Nucleon-Deuteron Scattering with Chiral Semilocal Coordinate Space and Momentum Space Regularized Interactions

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Abstract

The nucleon-deuteron elastic scattering reaction is investigated using two chiral models of the two-body interaction with semilocal regularization proposed by E. Epelbaum *et al.* and by P. Reinert *et al.* In particular, we give predictions for the differential cross section and the deuteron tensor analyzing power T_{22} at energies of the incoming nucleon up to 200 MeV. Both models deliver a qualitatively similar description of the nucleon-deuteron elastic scattering data. However, we find the model by P. Reinert *et al.* to be less sensitive to the values of regularization parameters used. For this interaction the long-standing problem of the cut-off dependence of three-nucleon predictions is practically absent, as the uncertainty of studied observables related to regularization parameters remains below 1% at most of the scattering angles. Only in the worst case of T_{22} at $E = 200$ MeV and for specific scattering angles this uncertainty amounts to 7%.

Keywords: *Nuclear forces; chiral effective field theory; nucleon-deuteron scattering; few-body systems*

1 Introduction

The Chiral Effective Field Theory (χEFT) has dominated studies of the nuclear forces since the beginning of the 21st century. The ideas introduced by S. Weinberg [1–3], J. Gasser and H. Leutwyler [4,5], C. Ordónež and U. van Kolck [6], and many others (see Refs. [7,8] for a historical background and a general introduction to the χEFT), resulted in various models of the nuclear interaction [9–14]. The two recent models from this group are the chiral interaction with semilocal regularization performed in the coordinate space (SCS) [12,13] and the chiral interaction with semilocal regularization applied in the momentum space (SMS) [14]. The nucleon-nucleon (NN) interaction has been derived completely up to the fifth order of the chiral expansion (N4LO) in both models. Additionally, in the SMS interaction also some terms from the sixth order of the chiral expansion have been incorporated. This, together with other improvements introduced to the SMS model, has resulted in the best NN data

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http://www.ntse.khb.ru/files/uploads/2018/proceedings/Skibinski.pdf.

description obtained so far, achieving $\chi^2/data \approx 1$ (see Ref. [14] for more specific values, which depend on the energy range, isospin channel, order of chiral expansion and values of regulator parameters).

The applications of both models beyond the two-nucleon system are ongoing. They are more advanced for the SCS interaction which has been already used to study the nucleon-deuteron scattering and properties of the light and medium mass nuclei [15–18] as well as to investigate the deuteron and ^{3}He photodisintegrations, the nucleon-deuteron radiative capture, and muon capture in the deuteron and ^{3}He [19]. Summarizing these studies one can conclude that the SCS interaction shows its good quality also in many-nucleon systems. Using only the two-body interaction, a nice convergence with respect to the chiral order is observed for all observables. The cut-off dependence, i. e., the dependence on the regulator parameter value, is small, however for the nucleon-deuteron scattering at higher energies and the energies of nuclear states it is not negligible. The three-nucleon interaction consistent with the SCS two-body force has been applied so far only up to the next-to-next-to-leading order (N2LO) to study elastic nucleon-deuteron scattering and structure of chosen nuclei. At this order of chiral expansion, where only the leading contributions to the three-nucleon interaction are present, the SCS model describes data with a precision similar to the semi-phenomenological models, like the combination of the AV18 NN force [20] with the Urbana IX three-nucleon interaction [21] discussed in detail, e. g., in Ref. [22]. Combining the chiral N2LO three-nucleon force with the N4LO two-nucleon interactions does not change that picture [18]. This shows that to improve the data description, the three-nucleon force at higher orders of chiral expansion has to be included. Unfortunately, an explicit regularization of the chiral three-nucleon force in the coordinate space is very challenging and has not been done beyond the N2LO order yet. The regularization in the momentum space gives more hope. While currently the three-nucleon force consistent with the NN interaction with the semilocal regularization in the momentum space is under construction, the application of the two-body interaction [14] is a first step towards obtaining a complete (i. e., $NN+3N+4N+...$) chiral interaction at higher orders of the chiral expansion.

