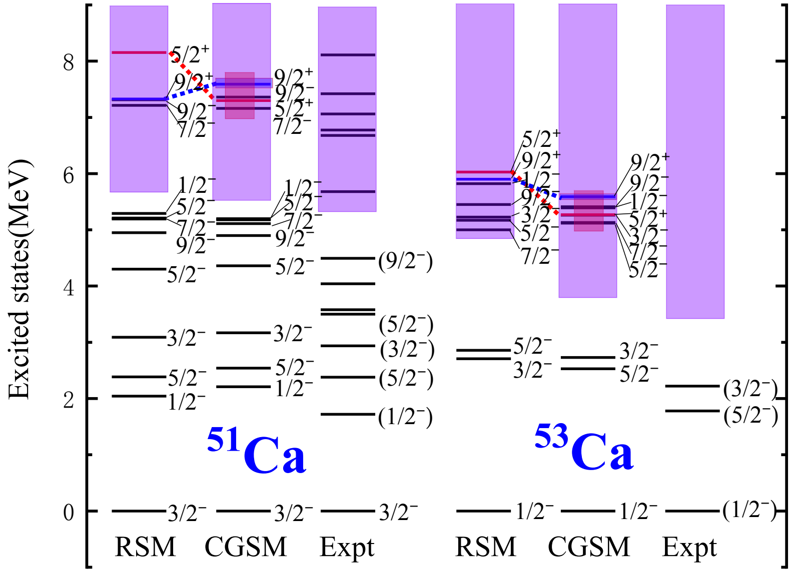
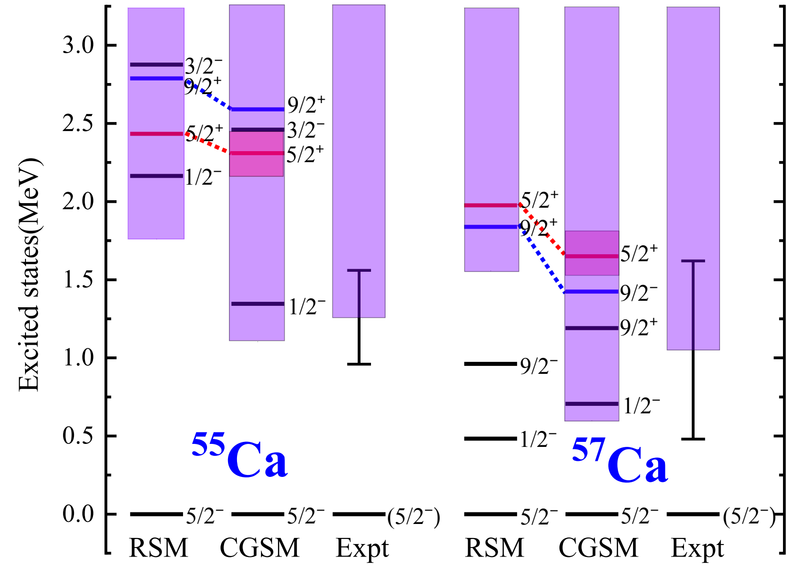
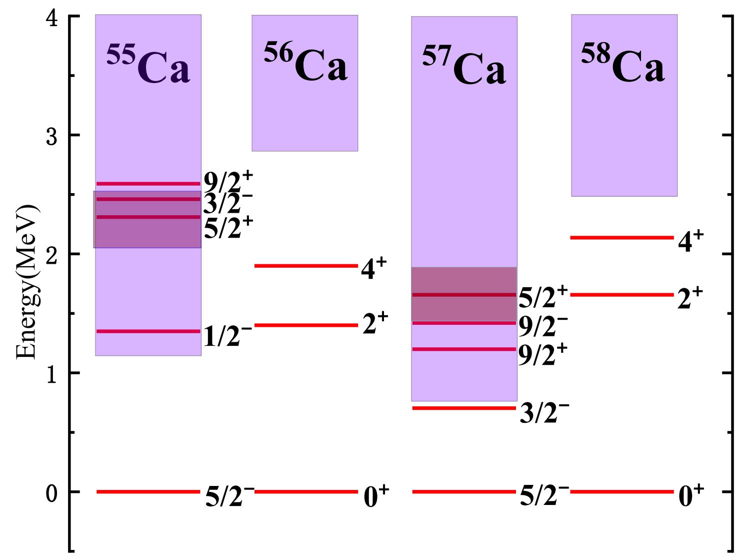




