Square-integrable basises in the few-body Coulomb scattering problem

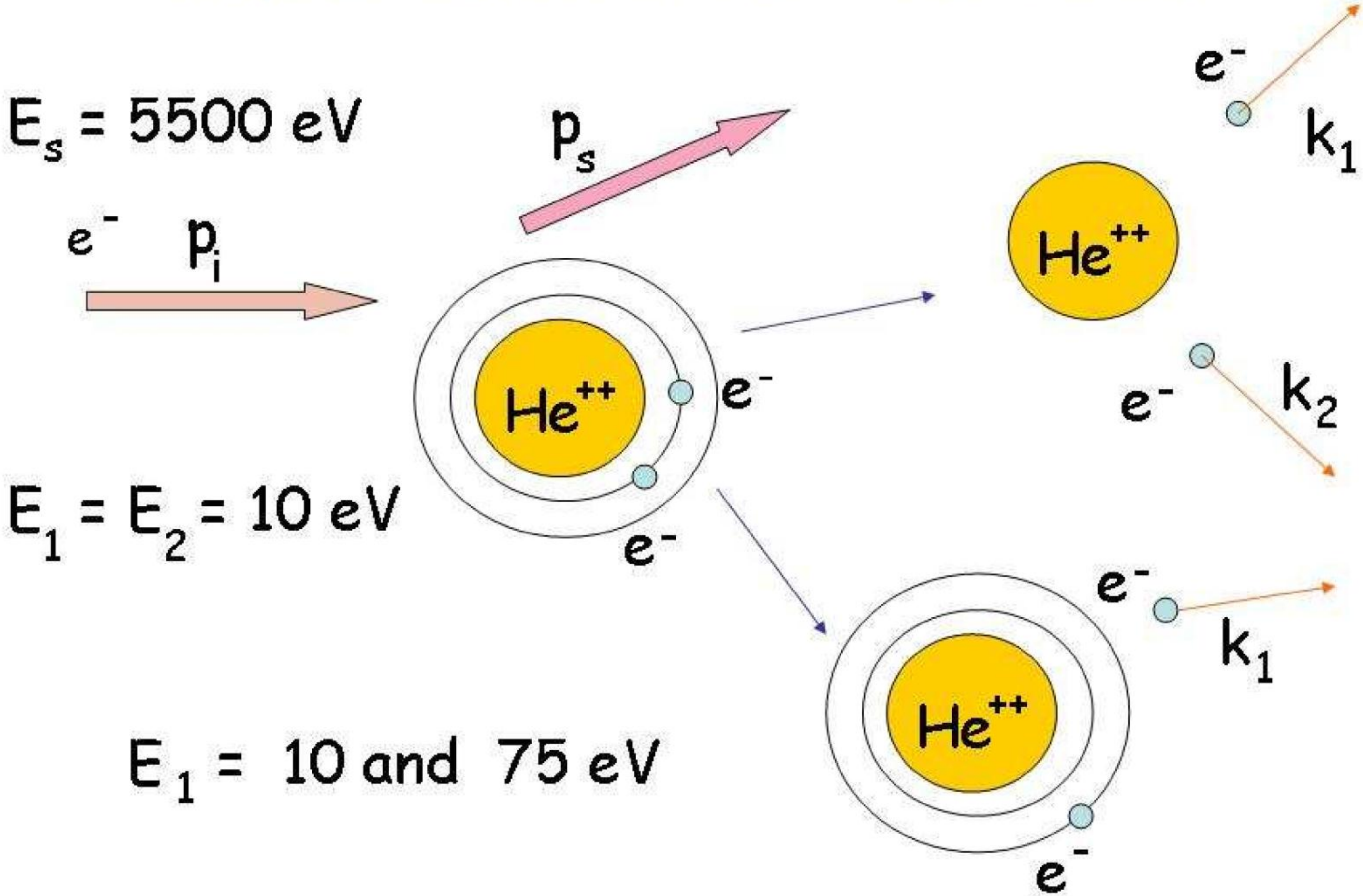
S. A. Zaytsev

Pacific National University, Khabarovsk, Russia

Collaboration

- Yu. V. Popov. Nucl. Phys. Inst., Moscow State University, Moscow, Russia
- B. Piraux. Inst. of Condensed Matter and Nanosciences, Universite catolique de Louvain, Louvain-la-Neuve, Belgium
- M. M. Silenou. CEPAMoQ, Faculty de Science, University of Douala, Douala, Cameroon
- M. G. Kwato. Faculty de Science, University of Douala, Douala, Cameroon
- G. Gasaneo. Departamento de Fisica, Universidad Nacional del Sur, Bahia Blanca, Buenos Aires, Argentina

Impact ionization of helium atom



Two-Electron Continuum (TEC)

- Convergent Close Coupling approach (CCC);
- Coulomb-Sturmian Separable Expansion method;
- J-Matrix method;
- R-Matrix method with Pseudo-States (RMPS);
- Exterior Complex Scaling approach (ECS)

