DEUTERON WAVE FUNCTION AND ELASTIC eD SCATTERING

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Introduction

Being the simplest nucleus, deuteron provides the most direct test of the various NN interaction models and the relevant degrees of freedom. In this context, the deuteron electromagnetic studies through electron or photon probes are the simplest in theoretical and experimental aspects. These studies provide picture of deuteron electromagnetic structure in terms of deuteron electromagnetic (EM) form factors (FFs). These FFs depend on the square of the 4-momentum transferred by a probe ($q^2 = -Q^2$).

During the last two decades a considerable advance has been made in the experimental knowledge of deuteron electromagnetic structure. On the other hand there is a substantial diversity of opinion regarding an appropriate theoretical general approach. Though, it seems natural as well as confirmed by the general data analysis [1] that in the space-like region of Q (corresponding to the elastic scattering) a successful theory may be obtained from a relativistic description of only the NN channel together with minor modifications of the short-range structure of the deuteron electromagnetic current (EMC) operator. Here again, there are different approaches concerning the relativistic description as well as the EMC operator structure [1,2].

It is customary to assume that most of the existing data of *eD* elastic scattering are described to high precision in the one-photon exchange approximation and by three electromagnetic deuteron FFs [1, 3-6]:

$$\frac{d\sigma}{d\Omega} \sim \left[A(Q^2) + B(Q^2) \tan^2 \frac{\theta}{2} \right] \cdot \left[1 + \sum \rho_{ij} t_{ij} \right]$$

$$A(Q^2) = G_C^2(Q^2) + \frac{2}{3}\eta G_M^2(Q^2) + \frac{8}{9}\eta^2 G_Q^2(Q^2), \qquad B(Q^2) = \frac{4}{3}\eta (1+\eta) G_M^2(Q^2)$$

$$t_{ij} = t_{ij}(Q^2, \theta) = \text{Bilinear forms of } G_C(Q^2), G_M(Q^2), G_Q(Q^2) \text{ and } \eta = Q^2/4M_D^2$$

The unpolarized differential cross section is

$$\frac{d\sigma}{d\Omega}\Big|_{NP} = \frac{\sigma_M}{1 + 2E\sin^2(\theta/2)/M_D} \left[A(Q^2) + B(Q^2)\tan^2\frac{\theta}{2} \right]$$

A separation of the form factors requires the measurement of A, B and at least one tensor polarization observable t_{ij} .

The one-photon exchange approximation assumes that the electron and deuteron exchange a single virtual photon. It is believed that for the most part this approximation must be valid to high precision because of the small value of the fine-structure constant. Therefore the elastic eD scattering allows to extract the deuteron EM FFs dependencies on the transferred 4-momentum Q in the space-like region. To extract these dependencies it is required to measure three independent observables of the eD elastic scattering in the region. Two of them (structure functions A and B) are extracted from the unpolarized differential cross section, and third one is extracted from polarization measurements.

The deuteron FFs are calculated from the deuteron wave function (S and D components) and from the nucleon FFs in the conventional model. Deuteron FFs may be chosen so as to be equal at Q =0 (static) limit to the deuteron charge, magnetic and quadruple momenta. The first two are described by the conventional nuclear model with only nucleon degrees of freedom. But deuteron static electric quadrupole moment is not reproduced well enough by the modern NN potential calculations. It is generally agreed that the meson exchange contributions must be taken into account for agreement with data.

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Rel/nonrel	$G_M(0) = \frac{M_D}{m_p} \mu_D$	$G_Q(0) = M_D^2 Q_D$
Exp	1.7148	25.83
NijmI	1.697/1.695	24.8/24.6
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JISP16	1.720/1.714	26.3/26.1
Moscow06	1.711/1.699	24.5/24.2

At low $Q \leq 4Fm^{-1}$ simple non rel calculations with realistic NN potentials (with NN channel only) agree well enough with one another and with the *eD* elastic scattering data. With rise of $Q > 4Fm^{-1}$ disagreement increases between various calculations and data.



This disagreement indicates that relativistic effects and effects of other channels may be essential at $Q > 4 Fm^{-1}$. It is obvious that relativistic corrections are requisite because 5 Fm^{-1} is approximately the nucleon mass. But what about other channels?

Non NN channels in eD

• This means virtual mesons and isobars usually and their effects are taken into account by additive corrections to the NN EM current operator:

$$J^{\nu} = J^{\nu}_{SA} + \Delta J^{\nu} = \left(J^{\nu}_{p} + J^{\nu}_{n}\right) + \Delta J^{\nu}$$

We do not discuss quark effects here because they can not be accounted for realistically in this region of nonpertubative QCD.

