Isospin-Symmetry Breaking Correction to Superallowed $0^+ \to 0^+$ β-Decay

Nadezda A. Smirnova[a] **and Latsamy Xayavong**[a,b]

[a] *CENBG (Université de Bordeaux - CNRS/IN2P3), 33175 Bordeaux-Gradignan, France*
[b] *Physics Department, Faculty of Sciences, National University of Laos, 7322 Dongdok, Vientiane Capital, Lao PDR*

Abstract

Superallowed $0^+ \to 0^+$ nuclear β-decay provides an important test of fundamental symmetries underlying the Standard Model of particle physics, namely, the conserved vector-current (CVC) hypothesis and the unitarity of the Cabibbo–Kobayasi–Maskawa (CKM) quark-mixing matrix. These applications require theoretical corrections to β-decay rates, accounting for electroweak interactions between emitted leptons and hadrons (*radiative* corrections) and for the loss of the analogue symmetry between the parent and the daughter states (*isospin-symmetry breaking* correction).

In this work, we present large-scale shell-model calculations of the isospin-symmetry breaking correction to $0^+ \to 0^+$ β-decay in the p, sd and pf-shell. We exploit accurate charge-dependent Hamiltonians and evaluate Fermi transition matrix elements using spherical Woods–Saxon (WS) radial wave functions. Calculations are performed beyond the closure approximation, which requires a large number of intermediate states. We address the question of convergence of nuclear charge radii and of the radial-overlap part of the correction as a function of intermediate states, testing, in particular, the splitting of the correction into two terms (the so-called separation ansatz). We show that the developed adjustment procedure, under available experimental constraints, leads to a rather consistent set of corrections for different shell-model interactions and WS parameterizations. The results are compared to previous studies within the shell model. Their implication for the CVC hypothesis of the Standard Model is discussed.

Keywords: *Nuclear shell model, isospin-symmetry breaking correction, superallowed beta decay, Standard Model*

1 Introduction

The superallowed $0^+ \to 0^+$ nuclear β-decay has been attracting attention of physicists already for a number of decades [1]. Since these transitions are governed uniquely by the vector part of the weak current, the constancy of the so-called absolute Ft-values for various emitters can serve to test the CVC hypothesis. These Ft values

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are obtained from the experimentally deduced ft values, where t is the experimental partial half-life of the transition and f is the statistical rate function, as given by the following master formula [1, 2]:

$$Ft^{0^+\to 0^+} \equiv ft^{0^+\to 0^+}(1+\delta'_R)(1+\delta_{NS}-\delta_C) = \frac{K}{|M_F^0|^2\, G_V^2(1+\Delta_R)}. \qquad (1)$$

Here $K = 2\pi^3\hbar \ln 2(\hbar c)^6/(m_e c^2)^5$, G_V is the vector coupling constant for a semi-leptonic decay, $|M_F^0| = \sqrt{T(T+1) - T_{zi}T_{zf}}$ is the absolute value of the Fermi matrix element in the isospin-symmetry limit, T and T_z are the isospin and its projection of the initial i and final f states. Besides, Eq. (1) contains Δ_R, δ'_R, δ_{NS}, which are transition independent, transition-dependent and nuclear-structure dependent *radiative* corrections, respectively, and δ_C is the *isospin-symmetry breaking* correction.

If the CVC hypothesis holds (Ft is obtained to be constant), from Eq. (1) one can deduce G_V and use it, combined with the fundamental Fermi coupling constant G_F extracted from a purely leptonic muon decay, to get the absolute value of the V_{ud} matrix element of the CKM matrix: $|V_{ud}| = G_V/G_F$. This is important for the unitarity tests of the CKM matrix, for example, one can check the normalization condition of its first row:

$$|V_u|^2 = |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2\,.$$

Other potential possibilities to extract V_{ud} are provided by the neutron decay, pion decay and transitions between mirror $T = 1/2$ partners [1, 3]. The neutron decay is free from the nuclear structure correction, however, one needs to determine the Gamow–Teller/Fermi branching ratio and thus one needs to measure an additional parameter, such as the β-asymmetry, ν-asymmetry or $e\nu$-correlation coefficient [1]. This increases experimental uncertainty on the extracted Ft value. Similar uncertainties arise for the mirror $T = 1/2$ decays, where, in addition to the correlation parameters, one has to evaluate the isospin-symmetry breaking correction (see Ref. [3] for details). The pion β-decay offers the possibility, but this branch is very weak and, thus, the experimental uncertainty is large. It follows that the $0^+ \to 0^+$ β-decay is by far more advantageous compared to other decays and this is why it is worth to put efforts into it.

At present, the ft-values of 14 best-known $T = 1$ emitters are obtained with a precision better than 0.4% [2]. They include ^{10}C, ^{14}O, ^{22}Mg, ^{26m}Al, ^{34}Cl, ^{34}Ar, ^{38m}K, ^{38}Ca, ^{42}Sc, ^{46}V, ^{50}Mn, ^{54}Co, ^{62}Ga and ^{74}Rb. The corresponding ft-values are deduced from the measured decay Q-values and partial half-lives. While nuclear masses are measured nowadays with high precision, the experimental uncertainty on the ft values is dominated by the error in the Fermi decay branching ratio [2]. In particular, the ft values of light $N = Z$ emitters are known with the best precision due to the fact that they are dominated by the Fermi branch at more than 90%.

In this contribution, we focus on the isospin-symmetry breaking correction, δ_C, which appears due to the lack of the analogue symmetry between the parent and daughter nuclear states. It is defined as a deviation of the realistic Fermi matrix element squared from its isospin-symmetry value:

$$|M_F|^2 = |M_F^0|^2(1-\delta_C). \qquad (2)$$

This correction should be evaluated within a nuclear-structure model able to account for the broken isospin symmetry in nuclear states. In spite of the recent progress

in microscopic many-body theory and nuclear forces, the precise description of the isospin-symmetry breaking is still a challenge. Existing predictions from various theoretical approaches are not in agreement (see Refs. [2, 4] and references therein). In particular, based on the compatibility with the CVC hypothesis, in their latest survey [2] Hardy and Towner retained only their own results for the $|V_{ud}|$ evaluation from Ref. [5] .

However, the fact that a calculation is consistent with the conservation of the vector current, does not provide any constraint onto the magnitude of the Ft value. Hence, alternative calculations would be of use. In addition, new experimental measurements or theoretical developments may emerge. For example, adopting a recently re-evaluated Δ_R [6], with a reduced uncertainty and slightly modified central value, and keeping all other values as in Ref. [2], the normalization condition for the CKM upper row clearly shows some tensions.

