Energy and Width of the Resonance in the 4n System

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Motivation

* **K. Kisamori** ***et al*****, Phys. Rev. Lett. 116, 052501 (2016)**
  proposed 4n resonance in ^{4}He + ^{8}He collisions
  $E_r = 0.83 \pm 0.65$ (stat) ± 1.25 (syst) MeV
  (above 4n disintegration threshold)
  $\Gamma \leq 2.6$ (MeV)
  Pure statistics: only 6 events

* Marques experiments of bound 4n in reaction
  ^{14}Be $\rightarrow$ ^{10}Be + 4n not confirmed

* There are no theoretical predictions 4n resonance in low-energy (about few MeV)

* We try to find 4n resonance using SS-HORSE method

SS-HORSE

- Phase shift $$\tan \delta_\ell(E_\nu) = -\frac{S_{\mathbb{N}+2,\ell}(E_\nu)}{C_{\mathbb{N}+2,\ell}(E_\nu)}$$

$$S_{\mathbb{N}+2,\ell}(q) = \sqrt{\frac{\pi n!}{\Gamma\left(n+\ell+\frac{3}{2}\right)}} q^{\ell+1} \exp\left(-\frac{q^2}{2}\right) L_n^{\ell+\frac{1}{2}}(q^2)$$

$$n = \frac{\mathbb{N}+2-\ell}{2}$$

$$C_{\mathbb{N}+2,\ell}(q) = (-1)^n \sqrt{\frac{\pi n!}{\Gamma\left(n+\ell+\frac{3}{2}\right)}} \frac{q^{-\ell}}{\Gamma\left(-\ell+\frac{1}{2}\right)} \exp\left(-\frac{q^2}{2}\right) {}_1F_1\left(-n-\ell-\frac{1}{2}, -\ell+\frac{1}{2}; q^2\right)$$

$$E = \frac{q^2}{2}\hbar\Omega$$

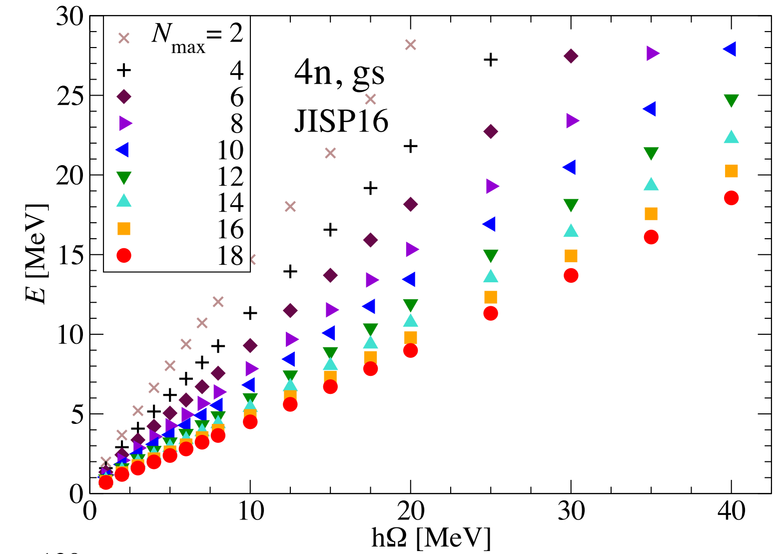
- E_ν is Shell Model Hamiltonian's eigenvalue with given quantum numbers of channel with $\mathbb{N}$ and $\hbar\Omega$ parameters
- By varying $\hbar\Omega$ and $\mathbb{N}$ we may obtain E_ν and δ_ℓ in some interval
- Scaling: phase shifts of relativity small energies depends from $s = \frac{\hbar\Omega}{\mathbb{N}+7/2}$
- In other hand, δ_ℓ can be parameterized in cases of resonance, bound, virtual, false states and it's various combinations
- Method gives reasonable results in $n\alpha$ elastic scattering with NCSM calculations ^{5}He and ^{4}He with JISP16 interaction, see ***arXiv:1608.05885 [nucl-th]***

