^{3}H and ^{3}He Bound State Calculations without Angular Momentum Decomposition

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Abstract

Recent advances in the so-called "three dimensional" ($3D$) formalism made it possible to perform numerically efficient calculations of the three-nucleon ($3N$) bound states that utilize $3N$ forces. In this paper we present results related to the ^{3}H bound state. We also discuss our way of incorporating the Coulomb forces in $3D$ calculations of the ^{3}He bound state.

Keywords: *Few-nucleon forces; three-nucleon system; bound states*

1 Introduction

An introduction to the $3D$ approach can be found in Ref. [1]. In this approach, few-nucleon calculations are carried out without using the partial wave decomposition and instead the three-dimensional momentum degrees of freedom of the nucleons are used. Some previous work related to the $3N$ bound state was reported in Refs. [2,3]. Below we provide a brief summary of the formalism used in these papers.

The starting point of the calculation is the operator form of the Faddeev component $|\,\psi\rangle$ of the $3N$ bound state $|\,\Psi\rangle$ [3,4]:

$$\langle \boldsymbol{p}\boldsymbol{q}; \Big(t\frac{1}{2}\Big)TM_T\,|\,\psi\rangle = \sum_{i=1}^{8} \psi_{tT}^{(i)}(p,q,\hat{\boldsymbol{p}}\cdot\hat{\boldsymbol{q}})\,\check{O}_i(\boldsymbol{p},\boldsymbol{q})\,|\,\chi^m\rangle. \tag{1}$$

In Eq. (1), $\psi_{tT}^{(i)}(p,q,\hat{\boldsymbol{p}}\cdot\hat{\boldsymbol{q}})$ are scalar functions that effectively define the Faddeev component $|\psi\rangle$, $\check{O}_i(\boldsymbol{p},\boldsymbol{q})$ are operators in the spin space of the $3N$ system (they are listed in Ref. [3]), $|(t\frac{1}{2})TM_T\rangle$ is a $3N$ isospin state in which the isospins of two nucleons are coupled to t and then further coupled with the isospin of the third particle to the total isospin T with projection M_T. Finally $\boldsymbol{p}$, $\boldsymbol{q}$ are Jacobi momenta and $|\chi^m\rangle$ is a $3N$ spin state (given explicitly in Ref. [3]).

The operator form (1) is plugged into the Faddeev equation (note that we use a version of the Faddeev equation without the two nucleon transition operator):

$$|\,\psi\rangle = \check{G}_0(E)\,\Big(\check{V} + \check{V}^{(1)}\Big)\big(\check{1} + \check{P}\big)\,|\,\psi\rangle, \tag{2}$$

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http://www.ntse.khb.ru/files/uploads/2018/proceedings/Topolnicki.pdf.

where $\check{V}$ is the two-nucleon ($2N$) potential acting between particles 2 and 3, $\check{V}^{(1)}$ is a part of the $3N$ potential that is symmetric with respect to the exchange of particles 2 and 3, $\check{G}_0(E)$ is the free propagator for energy E and finally $\check{P} = \check{P}_{12}\check{P}_{23} + \check{P}_{13}\check{P}_{23}$ is an operator composed from transpositions $\check{P}_{ij}$. After removing the spin dependencies from the resulting equations, Eq. (2) is transformed into

$$\check{A}(E)\,\psi = \psi, \tag{3}$$

where the energy-dependent operator $\check{A}(E)$ acts in a linear space spanned by the scalar functions ψ $\left(\equiv \psi^{(i)}_{tT}(p, q, \hat{\boldsymbol{p}} \cdot \hat{\boldsymbol{q}})\right)$ that appear in Eq. (1). In practice a slightly different equation is solved:

$$\check{A}(E)\,\psi = \lambda\psi, \tag{4}$$

and various values of the energy E are checked looking for the one that yields $\lambda = 1$, so the corresponding energy is the bound state energy. The resulting scalar functions ψ can be used to reconstruct the Faddeev component.