In this work we present calculations of the isospin-symmetry breaking correction to the $0^+ \to 0^+$ decay rates for a number of *p*-, *sd*- and *pf*-shell emitters, using the nuclear shell-model with isospin-nonconserving (INC) Hamiltonians and realistic WS radial wave functions. Performing large-scale computations beyond the closure approximation, we address the question of convergence of matrix elements of operators as a function of the number of intermediate states. In particular, we test the validity of the separation of δ_C into two parts (the isospin-mixing and radial overlap parts) by an exact calculation of the Fermi matrix element for lighter emitters. Investigation of the role of core-excited configurations is in progress and will be published elsewhere.

2 Formalism

Within the shell model, the nuclear eigenstates are obtained from diagonalization of an effective one- plus two-body Hamiltonian, in a many-body spherically-symmetric basis (typically, in the harmonic-oscillator basis). The Fermi matrix element can be expressed as a sum over valence space orbitals of one-body transition densities (OBTDs) times single-particle matrix elements of the isospin operator [7–9]:

$$M_F = \langle f|\hat{T}_+|i\rangle = \sum_{\alpha} \langle f|a^\dagger_{\alpha_n} a_{\alpha_p}|i\rangle \langle \alpha_n|\hat{t}_+|\alpha_p\rangle \,. \tag{3}$$

Here $|i\rangle$ and $|f\rangle$ are initial and final nuclear states obtained from the diagonalization of a shell-model Hamiltonian in a valence space, $\hat{T}_+ = \sum_k \hat{t}_+(k)$ is the isospin raising operator for valence nucleons, $a^\dagger_\alpha$ and a_α are the creation and annihilation operators for neutrons or protons, with $\alpha = (n_\alpha, l_\alpha, j_\alpha, m_\alpha)$ denoting a complete set of spherical quantum numbers.

If the nuclear Hamiltonian is isospin-invariant, then the Fermi matrix element given by Eq. (3) reduces to a model-independent isospin-symmetry value, M^0_F. The absolute value of the realistic Fermi matrix element is slightly smaller than $|M^0_F|$ because of the violation of the isospin symmetry. Usually two sources of the isospin-symmetry breaking are considered. First, the effective valence-space shell-model Hamiltonian should contain INC terms, such as the Coulomb interaction between protons and charge-dependent terms of nuclear origin. In this case, the initial and final nuclear states, obtained from the shell-model diagonalization in a chosen model space, will not be exactly analogue states. The resulting OBTDs, $\langle f|a^\dagger_{\alpha_n} a_{\alpha_p}|i\rangle$, will be different from their isospin-symmetry limit, denoted as $\langle f|a^\dagger_{\alpha_n} a_{\alpha_p}|i\rangle^T$.

Another source of the isospin-symmetry breaking arises from the single-particle matrix element, $\langle\alpha_n|\hat{t}_+|\alpha_p\rangle$, which is just an overlap between a proton and a neutron radial wave functions,

$$\langle\alpha_n|\hat{t}_+|\alpha_p\rangle = \int_0^\infty R_{k_\alpha,n}(r)\, R_{k_\alpha,p}(r)\, r^2 dr = \Omega_{k_\alpha} \equiv \Omega_\alpha, \tag{4}$$

with $k_\alpha = (n_\alpha, l_\alpha, j_\alpha)$. If the harmonic-oscillator basis is used, those overlaps are all equal to unity. However, they will differ from unity when the proton and neutron single-particle wave functions are obtained from a realistic single-particle potential with charge-dependent terms. We remark that as soon as we renounce the use of the harmonic-oscillator basis, the most right-hand-side expression for the Fermi matrix element in Eq. (3) becomes approximate in the valence space. As was recalled by Miller and Schwenk [10,11], acting on the single-particle bases, different for protons and neutrons, the exact isospin operator would connect single-particle states from the valence space to those which are outside the valence space. However, an existing calculation for ^{10}C decay within the No-Core Shell Model [12], which exploits the exact Fermi operator, does not report on essential differences with a phenomenological calculation in the valence space with an approximate isospin operator. As far as no evidence exists on the impact of the exact operator, we follow previous studies [1,2,13,14] and use the standard approximation for the isospin operator, truncating it to a given valence space.

It can be shown [7–9] that within the shell model, the δ_C correction can be split into two parts, according to the two sources of the isospin-symmetry breaking mentioned above. Let us denote by Δ_α a difference between OBTDs obtained from the INC and the isospin-symmetric shell-model Hamiltonians:

$$\Delta_\alpha = \langle f|a^\dagger_{\alpha_n} a_{\alpha_p}|i\rangle^T - \langle f|a^\dagger_{\alpha_n} a_{\alpha_p}|i\rangle. \tag{5}$$

Then, in the closure approximation, one can express the matrix element M_F in terms of Δ_α and Ω_α as

$$\begin{aligned} M_F &= \sum_\alpha \left(\langle f|a^\dagger_{\alpha_n} a_{\alpha_p}|i\rangle^T - \Delta_\alpha\right)\Omega_\alpha \\ &= \sum_\alpha \left(\langle f|a^\dagger_{\alpha_n} a_{\alpha_p}|i\rangle^T - \Delta_\alpha\right)[1-(1-\Omega_\alpha)] \\ &= M_F^0\left[1 - \frac{1}{M_F^0}\sum_\alpha \Delta_\alpha + \frac{1}{M_F^0}\sum_\alpha \Delta_\alpha(1-\Omega_\alpha)\right. \\ &\qquad \left. - \frac{1}{M_F^0}\sum_\alpha \langle f|a^\dagger_{\alpha_n} a_{\alpha_p}|i\rangle^T(1-\Omega_\alpha)\right]. \end{aligned} \tag{6}$$

Retaining only the leading-order (linear) terms in small terms, we can express M_F^2 as

$$|M_F|^2 = |M_F^0|^2\left[1 - \frac{2}{M_F^0}\sum_\alpha \Delta_\alpha - \frac{2}{M_F^0}\sum_\alpha \langle f|a^\dagger_{\alpha_n} a_{\alpha_p}|i\rangle^T(1-\Omega_\alpha) + \mathcal{O}(\zeta^2)\right], \tag{7}$$

where ζ denotes $(1-\Omega_\alpha)$ or Δ_α. Comparing Eq. (7) with Eq. (2), we observe that the total correction δ_C to a good approximation is given by a sum of two parts,

$$\delta_C = \delta_{IM} + \delta_{RO}, \tag{8}$$

which are the *isospin-mixing* part,

$$\delta_{IM} = \frac{2}{M_F^0} \sum_\alpha \Delta_\alpha, \tag{9}$$

and the *radial-overlap* part,

$$\delta_{RO} = \frac{2}{M_F^0} \sum_\alpha \langle f | a_{\alpha_n}^\dagger a_{\alpha_p} | i \rangle^T (1 - \Omega_\alpha). \tag{10}$$

Within the closure approximation this separation ansatz is thus almost exact, that can be easily verified numerically. The neglected terms are of the order of the product of two corrections and can be skipped with confidence. However, as we will explain later, in practice, the δ_{RO} is evaluated beyond the closure approximation. We discuss this issue in Section 5 and propose a numerical verification of the separation ansatz in small model spaces.

