International Workshop "Nuclear Theory in the Supercomputing Era"

### Application of the Monte Carlo shell model to the ab initio no core calculations

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> Pacific National University, Khabarovsk, Russia June 18-22, 2012

## Outline of this talk

- Motivation
- Monte Carlo Shell Model (MCSM)
- Benchmark s in p-shell nuclei
- Tests in sd-shell nuclei
- Application to K-computer
- Summary & perspective

## Current status of ab inito approaches

- Major challenge of the nuclear structure theory
	- Understand the nuclear structures from the first principle of quantum many-body theory by *ab-initio* calc w/ realistic nuclear forces
	- Standard approaches: GFMC, NCSM (up to A  $\sim$  12-14), CC (closed shell +/- 1,2), SCGF theory, IM-SRG, Lattice EFT, …
	- demand for extensive computational resources

*ab-initio*(-like) SM approaches (which attempt to go) beyond standard methods

- IT-NCSM, IT-CI: R. Roth (TU Darmstadt), P. Navratil (TRIUMF)
- Sp-NCSM: T. Dytrych, K.D. Sviratcheva, J.P. Draayer, C. Bahri, & J.P. Vary (Louisiana State U, Iowa State U)
- No-Core Monte Carlo Shell Model (MCSM)

Review: T. Otsuka , M. Honma, T. Mizusaki, N. Shimizu, Y. Utsuno, Prog. Part. Nucl. Phys. 47, 319 (2001)

# MCSM w/ a core

• MCSM (w/ a core) is one of the powerful shell model algorithms.





#### **Nuclear Landscape**

No-Core MCSM

UNEDF SciDAC Collaboration: http://unedf.org/

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F-process

terra incognita

Ab initio **Configuration Interaction Density Functional Theory** 

MCSM

 $\overline{\phantom{a}}$ 

neutrons

stable nuclei

**Contract Contract Co** 



Review: T. Otsuka , M. Honma, T. Mizusaki, N. Shimizu, Y. Utsuno, Prog. Part. Nucl. Phys. 47, 319 (2001)

### Monte Carlo shell model (MCSM)

• Importance truncation

#### Standard shell model



## SM Hamiltonian & MCSM many-body w.f.

• 2nd-quantized non-rel. Hamiltonian (up to 2-body term, so far)

$$
H = \sum_{\alpha\beta}^{N_{sps}} t_{\alpha\beta} c_{\alpha}^{\dagger} c_{\beta} + \frac{1}{4} \sum_{\alpha\beta\gamma\delta}^{N_{sps}} \bar{v}_{\alpha\beta\gamma\delta} c_{\alpha}^{\dagger} c_{\beta}^{\dagger} c_{\delta} c_{\gamma} \quad \bar{v}_{ijkl} = v_{ijkl} - v_{ijlk}
$$

• Eigenvalue problem

$$
H|\Psi(J,M,\pi)\rangle=E|\Psi(J,M,\pi)\rangle
$$

• MCSM many-body wave function & basis function<br>  $|\Psi(I, M, \pi)\rangle = \sum_{i}^{N_{basis}} (f_i) \Phi_i(I, M, \pi) \rangle$   $|\Phi(I, M, \pi)\rangle = \sum_{K} (g_K)^2 N_K P^{\pi} |\phi\rangle$ • Deformed SDs ( *c<sup>α</sup>* †*…* HO basis) These coeff. are obtained by Housholder/Lanczos methods. This coeff. is obtained by a stochastic sampling.

### Sampling of basis functions in the MCSM

• Deformed Slater determinant basis

$$
|\phi\rangle = \prod_{i}^{A} a_{i}^{\dagger} |-\rangle \qquad a_{i}^{\dagger} = \sum_{\alpha}^{N_{sps}} c_{\alpha}^{\dagger} D_{\alpha i} \qquad (\mathbf{c}_{\alpha}^{\mathbf{+}} ... \text{ HO basis})
$$

• Stochastic sampling of deformed SDs

$$
|\phi(\sigma)\rangle = e^{-h(\sigma)}|\phi\rangle
$$
  

$$
h(\sigma) = h_{HF} + \sum_{i}^{N_{AF}} s_i V_i \sigma_i O_i
$$



c.f.) Imaginary-time evolution & Hubbard-Stratonovich transf.

