Evgeny Epelbaum, RUB

High-precision nuclear forces from chiral EFT: Where do we stand?

Chiral EFT yields <u>CONSISTENT</u> nuclear forces and currents.

Bundesministerium für Bildung und Forschung
DFG Deutsche Forschungsgemeinschaft

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Chiral EFT yields CONSISTENT nuclear forces and currents.

…but what does this actually mean??

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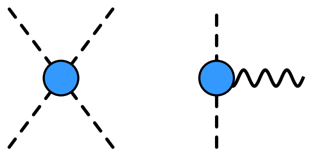
Chiral Effective Field Theory

GB dynamics

Weinberg, Gasser, Leutwyler, …

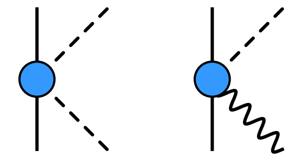


Chiral Perturbation Theory

$$Q = \frac{\text{momenta of particles or } M_\pi \sim 140 \text{ MeV}}{\text{breakdown scale } \Lambda_b}$$

πN dynamics

Bernard-Kaiser-Meißner et al.



Effective Lagrangian:

$$\mathcal{L}_\pi = \frac{F^2}{4}\mathrm{Tr}(\nabla^\mu U \nabla_\mu U^\dagger + \chi_+) + \ldots ,$$

$$\mathcal{L}_{\pi N} = \bar{N}(iv \cdot D + g_A u \cdot S)N + \ldots ,$$

$$\mathcal{L}_{NN} = -\frac{1}{2}C_S(\bar{N}N)^2 + 2C_T(\bar{N}SN)^2 + \ldots$$

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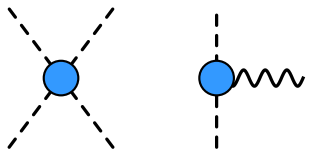

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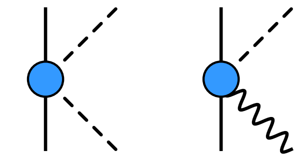

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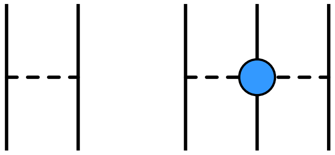
Nuclear forces

Weinberg, van Kolck, Kaiser, EGM, ...



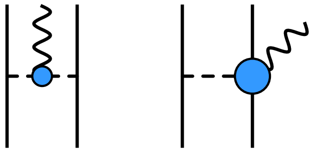
Auxilliary quantities (not observable):

More difficult to calculate than Feynman graphs

(renormalizability, off-shell consistency...)

Nuclear currents

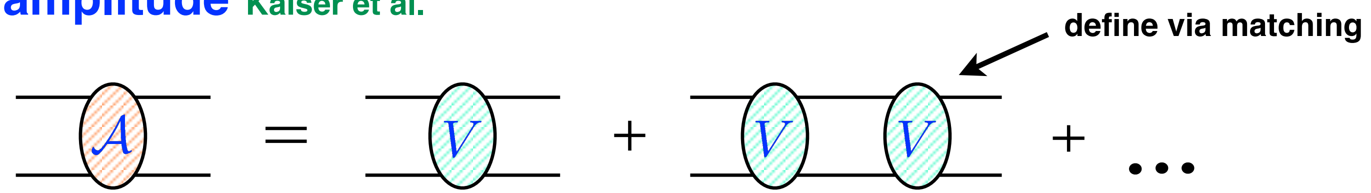
Park et al, Bochum-Bonn, JLab-Pisa



Derivation of the nuclear forces

Matching to the amplitude Kaiser et al.

Feynman graphs →



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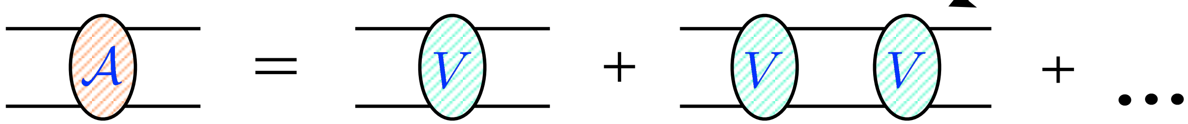
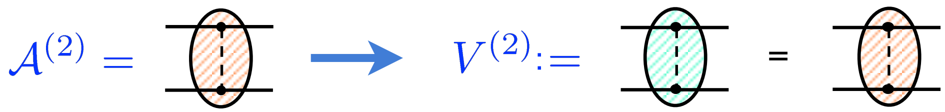
define via matching

Feynman graphs →





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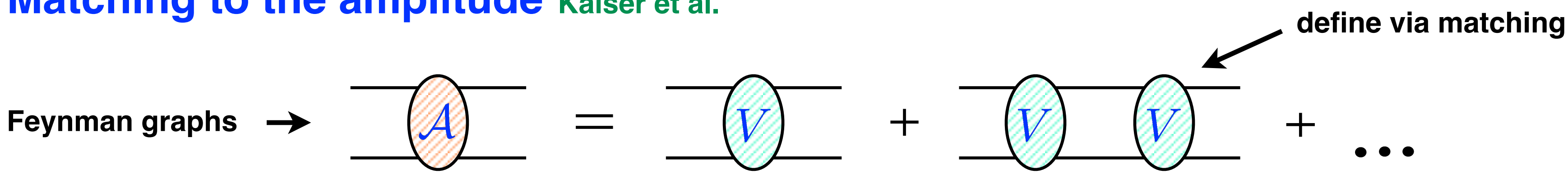

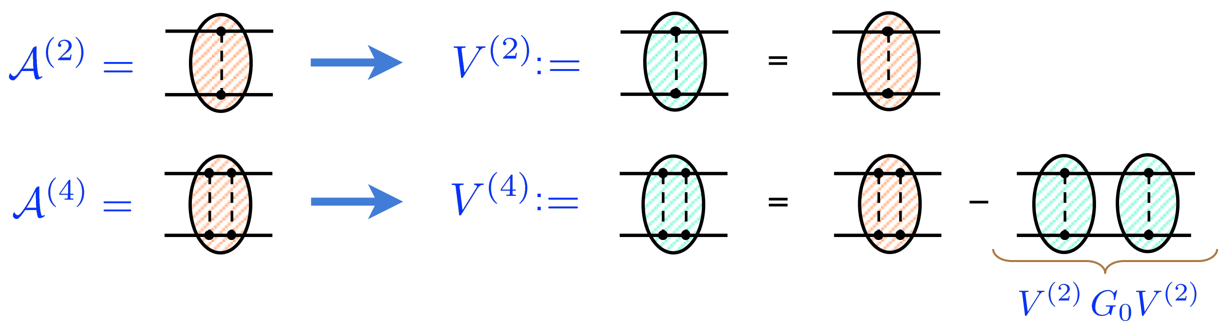

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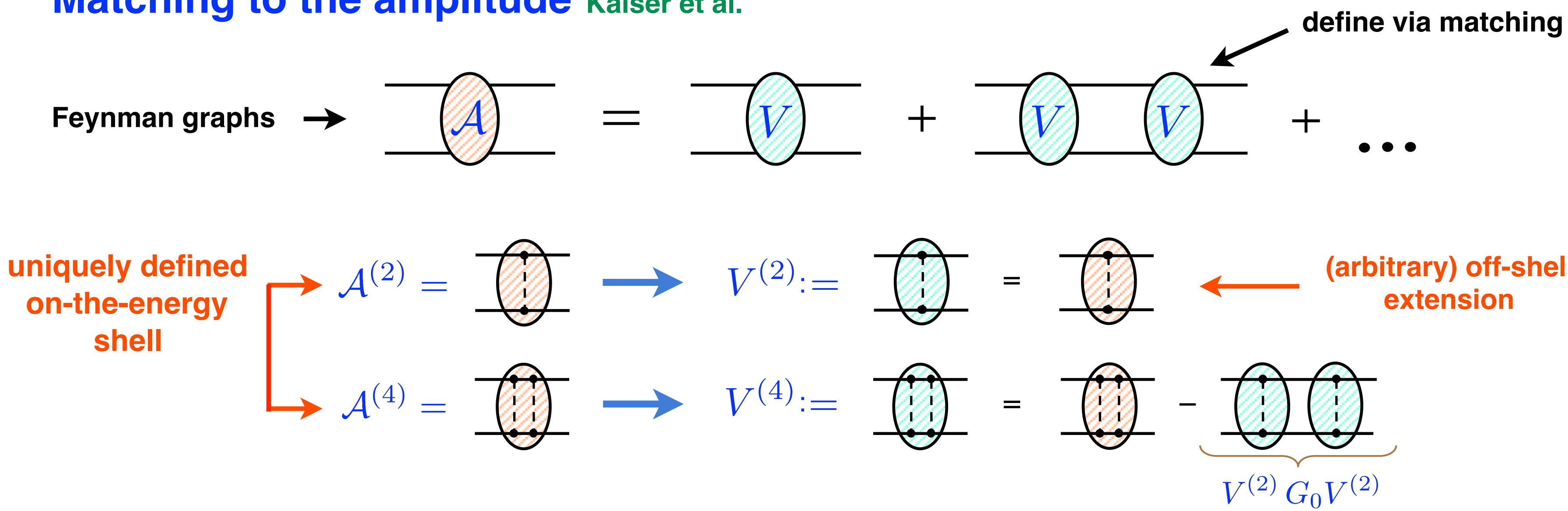

Higher-order terms in the Hamiltonian „know“ about the choice made for the off-shell extension (consistency...)

Derivation of the nuclear forces

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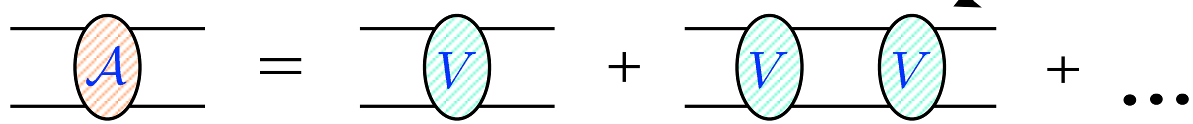
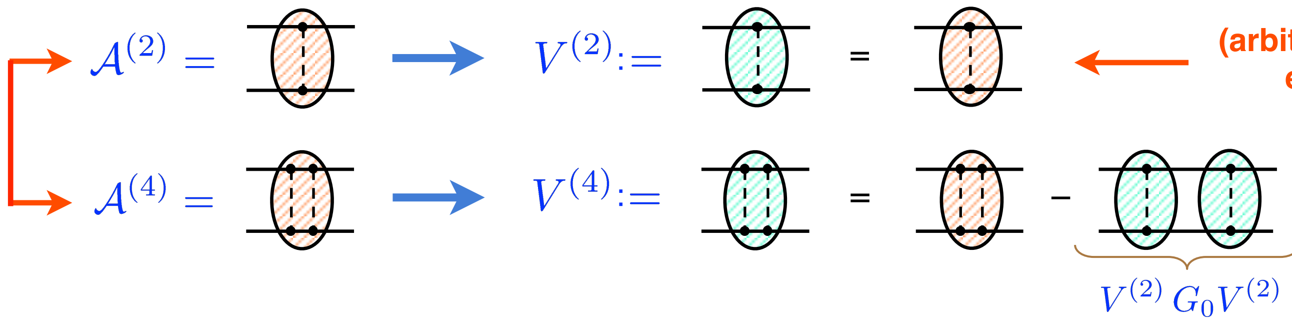


define via matching

uniquely defined on-the-energy shell

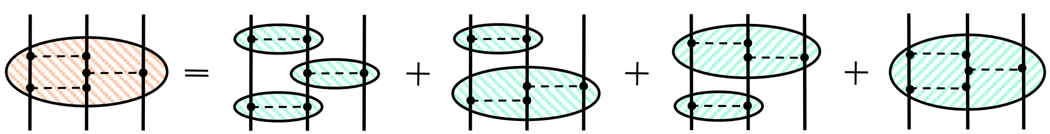


(arbitrary) off-shell extension

Higher-order terms in the Hamiltonian „know“ about the choice made for the off-shell extension (consistency...)

Are nuclear potentials well-defined (i.e. finite)?

