Recent Results with the Lorentz Integral Transform (LIT) Method for Inclusive Electron Scattering

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Abstract

A brief outline of the Lorentz Integral Transform method is given. Recent results for the inclusive electrodisintegration of ^{3}He and ^{4}He are discussed. The energy resolution that can be obtained with the LIT approach is studied and it is shown that the LIT method is a method with a controlled resolution. The final part discusses the role of the isoscalar monopole resonance of ^{4}He in (e, e') scattering.

Keywords: *Lorentz Integral Transform; inclusive electron scattering; few-body nuclei; three-nucleon force, isoscalar monopole resonance*

1 Introduction

Integral transforms are of common use in physics. In general they have the following form

$$\Phi(\sigma) = \int dE\, \mathcal{K}(E, \sigma)\, R(E), \tag{1}$$

where $\mathcal{K}(E, \sigma)$ is a well defined kernel and where $R(E)$ is an energy dependent response function of the system under consideration. Often it is very difficult or even impossible to determine $R(E)$ in a direct calculation, in particular when a many-body continuum wave function should be calculated. In such cases one may consider to determine directly the integral transform $\Phi(\sigma)$, i.e. without knowledge of $R(E)$. Then, the response function $R(E)$ can be obtained from the inversion of the integral transform.

In the following we will discuss the Lorentz integral transform (LIT) $L(\sigma)$ [1, 2]. In the past the LIT approach has been applied to a variety of inelastic electroweak reactions [2, 3]. Because of the specific form of the kernel and different from many other integral transforms, the LIT is an integral transform with a controlled resolution. The kernel $\mathcal{L}(E, \sigma)$ of the LIT is of Lorentzian shape:

$$\mathcal{L}(E, \sigma) = \frac{1}{(E - \sigma_R)^2 + \sigma_I^2} \tag{2}$$

($\sigma = \sigma_R + i\sigma_I$). It is evident that the parameter σ_I controls the width of the Lorentzian. A reduced value for σ_I leads to a higher energy resolution, however, at the same time one has also to increase the precision of the calculation. This point will be discussed in greater detail in Sect. 3.

The LIT $L(\sigma)$ is calculated by solving an equation of the form

$$(H - \sigma)\, \tilde{\Psi} = S, \tag{3}$$

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where H is the Hamiltonian of the system under consideration and S is an asymptotically vanishing source term. The solution $\tilde{\Psi}$ is localized. This a very important property, since it allows to determine $\tilde{\Psi}$ with bound-state methods, even in case that the direct calculation of the response function $R(E)$ constitutes a continuum state problem. Having calculated $\tilde{\Psi}$ one obtains the LIT from the following expression:

$$L(\sigma) = \langle \tilde{\Psi} | \tilde{\Psi} \rangle . \tag{4}$$

The response function $R(E)$ is determined from the calculated $L(\sigma)$ by inverting the equation

$$L(\sigma) = \int dE \, \frac{R(E)}{(E - \sigma_R)^2 + \sigma_I^2} \, . \tag{5}$$

A general discussion of the inversion and details about various inversion methods are given in Refs. [2, 4, 5].

An alternative way to write the LIT is given by

$$L(\sigma) = -\frac{1}{\sigma_I} \, Im \left(\left\langle S \left| \frac{1}{\sigma_R + i\sigma_I - H} \right| S \right\rangle \right) . \tag{6}$$

This reformulation is useful since it allows a direct application of the Lanczos algorithm for the determination of $L(\sigma)$ [6]. In fact the calculations discussed in the following sections are performed in this way by using expansions in hyperspherical harmonics (HH). The convergence is accelerated by introducing additional two-body correlations in case of three-nucleon applications (CHH), while for the four-body system an effective interaction approach is used (EIHH [7]).

2 Electron scattering off ^{3,4}He

In order to calculate a specific reaction one has to specify the source term S in Eqs. (3) and (6). In case of unpolarized inclusive electron scattering one has a longitudinal response function $R_L(q, \omega)$ and a transverse response function $R_T(q, \omega)$, where q and ω describe momentum and energy transfer of the electron to the nucleus. The source term S takes the following form:

$$|S\rangle = \theta |0\rangle , \tag{7}$$

where θ is a specific transition operator and $|0\rangle$ is the ground-state wave function of the nucleus. For the response functions $R_L(q, \omega)$ and $R_T(q, \omega)$ the transition operator θ corresponds to the nuclear charge and current operator, respectively.