The equation above means that virtual non *NN* channels are not included into the deuteron wave function.

Let's see now wave functions used in calculations.

The deuteron WFs in Configuration Space (r)



The deuteron WFs in Momentum Space (q)



All potentials are realistic meaning that they describe NN Partial Wave Analysis data and static deuteron properties to an extent.

Some of the potentials are meson exchange potentials. We see that all WFs are quite different from about q=2 /Fm. That means they require different exchange currents to describe eD data. Really there are procedures to extract exchange current operators for any realistic potential.

NN EM current operator – spectator approximation

$$J_{SA}^{\nu} = J_{p}^{\nu} + J_{n}^{\nu}$$

Each of the current operators may be written in the corresponding Breit system as

$$\begin{split} J_n^0 \Big|_{Breit} &\sim G_{en} \left(Q_n^2 \right), \qquad J_p^0 \Big|_{Breit} \sim G_{ep} \left(Q_p^2 \right) \\ \vec{J}_n \Big|_{Breit} &\sim G_{mn} \left(Q_n^2 \right) [\vec{q}_n \times \vec{s}_n], \\ &\qquad \qquad \vec{J}_p \Big|_{Breit} \sim G_{mp} \left(Q_p^2 \right) [\vec{q}_p \times \vec{s}_p]. \end{split}$$

You must note that Breit system for neutron, one for proton and one for deuteron are all different. All operators must be transformed properly to the system where a calculation is performed.

Nucleon EM FFs

We see that the important ingredients of the calculations are the nucleons FFs dependencies on the Q transferred to the individual nucleon. These dependencies are extracted experimentally.

Neutron and proton are nucleons and they are described as electromagnetic particles similarly, but our knowledge of their electromagnetic structure is not at similar level.



The proton FFs

are extracted from direct measurements with proton target.

Rosenbluth separation from ep elastic cs at different scat angles: Large uncertainties at large Q for Ge and at small Q for Gm. Akhiezer[1968]: Recoil polarization to improve the *FF accuracy*.

Nevertheless even in this case there is a notable discrepancy between the extracted values of the proton FFs ratio Gep/Gmp from polarization and those obtained from cross section experiments. The cross sections are necessary to extract the absolute value of *Gep* and *Gmp*, while polarization transfer measurements give only ration *Gep/Gmp*. The discrepancy begins at $Q = 5 Fm^{(-1)}$. It may be explained by the hard two-photon exchange process (TPE) and data show some evidences of the explanation.

It also should be noted that model calculations and analyses show that TPE significantly change values of *Gep*, while *Gmp* changes at the few percent level (3%). The latest analytical fit for the proton FFs is a simultaneous fit of the polarization and cross section data. The cross section data are corrected by an additive term assuming some phenomenological expressions of the TPE correction.

How the neutron FFs are measured

A free neutron target does not exist. The neutron FFs are extracted from measurements of eD or $e^{3}He$ scattering. Therefore, the data analysis is affected by uncertainties stemming from the nuclear theoretical model assumed to describe the target nucleus and the used reaction.

Extraction of Gen – procedure of Riordan et al,

Phys. Rev. Lett., 2010

For example in [13] the electric FF of the neuteron was measured up to $Q^2 = 3.4 \ GeV^2/c^2$ ($Q \approx$ 9 Fm^{-1}) using ${}^{3}\vec{He}(\vec{e},e'n)pp$ reaction. Details of extraction include calculations of the asymmetries in the quasi-elastic processes ${}^{3}\vec{He}(\vec{e},e'n)pp$ and ${}^{3}\vec{He}(\vec{e},e'p)np$. These calculations were performed using the generalized eikonal approximation (GEA), and included the spindependent final-state interactions and meson-exchange currents, and used the ${}^{3}He$ wave function that results from the AV18 potential. Finally, to extract G_{En} the linearly interpolated values of G_{Mn} from [14] are used.

Extraction of Gmn – procedure of Lachniet et al,

Phys. Rev. Lett., 2009

Procedure of [14] is a measurement of the ratio R of the cross sections for the ${}^{2}H(e,en)p$ and ${}^{2}H(e,ep)n$ reactions in quasielastic scattering on deuterium. To extract G_{Mn} from the R they use:

1) The cross section calculation using

the Plane Wave Impulse Approximation (PWIA) for Q² > 1.0 GeV²/c², the AV18 deuteron wave function, and Glauber theory for final-state interactions (FSI);
2) Calculation of the nuclear correction factor being the ratio of the full calculation to the PWIA without FSI;
3) The proton FFs (parameterization of [11]);
4) And finally parameterizations of G_{En} [15, 16].