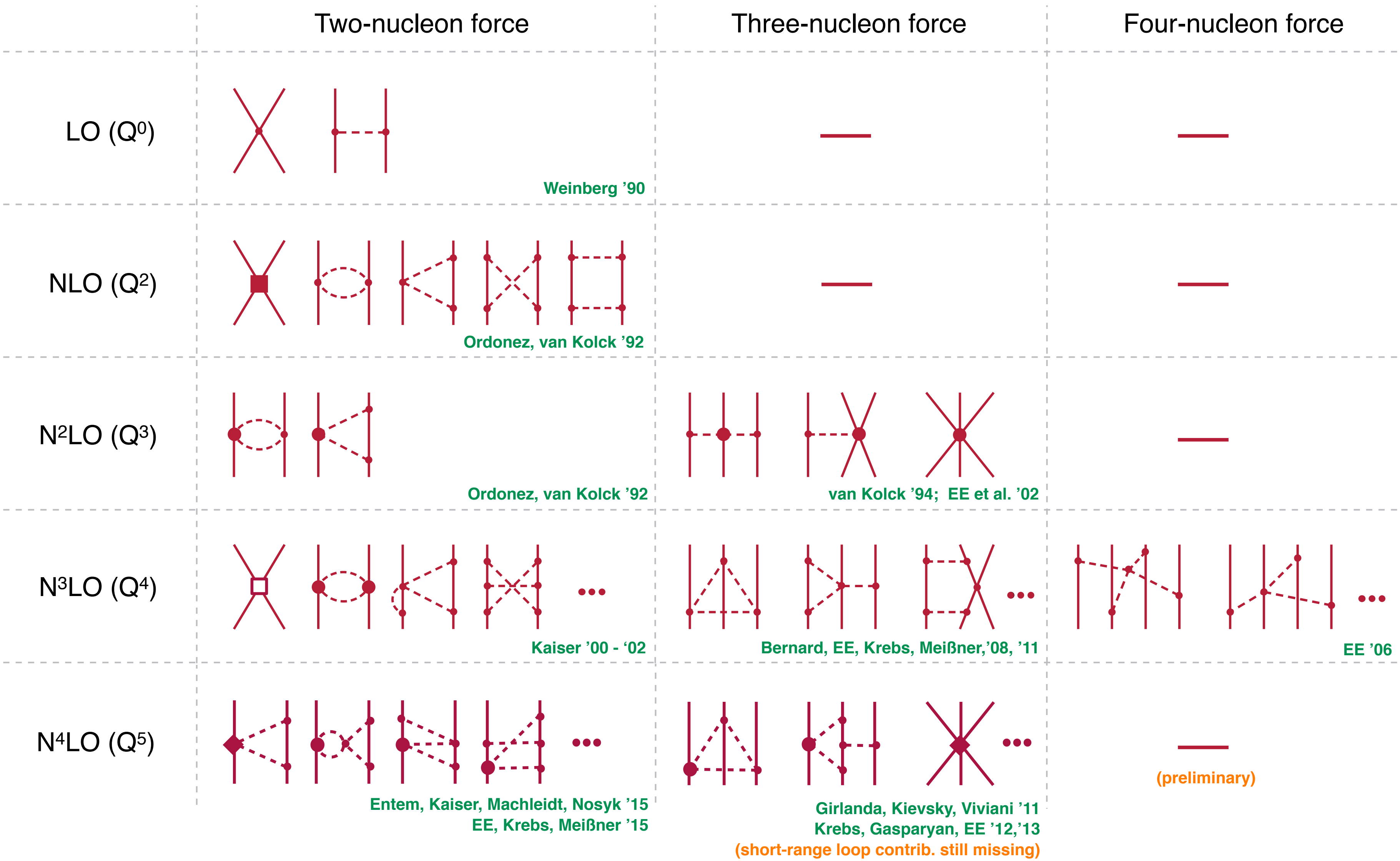
UV finite →



not necessarily UV finite

So far, it was always possible to renormalize nuclear forces by systematically exploiting their unitary ambiguity…

Chiral expansion of the nuclear forces [W-counting]



— Electromagnetic and weak currents worked out to N³LO **Krebs, Kölling, EE, Meißner; Baroni, Pastori, Schiavilla et al.**

— The derived forces and currents are consistent <u>provided one uses dim. reg. for all loop integrals!</u>

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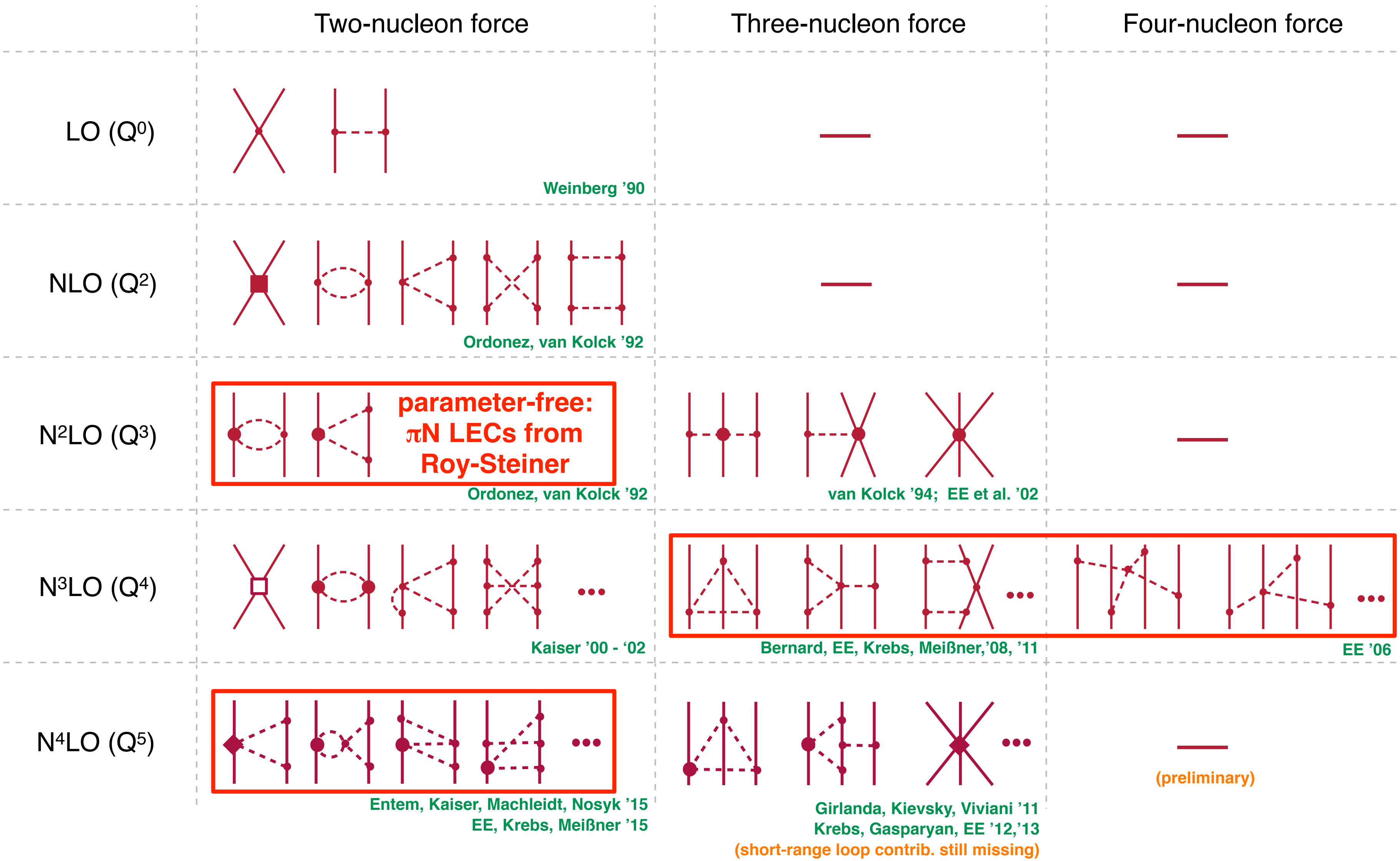

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Regularization of the long-range forces

The cutoff Λ has to be kept finite, $\Lambda \sim \Lambda_b$. In practice, low values of Λ are preferred:

— many-body methods require soft interactions,

— spurious deeply-bound states for $\Lambda > \Lambda^{crit}$ make calculations for $A > 3$ unfeasible…

⟶ it is crucial to employ a regulator that minimizes finite-Λ artifacts!

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EE, Glöckle, Meißner '04;
Entem, Machleidt '03;
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EE, Glöckle, Meißner '04;
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Local (implemented in coordinate space)

$$V_\pi(\vec r) \longrightarrow V_\pi(\vec r)\left[1-\exp(-r^2/R^2)\right]^n$$ used in **EE, Krebs, Meißner (EKM) '15**

— still an ad hoc procedure

— (technically) difficult to apply to 3NF and exchange currents

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[inspired by Thomas Rijken]

Reinert, Krebs, EE '18;

⟶ does not affect long-range physics **at any order** in $1/\Lambda^2$-expansion

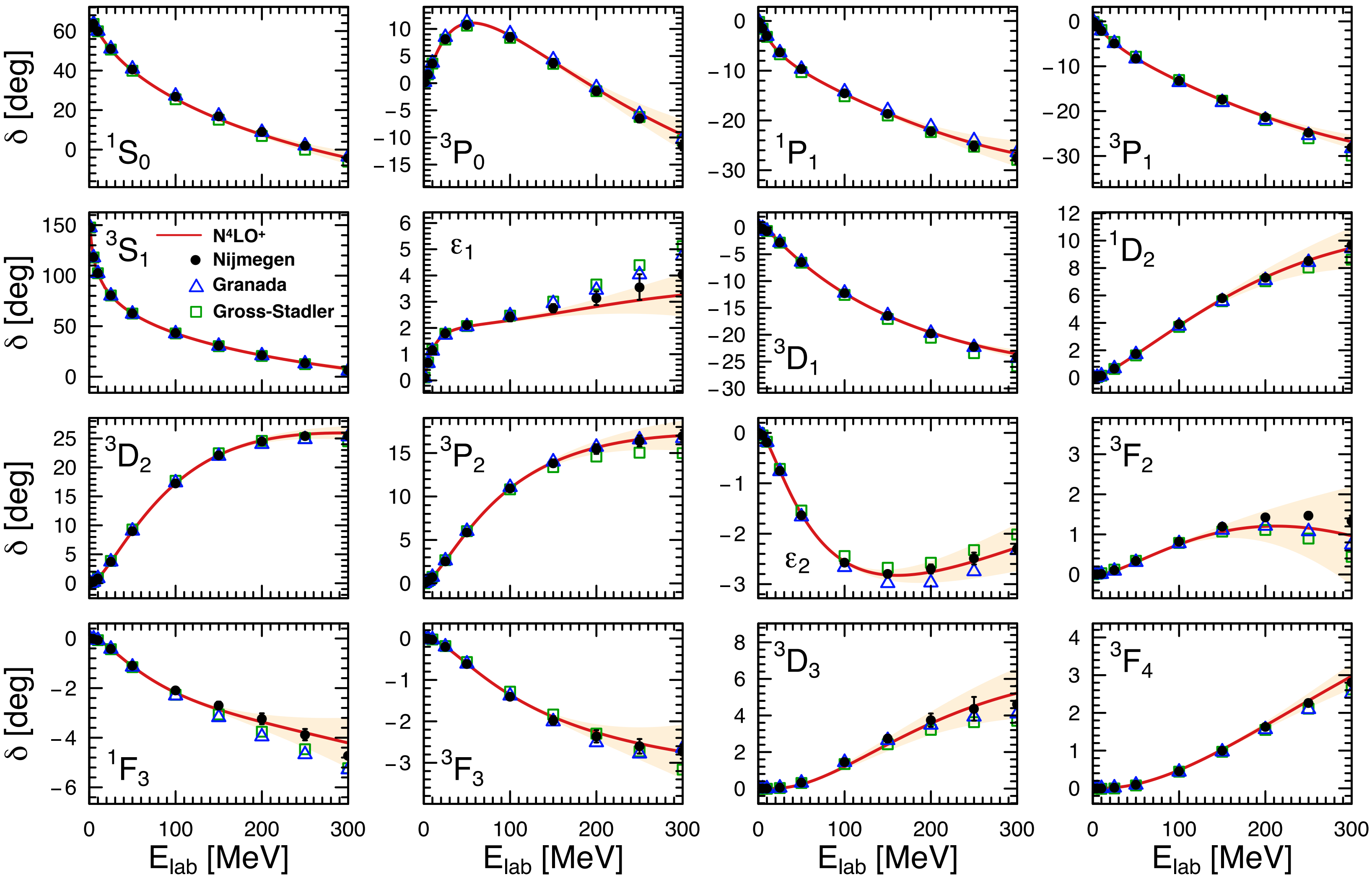
— Application to 2π exchange does not require re-calculating the corresponding diagrams:

$$V(q)=\frac{2}{\pi}\int_{2M_\pi}^\infty \mu\, d\mu \frac{\rho(\mu)}{q^2+\mu^2} + \ldots \quad \xrightarrow{\text{reg.}} \quad V_\Lambda(q) = e^{-\frac{q^2}{2\Lambda^2}}\frac{2}{\pi}\int_{2M_\pi}^\infty \mu\, d\mu \frac{\rho(\mu)}{q^2+\mu^2}\, e^{-\frac{\mu^2}{2\Lambda^2}} + \underbrace{\ldots}_{\text{polynomial in } q^2,\, M_\pi}$$

— Convention: choose polynomial terms such that $\Delta^n V_{\Lambda,\,\rm long}(\vec r\,)\big|_{r=0} = 0$

Partial wave analysis of NN data

P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88



— Contacts at N^{4}LO$^+$: 2 [Q^0] + 7 [Q^2] **+ 12** [Q^4] + 4 [F-waves, Q^6] + IB; Gauss regulator

— Clear evidence of the parameter-free chiral 2π exchange (Roy-Steiner LECs)!