In this contribution, we focus on the nucleon-deuteron elastic scattering process and would like to test the dependence of the predictions for the differential cross section and the deuteron tensor analyzing power T_{22} on the regulator values used in the SCS and the SMS interactions.

2 Formalism

The nucleon-deuteron scattering can be described within the formalism of Faddeev equations [23, 24]. In practical applications, we solve the Faddeev equation for an auxiliary state $T|\phi\rangle$,

$$T|\phi\rangle = tP|\phi\rangle + (1+tG_0)V_4^{(1)}(1+P)|\phi\rangle + tPG_0T|\phi\rangle + (1+tG_0)V_4^{(1)}(1+P)G_0T|\phi\rangle, \quad (1)$$

and from its solutions all physical observables for elastic nucleon-deuteron scattering and the deuteron breakup reaction are obtained [25]. The ingredients of Eq. (1) are the off-the-energy shell NN t-matrix t related to the NN interaction via the Lippmann–Schwinger equation, the three-body permutation operator P, the free $3N$ propagator G_0,, and the initial channel state $|\phi\rangle$ composed of a momentum eigenstate of the projectile nucleon and a deuteron. On top of the two-nucleon forces also a three-nucleon force is included, and $V_4^{(1)}$ is that part of it which is symmetrical under the exchange of nucleons 2 and 3. Equation (1) accounts for an infinite sequence of two-body and three-body rescattering processes with free propagations in between. In Ref. [23] the path to the cross section and spin observables is described in detail.

We solve Eq. (1) in the partial wave basis and take into account all two-body channels up to the $2N$ total angular momentum $j_{max} = 5$ and the $3N$ total angular momentum $J_{max} = \frac{25}{2}$. This number of partial waves is sufficient to achieve the convergence of predictions at both presented here energies. Since the consistent three-nucleon interaction is not available for the SMS model, we restrict ourselves only to the NN interaction and set $V_4^{(1)} = 0$. Thus we will not discuss the importance of the three-nucleon interaction for the nucleon-deuteron elastic scattering reaction.

3 Results

In the following we compare the cut-off dependence of our predictions for the differential cross section and the deuteron tensor analyzing power T_{22} obtained with two semilocal regularized interactions at N4LO. We choose two kinetic energies of the incoming nucleon in the laboratory (lab) system: $E = 65$ MeV and $E = 200$ MeV. While the first one is well within the range of applicability of chiral forces at the order used, for the second one it is expected that contributions from the higher orders of the chiral expansion can still play some role. We refer the reader to Ref. [26] for a more detailed discussion based on the analysis of the truncation errors.

The differential cross section at $E = 65$ MeV is shown in Fig. 1. It is clear that at