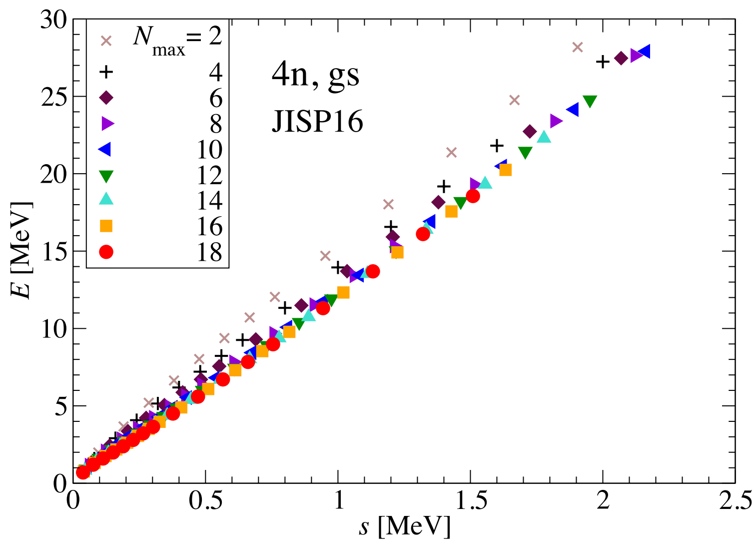
Generalization SS-HORSE for democratic 4n scattering

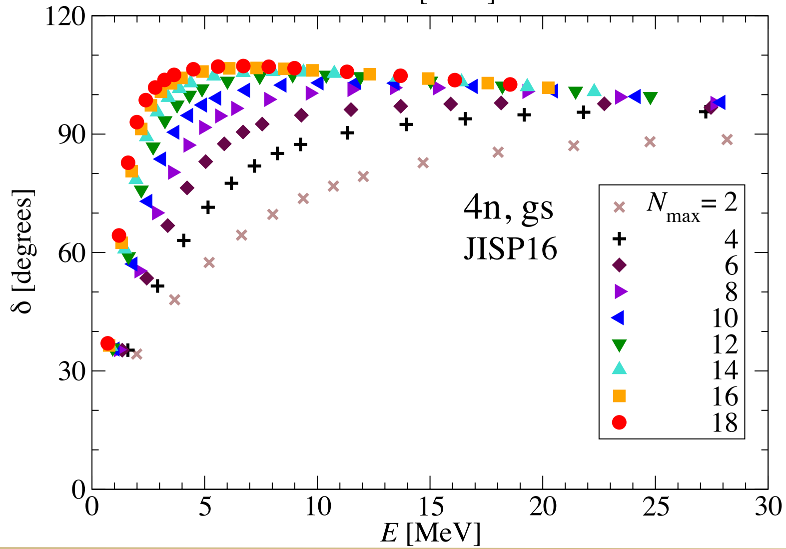
- Hyperspherical harmonic oscillator representation proposed in
  S. A. Zaitsev, Yu. F. Smirnov, A. M. Shirokov.
  Theor. Math. Phys. Vol. 117, Issue 2, pp 1291–1307 (1998)
- $\ell \to \mathcal{L} = K + \frac{3A-6}{2} = K + 3$
- K is hypermomentum, $K = K_{\min}, K_{\min}+2, \ldots$
- Partial waves with $K_{\min}+2, K_{\min}+4, \ldots$
  suppressed by large centrifugal barrier $\mathcal{L}(\mathcal{L}+1)/\rho^2$, $\rho^2 = \sum_{i=1}^{4}(\mathbf{r}_i - \mathbf{R})^2$
- $K = 2$
- 2 neutrons on s-shell, 2 neutrons on p-shell:
  $\mathbb{N} = N_0 + N_{\max} = N_{\max} + 2$, $N_{\max}$ is max excitation quanta

4n scattering with JISP16

- NCSM: lowest eigenvalues E_0, with $J = 0$
- E_0 (in continuum) increase as function of $\hbar\Omega$ for each $N_{\rm max}$
- Scaling par. $s = \hbar\Omega/(N_{\rm max} + 17/2)$
- $\tan\delta(E_0) = -\dfrac{S_{N_{\rm max}+4,5}(E_0)}{C_{N_{\rm max}+4,5}(E_0)}$