Partial wave analysis of NN data

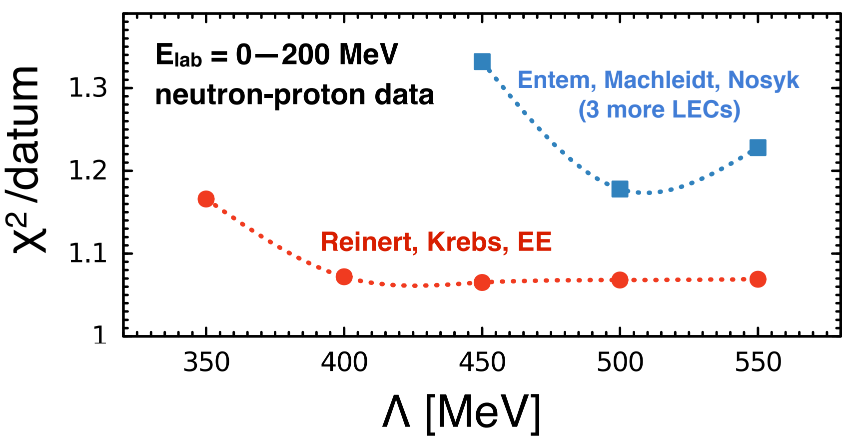
P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88

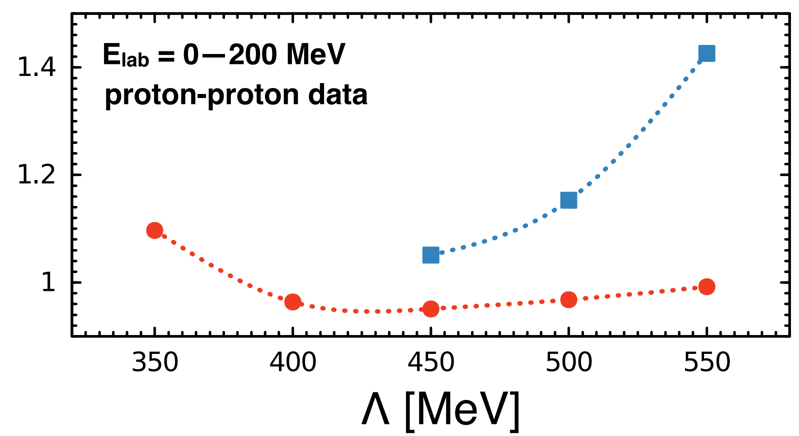
χ^2/datum for the description of the Granada-2013 database: χEFT vs. phenomenology

E_{lab} bin	CD Bonn$_{(43)}$	Nijm I$_{(41)}$	Nijm II$_{(47)}$	Reid93$_{(50)}$	N^{4}LO$^+$$_{(27+1)}$, this work
neutron-proton scattering data					
0 – 100	1.08	1.06	1.07	1.08	1.07
0 – 200	1.08	1.07	1.07	1.09	1.07
0 – 300	1.09	1.09	1.10	1.11	1.06
proton-proton scattering data					
0 – 100	0.88	0.87	0.87	0.85	0.86
0 – 200	0.98	0.99	1.00	0.99	0.95
0 – 300	1.01	1.05	1.06	1.04	1.00

For the first time, chiral EFT potentials qualify for being regarded as PWA!

N^{4}LO$^+$: semilocal (Reinert, Krebs, EE) vs. nonlocal (Entem, Machleidt, Nosyk)





Error analysis

P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88

Error analysis: statistical, truncation, πN LECs, fit energy. In most cases, the uncertainty is dominated by truncation errors. At N^{4}LO & low energies, other errors become comparable.

Example: deuteron asymptotic normalizations (relevant for nuclear astrophysics)

Our determination:

statistical error, truncation error, πN LECs, variation of E_{max}

$$A_S = 0.8847^{(+3)}_{(-3)}(3)(5)(1)\ \mathrm{fm}^{-1/2}$$

$$\eta \equiv \frac{A_D}{A_S} = 0.0255^{(+1)}_{(-1)}(1)(4)(1)$$

Exp: $A_S = 0.8781(44)\ \mathrm{fm}^{-1/2}$, $\eta = 0.0256(4)$

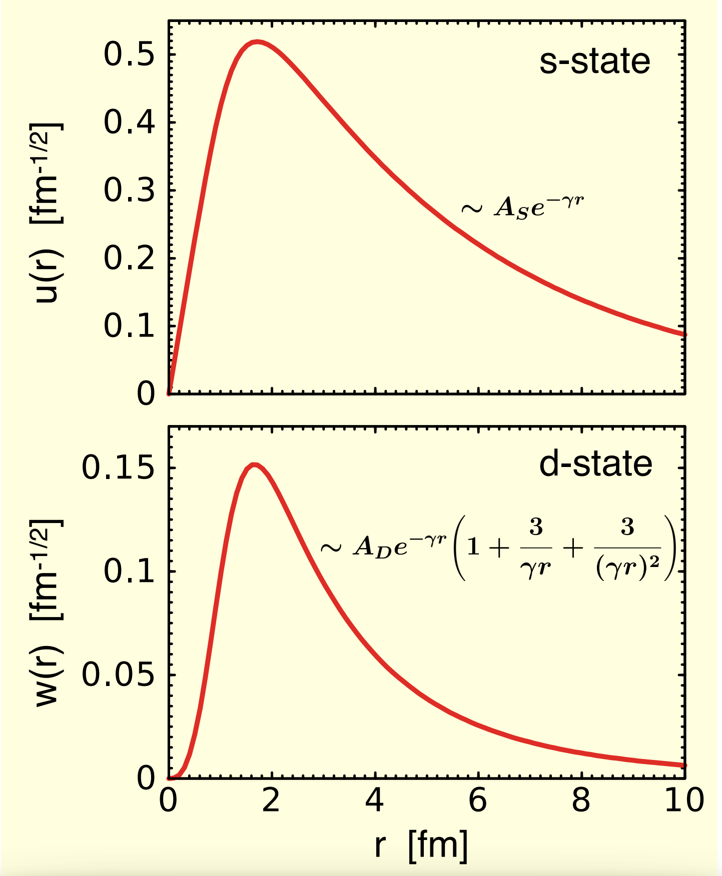
Borbely et al. '85 — Rodning, Knutson '90

Nijmegen PWA [errors are „educated guesses“] Stoks et al. '95

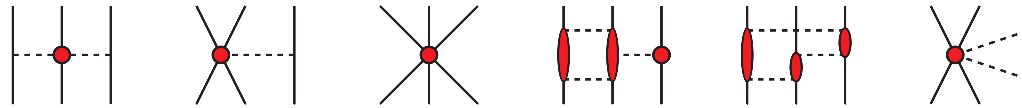
$$A_S = 0.8845(8)\ \mathrm{fm}^{-1/2}, \quad \eta = 0.0256(4)$$

Granada PWA [errors purely statistical] Navarro Perez et al. '13

$$A_S = 0.8829(4)\ \mathrm{fm}^{-1/2}, \quad \eta = 0.0249(1)$$



Beyond the two-nucleon system



N^{2}LO: tree-level graphs, 2 new LECs
van Kolck '94; EE et al '02

N^{3}LO: leading 1 loop, parameter-free
Ishikawa, Robilotta '08; Bernard, EE, Krebs, Meißner '08, '11

N^{4}LO: full 1 loop, almost completely worked out, several new LECs
Girlanda, Kievski, Viviani '11; Krebs, Gasparyan, EE '12,'13; EE, Gasparyan, Krebs, Schat '14

LENPIC: Low **E**nergy **N**uclear **P**hysics **I**nternational **C**ollaboration

Beyond the two-nucleon system

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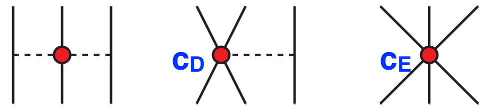

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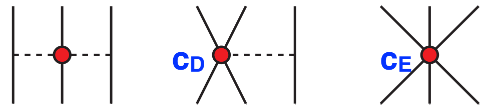
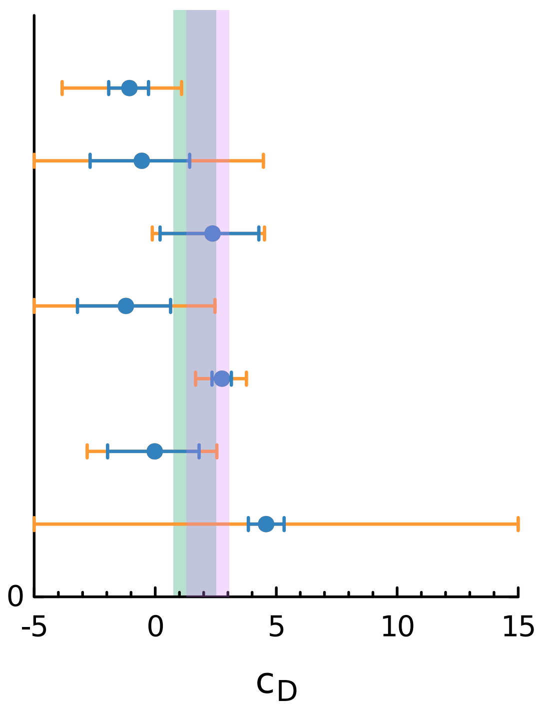


Determination of the LECs c_D, c_E

— Triton BE (c_D-c_E correlation)

— Explore various 3N observables and let theory and/or data decide…

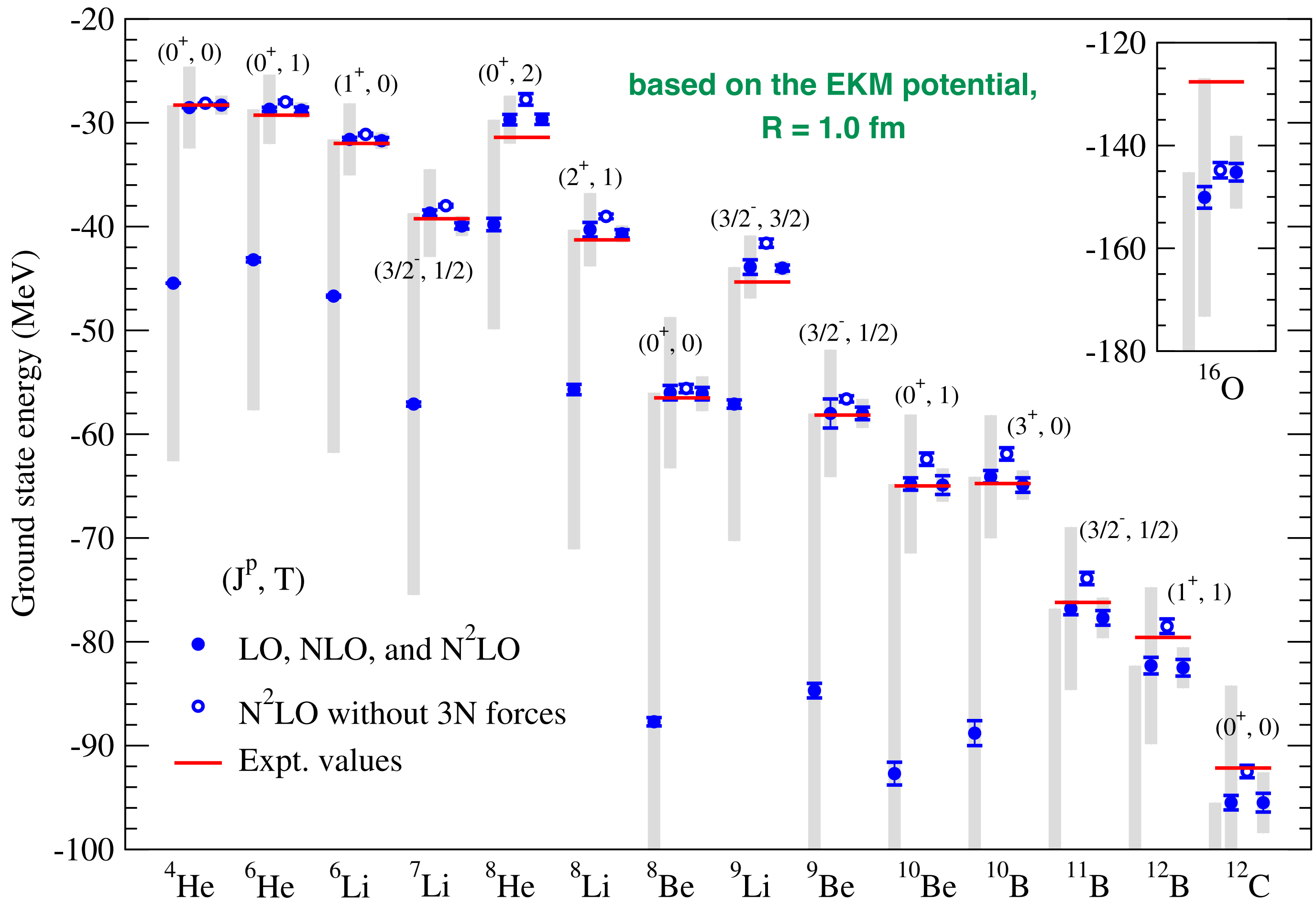


LENPIC, 1807.02848 [based on EKM, R = 0.9 fm]

LENPIC: Low **E**nergy **N**uclear **P**hysics **I**nternational **C**ollaboration

Light nuclei up to N²LO

EE et al. (LENPIC), arXiv:1807.02848



(c_D, c_E fixed from ^{3}H BE and Nd scattering)

More results in the talks by James Vary and Roman Skibinski

LENPIC: Low Energy Nuclear Physics International Collaboration

Intermediate summary

NN@N^{4}LO$^+$: accurate and precise

Intermediate summary

NN@N^4LO^+: accurate and precise

Few-N@N^2LO: accurate but imprecise

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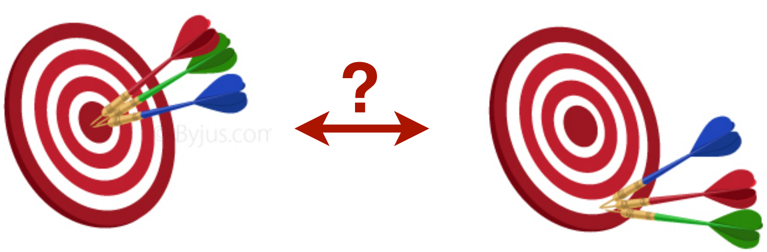
Few-N@N^{2}LO: accurate but imprecise