TEC wave function is approximated by a product of two fixed charge Coulomb waves

$$\Psi_{C2} = N_1 e_1^{i\mathbf{k_1}\cdot\mathbf{r}_1} F_1 \left[i\frac{Z}{k_1}, 1, -i\left(k_1 r_1 + \mathbf{k}_1 \cdot \mathbf{r}_1\right)\right]$$
$$\times N_2 e_1^{i\mathbf{k}_2\cdot\mathbf{r}_2} F_1 \left[i\frac{Z}{k_2}, 1, -i\left(k_2 r_2 + \mathbf{k}_2 \cdot \mathbf{r}_2\right)\right]$$

A long-range potential appears in the kernel of the corresponding Lippmann-Schwinger equation

J-matrix approach to electron-impact ionization of He

Schrödinger equation

$$\left[E + \frac{1}{2}\triangle_1 + \frac{1}{2}\triangle_2 + \frac{Z}{r_1} + \frac{Z}{r_2} - \frac{1}{r_{12}}\right]\Psi^{(-)}(\mathbf{k}_1, \mathbf{k}_2; \mathbf{r}_1, \mathbf{r}_2) = 0$$

$$\frac{Z}{r_i} = V_i^{(s)}(r_1, r_2) + V_i^{(l)}(r_1, r_2),$$

$$V_i^{(s)}(r_1, r_2) = \frac{Z}{r_i}\,\zeta(r_i, r_j), \quad V_i^{(l)}(r_1, r_2) = \frac{Z}{r_i}[1 - \zeta(r_i, r_j)]$$

$$\zeta(r_i, r_j) = 2/\left\{1 + \exp[(r_i/a)^\nu/(1 + r_j/b)]\right\}.$$

$$\Omega_0 :\; r_1 \sim r_2 \to \infty,\ \Omega_1 :\; r_1 \gg r_2 \text{ or } \Omega_2 :\; r_2 \gg r_1.$$

The final state wave function

$$\Psi^{(-)} = \frac{1}{\sqrt{2}}\left[\Psi_1^{(-)} + \Psi_2^{(-)}\right],\ \Psi_2^{(-)} = g\hat{P}_{12}\Psi_1^{(-)},\ g = \pm 1$$

$$\left[E + \frac{1}{2}\triangle_1 + \frac{1}{2}\triangle_2 + \frac{Z-1}{r_1} + \frac{Z}{r_2}\right]\Psi_1^{(-)} = V(\mathbf{r}_1, \mathbf{r}_2)\Psi_1^{(-)} \qquad (1)$$

$$V(\mathbf{r}_1, \mathbf{r}_2) = \frac{1}{r_{12}} - V_1^{(l)}(r_1, r_2) + \frac{Z-1}{r_1} - gV_2^{(s)}(r_1, r_2)\widehat{P}_{12}. \qquad (2)$$

$V(\mathbf{r}_1, \mathbf{r}_2)$ is short-range in the two-body scattering region Ω_1 $(r_1 \gg r_2)$

$$\hat{g}_1^{(-)}(E_1) = \left[E_1 + \frac{1}{2}\triangle_1 + \frac{Z-1}{r_1}\right]^{-1}$$

$$\hat{g}_2^{(-)}(E_2) = \left[E_2 + \frac{1}{2}\triangle_2 + \frac{Z}{r_2}\right]^{-1}$$

Lippmann-Schwinger type equation

$$\begin{aligned}\Psi_1^{(-)}(\mathbf{k}_1,\mathbf{k}_2;\mathbf{r}_1,\mathbf{r}_2) \;=\; & [\varphi^{(-)}(\mathbf{k}_2,\mathbf{r}_2;\,Z)\,\varphi^{(-)}(\mathbf{k}_1,\mathbf{r}_1;\,Z-1)\theta(k_1-k_2) \\ + \; & g\varphi^{(-)}(\mathbf{k}_1,\mathbf{r}_2;\,Z)\,\varphi^{(-)}(\mathbf{k}_2,\mathbf{r}_1;\,Z-1)\theta(k_2-k_1)] \\ + \; & \int\int d\mathbf{r}_1'\,d\mathbf{r}_2' G^{(-)}(\mathbf{r}_1,\mathbf{r}_2;\,\mathbf{r}_1',\mathbf{r}_2';\,E)\,V(\mathbf{r}_1',\mathbf{r}_2')\,\Psi_1^{(-)}(\mathbf{k}_1,\mathbf{k}_2;\,\mathbf{r}_1',\mathbf{r}_2').\end{aligned} \tag{1'}$$

Green's Function

$$G^{(-)}(\mathbf{r}_1,\mathbf{r}_2;\,\mathbf{r}_1',\mathbf{r}_2';\,E)=\frac{1}{2\pi\mathrm{i}}\int_{\mathcal{C}} d\mathcal{E} g^{(-)}(\mathbf{r}_1,\mathbf{r}_1';\,\mathcal{E}-\mathrm{i}0;\,Z-1)\;g^{(-)}(\mathbf{r}_2,\mathbf{r}_2';\,E-\mathcal{E}-\mathrm{i}0;\,Z).$$