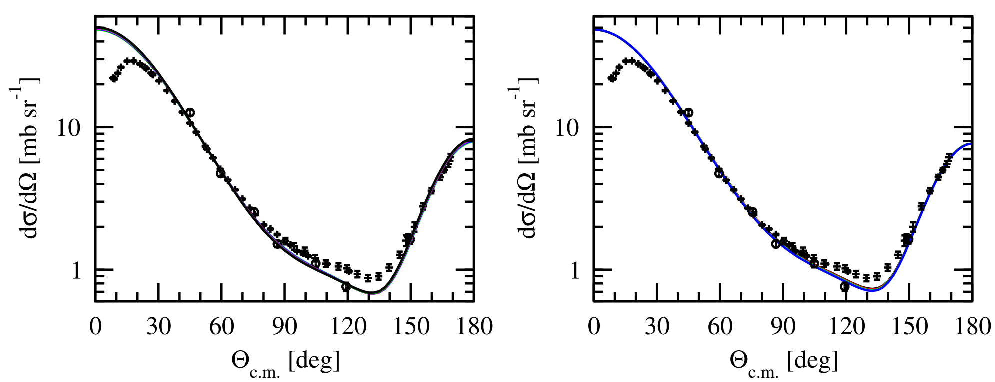

Figure 1: Differential cross sections for the elastic nucleon-deuteron scattering at the incoming nucleon energy in the lab system $E = 65$ MeV. The predictions have been obtained with the N4LO SCS potential (left) and the N4LO SMS force (right). In the left panel, curves are the predictions obtained with the following values of the regulator parameter R: 1.2 fm (black), 1.1 fm (dark green), 1.0 fm (magenta), 0.9 fm (blue), and 0.8 fm (green). In the right panel, curves are the predictions obtained with the following values of the regulator parameter Λ: 400 MeV (black), 450 MeV (red), 500 MeV (green), and 550 MeV (blue). The experimental data are taken from Refs. [27] (pluses) and [28] (circles).

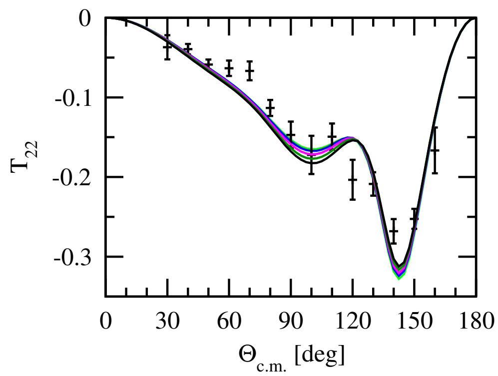

Figure 2: Deuteron tensor analyzing power T_{22} for the elastic nucleon-deuteron scattering at the incoming nucleon energy in the lab system $E = 65$ MeV. Curves are the same as in Fig. 1. The experimental data are taken from Ref. [29].

this energy the dependence of the predictions on the regulator parameters is insignificant for both potentials. All predictions are close to each other and the maximal difference between them for all scattering angles is smaller than the experimental uncertainties. The observed discrepancy with data in the minimum of the cross section stems from neglecting three-nucleon forces in calculations presented here.

Another picture is observed at the same energy, $E = 65$ MeV, for the deuteron tensor analyzing power T_{22}, see Fig. 2. Here the predicted magnitude of the T_{22} clearly depends on the value of the regulator R used for the SCS model. This dependence is especially strong at both minima of the T_{22} seen around scattering angles $\theta_{c.m.} = 100°$ and $\theta_{c.m.} = 145°$. The spread of predictions amounts up to 10% for the first minimum and up to 5% for the latter one and predictions obtained with $R = 0.8$ fm and $R = 1.2$ fm are the extreme ones for both minima. The SMS interaction predictions are much less sensitive to the value of the cut-off parameter. Here at the minimum around $\theta_{c.m.} = 100°$ the spreed of predicted values of T_{22} is 3% and at the minimum around $\theta_{c.m.} = 145°$ it amounts up to approximately 1.3%.

The cut-off dependence grows with energy. This is shown for the cross section at $E = 200$ MeV in Fig. 3 and for the deuteron tensor analyzing power T_{22} at the same energy in Fig. 4. At such a high energy, the cut-off dependence is seen already for the cross section when the SCS potential is used. Using various values of the regulator R leads to substantially different predictions not only near the minimum of the cross section but nearly for all scattering angles. The SMS model works much better at this energy and we observe only a tiny cut-off dependence in the minimum of the cross section. The SCS potential fails also for the T_{22}, especially at the scattering angles $60° \leq \theta_{c.m.} \leq 150°$. At $\theta_{c.m.} = 110°$ the spread of predictions exceeds 65%. The SMS potential delivers a much more stable description in this case. While the cut-off dependence of predictions is also seen in the range $60° \leq \theta_{c.m.} \leq 150°$, it is much smaller and at $\theta_{c.m.} = 110°$ predictions differ by less then 7%. Outside the $60° \leq \theta_{c.m.} \leq 150°$ range, the predictions based on various values of regulators remain practically the same for both models of the interaction used.