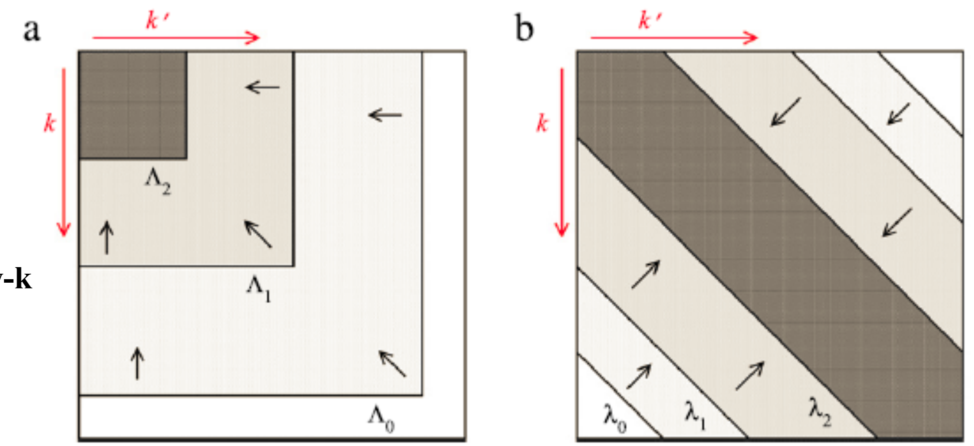
J^π	$E(^{51}Ca)$	$\Gamma(^{51}Ca)$	$E(^{53}Ca)$	$\Gamma(^{53}Ca)$	$E(^{55}Ca)$	$\Gamma(^{55}Ca)$	$E(^{57}Ca)$	$\Gamma(^{57}Ca)$
$5/2^+_1$	7.304	1.226	5.27	5.59	2.316	2.601	1.62	1.18
$9/2^+_1$	7.592	0.041	0.996	0.008	0.580	0.00	0.484	0.00



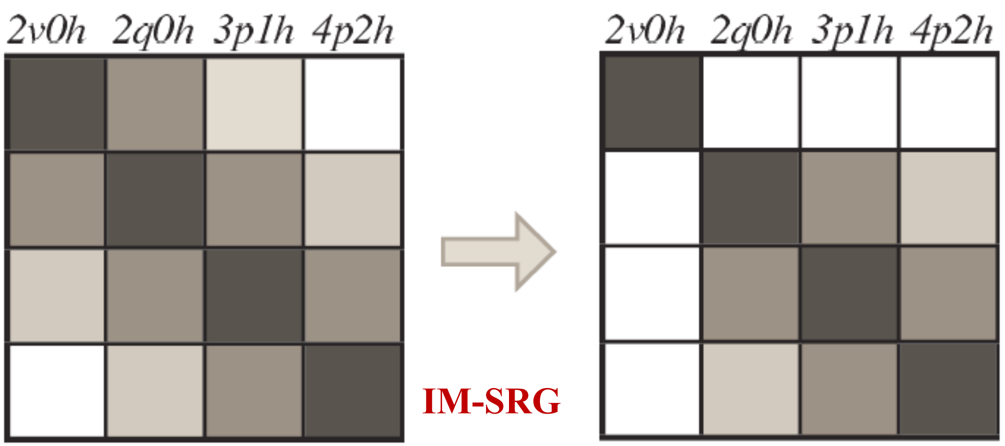
Gamow IM-SRG calculations

S.K. Bogner et al. / Progress in Particle and Nuclear Physics 65 (2010) 94–147

$V_{\text{low-k}}$



SRG



We solve IM-SRG in the complex-k space (Gamow IM-SRG)

Chiral NNLO$_{\text{opt}}$: A. Ekstrom *et al*., Phys. Rev. Lett. 110, 192502 (2013)

Gamow Hartree-Fock :

$$\langle k|h^{\text{HF}}|k'\rangle = (1-\frac{1}{A})\frac{\hbar^2}{2m}k^2\delta_{kk'} + \sum_{pq}\langle p|U_{\text{HF}}|q\rangle\langle k|p\rangle\langle q|k'\rangle$$

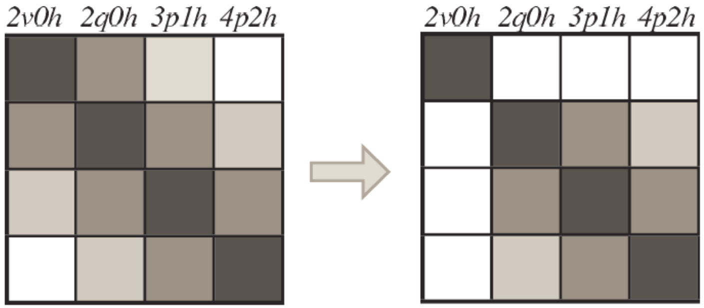
Bound, resonant and continuum HF basis states are obtained by diagonalizing the complex-energy HF Hamiltonian

