# Analysis of Resonant States within the SS HORSE Method

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# Plan:

- ✓ *HORSE: Harmonic Oscillator Representation of Scattering Equation.*
- ✓ *SS HORSE (Single State HORSE)*
- ✓ *SS HORSE: charged particles*
- ✓ *Low-energy phase shift parametrization based on analytical properties of the scattering amplitude*
  - A. M. Shirokov, A. I. Mazur, I. A. Mazur, and J. P. Vary arXiv:1608.05885 [nucl-th] (submit to PRC)
  - A. M. Shirokov,  G. Papadimitriou, A. Mazur,  I. Mazur,  R. Roth,  and J. P. Vary arXive: 1607.05631 [nucl-th] (submit to PRL)
  - L. D. Blokhintsev, A. Mazur, I. Mazur, A. Shirokov and D. A. Savin, submit to Phys. Atom. Nucl.
- ✓ *Approbation on model problem simulating nucleon-α scattering.*

# HORSE = *Harmonic Oscillator Representation of Scattering Equation.*

**Schrödinger equation**

$$H^l u_l(E,r) = E u_l(E,r).$$

**Wave function is expanded in infinite series of the oscillator functions**

$$u_l(E,r) = \sum_{n=0}^{\infty} a_{nl}(E) R_{nl}(r).$$

**Infinite set of algebraic equations**

$$\sum_{n'=0}^{\infty} (H^l_{nn'} - \delta_{nn'} E) a_{n'l}(E) = 0, \quad n = 0,1,\ldots$$

***n* is principal quantum number, *l* is orbital momentum, *μ* is reduced mass.**

$\hbar\Omega$ ***is oscillator parameters.***

$$R_{nl}(r) = (-1)^n \sqrt{\frac{2n!}{r_0 \Gamma(n+l+3/2)}} \left(\frac{r}{r_0}\right)^{l+1} \exp\left(-\frac{r^2}{2r_0^2}\right) L_n^{l+1/2}\left(\frac{r^2}{r_0^2}\right)$$

$r_0 = \sqrt{\hbar / \mu\Omega}$, $E = (q^2/2)\hbar\Omega$ – ***c.m. energy,***

$q = kr_0$ – ***dimensionless momentum.***

# HORSE

**Infinite Hamiltonian matrix**

$$H^l_{nn'} = T^l_{nn'} + V^l_{nn'}$$

**Non-zero kinetic energy matrix elements increased linearly with *n* :**

$$T^l_{nn'} \sim n, \ (n, n' \to \infty)$$

$$T^l_{nn} = \frac{\hbar\Omega}{2}(2n + l + 3/2)$$

$$T^l_{nn+1} = T^l_{n+1n} = -\frac{\hbar\Omega}{2}\sqrt{(n+1)(n+l+3/2)}$$

$$V^l_{nn'} - decrease \ with \ n, n' \to \infty$$

**Truncated potential matrix:**

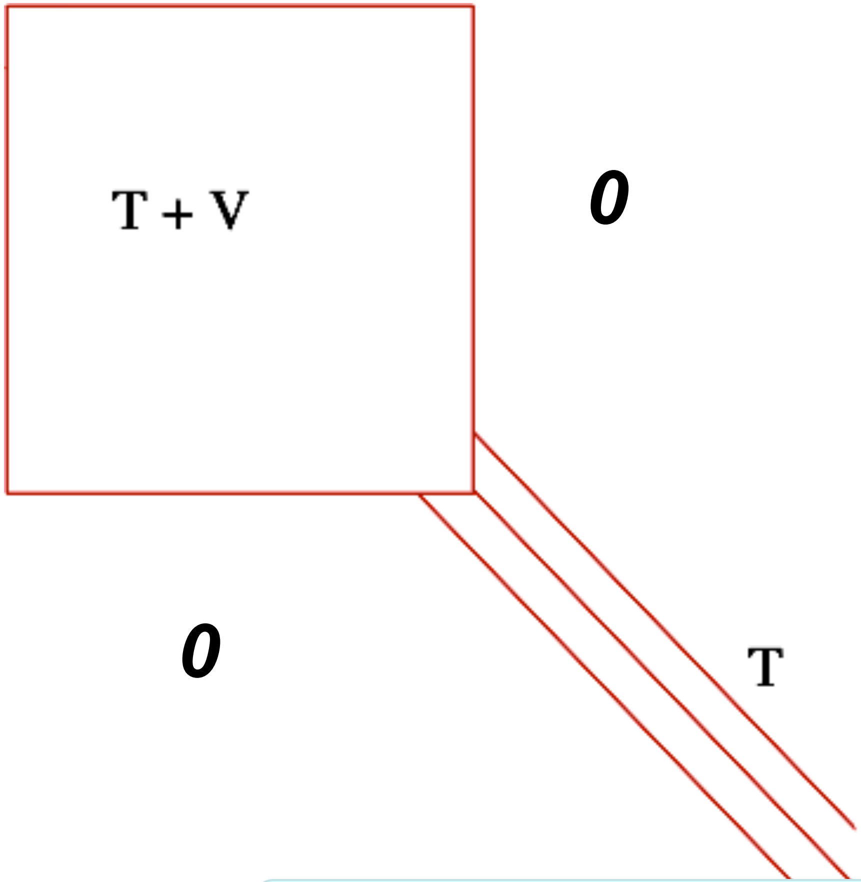
$$\tilde{V}^l_{nn} = \begin{cases} V^l_{nn'}, & if \ n \ and \ n' \leq N \\ 0, & if \ n \ or \ n' > N \end{cases}$$

# HORSE

**Structure of the infinite Hamiltonian:**

***Internal subspace*** **P:**
$n \leq N, \quad H = T + V$

***External subspace*** **Q**
$n > N, \quad H = T$




This is an exactly solvable algebraic problem!

**parameters:** $N, \ \hbar\Omega$

And this looks like
a natural extension of SM
where both potential
and kinetic energies are truncated

# HORSE

**Phase shift:**

$$\tan\delta(E) = -\frac{S_{Nl}(E) - G_{NN}(E)T^{l}_{N,N+1}S_{N+1,l}(E)}{C_{Nl}(E) - G_{NN}(E)T^{l}_{N,N+1}C_{N+1,l}(E)}$$

***Regular and irregular oscillator solutions***

$$S_{nl}(E) = \sqrt{\frac{2r_0 n!}{\Gamma(n+l+3/2)}}\; q^{l+1}\exp\left(-\frac{q^2}{2}\right)L_n^{l+1/2}(q^2)\ ,$$

$$C_{nl}(E) = \sqrt{\frac{2r_0 n!}{\Gamma(n+l+3/2)}}\; \frac{\Gamma(l+1/2)}{\pi\, q^l}\exp\left(-\frac{q^2}{2}\right)\Phi(-n-l-1/2,-l+1/2;q^2)\ .$$

$$\sum_{n=0}^{\infty} S_{nl}(E)R_{nl}(r) = \sqrt{\frac{2}{\pi}}kr\, j_l(kr),$$

$$\sum_{n=0}^{\infty} C_{nl}(E)R_{nl}(r) \to -\sqrt{\frac{2}{\pi}}kr\, n_l(kr).$$

$$E = \frac{q^2}{2}\hbar\Omega$$

# HORSE

**Phase shift**

$$\tan\delta(E) = -\frac{S_{Nl}(E) - G_{NN}(E)T^{l}_{N,N+1}S_{N+1,l}(E)}{C_{Nl}(E) - G_{NN}(E)T^{l}_{N,N+1}C_{N+1,l}(E)}$$

$$G_{NN}(E) = -\sum_{\nu=0}^{N}\frac{\langle N\,|\,\nu\rangle^2}{E_\nu - E}$$

$$E_\nu,\ \langle n\,|\,\nu\rangle\ \ (\nu = 0,1,\ldots,N)$$

*are eigenvalues and corresponding eigenvectors of the truncated Hamiltonian*

**P-*subspace:***

$$\sum_{n'=0}^{N} H^{l}_{nn'}\langle n'\,|\,\nu\rangle = E_\nu\langle n\,|\,\nu\rangle,\ \ n \le N$$

**So, to calculate phase shift within the HORSE we need to know the results of the full diagonalization of the Hamiltonian, what is impossible in the large scale NCSM calculations**

# Single State (SS) HORSE

$$\tan\delta(E) = -\frac{S_{Nl}(E) - G_{NN}(E)T^{l}_{N,N+1}S_{N+1,l}(E)}{C_{Nl}(E) - G_{NN}(E)T^{l}_{N,N+1}C_{N+1,l}(E)}$$

$$G_{NN}(E) = -\sum_{\nu=0}^{d}\frac{\langle N|\nu\rangle^{2}}{E_{\nu} - E}$$

**$E = E_\nu$ :**

$$\delta(E_\nu) = -\tan^{-1}\left(\frac{S_{N+1,l}(E_\nu)}{C_{N+1,l}(E_\nu)}\right)$$

**Important:**

1. *summation over the base states disappears*
2. *dependence from eigenvectors disappears*

***The phase shift is determined only by $E_\nu$***

*Calculating a set of eigenenergies $E_\nu$ with different ħΩ and N, we obtain a set of $\delta_l(E)$ values which we can approximate by a smooth curve at low energies.*

*SS HORSE can be used in all approaches that use the oscillator basis, including NCSM.*

# SS HORSE

## Scaling property of low-energy scattering

*From asymptotic behavior*
*of $S_{nl}$ and $C_{nl}$ under condition*

$$N >> \sqrt{2E/\hbar\Omega}$$

*one can obtain*

$$\delta_l \approx \tan^{-1}\left(\frac{j_l\left(2\sqrt{E_\nu/s}\right)}{n_l\left(2\sqrt{E_\nu/s}\right)}\right)$$

