A Study of Generalized Parton Distributions for the Proton in AdS/QCD

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Abstract

We have evaluated the generalized parton distributions (GPDs) from the electromagnetic form factors of the nucleons. The light front wave functions of the nucleons are obtained from soft wall model in AdS/QCD. We have considered a quark model with SU(6) spin-flavor symmetry. The GPDs in impact parameter space are compared with a phenomenological model.

Keywords: *Parton distributions; AdS/QCD; soft wall model; form factor; light front wave function*

1 Introduction

Generalized parton distributions (GPDs) encode more informations about the hadron than the ordinary parton distributions (PDFs). The GPDs are functions of three variables namely, longitudinal momentum fraction x of the quark or gluon, square of the total momentum transferred (t) and the skewness ζ which represents the longitudinal momentum transferred in the process and contain lot more informations about the nucleon structure and spin compared to the ordinary PDFs which are functions of x only. There are many good review articles on the GPDs [1]. The GPDs appear in the exclusive processes like Deeply Virtual Compton Scattering (DVCS) or vector meson productions and are expressed as off-forward matrix elements of bilocal light front currents. The GPDs reduce to the ordinary parton distributions in the forward limit and their first moments are related to the form factors and provide interesting informations about the spin and orbital angular momentum of the constituents as well as the spatial structure of the nucleons. Being off-forward matrix elements, the GPDs have no probabilistic interpretation. But for zero skewness, the Fourier transforms of the GPDs with respect to the transverse momentum transfer ($\Delta_\perp$) give the impact parameter dependent GPDs which satisfy the positivity condition and can be interpreted as distribution functions [2]. The impact parameter dependent GPDs provide us the information about partonic distributions in the impact parameter or the transverse position space for a given longitudinal momentum (x). The impact parameter $b_\perp$ gives the separation of the struck quark from the center of momentum. In the $t \to 0$ limit, Ji sum rule [3] relates the moment of the GPDs to the angular momentum contribution to the nucleon by the quark or gluon. Lot of experiments measured DVCS as well as vector meson production cross sections to gain informations about the GPDs [4]. Experiments will also be done in JLAB in near future.

Using AdS/QCD, one can extract the light front wave functions (LFWF) for the hadrons and thus provides an interesting way to calculate the GPDs. Polchinski and Strassler [5] first used the AdS/CFT duality to address the deep inelastic scattering. The AdS/QCD for meson and baryon sectors have been developed by several groups [6–8]. So far this method has been successfully applied to describe many hadron

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properties, e. g., hadron mass spectrum, parton distribution functions, meson and nucleon form factors, structure functions, etc. [9–11]. Recently it has been shown that the results for ρ meson electroproduction calculated with the light front wave functions derived from AdS/QCD are in agreement with experimental data [12]. Studies of the nucleon form factors with higher Fock sectors have been done in Ref. [13]. Vega *et al.* [14] proposed a prescription to extract GPDs from the form factors in AdS/QCD and they have done the GPD calculations using both the hard and soft wall models in AdS/QCD. Here we provide the results for GPDs using the LFWFs obtained from the AdS/QCD [15]. We use the formula for the nucleon form factors in the light front quark model with SU(6) spin flavor symmetry and compare the GPDs in the impact parameter space with a phenomenological model of the GPDs for the proton. The GPDs are related to the Dirac and Pauli form factor by sum rules and thus it is possible to extract the flavor form factors, i. e., individual quark contributions to the nucleon form factors. Recently, the flavor form factors calculated from the GPDs in this model are shown to agree remarkably with the experimental data [16].

2 GPDs in AdS/QCD

For the extraction of the nucleon wavefunctions in AdS/QCD we follow Brodsky and Teramond [6, 11]. We know that the AdS/CFT correspondence relates a gravitationally interacting theory in anti de Sitter space AdS_{d+1} with a conformal gauge theory in d-dimensions residing at the boundary. Since QCD is not a conformal theory, one needs to break the conformal invariance of the above duality to generate a bound state spectrum and to relate with QCD. There are two models in the literature to do so. One is the hard wall model in which the conformal symmetry is broken by introducing a boundary at $z_0 \sim 1/\Lambda_{QCD}$ in the AdS direction where the wavefunction is made to vanish. While in the soft wall model, the conformal invariance is broken by introducing a confining potential in the action of a Dirac field propagating in AdS_{d+1} space.