In the present contribution, we summarize results obtained for a series of *p*-, *sd*- and *pf*-shell nuclei, ^{10}C, ^{14}O, ^{22}Mg, ^{26m}Al, ^{26}Si, ^{30}S, ^{34}Cl, ^{34}Ar, ^{38m}K, ^{38}Ca, ^{46}V, ^{46}Cr, ^{50}Mn, ^{50}Fe, ^{54}Co, ^{54}Ni and ^{62}Ga. The shell-model calculations have been performed with NuShellX@MSU code [15], using the Cohen–Kurath effective Hamiltonians [16, 17] for $A = 10$, 14, the interactions of the ZBM-type [18–20] for $A = 14$, 22, USD/USDA/USDB [21, 22] for nuclei with $26 \leq A \leq 38$, KB3G [23], GXPF1A [24] and FPD6 [25] for nuclei with $A = 46$, 50, 54, and JUN45 [26] and MRG [27] for ^{62}Ga. There are strong indications [5] that decays of $A = 38$, 42, 46 have to be considered in the enlarged $s_{1/2}d_{3/2}f_{7/2}p_{3/2}$ model space. We work on this issue, however, in the present study we present preliminary results for $A = 38$ from the *sd*-shell and for $A = 46$ from the *pf*-shell.

The isospin-mixing part of the correction has been obtained from INC versions of the cited above Hamiltonians, as will be described in the next Section. The radial-overlap part has been estimated beyond the closure approximation using thoroughly adjusted WS radial wave functions. Then, we present calculations of δ_C obtained directly from the calculation of the Fermi matrix element beyond the closure approximation and confirm the use of the separation ansatz, at least, in small model spaces. In the last Section, we discuss the consistency of our preliminary results with the CVC hypothesis.

3 Isospin-mixing correction

To get the isospin-mixing part of the correction, δ_{IM}, according to Eq. (9), we have performed calculations with INC Hamiltonians designed for each model space. These Hamiltonians have been constructed using the method worked out in Refs. [28, 29]. One starts from a well-established isospin-conserving Hamiltonian, whose eigenstates form degenerate isobaric multiplets. Then, one introduces an INC term, V_{INC}, consisting of isovector single-particle energies, the two-body Coulomb interaction between protons and effective two-body charge-symmetry breaking and charge-independence breaking components of nuclear origin. Those two latter terms can be modeled by the Yukawa pion or rho-meson exchange potential or simply by a $T = 1$ term of the original isospin-conserving Hamiltonian. The resulting V_{INC} is thus a superposition

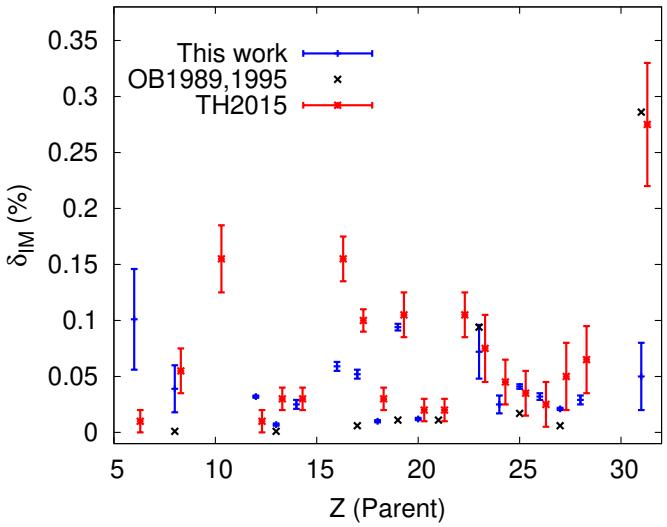

Figure 1: Isospin-mixing part of the correction for selected *sd*- and pf-shell nuclei: present results are compared with previous works of Towner and Hardy [2] and Ormand and Brown [14].

of an isoscalar, an isovector and an isotensor operators. Evaluating within perturbation theory the splitting of the isobaric multiplets due to the expectation values of the non-isoscalar terms, one gets for a mass excess an expression quadratic in T_z, known as the Wigner's isobaric-multiplet mass equation [30]:

$$M(\eta, T, T_z) = a(\eta, T) + b(\eta, T)T_z + c(\eta, T)T_z^2. \tag{11}$$

Here $\eta = (A, J^\pi, N_{exc}, ...)$ denotes all other quantum numbers (except for T), which are required to label a quantum state of an isobaric multiplet, whereas a, b and c are coefficients. The unknown strengths of the isovector and isotensor terms are found by a fit to experimental b and c coefficients for ground and excited states in a given model space (experimental databases can be found in Refs. [31, 32]). Having established the unknown strength parameters, we add V_{INC} to the original isospin-conserving effective Hamiltonian H: $H_{INC} = H + V_{INC}$. Obviously, H_{INC} does not commute with the many-body isospin operator anymore. We diagonalize it in the proton-neutron formalism and get states of a slightly mixed isospin.

Since the isospin is not conserved, calculations of β-decays between 0^+ isobaric analogue states show that a fraction of the Fermi strength is split among many non-analogue states in the daughter nuclei, producing contributions to δ_C from the model space (or δ_{IM}). One can distinguish two main sources of isospin impurities in the parent and daughter states [14]. The first is due to mixing with other non-analogue $0^+, T = 1$ states. Besides, in $N = Z$ nuclei, high-lying $0^+, T = 0$ states may provide certain admixtures as well. In the first order perturbation theory, the isospin-impurity amplitude is inversely proportional to the energy difference between the two admixed 0^+ states, ΔE. Therefore, the corresponding contribution to the isospin-mixing correction behaves as $\sim 1/(\Delta E)^2$. We estimate the isospin-mixing correction from the splitting of the Fermi states over about 100 excited 0^+ states. To improve the accuracy, we scale the theoretical values with experimental energy differences when available (typically, for a few lowest 0^+ states).