Here $j_l$ and $n_l$ are spherical Bessel and Neumann functions, and

$$s = \frac{\hbar\Omega}{\left(2N+l+7/2\right)}$$

– ***scaling parameter***

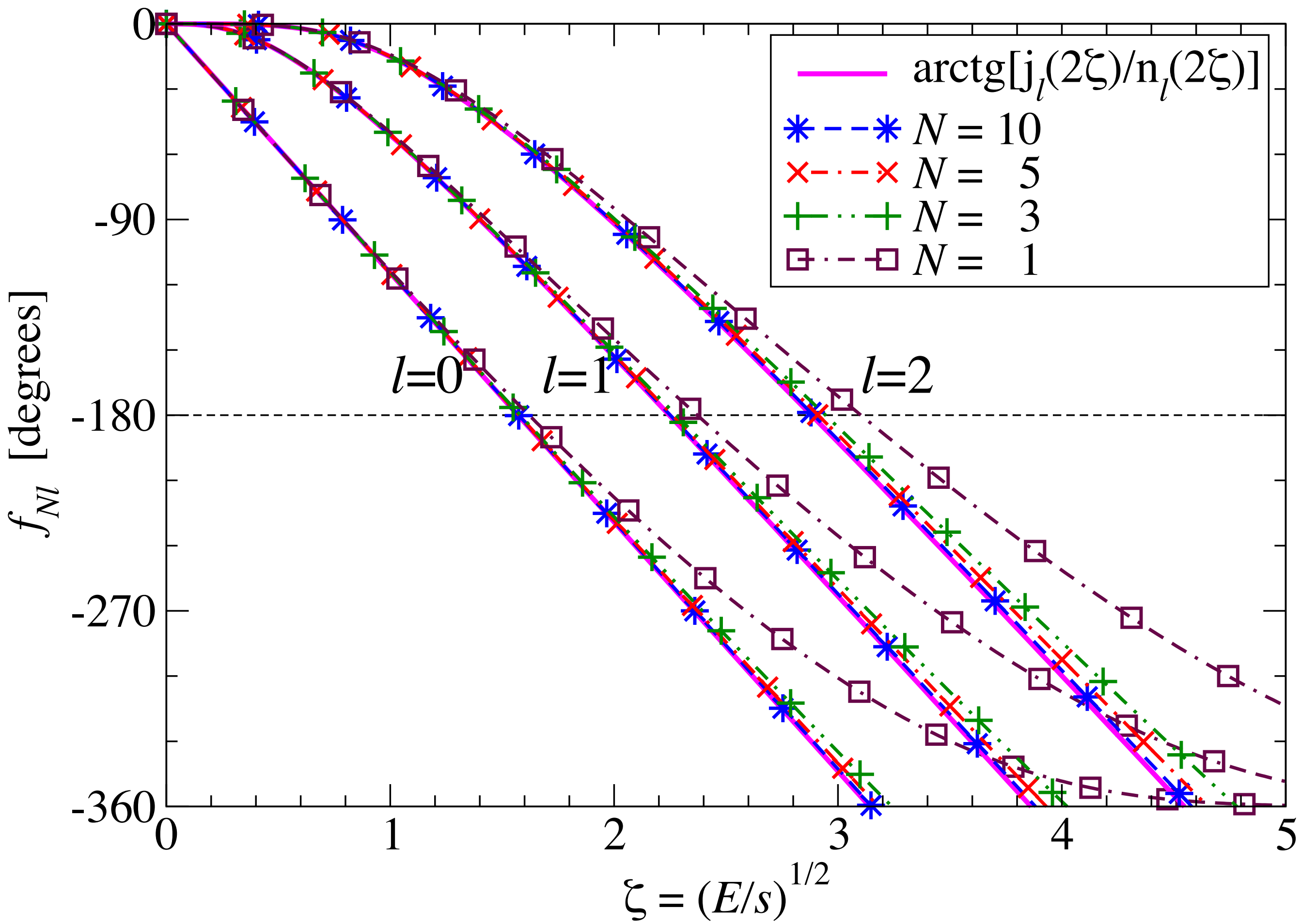
*Scaling:*
*phase shift depends not on the N and ħΩ alone, but depends on their combination s.*

# SS HORSE

*Scailing*

$$symbols: f_{Nl} = -\arctan\left(\frac{S_{N+1,l}(E)}{C_{N+1,l}(E)}\right)$$

$$solid: \arctan\left(\frac{j_l\left(2\sqrt{E/s}\right)}{n_l\left(2\sqrt{E/s}\right)}\right)$$



# HORSE: charged particle scattering

J. M. Bang, A. I. Mazur, A. M. Shirokov, Yu. F. Smirnov and S. A. Zaytsev, Ann. Phys. (N.Y.) 280, 299 (2000

*Auxiliary short-range potential*

$$V^{sh} = \begin{cases} V^{nucl} + V^{Cl}, \ r \le R', & R' > R^{nucl} \\ 0, \ r \le R' & \end{cases}$$

*Within HORSE one can calculate phase shift $\delta_l^{sh}$ .*

$$\tan\delta^{sh} = -\frac{S_{Nl} - G_{NN} T^l_{N,N+1} S_{N+1,l}}{C_{Nl} - G_{NN} T^l_{N,N+1} C_{N+1,l}}$$

*Extension HORSE*

$$\tan\delta_l = -\frac{W_{R'}(j_l, F_l) - W_{R'}(n_l, F_l)\tan\delta_l^{sh}}{W_{R'}(j_l, G_l) - W_{R'}(n_l, G_l)\tan\delta_l^{sh}}$$

*Quasi-Wronskian*

$$W_{R'}(j_l, F_l) = \left( \frac{d}{dr}\left[ j_l(kr) \right] F_l(\eta, kr) - j_l(kr) \frac{d}{dr}\left[ F_l(\eta, kr) \right] \right)\Bigg|_{r=R'}$$

$j_l(kr), \quad n_l(kr)$ — Bessel and Neumann functions;

$F_l(\eta, kr), \quad G_l(\eta, kr)$ — Coulomb functions;

$\eta = \dfrac{\mu Z_1 Z_2 e^2}{k}$ — Sommerfeld parameter.

# SS HORSE: charged particle

- **SS-HORSE**

*$E = E_\nu$ :*

$$\tan\delta_l = -\frac{W_{R'}(n_l, F_l) S_{N+1,l}(E_\nu) + W_{R'}(j_l, F_l) C_{N+1,l}(E_\nu)}{W_{R'}(n_l, G_l) S_{N+1,l}(E_\nu) + W_{R'}(j_l, G_l) C_{N+1,l}(E_\nu)}$$

***The phase shift is determined only by $E_\nu$***

- **Scaling at**

$$N+1 >> \sqrt{\frac{2E}{\hbar\Omega}} :$$

$$\delta_l(E_\nu) = -\tan^{-1}\left[\frac{F_l\left(\eta, 2\sqrt{E_\nu / s}\right)}{G_l\left(\eta, 2\sqrt{E_\nu / s}\right)}\right]$$

# Partial scattering amplitude

$$f_l(E) = \frac{S_l(E)-1}{2ik} = \frac{\exp\{2i\delta_l(E)\}-1}{2ik}$$
$$= \frac{k^{2l}}{k^{2l+1}ctg\delta_l(E) - ik^{2l+1}}$$

$$E = \hbar^2 k^2 / 2\mu$$

*The effective radius function is represented as Taylor series in $k^2$:*

$$k^{2l+1}ctg\delta_l(E) = -\frac{1}{a_l} + \frac{r_l}{2}k^2 - \frac{P_l}{2}k^4 + \ldots$$

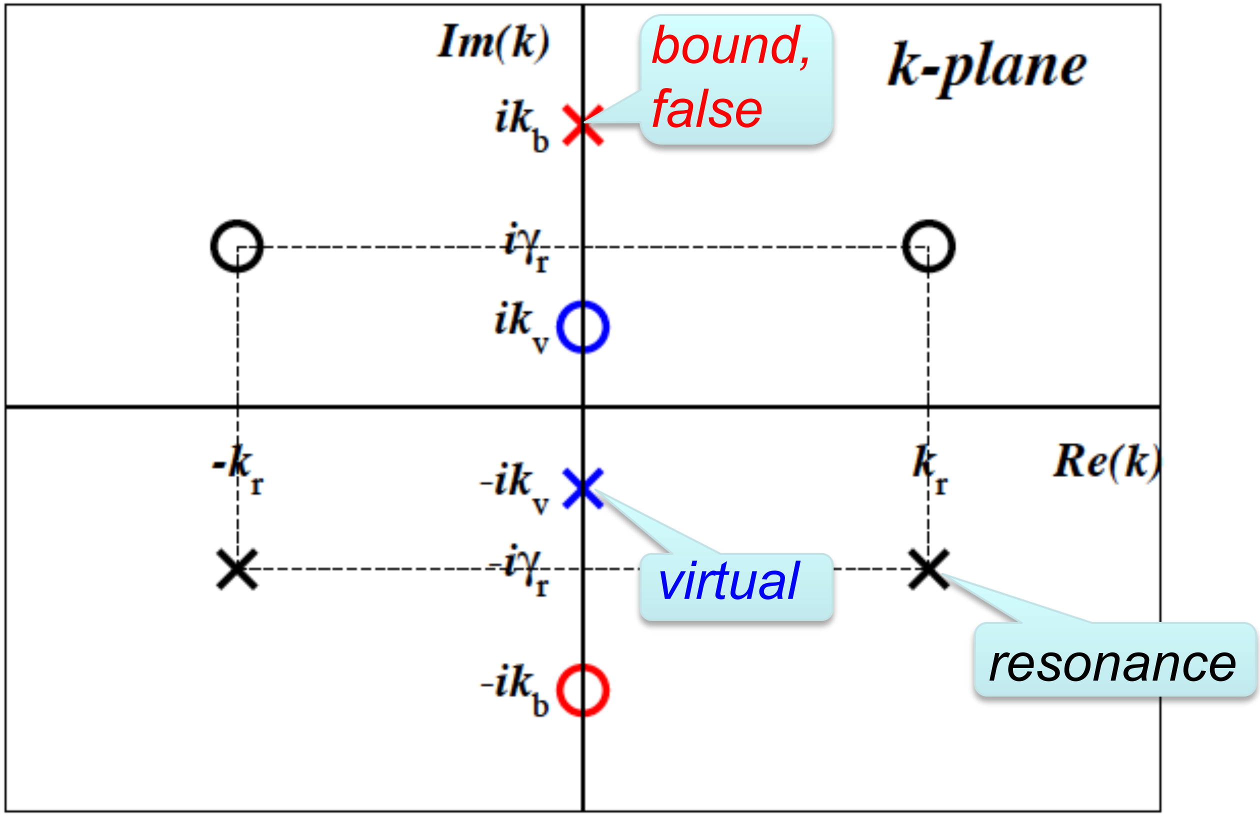
*Below we use the approximation with 3 terms*

$$k^{2l+1}ctg\delta_l(E) = -\frac{1}{a_l} + \frac{r_l}{2}k^2 - \frac{P_l}{2}k^4$$

# Partial scattering amplitude

$$f_l(E) = \frac{S_l(E) - 1}{2ik}$$

The poles
of the scattering
amplitude $f_l$
and of the $S$-matrix
are the same.