Refs on neutron extaraction

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Gen Data Analysis by Platchkov et al Nucl. Phys. A **508**, 343 (1990).



Comparison of the exp data for n and p FFs



The direct (for p) and indirect (for n) extractions give error bars of the same magnitude. It seems that error bars for n FF are underestimated. Probably systematic errors associated with NN potential used in analyses are not considered properly. Data are from review by Punjabi eta la, 2015; Lanchniet, 2009...

The objectives of the investigation

- 1. Different NN potentials correspond to different exchange current operators. The question is: Can the eD elastic scattering data and static D properties be explained without these currents?
- 2. Can we extract corresponding deuteron wave function and neutron FFs from the data?

From the microscopic view all realistic NN potentials are some projections into the NN channels and their extraction is not unambiguous.

We may assume that there is some "quasiunitary" transformation that minimizes matrix elements of the meson exchange currents for deuteron states.

We will take the necessary relativistic corrections.

RQM of systems with a fixed number of particles

- Direct generalization of the non-RQM.
- Invariance Poincare group (PG) instead of Galileo one. The WF of a system must transform according to a UR of the PG in some Hilbert space.
- Cluster separability (in Sokolov packing operators method). It means that symmetries and conservation laws that hold for a system of particles also hold for isolated subsystems.
- Direct interaction. No antiparticles and intermediate particles. But theory directly may be generalized to Lee model (Fuda[1990]) and to a quantum field theory (Fubini[1973].

Forms of RQM

• Where to insert interaction into the PG generators? Must it come into all of them?

$$[P^{\mu}, P^{\nu}] = 0, \qquad [M^{\mu\nu}, P^{\rho}] = -i(g^{\mu\rho}P^{\nu} - g^{\nu\rho}P^{\mu}),$$

$$[M^{\mu\nu}, M^{\rho\sigma}] = -i(g^{\mu\rho}M^{\nu\sigma} + g^{\nu\sigma}M^{\mu\rho} - g^{\mu\sigma}M^{\nu\rho} - g^{\nu\rho}M^{\mu\sigma})$$

Dirac[1949]: There are simpler ways. Kinematic generators – subgroup of Poincare group Dynamical ones – Hamiltonians • Instant Form, kinematic subgroup is:

$$\vec{P}, \quad \vec{M} = (M^{23}, M^{31}, M^{12})$$

Four Hamiltonians: Energy and Boosts

• Light Front, kinematic subgroup is:

 $P^{+} = \left(P^{0} + P^{Z}\right) / \sqrt{2}, P^{j}, M^{12}, M^{+-}, M^{+j}, j = 1, 2(x, y)$ Three Hamiltonians: P^{-}, M^{-j}

 Point Form, kinematic subgroup is: The Lorenz Group Four Hamiltonians: 4-momentum

All are unitary equivalent, Sokolov[1975,1978]:

by some unitary transformation a subgroup of PG may be made free of interaction

Point Form of RQM stands out because of

- It seems to be a simplest generalization of non-RQM: Hamiltonians are the 4-momenta operators, commuting. And states considered usually are their eigenstates.
- LG generators a free of interaction:
- 1. Spins and Orbital momenta are summed as in the non-RQM
- 2. Spectator approximation for transition operators is Lorenz covariant.
- 3. It becomes the non-RQM in the non-Rel limit without additional conditions.

How to insert interaction?

• Bakamjian-Thomas (BT) procedure [1953](two body) in point form: $M = M_0 + V$

$$P = MG, \qquad \mathbf{M} = \mathbf{l}(\mathbf{G}) + \mathbf{S}, \qquad \mathbf{N} = -iG^0 \frac{\partial}{\partial \mathbf{G}} + \frac{\mathbf{S} \times \mathbf{G}}{1 + G^0}$$
$$\mathbf{l}(\mathbf{G}) = \mathbf{i} \mathbf{G} \times \frac{\partial}{\partial \mathbf{G}}$$

Sokolov (packing operators) [1978] (many body)