Few-N@N^{3,4}LO (not yet available): precise, **hopefully also accurate**

→ **challenge the theory!**

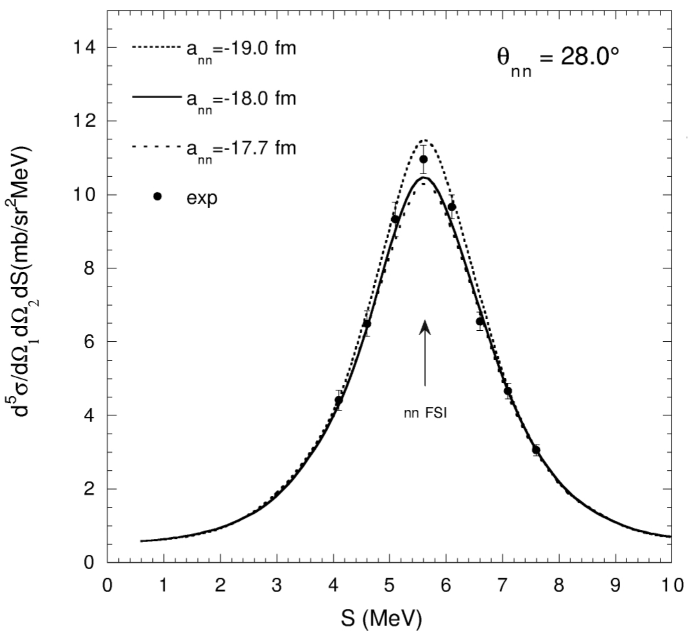
(consistency, error analysis)

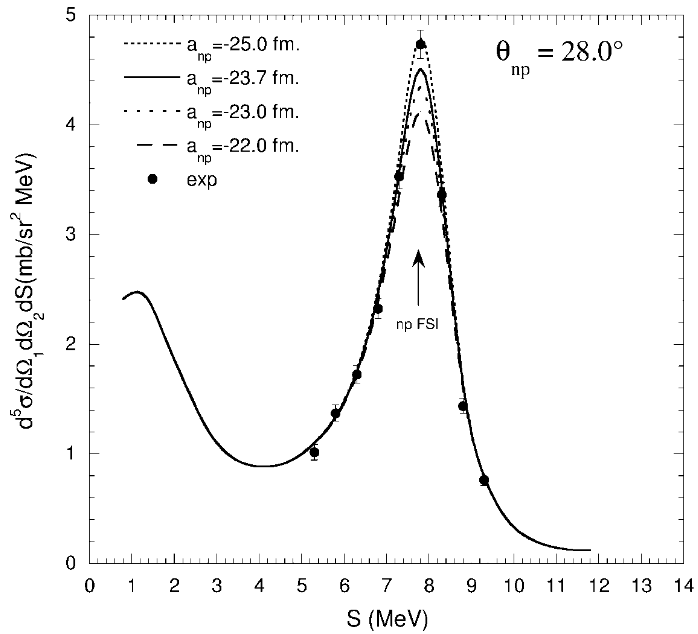
IB effects and precision few-N physics

- **Neutron-neutron scattering length from few-N reactions**

$\pi^- + {}^2\text{H} \rightarrow n + n + \gamma$ ⟶ $a_{nn} = -18.50 \pm 0.53$ fm — Howell et al. '98

$n + {}^2\text{H} \rightarrow n + n + p$ ⟶ $\begin{cases} a_{nn} = -18.7 \pm 0.6 \text{ fm} \\ a_{nn} = -16.3 \pm 0.4 \text{ fm} \end{cases}$ — Gonzales Trotter et al. '99; Huhn et al. '00





- **CSB nuclear forces and the BE difference of ^{3}H and ^{3}He**

Coulomb	Breit	K.E.	Two-Body	Three-body	Theory	Experiment
648	28	14	65(22)	5	760(22)	764

Friar et al., PRC 71 (2005) 024003

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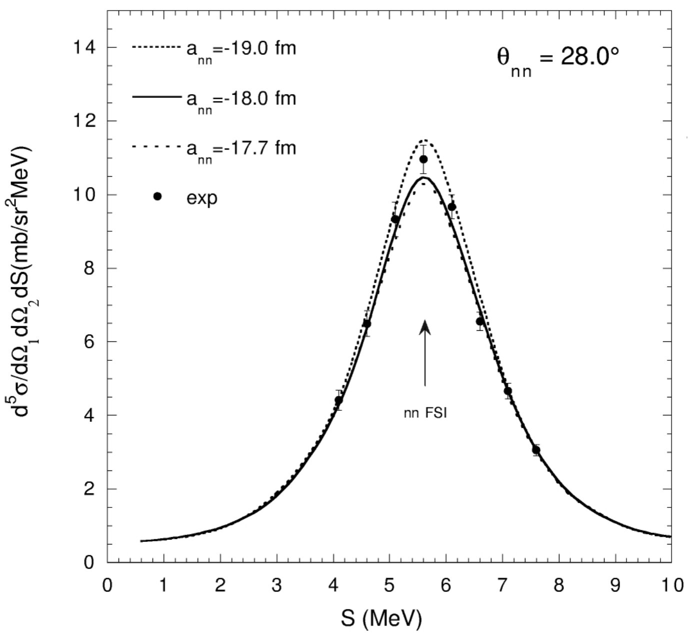

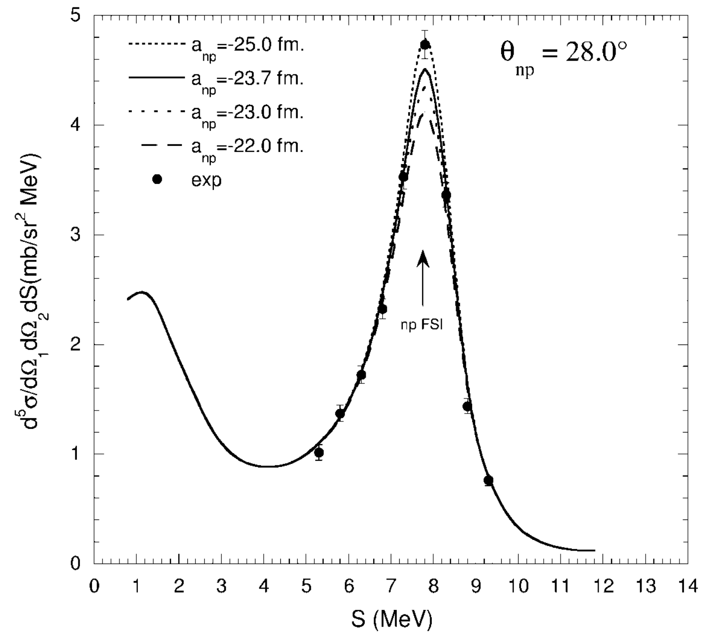

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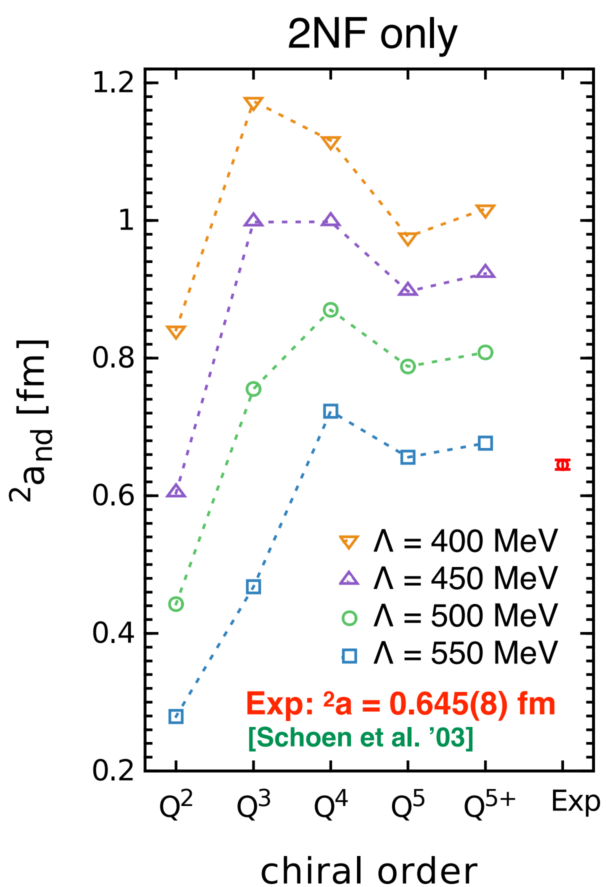

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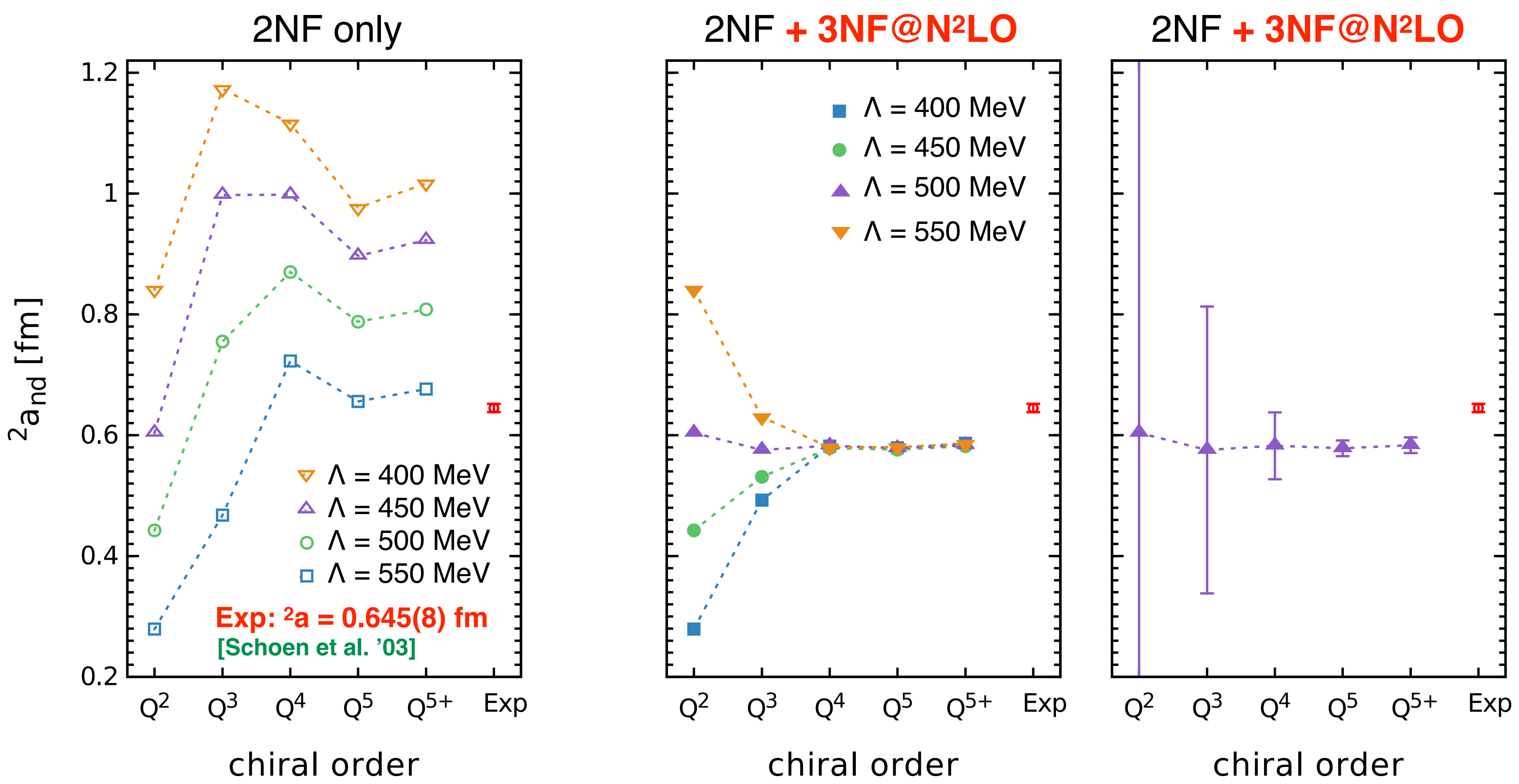