Summarizing, we can confirm that the newest chiral interaction derived by the Bochum group [14] preserves its high quality when moving from two- to three-nucleon

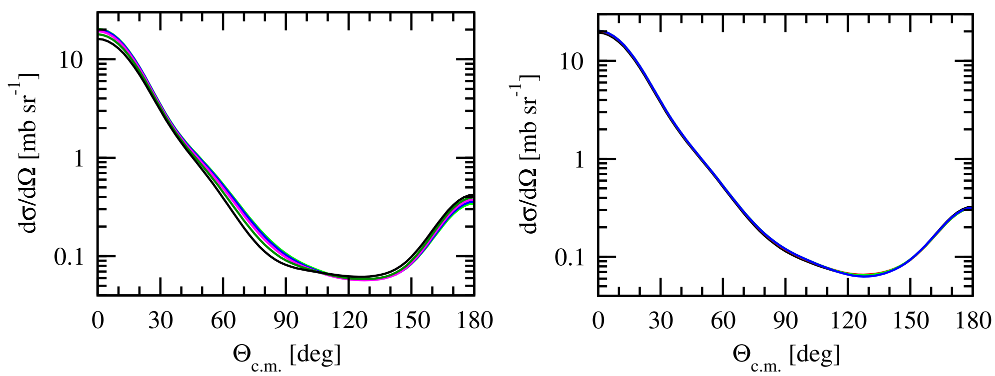

Figure 3: Differential cross section for the elastic nucleon-deuteron scattering at the incoming nucleon energy in the lab system $E = 200$ MeV. Curves are the same as in Fig. 1.

system. We found that for this model the predictions for the elastic nucleon-deuteron scattering practically do not depend (at N4LO) on the value of regulator used. This property eliminates one of the most important difficulties in practical applications of the chiral forces in many-nucleon systems and in the detailed analysis of the data and properties of nuclear forces. The lack of the cut-off dependence also reduces the theoretical uncertainties present in studying various secondary effects like, i. e., the role of many-nucleon forces. A study of the cut-off dependence of the three- and many-body observables including the consistent SMS three-nucleon force is in progress.

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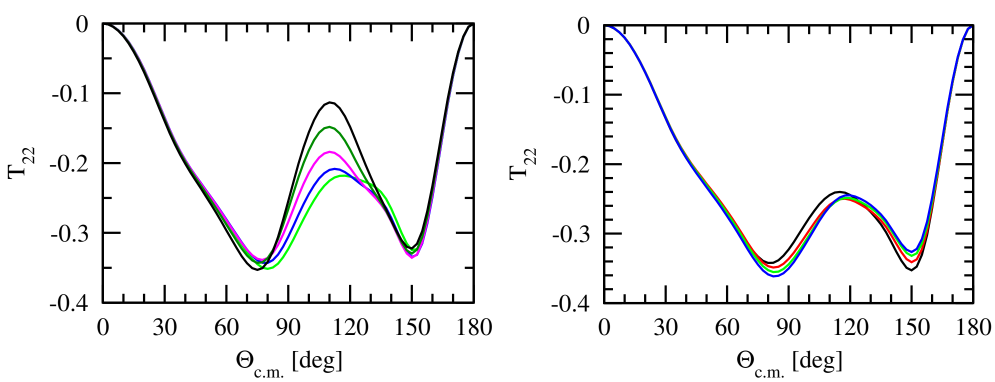

Figure 4: Deuteron tensor analyzing power T_{22} for the elastic nucleon-deuteron scattering at the incoming nucleon energy in the lab system $E = 200$ MeV. Curves are the same as in Fig. 1.

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