Partial decomposition

$$\Psi_1^{(-)}(\mathbf{k}_1, \mathbf{k}_2;\, \mathbf{r}_1, \mathbf{r}_2) = \frac{2}{\pi}\frac{1}{k_1\, k_2} \sum_{\substack{L\, l_0\, \lambda_0 \\ m_0\, \mu_0}} (l_0\, m_0\, \lambda_0\, \mu_0 | L\, M)\, i^{l_0+\lambda_0}$$

$$\Big[e^{-i(\sigma_{l_0}(k_1, Z-1)+\sigma_{\lambda_0}(k_2,Z))}\psi^{L\,M}_{l_0\,\lambda_0}(\mathbf{r}_1, \mathbf{r}_2; k_1, k_2) Y^*_{l_0\,m_0}(\widehat{\mathbf{k}}_1)\, Y^*_{\lambda_0\,\mu_0}(\widehat{\mathbf{k}}_2)\theta(k_1 - k_2) +$$

$$g e^{-i(\sigma_{l_0}(k_2, Z-1)+\sigma_{\lambda_0}(k_1,Z))}\psi^{L\,M}_{l_0\,\lambda_0}(\mathbf{r}_1, \mathbf{r}_2; k_2, k_1) Y^*_{l_0\,m_0}(\widehat{\mathbf{k}}_2)\, Y^*_{\lambda_0\,\mu_0}(\widehat{\mathbf{k}}_1)\theta(k_2 - k_1)\Big]\,.$$

Partial wave function

$$\psi^{L\,M}_{l_0\,\lambda_0}(\mathbf{r}_1, \mathbf{r}_2) = \sum_{l,\lambda,n,\nu} C^{L\,(l\,\lambda)}_{n\,\nu}(E)\ \langle \mathbf{r}_1, \mathbf{r}_2 | n\, l\, \nu\, \lambda;\ LM\rangle\,,$$

$$\langle \mathbf{r}_1, \mathbf{r}_2 | n\, l\, \nu\, \lambda;\ LM\rangle = \frac{\phi^l_n(r_1)}{r_1}\,\frac{\phi^\lambda_\nu(r_2)}{r_2}\mathcal{Y}^{LM}_{l\lambda}(\hat{\mathbf{r}}_1,\ \hat{\mathbf{r}}_2).$$

Sturmians

$$\phi_\nu^\lambda(r) = \left[\frac{(\nu-\lambda-1)!}{(\nu+\lambda)!}\right]^{1/2} (2\kappa r)^{\lambda+1} e^{-\kappa r} L_{\nu-\lambda-1}^{2\lambda+1}(2\kappa r), \quad \nu \geq 1+\lambda$$

$$\int_0^\infty \frac{dr}{r} \phi_\nu^\lambda(r) \phi_{\nu'}^\lambda(r) = \delta_{\nu,\nu'}$$

Coulomb partial spectral state

$$\varphi_l^\alpha(r;Z) = \sum_n \mathcal{S}_{nl}^\alpha(Z) \phi_n^l(r).$$

$$\mathcal{S}_{nl}(k;Z) = \left[\tfrac{(n+l)!}{(n-l-1)!}\right]^{1/2} 2^l \sin(\zeta)^{l+1} e^{-\pi t/2} \xi^{-it} \frac{|\Gamma(l+1-it)|}{(2l+1)!} \times$$
$$(\xi)^{-(n-l-1)} \, {}_2F_1(-n+l+1, l+1-it; 2l+2; 1-\xi^2).$$

$$t = \tfrac{Z}{k}; \;\; \xi = e^{i\zeta} = \tfrac{ik-\kappa}{ik+\kappa}.$$

Equation for coefficients

$$C_{n\,\nu}^{L(l\lambda)}(k_1,k_2)=\delta_{(l\lambda)(l_0\lambda_0)}\mathcal{S}_{nl_0}(k_1,Z-1)\mathcal{S}_{\nu\lambda_0}(k_2,Z)+$$

$$\sum_{n'\nu'n''\nu''}^{N}\left[\frac{1}{2\pi i}\int_{\mathcal{C}} d\mathcal{E}\; g_{nn'}^{(-)l}(\mathcal{E},Z-1)\; g_{\nu\nu'}^{(-)\lambda}(E-\mathcal{E},Z)\right]$$

$$\sum_{l''\lambda''}\; V_{n'\nu',\,n''\nu''}^{L(l\lambda)(l''\lambda'')}\; C_{n''\nu''}^{L(l''\lambda'')}(E).$$

$$V(\mathbf{r}_1,\mathbf{r}_2)=\frac{1}{r_{12}}-V_1^{(l)}(r_1,r_2)+\frac{Z-1}{r_1}-gV_2^{(s)}(r_1,r_2)\widehat{P}_{12}.$$

Potential matrix elements

$$V_{n\,\nu,\,n'\,\nu'}^{L(l\,\lambda)(l'\,\lambda')}=\langle n\,l\,\nu\,\lambda;\;LM|\,V(\mathbf{r_1},\,\mathbf{r_2})\,|n'\,l'\,\nu'\,\lambda';\;LM\rangle \quad (3)$$

Green's function matrix elements

$$g_{\nu\nu'}^{(\pm)\lambda}(E;Z) = -\frac{2}{p}\,\mathcal{S}_{\nu_<\lambda}(p,Z)\,\mathcal{C}_{\nu_>\lambda}^{(\pm)}(p,Z),$$

with $\nu_< = \min\{\nu,\nu'\}\,; \quad \nu_> = \max\{\nu,\nu'\}$ and $p=\sqrt{2E}$.