Simplest way – direct interaction

$$M |\chi\rangle_{\boldsymbol{i}} \equiv \left[\sqrt{\hat{\mathbf{q}}^2 + m_1^2} + \sqrt{\hat{\mathbf{q}}^2 + m_2^2} + V \right] |\chi\rangle_{\boldsymbol{i}} = M_{\boldsymbol{i}} |\chi\rangle,$$

$$\begin{bmatrix} \hat{\mathbf{q}}^2 + m \hat{V} \end{bmatrix} |\chi\rangle_{\mathbf{i}} = q^2 |\chi\rangle_{\mathbf{i}}$$
$$q^2 = \frac{M_{\mathbf{i}}^2}{4} - \frac{m_1^2 + m_2^2}{2} + \frac{(m_1^2 - m_2^2)^2}{4M_{\mathbf{i}}^2},$$

complications in case of not spinless particles

• The BT form of generators is now: $\Gamma^i = U_{12} \tilde{\Gamma}^i U_{12}^{-1}$ where the unitary operator from the internal Hilbert space to the Hilbert rep space of two particle state

$$U_{12} = U_{12}(G, \mathbf{q}) = \prod_{i=1}^{2} D[\mathbf{s}_i; \alpha(p_i/m)^{-1} \alpha(G) \alpha(q_i/m)]$$

With D[s;u] being the rep operator of the SU(2) corresponding to the u in SU(2) and generators s.

$$P, \chi \rangle = U_{12} | P \rangle \otimes | \chi \rangle$$

• Momenta of the particles in their c.m.f.

$$q_i = L[\alpha(G)]^{-1} p_i$$

$$q = q_1 = -q_2$$

- External part $\langle G|P'\rangle \equiv \frac{2}{M'}G'^0\delta^3(\mathbf{G}-\mathbf{G}')$
- Scalar product

$$\langle P''|P'\rangle = \int \frac{d^3 \mathbf{G}}{2G^0} \langle P''|G\rangle \langle G|P'\rangle$$

= $2\sqrt{M'^2 + \mathbf{P'}^2} \,\delta^3(\mathbf{P''} - \mathbf{P'})$

The above results for PF RQM may be summed up as follows:

- 1. All generators of the homogeneous Lorentz group are free of interactions:
- a. Generators of boosts and rotations are free of interaction
- b. The interaction terms (through mass operator) are present in all components of 4-momentum they are Hamiltonians.
- 2. As a manifest consequence of 1.a:
- a. In the c.m. frame, the relative orbital angular momentum and spins are coupled together as in the nonrelativistic case – moreover, most nonrelativistic scattering theory formal results are valid in case of two particles (Keister [1991]).
- b. The spectator approximation preserves its spectator character in any frame.
- 3. Interaction in Hamiltonians presents no special problem states considered are usually eigenstates of the 4-momentum.

Can we use the non-RQM NN-potentials in this theory?

In Schrodinger equation

$$m_1 \approx m_2 = m$$

$$q^2 = \frac{M_i^2}{4} - m^2$$

- Coester[1974]:
- In case of NN scattering states there is no corrections: $a^2/m = E^{-1/2}$

$$q^2/m = E_{lab}/2$$

In case of deuteron: q²/m = (M-2m)(1+(M-2m)/4m)
 "effective" energy -2.2233 MeV
 Instead of the experimental one -2.2246 MeV

EM current operator (CO):

- Siegert [1937]: It must depend on interaction being 4-vectors (for some generators are)
- Current conservation

Spectator model: EM CO of a system equals to the sum of the EM COs of the constituents.

Point Form: Lev [1994], Klink[1998] – equivalent, but different parameterization.

We use parameterization by F. Lev.

Lev construction of Spectator Approximation of

the EM CO for a two particle system

In a special frame:

$$G_i + G_f = 0,$$

 $G_i = P_i/M_i, G_f = P_f/M_f$
A vector-parameter:
 $\mathbf{h} = \mathbf{G}_f/\mathbf{G}_f^0$

 Using common properties of the covariant 4-vector operator we come to

 $\langle P_f, \chi_f | \hat{J}^{\mu}(x) | P_i, \chi_i \rangle = 4\pi^{3/2} \sqrt{M_i M_f} e^{i(P_f - P_i)x} \langle \chi_f | \hat{j}^{\mu}(\mathbf{h}) | \chi_i \rangle_{\text{n.r.}}$

Reduction to the inner space calculation

$$j^{\mu}(\mathbf{h}) = \sum_{i=1,2} (L^{i})^{\mu}_{\nu} D^{i}_{1} D^{i}_{2} j^{\nu}_{i}(\mathbf{h}) D^{i}_{3} K^{i} I_{i}(\mathbf{h})$$

One particle COs (s=1/2)

$$j_i^0(\mathbf{h}) = eF_e^i(Q_i^2),$$

 $\mathbf{j}_i(\mathbf{h}) = -\frac{ie}{\sqrt{1-\mathbf{h}_i^2}}F_m^i(Q_i^2)(\mathbf{h}_i \times \mathbf{s}_i)$