In the present work we used the charge-dependent versions of the above cited effective Hamiltonians in the p, $p_{1/2}d_{5/2}s_{1/2}$, sd, pf and $f_{5/2}pg_{9/2}$ shell-model spaces from Refs. [14, 15, 28]. The resulting δ_{IM} corrections are summarized in Fig. 1 in

Table 1: Preliminary values of δ_{IM}, δ_{RO} and δ_C with their uncertainties obtained in this work, in comparison with the results of Towner and Hardy from [2, 33]. The present results for $A = 38$ are obtained in the *sd*-shell, while the results for $A = 46$ are obtained in the pf-shell.

Parent	This work			Towner, Hardy (2015)		
	δ_{IM}(%)	δ_{RO} (%)	δ_C (%)	δ_{IM}(%)	δ_{RO} (%)	δ_C (%)
^{10}C	0.102 (45)	0.203 (24)	0.304 (51)	0.010 (10)	0.165 (15)	0.175 (18)
^{14}O	0.039 (21)	0.271 (25)	0.309 (33)	0.055 (20)	0.275 (15)	0.330 (25)
^{22}Mg	0.032 (1)	0.313 (30)	0.345 (30)	0.010 (10)	0.370 (20)	0.380 (22)
^{26}Al	0.007 (1)	0.249 (16)	0.255 (16)	0.030 (10)	0.280 (15)	0.310 (18)
^{26}Si	0.025 (4)	0.360 (28)	0.386 (28)	0.030 (10)	0.405 (25)	0.435 (27)
^{30}S	0.059 (4)	0.656 (31)	0.715 (31)	0.155 (20)	0.700 (20)	0.855 (28)
^{34}Cl	0.052 (4)	0.616 (30)	0.668 (30)	0.100 (10)	0.550 (45)	0.650 (46)
^{34}Ar	0.010 (1)	0.662 (52)	0.672 (52)	0.030 (10)	0.665 (55)	0.695 (56)
^{38}K	0.094 (3)	0.582 (99)	0.676 (99)	0.105 (20)	0.565 (50)	0.670 (54)
^{38}Ca	0.012 (1)	0.773 (83)	0.785 (83)	0.020 (10)	0.745 (70)	0.765 (71)
^{46}V	0.072 (24)	0.338 (46)	0.410 (52)	0.076 (30)	0.545 (55)	0.620 (63)
^{46}Cr	0.025 (8)	0.450 (70)	0.475 (70)	0.045 (20)	0.715 (85)	0.760 (87)
^{50}Mn	0.041 (2)	0.458 (16)	0.499 (16)	0.035 (20)	0.610 (50)	0.645 (54)
^{50}Fe	0.032 (3)	0.428 (76)	0.460 (76)	0.025 (20)	0.635 (45)	0.660 (50)
^{54}Co	0.021 (1)	0.632 (72)	0.652 (72)	0.050 (30)	0.720 (60)	0.770 (67)
^{54}Ni	0.029 (4)	0.601 (71)	0.629 (72)	0.065 (30)	0.725 (60)	0.790 (67)
^{62}Ga	0.050 (30)	0.961(181)	1.010 (184)	0.275 (55)	1.20 (20)	1.48 (21)

comparison with previous works [2, 14], while the numerical values can be found in Table 1. The uncertainties of our results stem from the use of different INC Hamiltonians for the same nucleus in the same model space. We observe that our results are in agreement with the calculations of Towner and Hardy [2] for a number of *sd*- and pf-shell emitters. The value of δ_{IM} for ^{10}C is much larger in our calculation than that of Towner and Hardy. This comes partly from a larger mixing matrix element and partly from a higher position of the second 0^+ state in ^{10}B. At the same time, our values for ^{22}Mg, ^{26}Al and ^{62}Ga are much smaller than the results of Towner and Hardy. For the latter one, we exceptionally use the pf shell for this calculation, because the construction of an accurate INC Hamiltonian in the model space beyond the ^{56}Ni core is still in progress. We notice that there is an agreement of both results for $A = 46$ and 50, although we have used the pf-shell model space, while Towner and Hardy used the $s_{1/2}d_{3/2}f_{7/2}p_{3/2}$ one.

We conclude by saying that, as seen from Fig. 1, δ_{IM} is very sensitive to the details of the shell-model interaction and, in particular, to the charge-dependent term. However, the δ_{IM} contribution is no more than 10% to the total correction, and hence it slightly affects final δ_C values.

4 Radial-overlap correction

The radial overlap correction, δ_{RO}, is the major part of the isospin-symmetry breaking correction. To go beyond the closure approximation of Eq. (10), one has to insert the summation over intermediate states in the $(A-1)$ nucleus [7, 8],

$$\delta_{RO} = \frac{2}{M_F^0} \sum_{\alpha,\pi} \langle f|a^\dagger_{\alpha_n}|\pi\rangle^T \langle i|a^\dagger_{\alpha_p}|\pi\rangle^T (1-\Omega^\pi_\alpha), \tag{12}$$

where the matrix elements $\langle f|a^\dagger_{\alpha_n}|\pi\rangle^T$ and $\langle i|a^\dagger_{\alpha_p}|\pi\rangle^T$ are related to the spectroscopic amplitudes for the neutron and proton pick-up, respectively. In the angular momentum coupled formalism, we can rewrite this expression as

$$\delta_{RO} = \frac{2}{M_F^0} \sum_{k,\pi} S^{T\pi}_{k_n} S^{T\pi}_{k_p} \, (1-\Omega^\pi_k), \tag{13}$$

with $k \equiv k_\alpha$ for simplicity and

$$S^{T\pi}_{k_n} = \frac{\langle \Psi(A) J_f || a^\dagger_{k_n} || \Psi(A-1) J_\pi\rangle^T}{\sqrt{2J_f+1}}, \qquad S^{T\pi}_{k_p} = \frac{\langle \Psi(A) J_i || a^\dagger_{k_p} || \Psi(A-1) J_\pi\rangle^T}{\sqrt{2J_f+1}}.$$

Again, the label T means that the nuclear eigenstates $\Psi(A)$ and $\Psi(A-1)$ are obtained in the isospin-symmetry limit. The radial integrals $\Omega^\pi_{k_\alpha} \equiv \Omega^\pi_k$ depend on π via

$$\Omega^\pi_k = \int_0^\infty R^\pi_{k_n}(r)\, R^\pi_{k_p}(r)\, r^2 dr. \tag{14}$$

The notations $R^\pi_k(r)$ means that these radial wave functions are extracted from a potential for which one nucleon separation energies correspond to $S_p + E_\pi$. Thus, for each excitation energy E_π we fit the potential in order to reproduce experimental proton and neutron separation energies separately. For every transition, we take up to 200 excited states of each spin and parity of the intermediate nucleus and check that the δ_{RO} is well converged. We also remark that the summation in Eq. (13) is incoherent, thus the consistency of signs of the spectroscopic amplitudes is very important.