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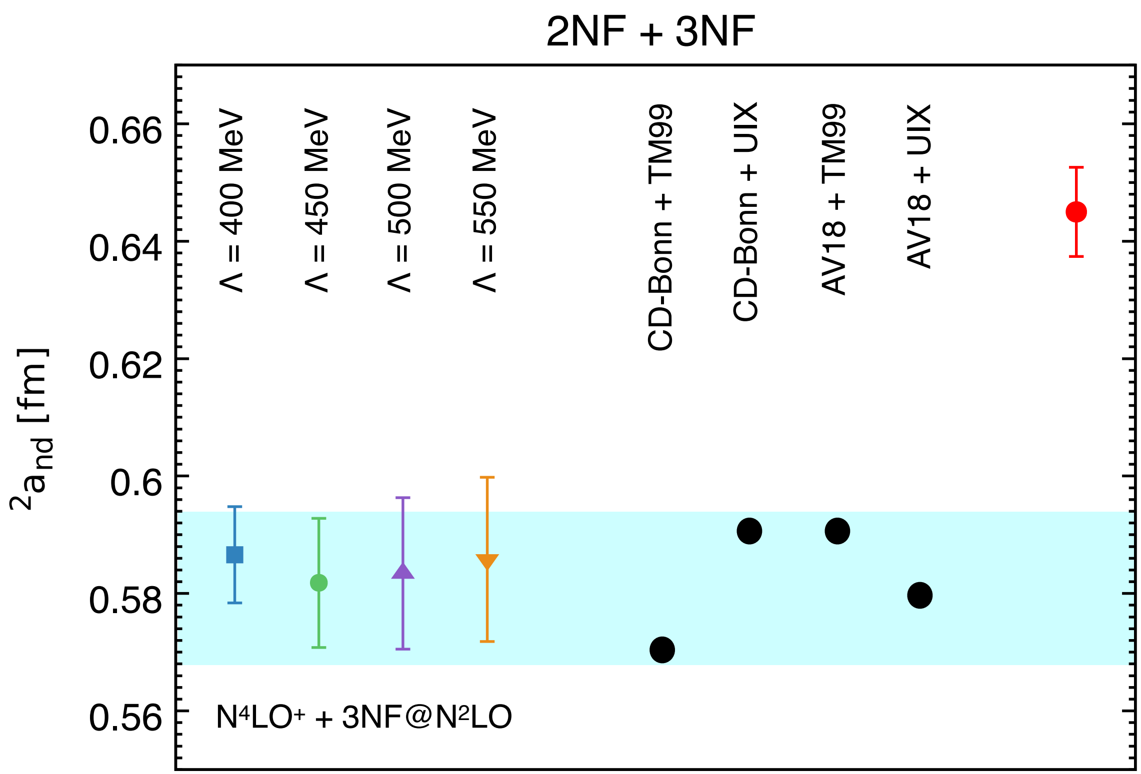

IB effects and precision few-N physics

- **Neutron-neutron scattering length from few-N reactions**

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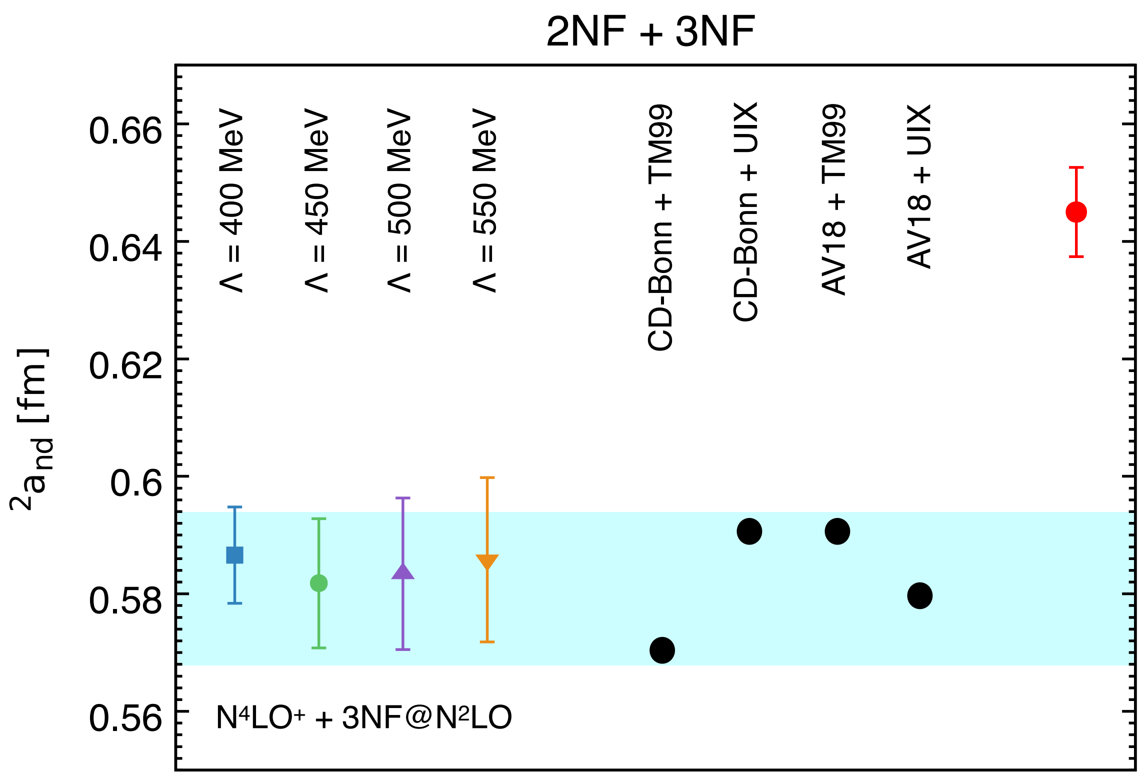

Can reproduce $^2a_{nd}$ using $a_{nn} \sim -16.5$ fm! Alternatives (3NF beyond N²LO, IB effects) need to be checked.
Can one then still understand the BE differences of mirror nuclei?

Radii of medium-mass nuclei: A smoking gun?

- **Point-proton radius of the deuteron** $\langle r^2 \rangle_{\rm pt} = \langle r^2 \rangle_{\rm ch} - \langle r^2 \rangle_{\rm ch}^{\rm p} - \langle r^2 \rangle_{\rm ch}^{\rm n}$

  All high-precision NN forces yield similar *matter* radii:

	RKE N^{4}LO$^+$	Granada PWA (δ-shell)	Nijm I	Nijm II	Reid93	CD-Bonn
$\sqrt{\langle r^2 \rangle_{\rm m}}$ (fm)	**1.965 . . . 1.968**	**1.965**	**1.967**	**1.968**	**1.969**	**1.966**

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- **$NNLO_{sat}$ and ΔNNLO reproduce r_p for ^{16}O:** A coincidence? E.g. matter radius of ^{2}H:

	$NNLO_{sat}$	$\Delta NNLO_{450}$	EMN $N^4LO^+_{450}$	EMN $N^4LO^+_{500}$	EMN $N^4LO^+_{550}$
$\sqrt{\langle r^2\rangle_{\rm m}}$ (fm)	**1.978** (+0.15%)	**1.982** (+0.35%)	**1.966** (−0.45%)	**1.973** (−0.1%)	**1.971** (−0.2%)

Towards consistent 3NF and MECs

Hermann Krebs, EE, in preparation

Regularization of the 3NF, 4NF and MEC at N^{3}LO and beyond is nontrivial!

Standard approach: Take expressions obtained in DR and multiply with some cutoff: finite-Λ artifacts are expected to be removed by contacts terms (adjusted to data). Is it true?

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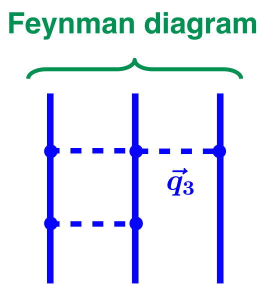

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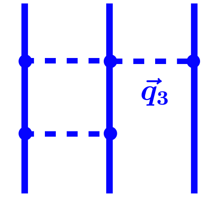
Feynman diagram



(finite in DR)

Linearly divergent: $\propto \Lambda \frac{q_3^a \, q_3^b}{\vec{q}_3^{\,2} + M_\pi^2}$

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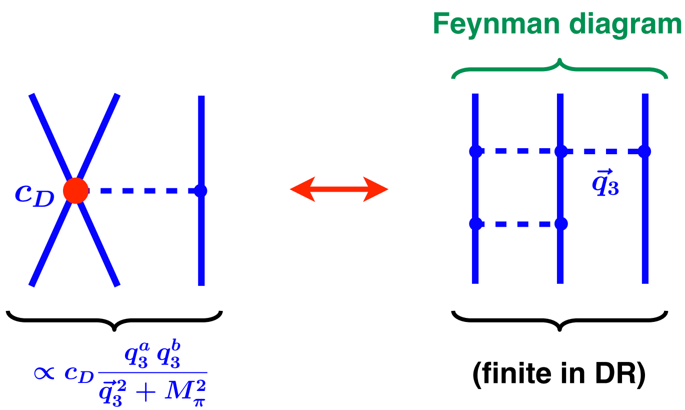

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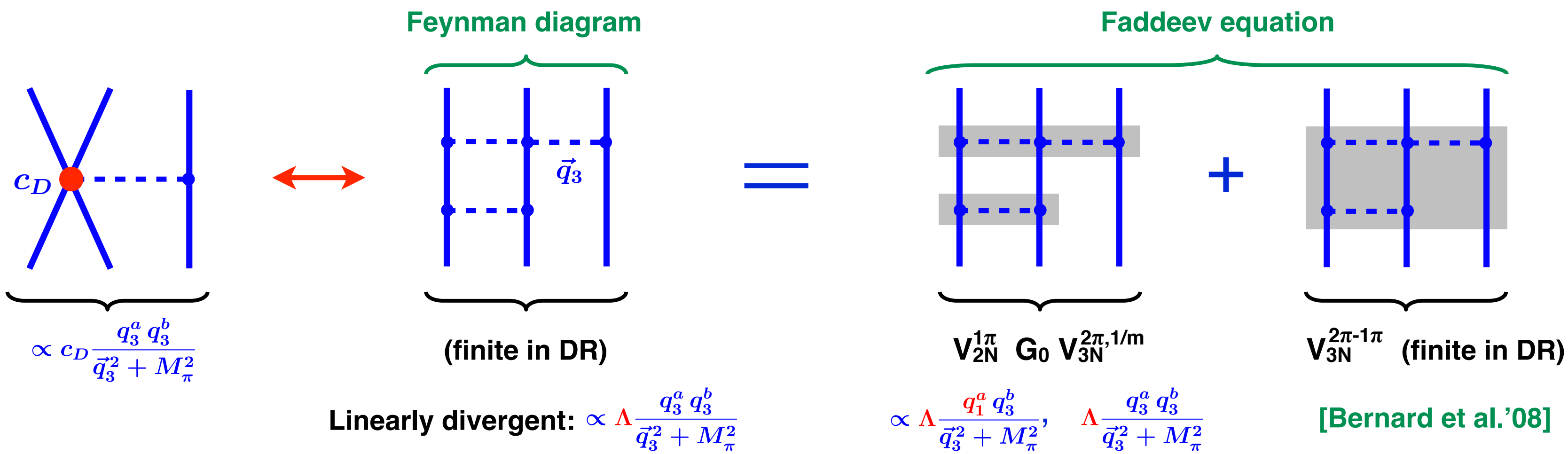

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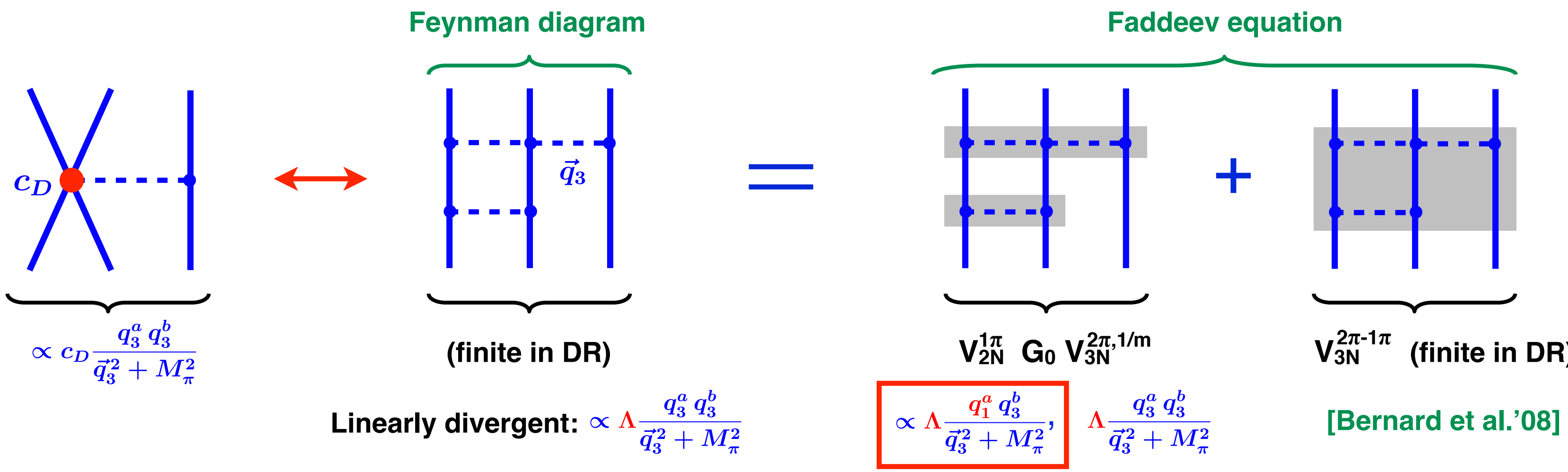

Renormalization of the iteration requires χ-symmetry breaking counter terms!