We will consider the soft model in this paper. The relevant action in soft model is written as [11]

$$S = \int d^4x\, dz\, \sqrt{g}\Big(\frac{i}{2}\bar\Psi\, e^M_A\, \Gamma^A\, D_M\, \Psi - \frac{i}{2}(D_M\bar\Psi)\, e^M_A\, \Gamma^A\, \Psi - \mu\, \bar\Psi\, \Psi - V(z)\, \bar\Psi\, \Psi\Big), \quad (1)$$

where $e^M_A = (z/R)\, \delta^M_A$ is the inverse vielbein and $V(z)$ is the confining potential, R is the AdS radius. The corresponding Dirac equation in AdS is given by

$$i\Big(z\, \eta^{MN}\, \Gamma_M\, \partial_N + \frac{d}{2}\Gamma_z\Big)\Psi - \mu\, R\, \Psi - R\, V(z)\, \Psi = 0. \quad (2)$$

With z identified as the light front transverse impact variable ζ which gives the separation of the quark and gluonic constituents in the hadron, it is possible to extract the lightfront wavefunctions for the hadron. In $d = 4$ dimensions, $\Gamma_A = \{\gamma_\mu, -i\gamma_5\}$. The form of the confining potential in the meson sector can be determined by introducing a dilaton background profile of the form $\phi(z) = e^{\pm\kappa^2 z^2}$. It generates an effective linear confining potential of $U(\zeta) = (R/\zeta)V(\zeta) = \kappa^2\zeta$ in the light front Dirac equation. For the baryon sector, the dilaton profile can be scaled away by redefinition of the fields [11]. In the baryon sector, the linear confining potential same as the meson sector is put in by hand. The nucleon wavefunctions in the soft wall model are given by [11]

$$\psi_+(z) = \frac{\sqrt{2}\kappa^2}{R^2}\, z^{7/2}\, e^{-\kappa^2 z^2/2}, \quad (3)$$

$$\psi_-(z) = \frac{\kappa^3}{R^2}\, z^{9/2}\, e^{-\kappa^2 z^2/2}. \quad (4)$$

The Dirac and Pauli form factors for the nucleons are related to the GPDs by the sum rules [17]

$$
\begin{aligned}
F_1^p(t) &= \int_0^1 dx \left[\frac{2}{3}H_v^u(x,t) - \frac{1}{3}H_v^d(x,t)\right], \\
F_1^n(t) &= \int_0^1 dx \left[\frac{2}{3}H_v^d(x,t) - \frac{1}{3}H_v^u(x,t)\right], \\
F_2^p(t) &= \int_0^1 dx \left[\frac{2}{3}E_v^u(x,t) - \frac{1}{3}E_v^d(x,t)\right], \\
F_2^n(t) &= \int_0^1 dx \left[\frac{2}{3}E_v^d(x,t) - \frac{1}{3}E_v^u(x,t)\right].
\end{aligned}
\tag{5}
$$

Here x is the fraction of the light cone momentum carried by the active quark and the GPDs for valence quark q are defined as $H_v^q(x,t) = H^q(x,0,t) + H^q(-x,0,t)$; $E_v^q(x,t) = E^q(x,0,t) + E^q(-x,0,t)$. The GPDs at $-x$ for quark are equal to the GPDs at x for antiquark with a minus sign.

A quark model with SU(6) spin-flavor symmetry is constructed by weighting the different Fock components in the nucleon state by the charge and spin-projections of the quarks as dictated by the symmetry [11]. The Dirac form factors for the nucleons in this model are given by

$$F_1^p(Q^2) = R^4 \int \frac{dz}{z^4}\, V(Q^2,z)\, \psi_+^2(z) \tag{6}$$

$$F_1^n(Q^2) = -\frac{1}{3}R^4 \int \frac{dz}{z^4}\, V(q^2,z)\, (\psi_+^2(z) - \psi_-^2(z)). \tag{7}$$

The Pauli form factors requires non-minimal electromagnetic coupling as proposed by Abidin and Carlson [10] and are given by

$$F_2^{p/n}(Q^2) \sim \int \frac{dz}{z^3}\, \psi_+(z)\, V(Q^2,z)\, \psi_-(z). \tag{8}$$

The normalization conditions are given by $F_1^{p/n}(0) = e_{p/n}$, where $e_{p/n}$ represents the electric charge of proton/neutron and $F_2^{p/n}(0) = \kappa_{p/n}$ where $\kappa_{p/n}$ is the anomalous magnetic moment of the proton/neutron. Using the the above mentioned wavefunctions ψ_+ and ψ_-, the Pauli form factors fitted to the static values are rewritten as

$$F_2^{p/n}(Q^2) = \kappa_{p/n} R^4 \int \frac{dz}{z^4}\, V(Q^2,z)\, \psi_-^2(z). \tag{9}$$

The bulk-to-boundary propagator for soft wall model is given by

$$V(Q^2,z) = \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right) U\left(\frac{Q^2}{4\kappa^2}, 0, \kappa^2 z^2\right), \tag{10}$$

where $U(a,b,z)$ is the Tricomi confluent hypergeometric function given by

$$\Gamma(a)\, U(a,b,z) = \int_0^\infty e^{-zx}\, x^{a-1} (1+x)^{b-a-1}\, dx. \tag{11}$$