SRG

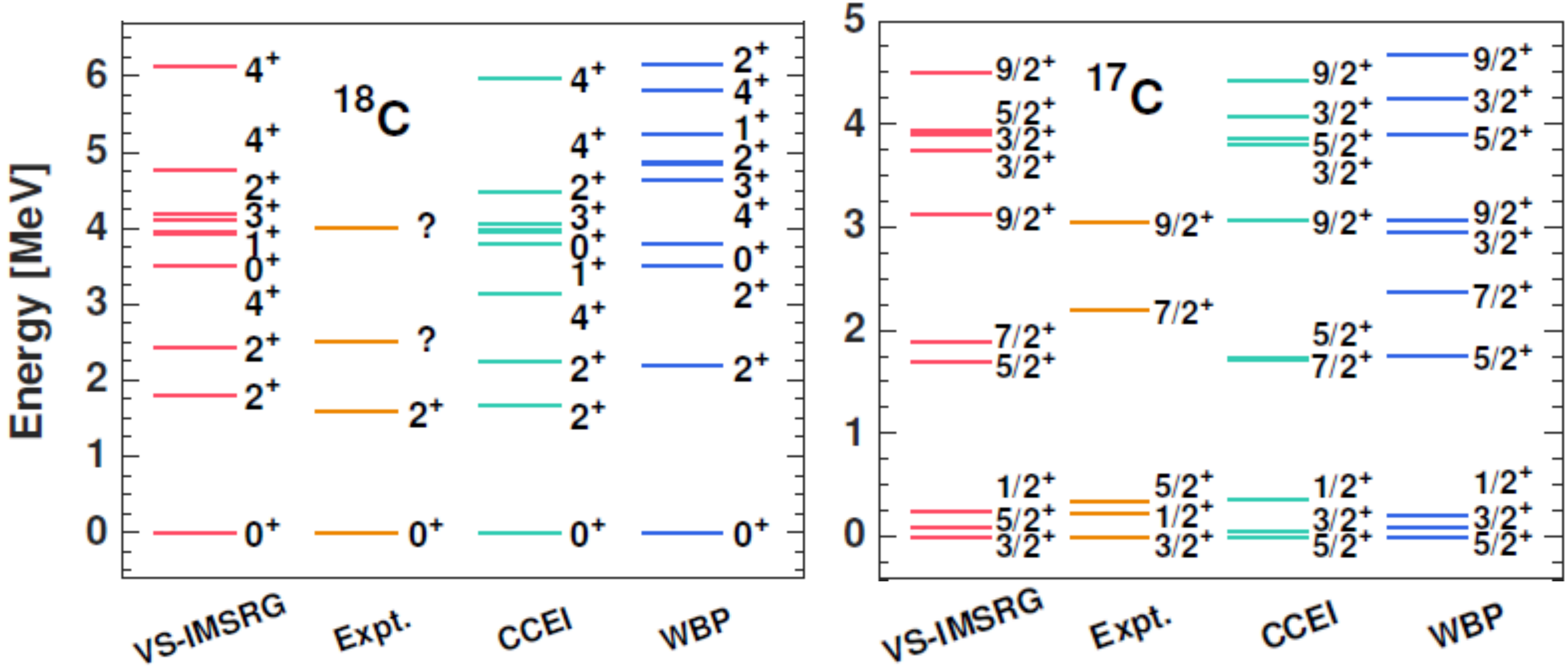
S. D. Glazek and K. G. Wilson, Phys. Rev. D 48, 5863 (1993) (quantum field theory);
F. Wegner, Ann. Phys. (Leipzig) 506, 77 (1994) (for condensed matter)

IM-SRG

K. Tsukiyama, S. K. Bogner, and A. Schwenk, Phys. Rev. Lett. 106, 222502 (2011).
H. Hergert, S. Bogner, T. Morris, A. Schwenk, and K. Tsukiyama, Physics Reports 621, 165 (2016)



Advantage: include more many-body correlations



Jansen, Engel, Hagen, Navratil, Signoracci, PRL **113**,142502 (2014).

Yuan, Suzuki, Otsuka, FRX, Tsunoda, PRC **85**, 064324 (2012).

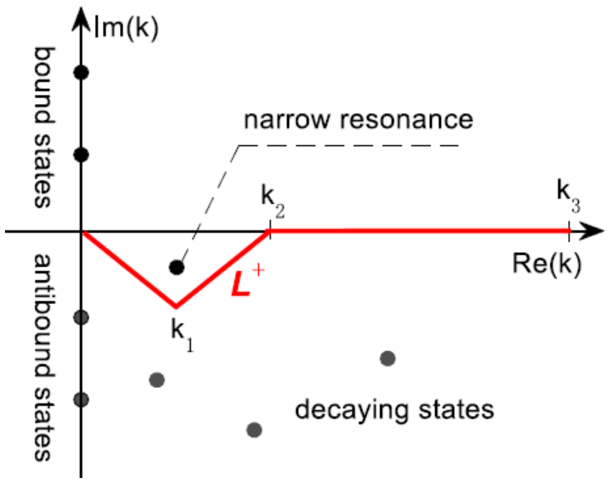
Gamow IM-SRG:

$$H(s) = U(s)H(0)U^{-1}(s) \qquad U(s)\cdot U^{-1}(s) = 1$$

Continuous similarity transformation

$$\frac{dH(s)}{ds} = [\eta(s), H(s)]$$ **Flow equation**

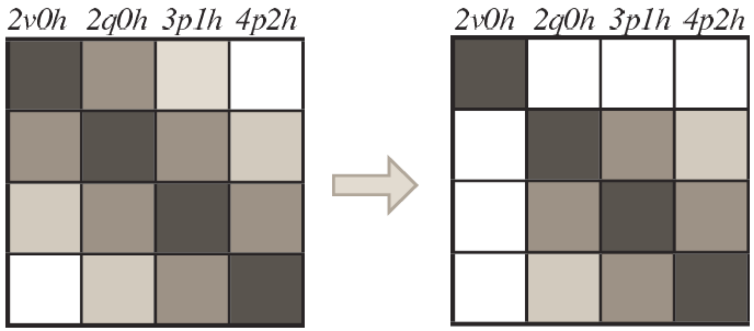
$$\eta(s) = \frac{dU(s)}{ds}U^{\dagger}(s) = -\eta^{\dagger}(s)$$