$$\mathcal{C}_{nl}^{(\pm)}(p,Z) = -\left[\tfrac{(n+l)!}{(n-l-1)!}\right]^{1/2}\,\frac{(n-l-1)!\;e^{\pi t/2}\xi^{it}}{(2\sin\zeta)^{l}}\,\frac{\Gamma(l+1\mp it)}{|\Gamma(l+1\mp it)|}\times$$
$$\times\frac{(\xi)^{\mp(n-l)}}{\Gamma(n+1\mp it)}\;\;{}_2F_1\left(-l\mp it, n-l; n+1\mp it; \xi^{\mp 2}\right).$$

$$\mathcal{C}_{nl}^{(+)}(p,Z) = \mathcal{C}_{nl}^{(-)}(p,Z) + 2i\mathcal{S}_{nl}(p,Z).$$

The five-fold differential cross-section of the (e, 3e) reaction

$$\frac{d^5\sigma}{d\Omega_s dE_1 d\Omega_1 dE_2 d\Omega_2} = \frac{4 p_s k_1 k_2}{p_i Q^4} \times$$
$$|\langle \Psi^{(-)}(\mathbf{k}_1, \mathbf{k}_2)| \exp(i\mathbf{Q}\mathbf{r}_1) + \exp(i\mathbf{Q}\mathbf{r}_2) - 2|\Psi_0\rangle|^2 .$$

Here $(E_i, \mathbf{p_i})$, $(E_s, \mathbf{p_s})$, $(E_1, \mathbf{k_1})$ and $(E_2, \mathbf{k_2})$ are energies and momenta of, respectively, the fast incident, fast scattered, and two slow ejected electrons; $\mathbf{Q}=\mathbf{p_i} - \mathbf{p_s}$ is the (little) transferred momentum,

Phys. Rev. A. 2011. V. 83. P. 052708-1-11.