$$
|\phi(\sigma)\rangle = \prod_{N_{\tau}} e^{-\Delta\beta h(\sigma)} |\phi\rangle
$$
  
\n
$$
e^{-\beta H} = \int_{-\infty}^{+\infty} \prod_{i} d\sigma_{i} \sqrt{\frac{\beta |V_{i}|}{2\pi}} e^{-\frac{\beta}{2}|V_{i}| \sigma_{i}^{2}} e^{-\beta h(\vec{\sigma})}
$$
  
\n
$$
h(\sigma) = \sum_{i}^{N_{AF}} (\epsilon_{i} + s_{i}V_{i}\sigma_{i}) O_{i}
$$
  
\n
$$
H = \sum_{i} \epsilon_{i} O_{i} + \frac{1}{2} \sum_{i} V_{i} O_{i}^{2}
$$

### Rough image of the search steps

- Basis search
	- HF solution is taken as the  $1<sup>st</sup>$  basis

Hamiltonian  
\nkernel  
\n
$$
H(\Phi, \Phi') =
$$

 $(n-1)*(n-1)$ matrix

fixed

Fix the n-1 basis states already taken

**n-th** (to be optimized)

– Requirement for the new basis: atopt the basis which makes the energy (of a many-body state) as low as possible by a stochastic sampling



### **Energy minimization by Conjugate Gradient method**

Evaluation of the energy variance is time consuming due to the four-body interaction.

$$
\left|\Psi(D)\right\rangle = \sum_{n=1}^{N_B} c_i \sum_{K=-J}^{J} g_K P_{MK}^{J,\Pi} \left| \phi(D^{(n)}) \right\rangle \; \left| \phi(D^{(n)}) \right\rangle = \prod_{\alpha=1}^{N_p} \left( \sum_{i=1}^{N_{sp}} c_i^{\dagger} D_{i\alpha}^{(n)} \right) - \rangle
$$

$$
E(D) = \langle \Psi(D) | H | \Psi(D) \rangle
$$

Minimize *E(D)* as a function of *D* utilizing Conjugate Gradient method

Step1 :Generate basis candidate by auxiliary field technique stochastically

$$
\ket{\phi(\sigma)}\!=\!\prod e^{\scriptscriptstyle{\Delta\beta\cdot h(\sigma)}}\!\!\cdot\!\ket{\phi^{\scriptscriptstyle{(0)}}}\! \qquad \qquad \text{Few Determinan}
$$

and select basis which lowers the energy

Step 2:Energy expectation value is taken as a function of D, and optimize it using Conjugate Gradient method (VAP)

Iterate these steps every basis till the energy converges

Few Determinant Approximation M. Honma, B.A.Brown, T. Mizusaki, and T. Otsuka Nucl. Phys. A 704, 134c (2002)  $\prod_{\alpha=1} \left( \sum_{i=1}^{n} C_i^{\dagger} D_{i\alpha}^{(n)} \right) \rightarrow$ <br>  $\prod_{\alpha=1}^{n} C_i^{\dagger} D_{i\alpha}^{(n)}$ <br>  $\vdots$  Pew Determinant Approxim<br>
M. Honma, B.A.Brown, T. Mizu<br>
Nucl. Phys. A 704, 134c (2002)<br>  $\vdots$  Hybrid Multi-Determinant<br>
G. Puddu, Acta P

Hybrid Multi-Determinant G. Puddu, Acta Phys. Polon. B42, 1287 (2011)

#### VAMPIR

K.W. Schmid, F. Glummer, M. Kyotoku, and A. Faessler



Conjugate gradient taken from wikipedia

#### **Energy minimization by Conjugate Gradient method**



### Recent developments in MCSM

• Acceleration of the computation of two-body matrix elements

$$
\langle \phi | \hat{V} | \phi' \rangle = \frac{1}{2} \sum_{i,k} \rho_{ki} \left( \sum_{j,l} v_{ijkl} \rho_{lj} \right) = \frac{1}{2} \sum_{(ki)} \rho_{(ki)} \left( \sum_{jl} v_{(ki),(lj)} \rho_{(lj)} \right)
$$