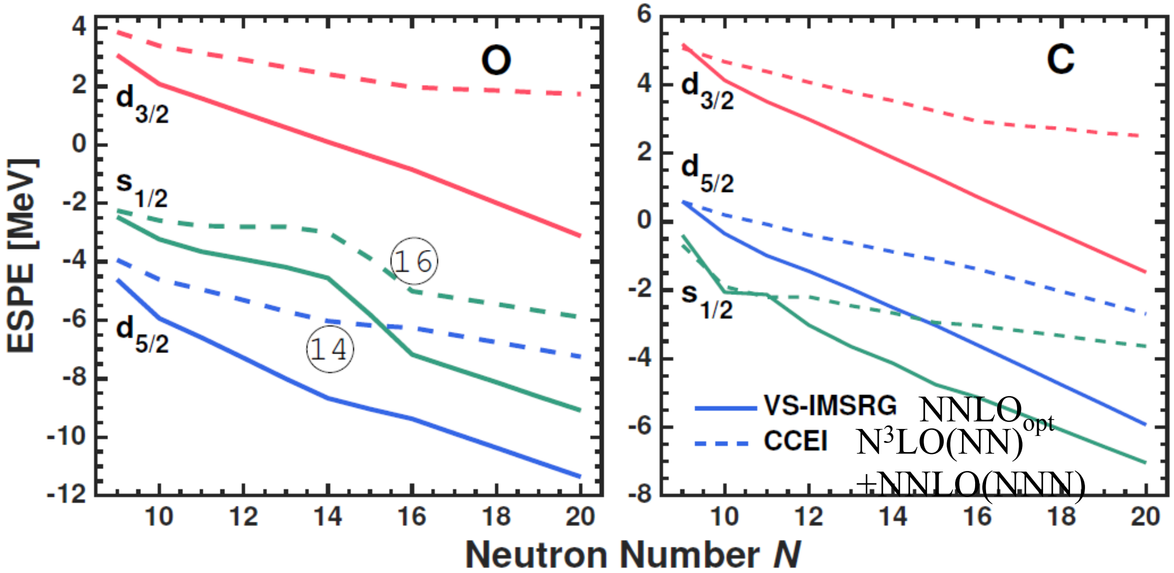


- ✓ **Directly gives g.s. energies for close-shell nuclei.**
- ✓ **For open shell or excited states, we use the Equation of Motion (EOM) Gamow IM-SRG + EOM :**

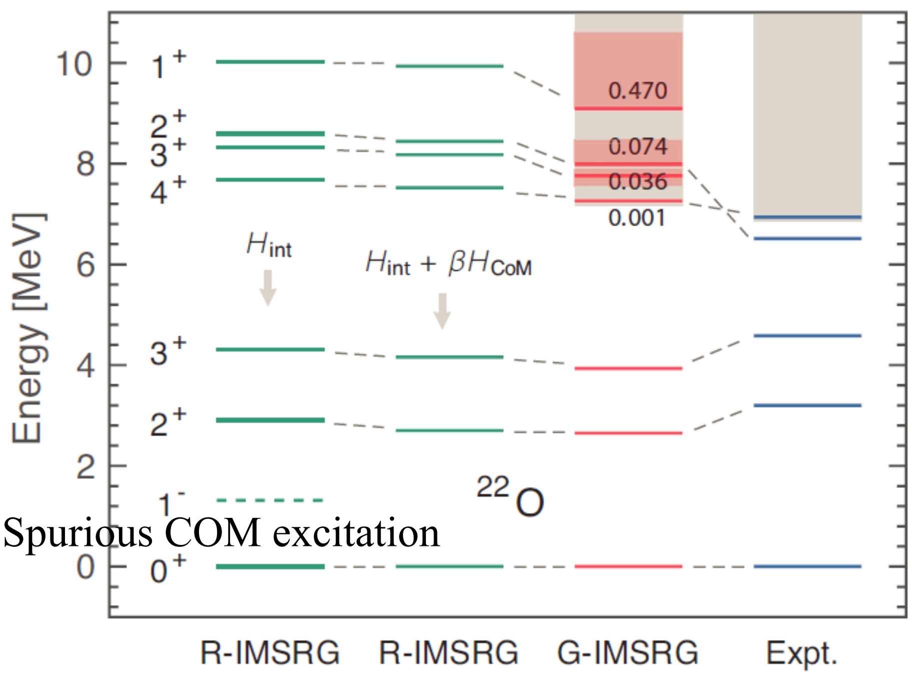
i) Gamow IM-SRG decouples the space from the closed-shell HF Fermi;