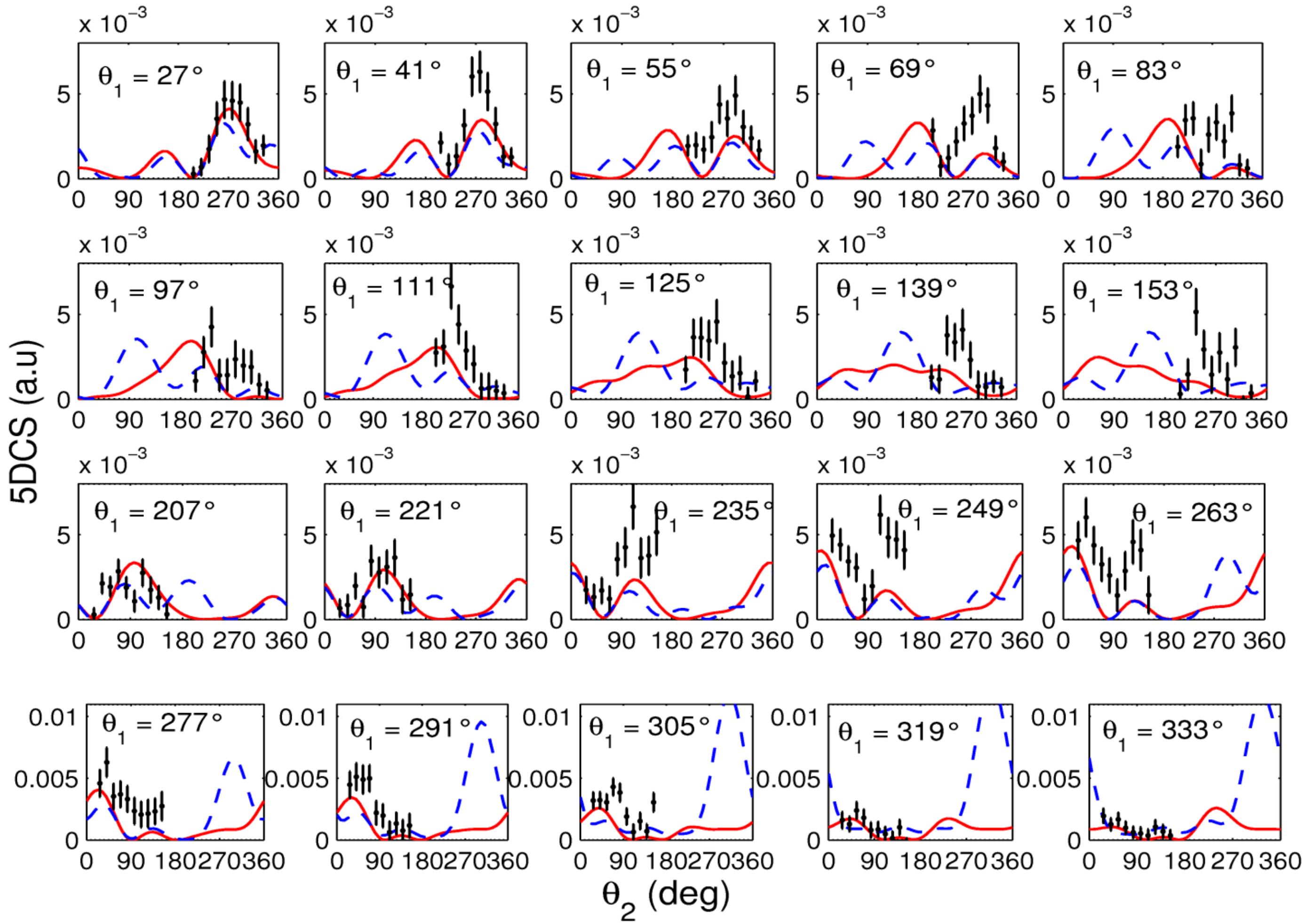
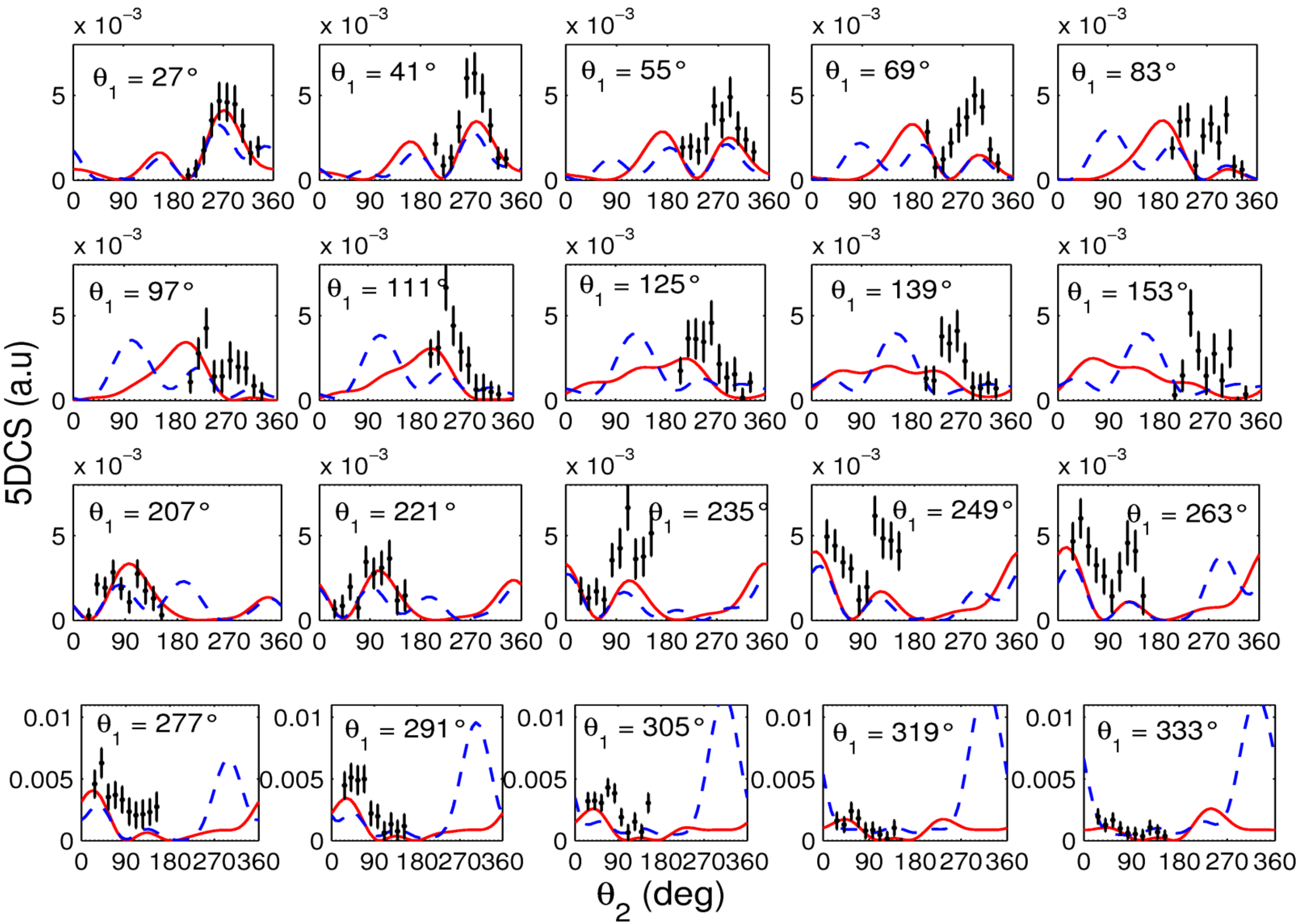


FIG. 1: (Color online) Fully five-fold differential cross section (5DCS) for electron impact double ionization reaction $He(e, 3e)He^{++}$. The incident energy is E_0=5599 eV and the energies of the slow ejected electrons are $E_1 = E_2 = 10$ eV. The scattering angle θ_s of the fast incident electron is fixed and equal to 0.45° while the angles of the ejected electrons are θ_1 and θ_2. One of these angles, θ_1 is fixed and the other varies. The blue dashed line is our result obtained by means of a zero order calculation.The red solid line is the result obtained by solving the Lippmann-Schwinger equation (11) for the double continuum wave function. The solid dots with error-bars are the absolute experimental data of Lahmam-Bennani [4].