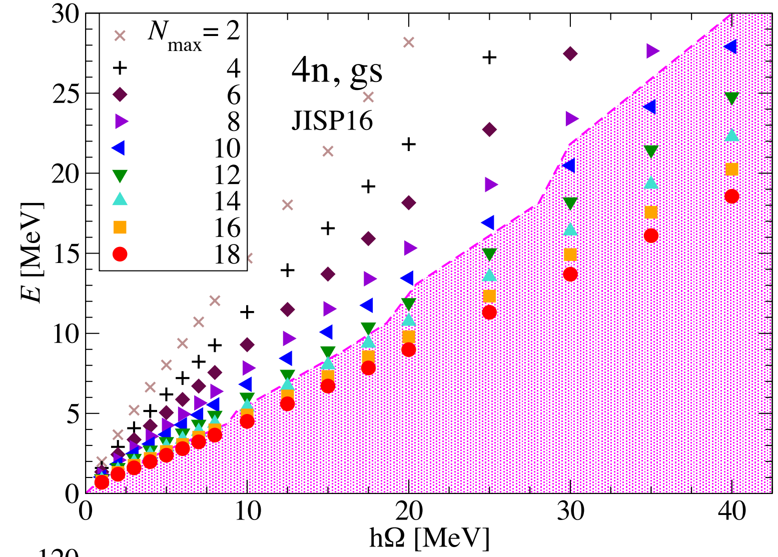


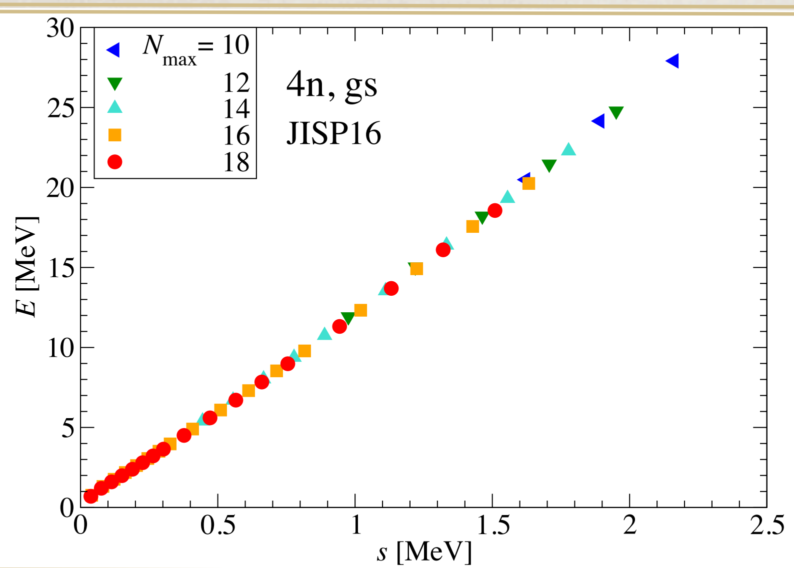


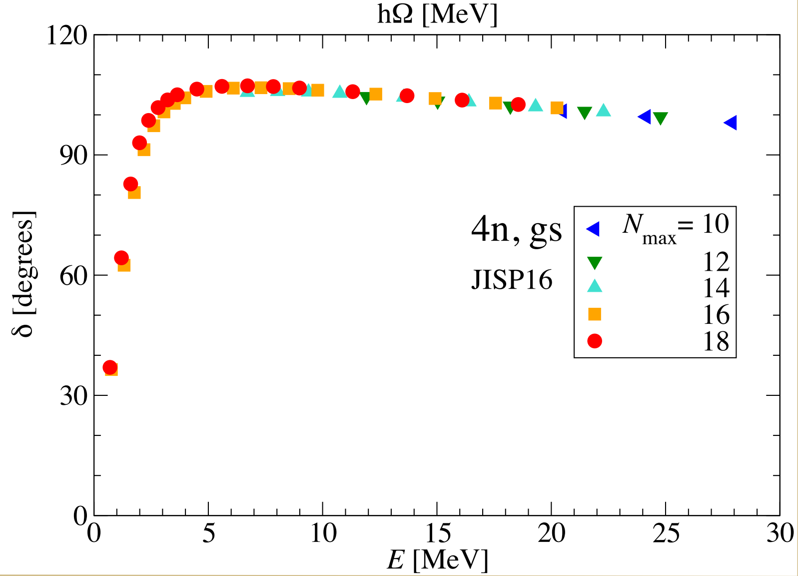


4n scattering with JISP16

* Scaling par. $s = \dfrac{\hbar\Omega}{N_{\max} + 17/2}$
* $\tan\delta(E_0) = -\dfrac{S_{N_{\max}+4,5}(E_0)}{C_{N_{\max}+4,5}(E_0)}$
* Selection criterion: forming smooth curves