2.1 Transverse response function $R_T(q, \omega)$ of ^{3}He in the quasi-elastic region

The inclusive transverse response function $R_T(q, \omega)$ of ^{3}He in inelastic electron scattering has recently been considered with the LIT method at momentum transfers ranging from 500 to 700 MeV/c [8]. Besides the usual non-relativistic nucleon one-body currents various additional current operators have been taken into account: meson exchange currents (MEC) [9, 10], isobar currents involving the Δ resonance (IC) [11], and relativistic corrections to the non-relativistic nucleon one-body currents [12]. In order to circumvent problems with special relativity the calculation is performed in the so-called active nucleon Breit (ANB) frame which moves with $-3\mathbf{q}/2$ with respect to the laboratory frame. In order to compare with experimental data the R_T result is then transformed to the laboratory system. As nuclear force a realistic nuclear interaction has been considered, which consists in the AV18 NN potential [13] and the UIX three-nucleon force [14].

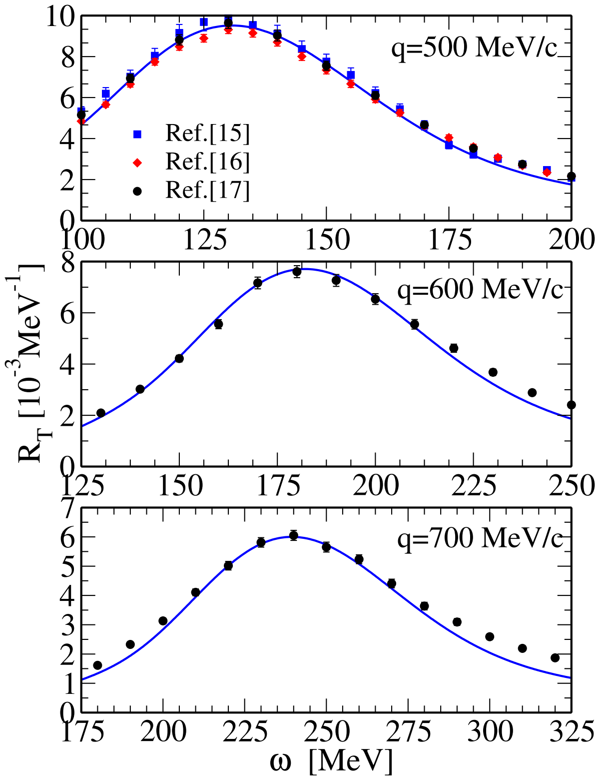

Figure 1: Transverse response function $R_T(q,\omega)$ of ^{3}He at $q = 500$, 600, and 700 MeV/c with force model AV18+UIX; experimental data from [15–17].

In Fig. 1 the resulting response function $R_T(q,\omega)$ is shown. One observes an excellent agreement with experimental data in the whole quasi-elastic peak region for all three considered momentum transfers. It should be pointed out that for the good agreement with experiment it is necessary to control, to some extent, problems due to special relativity (ANB frame) and to include both IC and relativistic corrections of the nucleon one-body current, whereas MEC are of less importance in the ^{3}He quasi-elastic peak region. The IC contribution is particularly interesting: (i) it cancels the effect of the three-nucleon force (3NF) in the peak region and (ii) in the isospin $T = 3/2$ channel of the disintegrated nucleus one finds an important IC contribution beyond the peak region; this isospin channel contributes exclusively to the three-body break-up of ^{3}He and thus IC should be included in the calculation of such reactions.

From the results in Fig. 1 it is evident that the LIT approach allows calculations of reactions up into the far many-body continuum. This is quite remarkable since no continuum wave functions are calculated and only bound-state methods are applied.