The above propagator can be written in a simple integral form [11, 18]

$$V(Q^2,z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2}\, x^{Q^2/(4\kappa^2)}\, e^{-\kappa^2 z^2 x/(1-x)}. \tag{12}$$

We use the integral form of the bulk-to-boundary propagator in the formulas for the form factors in AdS space to extract the GPDs using the formulas in Eq. (5). In

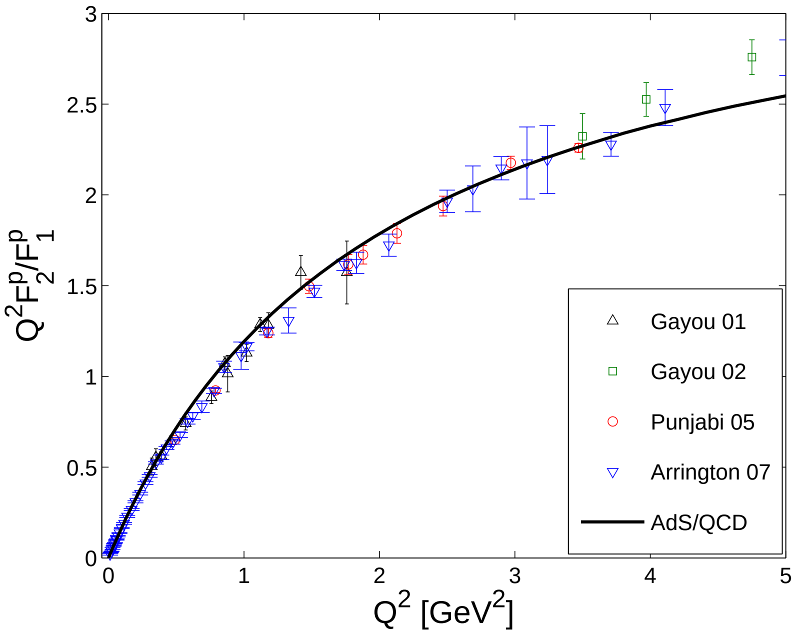


Figure 1: The ratio of the Pauli and Dirac form factors for the proton multiplied by $Q^2 = -t$. The experimental data are taken from Refs. [19–22].

Fig. 1, we show the fit of our result with the experimental proton form factor data. We found that the best fit to the form factors obtained for $\kappa = 0.4066$ GeV. All the calculations and plots presented here are done with this fixed value of κ.

In Figs. 2 (a) and (b) we have shown the GPD $H(x,t)$ as functions of x for different $-t$ values for up and down quarks. Except the fact that it falls off faster for d quark as x increases, the overall nature is the same for both u and d quarks. Similarly in Figs. 3 (a) and (b) we have shown the GPD $E(x,t)$ as a function of x for different $-t$ for u and d quark. Unlike $H(x,t)$, the fall off of the GPD $E(x,t)$ with increasing x is similar for both u and d quark.

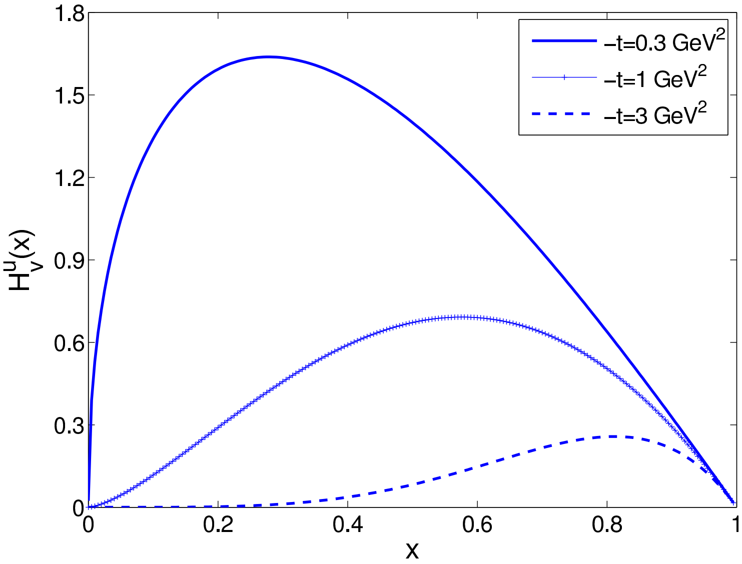
3 GPDs in impact parameter space

GPDs in transverse impact parameter space are defined as [23]:

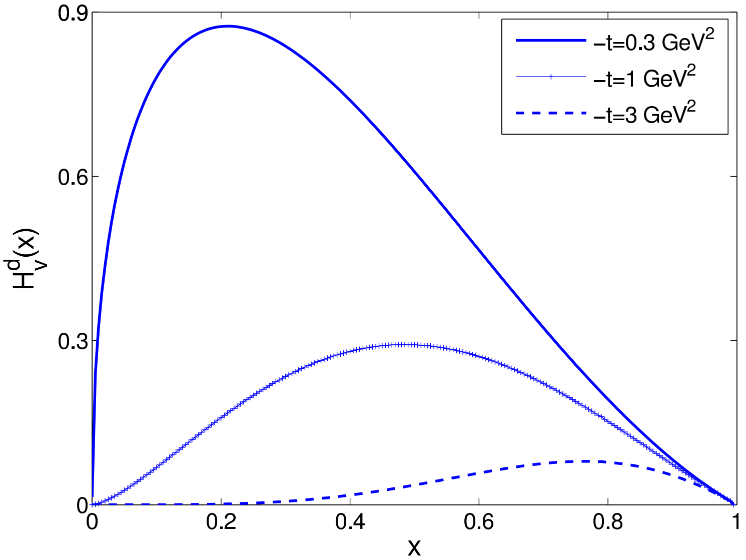
$$\begin{aligned} H(x,b) &= \frac{1}{(2\pi)^2}\int d^2\Delta\, e^{-i\Delta^\perp\cdot b^\perp} H(x,t), \\ E(x,b) &= \frac{1}{(2\pi)^2}\int d^2\Delta\, e^{-i\Delta^\perp\cdot b^\perp} E(x,t). \end{aligned} \tag{13}$$

The transverse impact parameter $b = |b_\perp|$ is a measure of the transverse distance between the struck parton and the center of momentum of the hadron and satisfies $\sum_i x_i b_i = 0$, where the sum is over the number of partons. An estimate of the size of the bound state can be obtained from the relative distance between the struck parton and the centre of momentum of the spectator system and is given by $\frac{b}{1-x}$ [17]. However as the spatial extension of the spectator system is not available from the GPDs, exact estimation of the nuclear size is not possible. In Figs. 4 (a) and (b), we have shown the behavior of $H^{u/d}(x,b)$ in the impact parameter space for fixed values of x and the similar plots for the GPD $E^{u/d}(x,b)$ are shown in Fig. 5.

We compare the AdS/QCD results for the GPDs in impact parameter space with those obtained from a phenomenological model for proton [24]. The GPDs in this



(a)



(b)

Figure 2: Plots of (a) $H^u(x,t)$ vs x for fixed values of $-t$. (b) same as in (a) but for d quark.

model are given by

$$H^q(x,t) = G_{M_x^q}^{\lambda^q}(x,t)\, x^{-\alpha^q - \beta_1^q(1-x)p_1 t}, \tag{14}$$

$$E^q(x,t) = \kappa_q G_{M_x^q}^{\lambda^q}(x,t)\, x^{-\alpha^q - \beta_2^q(1-x)p_2 t}, \tag{15}$$

where the first part is derived from spectator model and modified by Regge term to have proper behavior at low x. κ_q in the above equation is the quark contribution to the anomalous magnetic moment. The parameters are fixed by fitting the form factors. The details of the functional forms and the values of the parameters can be found in Ref. [24]. The impact parameter dependent GPDs from this model have been studied in Ref. [25]. One should remember here that the valence GPDs we have considered here in AdS/QCD are not exactly the same as GPDs in this model and so exact agreement is not expected. But one should expect that the valence GPDs will dominate the overall behavior for the proton GPDs and thus it is interesting to