 Matrix product is performed w/ bundled density matrices by DGEMM subroutine in BLAS level-3 library

Y. Utsuno, N. Shimizu, T. Otsuka, and T. Abe, arXiv:1202.2957 [nucl-th] (submitted to Comp. Phys. Comm.)

• Extrapolation method by the energy variance

$$
\langle \phi | \hat{V} | \phi' \rangle = \frac{1}{2} \sum_{i,k} \rho_{ki} \Bigg[ \sum_{j,l} v_{ijkl} \rho_{lj} \Bigg] = \frac{1}{2} \sum_{(ki)} \rho_{(ki)} \Bigg[ \sum_{jl} v_{(ki),(lj)} \rho_{(lj)} \Bigg]
$$
  
\nMatrix product is performed w/ bundle density matrices by DGEMM  
\nsubroutine in BLAS level-3 library  
\nY. Utsuno, N. Shimizu, T. Otsuka, and T. Abe, arXiv:1202.2957 [nucl-th] (submitted to Comp. Phys. Comm.)  
\n• Extrapolation method by the energy variance  
\n
$$
\langle H \rangle = E_0 + E_1 \langle \Delta H^2 \rangle + E_2 \langle \Delta H^2 \rangle^2 + \cdots \qquad \langle \Delta H^2 \rangle = \langle H^2 \rangle - \langle H \rangle^2
$$
  
\n
$$
\frac{\langle \phi | \hat{H}^2 | \psi \rangle}{\langle \phi | \psi \rangle} = \sum_{i < j, \alpha < \beta} \Bigg( \sum_{k < l} v_{ijkl} ((1 - \rho)_{k\alpha} (1 - \rho)_{l\beta} - (1 - \rho)_{l\alpha} (1 - \rho)_{k\beta}) \Bigg) \Bigg( \sum_{\gamma < \delta} v_{\alpha\beta\gamma\delta} (\rho_{\gamma i} \rho_{\delta j} - \rho_{\delta i} \rho_{\gamma j}) \Bigg) + \text{Tr}((t + \Gamma)(1 - \rho)(t + \Gamma)\rho) + \Big( \text{Tr}(\rho(t + \frac{1}{2}\Gamma)) \Bigg)^2 \qquad \Gamma_{ik} = \sum_{j,l} v_{ijkl} \rho_{lj}
$$
  
\n(naiively) 8-fold loops  $\rightarrow$  (effectively) 6-fold loops by the factorization

N. Shimizu, Y. Utsuno, T.Mizusaki, T. Otsuka, T. Abe, & M. Honma, Phys. Rev. C82, 061305(R) (2010) 13

# Hot spot of the MCSM calculation

• Evaluation of the Hamiltonian kernel btw. non-orthogonal SDs

$$
\mathcal{H}(q',q) = \mathcal{N}(q',q) \left( \sum_{l_1 l_2}^{N_s} t_{l_1 l_2} \rho_{l_2 l_1} + \frac{1}{2} \sum_{l_1 l_2 l_3 l_4}^{N_s} \rho_{l_3 l_1} \bar{v}_{l_1 l_2, l_3 l_4} \rho_{l_4 l_2} \right)
$$

$$
\langle V \rangle \equiv \sum_{l_1 l_2 l_3 l_4}^{N_s} \rho_{l_3 l_1} \, \bar{v}_{l_1 l_2, l_3 l_4} \, \rho_{l_4 l_2}
$$

## Computation of the TBMEs

- hot spot: Computation of the TBMEs c.f.) Indirect-index method (list-vector method)  $\frac{\langle \Phi'|V|\Phi\rangle}{\langle \Phi'|\Phi\rangle} = \frac{1}{2} \sum_{ijkl} \bar{v}_{ijkl} \rho_{ki} \rho_{lj}$  (w/o projections, for simplicity)<br>c.f.) Indirect-index meth
- Utilization of the symmetry