2.2 Longitudinal response function $R_L(q,\omega)$ of ^{4}He at lower momentum transfer

Up to present realistic LIT calculations for the ^{4}He electrodisintegration have been performed for R_L [18, 19] only, whereas for R_T a LIT calculation [20] with the central NN potential MTI/III [21] exists. The results for the longitudinal response are particularly interesting at lower momentum transfer since 3NF effects become quite important. Also at higher momentum transfer 3NF effects are non-negligible, but less important (below 10%). In Fig. 2 the ^{4}He $R_L(q,\omega)$ of [18] is shown at various

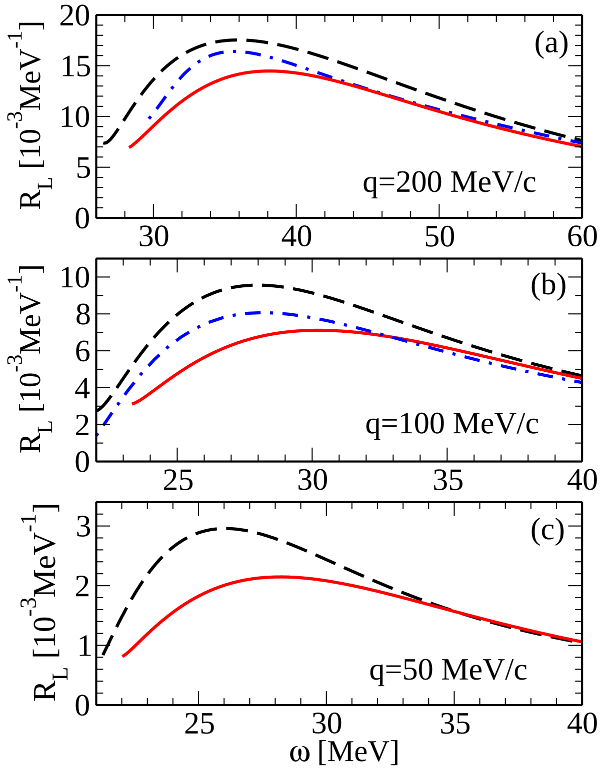

Figure 2: $R_L(q, \omega)$ of ^{4}He with force models AV18 (dashed), MTI/III (dash-dotted), and AV18+UIX (solid).

low momentum transfers for nuclear force models AV18 and AV18+UIX. In the low-energy region one finds a considerable decrease due 3NF which reaches almost 50% at $q = 50$ MeV/c. In Fig. 2 also a result with the MTI/III potential is depicted. Different from the realistic nuclear force models the MTI/III potential overestimates the ^{4}He binding energy by a few MeV. Nonetheless the MTI/III R_L lies between the AV18 and AV18+UIX results. This shows that the large 3NF effect cannot be caused just by a 3NF effect on the ^{4}He ground state, but that 3NF effects on the nuclear continuum wave function lead to essential contributions. In Ref. [19] also R_L results for force model AV18+TM' are included (TM' 3NF from Ref. [22]). In Fig. 3 we illustrate results from this reference for $q = 50$ MeV. Relatively large differences can be seen between the AV18+UIX and the AV18+TM' results, although both force models lead to almost equal ^{4}He binding energies.

3 Energy resolution with the LIT approach

It was already mentioned in the introduction that the LIT approach is a method with a controlled resolution. Here this aspect is illustrated in greater detail. A solution of the LIT equation (6) via an expansion on a basis with N basis functions can be understood as follows. One determines the spectrum of the Hamiltonian on this basis thus finding N eigenenergies E_n. Furthermore, the solution assigns to any eigenenergy a strength in form of a Lorentzian with height L_n and width σ_I. It should be noticed that the source term $|S\rangle$ affects only the height L_n. The LIT result then reads

$$L(\sigma) = \sum_{i=1}^{N} \frac{L_n}{(\sigma_R - E_n)^2 + \sigma_I^2} \,. \tag{8}$$

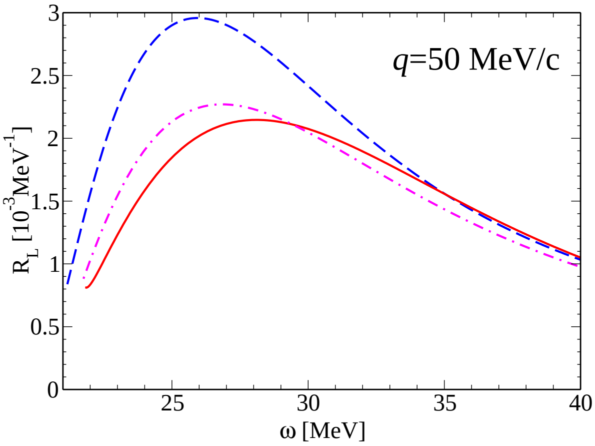

Figure 3: As Fig. 2 but for force models AV18 (dashed), AV18+TM' (dash-dotted), and AV18+UIX (solid).