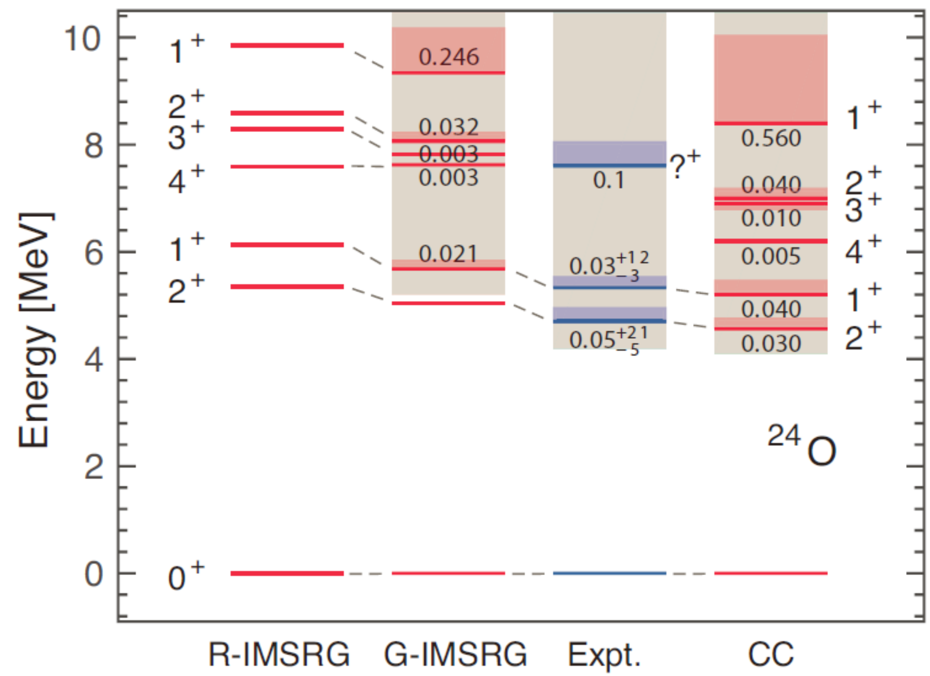
ii) Use EOM to calculate excited states



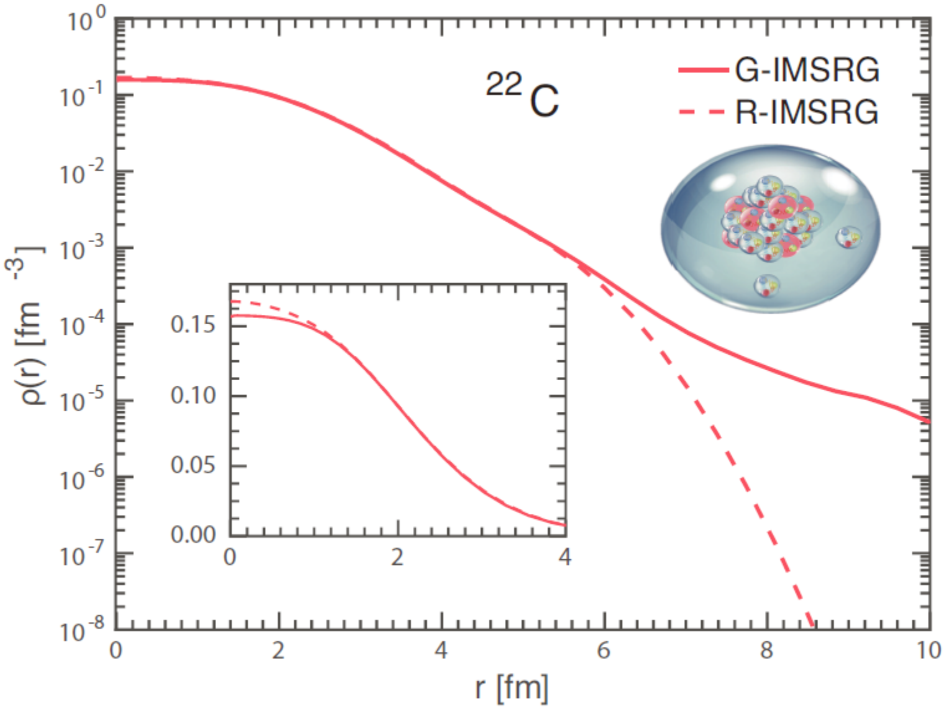


N=14 shell closure disappears in the Carbon chain





Hagen, *et al*., PRL. **108**, 242501 (2012)



1. Gamow IMSRG with discrete HF *s*-waves: 2.79 fm
2. Gamow IMSRG with continuum *s*-waves: 3.06 fm
3. Experimentally estimated matter radius: 3.44±0.08 [Y. Togano *et al.*, PLB **761**, 412 (2016)]