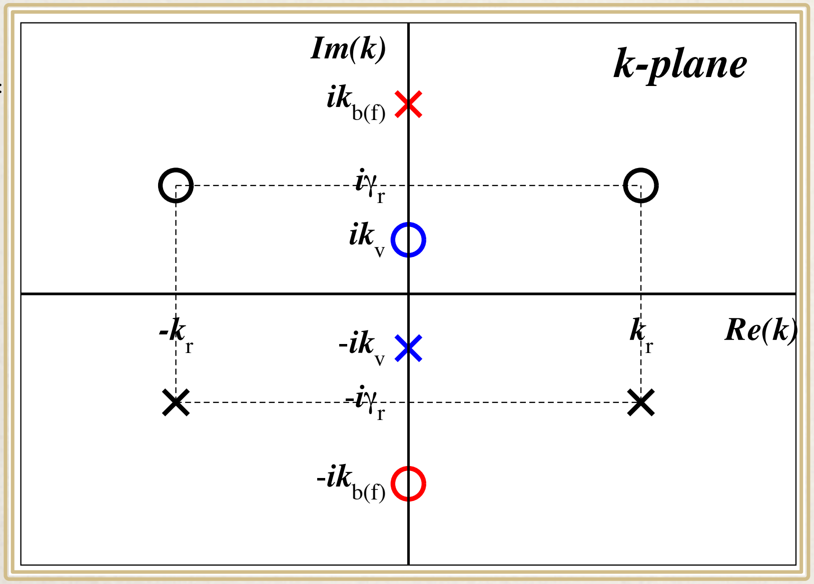


Phase shift parameterization

- Symmetry:
  $S(-k) = 1/S(k), \quad S(k^*) = [S(k)]^*$
- $S(k) = e^{2i\delta(k)}, \quad \delta(k) = -\delta(-k)$
- S-matrix expression:

$$S(k) = \Theta(k) S_p(k) \qquad \delta = \phi + \delta_p$$

background (→ $\Theta(k)$) pole (→ $S_p(k)$)



- Types of poles

Bound and false	Virtual	Resonance
$S_{b(f)} = \dfrac{k_{b(f)} - ik}{k_{b(f)} + ik}$	$S_v = \dfrac{k_v + ik}{k_v - ik}$	$S_r = \dfrac{(k - \kappa_r^*)(k + \kappa_r)}{(k - \kappa_r)(k + \kappa_r^*)}, \kappa_r = k_r - i\gamma_r$

Phase shift parameterization

- Parameterization $\delta = \delta_r + \phi$
- Resonance $\delta_r = -\arctan \frac{a\sqrt{E}}{E - b^2}$