Note that this result is related to the so-called Lanczos response $R_{\rm Lnczs}$ by

$$R_{\rm Lnczs}(E,\sigma_I) = \frac{\sigma_I}{\pi} L(E,\sigma_I). \tag{9}$$

In the limit $\sigma_I \to 0$ the Lanczos response is equal to the true response function $R(E)$. However, one often calculates $R_{\rm Lnczs}$ for a small but finite σ_I value and identifies the Lanczos response with the true response, which in general is an uncontrolled approximation. In the LIT approach one does not make such an identification of transform and response function. A proper treatment requires an inversion. From a practical point of view such a correct treatment is even advantageous, since the computational effort is much less. In fact, it allows to work with a not too small σ_I, thus with a relatively small number of basis functions N. Only in case of structures, which change rapidly with energy, e. g. resonances, one might need σ_I values of the order of the resonance width. To give a better understanding of the energy resolution with the Lorentz integral transform method also here deuteron photodisintegration in unretarded dipole approximation is considered as a simple example. The corresponding cross section is given by

$$\sigma_{\rm unret}(\omega) = 4\pi^2 \alpha \omega R_{\rm unret}(\omega), \tag{10}$$

where ω denotes the photon energy and α is the fine structure constant. The relevant transition operator for the calculation of $R_{\rm unret}(\omega)$ is the dipole operator $\theta = \sum_i z_i(1+\tau_{i,z})/2$, where z_i and $\tau_{i,z}$ are the z-components of the position vector and of the isospin operator of the ith nucleon, respectively. For the deuteron case the dipole operator allows only transitions to the following np final states: ${}^3P_0$, ${}^3P_1$, and ${}^3P_2$-${}^3F_2$. For simplicity in the following example only transitions to ${}^3P_1$ are considered. The following ansatz for the corresponding $\tilde\Psi$ is made:

$$|\tilde\Psi\rangle = \tilde\psi(r)\,|(l=1,S=1)j=1\rangle\,|T=1\rangle, \tag{11}$$

where r ($T=1$) is the relative distance (isospin) of the np pair. The resulting LIT equation can be easily solved by direct numerical methods or by expansions of $\tilde\psi(r)$ on a complete set. Since in case of nuclei with $A>2$ we are generally using expansions on hyperspherical harmonics, where the hyperradial part is expanded in Laguerre polynomials $L_n^{m+\frac{1}{2}}$ times an exponential fall-off, here a corresponding ansatz is made:

$$\tilde\psi(r) = \sum_{n=1}^{N} c_n\, r\, L_n^{1+\frac{1}{2}}(r/b)\, exp(-r/2b), \tag{12}$$

where c_n is a normalization factor and b a constant.

A comparison of results with the Lanczos response and inversion results was made in Ref. [2] for the simple example of deuteron photodisintegration in unretarded dipole approximation. In this case one can check the quality of the results by comparing with a conventional calculation, where np continuum wave function are calculated. The study of Ref. [2] has shown that within the LIT approach it is sufficient to use a rather large value of 10 MeV for σ_I and hence a basis with a rather low N. On the contrary for the Lanczos response, even when using $\sigma_I = 0.25$ MeV with a quite high number of basis states, it was not possible to reproduce the $R(E)$ sufficiently correctly.

In Figs. 4–6 LIT results for the $^3P_1$ channel are shown for various values of N and σ_I. To obtain the $^3P_1$ part of the unretarded deuteron photodisintegration cross section one has to invert these transforms. However, in order to make a reliable inversion $L(\sigma)$ should be sufficiently converged for a given σ_I. In particular isolated peaks of single Lorentzians should not appear, i. e. for any σ_R value one should have a significant contribution from various Lorentzians. The results of Figs. 4–6 show that the convergence pattern is quite different for the various σ_I. For the case with the lowest resolution ($\sigma_I = 2.5$ MeV) one obtains a sufficiently converged $L(\sigma)$ already with 30 basis functions ($N = 30$). For the case with $\sigma_I = 1$ MeV one is close to convergence with $N = 50$, whereas the LIT for the highest requested resolution ($\sigma_I = 0.1$ MeV) is quite far from convergence even with $N = 50$. For the latter case the number of basis functions should be increased considerably to obtain a converged $L(\sigma)$. It is evident that a higher resolution requires a higher computational effort. In an actual calculation one should check what is the lowest σ_I value with a sufficiently converged LIT. Structures which are considerably smaller than such a σ_I value cannot be resolved by the inversion. A helpful criterion is given in Ref. [23] (see discussion of Fig. 7 in Ref. [23]).