 $j_z(i)+j_z(j) = j_z(k)+j_z(l) \rightarrow j_z(i)-j_z(k) = -(j_z(j)-j_z(l)) \equiv \Delta m$ 

$$
\sum_{ijkl} \overline{v}_{ijkl} \rho_{ki} \rho_{lj} = \sum_{\Delta m} \left[ \sum_{a \in J_z(a) = -\Delta m} \tilde{\rho}_a \left( \sum_{b \in J_z(b) = \Delta m} \tilde{v}_{ab} \tilde{\rho}_b \right) \right]
$$

 $\rho_{ki} \rightarrow \tilde{\rho}_a \qquad \rho_{lj} \rightarrow \tilde{\rho}_b$  $\bar{v}_{ijkl} \rightarrow \tilde{v}_{ab}$ 

sparse dense

Schematic illustration of the computation of TBMEs

• Matrix-vector method

$$
\sum_{ijkl} \overline{v}_{ijkl} \rho_{ki} \rho_{lj} = \sum_{\Delta m} \left[ \sum_{a \in J_z(a) = -\Delta m} \widetilde{\rho}_a \left( \sum_{b \in J_z(b) = \Delta m} \widetilde{v}_{ab} \widetilde{\rho}_b \right) \right]
$$



Schematic illustration of the computation of TBMEs

• Matrix-matrix method



### Size of the off-diagonal dense matrix

$$
\sum_{ijkl} \bar{v}_{ijkl} \rho_{ki} \rho_{lj} = \sum_{\Delta m} \left[ \sum_{a \in J_z(a) = -\Delta m} \tilde{\rho}_a \left( \sum_{b \in J_z(b) = \Delta m} \tilde{v}_{ab} \tilde{\rho}_b \right) \right]
$$



Matrix size

Y. Utsuno, N. Shimizu, T. Otsuka, T. Abe, arXiv:1202.2957

# Tuning of the density matrix product



 $Nshell = 5$ 

The performance reaches 80% of the theoretical peak at hot spot.

SPARC64 requires large  $N_{\text{bunch}}$  in comparison to Xeon

Matrix product e.g.  $(390 \times 390) \times (390 \times 2N_{\text{bunch}})$ 

N<sub>bunch</sub> controllable tuning parameter chunk size

# Extrapolations in the MCSM

• Two steps of the extrapolation

1. Extrapolation of our MCSM (approx.) results to the FCI (exact) results in fixed model space

Energy-variance extrapolation

2. Extrapolation into the infinite model space Not applied in the MCSM, so far…

# Energy-variance extrapolation



### Why we need to extrapolate the energies

• Definition: (Correlation Energy)  $\equiv \langle \Psi | H |\Psi \rangle_{\rm JHF} - \langle \Psi | H |\Psi \rangle_{\rm Exact}$ 



NCSM wf w/ realistic NN int is more correlated (complicated) than SSM wf w/ effective int

22 Need energy-variance extrapolation for No-Core MCSM calc

# Energy-variance extrapolation

• Originally proposed in condensed matter physics

 Path Integral Renormalization Group method M. Imada and T. Kashima, J. Phys. Soc. Jpn 69, 2723 (2000)

• Imported to nuclear physics

Lanczos diagonalization with particle-hole truncation

T. Mizusaki and M. Imada Phys. Rev. C65 064319 (2002)

T. Mizusaki and M. Imada Phys. Rev. C68 041301 (2003)

single deformed Slater determinant

T. Mizusaki, Phys. Rev. C70 044316 (2004)

Apply to the MCSM (multi deformed SDs)