5DCS (a.u)
θ1 = 27°
θ1 = 41°
θ1 = 55°
θ1 = 69°
θ1 = 83°
θ1 = 97°
θ1 = 111°
θ1 = 125°
θ1 = 139°
θ1 = 153°
θ1 = 207°
θ1 = 221°
θ1 = 235°
θ1 = 249°
θ1 = 263°
θ1 = 277°
θ1 = 291°
θ1 = 305°
θ1 = 319°
θ1 = 333°
x 10^-3
0 90 180 270 360
θ2 (deg)

- Convergence of the results with increasing *N* is not as satisfactory; there are abrupt (up to three times) kinks in the cross section in certain angular regions even for a small variation of *N*.
- Calculations show that, at $\theta_1 = \theta_2$, the cross section grows with increasing *N*, whereas here it should vanish.

This is an obvious deficiency of the theoretical scheme with a truncated potential.

Three particles of masses m_1, m_2, m_3, charges Z_1, Z_2, Z_3 and momenta $\mathbf{k}_1$, $\mathbf{k}_2$, $\mathbf{k}_3$

Hamiltonian

$$\hat{H} = -\frac{1}{2\mu_{12}}\Delta_{\mathbf{R}} - \frac{1}{2\mu_3}\Delta_{\mathbf{r}} + \frac{Z_1 Z_2}{r_{12}} + \frac{Z_2 Z_3}{r_{23}} + \frac{Z_1 Z_3}{r_{13}}, \qquad (1)$$

Relative coordinates

$$\mathbf{r}_{ls} = \mathbf{r}_l - \mathbf{r}_s, \quad r_{ls} = |\mathbf{r}_{ls}|\,, \qquad (2)$$

Jacobi coordinates

$$\mathbf{R} = \mathbf{r}_1 - \mathbf{r}_2, \quad \mathbf{r} = \mathbf{r}_3 - \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2}{m_1 + m_2}. \qquad (3)$$

Reduced masses

$$\mu_{12} = \frac{m_1 m_2}{m_1 + m_2}, \quad \mu_3 = \frac{m_3\left(m_1 + m_2\right)}{m_1 + m_2 + m_3}. \qquad (4)$$

Schrödinger equation

$$\hat{H}\Phi = E\Phi$$

Energy E>0

$$E = \frac{1}{2\mu_{12}}\mathbf{K}^2 + \frac{1}{2\mu_3}\mathbf{k}^2,$$

Wave function

$$\Phi = e^{i(\mathbf{K}\cdot\mathbf{R}+\mathbf{k}\cdot\mathbf{r})}\Psi$$

Reduced wave function

$$\left[-\frac{1}{2\mu_{12}}\Delta_{\mathbf{R}} - \frac{1}{2\mu_3}\Delta_{\mathbf{r}} - \frac{i}{\mu_{12}}\mathbf{K}\cdot\nabla_{\mathbf{R}} - \frac{i}{\mu_3}\mathbf{k}\cdot\nabla_{\mathbf{r}} + \frac{Z_1Z_2}{r_{12}} + \frac{Z_2Z_3}{r_{23}} + \frac{Z_1Z_3}{r_{13}}\right]\Psi = 0 \qquad (5)$$

Generalized parabolic coordinates

$$\xi_1 = r_{23} + \hat{\mathbf{k}}_{23}\cdot\mathbf{r}_{23}, \quad \eta_1 = r_{23} - \hat{\mathbf{k}}_{23}\cdot\mathbf{r}_{23},$$

$$\xi_2 = r_{13} + \hat{\mathbf{k}}_{13}\cdot\mathbf{r}_{13}, \quad \eta_2 = r_{13} - \hat{\mathbf{k}}_{13}\cdot\mathbf{r}_{13},$$

$$\xi_3 = r_{12} + \hat{\mathbf{k}}_{12}\cdot\mathbf{r}_{12}, \quad \eta_3 = r_{12} - \hat{\mathbf{k}}_{12}\cdot\mathbf{r}_{12},$$

$\mathbf{k}_{ls} = \frac{\mathbf{k}_l m_s - \mathbf{k}_s m_l}{m_l + m_s}$ is the relative momentum, $\hat{\mathbf{k}}_{ls} = \frac{\mathbf{k}_{ls}}{k_{ls}}$ and $k_{ls} = |\mathbf{k}_{ls}|$

$$[D_0 + D_1]\,\Psi = 0. \qquad (5')$$

D_0 contains the leading term of the kinetic energy and the total potential energy:

$$\hat{D}_0 = \sum_{j=1}^{3} \frac{1}{\mu_{ls}(\xi_j+\eta_j)} \left[\hat{h}_{\xi_j} + \hat{h}_{\eta_j} + 2k_{ls}t_{ls}\right],$$

$$\text{for } j \neq l,\ s \text{ and } l < s,$$

$$\hat{h}_{\xi_j} = -2\left(\frac{\partial}{\partial \xi_j}\xi_j\frac{\partial}{\partial \xi_j} + ik_{ls}\xi_j\frac{\partial}{\partial \xi_j}\right),$$

$$\hat{h}_{\eta_j} = -2\left(\frac{\partial}{\partial \eta_j}\eta_j\frac{\partial}{\partial \eta_j} - ik_{ls}\eta_j\frac{\partial}{\partial \eta_j}\right).$$

$$t_{ls} = \frac{Z_l Z_s \mu_{ls}}{k_{ls}} \text{ and } \mu_{ls} = \frac{m_l m_s}{m_l+m_s}.$$

D_1 is the non-orthogonal part of the kinetic energy operator.