From the discussion above it is evident that the LIT approach is a method with a controlled resolution.

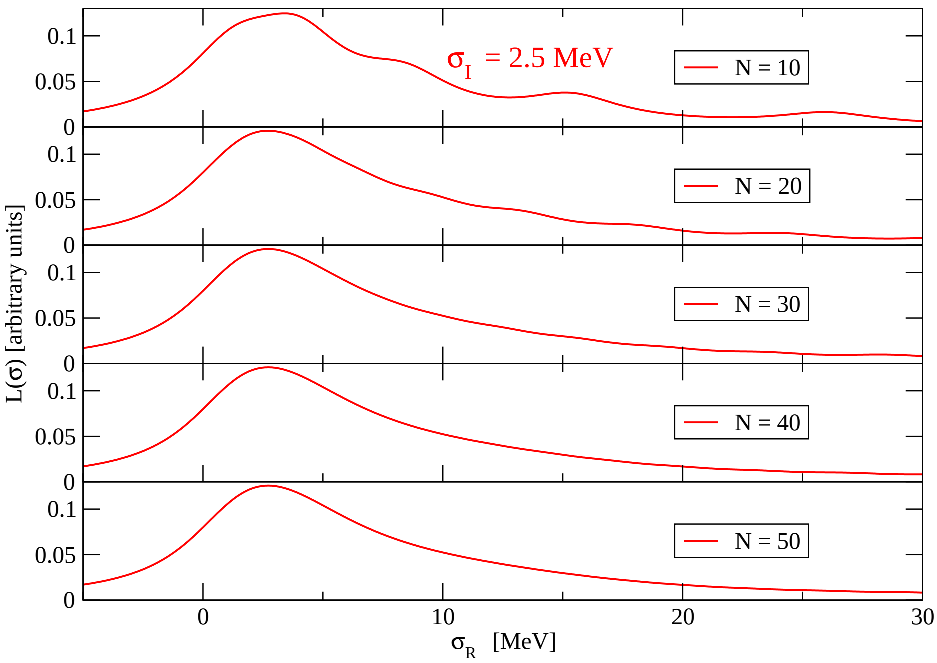

Figure 4: Deuteron photodisintegration in unretarded dipole approximation: LIT result for np channel $^3P_1$ with $\sigma_I = 2.5$ MeV.