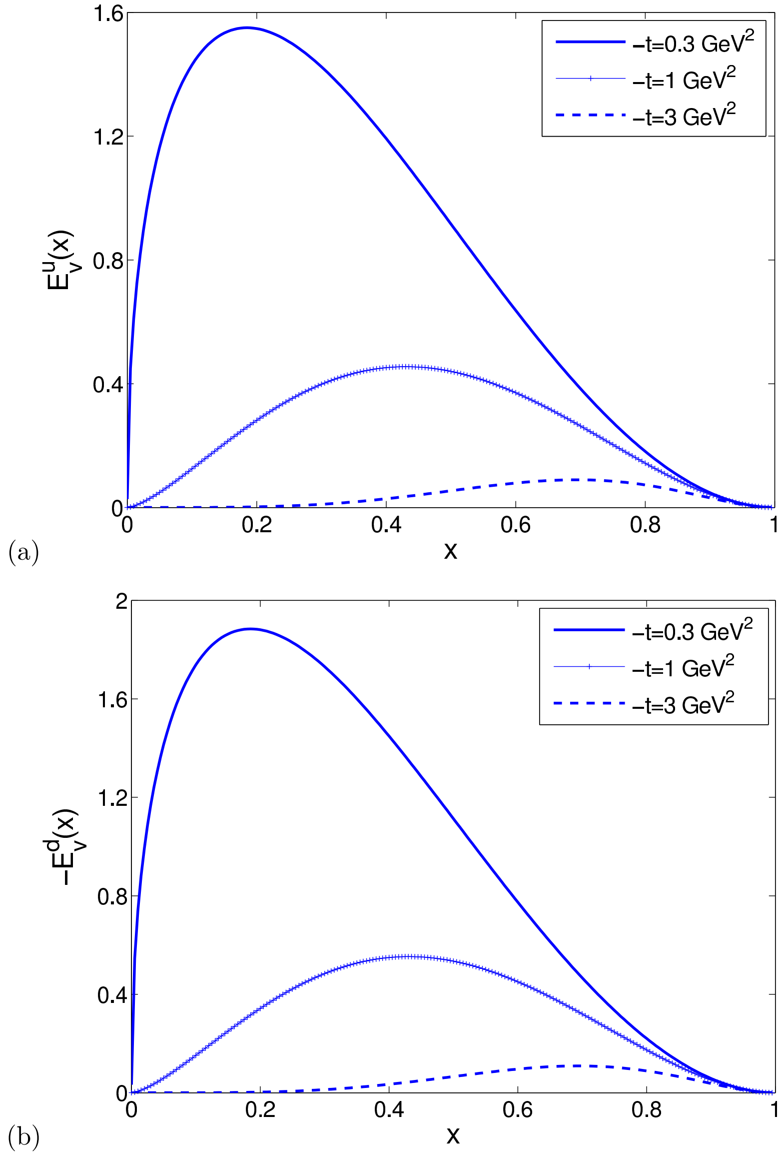
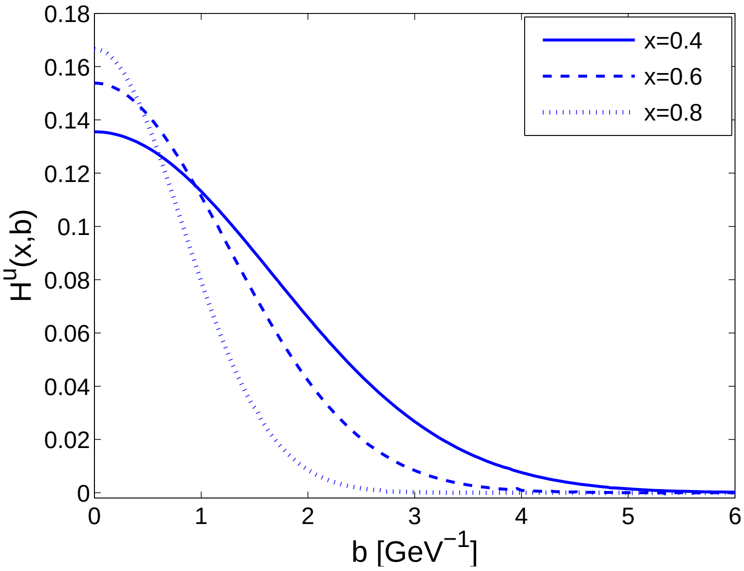


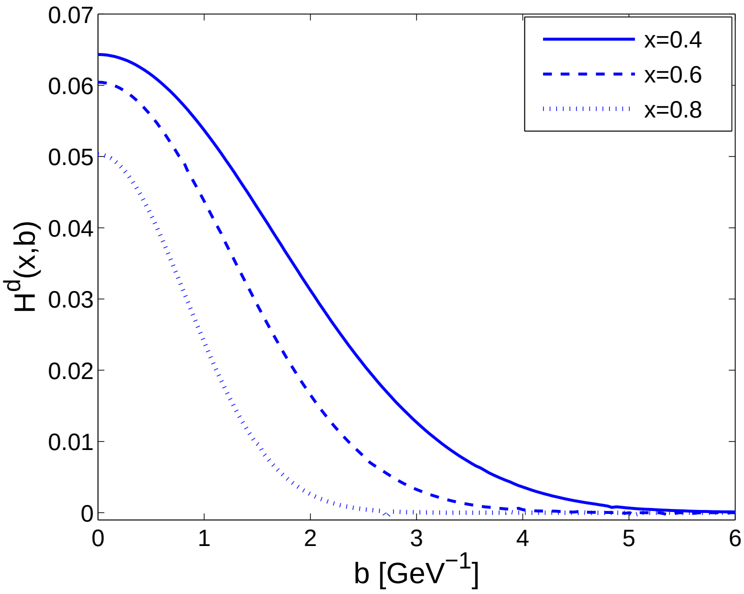
Figure 3: Plots of (a) $H^u(x,t)$ vs x for fixed values of $-t$. (b) same as in (a) but for d quark.

compare and contrast the GPDs from these two models.