The full bound state wave function of the $3N$ system $|\Psi\rangle$ is related to the Faddeev component $|\psi\rangle$ via

$$|\,\Psi\rangle = \left(\check{1} + \check{P}\right)|\,\psi\rangle. \tag{5}$$

The operator form (1) can also be used to represent the full bound state function $|\Psi\rangle$ where it will be defined by a different set of scalar functions Ψ $\left(\equiv \Psi^{(i)}_{tT}(p, q, \hat{\boldsymbol{p}}\cdot\hat{\boldsymbol{q}})\right)$. The operator forms of $|\Psi\rangle$ and $|\psi\rangle$ can be inserted into Eq. (5) and the spin dependencies can be removed. This results in the following equation:

$$\Psi = \check{B}\psi, \tag{6}$$

where the operator $\check{B}$ acts in a linear space spanned by the scalar functions that define the Faddeev component.

2 Numerical results

We show in Figs. 1 and 2 selected scalar functions ($\psi^{(1)}$, $\Psi^{(1)}$ and $\psi^{(2)}$, $\Psi^{(2)}$, respectively) for the Faddeev component $|\psi\rangle$ and the full bound state function $|\Psi\rangle$ of ^{3}H. These results were obtained using the same $2N$ and $3N$ potentials as in Ref. [3].

In order to verify our solutions, all plots related to the Faddeev component $|\psi\rangle$ also contain the functions

$$\beta = \check{A}\psi. \tag{7}$$

Since the functions ψ satisfy Eq. (4) with $\lambda \approx 1$, the two functions, ψ and β, should overlap. Additionally all plots related to the full $3N$ bound state function $|\Psi\rangle$ also contain functions

$$\frac{1}{3}\zeta = \frac{1}{3}\check{B}\,|\,\Psi\rangle. \tag{8}$$

Since the operator $\check{B}$ is directly related to $\check{1} + \check{P}$, acting with this operator on the Faddeev scalar functions twice results in a multiplying factor of 3. It can be expected that $\Psi = \frac{1}{3}\zeta$.

The calculations presented in Figs. 1 and 2 both benefit from the new method of implementing the $3N$ forces described in Ref. [5]. That paper also contains results related to the ^{3}He bound state with a screened Coulomb potential from Ref. [6]. The

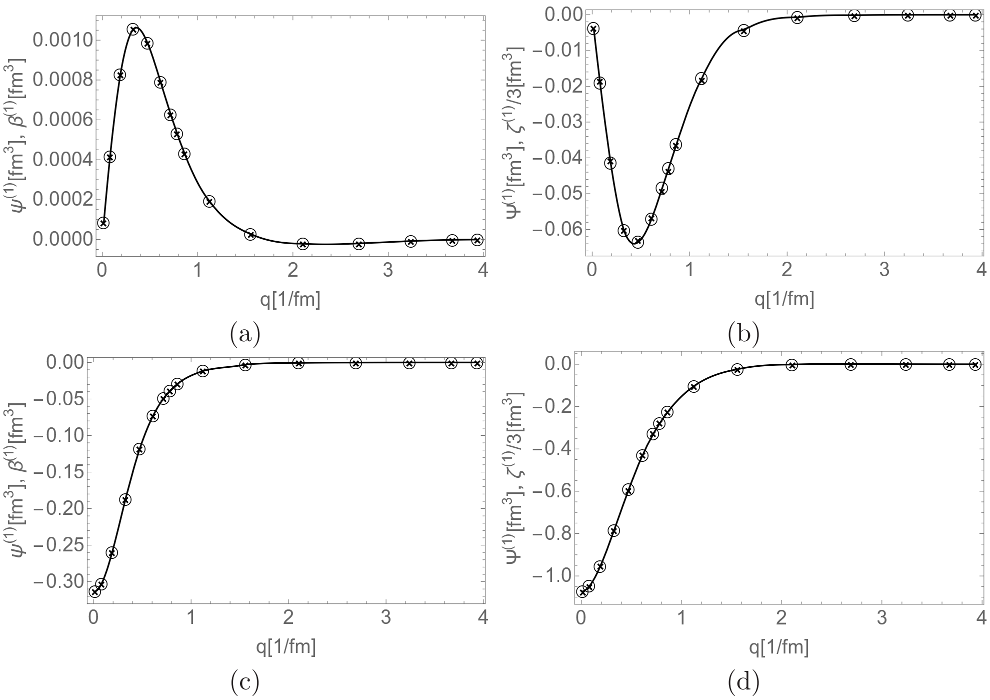

Figure 1: Selected scalar functions for the Faddeev component of the ^{3}H bound state are shown in the first column, panels (a), (c). The values of the ψ (circles) and β (crosses) functions practically overlap, verifying the solution. Chosen scalar functions for the full bound state of ^{3}H are shown in the second column, panels (b), (d). The values of the Ψ (circles) and $\frac{1}{3}\zeta$ (crosses) functions practically overlap, again verifying the obtained solution. All plots show the $t = 0$, $T = \frac{1}{2}$ components with $p = 0.355851$ fm^{-1}, $\hat{\boldsymbol{p}} \cdot \hat{\boldsymbol{q}} = -0.950676$. The presented calculations used an improved implementation of the $3N$ force from Ref. [5], and the resulting value of the bound state energy is -8.62 MeV.