$\mathrm{NNLO_{opt}}$ itself underestimates the radii of carbon isotopes

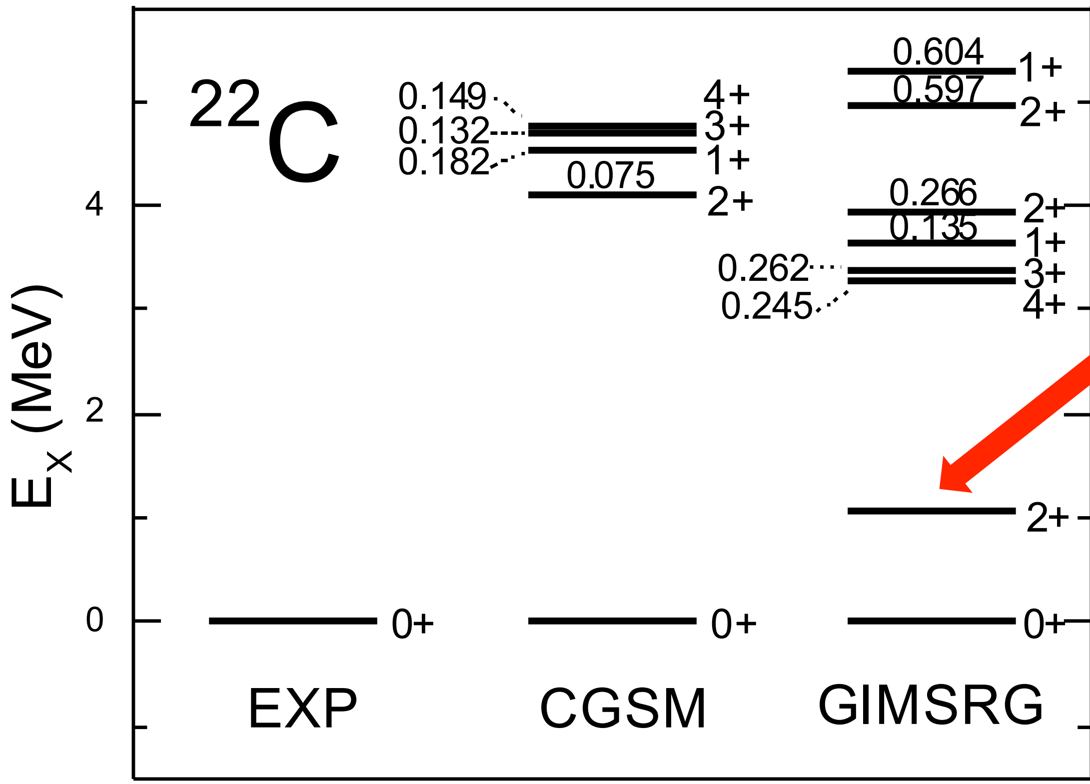
R. Kanungo *et al.*, Phys. Rev. Lett. 117, 102501 (2016)

TABLE I. Excited states for ^{22}C predicted by the Gamow EOM-IMSRG with the chiral NNLO$_{\text{opt}}$ interaction. Energies and widths are in MeV.

J^π	2_1^+	4_1^+	3_1^+	1_1^+	2_2^+	2_3^+	1_2^+
E_{IMSRG}	1.05	3.26	3.36	3.62	3.93	4.94	5.27
Γ_{IMSRG}	0.000	0.245	0.262	0.135	0.266	0.597	0.604

g.s. and 2_1^+ are bound

unbound resonances



Proton excitation in Gamow IM-SRG

PHYSICAL REVIEW C **98**, 044320 (2018)

Gogny-force-derived effective shell-model Hamiltonian

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$$V_{NN,12} = \sum_{i=1}^{2} e^{-(\vec{r}_1-\vec{r}_2)^2/\mu_i^2} \times (W_i + B_i P^\sigma - H_i P^\tau - M_i P^\sigma P^\tau) + t_3\delta(\vec{r}_1 - \vec{r}_2)(1 + x_0 P^\sigma)\left[\rho\left(\frac{\vec{r}_1+\vec{r}_2}{2}\right)\right]^\alpha + iW_0\delta(\vec{r}_1 - \vec{r}_2)(\vec{\sigma}_1 + \vec{\sigma}_2)\cdot\vec{k}'\times\vec{k},$$

$$e^{-(\vec{r}_1-\vec{r}_2)^2/\mu_i^2}$$

$$e^{-\vec{k}^2\mu_i^2/4}.$$

$$e_j = t_j + \frac{1}{2(2j+1)}\sum_{j^c}\sum_{JT}(2J+1)(2T+1) \times \langle jj^cJT|V|jj^cJT\rangle,$$

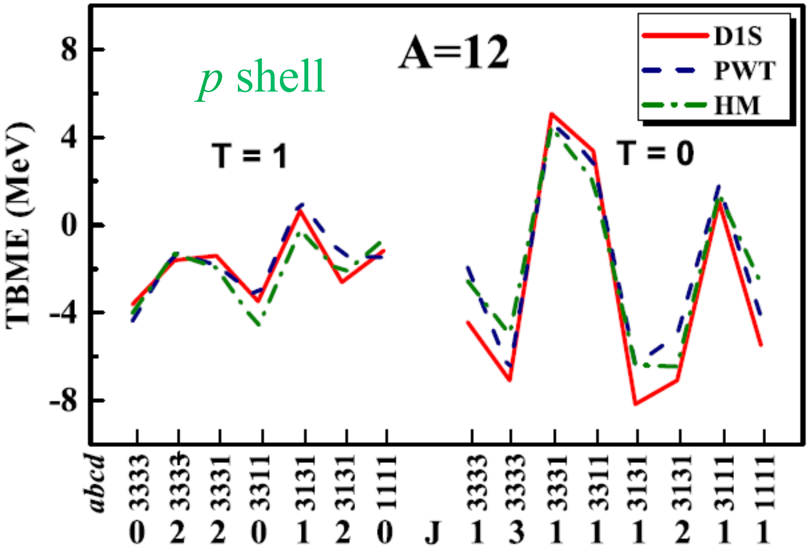
$$E_{\text{g.s.}} = E_v + E_{\text{Coul}} - t_{\text{COM}} + E_c$$