In our study, we have explored two different parameterizations of the WS potential: one being close to that of Ref. [34] (BM_m), while the other, referred to as SWV, being based on Ref. [35]. The details of these parameterizations and the role of each term are discussed extensively in Refs. [4, 9]. Briefly speaking, we use the WS potential of the standard form,

$$V(r) = V_0 f(r, R_0, a_0) + V_s \left(\frac{r_s}{\hbar}\right)^2 \frac{1}{r}\frac{d}{dr}[f(r, R_s, a_s)](\boldsymbol{l}\cdot\boldsymbol{\sigma}) + V_c(r), \tag{15}$$

where

$$f(r, R_i, a_i) = \frac{1}{1+\exp\left(\frac{r-R_i}{a_i}\right)}, \tag{16}$$

with i denoting either 0 for the central term or s for the spin-orbit term. The radius is modeled in a standard way as $R_i = r_i(A-1)^{1/3}$, while the diffuseness parameters, a_i, are kept fixed. In general, the spin-orbit length parameter, r_s, is smaller than

that of the volume term, r_0, because of a very short range of the two-body spin-orbit interaction [34]. The one-body Schrödinger equation is solved in relative coordinates for a particle of mass $\mu = m\,(A-1)/A$, where m is the nucleon mass and A is the mass number of the composite nucleus. The isovector terms of the WS potential, which provide the difference between the proton and neutron wave functions, are most crucial for the radial-overlap correction. Their global parameterization is not accurate enough. This is why we consider only the Coulomb term, while all other possible isovector contributions are taken into account by adjustment of a specific potential parameter (a potential depth or a surface term) to match either the proton or the neutron separation energy. The Coulomb radius parameter is related to the root-mean-charge radius of the parent nucleus. The WS length parameter is varied to insure that the charge density constructed from the proton radial wave functions yields a root-mean-charge radius in agreement with the experimental value measured by the electron scattering [36] or by the isotope-shift estimation [37].

Contrary to all previous works, we have calculated the nuclear radii beyond the closure approximation [4, 9]. The square of the charge radius (relative to the inert core) is given by the expectation value of the operator r^2_{sm} in the ground state of the parent nucleus:

$$\langle r^2\rangle_{sm} = \langle i|r^2_{sm}|i\rangle = \frac{1}{Z}\sum_{\alpha}\langle\alpha_p|r^2|\alpha_p\rangle\langle i|a^\dagger_{\alpha_p}a_{\alpha_p}|i\rangle. \tag{17}$$

Inserting the complete sum over intermediate states $\sum_\pi |\pi\rangle\langle\pi|$ into this equation, we convert the proton occupation numbers, $\langle i|a^\dagger_{\alpha_p}a_{\alpha_p}|i\rangle$, into a sum of the spectroscopic factors over intermediate states. Taking into account that the radial wave functions depend on π, we get the following expression:

$$\langle r^2\rangle_{sm} = \frac{1}{Z}\sum_{\alpha,\pi}\langle i|a^\dagger_{\alpha_p}|\pi\rangle^2\langle\alpha_p|r^2|\alpha_p\rangle^\pi, \tag{18}$$

with the single-particle matrix element being

$$\langle\alpha_p|r^2|\alpha_p\rangle^\pi = \int_0^\infty r^4|R^\pi_{k_p}(r)|^2\,dr. \tag{19}$$

The convergence of the expression (18) is quite fast. We found out that it results in slightly smaller values of the charge radii, requiring thus larger values of the length parameter r_0.

The adjustment procedure can be summarized as follows. For a given value of a, we vary two parameters, the potential depth V_0 and the length parameter, r_0, in a way to reproduce known experimental observables — the one-nucleon separation energies and the radius of the parent nucleus. The fit is performed either by a modification of the central part depth V_0 or by an addition of a separate surface term,

$$V_g(r) = \left(\frac{\hbar}{m_\pi c}\right)^2\frac{V_g}{a_s r}\exp\left(\frac{r-R_s}{a_s}\right)[f(r,R_s,a_s)]^2, \tag{20}$$

and by a modification of the V_g parameter. As we have found out for the *sd* shell [4], the procedure involving the variation of V_0 removes essentially the dependence on a particular parameterization: the BM_m and SWV results are very similar. The fit of

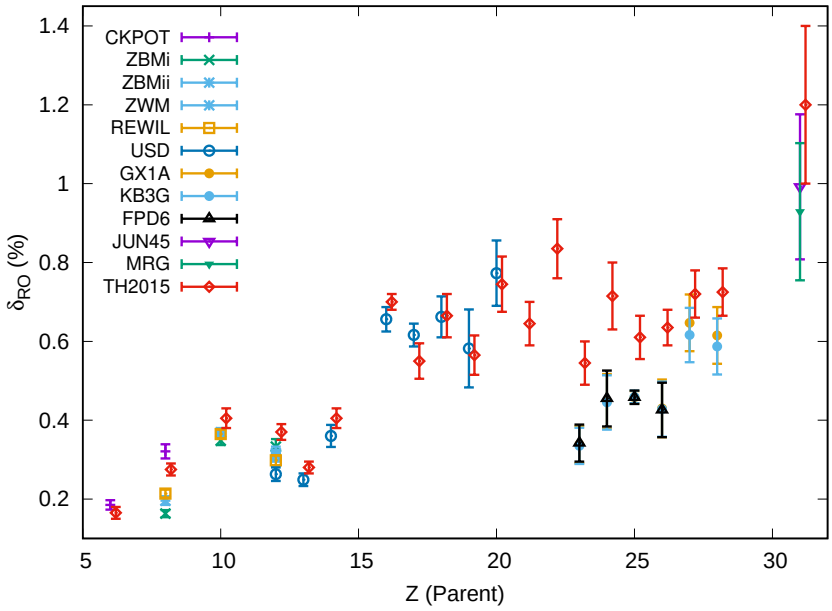

Figure 2: Radial-overlap corrections obtained from the shell model with WS radial wave functions (BM_m parameterizations). The results of Towner and Hardy [37] are shown for comparison.