implementation of the Coulomb interaction in $3D$ calculations is straightforward and requires only modifications in certain sets of integration points. The value of the bound state energy for ^{3}He obtained in Ref. [5] is -7.73 MeV.

3 Summary

The new implementation of the $3N$ force in $3D$ calculations described in Ref. [5] makes it possible to test a wide variety of nuclear interactions. Although the calculations presented in Ref. [5] were carried out with a small number of grid points for the scalar functions, the results are encouraging and we plan future calculations with a larger number of integration points. This will require adaptation of our code to an efficient use of the JURECA booster computer at the Jülich Supercomputing Center. This will increase the precision of the $3D$ calculations and allow us to test the newest versions of two- and three-nucleon forces.

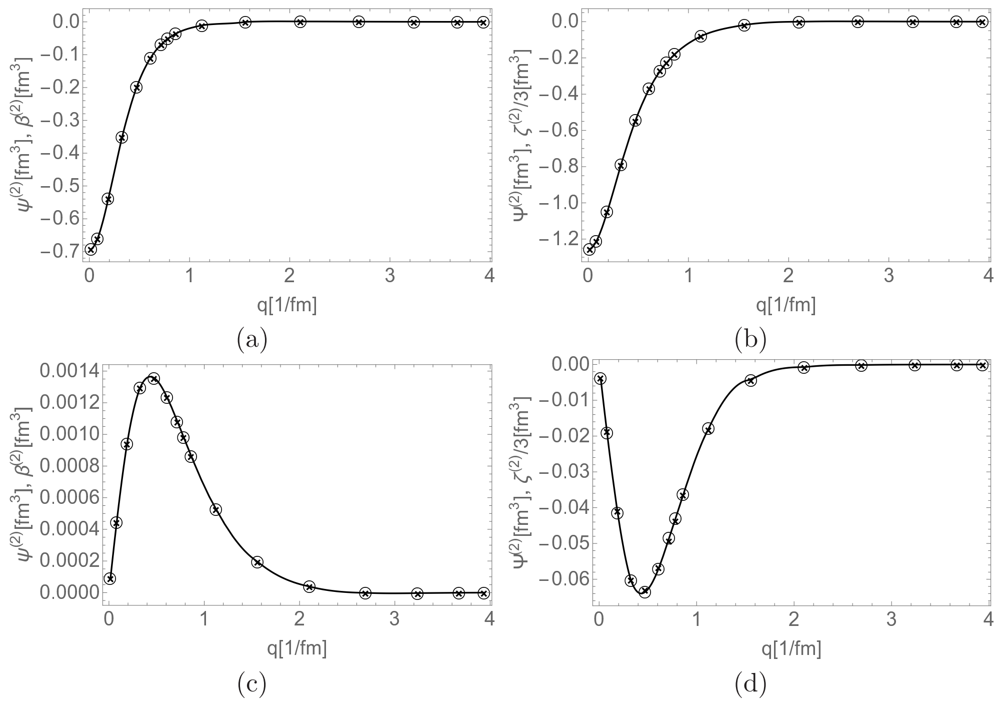

Figure 2: Selected scalar functions for the Faddeev component of the ^{3}H bound state are shown in the first column, panels (a), (c). The values of the ψ (circles) and β (crosses) functions practically overlap, verifying the solution. Chosen scalar functions for the full bound state of ^{3}H are shown in the second column, panels (b), (d). The values of the Ψ (circles) and $\frac{1}{3}\zeta$ (crosses) functions practically overlap, again verifying the obtained solution. All plots show the $t = 1, T = \frac{1}{2}$ components with $p = 0.355851$ fm^{-1}, $\hat{\boldsymbol{p}} \cdot \hat{\boldsymbol{q}} = -0.950676$. The presented calculations used an improved implementation of the $3N$ force from Ref. [5], and the resulting value of the bound state energy is -8.62 MeV.

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