In the case of (e^-, e^-, He^{++}) = (123) system with $m_3 = \infty$

$$\hat{D}_1 = \sum_{j=1}^{2} (-1)^{j+1} \left[\mathbf{u}_j^- \cdot \mathbf{u}_3^- \frac{\partial^2}{\partial \xi_j \, \partial \xi_3} + \mathbf{u}_j^- \cdot \mathbf{u}_3^+ \frac{\partial^2}{\partial \xi_j \, \partial \eta_3} + \mathbf{u}_j^+ \cdot \mathbf{u}_3^- \frac{\partial^2}{\partial \eta_j \, \partial \xi_3} + \mathbf{u}_j^+ \cdot \mathbf{u}_3^+ \frac{\partial^2}{\partial \eta_j \, \partial \eta_3} \right],$$

$$\mathbf{u}_j^{\pm} = \hat{\mathbf{r}}_{ls} \mp \hat{\mathbf{k}}_{ls}.$$

C3 wave function

$$\hat{D}_0 \Psi_{C3} = 0,$$

$$\Psi_{C3} = \prod_{j=1}^{3} {}_1F_1\left(it_{ls},\, 1;\; -ik_{ls}\xi_j\right).$$

Two-dimensional Coulomb Green's function

$$\hat{G}^{(\pm)} = \left[\mathfrak{h} + C\right]^{-1}.$$

$$\mathfrak{h} = \frac{1}{\mu(\xi+\eta)}\left[\hat{h}_\xi + \hat{h}_\eta + 2kt\right].$$

$$G^{(\pm)}\left(t,\,\mathcal{E};\;\xi,\,\eta,\,\xi',\,\eta'\right) = \mp\frac{i\gamma}{4}\,e^{\frac{i}{2}k(\xi'-\xi+\eta-\eta')}\,\mu(\xi'+\eta')\,\int_0^\infty dz\,\sinh(z)\,\left[\coth\left(\frac{z}{2}\right)\right]^{\mp 2i\tau}$$
$$\times e^{\pm i\frac{\gamma}{2}(\xi+\xi'+\eta+\eta')\,\cosh(z)}\,I_0\left(\mp i\gamma\sqrt{\xi\,\xi'}\sinh(z)\right)\,I_0\left(\mp i\gamma\sqrt{\eta\,\eta'},\sinh(z)\right),$$

$$C = \frac{1}{\mu}\left(\frac{k^2}{2} - \mathcal{E}\right),\ \mathcal{E} = \frac{\gamma^2}{2},\quad \tau = \frac{k}{\gamma}\,t$$

Six-dimensional Green's function

$$\hat{\mathcal{G}} = \hat{D}_0^{-1},$$

$$\hat{D}_0 = \hat{\mathfrak{h}}_1 + \hat{\mathfrak{h}}_2 + \hat{\mathfrak{h}}_3.$$

$$\mathcal{G}^{(+)}(X,\, X') = \frac{1}{\mu_{23}\,\mu_{13}} \int\limits_{\mathcal{C}_1}\int\limits_{\mathcal{C}_2} d\mathcal{E}_1\, d\mathcal{E}_2\, G^{(+)}\left(t_{23},\, \mathcal{E}_1;\; X_1,\, X_1'\right)$$

$$\times G^{(+)}\left(t_{13},\, \mathcal{E}_2;\; X_2,\, X_2'\right) G^{(+)}\left(t_{12},\, \mathcal{E}_3;\; X_3,\, X_3'\right),$$