$$E_r = b^2 - a^2/2, \quad \Gamma = a\sqrt{4b^2 - a^2}$$

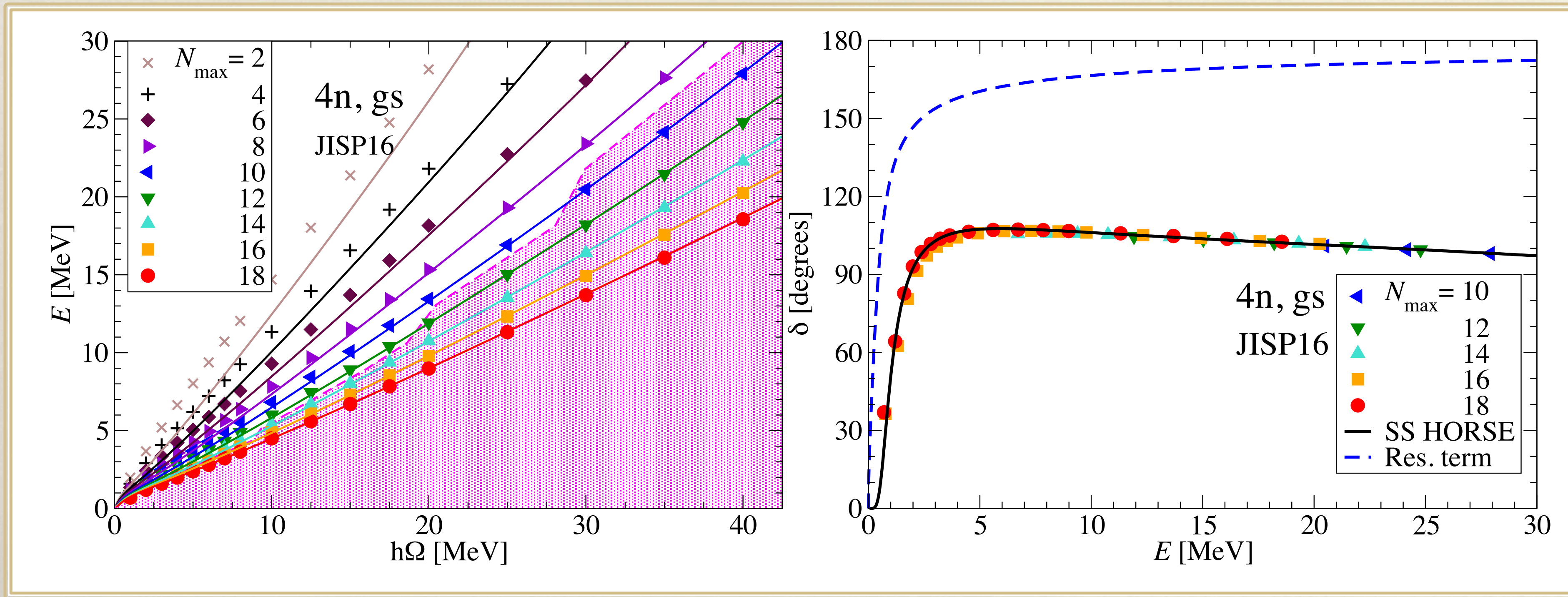
- Low energy limit $\delta \sim k^{2\mathcal{L}+1} \sim \left(\sqrt{E}\right)^{11}$
- Background $\phi = \frac{w_1\sqrt{E} + w_3\left(\sqrt{E}\right)^3 + c\left(\sqrt{E}\right)^5}{1 + w_2 E + w_4 E^2 + w_6 E^3 + dE^4}$

algebraic expressions of $w_i, \; i = 1, 2, 3, 4, 6$ provide right low energy limit