*resonance pole:*

$$E_p = E_r - i\frac{\Gamma}{2} \quad (E-plane) \quad or \quad k_p = k_r - i\gamma \quad (k-plane)$$

$$E_r = \frac{\hbar^2}{2\mu}\left(k_r^2 - \gamma^2\right), \qquad \frac{\Gamma}{2} = \frac{\hbar^2}{2\mu}2k_r\gamma$$

# Partial scattering amplitude

Scattering amplitude

$$f_l(E) = \frac{k^{2l}}{k^{2l+1}ctg\delta_l(E) - ik^{2l+1}}$$

$k \to k_p$

$(k_r - i\gamma)$

Auxiliary function:

L. D. Blokhintsev and D. A. Savin, Phys. Atom. Nucl. 79, 358 (2016)

$$F_l(E) \equiv R_l(E) + i \cdot I_l(E) = \frac{k^{2l+1}ctg\delta_l(E) - i(k_r - i\gamma)^{2l+1}}{E - \left(E_r - i\Gamma/2\right)}$$

*Auxiliary function has no singularities at $E = (E_r - i\,\Gamma/2)$*

*and can be expanded in a Taylor series in $k^2$ (energy)*

*as an effective radius function* $k^{2l+1}ctg\delta_l(E)$

# Partial scattering amplitude

For the real and imaginary parts of auxiliary function one can obtain

$$R_l(E) = \frac{\left(k^{2l+1} ctg\,\delta_l - Q_r\right)\left(E - E_r\right) - \frac{1}{2} Q_i \Gamma}{\left(E - E_r\right)^2 + \left(\Gamma/2\right)^2}$$

$$I_l(E) = \frac{\frac{1}{2}\left(k^{2l+1} ctg\,\delta_l - Q_r\right)\Gamma + Q_i\left(E - E_r\right)}{\left(E - E_r\right)^2 + \left(\Gamma/2\right)^2}$$

$$Q_r = \mathrm{Re}\left[i\left(k_r - i\gamma\right)^{2l+1}\right],$$

$$Q_i = \mathrm{Im}\left[i\left(k_r - i\gamma\right)^{2l+1}\right]$$

*Functions $R_l$ and $I_l$ can be expanded in a Taylor series in $k^2$ (energy), as an auxiliary function $F_l$.*

*Functions $R_l$ and $I_l$ are interrelated functions*

$$I_l(E)\left(E - E_r\right) = -\frac{1}{2} R_l(E)\Gamma - Q_i$$

*So we can fit only one function: $R_l$ or $I_l$.*
*But this relationship imposes certain restrictions on the coefficients in the Taylor series.*

# Partial scattering amplitude

*As appears, it is enough to approximate $R_l$ with help of second-order polynomial $R_l^{(2)}$ in powers of $(E-E_r)$*

$$R_l^{(2)} = -\frac{2}{\Gamma}\left[-Q_i + w_1\left(E-E_r\right) + w_2\left(E-E_r\right)^2\right]$$

*From this parameterization and the three term effective radius expansion one can obtain expressions for:*

$$k^{2l+1} ctg\delta_l(E) = -\frac{1}{a_l} + \frac{r_l}{2}k^2 - \frac{P_l}{2}k^4$$

*scattering length*

$$a_l = -\frac{\Gamma}{2}\left[Q_r\frac{\Gamma}{2} + Q_i E_r + \left(w_1 - w_2 E_r\right)\left(E_r^2 + \left(\Gamma/2\right)^2\right)\right]^{-1}$$

*effective radius*

$$r_l = \frac{\hbar^2}{\mu}\frac{2}{\Gamma}\left[-Q_i - 2w_1 E_r - w_2\left(3E_r^2 - \left(\Gamma/2\right)^2\right)\right]$$

# Phase shift parametrization

## Fitting procedure

*We have a set of selected lowest energies $E_0^{(i)}$ ( $i=1,2,3,\ldots,d$)*
*$E_0^{(i)}$ are calculated with different $N^{(i)}$ and $\hbar\Omega^{(i)}$.*

*From transcendent equation*

$$\frac{\left(\frac{C_{N+1,l}(E_0)}{S_{N+1,l}(E_0)}k^{2l+1}+Q_r\right)(E-E_r)+\frac{1}{2}Q_i\Gamma}{(E-E_r)^2+(\Gamma/2)^2}=\frac{2}{\Gamma}\left[Q_i-w_1(E-E_r)-w_2(E-E_r)^2\right]$$

*with fixed trial values of $w_1$, $w_2$, $E_r$ and $\Gamma$,*
*one can obtain a set of energies $\varepsilon^{(i)}$.*

*Final values of fitting parameters*
*$w_1$, $w_2$, $E_r$ and $\Gamma$ are determined*
*by minimizing the functional*

$$\Xi=\sqrt{\frac{1}{d}\sum_{i=1}^{d}\left(E_0^{(i)}-\varepsilon^{(i)}\right)^2}$$

# Phase shift parametrization

*From transcendent equation with fitted values of $w_1$, $w_2$, $E_r$ and $\Gamma$*

$$\frac{\left(-\frac{n_l\left(2\sqrt{E/s}\right)}{j_l\left(2\sqrt{E/s}\right)}k^{2l+1}+Q_r\right)\left(E-E_r\right)+\frac{1}{2}Q_i\Gamma}{\left(E-E_r\right)^2+\left(\Gamma/2\right)^2}=\frac{2}{\Gamma}\left[Q_i-w_1\left(E-E_r\right)-w_2\left(E-E_r\right)^2\right]$$

*one can calculate dependence E(s).*

*With help of equation*

$$k^{2l+1}ctg\,\delta_l=Q_r+\frac{R_l^{(2)}\left[\left(E-E_r\right)^2+\left(\Gamma/2\right)^2\right]+\frac{1}{2}Q_i\Gamma}{E-E_r}$$

*with fitted values of $w_1$, $w_2$, $E_r$ and $\Gamma$*
*we can calculate dependence δ(E).*

# Phase shift parametrization: charged particles

*In the case of the scattering of charged particles the procedure for phase shift parameterization is the same.*

*But instead of the usual scattering amplitude*

$$f_l(E) = \frac{k^{2l}}{k^{2l+1} ctg\delta_l(E) - ik^{2l+1}}$$

*we use Coulomb modified scattering amplitude*

$$\tilde{f}_l^N(E) = \frac{k^{2l}}{K_l(k^2) - 2\eta k^{2l+1} H(\eta)\left(c_{l\eta}\right)^{-1}}$$

*and instead usual effective radius function*

$$k^{2l+1} ctg\delta_l(E)$$

*we use Coulomb modified effective radius function.*

$$K_l(k^2) = k^{2l+1}\left(c_{l\eta}\right)^{-1}\left\{\frac{2\pi\eta}{\exp(2\pi\eta)-1}\left[ctg\ \delta_l(k) - i\right] + 2\eta H(\eta)\right\}$$

$$H(\eta) = \Psi(i\eta) + (2i\eta)^{-1} - \ln(i\eta)$$

# Model problem

***Scattering particle with reduced mass***

$$\mu = \tfrac{4}{5} m_{nucl}$$

***on Woods-Saxon potential***

$$V_{n\alpha} = \frac{V_0}{1+\exp\left[\left(r-R_0\right)/\alpha_0\right]} + \left(\vec{l}\cdot\vec{s}\right)\frac{1}{r}\frac{d}{dr}\frac{V_{ls}}{1+\exp\left[\left(r-R_1\right)/\alpha_1\right]}$$

$$V_0 = -43 \ MeV, \quad V_{ls} = -40 \ MeV \cdot fm^2, \quad R_0 = 2.0 \ fm, \quad \alpha_0 = 0.70 \ fm, \quad R_1 = 1.5 \ fm, \quad \alpha_1 = 0.35 \ fm$$