the surface term results in larger differences between the results from two parameterizations and we keep this in our analysis. Figure 2 shows our results for δ_{RO} obtained in the p, $p_{1/2}d_{5/2}s_{1/2}$, sd, pf and $f_{5/2}pg_{9/2}$ shell-model spaces, being the average from BM_m and SWV parameterizations and two different methods of the fit (volume or surface term). For comparison we show the latest calculations of Towner and Hardy [2,5]. Again, our sd-shell values for $A = 26$, 30 and 34 are in a good agreement with the results of Ref. [2]. The values obtained for $A = 38$ look to be close to the values of Towner and Hardy, however, we should remember that our calculation uses a small sd-shell model space. We also remark that Towner and Hardy [5] used different model spaces for $A = 46$ and 50, which causes major differences in these cases (our pf-shell values are systematically smaller). We remark that three different pf-shell interactions, KB3G, GX1A and FPD6, give almost identical values for $A = 46$ and 50. Calculations for $A = 38$, 42 and 46 in the model space around ^{40}Ca are in progress and will be published elsewhere. The dominant part of the uncertainty is systematic due to experimental uncertainties of nuclear radii (see Ref. [4] for details). The use of different shell-model interactions for a given model space is considered as a source of statistical uncertainty. Its contribution is minor. It is seen that, except for a few cases, our uncertainties are similar to those deduced by Towner and Hardy.

5 Beyond the separation ansatz

In this section we present calculations of the nuclear structure correction without the separation ansatz. First, let us derive the expression for the δ_C beyond the

closure approximation. In this case, the summation over intermediate states has to be included in the expression of the Fermi matrix element,

$$M_F = \sum_{\alpha} \langle f | a^{\dagger}_{\alpha_n} a_{\alpha_p} | i \rangle \langle \alpha_n | \hat{t}_+ | \alpha_p \rangle = \sum_{\alpha} \sum_{\pi} \langle f | a^{\dagger}_{\alpha_n} | \pi \rangle \langle i | a^{\dagger}_{\alpha_p} | \pi \rangle \Omega^{\pi}_{k_\alpha}. \tag{21}$$

In the J-coupled form, this expression becomes

$$M_F = \sum_{k,\pi} S^{\pi}_{k_p} S^{\pi}_{k_n} \Omega^{\pi}_{k}. \tag{22}$$

Denoting the deviations of the spectroscopic amplitudes from their isospin-symmetry limits as

$$S^{T\pi}_{k_p} - S^{\pi}_{k_p} = D^{\pi}_{k_p}, \tag{23}$$

$$S^{T\pi}_{k_n} - S^{\pi}_{k_n} = D^{\pi}_{k_n}, \tag{24}$$

we can rewrite the Fermi matrix element as

$$M_F = \sum_{k,\pi} \left(S^{T\pi}_{k_p} - D^{\pi}_{k_p} \right) \left(S^{T\pi}_{k_n} - D^{\pi}_{k_n} \right) \left[1 - (1 - \Omega^{\pi}_{k}) \right] \tag{25a}$$

$$= \sum_{k,\pi} \Bigg[\underbrace{S^{T\pi}_{k_p} S^{T\pi}_{k_n}}_{M^0_F} - \underbrace{S^{T\pi}_{k_p} S^{T\pi}_{k_n} (1 - \Omega^{\pi}_{k})}_{O(\zeta)} - \underbrace{\left(S^{T\pi}_{k_p} S^{T\pi}_{k_n} - S^{\pi}_{k_p} S^{\pi}_{k_n} \right)}_{O(\zeta)} \tag{25b}$$

$$+ \underbrace{\left(S^{T\pi}_{k_p} D^{\pi}_{k_n} + S^{T\pi}_{k_n} D^{\pi}_{k_p} \right) (1 - \Omega^{\pi}_{k})}_{O(\zeta^2)} - \underbrace{D^{\pi}_{k_p} D^{\pi}_{k_n} (1 - \Omega^{\pi}_{k})}_{O(\zeta^3)} \Bigg]. \tag{25c}$$

Line (25b) contains the Fermi matrix element in the isospin-symmetry limit and leading-order (linear) contributions to the correction δ_C expressed by $\delta_{RO} + \delta_{IM}$, while the terms shown on line (25c) represent the corrections of the second and third orders. Again, we can wish to keep only linear terms in small quantities and therefore to use an approximate expression:

$$M_F = M^0_F \Bigg[1 - \frac{1}{M^0_F} \sum_{k,\pi} \left(S^{T\pi}_{k_p} S^{T\pi}_{k_n} - S^{\pi}_{k_p} S^{\pi}_{k_n} \right) - \frac{1}{M^0_F} \sum_{k,\pi} S^{T\pi}_{k_p} S^{T\pi}_{k_n} (1 - \Omega^{\pi}_{k}) + O(\zeta^2) \Bigg]. \tag{26}$$

Taking this expression squared, we get

$$|M_F|^2 = |M^0_F|^2 \Bigg[1 - \frac{2}{M^0_F} \sum_{k,\pi} \left(S^{T\pi}_{k_p} S^{T\pi}_{k_n} - S^{\pi}_{k_p} S^{\pi}_{k_n} \right) - \frac{2}{M^0_F} \sum_{k,\pi} S^{T\pi}_{k_p} S^{T\pi}_{k_n} (1 - \Omega^{\pi}_{k}) + O(\zeta^2) \Bigg]. \tag{27}$$