$$E_c = t_c + V_c$$

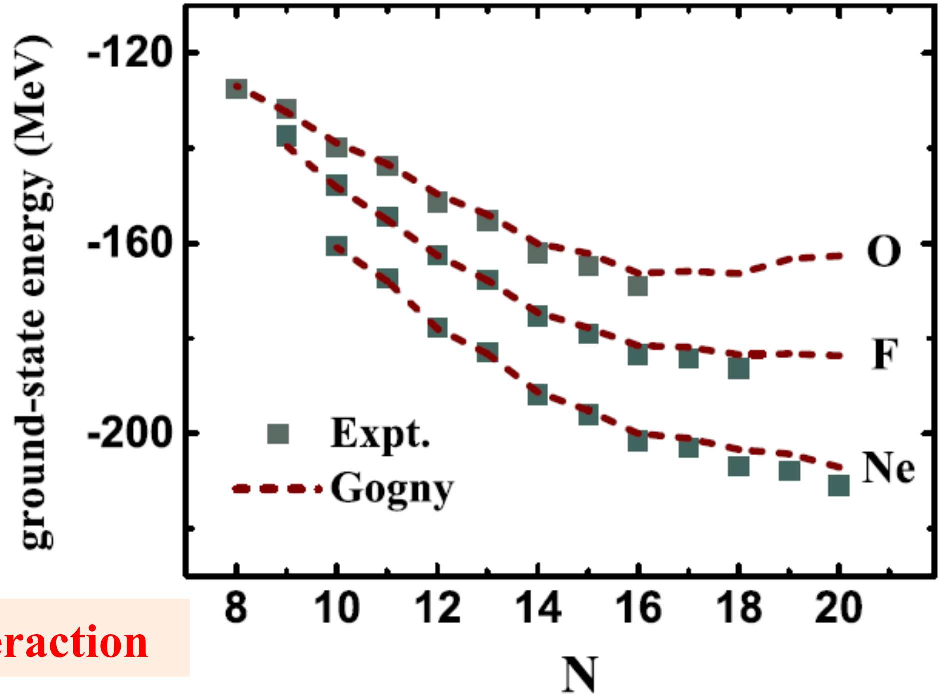
$$t_c = \sum_{j_a^c}(2T+1)(2J+1)\langle j_a^c|\hat{t}|j_a^c\rangle$$

$$V_c = \sum_{j_a^c\leqslant j_b^c}\sum_{JT}(2T+1)(2J+1) \times \langle j_a^c j_b^c JT|V|j_a^c j_b^c JT\rangle.$$

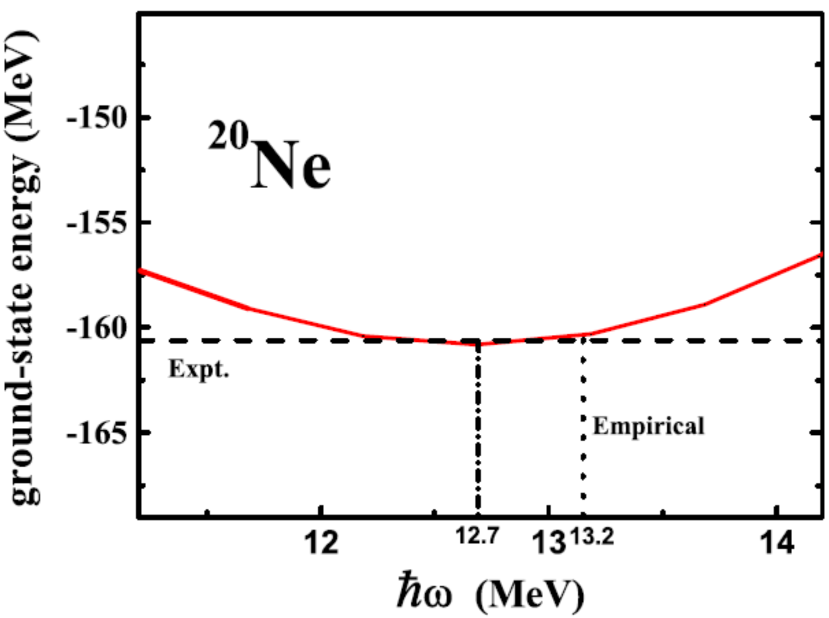
1. Easily go to cross-shell calculations
2. To heavy mass regions