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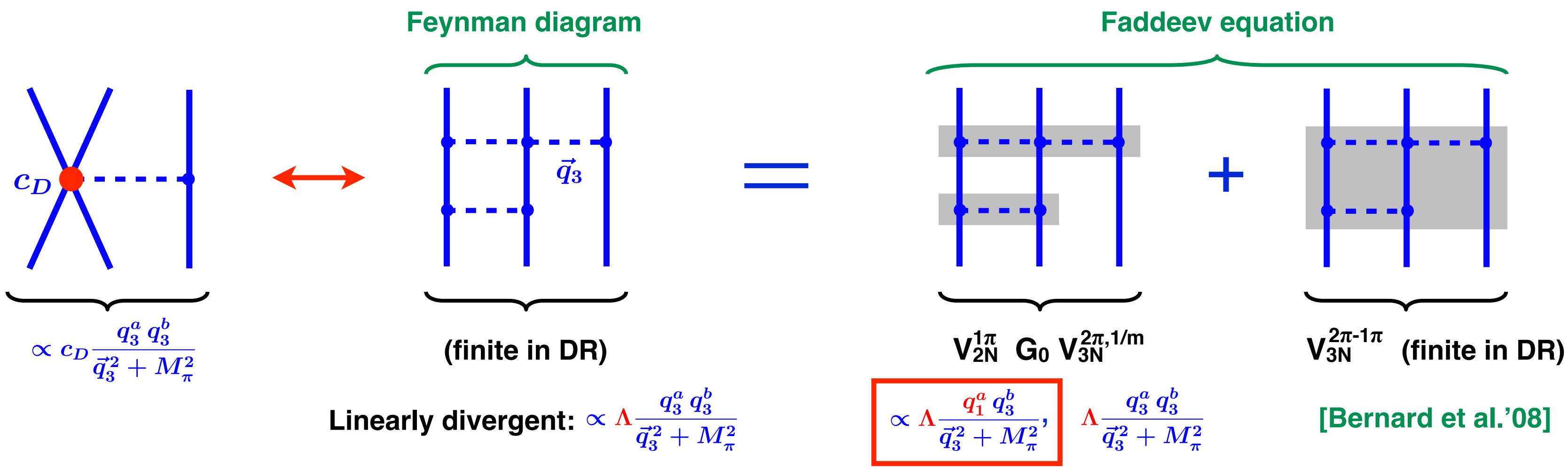

Renormalization of the iteration requires χ-symmetry breaking counter terms!

- The problematic divergence cancels out if $V_{3N}^{2\pi\text{-}1\pi}$ is calculated using cutoff regularization.
- Irrelevant for $\mathbf{V_{2N}}$: momentum dependence of 2N contacts is not constrained by χ-symm.
- Regularization of $\mathbf{V_{3N}}$ must be **consistent** to maintain matching (of finite pieces).
- Can one enforce renormalizability of V_{3N} (i.e. remove problematic divergences) by systematically exploiting unitary ambiguities? This indeed seems to be possible!

Regularization and the chiral symmetry

The same problems affect loop contributions to the exchange charge/current operators.

Is it enough to recalculate all loop contributions to the 3NF/exchange currents by modifying the pion propagators via $(\vec{q}^{\,2} + M_\pi^2)^{-1} \longrightarrow \exp[-(\vec{q}^{\,2} + M_\pi^2)/\Lambda^2]\,(\vec{q}^{\,2} + M_\pi^2)^{-1}$?

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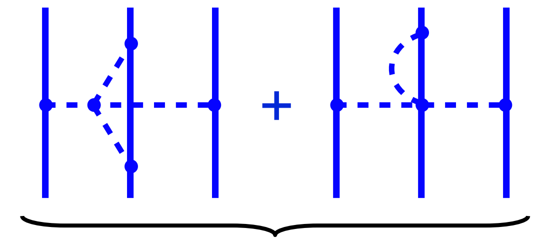
Not quite… Have to ensure that regularization maintains the chiral symmetry.

$$U(\vec{\pi}) = 1 + \frac{i}{F_\pi}\vec{\tau}\cdot\vec{\pi} - \frac{1}{2F_\pi^2}\vec{\pi}^{\,2} - \frac{i\alpha}{F_\pi^3}(\vec{\tau}\cdot\vec{\pi})^3 - \frac{8\alpha - 1}{8F_\pi^4}\vec{\pi}^{\,4} + \ldots$$



is independent on α in DR, but not of one uses (naive) cutoff regularization

All observables should be α-independent.

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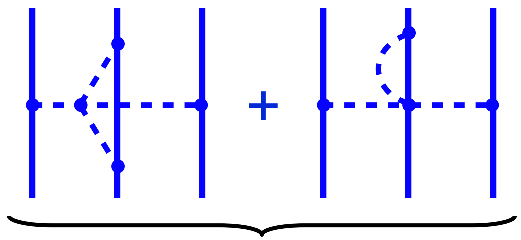

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All observables should be α-independent.

Solution: higher-derivative regularization [Slavnov, Nucl. Phys. B31 (1971) 301]

(designed to coincide with the employed local regularization in the NN sector)

$$\mathcal{L}_{\pi,\Lambda}^{(2)} = \mathcal{L}_\pi^{(2)} + \frac{F^2}{4}\mathrm{Tr}\left[\mathrm{EOM}\,\frac{1 - \exp\left(\frac{\mathrm{ad}_{D_\mu}\mathrm{ad}_{D^\mu} + \frac{1}{2}\chi_+}{\Lambda^2}\right)}{\mathrm{ad}_{D_\mu}\mathrm{ad}_{D^\mu} + \frac{1}{2}\chi_+}\,\mathrm{EOM}\right], \qquad \mathcal{L}_\pi^{(2)} = \frac{F^2}{4}\mathrm{Tr}\left[u_\mu u^\mu + \chi_+\right]$$

Hermann Krebs et al. (preliminary)

with $\mathrm{EOM} \equiv -[D_\mu,\, u^\mu] + \frac{i}{2}\chi_- - \frac{i}{4}\mathrm{Tr}(\chi_-)$ and $\mathrm{ad}_X Y \equiv [X,\, Y]$

Requires recalculation of the loop contributions to the 3NF/exchange currents (in progress)

Summary and outlook

- Precision calculations of few-N systems at $N^{3,4}LO$ will challenge chiral EFT! (especially in the 3N continuum — talk by Kimiko Sekiguchi)
- Naive regularization of 3NF and MECs, calculated using DR, should **NOT** be applied beyond N^2LO!
- Need to recalculate loop contributions to 3NF and MECs using regularization which **maintains the chiral symmetry** and **is consistent with the NN force** (in progress…)

Thanks to:

— my Bochum collaborators on these topics:
Vadim Baru, Arseniy Filin, Ashot Gasparyan, **Hermann Krebs**, Patrick Lipka, Daniel Möller, **Patrick Reinert**

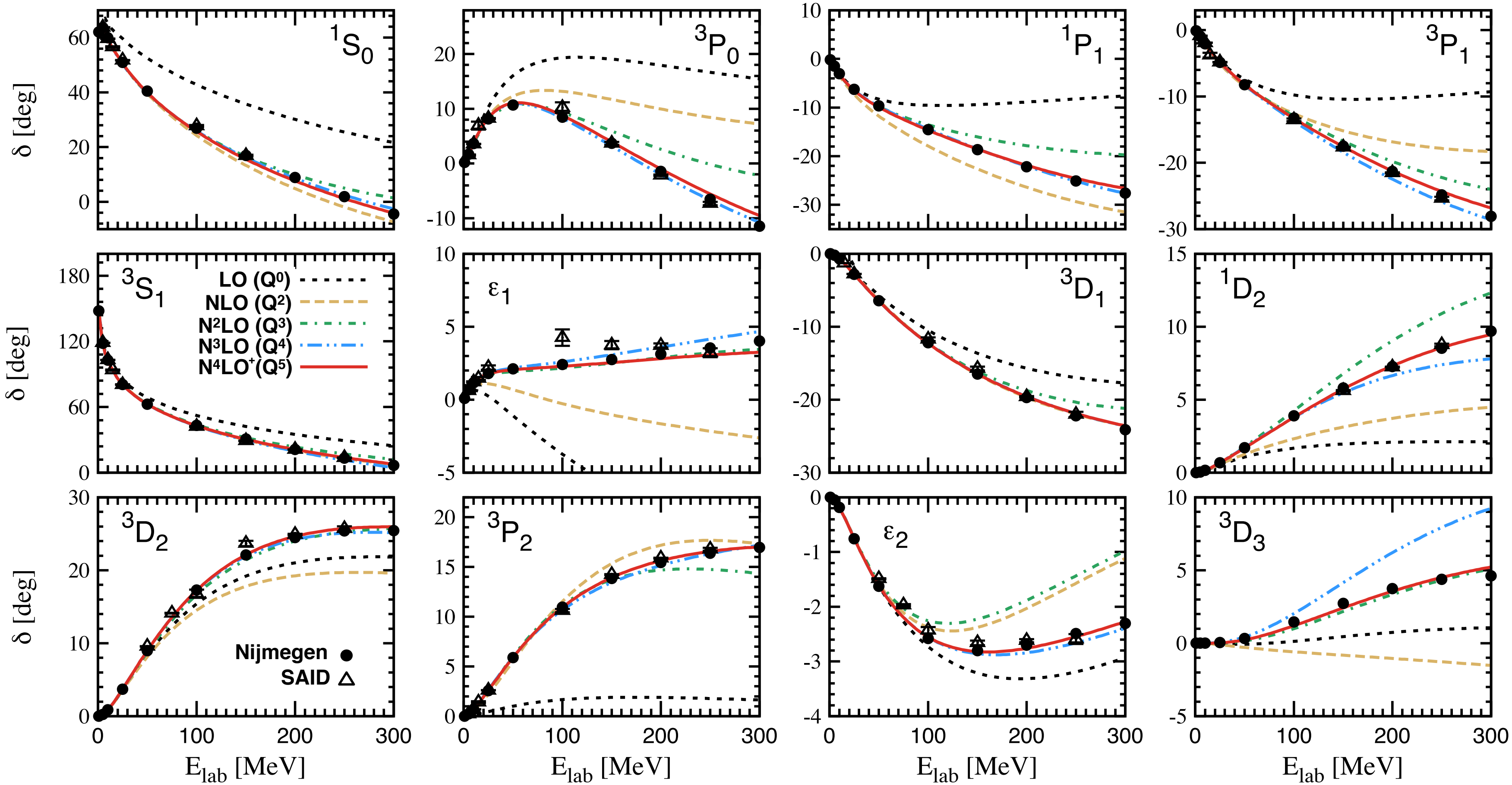
— **Ulf Meißner**, Andreas Nogga and the whole LENPIC

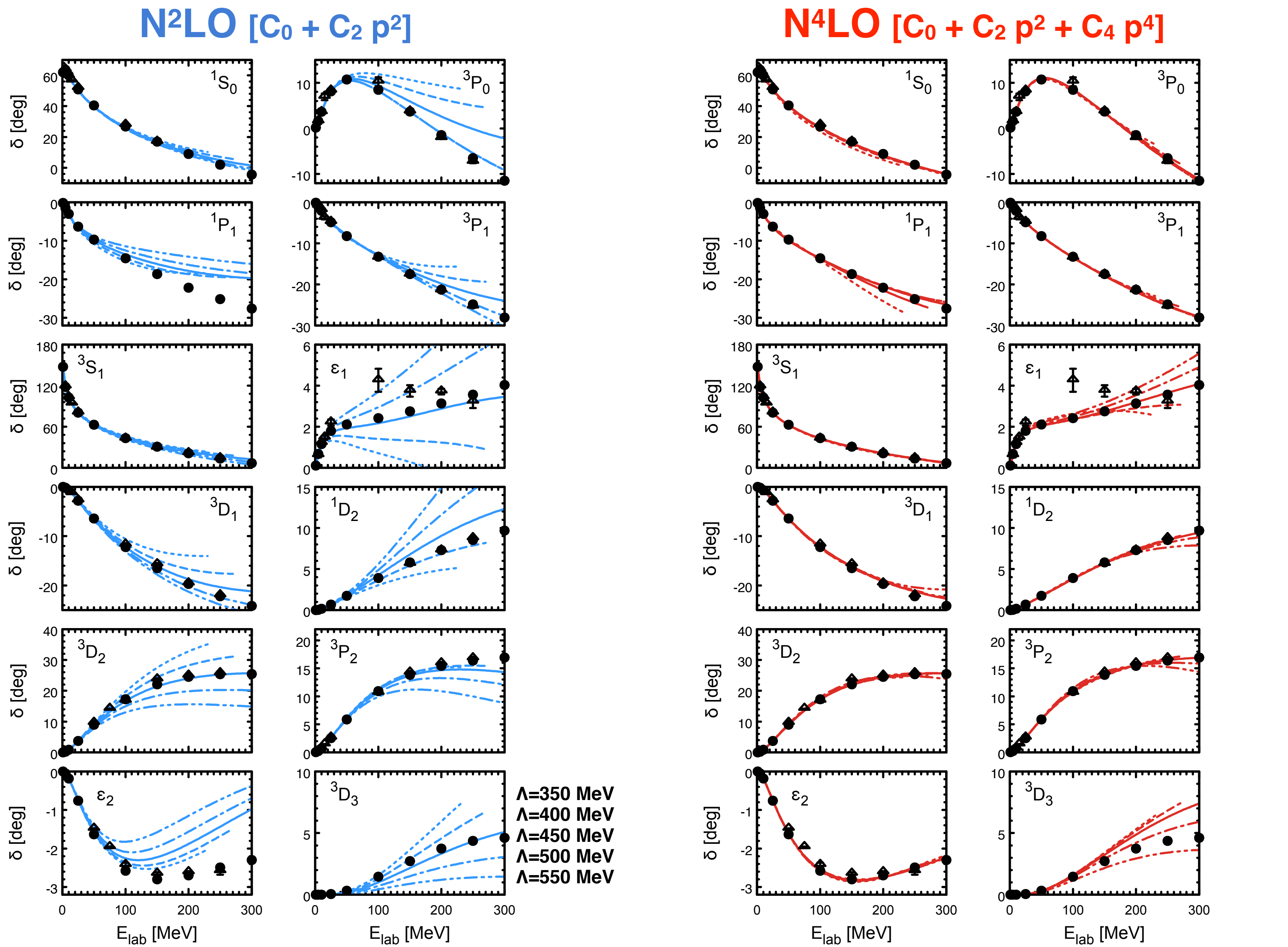
LENPIC: **L**ow **E**nergy **N**uclear **P**hysics **I**nternational **C**ollaboration