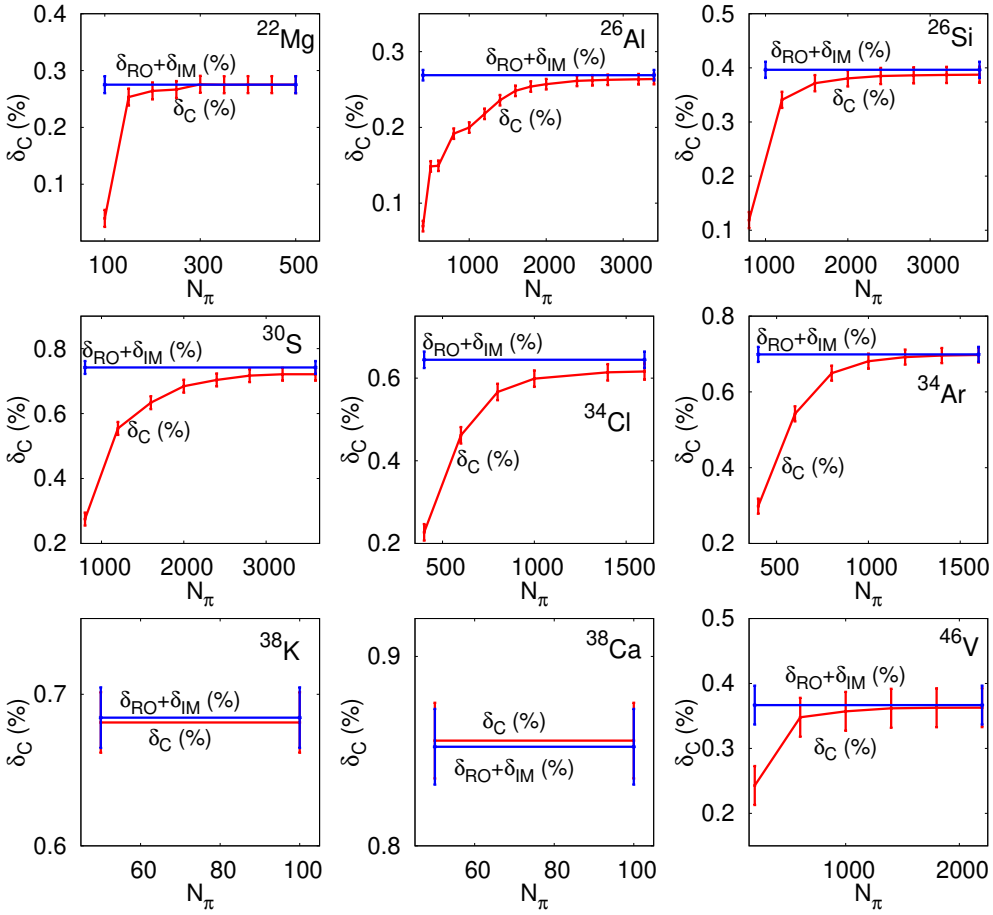

Figure 3: The isospin-symmetry breaking correction obtained from the exact calculation, δ_C, and from the sum $\delta_{IM} + \delta_{RO}$ as a function of the number of intermediate states of each angular momentum and parity.

Again, we observe that the correction splits into two parts, $\delta_C \approx \delta_{IM} + \delta_{RO}$, with δ_{RO} defined by Eq. (13) and the isospin-mixing part given by Eq. (9), which we can rewrite as

$$\delta_{IM} = \frac{2}{M_F^0} \sum_{k,\pi} \left(S_{k_p}^{T\pi} S_{k_n}^{T\pi} - S_{k_p}^{\pi} S_{k_n}^{\pi} \right). \tag{28}$$

Although the approximation seems to be good, we have performed the exact calculation of the Fermi matrix element using Eq. (21) and compared between the result and the approximation given by the sum $\delta_{IM} + \delta_{RO}$. The expression in Eq. (21) converges very slowly as a function of the number of intermediate states π: up to a few thousand intermediate states are often required, which is not easily doable in large model spaces. Thus, we have used for verification only the *sd*-shell emitters and ^{46}V in the *pf*-shell. Figure 3 shows the values of δ_C from the exact calculation of the Fermi matrix element and from the separation ansatz according to Eq. (27) as a function of the number of intermediate states of each angular momentum and parity. We remark

that, while the $\delta_{IM} + \delta_{RO}$ is converged when about 100 intermediate states are used, to get the δ_C converged we need at least 2000 excited states for nuclei from the middle of the *sd* shell. The results shown in Fig. 3 are obtained from the INC version of USD (for the *sd*-shell emitters) and the INC version of GXPF1A for $A = 46$ [15], but we checked that the results are similar for other interactions in the same model spaces. Uncertainties correspond to the systematic errors due to experimental uncertainties of the measured charge radii. For all cases considered, it is seen that the separation ansatz represents a robust approximation to the exact value of δ_C.

At the same time, the intermediate state procedure has still to be checked on the effect of orthogonality of the single-particle states, as well as possible center-of-mass contributions. These are open problems to be addressed in future.

6 Consistency with the CVC hypothesis

Our preliminary δ_C values obtained by now are summarized in Table I and can be also found in Fig. 4 in comparison with the latest results by Towner and Hardy [2].

To estimate the quality of our preliminary results, we perform the confidence-level test proposed recently by Towner and Hardy [38]. Based on the assumption that the CVC hypothesis is valid to at least $\pm 0.03\%$ and adopting the values of ft, δ'_R, δ_{NS} from Ref. [2], we check whether our set of corrections produce a statistically consistent set of $\mathcal{F}t$ values.

If we assume that the CVC hypothesis is satisfied ($\mathcal{F}t$ is constant), without regarding the CKM unitarity, we can convert the experimental ft values into experimental values for structure-dependent corrections and compare the results of the calculation. Defining pseudo-experimental values of the isospin-symmetry breaking correction as

$$\delta_C^{exp} = 1 + \delta_{NS} - \frac{\mathcal{F}t}{ft(1+\delta'_R)}, \tag{29}$$

we can test a set of δ_C for N superallowed transitions, using the method of least squares with $\mathcal{F}t$ as the adjustable parameter. To characterize the quality of the fit,

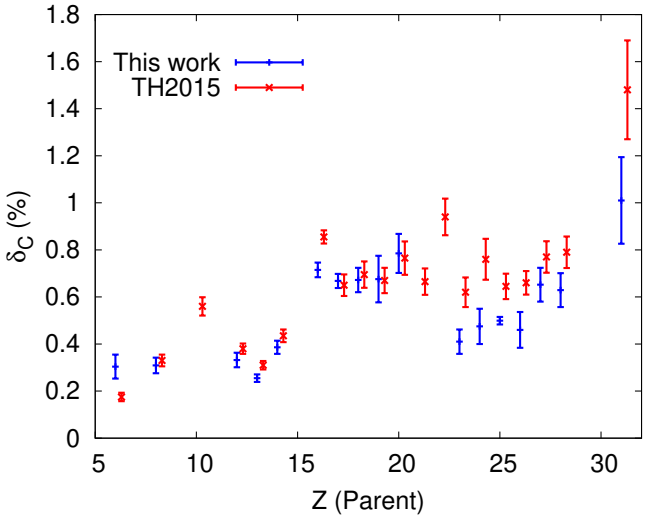

Figure 4: Isospin-symmetry breaking correction obtained from the INC shell-model with WS radial wave functions. The results of Towner and Hardy [2] are shown for comparison.

we use the optimized value for χ^2/ν ($\nu = N - 1$):

$$\chi^2/\nu = \frac{1}{N-1}\sum_{i=1}^{N}\frac{[\delta_C(i) - \delta_C^{exp}]^2}{\sigma_i^2}. \quad (30)$$

In the present test we consider 10 transitions: ^{10}C, ^{14}O, ^{22}Mg, ^{26m}Al, ^{26}Si, ^{34}Cl, ^{34}Ar, ^{50}Mn, ^{54}Co and ^{62}Ga. An improved measurement of the branching ratio of ^{26}Si decay has been reported recently [39], resulting in $ft = 3051.5(57)$ s, so we have included this transition in our analysis. However, we do not use our preliminary *sd*-shell results for $A = 38$ and *pf*-shell results for $A = 46$, since those decays should be evaluated in a more adequate *sdpf* shell-model space. The obtained optimized value of $\chi^2/\nu = 1.75$ is encouraging and the corresponding $\mathcal{F}t = 3074.0$ sec is in fair agreement with the latest evaluation by Towner and Hardy [2]. A more complete comparison will be done when we get results for all of 14 best known emitters.