In Fig. 6 we have compared the impact parameter dependent proton GPD $H(x,b)$ from AdS/QCD with the model mentioned above, for both u and d quarks. The GPDs are fatter in the AdS/QCD compared to the model when plotted against x, while in the impact parameter space they look almost same except the difference in the magnitudes. In Fig. 7 we have compared the two models for the proton GPD $E(x,b)$. The behavior in x for u quark is quite different in the two models while they agree better for d quark and again the GPDs from AdS/QCD are fatter compared to the other model. In the model, the behavior of $E(x,b)$ for u and d quarks is quite different when plotted against x for fixed values of impact parameter b whereas in the AdS/QCD, it shows almost same behavior for both u and d quarks. As a result, the GPD E in both models agrees better in impact parameter space for the d quark than for the u quark. It is interesting to note that in both cases, at small values of impact parameter b, the the GPD $H(x,b)$ is larger for u quark than for d quark whereas the



(a)



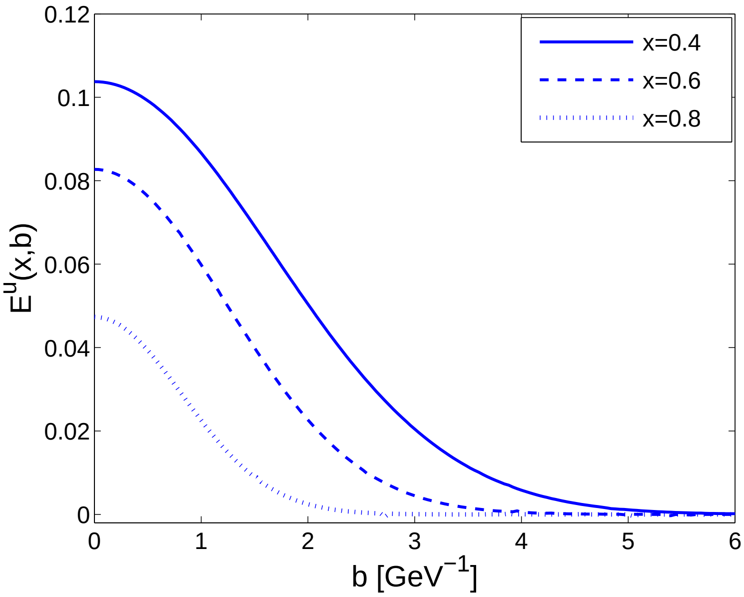
(b)

Figure 4: Plots of (a) $H^u(x,t)$ vs b for fixed values of x. (b) same as in (a) but for d quark.

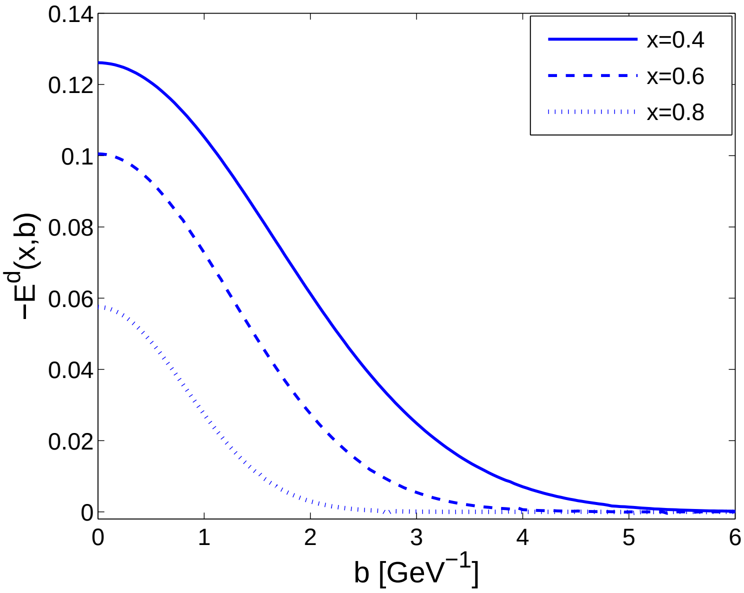
magnitude of the GPD $E(x,b)$ is marginally larger for d quark than the same for u quark and thus it is interesting to check with other models whether this is a model independent result.