NN data analysis

P. Reinert, H. Krebs, EE, arXiv:1711.08821[nucl-th]

Convergence of the chiral expansion for np phase shifts





N2LO [C0 + C2 p2]
N4LO [C0 + C2 p2 + C4 p4]
δ [deg]
1S0
3P0
1P1
3P1
3S1
ε1
3D1
1D2
3D2
3P2
ε2
3D3
Elab [MeV]
Λ=350 MeV
Λ=400 MeV
Λ=450 MeV
Λ=500 MeV
Λ=550 MeV

Description of the scattering data

E_{lab} bin	LO (Q^0)	NLO (Q^2)	N^2LO (Q^3)	N^3LO (Q^4)	N^4LO (Q^5)	N^4LO^+
neutron-proton scattering data						
0 − 100	**73**	**2.2**	**1.2**	**1.08**	**1.08**	**1.07**
0 − 200	**62**	**5.4**	**1.8**	**1.09**	**1.08**	**1.06**
0 − 300	**75**	**14**	**4.4**	**1.99**	**1.18**	**1.10**
proton-proton scattering data						
0 − 100	**2300**	**10**	**2.1**	**0.91**	**0.88**	**0.86**
0 − 200	**1780**	**91**	**33**	**2.00**	**1.42**	**0.95**
0 − 300	**1380**	**89**	**38**	**3.42**	**1.67**	**0.99**
	2 LECs	+ 7 + 1 IB LECs		+ 12 LECs	+ 1 LEC (np)	+ 4 LEC

	$\Lambda = 400$ MeV	$\Lambda = 450$ MeV	$\Lambda = 500$ MeV	$\Lambda = 550$ MeV	Empirical
A_S (fm$^{-1/2}$)	$0.8847^{(+3)}_{(-3)}(6)(4)(4)$	$0.8847^{(+3)}_{(-3)}(3)(5)(1)$	$0.8849^{(+3)}_{(-3)}(1)(7)(0)$	$0.8851^{(+3)}_{(-3)}(3)(8)(1)$	0.8846(8) [117]
η	$0.0255^{(+1)}_{(-1)}(1)(3)(2)$	$0.0255^{(+1)}_{(-1)}(1)(4)(1)$	$0.0257^{(+1)}_{(-1)}(1)(5)(1)$	$0.0258^{(+1)}_{(-1)}(1)(5)(1)$	0.0256(4) [118]

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Clear evidence of the (parameter-free) chiral 2π-exchange!

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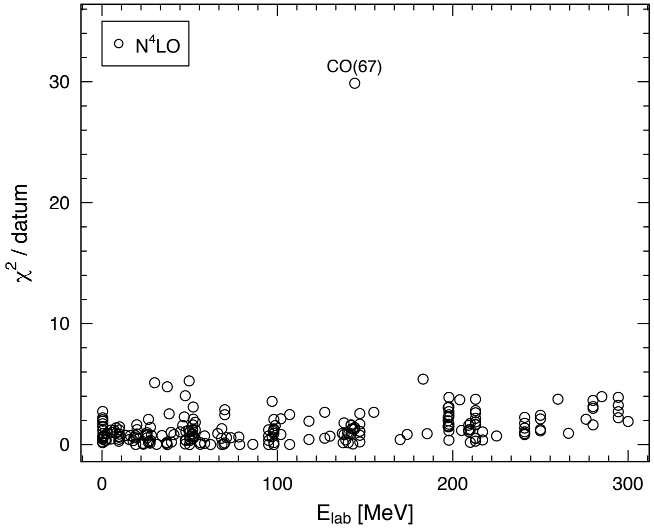
- ∃ some very precise pp data…



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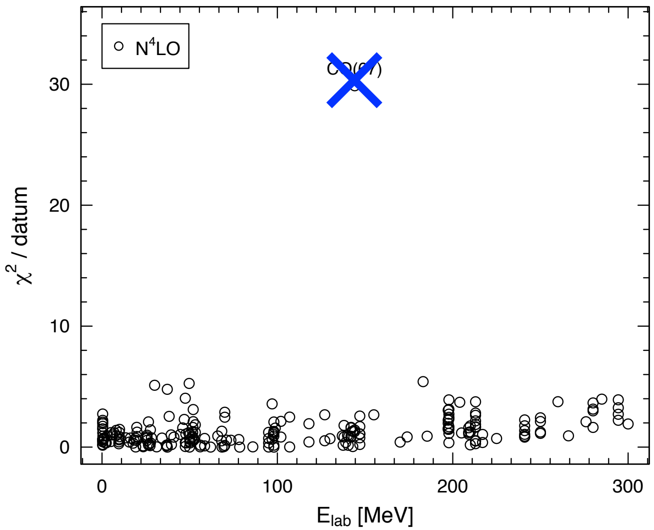

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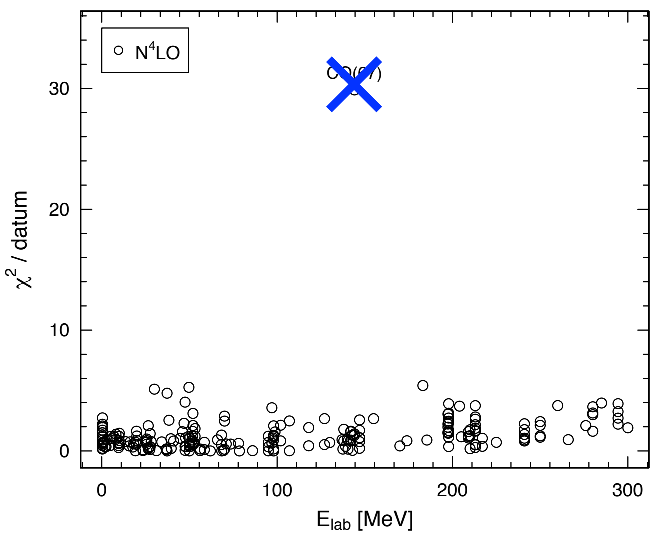

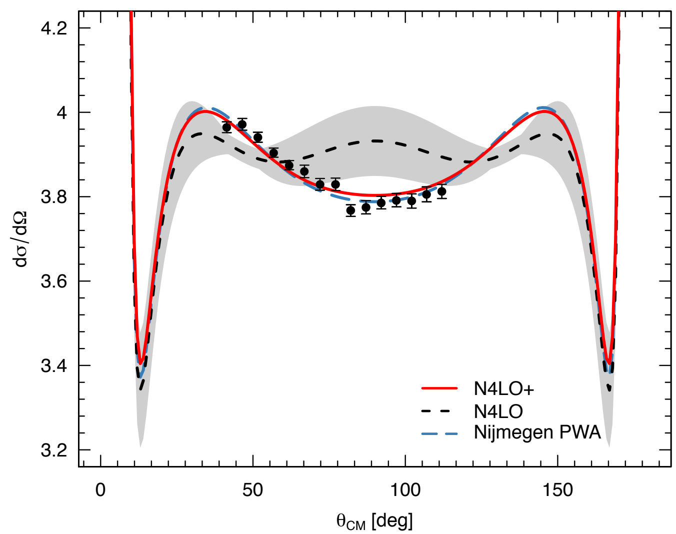

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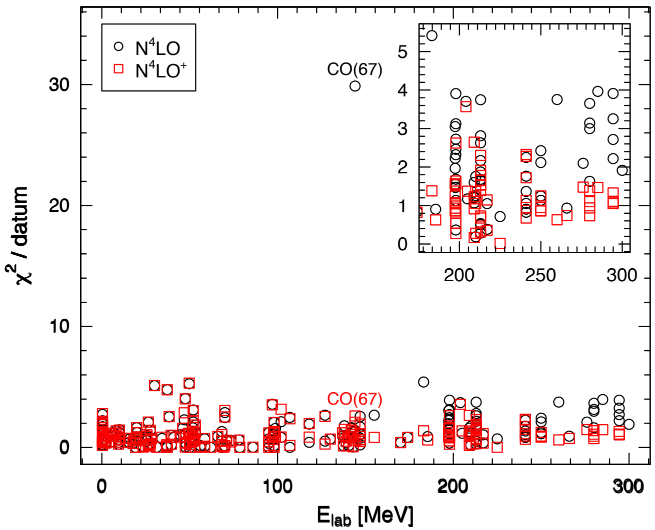
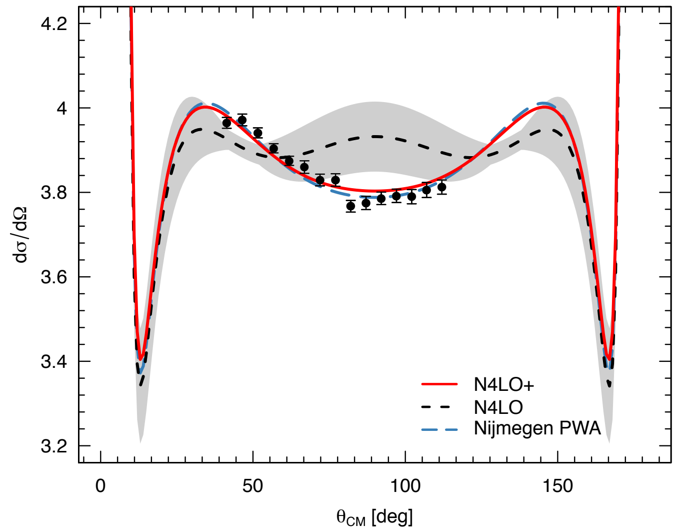




- much lower χ^2 per datum without the outliers

- probe $l > 2$ waves which are parameter-free at N⁴LO…

N⁴LO⁺: include N⁵LO contacts in $^3F_2$, $^1F_3$, $^3F_3$ and $^3F_4$

Step 4: Uncertainty quantification

1. Truncation error [use the algorithm of EE, Krebs, Meißner, EPJA 51 (2015) 53]

For any observable: $X^{(i)}(p) = X^{(0)} + \underbrace{\Delta X^{(2)}}_{\sim Q^2 X^{(0)}} + \ldots + \underbrace{\Delta X^{(i)}}_{\sim Q^i X^{(0)}}$ with $Q = \max(p/\Lambda_b, M_\pi/\Lambda_b)$

estimated from the error plots $\Lambda_b \sim 600$ MeV

Use the explicitly calculated $\Delta X^{(i)}$ to estimate the uncertainty $\delta X^{(i)}$ at order Q^i:

$$\delta X^{(0)} = Q^2 |X^{(0)}|,$$

$$\delta X^{(i)} = \max_{2 \le j \le i} \left(Q^{i+1} |X^{(0)}|, \, Q^{i+1-j} |\Delta X^{(j)}| \right)$$

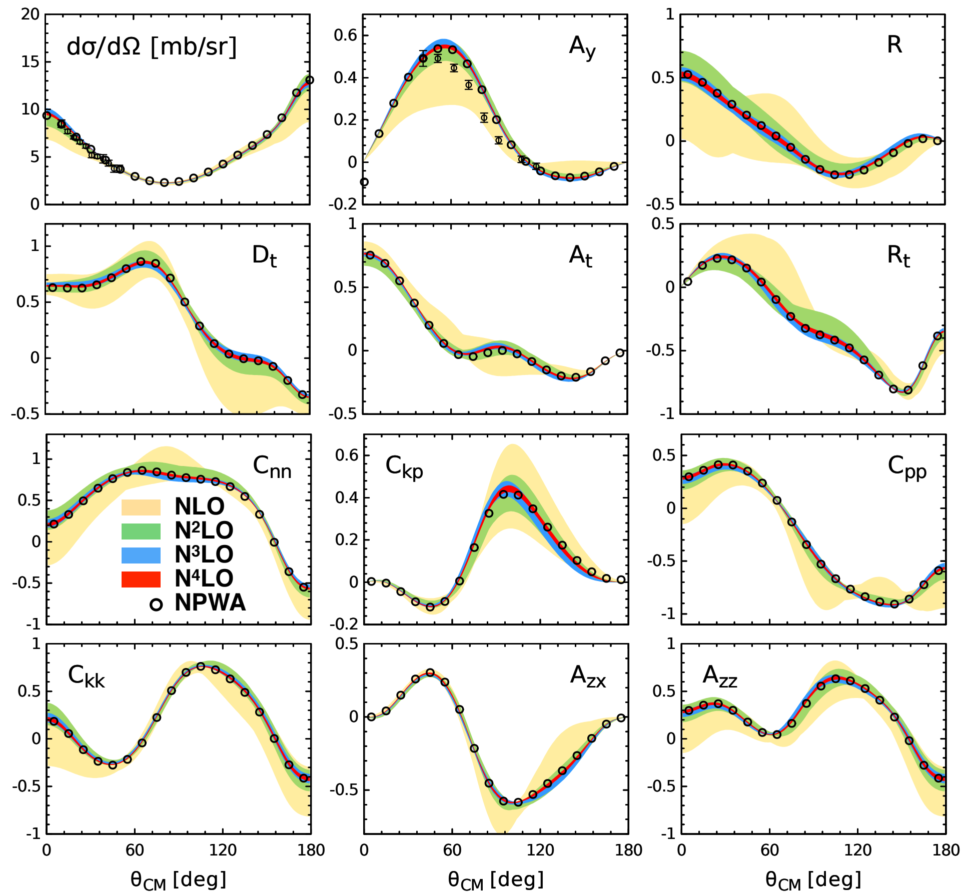
subject to the additional constraint

$$\delta X^{(i)} \ge \max_{j,k} \left(|X^{(j \ge i)} - X^{(k \ge i)}| \right).$$

- no reliance on the cutoff variation (not reliable)
- easily applicable to any observable (scattering, bound states, 3N, …)
- no reliance on experimental data
- for σ_{tot}, errors found to be consistent with 68% degree-of-belief intervals