7 Conclusions

In this contribution we present new shell-model calculations of the isospin-symmetry breaking correction to the superallowed $0^+ \to 0^+$ β-decay. The main goal of our study is to reexamine the previous work within the nuclear shell model making advantage of the remarkable progress in the state-of-the-art description of nuclear properties within a large-scale diagonalization technique. We find that the isospin-mixing part of the correction is very sensitive to a particular parameterization of the charge-dependent term of the Hamiltonian. However, representing a small part of the correction, it does not affect much the final values. The radial-overlap part of the correction is the dominant part. We have performed calculations for 13 emitters from *p*, $p_{1/2}sd_{5/2}$, *sd*, *pf* and $f_{5/2}pg_{9/2}$ model spaces without truncations and taking into account up to 200 excited states of each relevant spin and parity of the intermediate nucleus. Two different parameterizations of a spherical WS potential have been investigated. Our adjustment procedure, relying on the experimental nuclear radii and nucleon separation energies, largely removes the dependence of the results on the WS potential parameters. The calculation of nuclear radii have been performed beyond the closure approximation. In general, the results stay consistent with previous studies, although the final Ft value may deviate from the presently accepted value. A further work on nuclei of the masses $A = 38$, 42,46, 74 is needed to accomplish the study.

The preliminary implications of our new results for electroweak tests lead to a slightly different average Ft value with an overall good agreement with CVC. Calculations for a larger set of emitters are in progress.

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References

[1] I. S. Towner and J. C. Hardy, Rep. Prog. Phys. **74**, 046301 (2010).

[2] J. C. Hardy and I. S. Towner, Phys. Rev. C **91**, 025501 (2015).

[3] O. Naviliat-Cuncic and N. Severijns, Phys. Rev. Lett. **102**, 142302 (2009).

[4] L. Xayavong and N. A. Smirnova, Phys. Rev. C **97**, 024324 (2018).

[5] I. S. Towner and J. C. Hardy, Phys. Rev. C **77**, 025501 (2008).

[6] C.-Y. Seng, M. Gorchtein, H. H. Patel and M. Ramsey-Musolf, Phys. Rev. Lett. **121**, 241804 (2018).

[7] I. S. Towner, J. C. Hardy, M. Harvey, Nucl. Phys. A **284** 269 (1977).

[8] W. E. Ormand and B. A. Brown, Nucl. Phys. A **440**, 274 (1985).

[9] L. Xayavong, Ph. D. thesis, University of Bordeaux, Bordeaux, 2016.

[10] G. A. Miller and A. Schwenk, Phys. Rev. C **78**, 035501 (2008).

[11] G. A. Miller and A. Schwenk, Phys. Rev. C **80**, 064319 (2009).

[12] E. Caurier, P. Navrátil, W. E. Ormand and J. P. Vary, Phys. Rev. C **66**, 024314 (2002).

[13] W. E. Ormand and B. A. Brown, Phys. Rev. Lett. **62**, 866 (1989).

[14] W. E. Ormand and B. A. Brown, Phys. Rev. C **52**, 2455 (1995).

[15] B. A. Brown and W. D. M. Rae, Nucl. Data Sheets **120**, 115 (2014).

[16] S. Cohen and D. Kurath, Nucl. Phys. **73**, 1 (1965).

[17] S. Cohen and D. Kurath, Nucl. Phys. **101**, 1 (1967).

[18] A. P. Zuker, B. Buck and McGrory, Phys. Rev. Lett. **21**, 39 (1968).

[19] A. P. Zuker, Phys. Rev. Lett. **23**, 983 (1969).

[20] B. S. Reehal and B. H. Wildenthal, Part. Nucl. **6**, 137 (1973).

[21] B. H. Wildenthal, Prog. Part. Nucl. Phys. **11**, 5 (1984).

[22] B. A. Brown and W. A. Richter, Phys. Rev. C **74**, 034315 (2006).

[23] A. Poves, J. Sánchez-Solano, E. Caurier and F. Nowacki, Nucl. Phys. A **694**, 157 (2001).

[24] M. Honma, T. Otsuka, B. A. Brown and T. Mizusaki, Phys. Rev. C **69**, 034335 (2004).

[25] W. A. Richter, M. G. van der Merwe, R. E. Julies and B. A. Brown, Nucl. Phys. A **523**, 325 (1990).

[26] M. Honma, T. Otsuka, T. Mizusaki and M. Hjorth-Jensen, Phys. Rev. C **80**, 064323 (2009).

[27] E. Caurier, F. Nowacki, A. Poves and J. Retamosa, Phys. Rev. Lett. **77**, 1954 (1996).

[28] W. E. Ormand and B. A. Brown, Nucl. Phys. A **491**, 1 (1989).

[29] Y. H. Lam, N. A. Smirnova and E. Caurier, Phys. Rev. C **87**, 054304 (2013).

[30] E. P. Wigner, *in Proc. of the Robert A. Welch Foundation Conf. on Chemical Research, 1957*, ed. W. O. Milligan. Welch Foundation, Houston, 1958, Vol. 1, p. 86.

[31] Y. H. Lam, B. Blank, N. A. Smirnova, J. B. Bueb and M. S. Antony, At. Data Nucl. Data Tables **99**, 680 (2013).

[32] M. MacCormick and G. Audi, Nucl. Phys. A **925**, 61 (2014).

[33] I. S. Towner and J. C. Hardy, Phys. Rev. C **92**, 055505 (2015).

[34] A. Bohr and B. R. Mottelson, *Nuclear structure, vol. 1.* W. A. Benjamin, 1969.

[35] N. Schwierz, I. Wiedenhöver and A. Volya, arXiv:0709.3525 [nucl-th] (2007).

[36] I. Angeli and K. P. Marinova, At. Data Nucl. Data Tables **99**, 69 (2013).

[37] I. S. Towner and J. C. Hardy, Phys. Rev. C **66**, 035501 (2002).

[38] I. S. Towner and J. C. Hardy, Phys. Rev. C **82**, 065501 (2010).

[39] M. Bencomo *et al.*, Phys. Rev. C **100**, 015503 (2019).