# Numerical effort 8-folded loop = $\left(\sum_{ijkl\alpha\beta\gamma\delta}^{\infty}\right) \overline{v}_{ijkl}\overline{v}_{\alpha\beta\gamma\delta} \left[\frac{1}{4}(1-\rho)_{k\alpha}(1-\rho)_{l\beta}\rho_{\gamma i}\rho_{\delta j} + \rho_{\gamma\alpha}(1-\rho)_{l\beta}\rho_{ki}\rho_{\delta j} + \frac{1}{4}\rho_{ki}\rho_{lj}\rho_{\gamma\alpha}\rho_{\delta\beta}\right]$  $\frac{\langle\Phi'|V^2|\Phi\rangle}{\langle\Phi'|\Phi\rangle}$  $\frac{1}{4} \left( \sum_{i,j \in \beta} \left( \sum_{kl} \overline{v}_{ijkl} (1-\rho)_{k\alpha} (1-\rho)_{l\beta} \right) \left( \sum_{\gamma \delta} \overline{v}_{\alpha\beta\gamma\delta} \rho_{\gamma i} \rho_{\delta j} \right) \right)$ 6-folded loop  $+Tr(\Gamma(1-\rho)\Gamma\rho)+\frac{1}{4}[\text{Tr}(\rho\Gamma)]^2$  $^{\sim}O(Nsps^{6})$

$$
\rho_{\beta\alpha}=\frac{\langle\Phi'|c_{\alpha}^{\dagger}c_{\beta}|\Phi\rangle}{\langle\Phi'|\Phi\rangle}\ \ \Gamma_{ik}=\sum_{jl}\overline{v}_{ijkl}\rho_{lj}\ \frac{\langle\Phi'|V|\Phi\rangle}{\langle\Phi'|\Phi\rangle}=\frac{1}{2}\sum_{\alpha\beta\gamma\delta}\overline{v}_{\alpha\beta\gamma\delta}\rho_{\gamma\alpha}\rho_{\delta\beta}
$$

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N. Shimizu, Y. Utsuno, T.Mizusaki, T. Otsuka, T. Abe, & M. Honma, Phys. Rev. C82, 061305(R) (2010)

# Extrapolation of 12C Energy



#### Benchmark results

- Energy
- RMS
- Q-moment
- μ-moment

#### What we have calculated as Benchmark T. Abe, P. Maris, T. Otsuka, N. Shimizu, Y. Utsuno, J. P. Vary, arXiv:1204.1755

- Comparison btw MCSM & FCI (exact diag.) calc
- Nuclei (JP): s- & p-shell nuclei:
	- $-4He(0+)$
	- $6He(0+)$
	- $-6Li(1+)$
	- $-7$ Li $(1/2-$ ,  $3/2-$ )
	- 8Be(0+)
	- $-10B(1+, 3+)$
	- $-12C(0+)$
- Observables:
	- $-$  BF
	- Point-particle RMS radius (matter)
	- Electromagnetic moments (Q, μ)
- Our test set up:
- NN interaction: JISP16
- $-$  model space: Nshell = 2, 3, 4, (5)
- optimal hw selected for states & Nshell's
- w/o Coulomb
- w/o Gloeckner-Lawson prescription
- MCSM: Abe, Otsuka, Shimizu, Utsuno (Tokyo) T2K (Tokyo, Tsukuba), BX900 (JAEA)
- FCI: Maris, Vary (Iowa)

Jaguar, Franklin (NERSC, DOE)

#### JISP16:

A.M. Shirokov, J.P. Vary, A. I. Mazur, T.A. Weber, Phys. Lett. B644, 33 (2007) NCFC calc of light nuclei w/ JISP16: P. Maris, J.P. Vary, A.M. Shirokov, Phys. Rev. C 79, 014308 (2009)

#### Helium-4 & carbon-12 gs energies



### Energies of the Light Nuclei T. Abe, P. Maris, T. Otsuka, N. Shimizu, Y. Utsuno, J. P. Vary, arXiv:1204.1755



#### Convergence pattern of the 4He point-particle RMS radius w.r.t. MCSM basis dimension

 $N_{shell}$ 

.<br>.<br>. . .  $N_{shell} = 5$ . . .

• Comparison of MCSM (solid symbols) w/ FCI (dashed lines)  $\omega$  Nshell = 2 (sp), 3 (spsd), & 4 (spsdpf)

Good agreement w/ FCI within 0.001 fm up to Nshell = 4



### Point-particle RMS matter Radius

w/ energy-variance extrapolation by 1<sup>st</sup>-order polynomial



MCSM & FCI results are consistent within the size of symbols

#### Convergence pattern of the 6Li Q-moment w.r.t. MCSM basis dimension

 $N_{shell}$ 

.<br>.<br>. . .  $N_{shell} = 5$ . . .