***well simulates nα scattering***

*There are two resonant states:*

*broad $1/2^-$ ($E_r$=1.66 MeV, Γ=5.58 MeV)*

*narrow $3/2^-$ ($E_r$=0.837 MeV, Γ=0.780 MeV)*

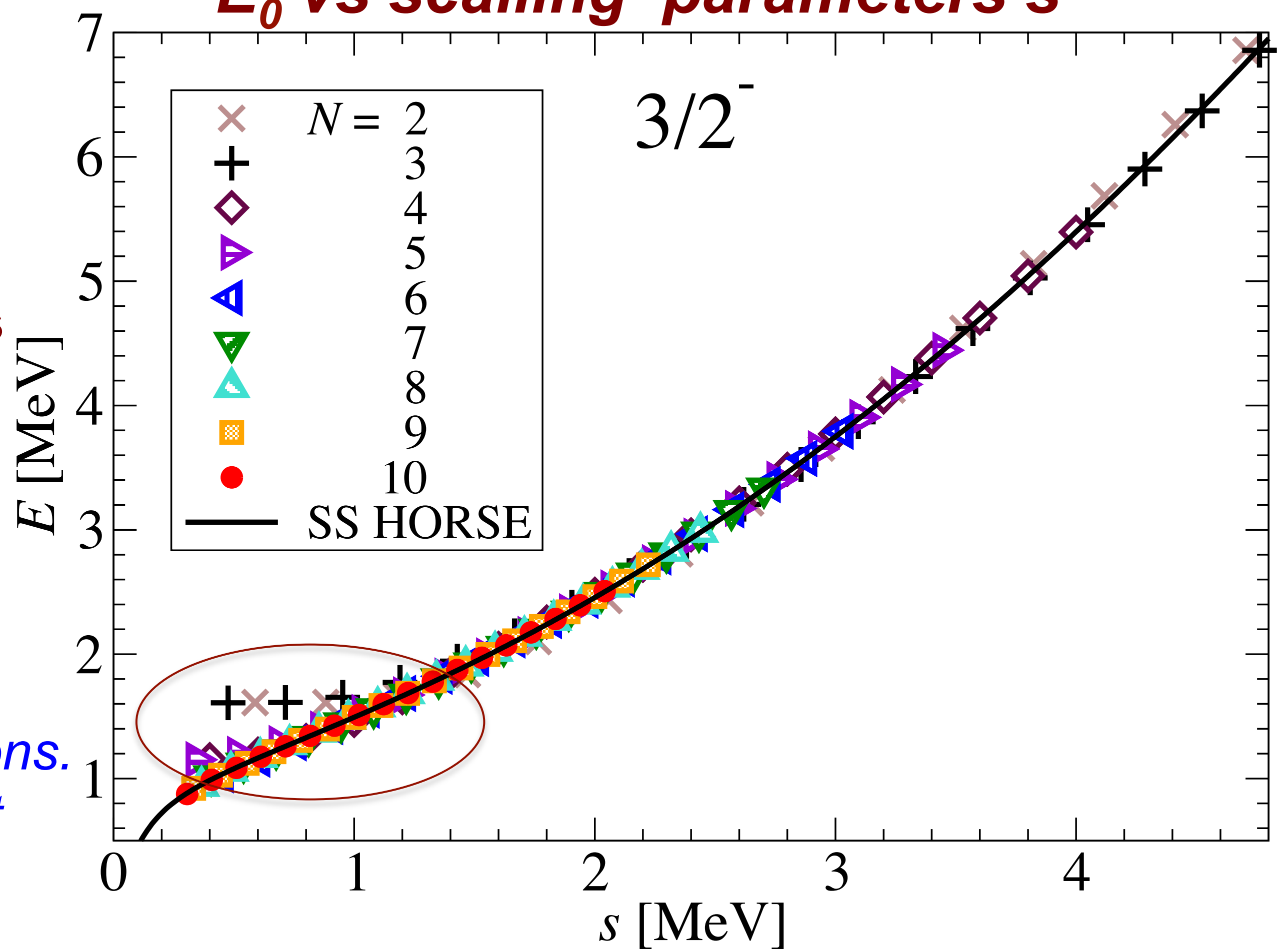
# 3/2⁻ resonance

***We calculated the energies of the lowest state $E_0$ by diagonalization Hamiltonian matrix in the oscillator basis with N = 2,3,4,…,10 and with values ħΩ from 2.5 MeV to 50 MeV (with step of 2.5 MeV).***

**SS HORSE results form a smooth line.**

**But some results (obtained in cases with small *N* at low energies (low *s*) significantly deviate from this smooth lines.**

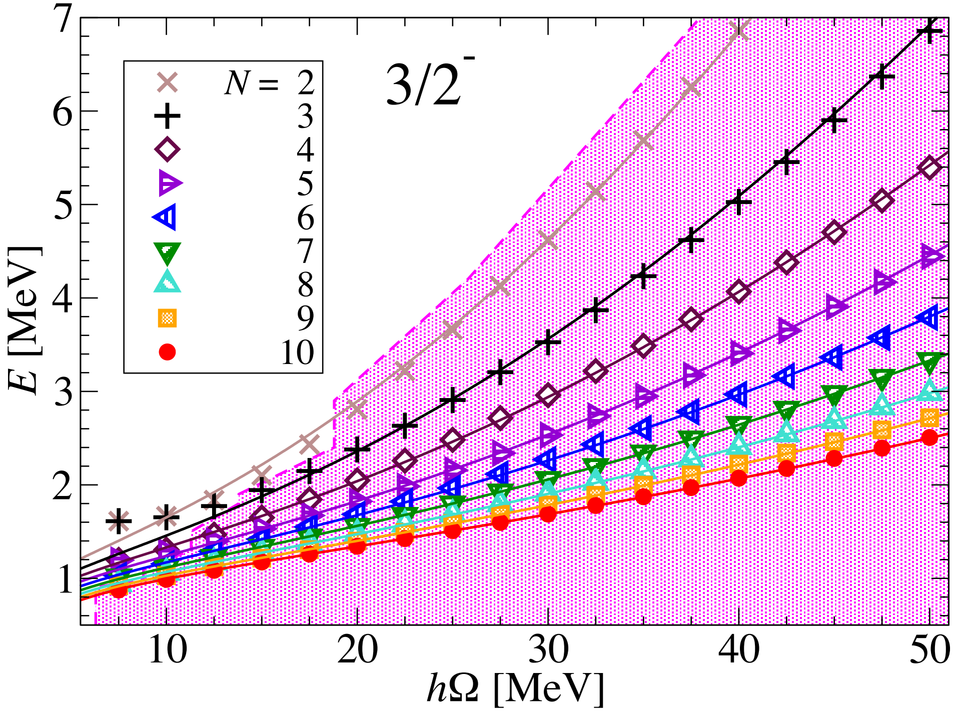
***$E_0$ vs scailing parameters s***




*Symbols correspond to SS HORSE calculations.*

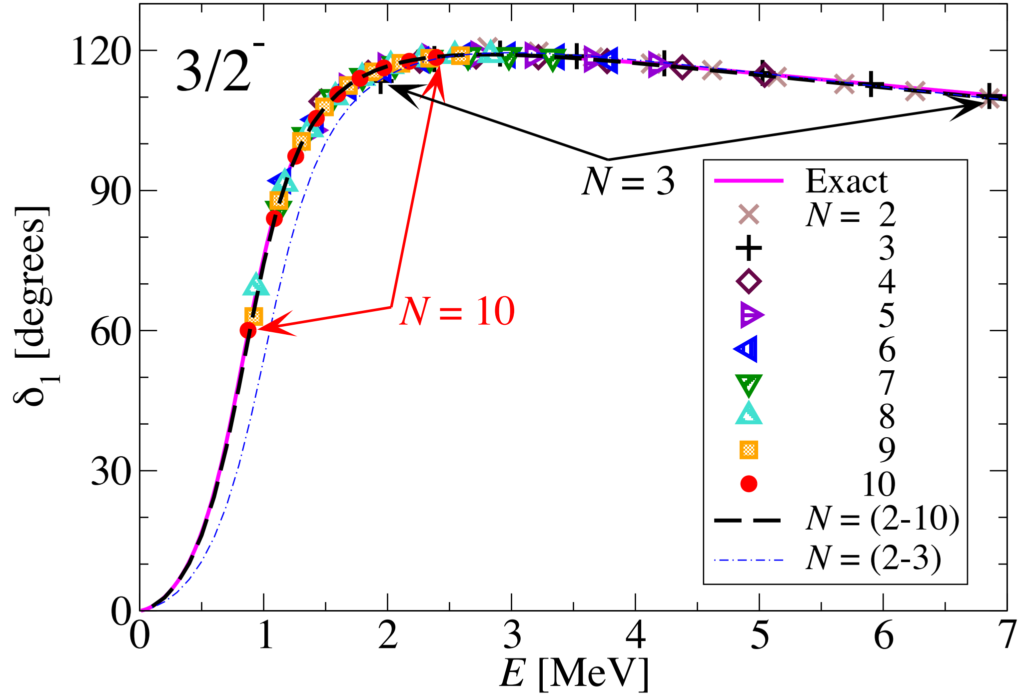
*Solid lines represent rezults of the fit.*

# 3/2⁻ resonance




**Selected data are in the shaded area**

**Many SS HORSE results are out of the resonance region. Curves for exact phase shift and fitted phase shift are indistinguishable**




*Symbols correspond to SS HORSE calculations of the lowest energy $E_0$.*

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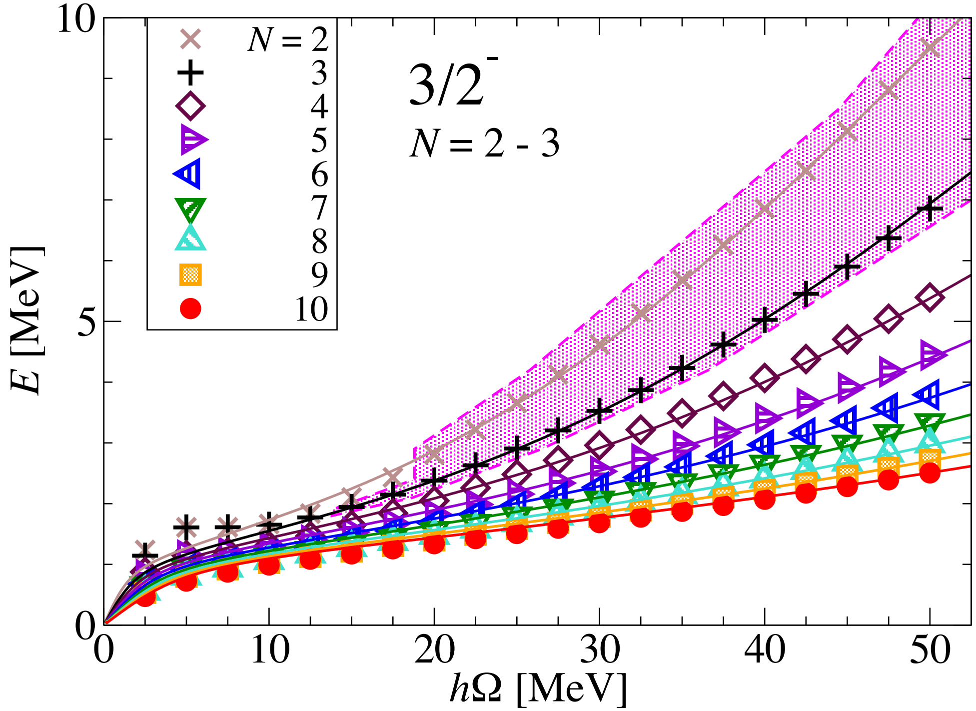
# 3/2⁻ resonance