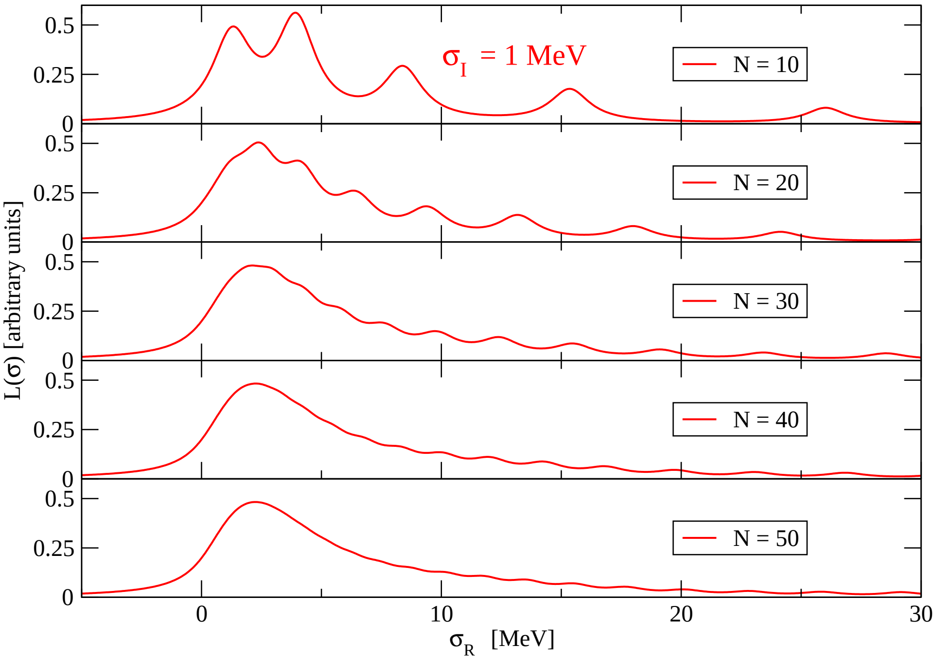

Figure 5: As Fig. 4 but with $\sigma_I = 1$ MeV.

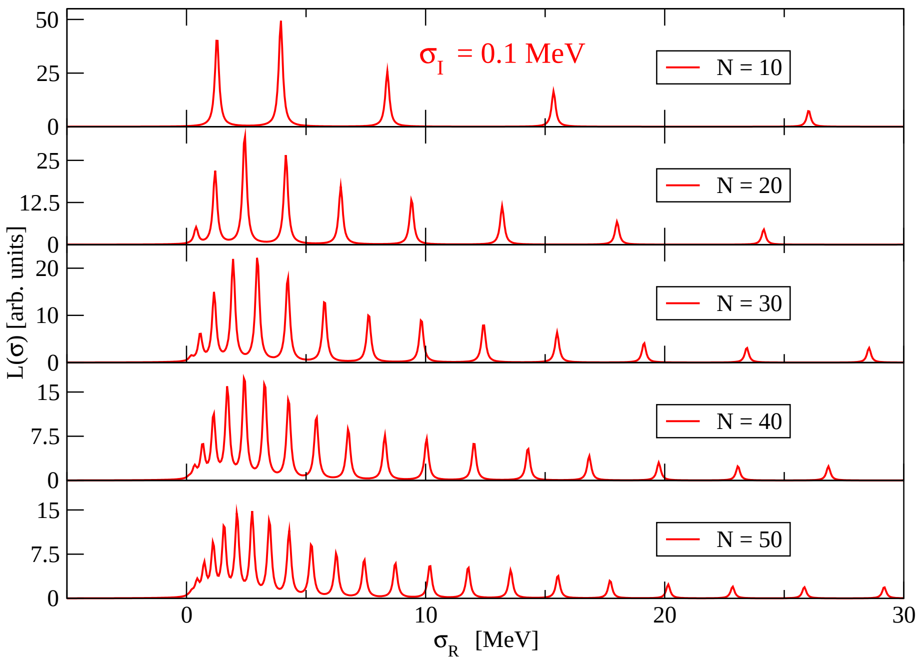

Figure 6: As Fig. 4 but with $\sigma_I = 0.1$ MeV.

4 Isoscalar monopole resonance of ^{4}He

The 0^+ resonance of ^{4}He can be studied in hadronic and electron scattering reactions. The signal of the resonance is much more pronounced in the latter case and thus electron scattering experiments of ^{4}He are the proper tool to study the resonance. In fact the pronounced cross section peak has been studied in various (e, e')

experiments [24–26]. There it has been found that the resonance is located between the two thresholds for the break-up in ^{3}H-p and ^{3}He-n and that the width is about 300 keV. In addition, the resonance strength has been measured over a rather large momentum transfer range.

In Ref. [27] a LIT calculation of the isoscalar monopole part of $R_L(q,\omega)$ of ^{4}He(e,e') has been performed using the nuclear force model AV18+UIX and a chiral nuclear force model with the Idaho N3LO NN potential [28] supplemented by a 3NF in N2LO in two different parameterizations. The calculation shows that both interaction models overestimate the resonance position by about 700 keV and sufficiently convergent LIT results could only be obtained for $\sigma_I \geq 5$ MeV. Such a resolution, much larger than the experimental width of 300 keV, is of course not sufficient to determine the detailed resonance structure. On the other hand it has been possible to separate the background strength from the resonance strength. For details of this separation I refer to Ref. [27]. Here it should only be mentioned that this is not a trivial task and that it has been achieved by an appropriate inversion procedure, which gave the energy distribution of the background strength and the total resonance strength.