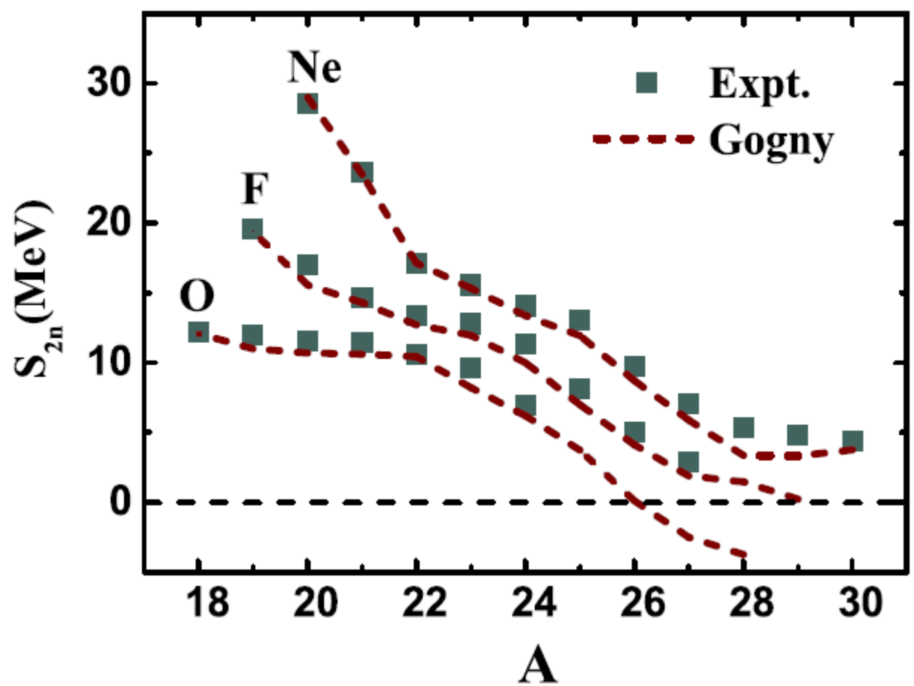
Hauge-Maripuu,

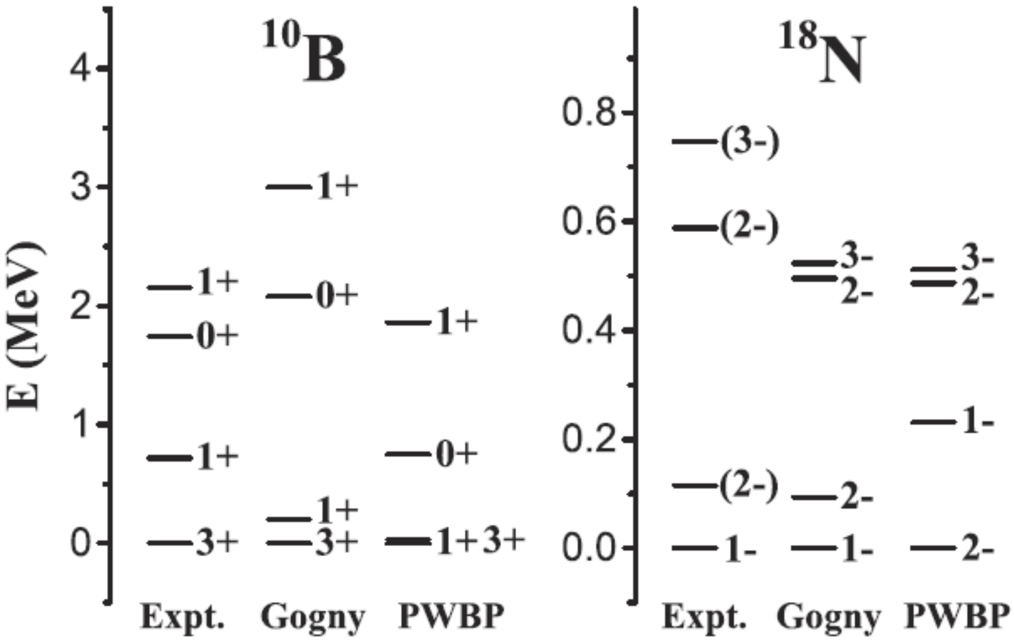


D1S interaction



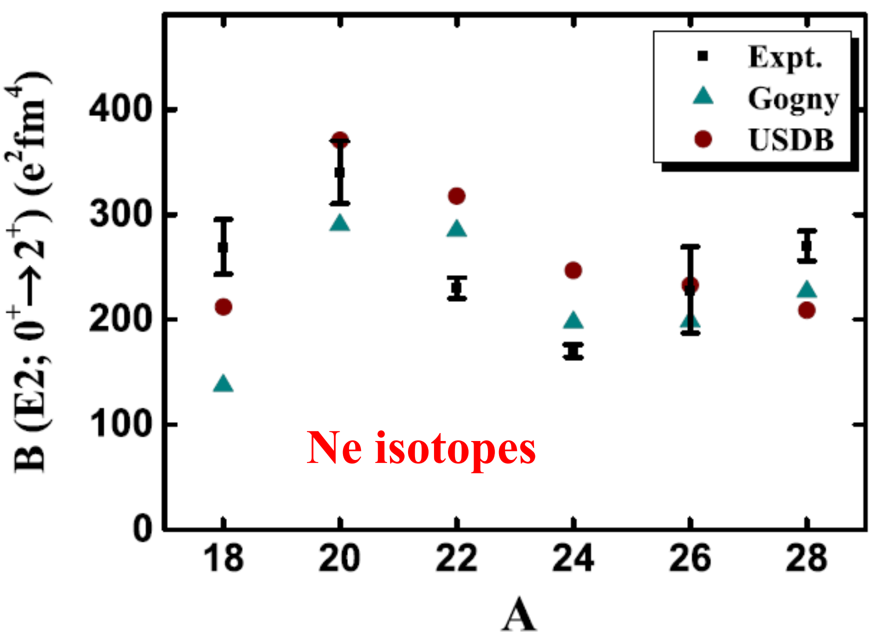
$$\hbar\omega = 45A^{-1/3} - 25A^{-2/3}$$





Important density dependence

$$t_3\delta(\vec{r}_1-\vec{r}_2)(1+x_0P^\sigma)\left[\rho\left(\frac{\vec{r}_1+\vec{r}_2}{2}\right)\right]^\alpha$$



Thank you for your attention

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