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**In the case N=2-3 the fitted phase shift is slightly shifted in compare with exact phase shift. All SS HORSE results are out of the resonance region.**

**Fitting in the case of *N*=2-3 predicts the correct behavior of the curves *E(ħΩ)* at *N*=4,5,…**

*Symbols correspond to SS HORSE calculations of the lowest energy $E_0$.*

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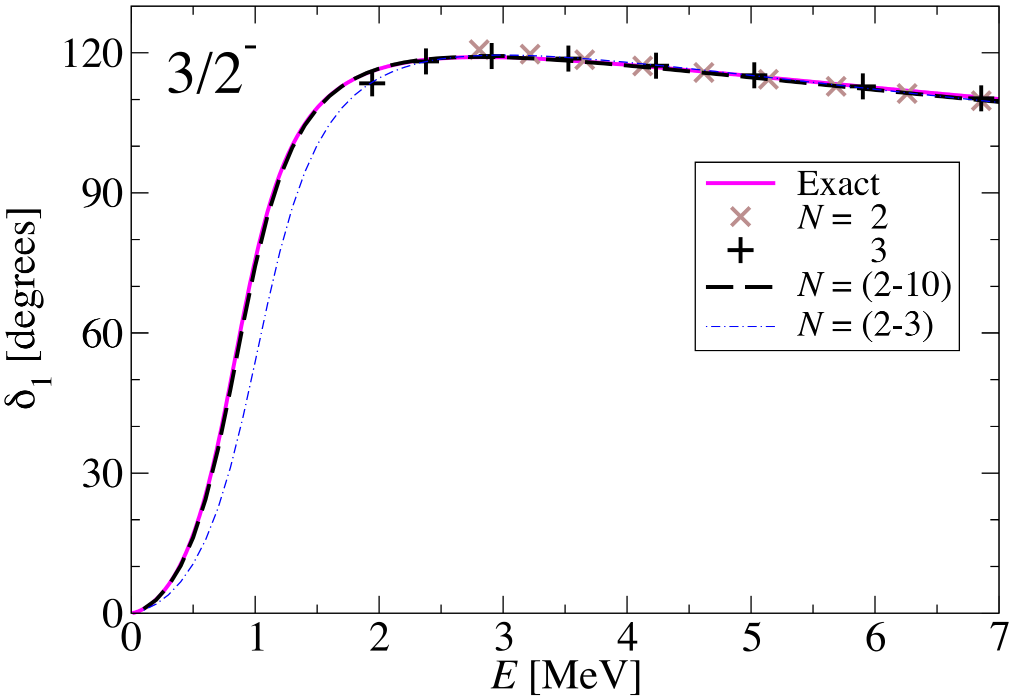
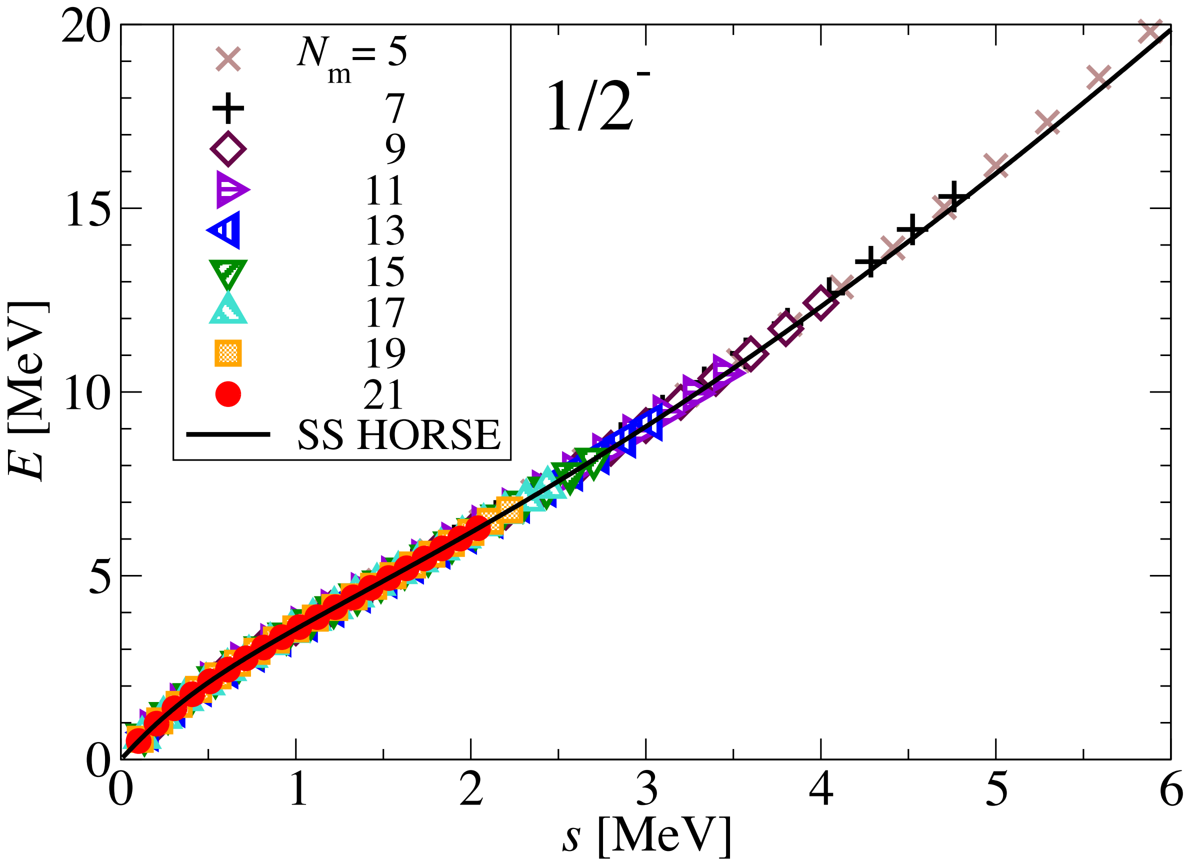


# 1/2⁻ resonance

**We calculated the energies of the lowest state $E_0$ by diagonalization Hamiltonian matrix in the oscillator basis with N = 2,3,4,…,10 and with values ħΩ from 2.5 MeV to 50 MeV (with step of 2.5 MeV).**

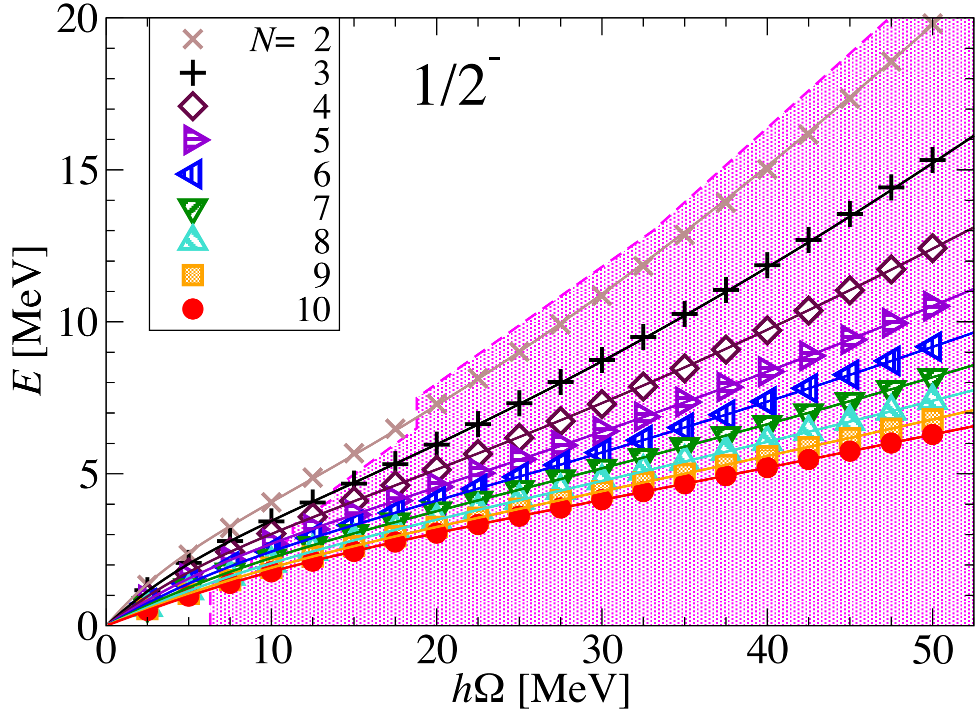
## $E_0$ vs scailing parameters s

**SS HORSE results form a smooth line.**



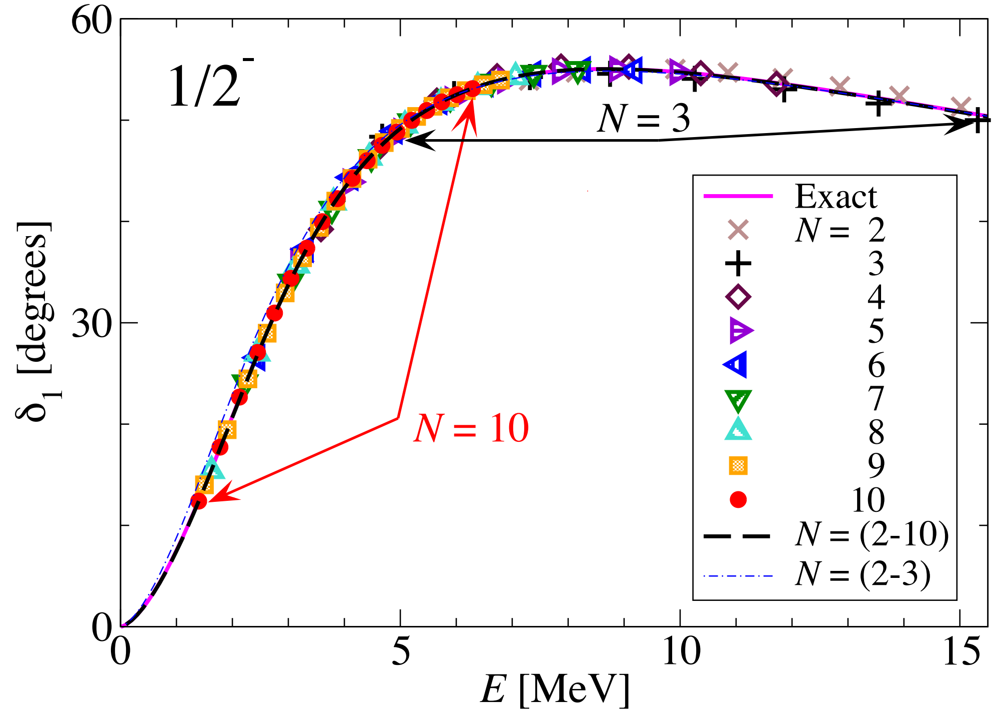

*Symbols correspond to variational calculations. Solid lines represent rezults of the fit.*