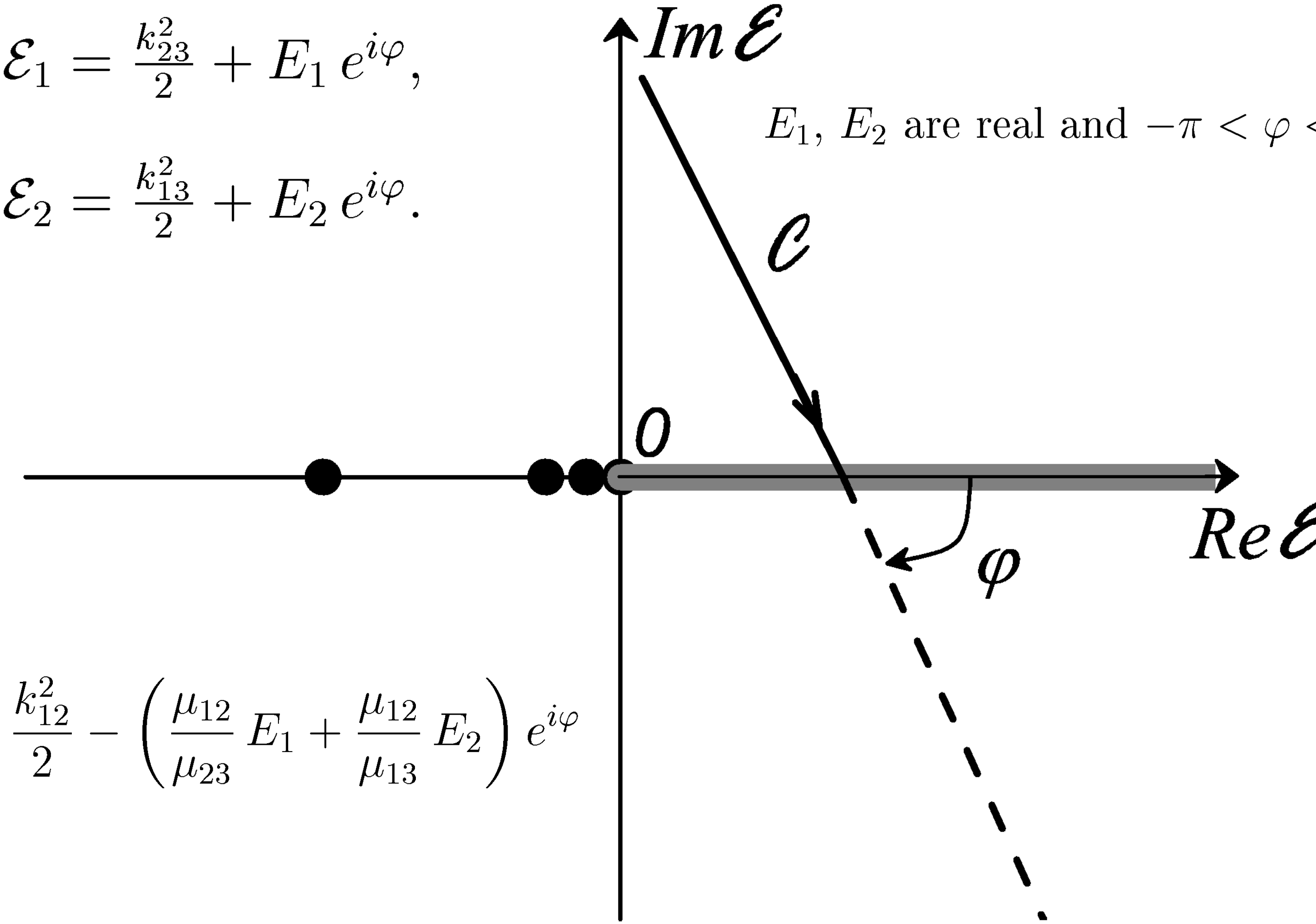
$$X = \left\{X_1,\, X_2,\, X_3\right\}, \quad X_j = \left\{\xi_j,\, \eta_j\right\},\; j = 1,\, 2,\, 3.$$

Path of integration

$$\mathcal{E}_1 = \frac{k_{23}^2}{2} + E_1\, e^{i\varphi},$$

$$\mathcal{E}_2 = \frac{k_{13}^2}{2} + E_2\, e^{i\varphi}.$$

$E_1,\ E_2$ are real and $-\pi < \varphi < 0.$



$$\mathcal{E}_3 = \frac{k_{12}^2}{2} - \left(\frac{\mu_{12}}{\mu_{23}}\, E_1 + \frac{\mu_{12}}{\mu_{13}}\, E_2\right) e^{i\varphi}$$

Lippmann-Schwinger type equation

$$\Psi = \Psi_{C3} - \hat{\mathcal{G}}\hat{D}_1\Psi.$$

The kernel of $\hat{\mathcal{G}}\hat{D}_1$ is non-compact

Square integrable parabolic basis functions

$$|\mathfrak{N}\rangle = \prod_{j=1}^{3} \phi_{n_j\, m_j}\left(\xi_j,\, \eta_j\right),$$

$$\phi_{n_j\, m_j}\left(\xi_j,\, \eta_j\right) = \psi_{n_j}\left(\xi_j\right)\psi_{m_j}\left(\eta_j\right),$$

$$\psi_n\left(x\right) = \sqrt{2b_j}\, e^{-b_j x} L_n(2b_j x).$$

$$\Psi = \Psi_{C3} - \sum_{j=1}^{3} \sum_{n_j=0}^{N_j-1} \sum_{m_j=0}^{M_j-1} [\underline{C}]_{\mathfrak{N}}\, \hat{\mathcal{G}}\, |\mathfrak{N}\rangle \qquad (6)$$

Wave function

$$\Psi = \Psi_{sc} + \Psi_{C3}$$

$$\Psi_{sc} \sim \sum a_{\mathfrak{N}} f_{\mathfrak{N}}, \qquad (6')$$

Quasi Sturmians $\{f_{\mathfrak{N}}\}$

$$f_{\mathfrak{N}} = \hat{\mathcal{G}} \left|\mathfrak{N}\right\rangle$$

Driven equation

$$\left[\hat{D}_0 + \hat{D}_1\right] \Psi_{sc} = -\hat{D}_1 \Psi_{C3}$$

- We expect that the resulting Sturmian functions to provide a basis of expansion for this kind of three-body Coulomb problem.

Thanks for attention!