Furnstahl et al., PRC 92 (2015) 024005

np scattering observables at E_{lab}=143 MeV



Step 4: Uncertainty quantification

2. Statistical uncertainties

Assume $\chi^2(c) \approx \chi^2_{\rm min} + \frac{1}{2}(c - c_{\rm min})^T H (c - c_{\rm min})$ where $H_{ij} = \left.\frac{\partial^2 \chi^2}{\partial c_i \partial c_j}\right|_{c=c_{\rm min}}$

Quadratic approximation is employed to propagate stat. errors in observables

$$O(c) = O(c_{\rm min}) + J_O(c - c_{\rm min}) + \frac{1}{2}(c - c_{\rm min})^T H_O (c - c_{\rm min})$$

see also: Carlsson et al., PRX 6 (16) 011019

3. Uncertainties due to πN LECs $c_{1,2,3,4}$, $d_{1,2,3,5,14,15}$ and $e_{14,17}$

Estimated based on the results using a different set of LECs (KH PWA of πN scattering)

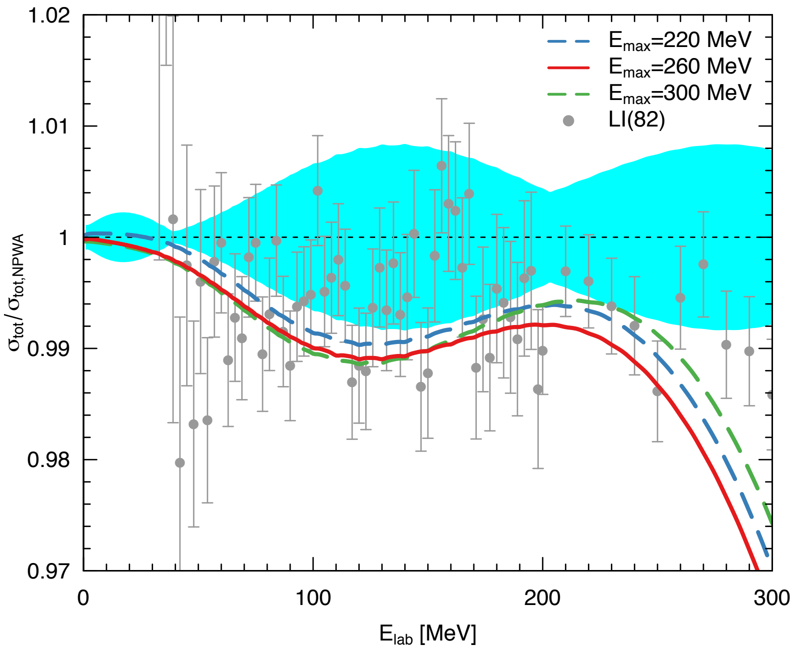
see EE, Krebs, Meißner, PRL 115 (15) 122301

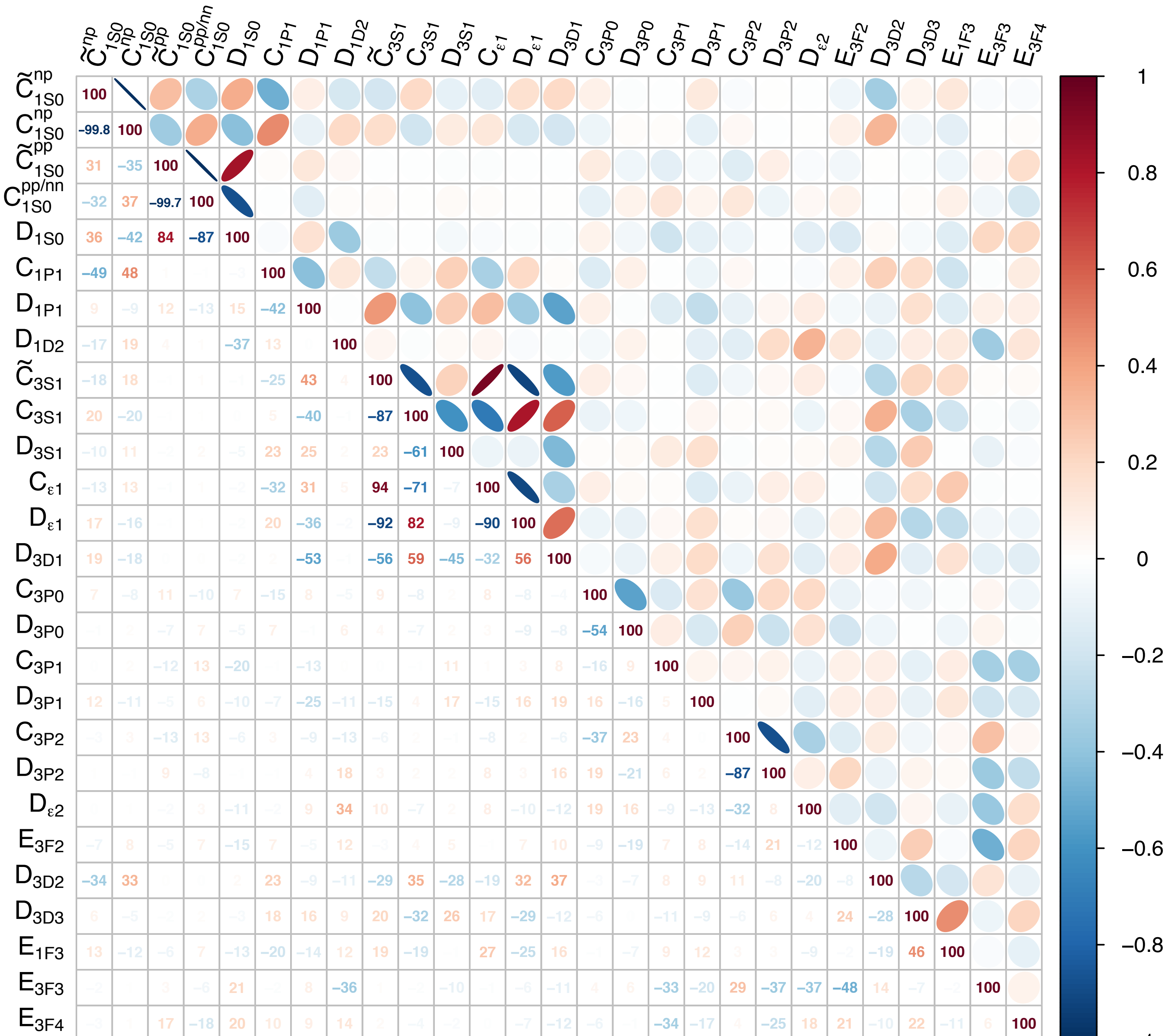
4. Choice of E_{max} in the fits

Uncertainty estimated at N⁴LO/N⁴LO⁺ by performing fits with E_{max} = 220…300 MeV

$E_{\rm lab}$ bin	220 MeV	260 MeV	300 MeV
neutron-proton scattering data			
0 – 100	1.07	1.07	1.08
0 – 200	1.06	1.07	1.07
0 – 300	1.10	1.06	1.06
proton-proton scattering data			
0 – 100	0.86	0.86	0.87
0 – 200	0.95	0.95	0.96
0 – 300	1.00	1.00	0.98

N⁴LO⁺, Λ = 450 MeV




$\widetilde{C}^{np}_{1S0}$
C^{np}_{1S0}
$\widetilde{C}^{pp}_{1S0}$
$C^{pp/nn}_{1S0}$
D_{1S0}
C_{1P1}
D_{1P1}
D_{1D2}
$\widetilde{C}_{3S1}$
C_{3S1}
D_{3S1}
$C_{\varepsilon 1}$
$D_{\varepsilon 1}$
D_{3D1}
C_{3P0}
D_{3P0}
C_{3P1}
D_{3P1}
C_{3P2}
D_{3P2}
$D_{\varepsilon 2}$
E_{3F2}
D_{3D2}
D_{3D3}
E_{1F3}
E_{3F3}
E_{3F4}
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1

	c_1	c_2	c_3	c_4	$\bar{d}_1+\bar{d}_2$	$\bar{d}_3$	$\bar{d}_5$	$\bar{d}_{14}-\bar{d}_{15}$	$\bar{e}_{14}$	$\bar{e}_{15}$	$\bar{e}_{16}$	$\bar{e}_{17}$	$\bar{e}_{18}$
Fit to the GW PWA [79]	−1.31	0.11	−2.54	1.85	1.43	−0.90	−0.16	−2.09	0.07	−3.44	1.65	−0.46	0.47
Statistical error	0.19	0.48	0.08	0.04	0.14	0.19	0.11	0.28	0.02	0.04	0.33	0.17	1.48
Fit to the KH PWA [80]	−1.35	−0.89	−2.19	1.63	2.08	−2.13	0.45	−3.69	−0.05	−6.59	7.22	−0.35	1.88
Statistical error	0.21	0.51	0.08	0.05	0.15	0.20	0.11	0.29	0.02	0.04	0.37	0.22	1.57

Eigenvalues of the covariance matrix:

29.2681
0.3481
0.2025
0.1225
0.0841
0.0144
0.0025
0.0009
0.0009
0.0004
0.0004
0.0001
0.0001

Eigenvalues of the covariance matrix

$$\Sigma = 2\frac{\chi^2}{N_{\rm dof}} H^{-1}$$

for LECs taken in natural units (N[4]LO[+], Λ = 450 MeV)

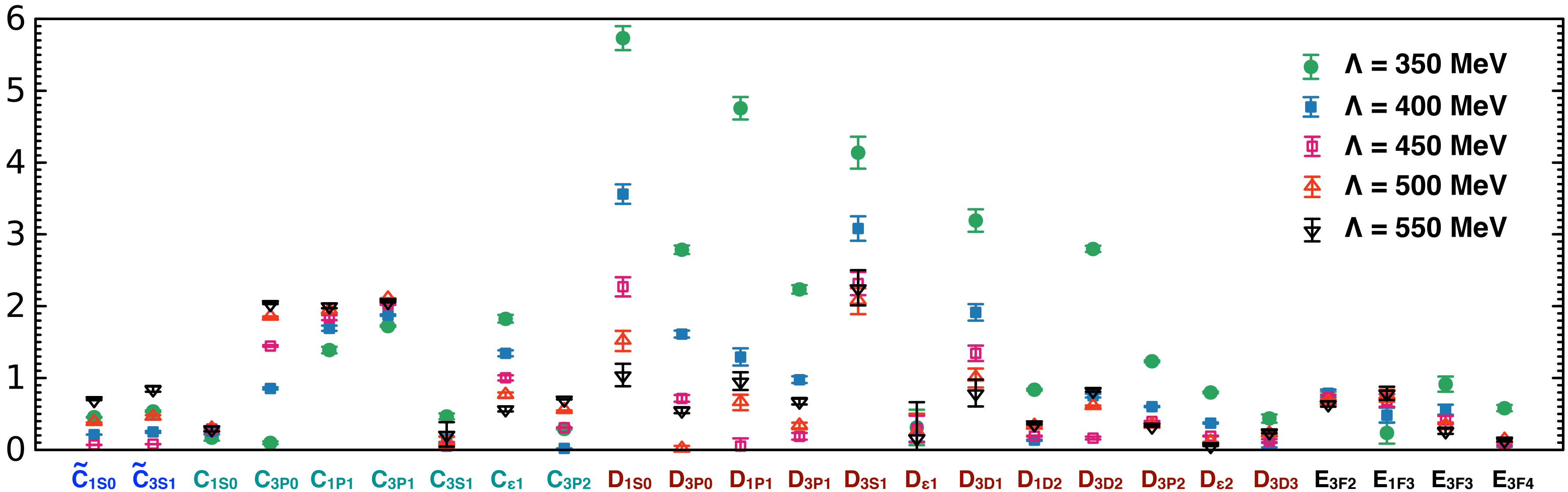
```
4.274396e-02
2.474783e-02
1.902965e-02
1.035190e-02
6.300807e-03
3.912243e-03
2.902483e-03
2.251440e-03
1.902579e-03
1.089075e-03
9.322493e-04
5.588222e-04
3.562153e-04
1.610448e-04
1.409259e-04
1.229603e-04
8.654795e-05
4.958497e-05
4.316301e-05
3.576713e-05
1.911708e-05
1.448694e-05
8.518138e-06
8.268942e-07
4.213655e-10
2.063609e-11
1.614358e-11
```

Natural units for the LECs according to NDA:

$$|\tilde{C}_i| \sim \frac{4\pi}{F_\pi^2}, \qquad |C_i| \sim \frac{4\pi}{F_\pi^2 \Lambda_b^2}, \qquad |D_i| \sim \frac{4\pi}{F_\pi^2 \Lambda_b^4}, \qquad |E_i| \sim \frac{4\pi}{F_\pi^2 \Lambda_b^6}$$

Assuming Λ_b = 600 MeV **[EE, Krebs, Meißner EPJA 51 (15) 53; Furnstahl, Klco, Phillips, Wesolowski, PRC 92 (15) 024005]**, all LECs come out of a natural size.

Absolute values of the LECs in natural units



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