# 1/2⁻ resonance




**Selected data are in the shaded area**

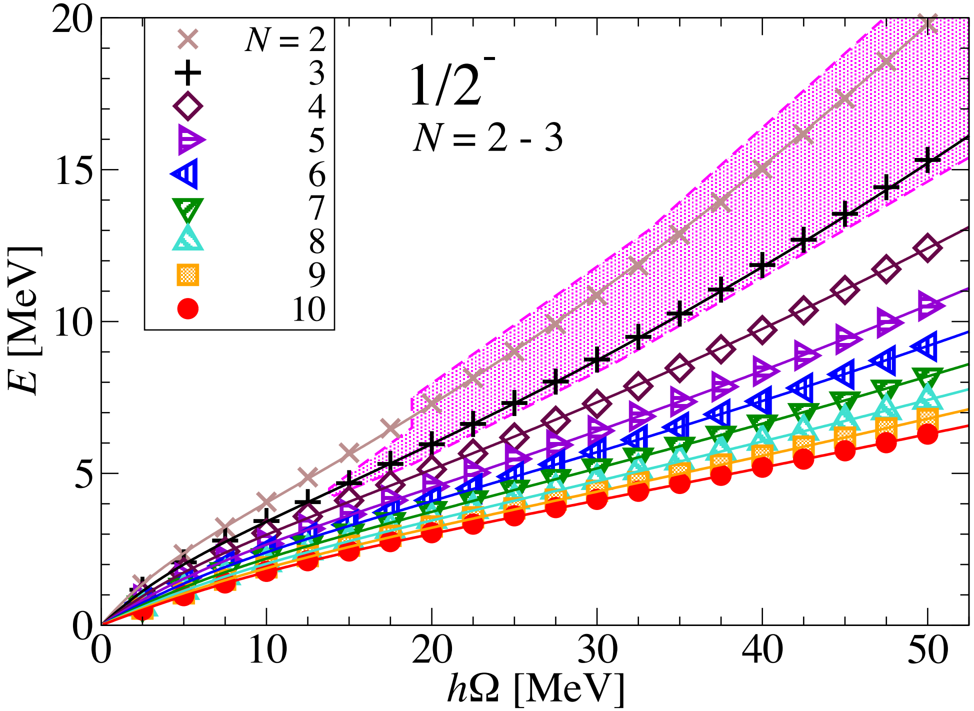
**Many SS HORSE results are out of the resonance region. Curves for exact phase shift and fitted phase shift are indistinguishable**




*Symbols correspond to SS HORSE calculations of the lowest energy $E_0$.*

*Solid lines represent rezults of the fit.*

# 1/2- resonance

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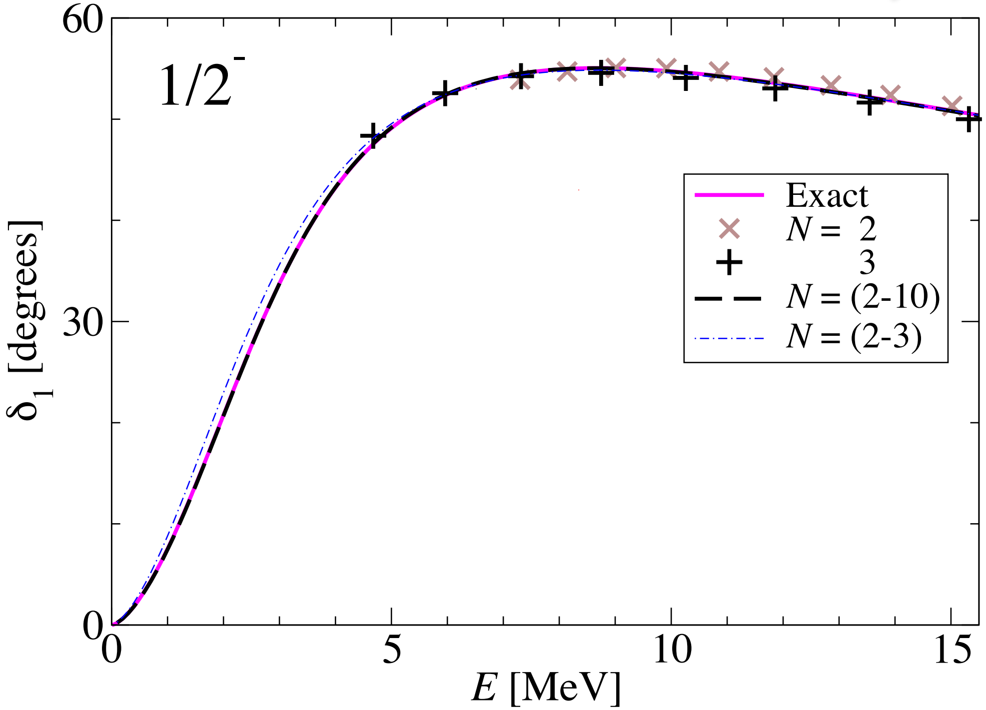



*Symbols correspond to SS HORSE calculations of the lowest energy $E_0$.*

*Solid lines represent rezults of the fit.*

# SS HORSE results

***3/2⁻ Resonance***

| | $E_r$ MeV | Γ MeV | $a_l$ fm$^3$ | $r_l$ fm$^{-1}$ | $w_1$ | $w_2$ | Ξ keV |
|---|---|---|---|---|---|---|---|
| N = (2-3) | 1.016 | 0.861 | -41.6 | -1.031 | $6.82\ 10^{-4}$ | $-2.13\ 10^{-5}$ | 121 |
| N = (2-4) | 0.888 | 0.787 | -53.6 | -0.881 | $4.63\ 10^{-4}$ | $-1.07\ 10^{-5}$ | 268 |
| N = (2-5) | 0.882 | 0.785 | -54.4 | -0.871 | $4.49\ 10^{-4}$ | $-0.98\ 10^{-5}$ | 232 |
| N = (2-10) | 0.848 | 0.777 | -59.6 | -0.806 | $3.50\ 10^{-4}$ | $-0.28\ 10^{-5}$ | 181 |
| exact | 0.837 | 0.780 | -61.7 | -0.777 | | | |

***NOTE:***

1. *fast convergence of the method;*
2. *even if we use a minimum set of data (case N=(2-3)) we get reasonable results.*

# SS HORSE results

## 1/2⁻ Resonance

| | $E_r$ MeV | Γ MeV | $a_l$ fm$^3$ | $r_l$ fm$^{-1}$ | $w_1$ | $w_2$ | Ξ keV |
|---|---|---|---|---|---|---|---|
| N = (2-3) | 1.37 | 5.38 | -20.0 | -0.126 | $3.56\ 10^{-3}$ | $4.88\ 10^{-5}$ | 121 |
| N = (2-4) | 1.75 | 5.53 | -15.5 | -0.362 | $4.71\ 10^{-4}$ | $1.82\ 10^{-5}$ | 268 |
| N = (2-5) | 1.72 | 5.56 | -15.8 | -0.330 | $4.57\ 10^{-4}$ | $2.45\ 10^{-5}$ | 232 |
| N = (2-10) | 1.69 | 5.56 | -16.1 | -0.309 | $4.46\ 10^{-4}$ | $2.81\ 10^{-5}$ | 121 |
| exact | 1.66 | 5.58 | -16.3 | -0.273 | | | |

# Model problem (charged particle)

**Scattering particle with reduced mass**

$$\mu = \tfrac{4}{5} m_{nucl}$$

**on the same Woods-Saxon potential**

$$V_{n\alpha} = \frac{V_0}{1+\exp\left[\left(r-R_0\right)/\alpha_0\right]} + \left(\vec{l}\cdot\vec{s}\right)\frac{1}{r}\frac{d}{dr}\frac{V_{ls}}{1+\exp\left[\left(r-R_1\right)/\alpha_1\right]}$$

$$V_0 = -43\ MeV,$$
$$V_{ls} = -40\ MeV\cdot fm^2,$$
$$R_0 = 2.0\ fm,\quad \alpha_0 = 0.70\ fm,$$
$$R_1 = 1.5\ fm,\quad \alpha_1 = 0.35\ fm$$

**with Coulomb potential of a uniformly charged sphere**

$$V^{Cl}(r) = \begin{cases} \dfrac{Z_1 Z_2 e^2}{r}, & if \quad r > R_1 \\[2ex] \dfrac{Z_1 Z_2 e^2}{2}\left(3 - \dfrac{r^2}{R_1^2}\right), & if \quad r \le R_1 \end{cases}$$