4 Conclusions

The main results of this work are the GPDs calculated in a quark model with SU(6) spin-flavor symmetry in AdS/QCD. The light front wave functions for the nucleons are evaluated from AdS/QCD. The parameter κ in the model is fixed by fitting the experimental data on proton form factors. The Pauli form factors require non-minimal electromagnetic coupling and are fitted to their static values. It was shown [11] that the electromagnetic form factors for proton and neutron calculated by using the



(a)



(b)

Figure 5: Plots of (a) $E^u(x,t)$ vs b for fixed values of x. (b) same as in (a) but for d quark.

AdS/QCD wave functions fit well with the experimental results. The Dirac and Pauli form factors for the nucleons are given by the first moments of the GPDs weighted with proper charge factors. Using these sum rules for the GPDs and exploiting the integral representation of the bulk-to-boundary propagator in AdS space we evaluate the GPDs for both up and down quarks. The Fourier transform of the GPDs with respect to the transverse momentum transferred give the GPDs in the impact parameter space. Though the GPDs don't have any density interpretation, the impact parameter dependent GPDs for zero skewness are positive definite and related with the charge and magnetization densities of the nucleons. We have compare the impact parameter dependent GPDs in the model with the GPDs obtained from a phenomenological model. It is found that the GPDs from AdS/QCD are fatter than the other model when compared the behaviors in x space for both u and d quarks. In

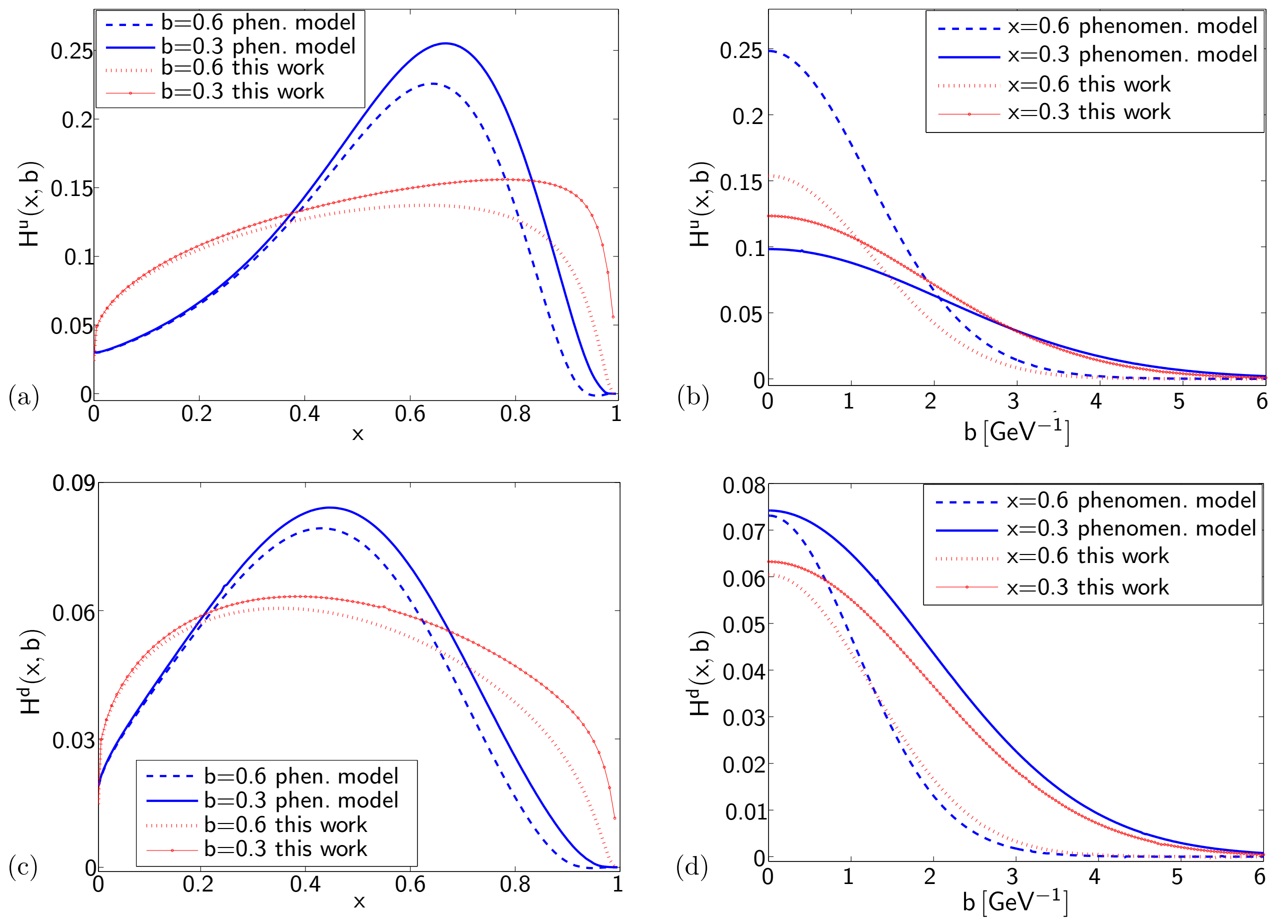


Figure 6: Plots of (a) $H^u(x,b)$ vs x for fixed values of $b = |b_\perp|$. (b) $H^u(x,b)$ vs b for fixed values of x. (c) same as in (a) but for d quark and (d) same as in (b) but for d quark.