In Fig. 7 the calculated resonance strength is compared to the above mentioned experimental data. One sees that the two realistic interaction models exhibit rather different results: the AV18+UIX force leads to a resonance strength which is about 20 % lower than that of the chiral force model. Thus the ^{4}He resonance strength turns out to be an observable which is very selective concerning force models. In Fig. 7 it can also be seen that even with force model AV18+UIX the experimental resonance strength is overestimated considerably. As discussed in detail in Ref. [27] it is not easy to understand what causes the difference of theoretical and experimental results (e. g., the calculated elastic ^{4}He form factor agree well with experimental data up to about $q^2 = 4$ fm^{-2} for both potential models). In Fig. 7 an additional theoretical result [29] is shown for a force model consisting in the AV8' NN potential and a simplistic 3NF. One observes a nice agreement with the experimental data. However, the calculation cannot be considered to be fully realistic (the not completely realistic potential model has led to a second 0^+ bound state and not to a resonance in the continuum).

One might ask how the width of the ^{4}He 0^+ resonance can be resolved with the

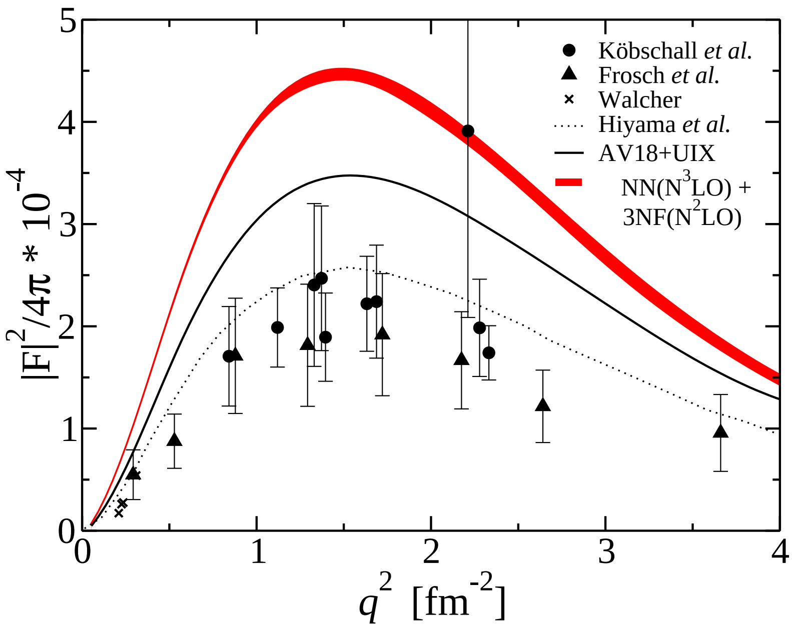

Figure 7: ^{4}He isoscalar monopole resonance strength $|F(q^2)|^2$.

LIT method. That the method is in principle capable to resolve a resonance with such a small width has been shown in Ref. [23] in a model study. In the present case one could increase the HH basis or increase the size of the box, but this might not lead to a much improved result. Probably it is better to describe the four-body system as 3+1 system with an HH expansion for the three-body part and a separate expansion for the relative motion of nucleon and residual system, of course, always with bound-state methods. Such an approach would be in close analogy to a scattering calculation for a two-body break-up.

5 Summary

An overview has been given on recent LIT applications for the inclusive electrodisintegration of ^{3}He and ^{4}He with realistic nuclear force models. The results for the transverse response function $R_T(q,\omega)$ of ^{3}He show (i) that an excellent agreement with experimental data is obtained in the quasi-elastic peak region at higher momentum transfers and (ii) that the LIT method can be applied also to reactions with energies far into the many-body continuum. For ^{4}He, results of the longitudinal response function $R_L(q,\omega)$ of ^{3}He have been reported. They exhibit strong 3NF effects at lower momentum transfers. In addition it has been discussed that a theoretical study for the isoscalar monopole part of the R_L of ^{4}He reveals (i) a strong dependence of the resonance strength on the nuclear force model and (ii) a considerable overestimation of the experimental resonance strength.

The energy resolution that can be obtained with the LIT method has also been discussed in greater detail. The discussion shows that the LIT approach is a method with a controlled resolution.

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