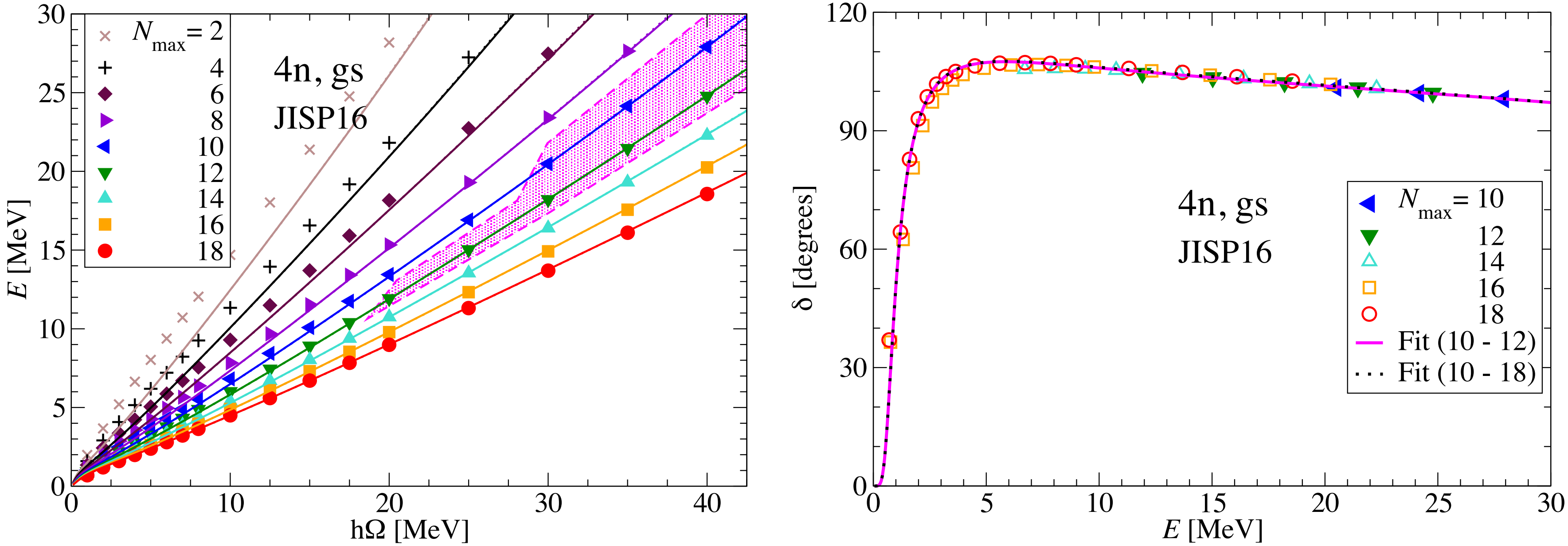
4n scattering with JISP16

$$\tan\left(\delta_r(E) + \phi(E)\right) = -\frac{S_{N_{\max}+4,5}(E)}{C_{N_{\max}+4,5}(E)}$$

Pole position		
E_r, MeV	Γ , MeV	δ inflection (MeV)
0.186	0.815	0.8



Convergence of results



Good convergence due to right limit to low energy

Assumption of false pole

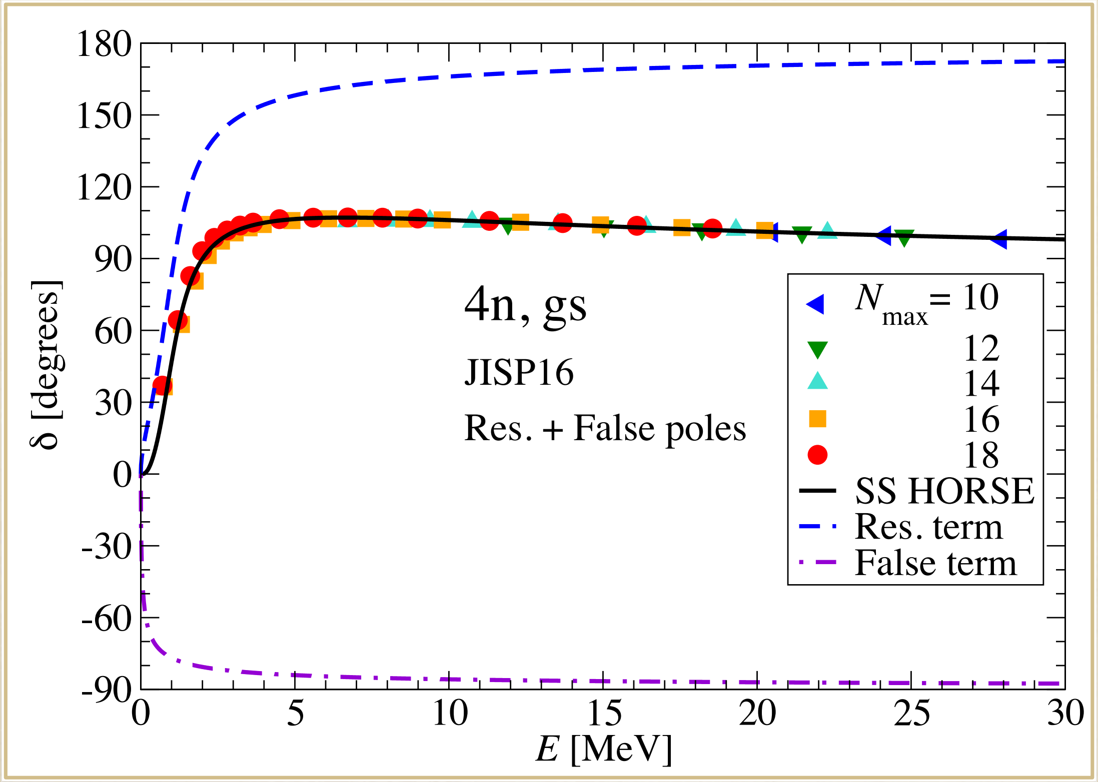
* False pole provides phase shift:

$$\delta_f = -\arctan\sqrt{\frac{E}{E_f}}$$

* Phase shift parameterization

$$\delta = \delta_r + \delta_f + \phi$$

E_r,MeV	Γ ,MeV	E_f ,keV
0.84	1.38	-54.9



Conclusions

* SS-HORSE method extended to democratic non-Coulomb scattering
* We found resonance in 4n system using SS-HORSE method with JISP16 *NN* interaction and calculate phase shift
* We propose resonance in 4n system with $E_r = 0.8$ MeV and $\Gamma = 1.4$ MeV

Thank you for attantion!