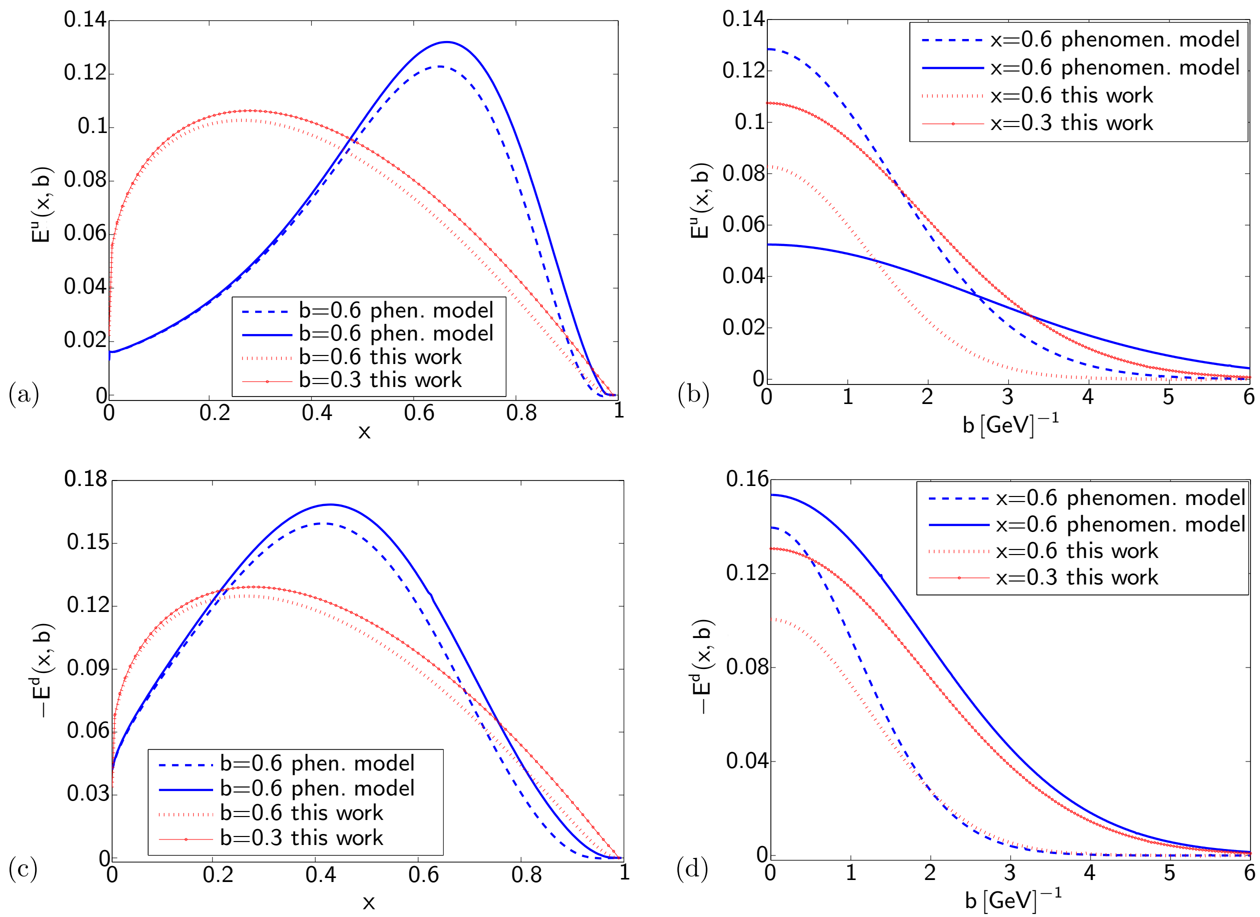


Figure 7: Plots of (a) $E^u(x,b)$ vs x for fixed values of $b = |b_\perp|$. (b) $E^u(x,b)$ vs b for fixed values of x. (c) same as in (a) but for d quark and (d) same as in (b) but for d quark.

the AdS/QCD we have only valence GPDs and as we expect that major contributions to proton GPDs should come from valence quarks, it is interesting to note that their behaviors in impact parameter space are quite similar to the phenomenological model for GPDs.

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