• Comparison of MCSM (solid symbols) w/ FCI (dashed lines)  $\omega$  Nshell = 2 (sp), 3 (spsd), & 4 (spsdpf)

Good agreement w/ FCI within 0.01 efm<sup>2</sup> up to Nshell = 4





MCSM & FCI results are consistent within the size of symbols

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#### Convergence pattern of the 6Li μ-moment w.r.t. MCSM basis dimension

 $N_{shell}=4$ 

.<br>.<br>. . .  $N_{shell} = 5$ . . .

• Comparison of MCSM (solid symbols) w/ FCI (dashed lines)  $@$  Nshell = 2 (s,p), 3 (s,p,sd), & 4 (s,p,sd,pf)

#### Good agreement w/ FCI within 0.01  $\mu_N$  up to Nshell = 4





μ moment

MCSM & FCI results are consistent with each other, and μ moments are well-reproduced even at small Nshell.

**MCSM** 

# Spurious CoM & Coulomb force

### Spurious CoM



### Spurious CoM



### Coulomb force

• Helium-4 gs energy @ Nshell=2



Coulomb force can be treated perturbatively at least  $\omega$  Nshell = 2.

# Tests in sd shell

# Test calculations in sd-shell nuclei

• Ground-state energies: Hint (MeV)

$$
H_{\beta} = H_{int} + \beta H_{c.m.}
$$

- For hw = 25 MeV w/ JISP16 w/o Coulomb
- MCSM results @ 100 basis dim. w/o energy-variance extrp.



# Application to K-computer



Japanese "K computer" got rank 1 in the world.

The construction will be completed at Nov. 2012.

SPARC64 VIIIfx 548352 cores What is the application program to run on it

# Strategic 5 Field

- Field 1: Computational Life Science and Application in Drug Discovery and Medical Development
- Field 2: Computational Materials Science Initiative (CMMI)
- Field 3: Projection of global change toward the mitigation of natural disasters
- Field 4: Next Generation Manufacturing
- Field 5: The origin of matter and the universe
	- Lattice QCD
	- Nucleus
	- Supernova Explosion
	- Early Star Formation

# Parallel efficiency @ K-computer

- Optimization of 15<sup>th</sup> basis dim. of the 4He (0+) w.f. in Nshell=5  $w/$ 100 CG iterations
- Optimization of 48th basis dim. of the 4He (0+) w.f. in Nshell=6 w/ 100 CG iterations



Note: it is a tentative result by early access to the K-computer at AICS, RIKEN. 45

Ratio to the peak performance  $\omega$  K computer (phase IV-1)

• Test case: Optimization of  $15<sup>th</sup>$  basis dim. of the w.f. in Nshell=5 w/ 100 CG iterations w/o preprocessing (MPI/OpenMP, 8 threads)



Note: it is a tentative result by early access to the K-computer at AICS, RIKEN.

### Strong scaling @ K computer (phase IV-1)

• Test case: Optimization of  $15<sup>th</sup>$  basis dim. of the w.f. in Nshell=5 w/ 100 CG iterations w/o preprocessing (MPI/OpenMP, 8 threads)



Definition (in this case):  $\alpha_{\text{strong}} = (T_{1920 \text{ nodes}}/T_{3840 \text{ nodes}})$  / (3840 nodes/1920 nodes)

Note: it is a tentative result by early access to the K-computer at AICS, RIKEN.

### Summary

- MCSM can be applied to the no-core calculations. Benchmarks for the p-shell & some tests for the sd-shell nuclei have been performed.
	- MCSM & FCI results for the p-shell nuclei are consistent with each other.

#### **Perspective**

- MCSM algorithm
	- Larger model spaces (Nshell = 5, 6, …), Nshell vs Nmax?
	- Inclusion of the (effective) 3-body force
	- Coupling to the continuum states
- Physics
	- Cluster(-like) states (12C Hoyle state, …)
	- Unnatural parity states
- Tuning of the MCSM code on the K Computer

# END