**well simulates pα scattering**

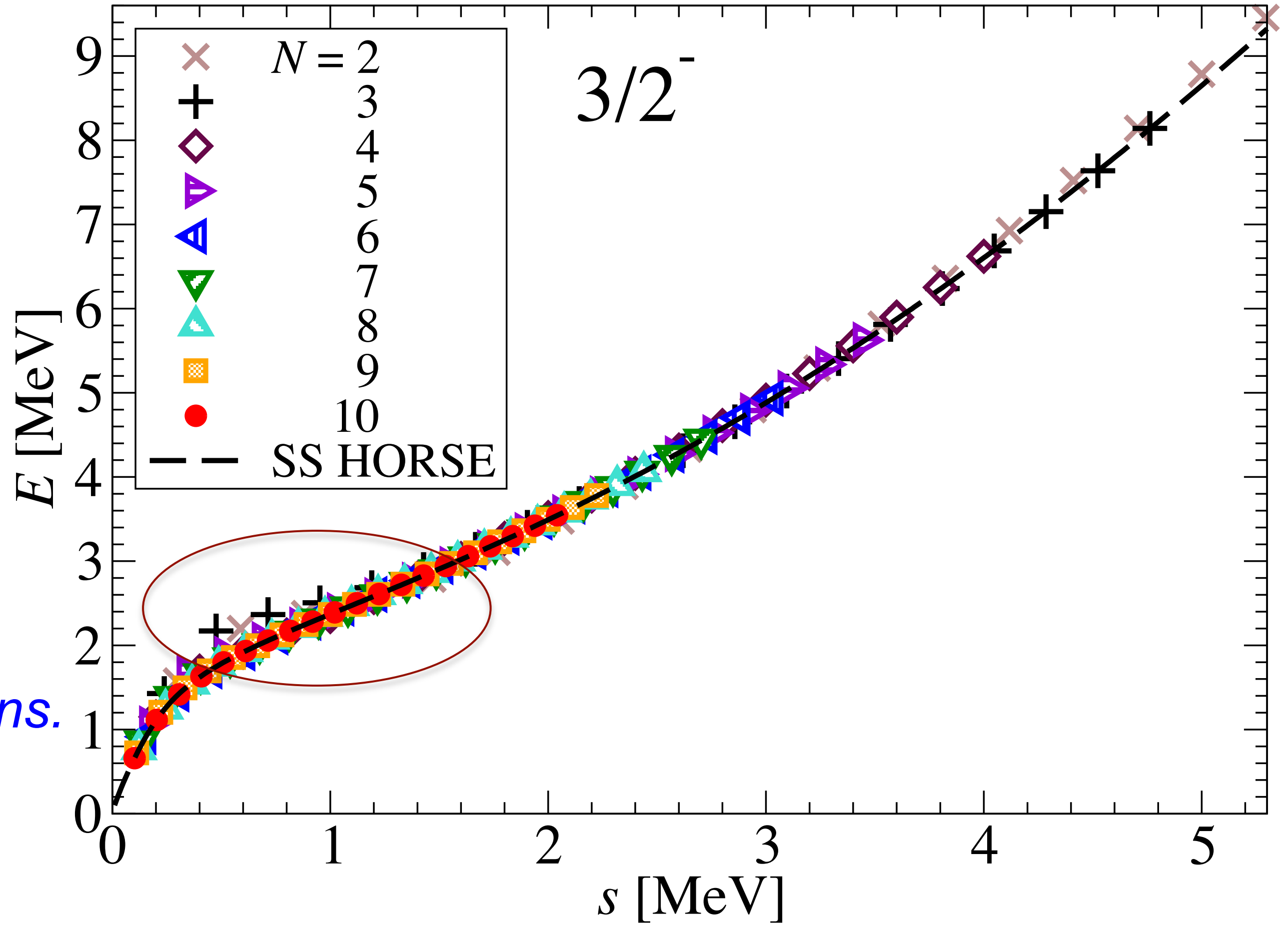
*There are two resonant states: broad 1/2⁻ and narrow 3/2⁻*

# Charged particle, 3/2⁻ resonance

***We calculated the energies of the lowest state $E_0$ by diagonalization Hamiltonian matrix in the oscillator basis with N = 2,3,4,…,10 and with values ħΩ from 2.5 MeV to 50 MeV (with step of 2.5 MeV).***

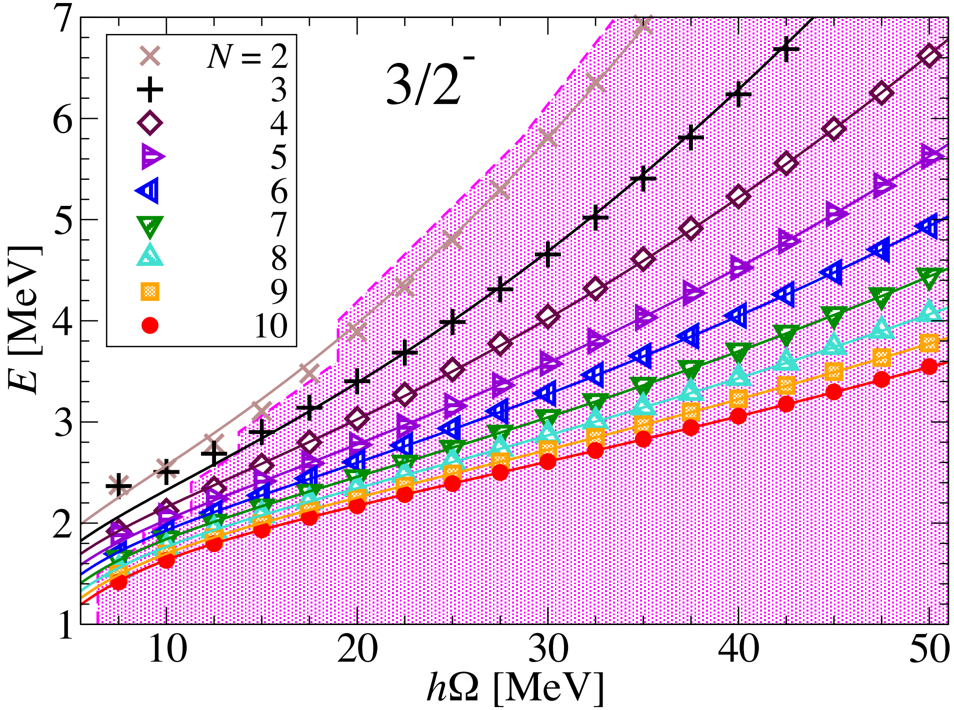
**SS HORSE results form a smooth line.**
**But some results (obtained in cases with small *N* at low energies (low *s*) significantly deviate from this smooth lines.**

***$E_0$ vs scailing parameters s***



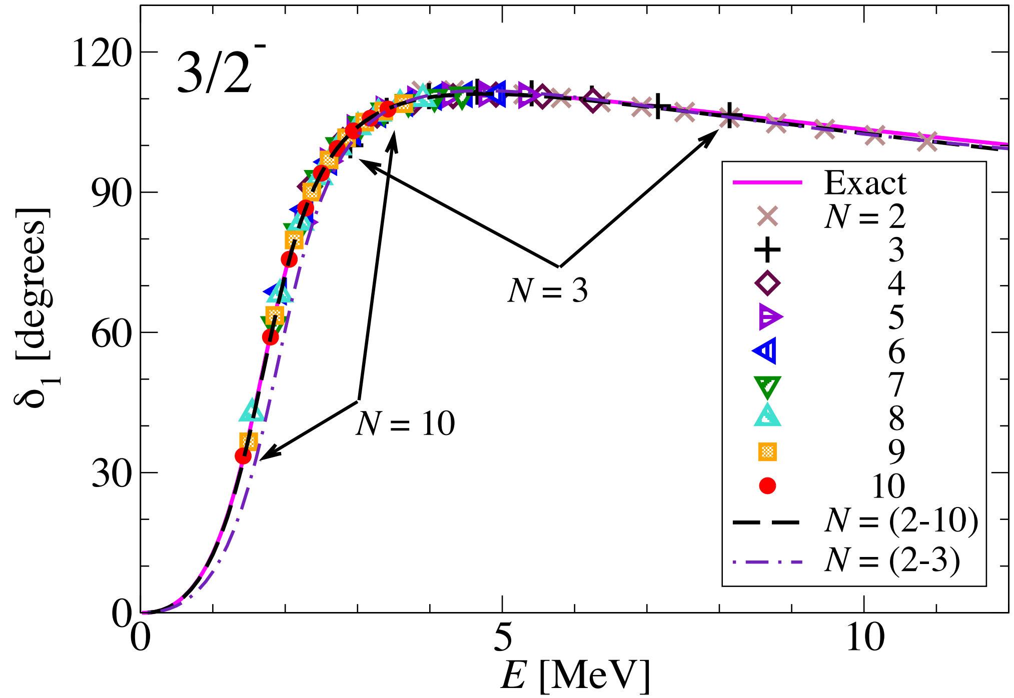

*Symbols correspond to SS HORSE calculations. Solid lines represent rezults of the fit.*

# 3/2- resonance




**Selected data are in the shaded area**

**Many SS HORSE results are out of the resonance region. Curves for exact phase shift and fitted phase shift are indistinguishable**