• And in PF RQM

$$Q_1^2 = -(q_1' - q_1)^2 = 16\left(m^2 + \mathbf{q}^2 - \frac{(\mathbf{q} \cdot \mathbf{h})^2}{h^2}\right)\frac{h^2}{(1 - h^2)^2} \neq Q^2$$

Details of calculation

Helicities of the deuteron states

$$\xi_i^{\Lambda} = \begin{cases} (0, \pm 1, -i, 0)/\sqrt{2}, & \Lambda = \pm \\ (-Q/2, 0, 0, P_0)/m_d = (-h, 0, 0, 1)/\sqrt{1 - h^2} & \Lambda = 0 \end{cases}$$

$$\xi_f^{\Lambda} = \begin{cases} (0, \mp 1, -i, 0)/\sqrt{2}, & \Lambda = \pm \\ (Q/2, 0, 0, P_0)/m_d = (h, 0, 0, 1)/\sqrt{1 - h^2} & \Lambda = 0 \end{cases}$$

And of the virtual photon

$$\epsilon^{\lambda} = \begin{cases} (0, \pm 1, -i, 0)/\sqrt{2}, & \lambda = \pm \\ (1, 0, 0, 0) & \lambda = 0 \end{cases}$$

Details of calculation

- And we come to helisity amplitudes (matrix elements) $j_{\Lambda_f\Lambda_i}^{\lambda} \equiv \langle \Lambda_f | \left(\epsilon_{\mu}^{\lambda} \cdot j^{\mu}(\mathbf{h}) \right) | \Lambda_i \rangle$
- And to form factors:

$$\begin{split} j^0_{00}(Q^2) &= G_C + \frac{4}{3} \frac{h^2}{1 - h^2} G_Q \\ j^0_{+-}(Q^2) &= j^0_{-+}(Q^2) = G_C - \frac{2}{3} \frac{h^2}{1 - h^2} G_Q \\ j^+_{+0}(Q^2) &= j^-_{-0}(Q^2) = j^-_{0+}(Q^2) = -\frac{h}{\sqrt{1 - h^2}} G_M. \end{split}$$
$$\eta &= Q^2 / 4m_d^2 = h^2 / (1 - h^2)$$

The deuteron WFs in CS (r)

$$u(r) = \sum_{i=0}^{m} a_i R_{i,0}(r) + \sum_{j=1}^{n} C_j e^{-m_j r}$$

$$w(r) = \sum_{i=0}^{m} b_i R_{i,2}(r) + \sum_{j=1}^{n} D_j e^{-m_j r} \left(1 + \frac{3}{m_j r} + \frac{3}{(m_j r)^2} \right)$$
To provide asymptotic: $m_j = \left(0.23145 + (j-1) \cdot 0.5 \right) Fm^{-1}$

$$R_{i,l}(r) = (-1)^n \sqrt{\frac{2n!}{r_0 \Gamma(n+l+3/2)}} \left(\frac{r}{r_0}\right)^{l+1} \times exp\left(-\frac{r^2}{2r_0^2}\right) L_i^{l+\frac{1}{2}}\left(\frac{r^2}{r_0^2}\right),$$

 L_i^{α} is the associated Laguerre polynomial, the oscillator radius $r_0 = 0.04 \ fm$.

The deuteron WFs in MS (q)

1 10

$$u(q) = c \cdot \sum_{i=0}^{m} a_i R_{i,0}(t \cdot q) + \left(\frac{2}{\pi}\right)^{1/2} \sum_{j=1}^{n} \frac{C_j}{q^2 + m_j^2}$$

$$w(q) = -c\sum_{i=0}^{m} b_i R_{i,2}(t \cdot q) + \left(\frac{2}{\pi}\right)^{1/2} \sum_{j=1}^{n} \frac{D_j}{q^2 + m_j^2}$$



Results: static FFs

Rel/nonrel	$G_M(0) = \frac{M_d}{m_p} \mu_d$	$\overline{G_Q(0)} = M_d^2 Q_d$
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Moscow06	1.711/1.699	24.5/24.2
Inverse	1.68/1.715	26.0/25.8

Results $\chi^2 \rightarrow \min(local)$







Results – neutron form factors



- Our results line, without exchange currents.
- Data are extracted with Argonne potential (mostly) + corresponding meson exchange currents

Results – proton form factors



- Our results line, without exchange currents.
- Data are extracted from ep scattering directly

Results: Deuteron wave functions



Conclusion

- Modest changes of the Deuteron wave function and of the nucleon form factor parametrizations may simulate effects of the meson exchange currents for the eD elastic scattering.
- Experimental estimates of the neutron form factors error bars are underestimated considerably, systematic errors of the NN interaction uncertainties are not properly accounted for.