*Symbols correspond to SS HORSE calculations of the lowest energy $E_0$.*

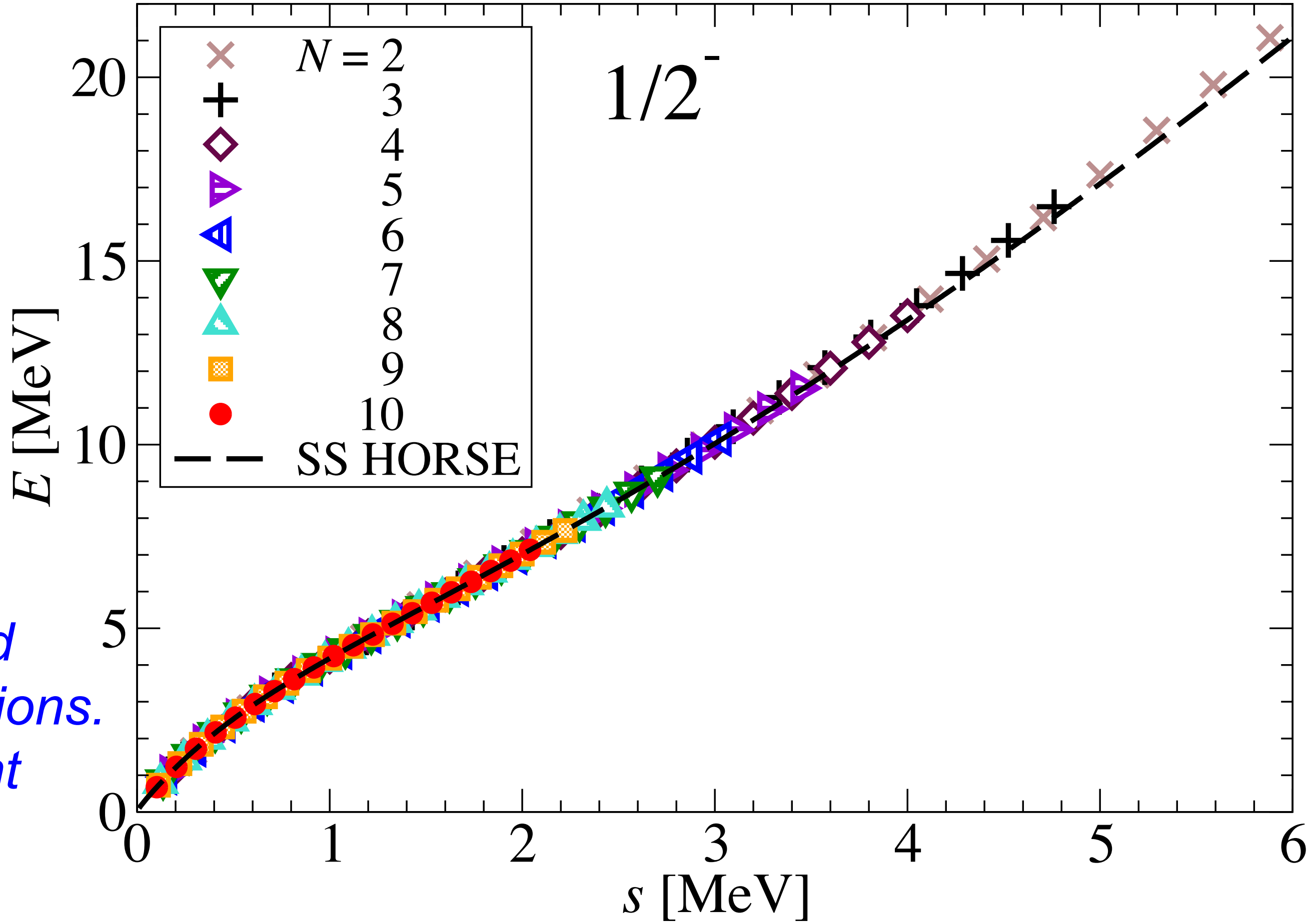
*Solid lines represent rezults of the fit.*

# Charged particle, 1/2⁻ resonance

**_We calculated the energies of the lowest state $E_0$ by diagonalization Hamiltonian matrix in the oscillator basis with N = 2,3,4,…,10 and with values ħΩ from 2.5 MeV to 50 MeV (with step of 2.5 MeV)._**

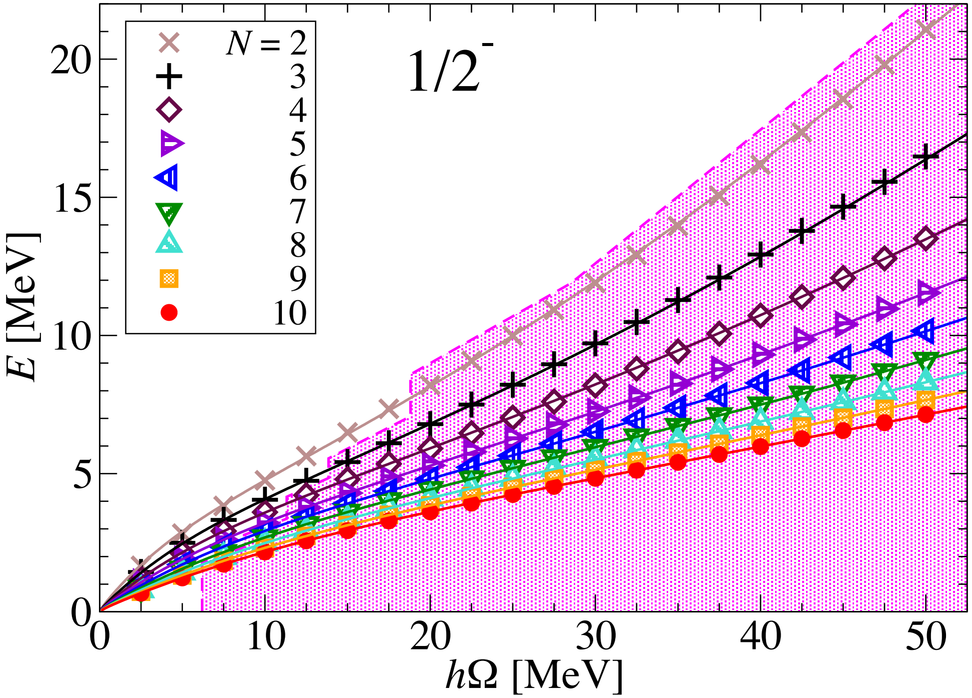
## $E_0$ vs scaling parameters s

**SS HORSE results form a smooth line.**



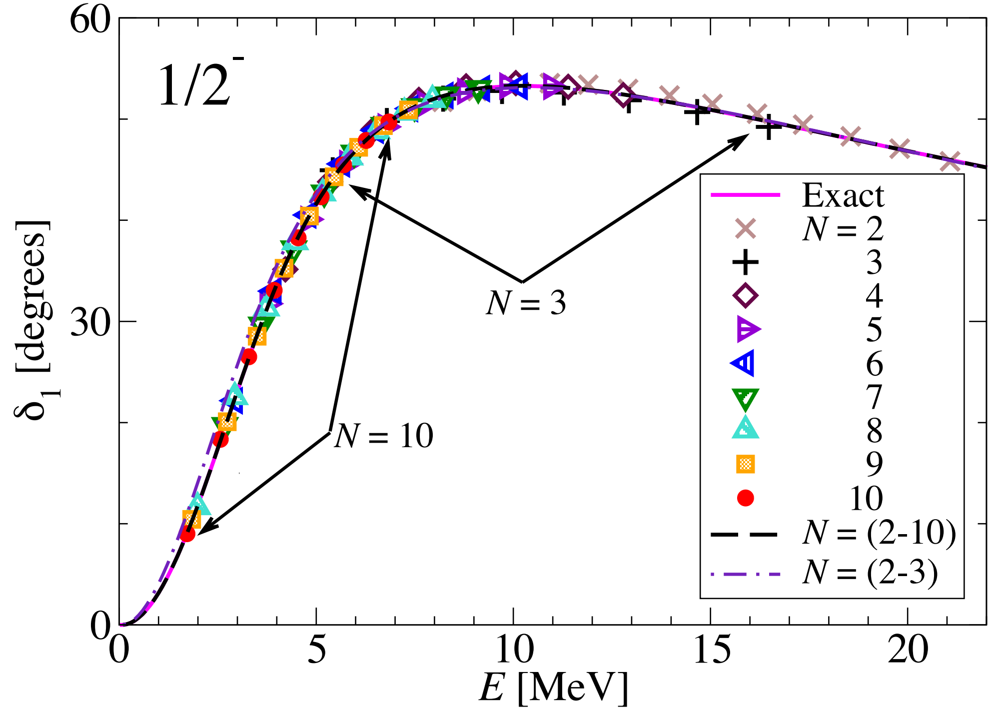

*Symbols correspond to SS HORSE calculations. Solid lines represent rezults of the the fit.*

# 3/2- resonance




**Selected data are in the shaded area**

**Many SS HORSE results are out of the resonance region. Curves for exact phase shift and fitted phase shift are indistinguishable**




*Symbols correspond to SS HORSE calculations of the lowest energy $E_0$.*

*Solid lines represent rezults of the fit.*

# SS HORSE results (charged particles)

| | $E_r$, MeV | $\Gamma$, MeV | $w_1$, $\text{fm}^{-3}\text{MeV}^{-1}$ | $w_2$, $\text{fm}^{-3}\text{MeV}^{-2}$ | $\Xi$ keV |
|---|---|---|---|---|---|
| Resonance $3/2^-$ | | | | | |
| $N=(2-3)$ | 1.89 | 1.46 | $9.88\cdot10^{-4}$ | $-2.54\cdot10^{-5}$ | 105 |
| $N=(2-4)$ | 1.74 | 1.42 | $7.26\cdot10^{-4}$ | $-1.15\cdot10^{-5}$ | 208 |
| $N=(2-5)$ | 1.73 | 1.42 | $6.88\cdot10^{-4}$ | $-9.11\cdot10^{-6}$ | 182 |
| $N=(2-10)$ | 1.68 | 1.43 | $5.60\cdot10^{-4}$ | $4.80\cdot10^{-7}$ | 134 |
| exact | **1.67** | **1.43** | | | |
| Resonance $1/2^-$ | | | | | |
| $N=(2-3)$ | 1.96 | 6.48 | $3.70\cdot10^{-3}$ | $5.95\cdot10^{-5}$ | 168 |
| $N=(2-4)$ | 2.47 | 6.47 | $4.90\cdot10^{-3}$ | $2.40\cdot10^{-5}$ | 148 |
| $N=(2-5)$ | 2.45 | 6.52 | $4.82\cdot10^{-3}$ | $2.89\cdot10^{-5}$ | 131 |
| $N=(2-10)$ | 2.43 | 6.53 | $4.73\cdot10^{-3}$ | $3.22\cdot10^{-5}$ | 85 |
| exact | | | | | |

***NOTE:***

***1. fast convergence of the method;***

***2. even if we use a minimum set of data (case N=(2-3)) we get reasonable results.***

# Summary

- SM states obtained at energies above thresholds can be interpreted and understood.
- Parameters of low-energy resonances (resonant energy and width) and low-energy phase shifts can be extracted from results of conventional Shell Model calculations


# *Thank you!*